

Question 04

Given that,

$$V_j = \sum_i w_{ji} y_i$$

$$e_j = d_j - y_j$$

$$\text{error, } E = \frac{1}{2} \sum_j e_j^2$$

Output from  $m$  layer neuron,  $y_m = \phi'(V_m)$

Here,

$$\begin{aligned} \delta_m &= \frac{\partial E}{\partial V_m} = \frac{\partial E}{\partial y_m} \frac{\partial y_m}{\partial V_m} \\ &= \frac{-\partial E}{\partial y_p} \phi'(V_m) \end{aligned}$$

$$\text{And, } E(n) = \frac{1}{2} \sum_{p \in c} e_p^2(n)$$

$$\begin{aligned} \Rightarrow \frac{-\partial E}{\partial y_m} &= \sum_{p \in c} e_p \frac{\partial e_p}{\partial y_m} \\ &= - \sum_{p \in c} e_p (-1) \phi(V_p) W_{pm} \\ &= \sum_{p \in c} \delta_p W_{pm} \end{aligned}$$

$$\begin{aligned} \frac{\partial e_p}{\partial y_m} &= \frac{\partial e_p}{\partial y_p} \frac{\partial y_p}{\partial V_p} \frac{\partial V_p}{\partial y_m} \\ &= (-1) \phi'(V_p) W_{pm} \end{aligned}$$

$$\therefore \delta_m = \phi'(V_m) \sum_{p \in c} \delta_p W_{pm}$$

$$\text{Finally, } \delta_m = \begin{cases} e_p \phi'(V_m) & [\text{When, } m \text{ is output neuron}] \\ \phi'(V_m) \sum_{p \in c} \delta_p W_{pm} & [\text{When, } m \text{ is hidden neuron}] \end{cases}$$

So, this is our computed local induced gradient,  $\delta_m$  (Ans)