Метод спектроскопии шума, основанный на преобразовании Фурье

Д. С. Морозова

Национальный исследовательский университет «МИЭТ»,
г. Москва, Россия
e-mail: DoshikMorozov@yandex.ru

Fourier transform method for noise spectroscopy

D. S. Morozova

National Research University of Electronic Technology,
Moscow, Russia
e-mail: DoshikMorozov@yandex.ru

Аннотация. Данная статья посвящена методу спектроскопии шума в квантовых устройствах, основанному на преобразовании Фурье. В статье описан принцип метода и приложены основные математические выкладки.

Annotation. This paper is dedicated to the Fourier transform method for noise spectroscopy for quantum systems. It describes the main principle of the method and basic mathematical calculations.

Ключевые слова: спектроскопия шума, спектр шума, преобразование Фурье, когерентность, затухание индукции, Spin Echo, функция фильтрации.

Key words: noise spectroscopy, noise spectrum, Fourier transform, coherence, Free Induction Decay, Spin Echo, filter function.

Introduction

Its is necessary to analyze the noise spectrum causing loss of coherence of qubits, for stable and correct work of quantum devices.

Coherence is the property of the system to keep the phase of wave function, e.g. the ability of the qubit to be in superposition. The higher time of coherence is, the more useful operations can be implemented.

Fourier transform noise spectroscopy or FTNS allows to determine noise spectrum for all kinds of conditions and at the same time significantly reduces the amount of required resources.

This method was first presented in May of 2024 by a group of 4 scientists from New South Wales University.

Этот метод был впервые представлен в мае 2024 учеными из университета Нового Южного Уэльса в журнале Nature.

FTNS description

FTNS implies using FID or Spin Echo as the basis for further spectrum analysis.

FID or Free Induction Decay is a process of observing induction decay after external influence that causes spin has been stopped. It possesses the information about spins' interactions with each other and external environment and respectively about time properties of coherence.

Spin Echo is a phenomenon, where after re-applying of short pulse to the system spin direction changes so that scattered phases resonate, e.g. creating an "echo". This effect is used to prolong observation time of quantum processes which is very useful for long coherence time.

At first change of qubit state over time is defined, it is either drop of coherence level or response of the system to re-applied pulse, FID or Spin Echo respectively.

Then, Fourier transform is applied to the received time signal. Thus, noise frequencies spectrum is obtained.

This spectrum can be analyzed in order to identify the nature and source of the noise and take optimal steps to reduce it.

Mathematics

Coherence function C(t) is defined by formula:

$$C(t) = e^{-\chi(t)} \tag{1}$$

Where χ(t) is an attenuation function, given by the overlap of the noise spectrum and a filter function that incorporates the effect of the pulses on the system.

$$\chi(t) = -\ln\big(C(t)\big) = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\omega\, S(\omega)F(\omega t) \tag{2}$$

Noise spectrum S(ω) is the Fourier transform of the equilibrium time correlation function S(t) of the environmental noise.

$$\mathrm{S}(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} S(t) \tag{3}$$

Filter function F(t) is given by physical properties of the system. It encodes the sign switching of the environmental fluctuations upon application of π-pulses.

For greater clarity, let's consider finding the noise spectrum for the free induction decay function:

$$F_{FID}(\omega t) = \frac{4}{\omega^2} sin^2\left(\frac{\omega t}{2}\right) \tag{4}$$

Substituting expression (4) for F_{FID} in equation (2) we obtain:

$$\chi(t) = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\omega\, S(\omega)\frac{4}{\omega^2} sin^2\left(\frac{\omega t}{2}\right) \tag{5}$$

Differentiating twice with respect to time:

$$\chi''(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega S(\omega)\cos(\omega t) \tag{6}$$

We Fourier transform both sides to find:

$$S(\omega) = \sqrt{2\pi}\mathcal{F}\big(\chi''(t)\big) \tag{7}$$

Noting that S(−ω) = S(ω). Thus, one-to-one map between the noise spectrum and FID function is visible.

