Solve the differential equation:

1. (x+2)dy-(2y)dx=0

Solution:

$$(x+2)dy-(2Y)dx \Rightarrow \frac{dy}{dx} = \frac{2y}{x+2} \Rightarrow y' = \frac{2y}{x+2}$$

```
In[ ]:= ClearAll["Global`*"]

In[ ]:= DSolve[y'[x] == 2 y[x] / (x + 2), y[x], x]
Out[ ]= {{y[x] → (2 + x)^2 C[1]}}
```

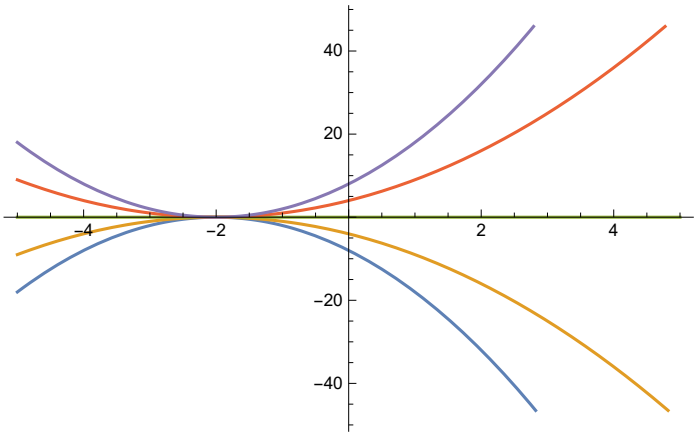
2. Plot the solutions for $\mathbb{C}_1$=-2,-1,0,1,2

```
In[ ]:= ClearAll["Global *"]

In[ ]:= y[x] = (2 + x)^2 c1
Out[ ]= c1 (2 + x)^2

In[ ]:= Plot[Evaluate[Table[y[x], {c1, -2, 2}]], {x, -5, 5}]
```

Out[ ]=



3. Solve y''+9y'=0,y'[0]=0,y[2]=1, Plot the solution.

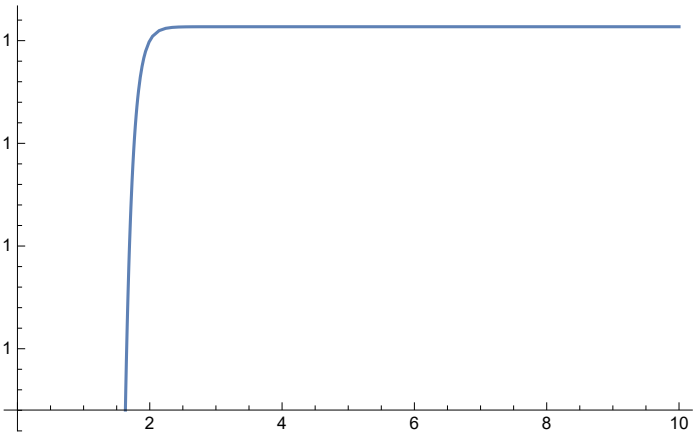
In[•]:=
```
ClearAll["Global`*"]
```

In[•]:=
```
sol1 = DSolve[{y''[x] + 9 y'[x] == 0, y'[0] == 4, y[2] == 1}, y[x], x]
```

Out[•]=
$$\left\{\left\{y[x] \to \frac{1}{9} e^{-18-9x}\left(-4 e^{18} + 4 e^{9x} + 9 e^{18+9x}\right)\right\}\right\}$$

In[•]:=
```
Plot[y[x] /. sol1, {x, 0, 10}]
```

Out[•]=



General: Further output of ReplaceAll::reps will be suppressed during this calculation.

ReplaceAll: {DSolve[{9 y′[0.204286] + y′′[0.204286] == 0, True, True}, y, 0.204286]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of DSolve::dsvar will be suppressed during this calculation.

DSolve: 0.20428591836734694` cannot be used as a variable.

ReplaceAll: {DSolve[{9. y′[0.000204286] + y′′[0.000204286] == 0., True, True}, y, 0.000204286]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

DSolve: 0.0002042857142857143` cannot be used as a variable.

ReplaceAll: {DSolve[{9 y′[0.000204286] + y′′[0.000204286] == 0, True, True}, y, 0.000204286]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

DSolve: 0.0002042857142857143` cannot be used as a variable.

