

The Scandal of Deduction and Aristotle's Method for Discovering Syllogisms at Prior

Analytics I. 27-31

Abstract

Call the following argument the 'scandal of deduction'. (1) If a deductive argument is valid, then the conclusion is not novel. (2) If the conclusion of an argument is not novel, the argument is not useful. So, (3) if a deductive argument is valid, it is not useful. Recent philosophers have tried to avoid (3) by denying (1) but have assumed (2). I claim that Aristotle would not have to accept (3) because he assumes that (2) is false. My argument that (2) is false for Aristotle has two premises. First, his method for discovering syllogisms, given in Prior Analytics I 27-31, finds premises for a given conclusion. Hence, some syllogisms do not have novel conclusions. Second, Aristotle considers the syllogisms generated by the method useful in a range of contexts. So, Aristotle thinks that at least some syllogisms are useful, despite not presenting novel conclusions. The contrast between certain contemporary approaches and Aristotle exposes a hidden pragmatic assumption that makes the scandal worrisome. Aristotle would deny that assumption.

I. Introduction

Philosophers have long been interested in a certain problem. The problem has two premises. (1) If a deductive argument is valid, then the conclusion is not novel. (2) If the conclusion of an argument is not novel, the argument is not useful. So, (3) if a deductive argument is valid, then it is not useful.¹ This problem has gone under various names, including the ‘paradox of inference’ and the ‘scandal of deduction’.² No one accepts the conclusion. But since the argument is valid, we must reject at least one of the premises in order to avoid the conclusion. This results in the following question for philosophers of logic: should we reject (1) or (2)?

20th Century philosophers have tried to give good reasons to reject (1) but have assumed premise (2). I argue here that Aristotle would deny (2). Aristotle acknowledges that sometimes a deductive argument is useful because of its novel conclusion. But Aristotle also thinks that at least some deductive arguments are useful, even though they present no new conclusions. Thus, Aristotle’s view contrasts with philosophers of logic in the 20th Century, who assume that all useful deductive arguments present a novel conclusion. I do not argue that Aristotle ‘solves’ the scandal: it was never posed in Aristotle’s time. Rather, I suggest that Aristotle does not face the problem which confronts some ancient and modern philosophers, because he assumes deductions can be useful, without presenting novel conclusions. Aristotle’s views of deduction tame the scandal.

¹ Since antiquity, some authors have challenged the epistemic value of deductive reasoning (e.g. Sextus Empiricus *Outlines of Pyrrhonism* II 195-197). Similar issues re-surfaced in the 19th Century in Mill (*System of Logic* II 3.2). For discussion of Mill see Walton (1977), Woods and Walton (1982, 92–97) and Woods (1999, 318–321). There are some similarities between the paradox I outline here and earlier challenges, but they are beyond the scope of this paper.

² Hintikka (1970, 135) coins the latter. I prefer Hintikka’s label, since there are other philosophical problems that also go under the label ‘the paradox of inference’. As will become clear below, I formulate the scandal as having two premises to expose what I take to be a hidden assumption that makes the scandal pressing. That hidden assumption is the connection, captured by (2), between novelty and usefulness.

Section II of this paper outlines in more detail the scandal of deduction and its tacit commitment to (2). Section III argues that Aristotle assumes (2) to be false. To do this, I show that Aristotle thinks that at least some syllogisms do not present a novel conclusion, but are nonetheless useful. My argument has two premises. First, Aristotle's method for discovering syllogisms given in *Prior Analytics* I. 27-31 finds at least one syllogism for a given conclusion. Because the method works 'backwards' from conclusion to premises, the conclusion is not novel. Second, Aristotle considers the syllogisms generated by the method useful in a range of contexts. In section III.1, I argue for my first premise. In section III.2, I argue for my second premise. In section IV, I draw some conclusions.

II. The Scandal of Deduction

One common way to motivate premise (1) of the scandal invokes the theory of semantic information suggested by Popper (1959, 96), Shannon and Weaver (1948) and Bar-Hillel and Carnap (1953). The theory posits an inverse relationship between probability and informativeness. An information bearer is informative just in so far as that information bearer is unlikely (Floridi 2006). Tautologies, however, are certain to hold so they carry no information. But, in classical logic, a sentence q is deducible from a set of premises p_1, \dots, p_n iff the conditional $p_1 \& \dots \& p_n \rightarrow q$ is a tautology. Since tautologies carry no information, no deductive

inference can yield an increase in information (D'Agostino and Floridi 2009, 3; D'Agostino 2013a, 40).³ So if an argument is classically valid, it cannot present a novel conclusion.

I will discuss two ways in which 20th Century philosophers of logic have approached the scandal. The first is the 'psychological novelty' approach, put forward by Hempel. The second is the 'objective novelty' approach, first put forward by Hintikka.⁴ Both approaches reject (1) by trying to show how an argument can be both valid and conclude something new. Because both attempts to address the scandal try to deny (1), they tacitly assume (2). I begin with Hempel.⁵

Hempel claims that the conclusion of a deduction may be psychologically new. Some agent may not realize that a particular conclusion follows from the premises. Although the conclusion is 'contained' within the premises, that conclusion can be novel to me. If an agent were logically omniscient, then she would believe all the consequences of her beliefs. But, Hempel points out, since we are not logically omniscient, a deductive argument might still be useful. Deduction is useful for drawing novel beliefs out of beliefs we already hold. The drawbacks with Hempel's approach are well articulated by Hintikka (1970, 222) and have been widely accepted since. But I want to stress that Hempel assumes that the usefulness of deduction

³ Note that there are other ways, arguably independent of the theory of semantic information, to motivate (1). Wittgenstein's *Tractatus* (e.g. Sections 6.125-6.127) prominently features the idea that logical proofs are tautologies and the results cannot be surprising, without discussing semantic information. Others motivate (1) in a more informal way (e.g. Cohen and Nagel 1934, 173; Hempel 1945; Hintikka 1970, 135; Dummett 1993, 195 and Dutilh-Novaes 2010, 1). Aristotle himself seems, at least in places, sympathetic to (1). *Prior Analytics* II 21 67a5-67b11 and *Posterior Analytics* I 1 71a17-71b8 both suggest that we already know, in some sense of 'know', the conclusion of an argument when we know the premises.

⁴ More recent philosophers have developed approaches that don't fit neatly into either camp (e.g. Sequoiah-Grayson 2008; Primiero 2008, chap. 2; Sillari 2008; D'Agostino and Floridi 2009; Duží 2010 and Jago 2013).

⁵ Hempel (1945) cited by D'Agostino and Floridi (2009, 7). Cf. Dummett (1993, 195).

consists in producing a certain kind of novelty: psychological novelty. Hempel assumes a connection between novelty and usefulness.

