LAKSHYA

FOR JEE

LECTURE–06

RELATIONS AND FUNCTIONS

SACHIN JAKHAR

TODAY's GOAL

Question Practice,
Introduction to Functions,
Different Types of Functions.

Q Relation R in the set A of human beings in a town at a particular time given by

(a) R = {(x, y) : x and y work at the same place}

(b) R = {(x, y) : x and y live in the same locality}

(c) R = {(x, y) : x is exactly 7 cm taller than y}

(d) R = {(x, y) : x is wife of y}

(e) R = {(x, y) : x is father of y}

[NCERT]

Equiv. ✓

Equiv.

(c)
Ref ×
Symm ×
Trans ×

(d) $\left(\underset{W}{x}, \underset{H}{y}\right)$
Ref ×
Symm ×
Trans ×

Wrong

Transitive ✓ (check nhi kr payenge)

(e) $\left(\overset{\times}{R}\ \overset{\times}{S}\ \overset{\times}{T}\right)$

$R = \left\{ \left(\overset{W}{x}, \overset{H}{y}\right) \left(\overset{W}{z}, \overset{H}{p}\right) \left(\overset{W}{a}, \overset{H}{b}\right) \dots \right\}$

x is f of y.

z is father of x.

(e) $R = \{ \underbrace{(x, y)}_{} \; (z, x) \; (z, y)$

f

Son/D

×

Trans. X

(Notes mein Teacher ki boli gyi har baat apni language mein likhe)

Q If R_1 and R_2 are two non-empty relations in a set A. Which of the following is not true?

Wrong statement.

Sochne Wala Question

(A) If R_1 and R_2 are transitive, then $R_1 \cup R_2$ is transitive

#H.W.

(B) If R_1 and R_2 are transitive, then $R_1 \cap R_2$ is transitive

Try to think, $\check{R}_1 \cap \check{R}_2 = \overset{\times}{T}$

True.

(C) If R_1 and R_2 are symmetric, then $R_1 \cup R_2$ is symmetric

True.

(D) If R_1 and R_2 are reflexive, then $R_1 \cap R_2$ is reflexive

True

Option (A)

$$\begin{cases} R_1 = \{(3,2)\} \to T \\ R_2 = \{(2,3)\} \to T \end{cases}$$

$$R_1 \cup R_2 = \{(3,2)(2,3)\} \to T \times$$

Option (B) :

$$R_1 \cap R_2 = \{(1,2)\ (2,3)\} \xrightarrow{(1,3)} \text{Transitive}$$

$\rightarrow$ non-Transitive

$\checkmark$ T $\leftarrow$ $R_1 = \{(1,2)\ (2,3)\ (1,3)\ (2,2)\}$

$\checkmark$ T $\leftarrow$ $R_2 = \{(1,2)(2,3)\ (1,3)\ (1,1)\}$

NOTE:

If R_1 & R_2 are two Transitive Relations on any set then:

1. $R_1 \cup R_2$ may or may not be Transitive.

2. $\boxed{R_1 \cap R_2 \text{ is always Transitive.}}$

⇒ "Remember".

Q Consider the following relation : R = {(x, y) | x, y are real numbers and x = wy for some rational number w}; $S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \mid$ m, n, p and q are integers such that n, q $\neq$ 0 and qm = pn} Then [AIEEE-2010]

(A) R is an equivalence relation but S is not an equivalence relation.

(B) Neither R nor S is an equivalence relation.

(C) S is an equivalence relation but R is not an equivalence relation.

(D) R and S both are equivalence relations.

$\boxed{x = wy}$

Wrong!

$R = \{(5,2)\ (0,0) \ldots\}$ → Eq. ✗

$S = \left\{\left(\frac{4}{1}, \frac{12}{3}\right) \ldots\right\}$

↳ R ✓ S ✓ T ✓ ≡ E ✓

Correct Solution:

$$R = \left\{ (x, y) : \boxed{x = wy} \right\}$$

Reflexive ✓ $\Rightarrow \{ (0,0)\ (1,1)\ (\sqrt{2}, \sqrt{2}) \ldots\ldots (\pi, \pi) \ldots \}$

Symmetric ✗ $\Rightarrow \{ (0,1)\ (1,0)$

(Equiv.) ×

NUMBER OF RELATIONS

If $n(A) = p$ then: # $n(A \times A) = p^2$

(i) Total Number of relations on set A = $2^{(p^2)}$

(ii) Number of Identity Relations = 1.

(iii) ** Number of Reflexive Relations = $2^{(p^2 - p)}$

(iv) ** Number of Symmetric Relations = $2^{\frac{p(p+1)}{2}}$

} Memorise.

(Is formula mein { } bhi count hua h)

(v) **For Transitive Relations:**

If $n(A) = 1 \Rightarrow$ Number of Transitive relations = 2

If $n(A) = 2 \Rightarrow$ Number of Transitive relations = 13

If $n(A) = 3 \Rightarrow$ Number of Transitive relations = 171

If $n(A) = 4 \Rightarrow$ Number of Transitive relations = 3994

Koi puchega nahi

NO-GENERAL FORMULA

(vi) **For Equivalence Relations:**

If $n(A) = 1 \Rightarrow$ Number of equivalence relation = 1

If $n(A) = 2 \Rightarrow$ Number of equivalence relation = 2

If $n(A) = 3 \Rightarrow$ Number of equivalence relation = 5

If $n(A) = 4 \Rightarrow$ Number of equivalence relation = 15

Ex. If $n(A) = 3$ and

l = number of Identity relations on set - A

m = number of reflexive relations on set - A

n = number of non-empty symmetric relations on set A

p = number of transitive relations on set A

Then value of $(l + m) - (n + p)$ is ______ $(1+64)-(63+171) = 2-171 = -169.$

H.W.

