Week-5 (Assignment 4)

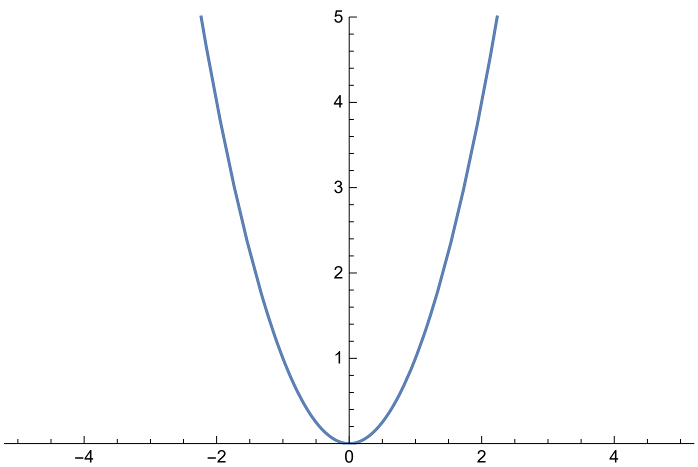
Parametric Equations

In[28]:= `y = x^2`

Out[28]= x^2

In[32]:= `Plot[x^2, {x, -5, 5}, PlotRange → {0, 5}]`

Out[32]=



say $x = t$. Then $y = t^2$

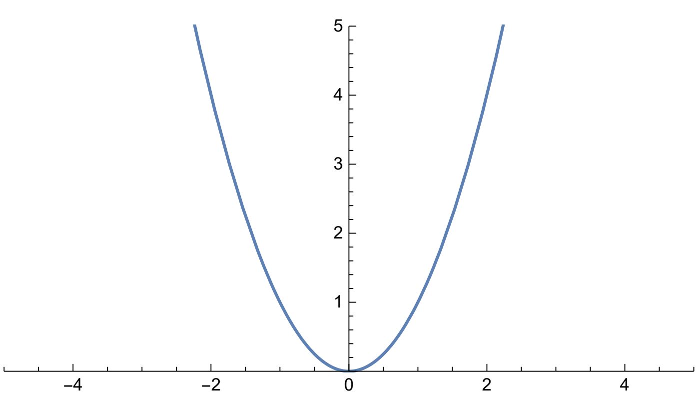
`ParametricPlot[{t, t^2}, {t, -5, 5}, PlotRange → {{-5, 5}, {0, 5}}]`

Set: Tag Times in t^4 t.Then is Protected.

Set: Tag Times in say t^2 is Protected.

Out[41]= t^2

Out[42]=



we have to graph the curve $x = y^4 - 3y^2$

Out[71]= $-3\left(-3t^2+t^4\right)^4+\left(-3t^2+t^4\right)^8$

Set: Tag Times in curve graph have t^2 the to we is Protected.

In[73]:= `Solve[x == y^4 - 3 y^2, y, Reals]`

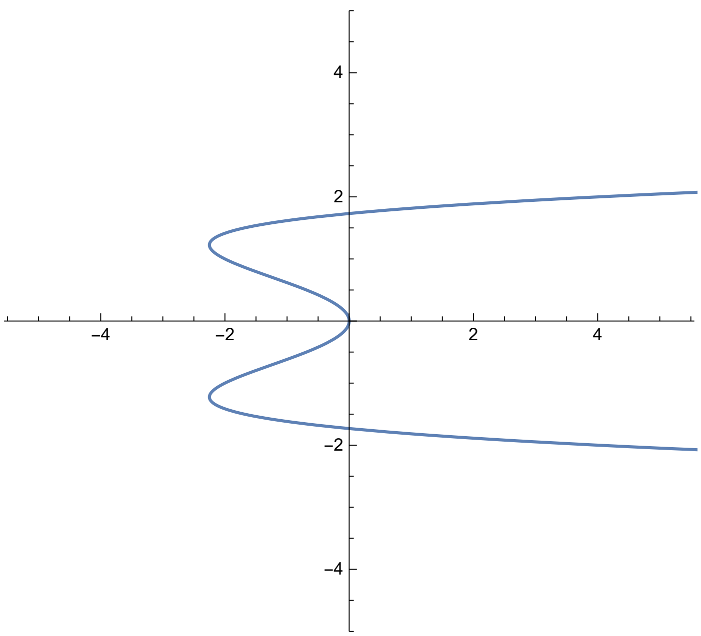
Solve: $(-3t^2+t^4)^2$ is not a valid variable.