The ‘objective novelty’ approach to the scandal reveals a similar pattern. Hintikka (1970) tries to show that deduction can be valid and produce novel conclusions.⁶ Hintikka holds the novelty to be objective, but shares with Hempel the view that novelty is necessary for usefulness. Hintikka distinguishes between ‘surface’ and ‘depth’ information. Both sorts of information are objective. Depth information is a sort of potential information and so cannot increase with deductive inference. Surface information, on the other hand, can grow with deductive inference since it is, so to speak, actualized information. Hintikka’s distinction between two sorts of information is only present in non-decidable logics, so only such logics can ‘increase’ information, by actualizing the potential information. In a decidable logic, all the potential information is actual.⁷

Hintikka’s approach has been criticized on various grounds (see Sequoiah-Grayson 2008). For now, I stress that Hintikka cannot say how propositional logic, monadic predicate calculus and, indeed, syllogistic logic increase information, since they are all decidable. In other words, Hintikka’s solution is not general. But what interests me here is Hintikka’s strategy. In response to the scandal he denies (1) by trying to give a sense in which a deductive argument, in this case, an argument in predicate calculus with embedded quantification, can be both valid and

⁶ Hintikka (1973) develops the philosophical consequences of this perspective.

⁷ See D’Agostino and Floridi (2009, 9) for this reading of Hintikka’s approach.

have a conclusion with more objective information than the premises.⁸ Like Hempel, then, he assumes a connection between validity and usefulness.

Rather than rehearse merits of existing solutions, I want to suggest an approach to the scandal that is not yet well developed.⁹ Both the psychological and objective approaches try to show that a conclusion can be novel, even though a deduction is valid. That is, they deny (1). But premise (2) is what makes the scandal troubling. Without (2), an assumed connection between novelty and usefulness, the worst that we can conclude is that valid arguments do not yield new knowledge. But, in itself, a lack of novelty is not worrying. Lack of novelty is only worrying in a context where we think deduction yields new knowledge, like mathematics or science. In short, there is a hidden pragmatic assumption, connecting novelty with usefulness, at play in the scandal, which I made explicit in (2). Without (2) the scandal of deduction is not scandalous at all. In the next section, I will discuss Aristotle's attitude towards the usefulness of deduction. I will show that for Aristotle, novelty is not necessary for usefulness. In this way, he has a different view of the usefulness of deduction to 20th Century philosophers of logic and because he assumes that (2) is false, he would not face the scandal of deduction.

III. Aristotle on the Novelty and Usefulness of Deduction

Aristotle does not hold that, in deduction, usefulness necessitates novelty. In some cases, a useful deduction does not need a novel conclusion. I do not need to prove the stronger claim that, for

⁸ D'Agostino and Floridi (2009) develop an approach, distinct from Hintikka's. See also D'Agostino (2013b).

⁹ For a good overview of the available approaches, see D'Agostino (2013a, 39–44).

Aristotle, the usefulness of a deduction is never the novelty of its conclusion. Indeed, Aristotle sometimes locates the usefulness of a deduction in the psychological novelty of the conclusion.¹⁰ But it will become clear that in many cases, a deduction is useful, for Aristotle, despite lacking a novel conclusion.

My argument for this has two premises. First, Aristotle thinks some deductions lack a novel conclusion. Second, those deductions are nevertheless useful, in certain contexts. I argue for the first premise on the basis of an important, but neglected, stretch of *Prior Analytics I*, where Aristotle presents his method for discovering syllogisms.¹¹ The method moves ‘backwards’ to discover premises for certain conclusions. Hence, the conclusions are not novel. I argue for the second premise by showing that Aristotle thinks the method discovers syllogisms that are useful in a range of contexts. But in none of these contexts does their usefulness depend on producing novel conclusions.¹²

¹⁰ See, for example, *Posterior Analytics I* 1 71a25-71b11 and *Prior Analytics II* 21. These passages invoke an epistemic sense in which the conclusion of a deductive argument is novel. *Topics VIII* 155b1-15 suggests that a psychologically novel conclusion might be useful in dialectic.

¹¹ I use the term ‘discovery’ and cognates, rather than ‘heuristic’, to distinguish the method I identify from the widely discussed ‘heuristic’ use of teleology in Aristotle’s science discussed by Wieland (1975a). Some think that teleology is heuristic in the sense that it is a means to discover the real (material and efficient) causes of natural phenomena. When this is done, teleological explanations are discarded. In a variation on this, Leunissen (2010, 125–7) thinks that certain teleological principles, such as ‘nature does nothing in vain’, are heuristic, but she differs from Wieland in that she holds that teleological causes are also real causes.

¹² A terminological note: Aristotle uses the expression ‘syllogismos’ in a range of ways. Sometimes he means roughly what we mean by ‘a deduction’, namely, an indefeasible argument of any sort (e.g. *Prior Analytics I* 24b18). At other times he uses it to mean an argument with two categorical premises connected by a middle term and a conclusion in one of his fourteen valid moods. Call the former ‘broad syllogisms’ and the latter ‘narrow syllogisms’. Of course, all narrow syllogisms are also broad syllogisms, so all narrow syllogisms are deductions. When discussing Aristotle, I use the term ‘deduction’ to mean an indefeasible argument and ‘syllogism’ to mean a narrow syllogism.

III.1 Some deductions lack novel conclusions

Aristotle gives us a tripartite division of the project of Prior Analytics I (Prior Analytics I 32 47a1-6 cf. I 27 43a20-24 and II 1 52b38-53a3). Chapters 1-22 comprise the first part and concern the proofs of the syllogistic moods. Chapters 23-31, which make up the second part, discuss indirect arguments and how to find syllogisms for a given conclusion. The third part, chapters 32-44, discusses how to express informal arguments in Aristotle's regimented system. The bulk of the second part is what I call, following Aristotle, a discovery method for syllogisms.

Since Aristotle's discovery method depends on some formal properties of his narrow syllogisms, I need to expound some details. Syllogisms are Aristotle's celebrated arguments, with two premises and a conclusion. Each premise and conclusion is in the categorical subject-predicate form, where the predicate (A) and subject (B), known as 'terms', relate in one of four ways:

1. A belongs to every B (the 'a' relation, written as AaB);
2. A belongs to some B (the 'i' relation, written AiB);
3. A belongs to no B (the 'e' relation, written AeB);
4. A does not belong to some B (the 'o' relation, written AoB).

Valid syllogistic forms, the moods, are known by their medieval mnemonic names, Barbara, Celarent, Darii, Ferio and so on. Barbara, for instance, has the form AaM, MaB, AaB. An example of an argument in Barbara would be: mortal belongs to all humans; human belongs to

all Athenians; so, mortal belongs to all Athenians. Key to giving a valid syllogistic argument is the middle term, labeled above as ‘M’. This term occurs in each premise, but not in the conclusion. In our example, ‘human’ is the middle term. Each and every valid syllogism has a middle term, related in a certain way to the subject and predicate of the conclusion. The discovery method finds, for a given conclusion, a middle term to prove that conclusion. Once we have the middle term, along with the other two terms which feature in the conclusion, we can determine the argument we need to prove that conclusion.