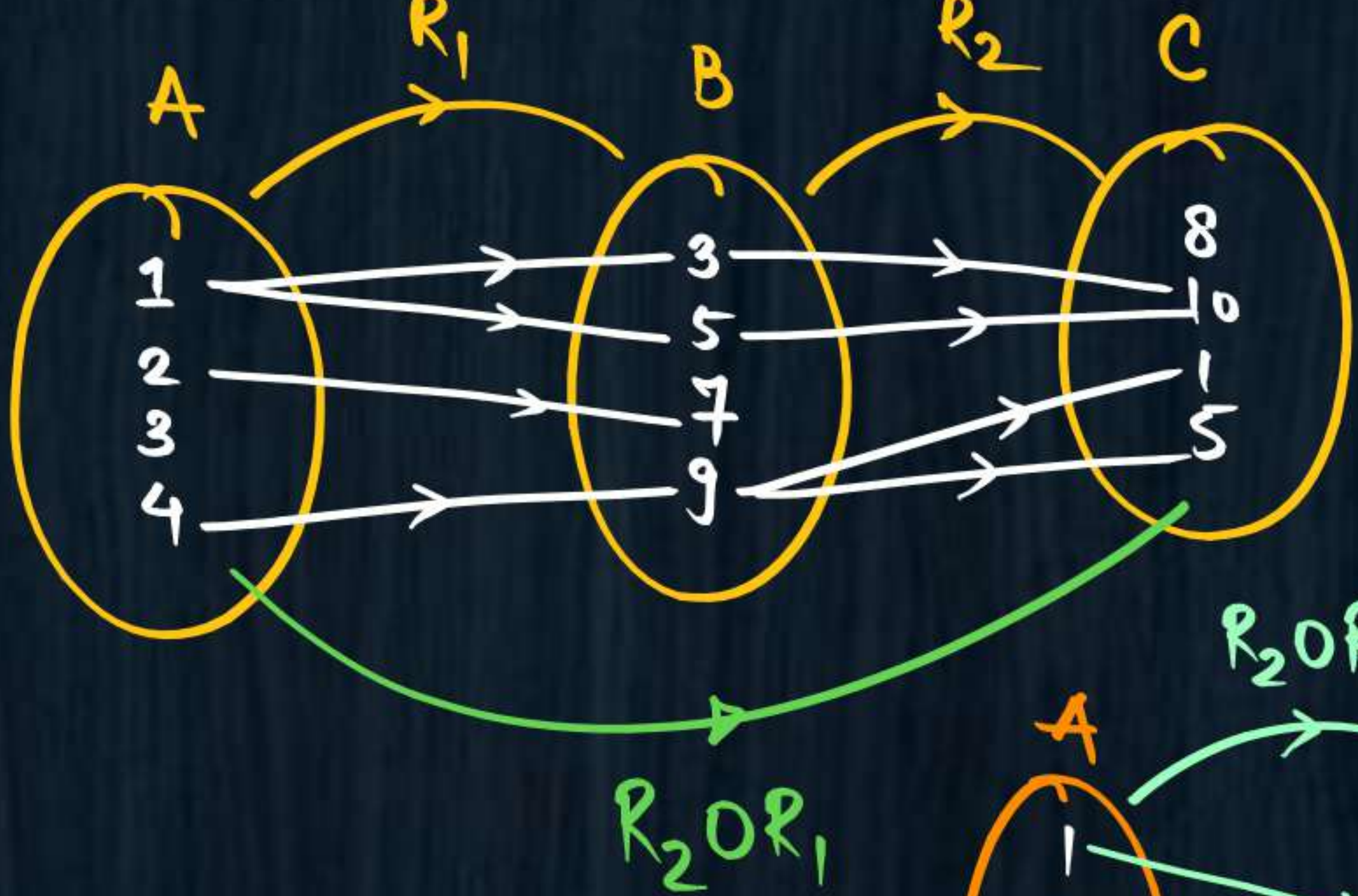
$l = 1.$

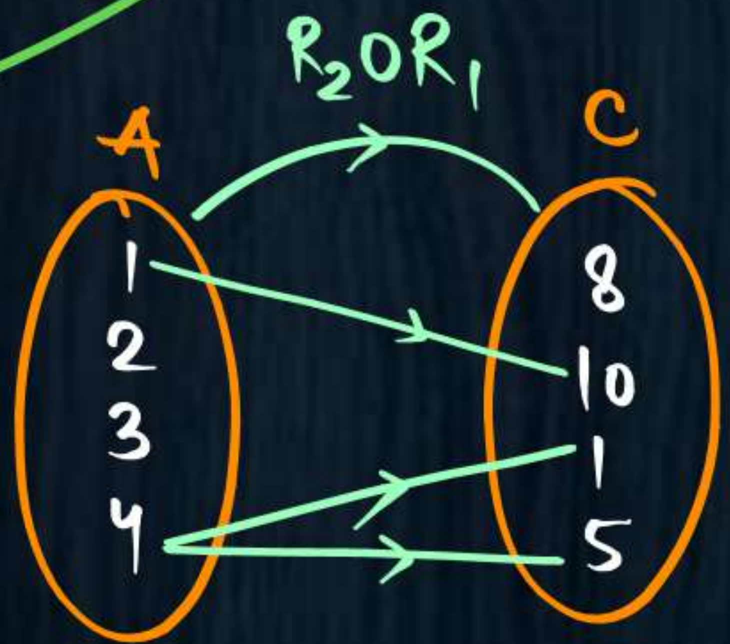
$\boxed{p = 171}$

$m = 2^{p^2-p} = 2^{9-3} = 2^6 = 64.$

$n = \left(2^{\frac{p(p+1)}{2}}\right) - \underset{\{\}}{1} = 2^6 - 1 = 63.$

COMPOSITE RELATION