DSolve: Equation or list of equations expected instead of True in the first argument {9 y′[x] + y′′[x] == 0, True, True}.

Syntax: Expression "sol1 = DSolve {(y ''[x]) + (9 y '[x]) == 0, y '[0] == 4, y[2] == 1}, y, x]" has no opening "[".

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

Solve: $-4 + y'[0] == 0 \&\& -1 + y[2] == 0$ /. DSolve`DSolveFirstOrderODEDump`LinearFirstOrderODE[$c1\,(2 + x)^2$, 0, −9, c_1] == 0 is not a quantified system of equations and inequalities.

Solve: $-4 + y'[0] == 0 \&\& -1 + y[2] == 0$ /. DSolve`DSolveFirstOrderODEDump`LinearFirstOrderODE[$c1\,(2 + x)^2$, 0, −9, c_1] == 0 is not a quantified system of equations and inequalities.

DSolve: For some branches of the general solution, unable to solve the conditions.

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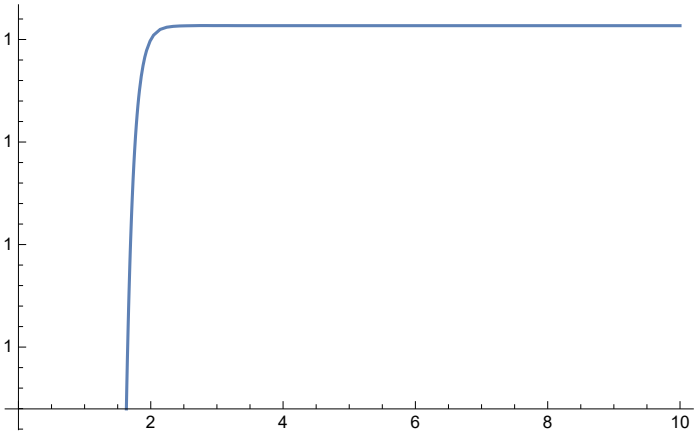
4. Plot the numerical solution of y '' + 9 y ' = 0, y '[0] = 0, y[2] = 1

In[•]:=
```
sol2 = NDSolve[{y''[x] + 9 y'[x] == 0, y'[0] == 4, y[2] == 1}, y, {x, 0, 10}]
```

Out[•]=
```
{{y → InterpolatingFunction[ ⊞ Domain: {{0., 10.}}
                             Output: scalar ]}}
```

In[•]:=
```
Plot[y[x] /. sol2, {x, 0, 10}]
```

Out[•]=



Forced harmonic oscillator: $\frac{d^2y}{dt^2}+p\frac{dy}{dt}+qy=\cos(\omega t)$

We can split this equation into a system:

$\frac{dy}{dt}=v,$

$\frac{dv}{dt} = -qy - pv + \cos(\omega t)$

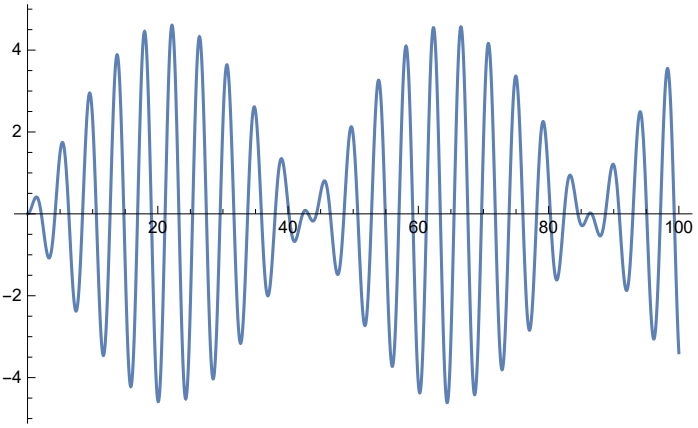
In[•]:=
```
ClearAll["Global`*"]
```

In[•]:=
```
sol3 = DSolve[{y'[t] == v[t], v'[t] == -2 y[t] + Cos[1.56 t], y[0] == 0, v[0] == 0}, {y, v}, t]
```

