Aristotle's Theory of Deduction and Paraconsistency

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Abstract

In the *Organon* Aristotle describes some deductive schemata in which inconsistencies do not entail the trivialization of the logical theory involved. This thesis is corroborated by three different theoretical topics by him discussed, which are presented in this paper. We analyse inference schemata used by Aristotle in the *Protrepticus* and the method of indirect demonstration for categorical syllogisms. Both methods exemplify as Aristotle employs classical *reductio ad absurdum* strategies. Following, we discuss valid syllogisms from opposite premises (contrary and contradictory) studied by the Stagerian in the *Analytica Priora* (B15). According to him, the following syllogisms are valid from opposite premises, in which small Latin letters stand for terms such as subject and predicate, and capital Latin letters stand for the categorical propositions such as in the traditional notation: (*i*) in the second figure, $Eab, Aab \vdash Ebb$ (Cesare), $Aab, Eab \vdash Ebb$ (Camestres), $Eab, Iab \vdash Obb$ (Festino), and $Aab, Oab \vdash Obb$ (Baroco); (*ii*) in the third one, $Eac, Aac \vdash Oaa$ (Felapton), $Oac, Aac \vdash Oaa$ (Borcardo) and $Eac, Iac \vdash Oaa$ (Ferison). Finally, we discuss the passage from the *Analytica Posteriora* (A11) in which Aristotle states that the Principle of Non-Contradiction is not generally presupposed in all demonstrations (scientific syllogisms), but only in those in which the conclusion must be proved from the Principle; the Stagerian states that if a syllogism of the first figure has the major term consistent, the other terms of the demonstration can be each one separately inconsistent. These results allow us to propose an interpretation of his deductive theory as a broad sense paraconsistent theory. Firstly, we proceed to a hermeneutical analysis, evaluating its logical significance and the interplay of the results with some other points of Aristotle's philosophy. Secondly, we point to a logical interpretation of the Aristotelian results as the antilogisms proposed by Christine Ladd-Franklin (1883) and the ones in da Costa's paraconsistent logics $C_n, 1 \le n \le \omega$ (da Costa (1963, 1974)). These two issues seem having not yet been analysed in detail in the literature.

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Introduction

Aristotle's contribution to the foundations of logic and of the scientific method is broadly recognized. His theory of syllogism is the most ancient known logical system. In the *Organon*, the Stagerian describes some deductive schemata in which a contradiction does not imply trivialization of the logical theory involved. This thesis is corroborated by three different theoretical topics by him discussed, which are presented in the following three sections of this paper. In the first section, we analyse inference schemata utilized by Aristotle in the *Protrepticus* and the method of indirect demonstration for categorical syllogisms; both situations exemplify as Aristotle employs classical *reductio ad absurdum* strategies. In the second one, we discuss valid syllogisms from opposite premises (contrary and contradictory) studied by the Stagerian in the *Analytica Priora* (B15). In the third section, we discuss the passage from the *Analytica Posteriora* (A11), in which Aristotle states that the Principle of Non-Contradiction is not generally presupposed in all demonstrations (scientific syllogisms), but only in those in which the conclusion must be proved from the Principle. Stating these results, the Stagerian challenges the scholars to explain how these theses can be put together in order to produce a coherent understanding of his deductive theory, in face of vigorously supported positions as that ones defended in Book Γ of *Metaphysica*. In this book, he strongly argues in favour of the Principle of Non-Contradiction what, at first sight, seems incompatible with some of the results mentioned above. We intend to demonstrate, through some logical-hermeneutical methods, how all these Aristotelian logical results can be understood to produce a coherent and reasonable interpretation. We try to connect, by means of a hermeneutical analysis, the earlier mentioned results exposed by Aristotle with other issues of his logic and philosophy. We also propose a contemporary interpretation of these results, showing that the two former ones can be interpreted as a broad sense paraconsistent theory and the later one can be formalized in a strict sense paraconsistent logic.

The *ex falso* is a thesis, related to the Principle of Non-Contradiction, that leads a logical system in which it holds to triviality. This is the case, for instance, of classical and intuitionistic logic. It is propositionally denoted[1] by

$$(\mathbf{A} \wedge \neg \mathbf{A}) \to \mathbf{B} \tag{1}$$

and it states that, from a contradiction, every formula of the logical language is demonstrated. However, in any paraconsistent logic it is not possible, in general, to deduce all formulae from a formula **A** and its negation ¬**A**.

Paraconsistent logic separates inconsistency and triviality. There are two general conceptions of paraconsistency. The first one is a *broad sense paraconsistency*, that applies to paraconsistent theories in which the *ex falso* is only restricted; in such theories from a contradiction, only specific kinds of formulae are deducible – it is the case of Kolmogorov-Johansson's minimal intuitionistic logic.[2] The second one is a *strict sense paraconsistency*, that applies to paraconsistent theories in which the *ex falso* does not hold in general; for instance, da Costa's paraconsistent logics $C_n, 1 \le n \le \omega$.[3]

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[1] We use bold capital Latin letters standing for metavariables for formulae. The context in which they occur will help to choose if they stand for propositional variables.

[2] *Vide* Kolmogorov (1925) and Johansson (1936).

[3] *Vide* da Costa (1963, 1974).

All the mentioned Aristotelian passages are well-known in the history of logic literature. Bocheński (1961) quotes them in his discussion about pre-Aristotelian logic and about the Principle of Non-Contradiction in Aristotle. He also quotes a paper by Isaac Husik (1906) that analyzes Chapter 11 of the First Book of the *Analytica Posteriora*, which is also mentioned by Jan Łukasiewicz (1910b, 1910a). Priest (2005, p. 5–6) concludes from the *Analytica Priora* (B15) that the syllogistic is paraconsistent; however, in spite of his categorical conclusion, we consider that some aspects of the discussion must be clarified. Priest believes that the broad sense paraconsistency is an insufficient paraconsistent logic approach; he recognizes that Kolmogorov-Johansson's minimal intuitionistic logic is paraconsistent, but he considers that "it is clearly antithetical to the *spirit* of paraconsistency, if no the letter" (2005, p. 4). Thus we consider his conclusion does not match with ours because the kind of paraconsistency we interpret in Aristotle's theory of deduction is exactly a broad sense paraconsistency, similar to that one of Kolmogorov-Johansson's minimal intuitionistic logic.