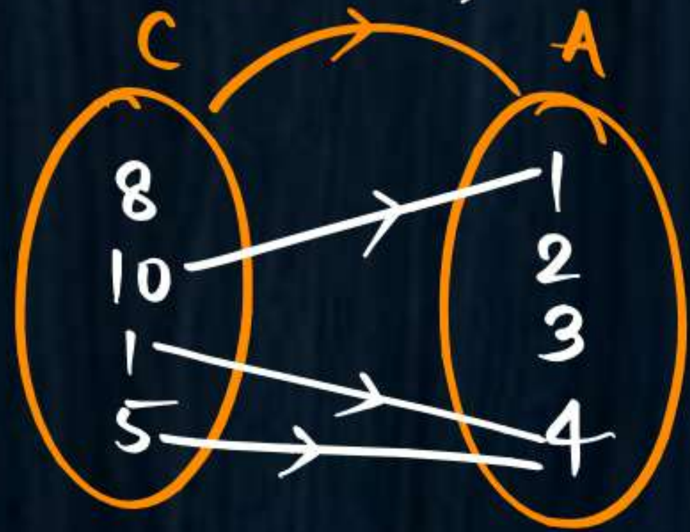


$\boxed{R_2 O R_1}$

R_1 → Ist

R_2 → II^{nd}

$(R_2 O R_1)^{-1}$



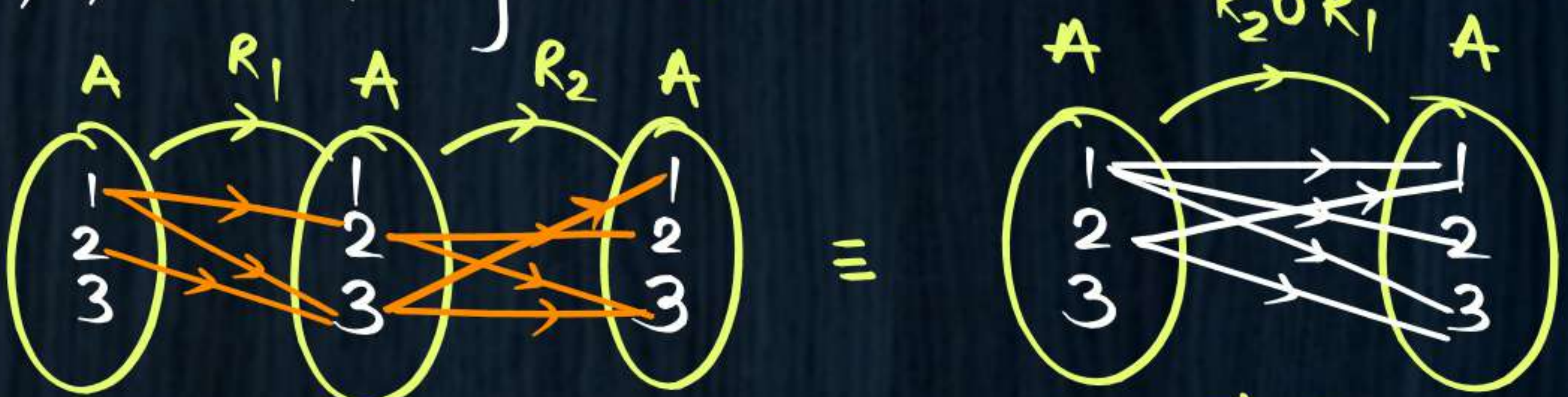
Ex:

$R_1 = \{(1,3)\ (2,3)\ (1,2)\}$

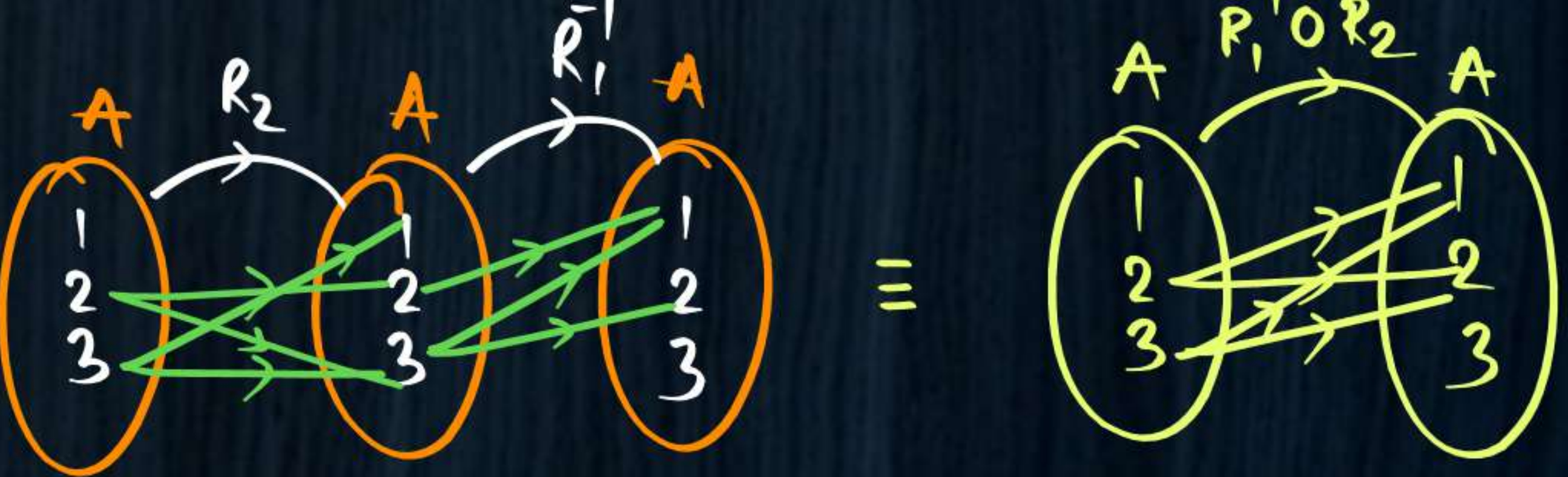
$R_2 = \{(2,2)\ (3,3)\ (3,1)\ (2,3)\}$

on set A.

