Testing singleSlipPlane.xml

The goal of this test is to examine slip planes that are both parallel and at some angle (45 degrees) to the loading. In this notebook, we examine slip along a plane that is rotated to the global axes. We seek to determine if the character of Fp is aligned with our implementation. We recently found a bug in the transformation matrix (orientation) in the crystal plasticity implementation. This notebook verifies the current implementation for the character of Fp.

```xml
<ParameterList name="Forty Five Degree Rotated Lattice">

   <ParameterList name="Material Model">
     <Parameter name="Model Name" type="string" value="CrystalPlasticity"/>
   </ParameterList>

   <!-- Crystal Elasticity moduli and orientation -->
   <ParameterList name="Crystal Elasticity">
     <Parameter name="C11" type="double" value="168.4"/>
     <Parameter name="C12" type="double" value="121.4"/>
     <Parameter name="C44" type="double" value="75.4"/>
     <Parameter name="Basis Vector 1" type="Array(double)" value="{ 0.707106781186548, 0.707106781186548, 0.0}"/>
     <Parameter name="Basis Vector 2" type="Array(double)" value="{-0.707106781186548, 0.707106781186548, 0.0}"/>
     <Parameter name="Basis Vector 3" type="Array(double)" value="{ 0.0,          0.0,          1.0}"/>
   </ParameterList>

   <!-- Crystal Plasticity Slip System -->
   <Parameter name="Number of Slip Systems" type="int" value="1"/>
   <ParameterList name="Slip System 1">
     <Parameter name="Slip Direction" type="Array(double)" value="{1.0, 0.0, 0.0}"/>
     <Parameter name="Slip Normal" type="Array(double)"    value="{0.0, 1.0, 0.0}"/>
     <Parameter name="Tau Critical" type="double" value="0.6"/>
     <Parameter name="Gamma Dot" type="double" value="1.0"/>
     <Parameter name="Gamma Exponent" type="double" value="1.0"/>
   </ParameterList>
```

State basis for material coordinate system

In[27]:=
```
basis = {{1 / Sqrt[2], 1 / Sqrt[2], 0}, {-1 / Sqrt[2], 1 / Sqrt[2], 0}, {0, 0, 1}};
MatrixForm[basis]
```

Out[28]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Derive direction cosine matrix for coordinate transformations

```
In[29]:= orientation = Transpose[basis];
         MatrixForm[orientation]
```

Out[30]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transform slip and normal directions to global coordinate system

```
In[31]:= slipdirectionGlobal = orientation . {{1}, {0}, {0}}
         slipnormalGlobal = orientation . {{0}, {1}, {0}};
         MatrixForm[slipdirectionGlobal]
         MatrixForm[slipnormalGlobal]
```

Out[31]= $\left\{\left\{\frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}\right\}, \{0\}\right\}$

Out[33]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

Out[34]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

Create Schmid tensor

```
In[39]:= projection = Outer[Times,
             Transpose[slipdirectionGlobal][[1]], Transpose[slipnormalGlobal][[1]]];
         MatrixForm[projection]
```

Out[40]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Define velocity gradient Lp_{n+1}

In[51]:=
```
Lpnplus1 = gammaDot * projection
```

Out[51]= $\left\{\left\{-\frac{\text{gammaDot}}{2}, \frac{\text{gammaDot}}{2}, 0\right\}, \left\{-\frac{\text{gammaDot}}{2}, \frac{\text{gammaDot}}{2}, 0\right\}, \{0, 0, 0\}\right\}$

Find Fp_{n+1} through exponential map, Fp_{n} = Identity

This is the form of Fp.

In[55]:=
```
Fpnplus1 =
  MatrixExp[Δt * Lpnplus1] . {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} /. {Δt * gammaDot → Δγ}
```

Out[55]= $\left\{\left\{1-\frac{\Delta\gamma}{2}, \frac{\Delta\gamma}{2}, 0\right\}, \left\{-\frac{\Delta\gamma}{2}, 1+\frac{\Delta\gamma}{2}, 0\right\}, \{0, 0, 1\}\right\}$

Results from LCM

Given the resulting Fpnplus1 from the entire simulation, we can determine if the derived Fp is consistent with the implementation. Because there is only one slip system, we can assume that that the load will not be shed to other slip systems. Consequently, we can do this in a single increment. Because the norm of the difference is machine precision, we have confidence.

In[71]:=
```
FpLCM = {{1.010351560423953, -0.01035156042395333, 0},
   {0.01035156042395335, 0.9896484395760466, 0}, {0, 0, 1}};
```

In[76]:=
```
deltaGammaEval = Solve[Fpnplus1[[1, 1]] - FpLCM[[1, 1]] == 0, Δγ][[1]];
Fpnplus1Eval = Fpnplus1 /. deltaGammaEval ;
```

In[79]:=
```
Norm[Fpnplus1Eval - FpLCM]
```

Out[79]= 5.26879×10^{-16}