Let's assume that initial spectrum has form of:

$$S(\omega) = Ae^{-\left(\frac{\omega}{\sigma}\right)^2} \tag{8}$$

For this spectrum the coherence function will have the form of:

$$C(t) = \exp\left(-\frac{A}{\sigma}\left(\frac{t\sigma}{2}E\left(\frac{t\sigma}{2}\right) + \frac{e^{-\frac{t^2\sigma^2}{4}} - 1}{\sqrt{\pi}}\right)\right) \tag{9}$$

Where E(t) is the error function. Based on equations (6) and (9) we obtain:

$$\chi''(t) = \frac{A}{\sqrt{2\pi}}e^{-\frac{t^2\sigma^2}{4}} \tag{10}$$

Substituting this value in equation (7):

$$S(\omega) = \sqrt{2\pi}\mathcal{F}\left(\frac{A}{\sqrt{2\pi}}e^{-\frac{t^2\sigma^2}{4}}\right) = Ae^{-\left(\frac{\omega}{\sigma}\right)^2} \tag{11}$$

Thus, the obtained noise spectrum from equation (11) is equal to initial one from equation (8). This example shows the correct work of FTNS method.

Figures 1 and 2 depict comparisons of restored spectrums, obtained by different dynamic methods and FTNS method.

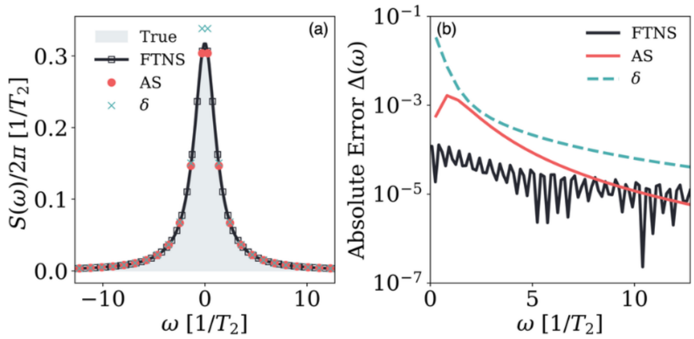

Figure 1 — Comparison of a simple noise spectrum reconstruction between the FTNS and DDNS methods

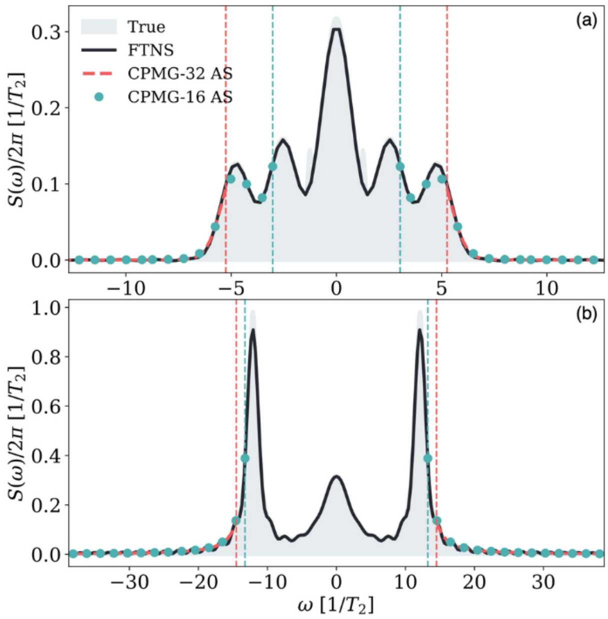

Figure 2 — Comparison of structured noise spectra reconstructions between the FTNS and DDNS methods.

Библиографический список

1. Arian Vezvaee, Nanako Shitara, Shuo Sun & Andrés Montoya-Castillo https://www.nature.com/articles/s41534-024-00841-w

2. Joseph P. Hornak. "The Basics of MRI". Rochester Institute of Technology. Chapter 4: NMR SPECTROSCOPY https://www.cis.rit.edu/htbooks/mri/chap-4/chap-4.htm#4.1

3. Андрей Коняев. Квантовая азбука: «Когерентность». Можно ли потревожить квантовую систему чуть-чуть, а потом вернуть все обратно? https://nplus1.ru/material/2016/04/12/coh