Out[73]= Solve[True, $(-3t^2+t^4)^2$, ℝ]

In[74]:= **Let y = t, Then $x = t^4 - 3t^2$**

In[74]:= `ParametricPlot[{t^4 - 3 t^2, t}, {t, -5, 5}, PlotRange → {-5, 5}]`

Out[74]=



Set: Tag Times in $(-3t^2+t^4)$ t.Then is Protected.

Set: Tag Times in Let $(-3t^2+t^4)^2$ is Protected.

Out[69]= $-3t^2+t^4$

Set: Tag Times in curve graph have $(-3t^2+t^4)$ the to we is Protected.

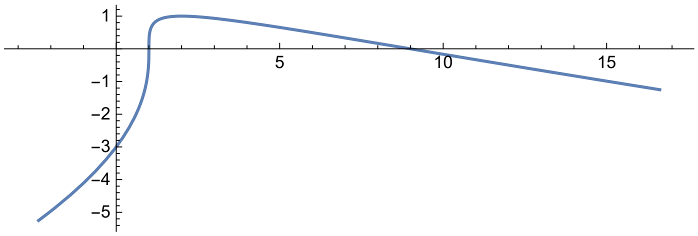
Out[57]= { { } }

Set: Tag Times in curve graph have $(-3t^2+t^4)$ the to we is Protected.

Find the area enclosed by the curve $x = t^3 + 1$, $y = 2t - t^2$ and the x axis.

In[75]:= `ParametricPlot[{t^3 + 1, 2 t - t^2}, {t, -1.5, 2.5}]`

Out[75]=



Say, $x = f(t) = t^2 + 1$ and $y = g(t) = 2t - t^2$.

Area$=\int_a^b y\, dx$

$\frac{dx}{dt} = 3t^2 => \mathrm{dx} = 3t^2\, \mathrm{dt}$

In[76]:= `Solve[2 t - t^2 == 0, t, Reals]`

Out[76]= `{{t → 0}, {t → 2}}`

In[77]:= $\text{Area} = \int_0^2 (2t - t^2)(3t^2)\, dt$

Set: Symbol Area is Protected.

Out[77]= $\frac{24}{5}$

Find the length of the curve $x = t^2 + 1$, $y = 2t - t^2$ from t=0 to t=2.

In[78]:= $L = \int_a^b \sqrt{\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)}\, dt$

Integrate: Unmatched differential operator dt found in the integrand body of $\int_a^b \sqrt{\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)}\, dt$. There may be too many differential operators or they may not appear at the end of the integral.

In[78]:= `D[t^3 + 1, t]`

Out[78]= $3t^2$

In[80]:= `D = [2 t = t^2, t]`

In[81]:= $L = \int_0^2 \sqrt{\left((3t^2)^2 + (2 - 2t)^2\right)}\, dt$ // N

Out[81]= 8.7891

Parametric equations of a semicircle are x=r cos(t) and y=r sin(t). Rotate the semicircle about x-axis and find the area of the surface.

about x axis.

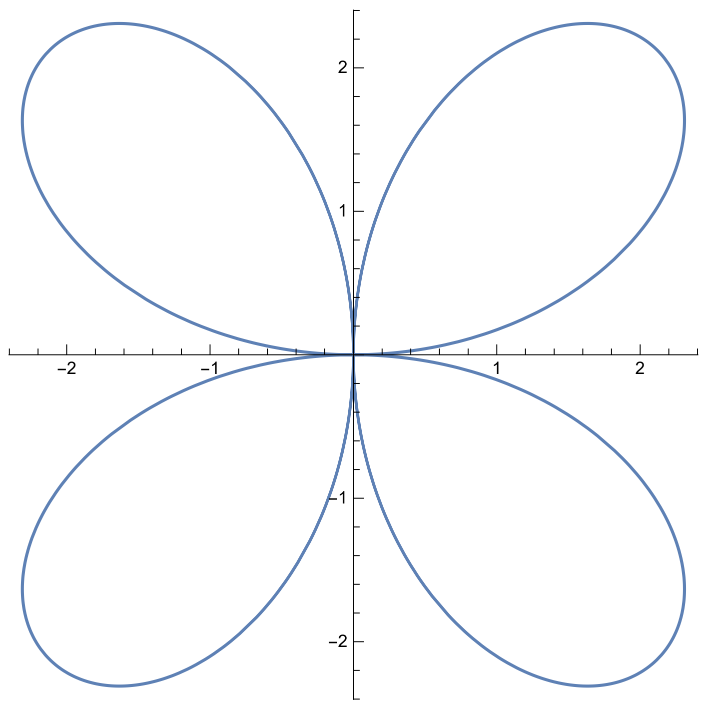
S=$\int_a^b 2\,\text{Pi}\, y\, ds$ where ds=$\sqrt{\left(\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right)}$

In[82]:= $\int_0^{\text{Pi}} 2\,\text{Pi}\,r\,\text{Sin}[t]\,\sqrt{\left(r^3\,(-\text{Sin}[t])^2+r^3\,(\text{Cos}[t])^2\right)}\,dt$

Out[82]= $4\,\pi\,r\,\sqrt{r^3}$

Graph r = 3 Sin[2 θ] for 0 ≤ θ ≤ 2 π.