Is a paraconsistent syllogistic system logically possible? The most ancient indication into this direction comes from da Costa, Beziau & Bueno (1998), who propose a paraconsistent syllogistic built on the well-known paraconsistent first-order logic C_1^* (da Costa (1963, 1974)). Of course, their logical approach is correct. However, it is based on a logical point of view, and does not come either from a historical or from a philosophical point of view, as we propose in this paper.

1   Apagogical inference schemata and non-triviality

Aristotle conceived and described the apagogical inference schemata (or by *reductio ad absurdum*) in his deductive theory. We begin our exposition of these rules, discussing that one utilized by the Stagerian in the beginning of his philosophical career. In this period, he still uses apagogical standards of inference very similar to that found in Zeno of Elea, in the sophists and in Plato. The Aristotelian demonstration of the necessity of philosophy in the *Protrepticus* exemplifies his usage of these inference schemata. According to Bocheński (1957, p. 16) tradition is a bit confusing in describing his use of that rule, because it is reported with slight differences in some ancient commentators. There are different statements in Alexander of Aphrodisias, Lactantius and Olympiodorus of Alexandria, for example. However, such variations are not an obstacle to the study of this logical rule. Alexander of Aphrodisias reports:

> There are cases in which, whatever view we adopt, we can refute on that ground a proposition under consideration. So for instance, if someone was to say that it is needless to philosophize: since the enquiry whether one needs to philosophize or not involves philosophizing, as he [Aristotle] has himself said in the *Protrepticus*, and since the exercise of a philosophical pursuit is itself to philosophize. In showing that both positions characterize the man in every case, we shall refute the thesis propounded. In this case one can rest one's proof on both views.[4]

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[4]Alexander, *Commentarius in Topica* 149.11-15, Rose Fragm. 51 (3rd ed.) *apud* Bocheński (1961,

An anonymous scholiast reports the following version of the same demonstration:

> Of the same kind is the Aristotelian dictum in the *Protrepticus*: whether one has to philosophize or not, one must philosophize. But either one must philosophize or not; hence one must in any case philosophize.[5]

Bocheński (1961, p. 32–33) suggests that the following rule of inference underlies Alexander's report:

> Suppose that if **A** belongs to **x**, **A** belongs to **x** and, if **A** does not belong to **x**, **A** belongs to **x**, then **A** belongs to **x**.

The historian also considers that the report of the anonymous scholiast shows a rule of inference still more complete. In fact, such inference schema is not an apagogical one, but a proof by cases.

> If **A** belongs to **x**, then **A** belongs to **x**; if **A** does not belong to **x**, then **A** belongs to **x**; either **A** belongs to **x** or **A** does not belong to **x**; therefore, **A** belongs to **x**.

According to this Aristotelian argument, philosophizing is so peculiar – absolutely necessary – that even if somebody denies doing it, such person will necessarily perform it. From a logical point of view, the underlying schema of inference used in the proof is related to a strict kind of *reductio*, what preserves classical logic from triviality. For if a proposition implies its own negation, then it is false; and it is just its negation that is affirmed. In our notation, we can point out that inference schema by the formula

$$(\neg \mathbf{A} \to \mathbf{A}) \to \mathbf{A} \tag{2}$$

that is known as *consequentiae mirabilis*. Thanks to Łukasiewicz, this rule has become known as 'Clavius law', in honour of a jesuit of the second half of the 16th century that called attention to this rule in his edition of the Euclid's *Elements*.[6] Łukasiewicz also notes that this rule was already known by stoics.[7] Bocheński (1957, p. 16) reports that H. Scholz also identified a similar rule in a different fragment of the Aristotle's *Protrepticus*[8], slightly different, though analogous to the *consequentiae mirabilis*:

$$((\mathbf{Ax} \to \mathbf{Ax}) \wedge (\neg \mathbf{Ax} \to \mathbf{Ax})) \to \mathbf{Ax} \tag{3}$$

A plausible interpretation of these inference schemata is that a hypothesis that leads into a contradiction is necessarily false. When hypothesis ¬**A** leads

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p. 30–31). *Vide* also Aristotle (1985, vol. 2, p. 2404).

[5]Rose Fragm. *apud* Bocheński (1961, p. 31). *Vide* also Elias, *Prolegomena Philosophiae* 3.17-23, Rose Fragm 51 (3rd ed.) *apud* Aristotle (1985, vol. 2, p. 2416).

[6]Kneale (1966, p. 62–63) reports that Christophorus Clavius (1538–1612) learned in mathematics and astronomy, who propounded modern Gregorian calendar, recognized this pattern of inference in the proposition IX, 12 in a note. Bocheński (1957, p. 16) attributes to Vailati the discovery of this rule, in relation to the same proposition of the *Elements*. *Vide* also Blanché (1996, p. 15).

[7]*Vide* Kneale (1966, p. 63) and Blanché (1996, p. 14, note 1).

[8]Rose Fragm. 51 *apud* Bocheński (1957, p. 16).

into a contradiction, we conclude that its negation is true, therefore **A**. If a hypothesis **A** implies a contradiction, then its negation ¬**A** is what we must derive. Thus, it is permitted to carry out an uniform substitution in (2) in order to get the following instance of the *consequentiae mirabilis*:

$$(\mathbf{A} \to \neg\mathbf{A}) \to \neg\mathbf{A} \tag{4}$$

that shows clearly that if a proposition implies its own contradiction, then it is false. So, such rule or inference schema determines that the negation of the hypothesis must be derived from such contradiction, in order to avoid the triviality of the system.

The implication underlying the *consequentiae mirabilis* rule is classical and incompatible with another different kind of implication used by Aristotle in a demonstration of the *Analytica Priora* (B4, 57a36–57b17). Such implication is known in the literature as connexive implication and states that no proposition implies or is implied by its own negation.[9] That implication can be expressed by the following formula

$$\neg(\neg\mathbf{A} \to \mathbf{A}) \tag{5}$$

that is called Aristotle's thesis in virtue of the use he has made of it in the mentioned demonstration. Kneale (1966, p. 65–66) recognizes the incompatibility between these implications and suggests that the *consequentiae mirabilis* would only hold to Aristotle when a proposition in question is necessary.[10] Mortensen (1984) shows sufficient conditions to a logic in which (5) holds being consistent and that there is also a class of inconsistent non-trivial logics all containing Aristotle's thesis.[11] This class of logics, of course, is one of paraconsistent logics.