$R_2 \circ R_1 \Rightarrow$



$R_1^{-1} \circ R_2 \Rightarrow$



Q Consider three sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8, 9\}$ and R_1 is defined from A to B such that $R_1 = \{(x, y), 2x = y, x \in A, y \in B\}$. Similarly R_2 is defined from B to C such that $R_2 = \{(x, y) : \text{'x divides y'}, x \in B \text{ and } y \in C\}$, then:

$\Downarrow$

$\boxed{\dfrac{y}{x} = \text{Integer.}}$

(i) $R_2 \, o R_1$

(ii) $R_1^{-1} o R_2^{-1}$

ANTI-SYMMETRIC : (not in JEE)**

Definition: $\boxed{\text{If } (a,b)\in R \text{ and } (b,a)\in R \text{ only if } a=b}$

OR

Agar (a, b) relation (R) mein ho to (b, a) nhi hona chahiye, but (a, a) allowed hai.

{ } → I ×
→ R ×
→ S ✓
→ T ✓
→ AS ✓

$\boxed{\text{Anti-symm.} \equiv \text{Symm. na ho}}$ → # Wrong

Example: Check following Relation on set $\boxed{A = \{1,2,3\}}$ for Anti-Symm.

(i) $R_1 = \{(1,2)\ (1,3)\ (2,3)\} \longrightarrow \overset{\checkmark}{A.S.},\ \overset{\times}{S}$

(ii) $R_2 = \{(1,2)\ (3,3)\ (1,1)\} \longrightarrow \overset{\checkmark}{A.S.},\ \overset{\times}{S}$

(iii) $R_3 = \{(2,3)\ (3,2)\} \longrightarrow \overset{\times}{A.S.},\ \overset{\checkmark}{S}$

(iv) $R_4 = \{(1,1)\ (2,2)\} \longrightarrow \overset{\checkmark}{A.S.},\ \overset{\checkmark}{S}$

(v) $R_5 = \{(1,2)(2,3)(3,3)(2,1)\} \longrightarrow \overset{\times}{A.S.},\ \overset{\times}{S}$

$\boxed{\text{JEE - MAINS}}$

$\boxed{\text{Relation}}$

R, S, T, I.

no. of Relations.

C.P.

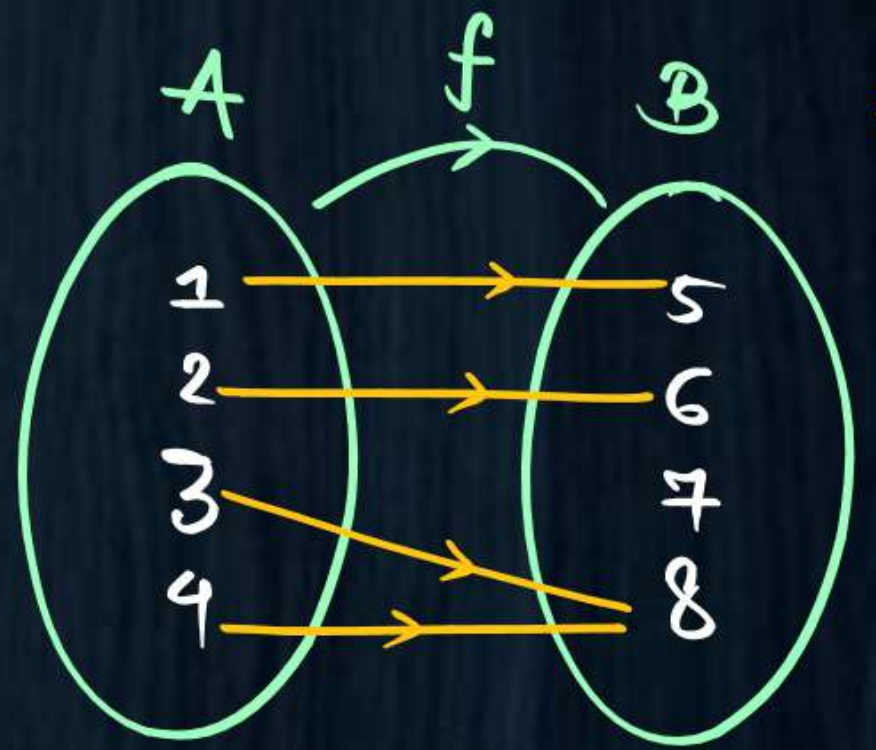
FUNCTIONS

Functions (Mapping) is a relation defined from set A to set B ($f : A \rightarrow B$) such that all "elements of set A" related to only single (one / unique) element in set B.

$$\boxed{\text{Relation}} \xrightarrow[\text{Cond}^n]{\text{Two}} \boxed{\text{Functions}}$$

(i) left set k saare elem. relate hone chahiye.

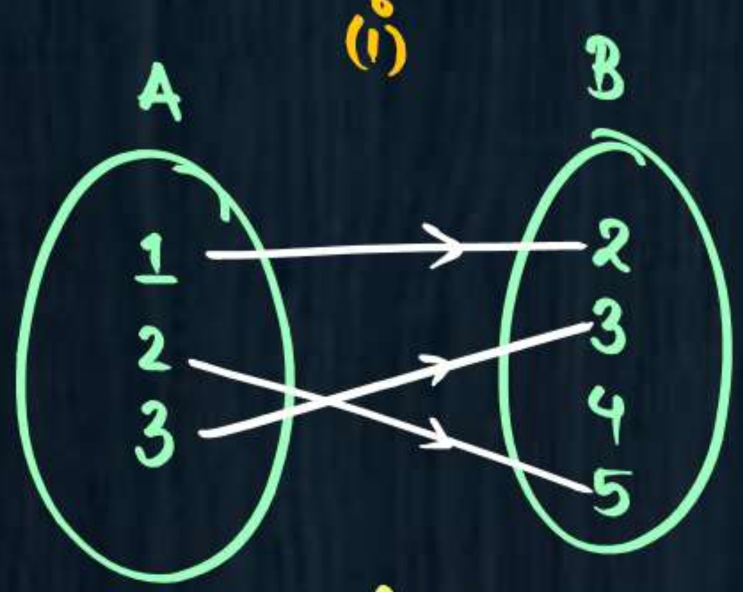
(ii) (ek hi elem. of set (B) se relate hoskte h.)