As Aristotle describes it, the method has two steps. The first step develops lists of terms and how those terms relate to each other. The second step applies a mechanical procedure, a procedure which requires no insight, to those lists. The procedure gives us a middle term we need to argue for a given conclusion. At *Prior Analytics* I 27 43b1-11, Aristotle describes the first part of the method. He tells us to set down the target term, call it ‘T’.¹³ We then gather lists of terms that relate to T in three categories: what follows T, what is followed by T and what cannot belong to T. This ‘follows’ relationship picks out predication: the Greek equivalents of ‘A follows B’, ‘A belongs to B’ and ‘A is predicated of B’ all mean the same in this context. Aristotle tells us to collect basic information about which terms relate to T, but also information about the kind of relation. Is the predication definitional, peculiar or accidental? Is it true or reputable? I will return to the way that the tables are organized in section III.2, when I discuss how the resulting syllogisms are useful. Below I give a table containing a sample of the lists.

	Term	... is subject of...	... is predicate of...	Terms that cannot belong...
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¹³ Cf. Smith (1989, 150) and Striker (2009, 192). Striker interprets ‘T’ as being only the terms in the conclusion that we are interested in establishing. But that cannot be correct, since we are still gathering information into the tables at this stage: we do not yet know what we will be arguing for, hence do not know what terms will be in the conclusion. As I read Aristotle, ‘T’ is just any term that we are interested in adding to the table.

	biologist	rational, human, scientist	geneticist, zoologist	stone, dog
	scholar	rational, human	geneticist, physicist, scientist	stone, dog

Table 1: Example lists for target terms 'biologist' and 'scholar'

Aristotle gives two more requirements on the basic lists. First, the universal predication requirement: the lists should contain only predicates of T that belong to all T and subjects of T such that T belongs to all of the subject (Prior Analytics I 27 43a11-13). Although Aristotle introduces it without explanation, the syllogistic system justifies the requirement. Each valid mood involves at least one universal predication: no valid mood has two particular premises. So, in so far as we want to generate syllogisms, we need to guarantee that at least one premise involves a universal predication. We can ensure this by recording only universal predications in our table.

Aristotle also states, but does not explain, the specificity requirement: do not include in the lists terms that apply to every item (terms such as 'thing' or 'self-identical') (Prior Analytics I 27 43b35-39). Again, this requirement is justified. If terms that apply to every item were included in the lists, then the method might generate invalid arguments. Take an arbitrary 'i' conclusion, AiB . I can prove AiB using the third figure Darapti (MiA , MiB , so, AiB), as long as the middle term, M , is predicated of every A and every B . If that middle term happens to belong to everything, then I can prove any AiB , including a false one, from two true premises. Imagine I recorded 'self-identical' in my list of terms. The method then might discover the following invalid argument: self-identical belongs to every scientist; self-identical belongs to every stone; so, scientist belongs to every stone. Without the specificity requirement, the method could discover invalid arguments.

Aristotle includes the method for discovering syllogisms for practical purposes. An individual in a discursive context could use the method. Despite the view of Smith (1989, 150), Aristotle is clear that the lists record neither all the predications nor only true ones involving the target term T. First, an exhaustive collection of predications would be impossible for any actual agent. Moreover, Aristotle implicitly denies that the lists of terms will be exhaustive: ‘the more such terms one has available, the faster one will hit upon a conclusion and the more such terms one has available, the more one will demonstrate’ (Prior Analytics I 27 43b9-11). If Aristotle conceived of the lists as exhaustive, his advice to get as many terms on the list as possible would be irrelevant. So the lists of predication terms can be non-exhaustive. Second, the recorded predications need not be true, since Aristotle tells us that we should record which predications hold ‘only as a matter of opinion and which according to truth’ (Prior Analytics I 27 43b8-10). This remark would be moot if Aristotle held that the lists record only true predications. So Aristotle does not think the tables collect all the true predications nor only the true ones.

Generating such tables is the first step in the discovery method. The second part of the method is the mechanical procedure for finding the middle term (and hence premises) for a given conclusion. Aristotle gives us four rules, one corresponding to each form the conclusion can take (a, i, e or o). Prior Analytics I 28 43b39-44a1 describes the rule for finding the middle term for an a-conclusion. I will call this the ‘a-rule’:

- (A) If one wants to establish a conclusion of the form ‘A belong to all B’, then (i) take the list of terms that A is the predicate of, and (ii) take the list of terms B is the subject of. Any term on both lists will be a middle term for the conclusion A belongs to all B.

This procedure finds a middle term for the ‘a’ conclusion. Barbara is the only mood that can establish a universal affirmative conclusion. Barbara has the form: ‘A belongs to all M, M belongs to all B, so, A belongs to all B’. The procedure tells us to take both lists of terms: those that A is predicate of and those that B is subject of. Any item on both lists will be a middle term for the conclusion. The procedure may, of course, find more than one possible middle term for the conclusion.

Let me illustrate the mechanical procedure with an example. Suppose we are trying to find an argument for the conclusion that scholar belongs to every biologist. We can extend the table given above to note the role each target term plays in the conclusion (i.e. whether it is a subject or a predicate). This gives the following table:

Place in the conclusion:	Term	... is subject of...	... is predicate of...	Terms that cannot belong...
Subject	biologist	rational, human, scientist	geneticist, zoologist	stone, dog
Predicate	scholar	rational, human	geneticist, physicist, scientist	stone, dog

Table 2: Example lists for target terms 'biologist' and 'scholar' organized to find middle term for 'scholar belongs to every biologist'

Since the conclusion is a universal affirmative, we apply the a-rule. First, we look at the list of terms which the predicate of the conclusion, ‘scholar’, is predicate of. We find there the terms ‘geneticist’, ‘physicist’ and ‘scientist’. Next we look at all the terms that the subject of the conclusion, ‘biologist’, is subject of. We find ‘rational’, ‘human’ and ‘scientist’. We now see that ‘scientist’ is a term on both lists. According to the mechanical procedure, ‘scientist’ will be a middle term for a syllogism with ‘scholar belongs to every biologist’ as the conclusion. Indeed it is: scholar belongs to every scientist; scientist belongs to every biologist; so, scholar belongs to

every biologist. Note that the procedure is an existential search: it looks for one possible middle term for the conclusion. The procedure may not, and need not, find every possible middle term for the conclusion.