Aristotelian rules hitherto showed reveals a clear propositional approach. They witness that Aristotle has a good knowledge of a propositional logic, for such rules are perfectly expressible in a propositional logic or in a logic of terms. As we have just seen in the passages, the Stagerian deals with propositions as a non-analysed whole, as it is typical in propositional logic; when some individual has to be denoted, propositional schemata is enriched in order to express terms applied to individuals. Thus, we have a logic of terms adjacent to a propositional logic. Although Corcoran (1972, *passim*) does not recognize that Aristotle has formulated a propositional logic, we believe that we cannot discard that he has a good knowledge of it. In several passages of the *Organon* he uses, discusses and proposes rules or propositional inference schemata. Mulhern (1972, p. 135) suggests that "The evidence points rather to Aristotle's awareness of propositional logic but his rejection of it as an instrument unfit for the purposes he intended."[12] Bocheński (1957, p. 13) believes that Aristotle was convinced that the propositional and predicate approaches to logic were not incompatible, on the contrary, they are necessary to a complete description of the logic that could not be reached without laws and rules of both. Aristotle,

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[9]*Vide* McCall (1966).

[10]Kneale (1966, p. 66) suggests that "for the property of being demonstrable by the *consequentiae mirabilis* is confined to absolutely truths, which Saccheri called *primae veritates*".

[11]In his paper Mortensen refers to the formula ¬($A \to \neg A$), instead of ¬($\neg A \to A$).

[12]Mulhern considers "that Aristotle could have elaborated a system of propositional logic, but that the theory of demonstrative science which he envisioned required a system of analyzed propositions, in which the modality of predications could be clearly shown. Thus he rejected a logic of unanalyzed propositions in favor of syllogistic." (1972, p. 135–136).

Bocheński writes, explicitly recognized the legitimacy of rules corresponding to the following law

$$((A \wedge B) \rightarrow C) \rightarrow ((A \wedge \neg C) \rightarrow \neg B)) \tag{6}$$

such rule or inference schemata clearly belongs to propositional logic, but it can be expressed in a logic of terms.

As we can see so far, following Bocheński (1961, p. 31–32) analysis, we consider that (2), (3) and (4) can be interpreted as propositional logic rules or inference schemata. Moreover, they seem to reinforce the thesis that Aristotle adopted refutation strategies similar to that ones found in pre-Aristotelian dialectics.

The general feature of Aristotelian logic has been well summed up by Corcoran (1972, p. 109): "Aristotle's theory of deduction is his theory of perfecting syllogisms". In the *Analytica Priora* the Stagerian classifies syllogisms in perfect and imperfect ones. A syllogism is perfect 'if it stands in need of nothing else besides the things taken in order for the necessity to be evident" (A1, 24b23).[13] This is the case of the valid moods Barbara, Celarent, Darii and Ferio of the first figure. A syllogism is imperfect "if it still needs either one or several additional things which are necessary because of the terms assumed, but yet were not taken by means of premises" (A1, 24b24–6). Imperfect syllogisms are, however, perfectible. These ones can be proved in two ways: (a) direct or ostensively, or (b) indirectly. To demonstrate a syllogism directly, several deductive tools are required. Beyond the perfect syllogism from the first figure (playing the role of logical laws of the theory), other inference rules as conversion, repetition and interpolation can be used.[14] The imperfect syllogisms are indirectly demonstrated *through impossibility* (εἰς τὸ ἀδύνατον). In the *Analytica Priora* (A23, 41a23–31) Aristotle explains how a syllogism is proved indirectly:

> For all those which come to a conclusion through an impossibility *deduce* the falsehood, but *prove* the original thing from an assumption when something impossible results when its contradiction is supposed, <proving,> for example, that the diagonal is incommensurable because if it is put as commensurable, then odd numbers become equal to even ones. It *deduces* that odd number become to even ones, then, but it *proves* the diagonal to be incommensurable from an assumption since a falsehood results by means of its contradiction. For this is what deducing through an impossibility was: proving something impossible by means of the inicial assumption (τοῦτο γὰρ ἦν τὸ διὰ τοῦ ἀδυνάτου συλλογίσασθαι, τὸ δεῖξαί τι ἀδύνατον διὰ τὴν ἐξ ἀρχῆς ὑπόθεσιν).

Note that Smith translates *syllogism* (συλλογισμός) by *deduction* and *syllogize* (συλλογίζεσθαι) *'to prove by syllogism'* by *to deduce*. This method of proof always deduces something 'proving something impossible by means of the inicial assumption'. Of course, the hypothesis – the contradictory of the conclusion – is carefully chosen. However, only the valid syllogisms can be proved thanks to effectiveness of the method supported by the consistency of the logical system

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[13] Translation of *Analytica Priora* by Robin Smith.
[14] *Vide An. Pr.* A7, 29a30-40.

in which it works. In any case, only a valid conclusion could be derived, if there were one. If the syllogism in question is valid, then the hypothesis joint with the premises of the syllogism will lead inevitably to something impossible, in this case, a contradiction. Hence we can conclude that the hypothesis cannot be the case, step in which the negation of it is derived. Note that the proving scheme is negative: it always concludes with the negative of the hypothesis. The following schema explains this inference:

> Suppose **A**; let **B** be a premise; from this follows **C**, what is impossible; therefore, ¬**A**.

In the *Analytica Priora* (B14, 62b29–35; 37–38) Aristotle compares indirect to direct demonstration:

> A demonstration <leading> into an impossibility (εἰς τὸ ἀδύνατον) differs from a probative demonstration in that it puts as a premise what it wants to reject by leading away into an agreed falsehood, while a probative demonstration begins from agreed positions. More precisely, both demonstrations take two agreed premises, but one takes the premises which the deduction is from, while the other takes one of these premises and, as the other premise, the contradictory of the conclusion.

And he concludes:

> It makes no difference whether the conclusion is an affirmation or a denial, but rather it is similar concerning both kinds of conclusion.