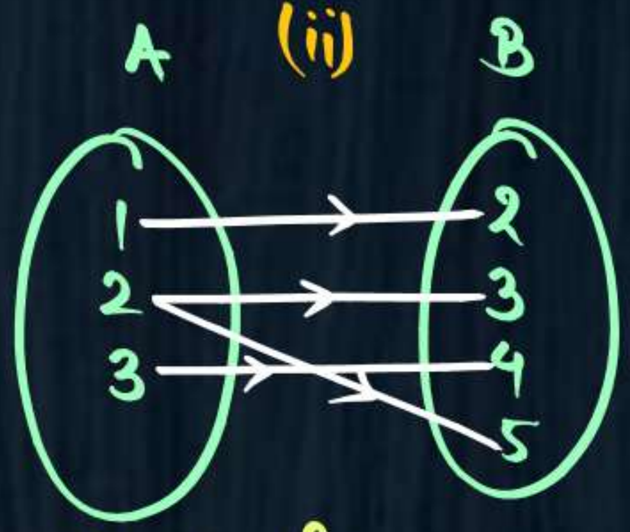
Relation ✓ Func^n ✓

Func^n $\subset$ Rel.

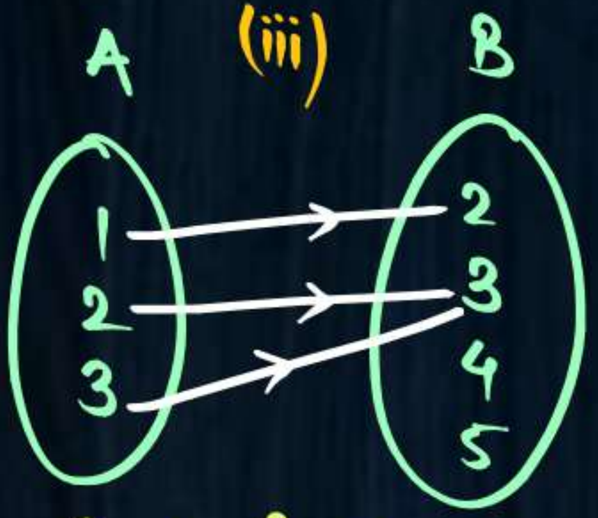
Ex: Which of the following arrow diagram represents a function?