Out[•]=
```
{{v → Function[{t}, (-2.79346 × 10^-16 + 7.99207 × 10^-16 i) Cos[1.41421 t] +
      (2.93744 × 10^-16 - 8.40401 × 10^-16 i) Cos[0.145786 t] Cos[1.41421 t] -
      (1.43984 × 10^-17 - 4.11938 × 10^-17 i) Cos[1.41421 t] Cos[2.97421 t] +
      (3.42967 - 9.51926 × 10^-17 i) Cos[1.41421 t] Sin[0.145786 t] -
      (3.26156 + 1.28024 × 10^-16 i) Sin[1.41421 t] + (3.42967 + 1.34623 × 10^-16 i) Cos[0.145786 t]
       Sin[1.41421 t] - (0.168112 + 6.59877 × 10^-18 i) Cos[2.97421 t] Sin[1.41421 t] -
      (6.97612 × 10^-16 - 8.40401 × 10^-16 i) Sin[0.145786 t] Sin[1.41421 t] +
      (0.168112 - 4.66604 × 10^-18 i) Cos[1.41421 t] Sin[2.97421 t] -
      (3.41947 × 10^-17 - 4.11938 × 10^-17 i) Sin[1.41421 t] Sin[2.97421 t]], y →
   Function[{t}, (-2.42515 - 1.00967 × 10^-16 i) ((-0.950983 + 5.08256 × 10^-33 i) Cos[1.41421 t] +
      (1. + 0. i) Cos[0.145786 t] Cos[1.41421 t] - (0.0490168 + 3.1766 × 10^-34 i) Cos[1.41421 t]
       Cos[2.97421 t] - (1.57009 × 10^-16 - 1.57009 × 10^-16 i) Cos[1.41421 t] Sin[0.145786 t] +
      (2.63951 × 10^-17 - 2.11161 × 10^-16 i) Sin[1.41421 t] - (2.77556 × 10^-17 - 2.22045 × 10^-16 i)
       Cos[0.145786 t] Sin[1.41421 t] + (1.36049 × 10^-18 - 1.08839 × 10^-17 i) Cos[2.97421 t]
       Sin[1.41421 t] - (1. - 5.55112 × 10^-17 i) Sin[0.145786 t] Sin[1.41421 t] -
      (7.69609 × 10^-18 - 7.69609 × 10^-18 i) Cos[1.41421 t] Sin[2.97421 t] -
      (0.0490168 - 2.72098 × 10^-18 i) Sin[1.41421 t] Sin[2.97421 t])]}}
```

In[•]:=
```
Plot[y[t] /. sol3, {t, 0, 100}]
```

Out[•]=



In[•]:=
```
sol03 = DSolve[{y'[t] == v[t], v'[t] == -2 y[t] + Cos[156/100 t], y[0] == 0, v[0] == 0}, {y, v}, t]
```