Comparing inference schemata of *Protrepticus* with the indirect demonstration above, we realize that Aristotle's description of the inference from *reductio ad absurdum* is very coherent. The main idea is that not every proposition can be deduced from a contradiction, but only the negative of the hypothesis. As Aristotle pointed above, it makes no difference whether the conclusion proved is either affirmative or negative. Such Aristotelian schema is analogous to the form of *reductio* found in the pre-Aristotelian period of Greek logic history. In a dialectical debate, the target was to destroy opponent's thesis. Such inferences always led to negative conclusions. To sum up, we are again facing an inference schema that does not trivialize the logic in which it works from a contradiction. It does not allow that any proposition or sentence may be derived.

2   Valid syllogisms from opposite premises

In the *Analytica Priora* (Book B, Chapter 15) Aristotle states some results that allow us to interpret his theory of syllogism as being a broad sense paraconsistent theory. The subject of this text seems not to have been completely examined as to its logical-philosophical meaning. Smith (*in* Aristotle 1989, p. 202) affirms that the subject of Chapter 15 in Book B is very independent, either relatively to the chapters preceeding it (1–14) or to those following it (16–21). He also considers that Aristotle's actual motivation in this chapter is not clear. Corcoran (1972, p. 99) proposes that this text was written later and that examples

inside it would have ample grounds urging for its extrasystematic character. Despite such hermeneutical position, we seek to detach some motivations and also show how the results presented by Aristotle in this chapter can support our conclusion, in the sense that in Aristotle's logic we cannot prove any categorical proposition from opposite premises. Once this fact has been put, we can conclude that the *ex falso* does not hold in his theory of syllogism, what preserves this deduction theory of trivialization in face of contradictions, what characterizes the broad sense paraconsistency. We still believe that the *apparent detachment* of this chapter in relation to the other in the *Analytica Priora* only enforces that Aristotle would have expanded his own deductive system in order to deal with most of the possible deductive situations. One of them, as we understand in the content of this chapter, was a specific subsystem to deal with opposite premises. Such a kind of subsystem, inside the Aristotelian deductive theory, can be understood as a broad sense paraconsistent theory.

According to Aristotle, the following syllogisms are valid from opposite (contradictory and contrary) premises, in which small Latin letters stand for terms such as subject and predicate and capital Latin letters stand for the categorical propositions such as in the traditional notation. We also put the predicate first, then subject, as Aristotle did in *Analytica*. In the second figure,

$$Aab, Oab \vdash Obb \quad \text{(Baroco)} \tag{7}$$

$$Aab, Eab \vdash Ebb \quad \text{(Camestres)} \tag{8}$$

$$Eab, Aab \vdash Ebb \quad \text{(Cesare)} \tag{9}$$

$$Eab, Iab \vdash Obb \quad \text{(Festino)} \tag{10}$$

In the third one,

$$Eac, Aac \vdash Oaa \quad \text{(Felapton)} \tag{11}$$

$$Oac, Aac \vdash Oaa \quad \text{(Borcardo)} \tag{12}$$

$$Eac, Iac \vdash Oaa \quad \text{(Ferison)} \tag{13}$$

Let us check the Aristotelian ground concerning these results. According to him, only in the second and in the third figures we can (*a*) affirm and deny a subject to a same predicate, because in the second figure the middle term is a predicate in both premises; and, (*b*) affirm and deny a predicate belonging to a same subject, because in the third figure the middle term is a subject in both premises. Such conditions completely work if we have only two terms involved. For this reason, according to Aristotle (*An. Pr.* B15, 63b31–39), in the first figure it is impossible to obtain a syllogism from opposite premises, whether being affirmative or negative. The Aristotelian theory of syllogism reflects, of course, his theory of predication, that avoids self-predication because something must be predicated of something different such as a species of a genus.[15] As Mulhern says (1972, p. 144) "predicates must be of a higher order than their arguments". In fact, we do not find in the *Analytica* any categorical proposition of the form '*Aaa*' ('*a* belongs to all *a*').[16] Chapter 15, in the second book of the *Analytica Priora*, is the only place in which negative self-predication appears in all the

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[15]*Vide Cat.* 1b9–15; 2b19–22.

[16]For this reason, Corcoran (1972, p. 99) proposes in his mathematical model for categorical syllogism that "self-predication is here avoided because Aristotle avoids it in the system of the *Prior Analytics*".

Analytica. Corcoran (1972, p. 99) explains that "In this passage the sentences 'No knowledge is knowledge' and 'Some knowledge is not knowledge' appear as conclusions of syllogisms with contradictory premises and there are ample grounds urging for the extrasystematic character of the examples. In any case, no affirmative self-predications occur at all". For this reason, such passages have been seen as having few systematic values. However, Mulhern (1972, p. 144) shows that the reference to the self-predication is not completely absent in Aristotle's *corpus*. For instance, to define truth in Book Γ of *Metaphysics*, he uses a certain kind of identity relation that expresses corresponding character of his truth notion. The Stagerian states: "To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true" (*Met.* Γ7, 1011b26–8).[17] There is evidence that Aristotle has considered suspicious proofs built upon identity relations because they could not express authentic predications. In the *Analytica Posteriora* (A3; esp. 72b25–73a6), when he analyses inadequacy of circular demonstrations, he concludes: "Consequently the upholders of circular demonstration are in the position of saying that if *A* is, *A* must be – a simple way of proving anything".[18] Thus, for Aristotle proving is to exhibit a predication, not an identity relation, because a tautology could prove everything. Mulhern (1972, p. 144) explains that "The answer seems to be connected with the fact that, for Aristotle, identifications are not predications: on his view, there is no predication unless something is said of something else".

By analysing the allegation that syllogisms from opposite premises have an extrasystematic character, we could remark two points in order to counterpoise this reading. First, we believe that Aristotle studies a kind of deductive subsystem proper to deal with opposite propositions in this chapter, what is suitable to several logical situations and, for this reason, could be assimilated in the theory of syllogism exposed in the *Analytica Priora*. Thus, the result corroborates the effectiveness of the deductive theory and its capacity of distinguish valid from invalid deductions or arguments, even if it works with inconsistent material. Second, we believe that this chapter is relegated to a second level, because it causes perplexity, once its author, one of the founders of classical logic, shows that we can have valid syllogisms from contradictory and contrary premises without destroying or colapsing the logical consequence relation. This seems to be attributed to Aristotle's logical and intelectual capacity that, as we will see, anterpates an approach typical of nowadays paraconsistent logic. On the other hand, we also believe that this interpretation of the results shows that the traditional reading of them is strongly attached to a preferencial reading of Aristotle – classical from the logical and ontological points of view – ignoring what seems to contradict this hermeneutical paradigm.