There are a few points to note about the procedure. First, it relies on the lists of terms generated in the first step of the discovery method. The procedure cannot do anything without the lists telling us how terms truly or apparently relate, because the procedure operates on the lists. Second, the procedure is mechanical in the sense that it gives detailed instructions for finding a syllogism for a particular conclusion, which can be applied without insight. The conclusion fully determines which rule to apply; if the tables contain sufficient information, the rule determines at least one middle term that will give that conclusion. Facts about the syllogistic system determine how to relate the middle term to the others to produce a valid syllogism. In contrast, generating the lists of terms is not obviously a mechanical procedure: it seems to require insight into, at least, the meanings of the terms.¹⁴

I have given just one simple example, using the ‘a-rule’. Here are the other three rules that Aristotle gives:

- (I) If one wants to establish a conclusion of the form ‘A belongs to some B’, then (i) look at the terms that A is the predicate of, and (ii) look at the terms B is the predicate of. Any

¹⁴ Topics I. 14 suggests a similar, but relevantly different, method. In the Topics passage, Aristotle suggests that we draw up tables to help us play the dialectical game. The similarities, I think, are superficial. In the Topics, Aristotle wants the reader to draw up lists of ‘premises’, not lists of terms. In the Topics context, these premises are the starting points of debates, which have to be proved or refuted, rather than the elements which go into making a premise. Moreover, in the Topics, Aristotle does not give us a method for using these lists, unlike in the Prior Analytics, where the mechanical part of the procedure is key.

term on both lists will be a middle term for the conclusion A belongs to some B (Prior Analytics I 28 44a1-3).¹⁵

(E) If one wants to establish a conclusion of the form ‘A belongs to no B’, then (i) look at the terms that cannot belong to A, and (ii) look at the terms B is the subject of. Any term on both lists will be a middle term for the conclusion A belongs to no B (Prior Analytics I 2844a4-7).¹⁶

(O) If one wants to establish a conclusion of the form ‘A does not belong to some B’, then (i) look at the terms that cannot belong to A, and (ii) look at the terms that B is the predicate of. Any term on both lists will be a middle term for the conclusion A does not belong to some B (Prior Analytics I 28 44a8-12).¹⁷

Each rule is a conditional, which tells us what to do given that we want to find a certain conclusion. When we apply any of the rules, comparing the two lists may not yield any terms. If so, we cannot find a syllogism for the conclusion using the information contained in the tables we have. But if comparing the lists does yield at least one term, there will be a middle term for that conclusion. If the two lists share more than one term, then a further step is required, namely

¹⁵ Given that the lists only include terms that belong to all other terms, this rule will only generate syllogisms in the third figure Darapti. There are other syllogisms with ‘i’ conclusions, but you cannot find them using the discovery method. This is further evidence that the method is for discovering some syllogism for a given conclusion, not every syllogism.

¹⁶ This rule will give us a syllogism in Cesare. But, of course, there are other syllogisms with an ‘e’ conclusion. Aristotle gives us an alternative rule, where we can look at terms that cannot belong to the subject term, but which belong to the predicate term. If we do this, we get a second figure syllogism, Camestres. But again, this leaves out other ways of proving an ‘e’ conclusion, such as Celarent.

¹⁷ This rule can only produce syllogisms in Felapton. But, again, there are other ways to prove the ‘o’ conclusion, e.g. Ferio, Festino, Baroco, Bocardo and Ferison.

to select one of the terms to be a middle term for this particular conclusion. I will discuss below the reasons one might have to select one middle term rather than another.¹⁸

It is now easy to see that, according to Aristotle, some arguments do not present novel conclusions. The discovery method discovers deductive arguments of a certain sort, namely, syllogisms. The conclusions of those syllogisms are not novel. Each of the four rules begins with the conclusion and tells how to find an argument to establish it. The conclusion that one agent wants to establish or convince someone of has already been identified. So the conclusion cannot be novel, in either the psychological sense or in the objective sense. The conclusion cannot be novel in the psychological sense, since the discoverer already knows what the conclusion is before she discovers the syllogism. The conclusion is not novel in some objective sense either. The conclusion contains no more (objective) information than the premises built around the middle term, whatever this notion of objective information might be. Aristotle, then, happily admits that at least some syllogistic arguments, in certain situations, do not present novel conclusions. In the next section, I argue that Aristotle, nonetheless, considers such arguments

¹⁸ I should mention an alternative reading of the discovery method. Smith takes the method as comparable to a ‘decision procedure’ in modern logic. Smith (1981, 154) attributes to Aristotle the claim that the ‘procedure will find a deduction if and only if a deduction is possible’. If this bi-conditional holds, the method will find all and only possible deductions, which would be sufficient to determine what can be proved from a set of truths. If Aristotle thinks that the bi-conditional holds, then Aristotle thinks the method will find all conclusions that can be proved, not just from information in the tables, but in general. But Aristotle never asserts the biconditional and in fact it is false. One direction of the bi-conditional holds trivially: if the method finds a deduction then the deduction is possible. But the other direction is frighteningly strong: if a deduction is possible, then the method will find it. The method, of course, cannot find all possible deductions, but only a very small class of deductions, if any: those deductions that will yield a given conclusion, based on information contained within a certain set of tables.

useful. This will show that, for Aristotle, some valid arguments don't present novel conclusions, but are nonetheless useful, i.e. that Aristotle holds (2) to be false.¹⁹

III.2 How the syllogisms discovered are useful

Whether something is useful depends on what we intend to use it for. The way Aristotle sets out the discovery method points to two contexts of use for the discovered syllogisms. The tables record information about predications. But Aristotle also suggests that we group that information according to whether those predications are true or reputable (endoxon) (Prior Analytics I 27 43b2-4). The dichotomy between true and reputable predications recalls the beginning of the Prior Analytics, where Aristotle divides syllogistic premises, and hence syllogisms, into demonstrative and dialectical sorts at Prior Analytics I 1 24a30-24b11 (Cf. Topics I 100a27-30). Aristotle says that a premise is demonstrative if 'it is true and accepted on the basis of the starting points' (Prior Analytics I 1 24a30), while a dialectical premise is the taking of something 'apparent and reputable (endoxon)' (Prior Analytics I 1 24b10). This dichotomy suggests, again, that the method is useful in the context of dialectic, but also in the context of demonstration.

When he reflects on the application of the discovery method, Aristotle confirms that there are two contexts of use, demonstration and dialectic, for the syllogisms discovered by the method:

¹⁹ David Lee raised the following objection in discussion of this paper. Syllogisms do not produce novel conclusions because the syllogistic is a decidable logic and no decidable logic can increase information yield (only non-decidable logics can). Lee's point here is a special case of Hintikka's idea that only non-decidable logics can yield an increase in surface information. My response is to welcome Lee's point. In fact, Lee's claim that syllogisms cannot present novel conclusions entails the premise I need for my argument, namely, that syllogisms discovered by the method do not present novel conclusions. Of course, my premise assumes less than Lee's. In particular, my premise does not commit me to a view about decidability and information yield.