In[84]:= `PolarPlot[3 Sin[2 θ], {θ, 0, 2 π}]`

Out[84]=


The path of the particle is given by:

x=sin(t)

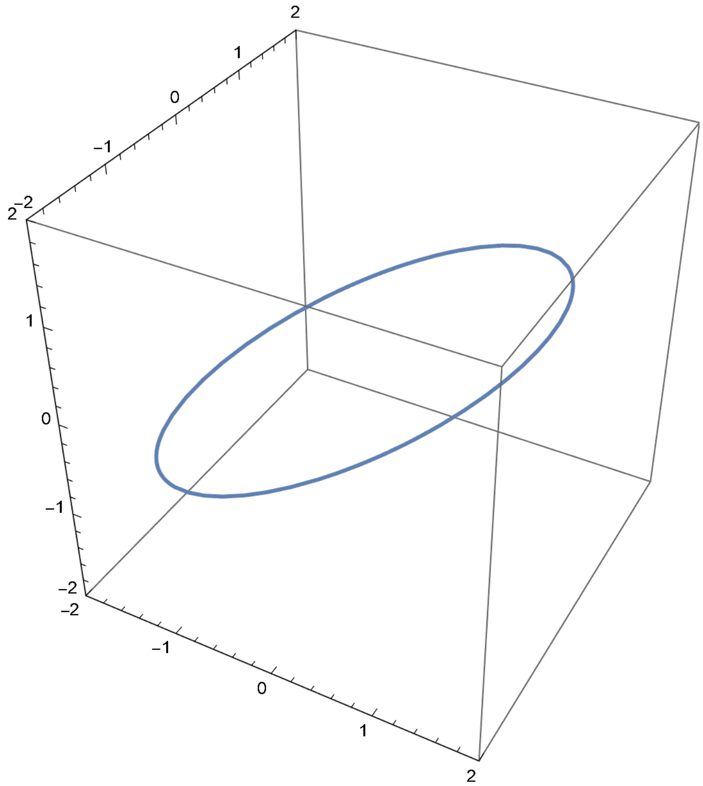
y=2Sin[t]

z=cos(t)

Plot the path of the particle for $0 \le t \le 2\pi$

In[86]:=
```
ParametricPlot3D[{Sin[t], 2 Sin[t], Cos[t]},
 {t, 0, 2 π}, PlotRange → {{-2, 2}, {-2, 2}, {-2, 2}}]
```

Out[86]=



In[103]:= $x = 4 \, Sin[\phi] \times Cos[\theta]$

Out[103]= $4 \, Cos[\theta] \times Sin[\phi]$

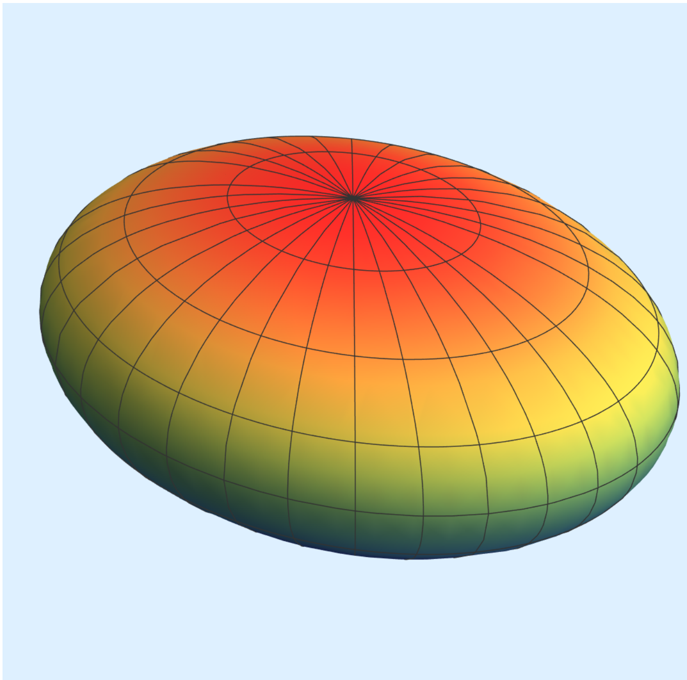
In[104]:= $y = 3 \, Sin[\phi] \times Sin[\theta]$

