

WEEK6_L-1:

Evaluate the taylor expansion of $f(x) = \sin x$ and the graph the first 10 taylor polynomials along with $f(x)$.

In[31]:= **Series[Exp[x], {x, 0, 7}] // Normal**

Out[31]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040}$

In[30]:= **Series[f[x], {x, a, 3}]**

Out[30]= $f[a] + f'[a](x-a) + \frac{1}{2}f''[a](x-a)^2 + \frac{1}{6}f^{(3)}[a](x-a)^3 + O[x-a]^4$

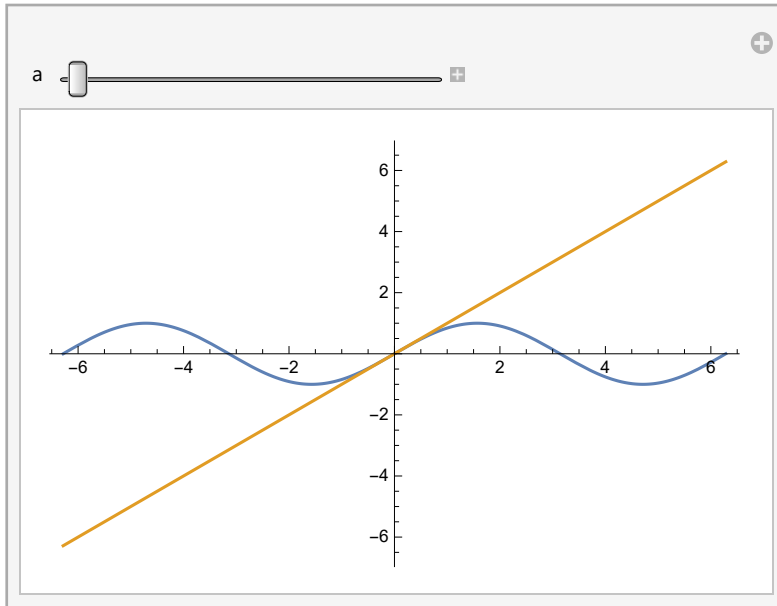
In[29]:= **TableForm[Table[Normal[Series[Sin[x], {x, 0, a}]], {a, 1, 10}]]**

Out[29]//TableForm=

| |
|---|
| x |
| x |
| $x - \frac{x^3}{6}$ |
| $x - \frac{x^3}{6}$ |
| $x - \frac{x^3}{6} + \frac{x^5}{120}$ |
| $x - \frac{x^3}{6} + \frac{x^5}{120}$ |
| $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ |
| $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ |
| $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880}$ |
| $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880}$ |

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In[28]:= Manipulate[Plot[{Sin[x], Evaluate[Normal[Series[Sin[x], {x, 0, a}]]]}, {x, -2 Pi, 2 Pi}],
  {a, 1, 30, 2}]
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Out[28]=



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In[27]:= DSolve[{f'[x] == 3 x + 4}, f[x], x]
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Out[27]= $\left\{ \left\{ f[x] \rightarrow 4x + \frac{3x^2}{2} + c_1 \right\} \right\}$

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In[39]:= Clear[x]
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... DSolve: x cannot be used as a function.

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In[38]:= DSolve[{g'[y] + 5 * g'[y] + 2 * g[y] == 0}, g[y], y]
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Out[38]= $\left\{ \left\{ g[y] \rightarrow e^{\left(-\frac{5}{2} - \frac{\sqrt{17}}{2}\right)y} c_1 + e^{\left(-\frac{5}{2} + \frac{\sqrt{17}}{2}\right)y} c_2 \right\} \right\}$

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In[41]:= DSolve[{g'[y] + 5 * g'[y] + 2 * g[y] == 0, g[0] == 5, g'[0] == 10}, g[y], y]
```

Out[41]= $\left\{ \left\{ g[y] \rightarrow -\frac{5}{34} \left(-17 e^{\left(-\frac{5}{2} - \frac{\sqrt{17}}{2}\right)y} + 9 \sqrt{17} e^{\left(-\frac{5}{2} - \frac{\sqrt{17}}{2}\right)y} - 17 e^{\left(-\frac{5}{2} + \frac{\sqrt{17}}{2}\right)y} - 9 \sqrt{17} e^{\left(-\frac{5}{2} + \frac{\sqrt{17}}{2}\right)y} \right) \right\} \right\}$

In[43]:= **DSolve**[{g''[y] + 5 * g'[y] + 2 * g[y] == 0, g[1] == 5, g'[6] == 10}, g[y], y]

Out[43]= $\left\{ \left\{ g[y] \rightarrow \frac{\left(5 e^{-\frac{\sqrt{17}}{2}} \left(-4 e^{15 + \frac{7\sqrt{17}}{2} + \left(-\frac{5}{2} - \frac{\sqrt{17}}{2} \right) y} - 5 e^{\frac{5}{2} + 6\sqrt{17} + \left(-\frac{5}{2} - \frac{\sqrt{17}}{2} \right) y} + \sqrt{17} e^{\frac{5}{2} + 6\sqrt{17} + \left(-\frac{5}{2} - \frac{\sqrt{17}}{2} \right) y} + 5 e^{\frac{5}{2} + \left(-\frac{5}{2} + \frac{\sqrt{17}}{2} \right) y} + \sqrt{17} e^{\frac{5}{2} + \left(-\frac{5}{2} + \frac{\sqrt{17}}{2} \right) y} + 4 e^{15 + \frac{5\sqrt{17}}{2} + \left(-\frac{5}{2} + \frac{\sqrt{17}}{2} \right) y} \right) \right)}{\left(5 + \sqrt{17} - 5 e^{5\sqrt{17}} + \sqrt{17} e^{5\sqrt{17}} \right)} \right\} \right\}$