When it is a question of truth [one must select] from the lists of those things that belong truly, but when in a dialectical syllogism [one must select] premises from the premises according to opinion (Prior Analytics I 30 46a8-10).

Notice that Aristotle does not say that the discovery method itself is useful in the context of demonstration and the context of dialectic. Rather, the resulting syllogism is useful: we select from the list of true predications when we want to find demonstrations, and from the list of opinion predications when we want a dialectical syllogism. This is important, since I have claimed that the usefulness of deductions, not the usefulness of some method for finding syllogisms, is what is at stake in the scandal of deduction. Aristotle here confirms that he is interested in how the resulting deductions are useful.

Aristotle suggests demonstration and dialectic as two broad contexts of use for syllogisms discovered by the method. The key difference between these two contexts is that dialectic contexts are necessarily multi-agent, while demonstrative contexts need not be. This is clear from Prior Analytics I 1 24a22-25, where Aristotle says that in a demonstration premises are assumed, while in dialectic they must be asked for from an interlocutor. Since in dialectic, premises must be asked for, at least two agents are involved in the argument, but the same does not hold of demonstration (cf. also Topics I 1 100a27-30 and Sophistical Refutations I 164a37-165b9). This formal difference, however, does not affect the underlying logic of the argument (Prior Analytics I 1 24a25-28), so the syllogisms found by the discovery method are available for use in both cases. So I must now show how the syllogisms produced by the discovery method,

which, necessarily do not present novel conclusions, are useful in mono- and multi-agent contexts, as Aristotle conceives of them. I begin with mono-agent contexts of demonstration.

The first use of a syllogism without a novel conclusion is simply to be certain that the conclusion is the case. A syllogism is an indefeasible, necessarily truth-preserving argument: if the premises are true, then the conclusion must be true. Syllogisms are a powerful epistemic tool because they preserve truth. Since they preserve truth, I can be certain that the conclusion is true, if the premises are true. Aristotle's remarks in *Topics* I 1 100b18, which stress that demonstrations have true premises, suggest this use of demonstration. I might think that, for example, all biologists are scientists, but I might wish to convince myself that all biologists are scientists. If I have sufficiently comprehensive tables of true predications concerning the practitioners of science, then I will be able to discover whether all biologists are scientists. Then, given that the premises are true, I can be certain that the conclusion is true, since the syllogisms discovered by the method have the property of necessary truth-preservation.

There is second use for syllogisms in demonstrative contexts, independent of the novelty of their conclusions. A syllogism of a certain sort might help us to understand the conclusion by showing us how it is integrated into a systematic body of knowledge, such as one of the special sciences.²⁰ We may, of course, wish to integrate a conclusion which we already believe or know to be true. *Posterior Analytics* 71b17-22 claims that demonstrations are deductions of a certain kind. Demonstrations are deductions which give rise to understanding.²¹ Aristotle lists six

²⁰ Cf. *Physics* I 1 184a16-20, *Posterior Analytics* I 1 71a1-30 and *Metaphysics* II 9 992b29-993a1. In these passages, Aristotle suggests that we start with propositions that are better known to us, and progress to those that are better known 'in themselves'. Wieland (1975b, 129–130) interprets this as Aristotle pointing out that we begin with one sort of knowledge and 'upgrade' it to another. This is precisely the process we see in the discovery method, as I describe it.

²¹ I follow Barnes (1994) in translating *episteme* and cognates as 'understanding', who himself follows Burnyeat (1981, 97–108). Roughly, Burnyeat's insight is that there is something holistic

individually necessary conditions on a demonstrative syllogism giving understanding. Demonstrative syllogisms are (a) true, (b) primitive, (c) unmediated, (d) better known than, (e) prior to and (f) explanatory of the conclusion.²² I focus here on the condition that the starting points are ‘better known’ than the conclusion. I argue that demonstrations are useful, according to Aristotle, because they make the conclusions better known. So how should we understand Aristotle’s idea that the premises are ‘better known’ than the conclusion?²³

Topics VI 6 141b30-34 illuminates this question. Aristotle explains that both genus and differentia are better known than the species. Aristotle’s analysis of being better known is in modal and epistemological terms. In his example, ‘man’ names the species, ‘animal’ the genus and ‘footed’ the differentia. Aristotle’s explains that an agent can know the genus, being an animal, without necessarily knowing the species, being a man. Likewise with the differentia, footed. But it is not possible to know the species without knowing the differentia. We could formulate Aristotle’s idea of being better known this way:

about episteme in Aristotle’s Greek that is not captured by our noun ‘knowledge’. That holism is, incidentally, captured by our adjective ‘knowledgeable’.

²² If (a) to (f) are conditions on the principles appropriate to some domain of investigation, it does not follow that every premise in every proof in that domain must meet all six conditions. The six conditions Aristotle gives on ‘demonstration’ do not apply to the premises of each and every deduction; rather they ‘properly characterize the principles of axioms of a demonstrative science’ (Barnes 1994, 93).

²³ Aristotle also mentions that a demonstration can fail because the premises are not ‘prior’ to the conclusion (Prior Analytics II 21 64b33-4), but in Posterior Analytics I 2 71b31-72a5, Aristotle seems to identify the being better known and priority requirements. In any case, discussing being better known is sufficient for the point I wish to make. This brief explanation is also given in [author’s publication A].

(BK) X is better known to a than Y iff it is possible that (a knows X and a does not know Y).²⁴

If (BK) states Aristotle's principle, the 'better known than' relation is irreflexive. When we replace 'X' and 'Y' in the schema with the same expression, the right-hand side of the resulting biconditional is false. For (BK) to hold, whatever replaces 'X' cannot also replace 'Y'. In the case where we better know the premises, it follows from BK that we do not know the conclusion. In that case, at least, there is a hierarchical structure between the things a demonstration proceeds from (the premises) and what it proceeds to (the conclusions): the premises are different from the conclusion and better known than it.

But we 'upgrade' our cognition of the conclusion to understanding by showing how it relates to this hierarchy of better-known propositions. Aristotle's idea here, then, is that we know p better, once we have shown how it relates to other propositions, q, and r, which are different from p. Suppose q and r are better known than p: that is it is possible to know q and r but not p. Assuming that deduction is closed under known implication, if we know q and r, p can become known once we know how to relate it deductively to q and r. We get to know p by integrating p into the systematic structure of knowledge we already possess. But to do this we already have p available; so, p is not novel. What we need is a deduction of p. That deduction will be useful, as it will help us understand p by learning how p fits with propositions we know. But p need not

²⁴ The brackets in this formulation indicate that the possibility operator has a wide scope. Aristotle does not, in general distinguish between individuals, terms and propositions when discussing knowledge. Hence 'X' and 'Y' can range over any of these classes.

be novel. So a deduction can be useful for increasing understanding, but without producing a novel conclusion.