If an affirmative self-predication devastates the Aristotelian theory of syllogism, leading consequence relation to collapse, the negative self-predication is just the result of a refutation. Corcoran (1972, p. 99) suggests that "Perhaps further slight evidence that Aristotle *needed* to exclude them [self-predications] can be got by noticing that the mood *Barbara* with a necessary major and nec-

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[17]Translation of *Metaphysica* by W. D. Ross. Mulhern (1972, p. 144) taking Corcoran statement, saying that Aristotle does not have a pure theory of logical truth in consideration, pondering that the Aristotelian theory of truth and identity is very different of modern ones. The main difference is that for Aristotle predication and not identity relation is the fundamental ground.

[18]Translation of the *Analytica Posteriora* by G. R. G. Mure.

essary conclusion (regarded as valid by Aristotle) is absurdly invalid when the predicate and middle are identical". The case that Corcoran alludes is the following:

$$Aaa, Aab \vdash Aab \tag{14}$$

that stands for: '*a* belongs to all *a*' and '*a* belongs to all *b*', so '*a* belongs to all *b*'. This syllogism is invalid because it violates the Aristotelian definition of logical consequence.[19] According to it, consequence relation is not reflexive. In a categorical syllogism, premises and the conclusion have to be distinct. In (15), nevertheless, the minor premise and the conclusion are identical.

If we consider, in addition, other syllogism with self-predicative premises, we can see that self-predication makes things worst. Consider the following Barbara:

$$Aaa, Abb \vdash Aab \tag{15}$$

such syllogism is completely invalid and syntactically incomplete. In fact, it is not really a syllogism; self-predicative premises makes the middle term disappear and the conclusion cannot be stated.

The same cannot be said of syllogism from opposite premises, that are recognized and correctly justified by Aristotle as valid deductions in the remaining figures. The Stagerian explains:

> But in the middle figure, it is possible for a deduction to come about both from opposite and from contrary premises. Let A stand for good and B and C for science. Now, if someone took every science to be good, and also no science to be good, then A belongs to every B and to no C, so that B belongs to no C: no science, therefore, is a science. (B15, 63b40–64a4)

Above Aristotle spells out how to build up the contradictory Camestres as follows:

> Good is predicate of all science,
> and good is predicate of no science.
> Hence, science is predicate of no science.

We call attention to the fact that the Stagerian always uses contingent terms when he intends to emphasize the logical form in spite of the material content involved.[20] This syllogism is denoted as

$$Aba, Ebc \vdash Eac$$

but, considering that *a* and *c* stand for *science*, we can replace them by *g*:

$$Abg, Ebg \vdash Egg$$

Thus, a proof of contradictory Camestres is similar to that of the standard one:

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[19] *Vide Analytica Priora* A1, 24b 18–22.
[20] *Vide* Correia (2002, p. 24).

1	*Abg*	
2	*Ebg*	
3	*Egb*	2 simple conversion
4	*Egg*	2, 3 Celarent

Proofs for Baroco, Cesare and Festino are analogous. Concerning the third figure Aristotle explains:

> In the third figure, an affirmative deduction will never be possible from opposite premises for the reason also stated in the case of the first figure, but a negative deduction will be possible both when the terms are universal and when they are not universal. For let B and C stand for science and A for medical knowledge. If, therefore, someone should take all medical knowledge to be science and no medical knowledge to be a science, then he has taken B to belong to every A and to no C; consequently, some science will not be a science. (B15, 64a20–27)

The syllogism that Aristotle indicates above is an instance of the valid mode Felapton:

> Science is predicate of no medical knowledge,
> and science is predicate of all medical knowledge.
> Hence, science is not predicate of some science.

Thus, we have in our notation

$$Eba, Aca \vdash Obc$$

but *a* and *c* stand for the same term *j*. So we have

$$Eja, Aja \vdash Ojj$$

A proof of contradictory Felapton is also similar to that stardard one:

1	*Eja*	
2	*Aja*	
3	*Iaj*	2 accidental conversion
4	*Ojj*	2, 3 Ferio

Proofs for contradictory Bocardo and Ferison are analogous. The results showed by Aristotle in the *Analytica Priora* (B15) can be summarized as suggested by Thom (1981, p. 196):

Theorem 1 (Theorem for syllogisms from opposite premises) *Let **A** and **B** be metavariables for any Aristotelian categorical propositions. If **A** and ∼ **A** are contradictory premises, then the conclusion **B** has Oaa form; if **A** and ¬**A** are contrary premises, then **B** has Eaa or Oaa form.*

An interesting thing in the Aristotelian approach to syllogism from opposite premises is how to interpret its truth. He concludes:

> It is also evident that while it is possible to deduce a true conclusion from falsehoods (as was explained earlier)[21], it is not possible to do so from opposite premises. For the deduction always come about contrary to the subject (for instance, if it is good, the deduction is that it is not good, or if it is an animal, the deduction is that it is not an animal), because the deduction is from a contradiction (and the subjects terms are either the same or one is a whole and the other a part). (B15, 64b7–12)

Firstly, we must emphasize that in this excerpt Aristotle seems to refer to a very clear distinction between logical validity and factual truth. In fact, a syllogism from opposite premises is logically possible because it can be logically sound. Its validity is grounded in the Aristotelian theory of syllogism (or deduction) exposed by him in Book A of the *Analytica Priora*. However, semantically, his logical theory of deduction respects the Principle of Non-Contradiction. Thus, conclusions that have been drawn from it cannot be true. In this sense, such syllogisms are a refutation method suitable for dialectical debate contexts. This reading is supported by the following passage:

> And it is clear that in trick arguments nothing prevents the contradictory of the assumption following (for instance, that it is not odd if it is odd). For a deduction from opposite premises was contrary: thus, if one takes such premises, then the contradictory of the assumption will result. (B15, 64b13–17)

In fact, there is an 'absent' contradictor in whole Chapter 15, whose opposition is pressuposed to build the opposite premises posed in each Aristotelian instance. This explains why the previous syllogisms can be considered a method of refutation for the dialectical debate, as indicated by Smith (*in* Aristotle 1989, p. 2002). Similar motivation led Jaśkowski to conceive discussive logic D_2.[22] This interpretation is also supported by other passages of the *Analytica Priora*:

> Since we know when a deduction comes about, i. e. with what relations of the terms, it is also evident both when a refutation (ελενχος) will be possible and when it will not. [...] Consequently, if what is proposed is contrary to the conclusion, then it is necessary for a refutation to come about (for a refutation is a deduction of a contradiction (ὁ γὰρ ἔλεγχος αντιφασις συλλογισμός)). (B20, 66b 4–6; 9–12)

In the previous excerpt, Smith (*in* Aristotle 1989, p. 212) considers that 'what is proposed', it is to be refuted and explains that the topic of debate in this chapter "gives a further application of Aristole's deductive theory to argumentative practice (the assimilation of refutations to deductions indicates his aim of generalizing as far as possible)".

