Applied Estimation(EL2320) Lab 1 EKF

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Linear Kalman Filter:

1. What i s t he difference between a ' control' $\mathbf{u}_t$, a ' measurement' $\mathbf{z}_t$ and t he state $\mathbf{x}_t$? Give examples of each?
   control u is the input of the system, measurement z is observations of the state of the system , and state x contains current information about the system.
   For example, if we want to control a car, u iscontrol signal of gas and steering wheelm, z is Odometer and GPS of the car, and x contain Speed, heading informations.

2. Can the uncertainty in the belief increase during an update? Why (or not)?
   The uncertainty will decrease. During update Σt= ¯Σt−KtCt¯Σt, so it will decrease.

3. During update what is it that decides the weighing between measurements and belief?

$$\mu_t = (I - W)\bar{\mu}_{\mathbf{t}} + W\mu_{\mathbf{z}}$$
$$W = \Sigma_t C_t^T Q_t^{-1} C_t$$
$$\text{and } C_t \mu_{\mathbf{z}} = (\mathbf{z}_t - \bar{\mathbf{z}}_{\mathbf{t}} + C_t \bar{\mu}_{\mathbf{t}})$$

   The relation between Σt(uncertainty) and Qt (measurement error) after the update is done decides the weighting between the measurements and the belief.

4. What would be the result of using a too large a covaraince (Q matrix) for the measurement model?
   Estimates would be pessimistic (conservative) and convergence would be slower.

5. What would give t he measurements an i ncreased effect on t he updated state estimate?
   A larger Kalman gain, the small covariance matrix for the measurement error model.

6. What happens to the belief uncertainty during prediction? How can you show that?
   It will increase. According to ¯Σt=Qt+AtΣt−1AT, Each predict will introduce noise, and uncertainty is often growing.

7. How can we s ay t hat t he Kalman filter i s t he optimal and minimum least square error estimator i n t he case of i ndependent Gaussian noise and Gaus-sian priori distribution? (Just describe t he reasoning not a f ormal proof.)

   The Kalman filter gives the true posterior distribution for a linear Gaussian system. So, suppose µbetter for the state has lower expected square error than the Gaussian mean µ.

$$\begin{aligned} & \int_{-\infty}^{\infty} (x - \mu_{better})^2 G(x, \mu, \Sigma) dx \\ \Leftrightarrow \quad & \int_{-\infty}^{\infty} (x^2 - 2\mu_{better} x + \mu_{better}^2) G(x, \mu, \Sigma) dx \\ \Leftrightarrow \quad & \Sigma + \mu^T \mu - \mu^T \mu_{better} - \mu_{better}^T \mu + \mu_{better}^T \mu_{better} \end{aligned}$$

Minimizing the equation by differentiating and we have:

$$-2\mu + 2\mu_{better} = 0$$
$$\mu_{better} = \mu$$

So, the Kalman filter is the optimal and minimum least square error estimator.

8.In the case of Gaussian white noise and Gaussian priori distribution, is the Kalman Filter a MLE and/or MAP estimator?
It's a MLE estimator.

Extended Kalman Filter:

9. How does the extended Kalman filter relate to the Kalman filter?
   The extended Kalman filter applies the Kalman filter to the linearized nonlinear system. Furthermore, Atxt−1 + Btut is replaced by g(ut, xt−1), At by Gt and Ct by Ht.

10. Is the EKF guaranteed to converge to a consistent solution?
    NO, the mean value of the nonlinear function is approximately linear at the estimation point, and the update depends on this linearization. If the linearization accuracy is poor, the convergence result will be affected.

11. If our filter seems to diverge often can we change any parameter to try and reduce this?
    Yes. If divergence occurs on update, we can change modeled uncertainties Q and R and increase the relative size of the measurement covariance Q. If the divergence is due to bad data association, we can change the matching threshold.

Localization:

12. If a robot is completely unsure of its location and measures the range r to a know landmark with Gaussain noise what does its posterior belief of its location $p(x, y, \theta|r)$ look like? So a formula is not needed but describe it at least.
    It will have a uniform distribution over the heading θ between−π and π. The position will be a sort of donut/ring. So a Gaussian on ρ in radial coordinate with uniform distribution on the angle φ
    So, x = ρ cos φ, y = ρ sin φ, and $e^{-\frac{(\rho-r)^2}{2\sigma_r^2}}$

13. If the above measurement also included a bearing how would the posterior look?
    The same except that the heading and angle around the ring would be Gaussian with a completely correlated covariance.

$$e^{-\left[\frac{(\rho-r)^2}{2\sigma_r^2}+\frac{(b-\phi+\theta)^2}{2\phi_b^2}\right]}$$

14. If the robot moves with relatively good motion estimation (prediction error is small) but a large initial uncertainty in heading θ how will the posterior look after traveling a long distance without seeing any features?
    It will look like a cresent/arc/C shape. The heading θ will be correlated with position along the arc.

15. If the above robot then sees a point feature and measures range and bearing to it how might the EKF update go wrong?
    The new feature will prevent the update from following the original trajectory, making it easy to diverge and cause errors.