1 Nov 2020 500 math test

1) C 2) A 3) D apparently you need to show work

4) C 5) B? 6) $2^2 * 3^3 * 7^2 * 11$

7) $1848 = 2^3 * 3 * 7 * 11$ 8) $x = -3$ $y = -2$ 9) 15 10) 30030

~~11) ax+by = gcd(a,b) ⇒ ax~~

17) We always want to visit vis[i * prime[j]] when prime[j] is the smallest factor in i* prime[j]. If this holds, then each number will only be visited once because each number has exactly one smallest prime factor.

Suppose for the sake of contradiction that prime[j] is not the smallest factor of i* prime[j]. The smallest factor must be a prime, because if it were composite then that composite would have prime factors which are smaller. prime array contains all primes less than prime[j], so a previous iteration of the inner loop must have ~~been the smallest factor~~ had prime[j] = smallest factor

because prime[j] is not smallest factor, smallest factor must divide i. however, when the previous iteration had prime[j] = smallest factor, the loop would have hit the break condition.

thus, prime[j] must be the smallest factor of i* prime[j]

thus each number is only visited once, so the time complexity is linear

18.1 mu[i] = 1 (only one prime factor → odd)

18.2 mu[x] = 0 (prime[j]^2 is a factor)

18.3 mu[x] = -1* mu[i] (flip parity)

x = i* prime[j]

cont

13) $x = \Sigma\, a_i M_i y_i$ where $M_i y_i \equiv 0 \mod \cancel{\{m_1 \ldots m_n\}} \setminus m_i$

$M_1 = 3*5$ $\quad M_2 = 2*5$ $\quad M_3 = 2*3$

$Y_i = M_i^{-1} \mod m_i$ $\quad 15\%2 = 1$ $\quad 10\%3 = 1$ $\quad 6\%5 = 1$

$Y_1 = 1$ $\quad Y_2 = 1$ $\quad Y_3 = 1$??

$a_1 M_1 Y_1 + a_2 M_2 y_2 + a_3 M_3 Y_3$

$= 1(15)1 + 2(10)1 + 3(6)1$

$= 15 + 20 + 18 = \boxed{53}$ uh, there should be a smaller one?

14) $3^{290} \mod 360$

3^1 3

3^2 9

3^3 27

3^4 81

3^5 243

3^6 729

3^7 2187

3^8 6561

3^9 19683

wait what am I doing

3^{10} 59049

$3^{290} = 3^{10^{29}}$

$3^{10^{29}}$

$= 9^{29} = 9(9^{28}) = 9(9^{4^7})$

$= 9(3^{2^{4^7}})$

$= 9(6561^7)$

$= 9(81^7) = 9(81)(81^{2^3})$

$= 9(81^{2^3}) \equiv 9(81)$??

$\equiv \boxed{9}$?

cont

11. $313(0) + 1237(1) = 1237$ a

$313(1) + 1237(0) = 313$ b

$q = 1237/313 = 3$

$313(-3) + 1237(1) = 298$ a%b

$313/298 = 1$

$313(4) + 1237(-1) = 15$

$298/15 = 19$

$313(-79) + 1237(80) = 13$

$15/13 = 1$

$313(83) + 1237(-21) = 2$

$13/2 = 6$

$313(-577) + 1237(146) = 1 \rightarrow$ break

$2/1 = 2$

$-577 + 1237 = \boxed{660} \equiv 313^{-1} \pmod{1237}$

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16. $\gcd(a,b) = \cancel{p_1^{e_1} p_2^{e_2} p_3^{e_3}}$

$$p_1^{\min(a_1,b_1)} \; p_2^{\min(a_2,b_2)} \cdots$$

$$\operatorname{lcm}(a,b) = p_1^{\max(a_1,b_1)} \; p_2^{\max(a_2,b_2)} \cdots$$

$\min(a,b) + \max(a,b) = a+b$

because if $a \neq b$ then min will chose the number that max didn't, and if $a = b$ then it doesn't matter.

so $\gcd(a,b) \cdot \operatorname{lcm}(a,b)$

$$= p_1^{\min(a_1,b_1)+\max(a_1,b_1)} \; p_2^{\min(a_2,b_2)+\max(a_2,b_2)}$$

$$= p_1^{a_1+b_1} \; p_2^{a_2+b_2} \cdots$$

$$= \left(p_1^{a_1} p_2^{a_2} \cdots\right)\left(p_1^{b_1} p_2^{b_2} \cdots\right) = a * b$$

15. is there a better way to do this?

$40 + 20 + 13 + 10 + 8 + 6 + 5(2) + 4(2)$

1 2 3 4 5 6 7 8 9 10

$3(3) + 2(7) + 1(20)$

11..13 14..20

$\boxed{158}$

8) scratch work

$60x + 96y = 12$

$60(0) + 96(1) = 96$

$60(1) + 96(0) = 60$

$96/60 = 1$

$-1 \quad 1 = 36$

$60/36 = 1$

$2 \quad -1 = 24$

$36/24 = 1$

$-3 \quad -2 = 12$ gcd! break

~~24/12 = 2~~

scratch

1. $7\overline{)247}$ 03, 21, 37 $13\overline{)247}$ 01, 13, 117 $\sum_{d|n} \mu(d) f(\frac{n}{d}) = \varepsilon$

8 0

~~11~~

12

13 0

~~17~~

~~18~~

19

20

$7\overline{)179}$ 02, 14, 39

$130 - 13 = \boxed{117}$

$7\overline{)167}$ 02, 14, 47

$2^{4} \times 2^{2} \times 1 \times 1 \times 1 \times 1$

$7\overline{)133}$ 01, 7, 63

mod 3

2^{-1}

$7\overline{)154}$ 02, 14, 14, ~~133~~

$80 = 2^4 \times 5$

$50 = 2 \times 5^2$

$30 = 2 \times 3 \times 5$

$24 = \cancel{2^3 \times 3}$

$24 = \cancel{3} * \cancel{2^3}$

$33 = 3 * 11$

$56 = 7 * 2^3$

$22 * 7 = 14 + 140 = 154$

$ax + by = 1$

$ax = 1 - by$

$(a+b) \% n = a\% n + b\% n$

$2x + 3y = 1$

$2(-1) + 3($

$2/3 = 0$

$3/2 = 1$

$2 = 3 = 2$

$2(0) + 3(0)$

$2(0) + 3(-1)$

$2(-1) + 3(1)$

$0 + 1 + 2 + 4$

$\sum_{d|8} \varphi(d) =$ 1 2 3 5 6 10

1 2 4 8 16

5 10

$2^2 \times 3^3 \times 7^2 \times 11$

$\sum a_i M_i y_i \equiv P - ix \pmod{p}$