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[21]*Vide An. Pr.* B2–4, 53b5–58a18.
[22]*Vide* Jaśkowski (1949).

If we compare the Aristotelian and contemporary perspectives about paraconsistent syllogisms, we will realize an important difference. The Aristotelian syllogism from opposite premises does not match with modern paraconsistent interpretation of syllogistic, as proposed by da Costa and Bueno.[23] These authors propose a paraconsistent interpretation of traditional syllogistic in monadic paraconsistent first-order logic C_1^*, in which: (*a*) all modes are demonstrated from the first and the third figures of syllogism; (*b*) none valid mode is obtained from the second one; and, (*c*) in the fourth figure, only Bramantip and Dimaris are proved. They still suggest that by using the strong negation of C_1^*, that corresponds to the classical negation, the classical theory of syllogism is obtained, although those theories are not equivalent. One reason seems to be related to different conceptions of paraconsistency underlying Aristotle and da Costa's approaches. Whereas the Aristotelian theory of syllogism seems to be a broad sense paraconsistent theory, da Costa's is paraconsistent syllogistic is a strict sense paraconsistent theory. Another reason is related to the significant differences between the languages and deductive systems underlying each formulation of the theory of syllogism.

3   Demonstration with inconsistent terms

In Book Γ of *Metaphysica*, Aristotle claims that every demonstration needs the Principle of Non-Contradiction. He claims:

> It is for this reason that all who are carrying out a demonstration reduce it to this as an ultimate belief; for this is naturally the starting-point even for all the other axioms (φύσει γὰρ ἀρχὴ καὶ τῶν ἄλλων ἀξιωμάτων αὕτη πάντων). (*Met.* Γ3, 1005b 33–34)

The ultimate belief seems to be the following version of the Principle of Non-Contradiction:

> For it is impossible for any one to believe the same thing to be and not to be, as some think Heraclitus says. (*Met.* Γ3, 1005b 23–25)

However, there is a situation in which the Principle of Non-Contradiction does not hold:

> For 'that which is' has two meanings, so that in some sense a thing can come to be out of that which is not, while in some sense it cannot, and the same thing can at the same time be in being and not in being – but not in the same respect. For the same thing can be potentially (δυνάμει) at the same time two contraries, but it cannot actually (ἐντελεχεία). (*Met.* Γ5, 1009a 32–35)

Aristotle has described the only way in which contradiction can be tolerated in his metaphysics. However, there is a logical situation in which he also allows contradiction – if the major term of a demonstration is consistent. He states:

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[23]*Vide* da Costa, Béziau and Bueno (1998, p. 142–150). Recently, these ideas have been published in English by da Costa, Krause and Bueno (2007, p. 828–829).

The law that it is impossible to affirm and deny simultaneously the same predicate of the same subject is not expressly posited by any demonstration except when the conclusion also has to be expressed in that form[24]; in which case the proof lays down as its major premiss that the major is truly affirmed of the middle but falsely denied (δείχνυται δὲ λαμβοῦσι τὸ πρῶτον κατὰ τοῦ μέσον, ὅτι ἀληθές, ἀποφάναι δ᾽ οὐκ ἀλνθές).

It makes no difference, however, if we add to the middle, or again to the minor term, the corresponding negative.

For grant a minor term of which it is true to predicate man – even if it be also true to predicate not-man of it – still grant simply that man is animal and not not-animal, and the conclusion follows: for it will still be true to say that Callias – even if it be also true to say that not-Callias – is animal and not not-animal. (*An. Post.* A11, 77a10–18)

H. Maier (1896-1900)[25] and Isaac Husik (1906) were the first to call attention to this passage, indicating that it could be used to support an argument in which the Principle of Non-Contradiction in Aristotle's logical theory was not absolute. Łukasiewicz (1910b) shows a very interesting analysis to this excerpt. He built the following syllogism to the previous passage:

B is A	Man is an animal.
C is B	Callias is a man.
———	—————————
C is A	Callias is an animal.

In which we have (*i*) the major term 'is truly affirmed of the middle but falsely denied'; and (*ii*) 'it makes no difference, however, if we add to the middle, or again to the minor term, the corresponding negative'. Moreover, the syllogism proposed is built in the first figure, what is a formal requirement for a syllogism to become a demonstration as the Stagerian states in the *Analytica Posteriora* (A14). From the Aristotelian predication theory, we can propose the following syllogism based upon predicables schemata replaced in the scientific syllogism above.

A species belongs to a genus,
and an individual belongs to a species.
Hence, an individual belongs to a genus.