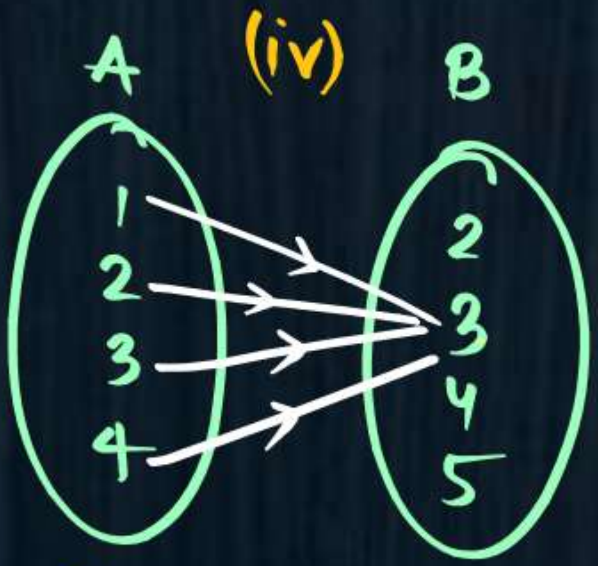
R✓ f✓



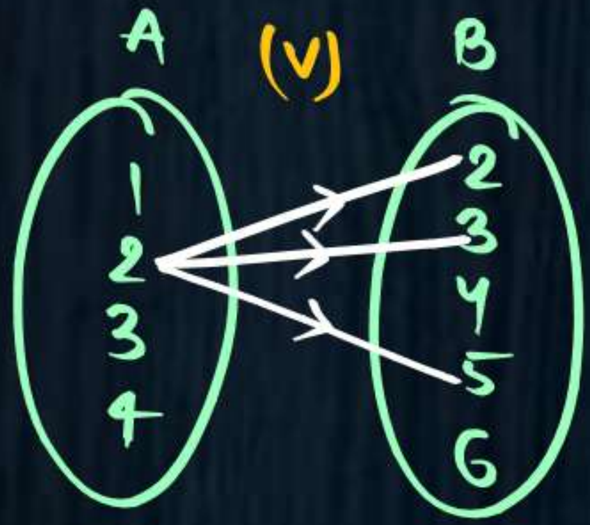
R✓ f✗



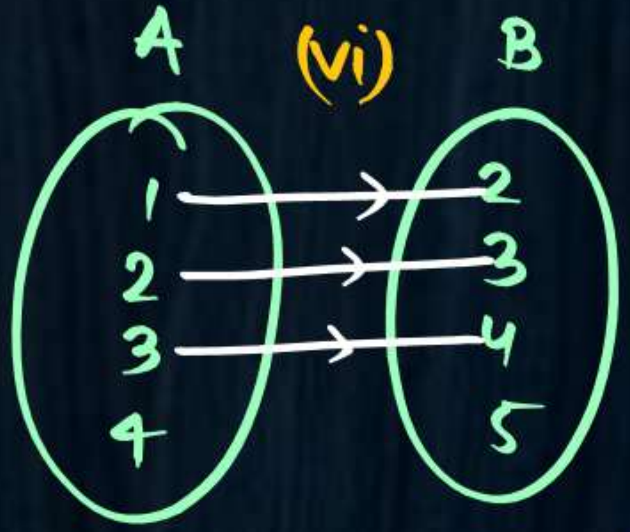
R✓ f✓



R✓ f✓



R✓ f✗



f✗ R✓

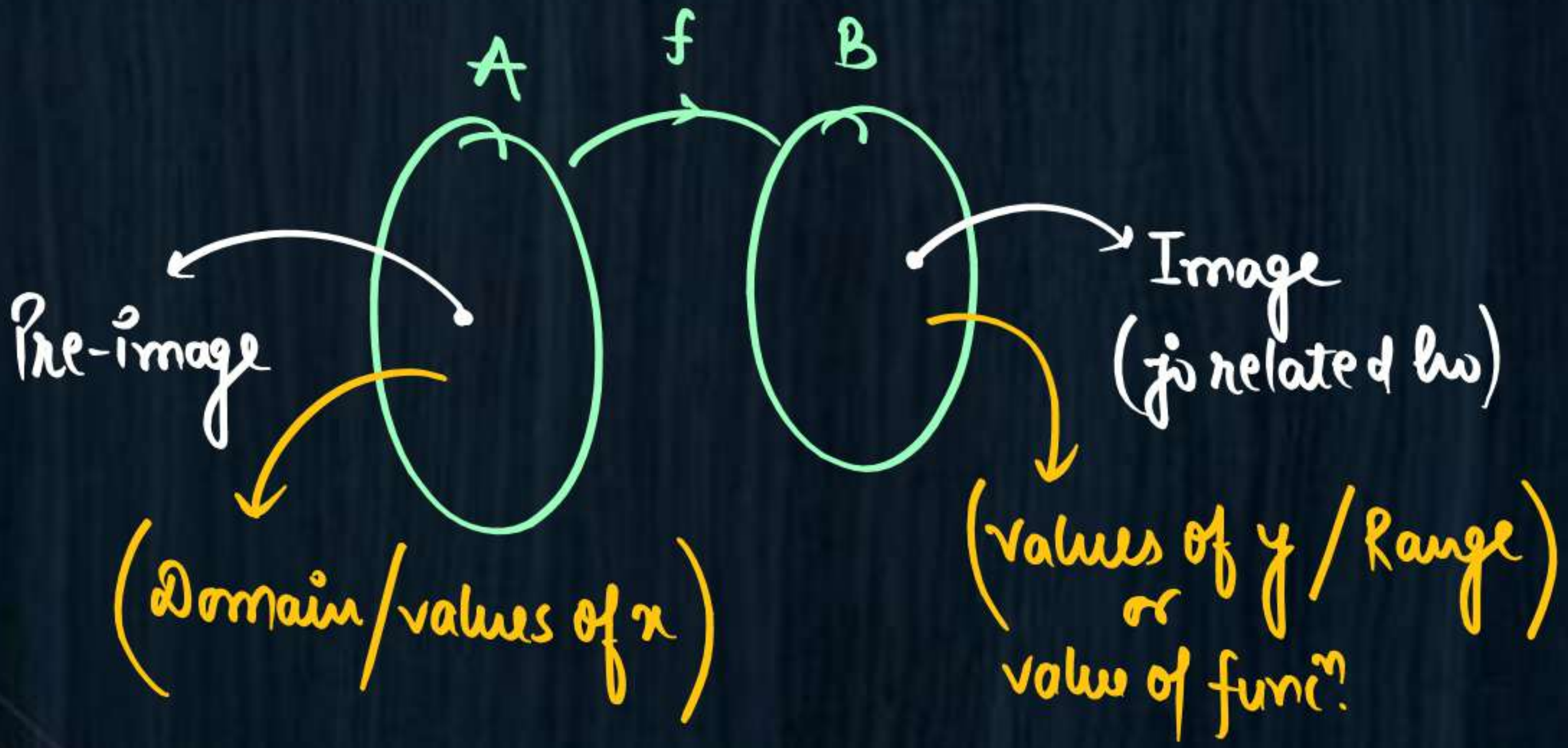
VERTICAL LINE TEST

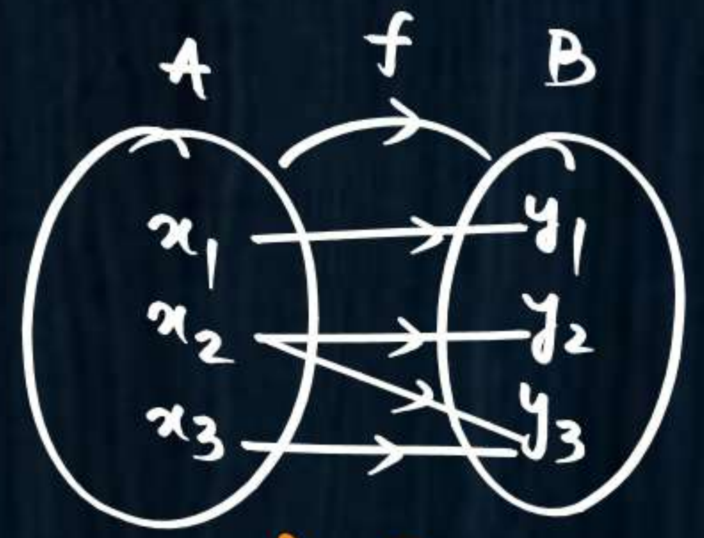
\# for graphs.

 # In given graph:

Draw all possible lines parallel to Y-axis.

If any line cuts the graph at two or more than two points then it is **NOT A FUNCTION.**