The discovery method can help integrate a given, previously known, conclusion into a wider body of knowledge. Aristotle tells us as much at *Prior Analytics* 46a18-27. A brief example will prove his point. Suppose we have extensive tables of predicates in the domain of geometry. I want to understand why it is the case that:

(C) This right-angled triangle of sides 3 unit-lengths and 4 unit-lengths has a hypotenuse of 5 unit-lengths.

I have a true belief that (C) but I want to upgrade my cognitive achievement to understanding. Using my tables and discovery method I can discover a syllogism to (C) in Darapti, involving the following two premises:

(P1) All right-angled triangles have sides whose squares sum to the square on the hypotenuse;

(P2) All right-angled triangles of sides 3 unit-lengths and 4 unit-lengths have sides whose square sums to the square on the hypotenuse.

It is clear, given (BK), that (P1) and (P2) are better known than (C): I can know each of (P1) and (P2) without knowing (C). But once I have a syllogism from (P1) and (P2) to (C), (C) becomes as well-known as them, because I cannot know (P1) and (P2) together without knowing (C), again assuming that knowledge is closed under known implication. Here is a syllogism that results from the discovery method and so cannot have a novel conclusion. But the deduction in question is still useful in Aristotle's epistemic framework, since it allows me to relate a conclusion to other propositions I know and hence to better know the conclusion.²⁵

The syllogisms discovered by the method, then, are useful in demonstrative contexts, even though they do not present novel conclusions. The same is true of some multi-agent contexts. At *Sophistical Refutations* I 164a37-165b9, Aristotle distinguishes four sorts of multi-agent encounter. The first, didactic, is the context of teaching. The second, dialectic in a narrow sense, is a game to drive one's interlocutor into a contradiction, starting with reputable opinions (*endoxa*).²⁶ The third is *peirastic*, roughly, Socratic-style questioning. The fourth, is *eristic*,

²⁵ Although the method allows us to understand the conclusion through a demonstration, other conditions Aristotle puts on demonstration cannot reliably be met by the deductions discovered in the discovery method. Condition (f), for example, cannot reliably be met by the method. The method discovers some middle term through which a given conclusion can be demonstrated. The method may not find terms that explain a conclusion, merely those that can be used to deduce it. For example, scholar belongs to all geneticist can be deduced using two different middle terms: scientist or lab-worker. But only the former middle term explains why scholar belongs to all geneticist: geneticists are scholars because they are scientists, not because they are lab workers. This should not concern us: Aristotle is well aware that there is a difference between demonstrations that something is the case and demonstrations which explain why something is the case. For the distinction between 'factual' and 'explanatory' syllogisms see *Posterior Analytics* I 13 78b35-79a6; *Posterior Analytics* II 2 89b44-90a34; *Progression of Animals* 1 704b9-10; *Parts of Animals* II 1 646a8 and *Prior Analytics* II 2 53b9.

²⁶ By 'narrow' dialectical contexts, I mean the dialectical game that Aristotle describes in the *Topics*. I take it that 'broad' dialectical contexts are all those that involve reasoning from *endoxa*. There is a well-known debate about whether dialectic, in what I call the broad sense, somehow grounds enquiry in Aristotle. Defenders of this view include Irwin (1989) and Nussbaum (1986 Chapter 8). Criticism of this view is found in Hamlyn (1990), Bolton (1990) and Smith (1993).

contexts of exclusively adversarial argument. In this paper, I discuss the first two sorts of encounter and show how the arguments in such encounters need not present novel conclusions, although the arguments are useful. Remember that, to prove my case, I do not need to show that every useful argument lacks a novel conclusion, but only that some such arguments lack a novel conclusion.

I begin with narrow dialectical contexts. The Topics says that two agents participate in a dialectical syllogism and they proceed by question-and-answer (see Smith 1993, 337). The two participants are known as the questioner and the answerer. The syllogisms proceed from reputable opinions. The questioner aims to drive the answerer into a contradiction.²⁷ The questioner presents a problem (*problema*) to the answerer. Where ‘X’ and ‘Y’ can range over individuals or terms, all problems are questions of the form ‘is X Y or not?’, offering the answerer the choice of one of a pair of contradictories. For example, ‘Are knowledge and perception the same or not?’ (Topics I 5 102a8). The answerer selects one of the contradictories. This is the ‘starting point’ (*to en archêi*). I will label it *p*. The questioner tries to compel the answerer, by means of ‘yes or no’ questions, to concede the contradictory of the starting point, namely, not-*p*.²⁸ Each statement that the answerer accepts along the way is called a ‘proposition’ or ‘premise’ (*protasis*). Eventually, the answerer is forced to accept the conclusion

What I say here should be compatible with either view about the relation of broad dialectic to inquiry in Aristotle.

²⁷ For details of the structure of the Topics game, see Moraux (1968), Slomkowski (1997). Castelnérac and Marion (2009) suggest a rather different ‘rational reconstruction’ of the Topics game, in light of Plato’s Socratic dialogues.

²⁸ Cf. Bolton (1994, 103). Bolton wrongly suggests that the conclusion is always concealed in dialectic. The rules imply that both questioner and answerer know what the overall conclusion should be, and the skill is in forcing or avoiding this conclusion. Some intermediate conclusions, which are needed as part of a larger argument for the overall conclusion, may be concealed. Owen (1968, 107) claims that the tactics for concealment discussed in the Topics are recommended only in eristic contexts.

(sumperasma), won by a series of such questions.²⁹ We can see in the example below that (1) represents the problema, presented by the questioner, and starting point, chosen by the answerer, (2)-(6) the protaseis and (7) the sumperasma:

1. Q: Are knowledge and perception the same, or not? A: They are the same.
2. Q: Is it possible to both know and not know the same thing at the same time? A: No.
3. Q: If I remember something, do I know it? A: Yes.
4. Q: If I remember something, do I perceive it? A: No.
5. Q: So, if I remember something, I both know it and do not perceive it? A: Yes
6. Q: But knowing and perceiving are the same? A: Yes
7. Q: So if I remember something, I both know and do not know it. Which you agreed is impossible. A: Oh....
8. Q: So knowledge and perception are not the same.³⁰

It is clear that the conclusion is not psychologically or objectively novel. Both parties, the questioner and the answerer, know where the argument is heading. Because of the rules of the game, the questioner has to force the answerer to accept the contradictory of the proposition the answerer chose as the ‘starting point’ (p). But, also because of the rules of the game, both parties know what p is and what the aim of the game is. There is no question of this game producing a

²⁹ Ryle points out that neither competitor is committed to the truth or falsehood of p in propria persona. It is accepted or denied merely for the sake of argument (see Ryle 1968, 74–75; Moraux 1968, 302 and Slomkowski 1997, 107ff).