Of course, in this instance, the major term corresponds to a genus, the middle one to a species, and the minor term to an individual. What Aristotle explained

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[24] As Aristotle claims right after this passage: "The law that every predicate can be either truly affirmed or truly denied of every subject is posited by such demonstration as uses *reductio ad impossibile*, and then not always universally, but so far as it is requisite; within the limits, that is, of the genus – the genus, I mean (as I have already explained), to which the man of science applies his demonstrations." (*An. Post.* A11, 77a22–26)

[25] *Vide* Raspa (1999). He quotes: Maier, Heinrich (1896-1900), *Die Syllogistik des Aristoteles*, 3 Bde., Tübingen: Verlag der H. Lauppschen Buchhandlung.

is that if this predicable structure is sound, including a genus well-definition (consistent or not-contradictory), there is no problem if the middle term or minor term can each one be separately inconsistent. Because, as Aristotle explains,

> The reason is that the major term is predicable not only of the middle, but of something other than the middle as well, being of wider application; so that the conclusion is not affected even if the middle is extended to cover the original middle term and also what is not the original middle term. (*An. Post.* A11, 77a19–21)

From all these Aristotelian conditions, Łukasiewicz (1910b) proposes the following two schemata for the corresponding syllogism, in which *A* stands for the major term 'animal', *B* stands for the middle term 'man' and *C* stands for the minor term 'Callias':

(*α*)

B is *A* (and is not not-*A* at the same time)
C is *B* and is not *B*

———————————

C is *A* (and is not not-*A* at the same time)

(*β*)

B is *A* (and is not not-*A* at the same time)
C, that is not *C*, is *B*

———————————

C is *A* (and is not not-*A* at the same time)

Łukasiewicz explains that (*α*) and (*β*) are sound because *C* is *B*. As explained earlier by Aristotle, this is a consequence of the extension of major term *A* in the example (*α*). Its extension is suficient to include the term *B* as well as the term not-*B*. And in the case of example (*β*), the term *B* has extension enough to include the term *C* as well as the term not-*C*. Raspa (1999) calls atention to the fact of which Łukasiewicz himself was conscious: both these syllogisms are possible, but not necessary. We remark the Aristotelian result exposed in the *Analytica Posteriora* (A11) showing it as a theorem.

Theorem 2 (Theorem for demonstrations with inconsistent terms) *If a syllogism is a demonstration and, if the major term is consistent (not-contradictory, or well-behaved), then the minor or middle terms can be each one separetely inconsistent.*

4   Logical interpretation of the Aristotelian results

In this section we explore two possible interpretations for the results showed in the Aristotelian texts that we have studied in the previous second and third sections. First, we show how to build antilogisms corresponding to the syllogisms from opposite premises, as proposed by Ladd-Franklin; this method is a classical one. Next, we show, based on da Costa's paraconsistent first-order logic C_1^*, how to build proofs corresponding to the demonstrations with inconsistent terms as exposed in the earlier section.

4.1   The antilogisms approach

Christine Ladd-Franklin (1883) proposes as the fundamental relation in her algebra of logic a kind of negated copula, denoted by

$$a \,\overline{\vee}\, b \qquad\qquad (16)$$

which stands for '*a* is excluded of *b*', and its negation

$$a \vee b \qquad\qquad (17)$$

which stands for '*a* is partially in *b*' ('*a* is not totally excluded of *b*'). Using such negated copula, she proposes the following representation of categorical propositions in her algebra of logic:

CATEG. PROPOSITION	TRADITIONAL FORM	LADD-FRANKLIN FORM
A	'all *a* is *b*'	$a \,\overline{\vee}\, -b$
E	'no *a* is *b*'	$a \,\overline{\vee}\, b$
I	'some *a* is *b*'	$a \vee b$
O	'some *a* is not *b*'	$a \vee -b$

She also proposes that any valid syllogism can be described by an inconsistent triad that presents the following configuration:

> Take the contradictory of the conclusion and see that universal propositions are expressed with a negative copula and particular propositions with an affirmative copula. If two of the propositions are universal and the other particular, and if that term only which is common to the two universal propositions has unlike signs then, and only then, the syllogism is valid. (Ladd-Franklin, *The algebra of logic*, 1883, p. 33 *apud* Russinoff 1999, p. 462)

The rule of syllogism proposed by Ladd-Franklin catches, according to her, the fundamental issue of syllogism: the elimination of middle term. If we take the contradictory of the conclusion, the repeated term in the two universal propositions has to show different signals; so it can be algebrically eliminated. Actually, Ladd-Franklin method of antilogisms can be seen as a generalization of the Aristotelian method of proof *through impossibility*.[26] Here we apply her method to verify the syllogisms from opposite premises, as stated by Aristotle. We could realize that such syllogisms constitute each one an inconsistent triad, in which the rule of syllogism for antilogism works. Consider the Aristotelian instance bellow:

> All science is good,
> and some science (medical knowledge) is not good.
> Hence, some science (medical knowledge) is not science.

This is the previous syllogism showed, the contradictory Baroco:

$$Abg, Obg \vdash Ogg$$

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[26]*Vide An. Pr.* A7, 29a35-40.

that is validated by the antilogism method. It corresponds to

$$(g\overline{\vee} - b)\, (g \vee -b)\, (g\overline{\vee} - g)\, \overline{\vee} \tag{18}$$

We observe, as pointed out by Russinoff (1999, p. 460, note 15), that Ladd-Franklin uses, sometimes in the same formula, the same symbol denoting distinct inconsistency relations. Consider the formula $a\overline{\vee}b$; when a and b are terms or classes, the symbol '$\overline{\vee}$' spells out that 'no a is b' as previously explained; however, if a and b are categorical propositions, the symbol '$\overline{\vee}$' denotes that a and b are inconsistent. In the formula (18), that corresponds to an antilogism, the external '$\overline{\vee}$' means that the three formulae, corresponding to propositions, are inconsistent at all.

The remaining antilogisms corresponding to the Aristotelian valid syllogisms from opposite premises are presented bellow.

OPPOSITE PREMISES SYLLOGISMS	OPPOSITE PREMISES ANTILOGISMS
$Abg, Ebg \vdash Egg$ (Camestres)	$(g\overline{\vee} - b)\, (g\overline{\vee}b)\, (g \vee g)\, \overline{\vee}$
$Ebg, Abg \vdash Egg$ (Cesare)	$(g\overline{\vee}b)\, (g\overline{\vee} - b)\, (g \vee g)\, \overline{\vee}$
$Ebg, Ibg \vdash Ogg$ (Festino)	$(g\overline{\vee}b)\, (g \vee b)\, (g\overline{\vee} - g)\, \overline{\vee}$
$Ejb, Ajb \vdash Ojj$ (Felapton)	$(b\overline{\vee} j)\, (b\overline{\vee} - j)\, (j\overline{\vee} - j)\, \overline{\vee}$
$Ojb, Ajb \vdash Ojj$ (Bocardo)	$(b \vee -j)\, (b\overline{\vee} - j)\, (j\overline{\vee} - j)\, \overline{\vee}$
$Ejb, Ijb \vdash Ojj$ (Ferison)	$(b\overline{\vee}j)\, (b \vee j)\, (j\overline{\vee} - j)\, \overline{\vee}$

We must remark that antilogism is a classical procedure for decision, based on refutation of the 'middle opposite' term. In this case, its adequacy to verify the validity of syllogisms from opposite premises is related, we suppose, to the fact that this Aristotelian syllogism constitutes a broad sense paraconsistent theory, which preserves all the results of the classical theory of syllogism, in particular, these ones.