Out[104]= $3 \, Sin[\theta] \times Sin[\phi]$

In[105]:= $z = 2 \, Cos[\phi]$

Out[105]= $2 \, Cos[\phi]$

In[108]:=
```
ParametricPlot3D[{x, y, z}, {θ , 0, π}, {ϕ, 0, 2 π},
 ColorFunction → "Rainbow", Background -> LightBlue, Axes -> False, Boxed -> False]
```

Out[108]=


Positions of two particles at time t are given by:

x1 = Sin[t], y1 = t

x2 = Cos[t], y2 = π/4 - t

Do their paths intersect? Do they collide too?

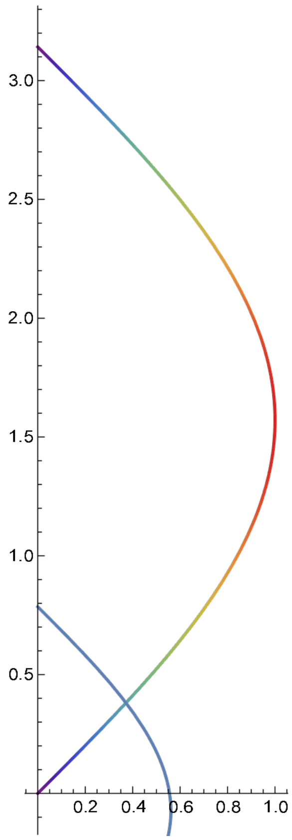
In[119]:=
```
ClearAll["Global *"]
```

In[113]:=
```
a = ParametricPlot[{Sin[t], t}, {t, 0, π}, ColorFunction → "Rainbow"];
```

In[114]:=
```
b = ParametricPlot[{Cos[t] t, π / 4 - t}, {t, 0, π}];
```

In[117]:=
```
Show[a, b]
```

Out[117]=



In[118]:=
```
Solve[{Sin[t] == Cos[2 t] && 0 ≤ t ≤ π, t == π / 3 - t}, t, Reals]
```

Out[118]= $\left\{\left\{t \to \frac{\pi}{6}\right\}\right\}$

A quantum particle has the initial wave function:

$$\Psi(x,0)=\begin{cases} \mathrm{Sin}\left[\frac{\pi x}{a}\right]^3 & 0 \le x \le \frac{a}{2} \\ \frac{5a}{6} - x & \frac{a}{2} \le x \le \frac{5a}{6} \end{cases}$$

1)Use the normalizing condition $\int_0^a (\Psi(x, 0))^2 \, dx = 1$ to find A.

2) Graph Ψ(x,0) for a=3.

In[124]:=
```
ClearAll["Global`*"]
```

In[125]:=
```
ψ[x_] = Piecewise[{{Sin[π x / a]^3, 0 ≤ x ≤ a/2}, {5 a/6 - x, a/2 ≤ x ≤ 5 a/6}}]
```

Out[125]=

$$\begin{cases} \mathrm{Sin}\left[\frac{\pi x}{a}\right]^3 & 0 \le x \le \frac{a}{2} \\ \frac{5a}{6} - x & \frac{a}{2} \le x \le \frac{5a}{6} \\ 0 & \mathrm{True} \end{cases}$$

In[129]:= $$\int_0^{a/2}\left(A\,\mathrm{Sin}\left[\frac{\pi x}{a}\right]^3\right)^2 dx+\int_{a/2}^{a}\left(A\left(\frac{5a}{6}-x\right)\right)^2 dx$$

Out[129]= $$\frac{5\,a\,A^2}{32}+\frac{a^3\,A^2}{72}$$

In[130]:= $$\mathrm{Solve}\left[\frac{5\,a\,A^2}{32}+\frac{a^3\,A^2}{72}==1,\ A,\ \mathrm{Reals}\right]$$

Out[130]= $$\left\{\left\{A\to\boxed{-12\sqrt{2}\sqrt{\frac{1}{45\,a+4\,a^3}}\quad \text{if}\quad a>0}\right\},\left\{A\to\boxed{12\sqrt{2}\sqrt{\frac{1}{45\,a+4\,a^3}}\quad \text{if}\quad a>0}\right\}\right\}$$

In[131]:= a = 3;

In[132]:= $$A=12\sqrt{2}\sqrt{\frac{1}{45\,a+4\,a^3}};$$

In[134]:= Plot[ψ[x], {x, 0, a}]

Out[134]=