³⁰ I loosely base this fictional dialectical game on one of Socrates’ refutations of the ‘Knowledge is perception’ thesis in Plato’s Theaetetus 161-3. This description of the narrow dialectical game also appears in my [author’s paper A].

new conclusion; the only question is whether the questioner can successfully force the answerer to accept the *sumperasma*. The game is only worth playing if both parties already know what the *sumperasma* is, at least after the first move at (1). A Topics game will, therefore, never head towards a novel conclusion.³¹

But is the syllogism useful? Specifically, is a syllogism generated by the discovery method, which we know does not present a novel conclusion, useful in the context of dialectic? Notice first that Aristotle thinks that the underlying logical structure, the syllogism, is the same in a dialectical argument as in a mono-agent argument. He says as much at *Prior Analytics* I 1 24a25-8. Further evidence that the structure is the same is the way Aristotle distinguishes the contexts of didactic, dialectic, *peirastic* and *eristic* at *Sophistical Refutations* I 164a37-165b9. Each context involves syllogisms. The difference between these syllogisms is not logical; it is epistemic. Dialectical syllogisms have merely reputable opinions as premises and need not have true ones; didactic syllogisms differ from the other kinds of syllogisms because their premises are ‘principles appropriate to each branch of learning’. Only the epistemic status of the premises differs. The logical structure underlying each sort of argument is the same. But because the logical structure is the same, the method can deliver syllogisms that are useful in each of the contexts mentioned.

In dialectic, the questioner can use the method to discover the premises she needs in order to force the answerer to accept a particular conclusion. Moreover, the method will tell the questioner the doxastic status of the premises needed (i.e. whether they are true, reputable or

³¹ For the view that these games are the conceptual or historical antecedents of deductions for Aristotle, see Kapp (1942, 14–16) and Kapp (1975, 40).

hold for the most part).³² From this, the questioner can tell which ones the answerer is more likely to grant.³³ To see this, look again at the example game above. (1) is the starting point, and ‘all known things are perceived’ follows directly from it. Where ‘K’ stands for ‘known thing’ and ‘P’ stands for ‘perceived thing’, we write this as ‘PaK’.³⁴ The questioner knows that, due to the rules of the game, she must refute PaK. She knows that to do this, she can prove the contradictory, namely PoK (i.e. perceived thing does not belong to some known thing).

Allow that the questioner has access to extensive tables of predications in the domain of epistemology. She needs to prove an o-conclusion, so she applies the o-rule to the tables. She discovers that ‘memory’ (‘M’) will be the middle term she needs for a syllogism in Bocardo: PoM, KaM, so, PoK. Incidentally, there will also be a syllogism in Felapton to the same conclusion: PeM, KaM, so PoK. Neither of these syllogisms give a novel conclusion: the conclusion was already known to the questioner. In fact, the rules of the dialectical game determine what the conclusion has to be. But the questioner has discovered which premises she needs to force the answerer to admit PoK, and hence refute the answerer’s starting-point of PaK. In particular, the questioner needs the answerer to admit PoM and KaM. The questioner could also derive the conclusion she needs with the admission of PeM and KaM. The syllogism discovered does not present a novel conclusion, but the syllogism is useful because it can force the answerer to accept a certain conclusion. This is how a syllogism without a novel conclusion can be useful in a narrow dialectical case.

³² It is not the always case that any reputable opinion will be admitted by the answerer: it depends on the dialectical context (see Topics VIII 5 159a37-b25 on this point). But, whatever the attitude of the answerer towards reputable opinions, it is important information for the answerer to have available.

³³ See Topics VIII 1 155b1-15 for these sorts of consideration.

³⁴ Recall that the predicate term is first in this notation.

Not all question-and-answer interactions aim at driving the answerer into contradiction but even so syllogisms without novel conclusions are useful. Didactic discussions aim at one party, the answerer, increasing their knowledge. Didactic arguments have premises that are better known to the learner/ answerer than the conclusion, which is less well known (Topics VIII 3 159a10-14).³⁵ Didactic discussion is not spelled out in just one location in Aristotle's corpus, but it seems that (i) didactic arguments proceed to truths, because no one would teach a falsehood (Topics VIII 4 159a29-30; cf. Topics VIII 7 161a25); (ii) the starting points for didactic are proper to the branch of science in question (Sophistical Refutations I 165b2-3). Moreover, (iii) the learner/answerer should always agree to reputable opinions, which are moves on the way to the conclusion (Topics 159a30). But clearly, the central feature of didactic is connecting the conclusion, which is less well known to the learner/answerer, to the premises, which are better known to her.

Syllogisms discovered by the method will be useful for teaching, even though the conclusion is already known.³⁶ Suppose I am trying to teach someone that (C) all triangles have angles that sum to two right-angles. (C) could be thought of as the conclusion of a certain syllogism. But that conclusion is not novel to me (and, although it may be novel to my student, it need not be: she may be able to parrot it, for example, without having understood it). As long as the pupil accepts the axioms of geometry, she should become convinced of (C) on the basis of the syllogisms which connect the axioms to the conclusion. The syllogism which has (C) as its conclusion is useful because it enables the pupil to see not only that (C) holds, but also why (C)

³⁵ Cf. Smith (1997, 127-8).

³⁶ This account of the usefulness of deduction is independent of Barnes' well-known view that the Posterior Analytics presents a 'formal model' for how teachers should present a science to their students (Barnes 1975).

holds, which is one necessary part of learning (C). So the discovery method would discover a didactically useful syllogism which does not present a novel conclusion.

In many cases, as we have seen for Aristotle, syllogisms are useful although the conclusion is not novel. But sometimes, for Aristotle, the usefulness of a deduction does depend on the conclusion being novel, at least, novel to one of the players. Topics VIII 1 155b1-15 points out that in multi-agent arguments the answerer may be ignorant about what will result from the deduction. The answerer is not logically omniscient: they do not know all the consequences of their commitments. In certain situations, Aristotle says, the questioner should exploit this ignorance for a strategic advantage. For example, in an eristic encounter, the questioner may ask for premises that will lead the answerer to a contradiction, but not ask them one after the other. This is to conceal the questioner's strategy (Topics VIII 1 155b20-25). But this means that the psychological novelty of the conclusion is precisely why a certain deduction is useful: since the answerer does not know what the conclusion will be, the questioner can extract admissions which he needs to force the answerer to admit the conclusion. These cases, of course, do not show that Aristotle endorses the second premise of the scandal (if a conclusion of an argument is not novel, the argument is not useful). For Aristotle, sometimes deduction is useful because its conclusion is novel, sometimes it useful even though the conclusion is not novel. This is sufficient to show that Aristotle would reject the second premise of the scandal.

IV. Conclusion

I began with a certain problem that has occupied philosophers of logic in the 20th century. (1) If a deductive argument is valid, then the conclusion is not novel. (2) If the conclusion of an argument is not novel, the argument is not useful. So, (3) if a deductive argument is valid, then it is not useful. Certain philosophers in the 20th Century responded by denying the first premises of the scandal. This strategy betrays their assumption about what constitutes a useful deduction, an assumption captured by premise (2) of the argument.