4.2   Da Costa's paraconsistent logic approach

As it is well known, da Costa introduced in 1963 his hierarchies of paraconsistent propositional logics C_n, $1 \le n \le \omega$, of paraconsistent first-order logics C_n^*, $1 \le n \le \omega$, of paraconsistent first-order logics with equality $C_n^=$, $1 \le n \le \omega$, of paraconsistent description calculi D_n, $1 \le n \le \omega$, and paraconsistent set theories NF_n, $1 \le n \le \omega$.[27]

We propose that the Aristotelian argument in the *Analytica Posteriora* (A11) can be logically interpreted using as underlying logic da Costa's paraconsistent first-order logic C_1^*, the first predicate calculus of the hierarchy C_n^*, $1 \le n \le \omega$.

In da Costa's paraconsistent logics, in particular in the language of the system C_1^*, a unary operator '$\circ$' is introduced by definition:

$$\mathbf{A}^{\circ} =_{def.} \neg(\mathbf{A} \wedge \neg\mathbf{A}) \tag{19}$$

where '$\neg$' is the primitive paraconsistent negation of the language.

The formula $\mathbf{A}^{\circ}$ is read as "*A* is a well-behaved formula".

As we intend to show, the demonstrative strategy explained by Aristotle in the *Analytica Posteriora* (A11) can be correctly formalized in C_1^*.

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[27]*Vide* da Costa (1963, 1974).

In order to develop it, we will use the first system DNC_1^* of the hierarchy of natural deduction systems DNC_1^*, $1 \leq n \leq \omega$, introduced by Castro (2004), which are equivalent to the corresponding systems of da Costa's hierarchy C_n^*, $1 \leq n \leq \omega$.

Let us recall the Aristotelian argument:

Man is an animal.
Callias is a man.
─────────────
Callias is an animal.

Let x be an individual variable and c an individual constant corresponding to 'Callias'. Let A and M be monadic predicate symbols corresponding to 'being animal' and 'being man', respectively. In Castro's DNC_1^*, considering the Aristotelian major term 'animal' as a 'well-behaved' formula, that is, the formula $(Ax)°$, the Aristotelian argument can be formalized as:

1	$\forall x(Mx \rightarrow (Ax)°)$	Major premise
2	Mc	Minor premise
3	$Mc \rightarrow (Ac)°$	1, Castro's Rule of Elimination of the $\forall$
4	$(Ac)°$	2, 3 MP

The proof above corresponds to the general schema of the Aristotelian demonstration.

However M and c can be not well-behaved, that is, they can be contradictory or inconsistent.

If M ('man') is not well-behaved, we suppose a new predicate letter S denoting 'species', which includes Mx as well $\neg Mx$. Such formal procedure is correct because, as Aristotle claims, the term 'man' can be inconsistent, however, it must be restricted to the major term 'animal'. In other words, for instance, 'not-man' cannot be a species out of the class corresponding to the genus 'animal'. For this reason, Aristotle demands that the term animal be closed under double negation, when he claims that 'man is animal and not not-animal', in the passage quoted in the previous section. Thus, we have the following proof, where $\neg\neg S$ corresponds to the Aristotelian condition just explained.

1	$\forall x(Sx \rightarrow (Ax)°)$	Major premise
2	$\neg\neg Sc$	Minor premise
3	Sc	Castro's Rule of Elimination of Double Negation
4	$Sc \rightarrow (Ac)°$	3, 1, Castro's Rule of Elimination of the $\forall$
5	$(Ac)°$	2, 3 MP

This argument concludes that Callias is an animal.

Now, if Callias c is not consistent, what we will denote by the new constant $c_\neg$, we will have the following proof:

1	$\forall x(Mx \to (Ax)^\circ)$	Major premise
2	Mc	Minor premise
3	$Mc_\neg$	Minor premise with constant c not consistent
4	$Mc \to (Ac)^\circ$	1, Castro's Rule of Elimination of the $\forall$
5	$(Ac)^\circ$	4, 6 MP
6	$Mc_\neg \to (Ac_\neg)^\circ$	1, Castro's Rule of Elimination of the $\forall$
7	$(Ac_\neg)^\circ$	5, 8 MP
8	$(Ac)^\circ \wedge (Ac_\neg)^\circ$	7, 9 Castro's Rule of Introduction of the $\wedge$

We have just proved that Callias (c) and not-Callias ($c_\neg$) are animals. In fact, there are several restrictions on interpreting an Aristotelian argument schema into a modern logical language such as da Costa's paraconsistent first-order logic C_1^*. We have showed that such an interpretation is possible, but that it is also an approximation to the Aristotelian argument, due to the different paraconsistent logical grounding employed.

5   Concluding remarks

We would like to remark the following points concerning the Aristotelian results here discussed and their relation to paraconsistency. First, we propose that the Stagerian knew that, under special conditions, his theory of deduction does not prove everything in face of contradiction, or becomes trivial as we say today. His avoidance of self-predication and circular demonstration also witnesses his awareness of the risk of triviality. Second, Aristotle distinguishes between 'well' and 'bad' behaved categorical propositions, using this notion to delimit the extension of the Principle of Non-Contradiction in the *Analytica Posteriora* (A11). The logic that underlies the 'bad' behaved terms can be a strict sense paraconsistent theory. Third, it seems to be possible to interpret the syllogisms from opposite premises as a broad sense paraconsistent theory, because from such premises not every categorical proposition can be proved. Fourth, apagogical inference schemata used by Aristotle also show a broad sense paraconsistency, because they do not allow that a contradiction entails everything; in fact, only the negative statements can be proved from initial assumptions. Fifth, as well as the antilogisms, da Costa's paraconsistent systems C_n^*, $1 \le n \le \omega$, allow interpreting the results exposed. To sum up, the role of Aristotle in the pre-history of paraconsistent logic seems to be much more important than it is usually admitted.

6   Acknowledgements

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