Out[•]=

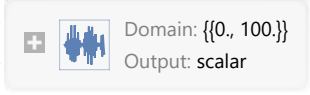
$$\left\{\left\{v \to \text{Function}\left[\{t\},\right.\right.\right.$$
$$\frac{25}{542}\left(-50\sqrt{2}\,\text{Sin}\left[\sqrt{2}\,t\right]+39\,\text{Cos}\left[\left(-\frac{39}{25}+\sqrt{2}\right)t\right]\text{Sin}\left[\sqrt{2}\,t\right]+25\sqrt{2}\,\text{Cos}\left[\left(-\frac{39}{25}+\sqrt{2}\right)t\right]\right.$$
$$\text{Sin}\left[\sqrt{2}\,t\right]-39\,\text{Cos}\left[\left(\frac{39}{25}+\sqrt{2}\right)t\right]\text{Sin}\left[\sqrt{2}\,t\right]+25\sqrt{2}\,\text{Cos}\left[\left(\frac{39}{25}+\sqrt{2}\right)t\right]\text{Sin}\left[\sqrt{2}\,t\right]-$$
$$39\,\text{Cos}\left[\sqrt{2}\,t\right]\text{Sin}\left[\left(-\frac{39}{25}+\sqrt{2}\right)t\right]-25\sqrt{2}\,\text{Cos}\left[\sqrt{2}\,t\right]\text{Sin}\left[\left(-\frac{39}{25}+\sqrt{2}\right)t\right]+$$
$$\left.39\,\text{Cos}\left[\sqrt{2}\,t\right]\text{Sin}\left[\left(\frac{39}{25}+\sqrt{2}\right)t\right]-25\sqrt{2}\,\text{Cos}\left[\sqrt{2}\,t\right]\text{Sin}\left[\left(\frac{39}{25}+\sqrt{2}\right)t\right]\right)\Big],$$
$$y \to \text{Function}\left[\{t\},-\frac{1}{1084}\times 25\left(-100\,\text{Cos}\left[\sqrt{2}\,t\right]+50\,\text{Cos}\left[\sqrt{2}\,t\right]\text{Cos}\left[\left(-\frac{39}{25}+\sqrt{2}\right)t\right]+\right.\right.$$
$$39\sqrt{2}\,\text{Cos}\left[\sqrt{2}\,t\right]\text{Cos}\left[\left(-\frac{39}{25}+\sqrt{2}\right)t\right]+50\,\text{Cos}\left[\sqrt{2}\,t\right]\text{Cos}\left[\left(\frac{39}{25}+\sqrt{2}\right)t\right]-$$
$$39\sqrt{2}\,\text{Cos}\left[\sqrt{2}\,t\right]\text{Cos}\left[\left(\frac{39}{25}+\sqrt{2}\right)t\right]+50\,\text{Sin}\left[\sqrt{2}\,t\right]\text{Sin}\left[\left(-\frac{39}{25}+\sqrt{2}\right)t\right]+$$
$$39\sqrt{2}\,\text{Sin}\left[\sqrt{2}\,t\right]\text{Sin}\left[\left(-\frac{39}{25}+\sqrt{2}\right)t\right]+50\,\text{Sin}\left[\sqrt{2}\,t\right]\text{Sin}\left[\left(\frac{39}{25}+\sqrt{2}\right)t\right]-$$
$$\left.\left.\left.39\sqrt{2}\,\text{Sin}\left[\sqrt{2}\,t\right]\text{Sin}\left[\left(\frac{39}{25}+\sqrt{2}\right)t\right]\right)\right]\right\}\right\}$$

In[•]:=
```
sol4 = NDSolve[
  {y'[t] == v[t], v'[t] == -2 y[t] + Cos[1.56 t], y[0] == 0, v[0] == 0}, {y, v}, {t, 0, 100}]
```

Out[•]=

$$\left\{\left\{y \to \text{InterpolatingFunction}\left[\right.\right.\right.$$ 

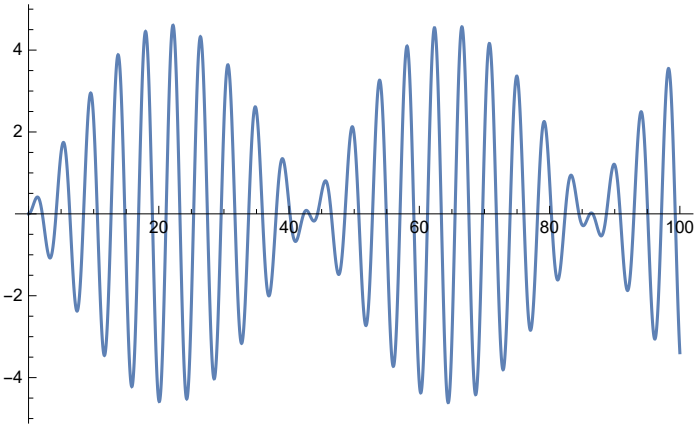
$$\Big],$$

$$v \to \text{InterpolatingFunction}\left[\right.$$ 

$$\left.\left.\right]\right\}\right\}$$

In[•]:=
```
Plot[y[t] /. sol4, {t, 0, 100}]
```

Out[•]=



NDSolve: Endpoint Null in {t, 100., Null} is not a real number.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

ReplaceAll: {NDSolve[{9 y′[0.204286] + y′′[0.204286] == 0, True, True}, y, {0.204286, 0, 10}]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of NDSolve::dsvar will be suppressed during this calculation.

NDSolve: 0.20428591836734694` cannot be used as a variable.

ReplaceAll: {NDSolve[{9. y′[0.000204286] + y′′[0.000204286] == 0., True, True}, y, {0.000204286, 0., 10.}]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

NDSolve: 0.00020428571428571433` cannot be used as a variable.

ReplaceAll: {NDSolve[{9 y′[0.000204286] + y′′[0.000204286] == 0, True, True}, y, {0.000204286, 0, 10}]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

NDSolve: 0.00020428571428571433` cannot be used as a variable.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.