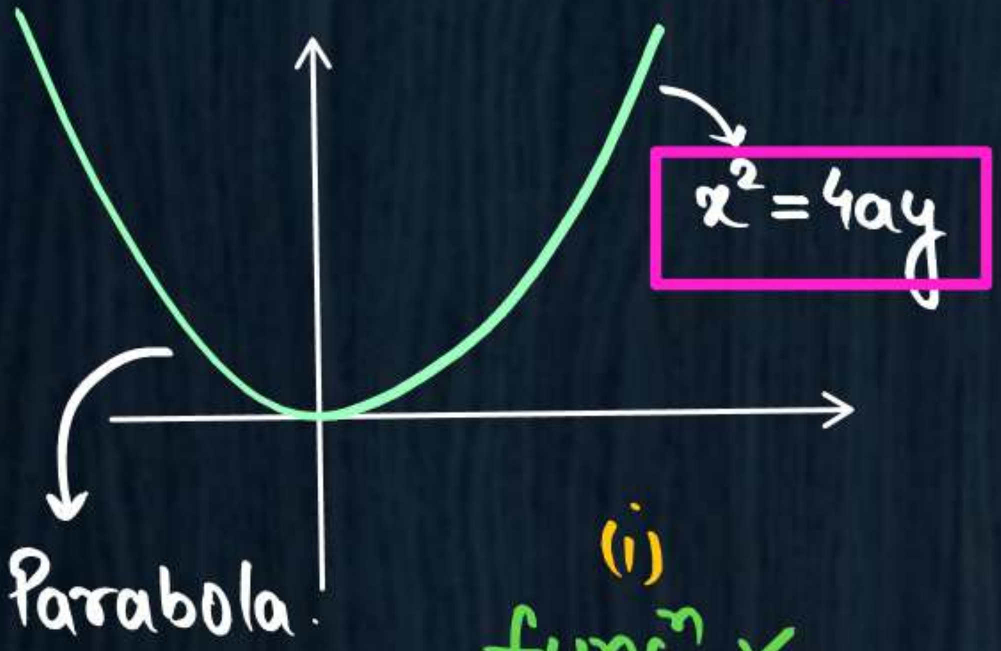
func^n ✗

$\boxed{\text{ek 'x' pr do 'y' mile}}$

$\Downarrow$

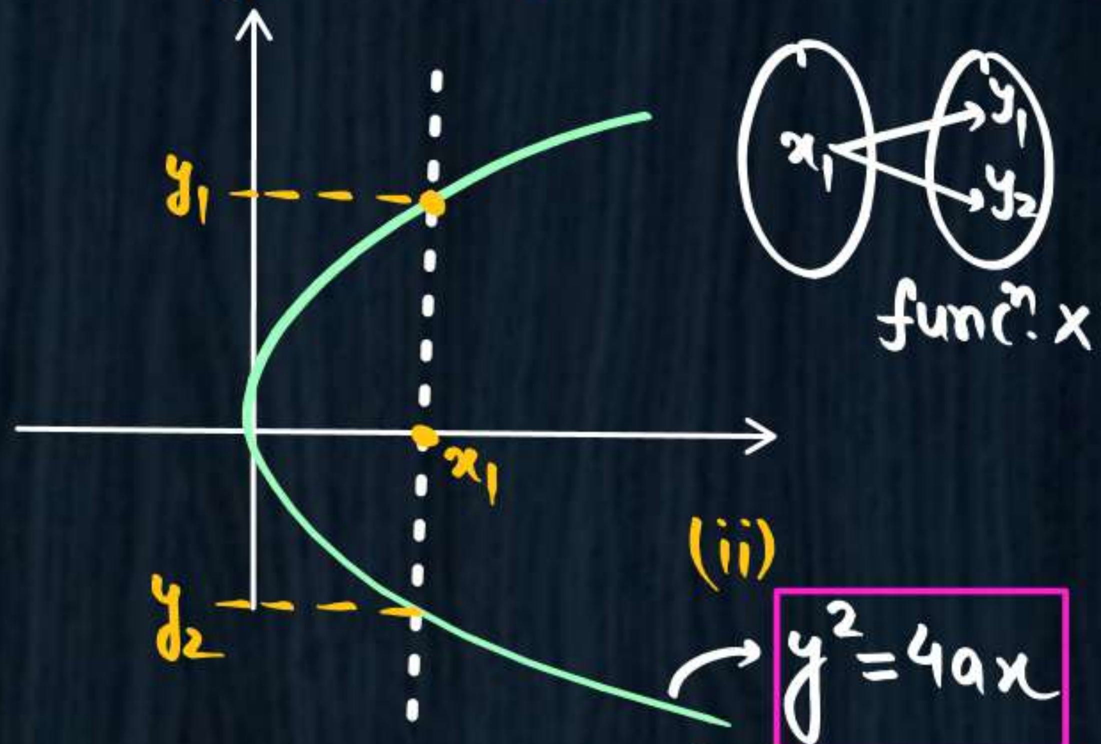
func^n ✗

Ex: Which of following Graph is graph of Function?



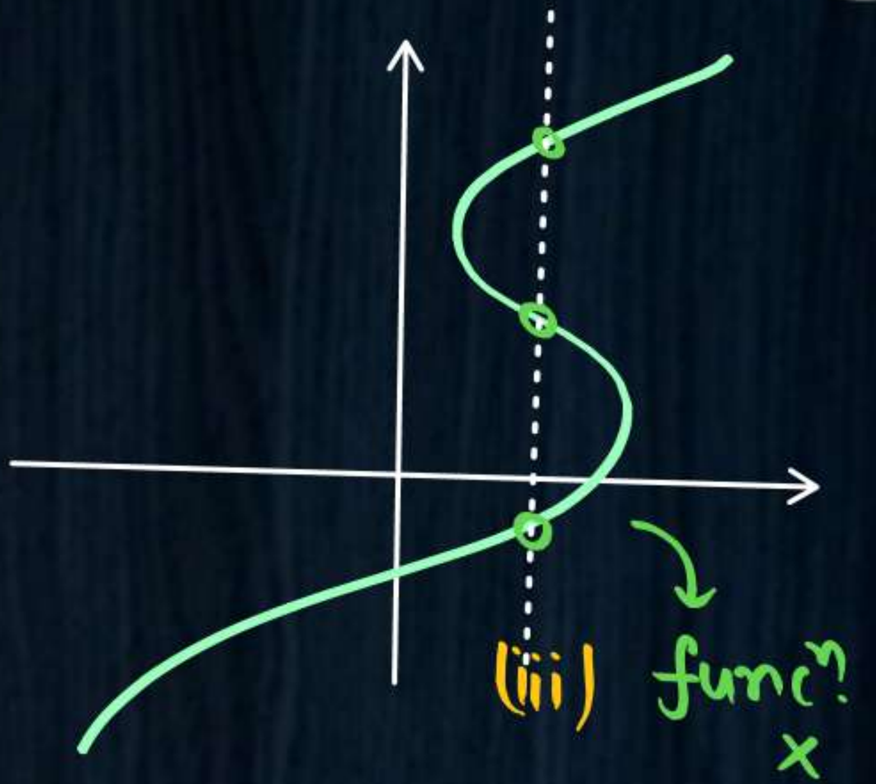
(i)