Aristotle does not assume that a deduction is useful just in so far as its conclusion is novel. In many contexts Aristotle considers, a syllogism, a sort of deduction, is useful, despite not presenting a novel conclusion. Aristotle's method discovers middle terms for syllogisms based on the conclusion of that syllogism. If this is the case, the syllogism discovered cannot have a novel conclusion. But, as I went on to argue, in the demonstrative and dialectical contexts considered by Aristotle, these deductions are still useful. Aristotle does not assume that having a novel conclusion is necessary for the usefulness of a syllogism. This shows that Aristotle would reject (2). (2) is a hidden assumption about the use of deduction. Aristotle does not face the scandal because he makes different assumptions about the uses of deduction.³⁷

References

- Bar-Hillel, Y., and Carnap, R. 1953. "Semantic Information." *The British Journal for the Philosophy of Science* 4 (14): 147–157.
- Barnes, J. 1975 "Aristotle's Theory of Demonstration". In J. Barnes, M. Schofield and R. Sorabji eds. *Articles on Aristotle* 1 (pp. 65-87).
- . 1994. *Posterior Analytics*. Oxford University Press, USA.
- Bolton, R. 1990. "The Epistemological Basis of Aristotelian Dialectic." *From Puzzles to Principles? Essays on Aristotle's Dialectic*: 57–105.

³⁷ [Acknowledgements removed for blind review]

- . 1994. "The Problem of Dialectical Reasoning (Συλλογισμός) in Aristotle". *Ancient Philosophy* 14: 99-132.
- Burnyeat, M. 1981. "Aristotle on Understanding Knowledge." In *Aristotle on Science: The Posterior Analytics*, by Berti, E., 97–140. Padua: Antenore.
- Castelnérac, B., and Marion, M. 2009. "Arguing for Inconsistency: Dialectical Games in the Academy." *Acts of Knowledge: History, Philosophy and Logic*. London: College Publications.
- Cohen, M. R., and Nagel, E. 1934. *An Introduction to Logic and Scientific Method*. Harcourt, Brace and company.
- D'Agostino, M., and Floridi, L. 2009. "The Enduring Scandal of Deduction." *Synthese* 167 (2): 271–315.
- D'Agostino, M. 2013a. "Semantic Information and the Trivialization of Logic: Floridi on the Scandal of Deduction." *Information* 4: 33–59.
- . 2013b. "Analytic Inference and the Informational Meaning of the Logical Operators." *Logique Et Analyse*.
- Dummett, M. 1993. *The Logical Basis of Metaphysics*. Cambridge, Mass.: Harvard University Press.
- Dutilh Novaes, C. 2010. "Surprises in Logic." In *LOGICA Yearbook 2009.*, by M. Palis. London: College Publications.
- Duží, M. 2010. "The Paradox of Inference and the Non-Triviality of Analytic Information." *Journal of Philosophical Logic* 39 (5) (October 1): 473–510.
- Floridi, L. 2006. "The Logic of Being Informed." *Logique Et Analyse*.
- Hamlyn, D. W. 1990. "Aristotle on Dialectic." *Philosophy* 65 (254): 465–476.
- Hempel, C. G. 1945. "Geometry and Empirical Science." *The American Mathematical Monthly* 52 (1): 7–17.
- Hintikka, J. 1970. "Information, Deduction, and the A Priori." *Nous* 4 (2): 135–152.
- . 1973. *Logic, Language-games and Information: Kantian Themes in the Philosophy of Logic*. Clarendon Press Oxford.
- Irwin, T. 1989. *Aristotle's First Principles*. Oxford University Press.
- Jago, M. 2013. "The Content of Deduction." *Journal of Philosophical Logic*: 1–18.
- Kapp, E. 1942. *Greek Foundations of Traditional Logic*. New York: Columbia University Press.
- . 1975. "Syllogistic." In J. Barnes, M. Schofield and R. Sorabji eds. *Articles on Aristotle* 1 (pp.35–49).
- Leunissen, M. 2010. *Explanation and Teleology in Aristotle's Science of Nature*. Cambridge: Cambridge University Press.
- Morau, P. 1968. "La Joute Dialectique D'après Le Huitième Livre Des Topiques." *Aristotle on Dialectic*: 302.
- Nussbaum, M. C. 1986. *The Fragility of Goodness: Luck and Ethics in Greek Tragedy and Philosophy*. Cambridge: Cambridge University Press.
- Owen, G.E.L. 1968. 'Dialectic and Eristic in the Treatment of the Forms'. In G.E.L. Owen (ed.) *Aristotle on Dialectic*. Oxford: Clarendon (pp. 103-125)
- Popper, K. R. 1959. *The Logic of Scientific Discovery*. London: Routledge.
- Primiero, G. 2008. *Information and Knowledge: A Constructive Type-theoretical Approach*. Springer.
- Ryle, G. 1968, 'Dialectic in the Academy'. In G. E. L. Owen (ed.) *Aristotle on Dialectic*. Oxford: Clarendon (pp. 69–79).

- Sequoiah-Grayson, S. 2008. "The Scandal of Deduction." *Journal of Philosophical Logic* 37 (1): 67–94.
- Shannon, C. E. and Weaver, W. 1948. *A Mathematical Theory of Communication*. American Telephone and Telegraph Company.
- Sillari, Giacomo. 2008. "Quantified Logic of Awareness and Impossible Possible Worlds." *Review of Symbolic Logic* 1 (4): 514–529.
- Slomkowski, P. 1997. *Aristotle's Topics*. Leiden: Brill.
- Smith, R. 1989. *Prior Analytics*. Indianapolis: Hackett.
- . 1993. "Aristotle on the Uses of Dialectic." *Synthese* 96 (3): 335–358.
- . 1997. *Topics Books I and VIII*. Oxford: Clarendon Press.
- Striker, G. 2009. *Aristotle: Prior Analytics, Book I*. Oxford: Clarendon Press.
- Walton, D. 1977. "Mill and DeMorgan on Whether the Syllogism Is a Petitio." *International Logic Review* 8: 57–68.
- Wieland, W. 1975a. "The Problem of Teleology." In J. Barnes, M. Schofield and R. Sorabji eds. *Articles on Aristotle 1* (pp. 141–60).
- . 1975b. "Aristotle's Physics and the Problem of Inquiry into Principles'." In J. Barnes, M. Schofield and R. Sorabji eds. *Articles on Aristotle 1* (pp. 127–40).
- Woods, J. 1999. "John Stuart Mill (1806–1873)." *Argumentation* 13 (3): 317–334.
- Woods, J. and Walton, D. 1982. "The Petitio: Aristotle's Five Ways." *Canadian Journal of Philosophy*: 77–100.