func^n ✓

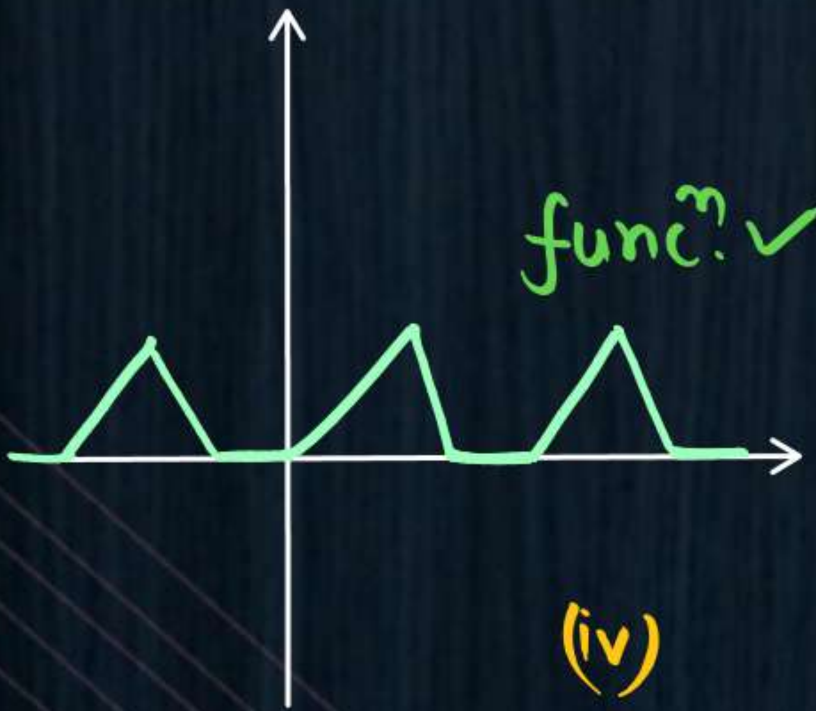


(ii)

Parabola

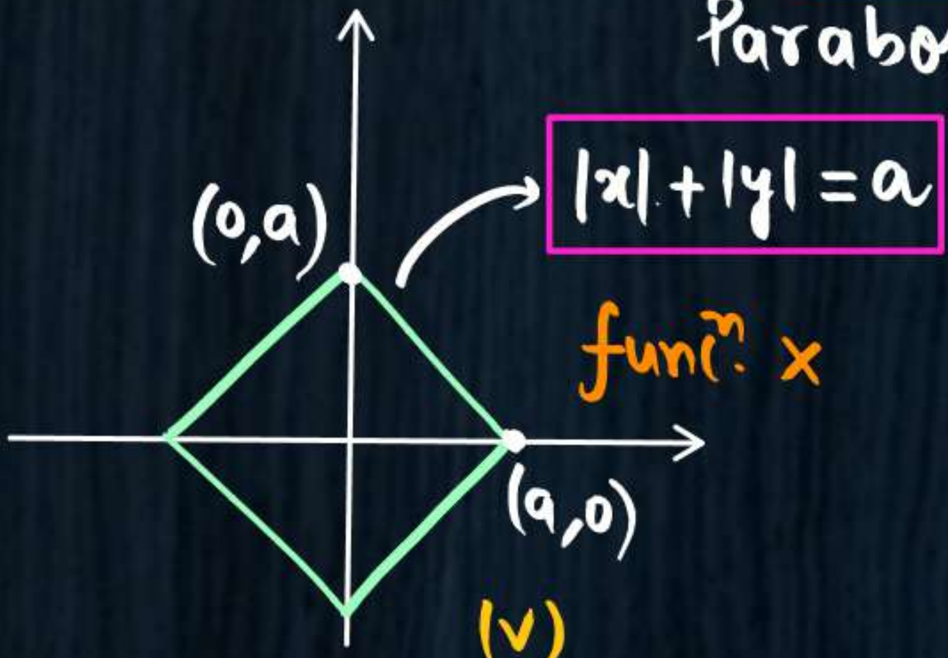


(iii) func^n ×



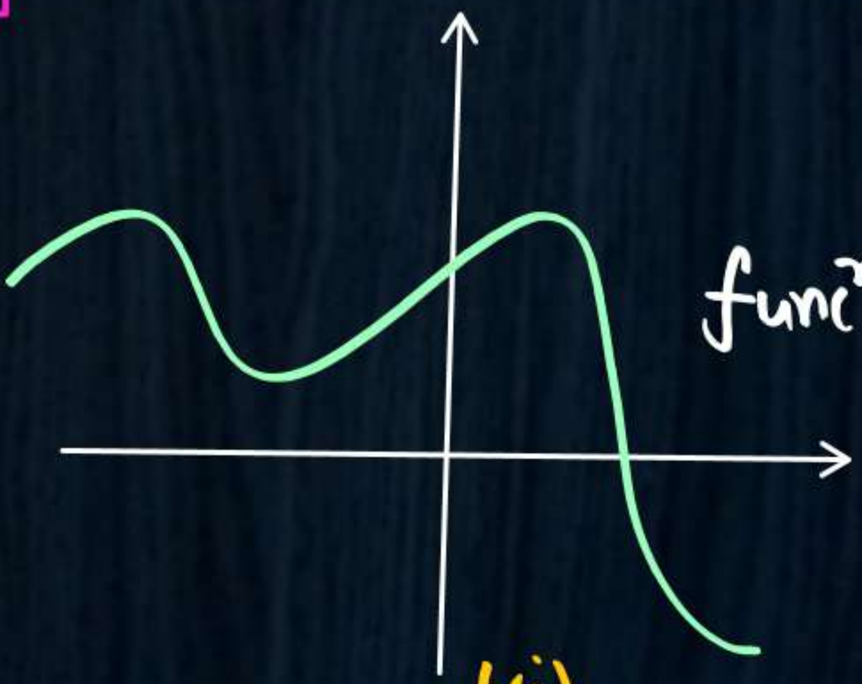
func^n ✓

(iv)



func^n ×

(v)



func^n ✓

(vi)

Ex: Which of the following is a function?

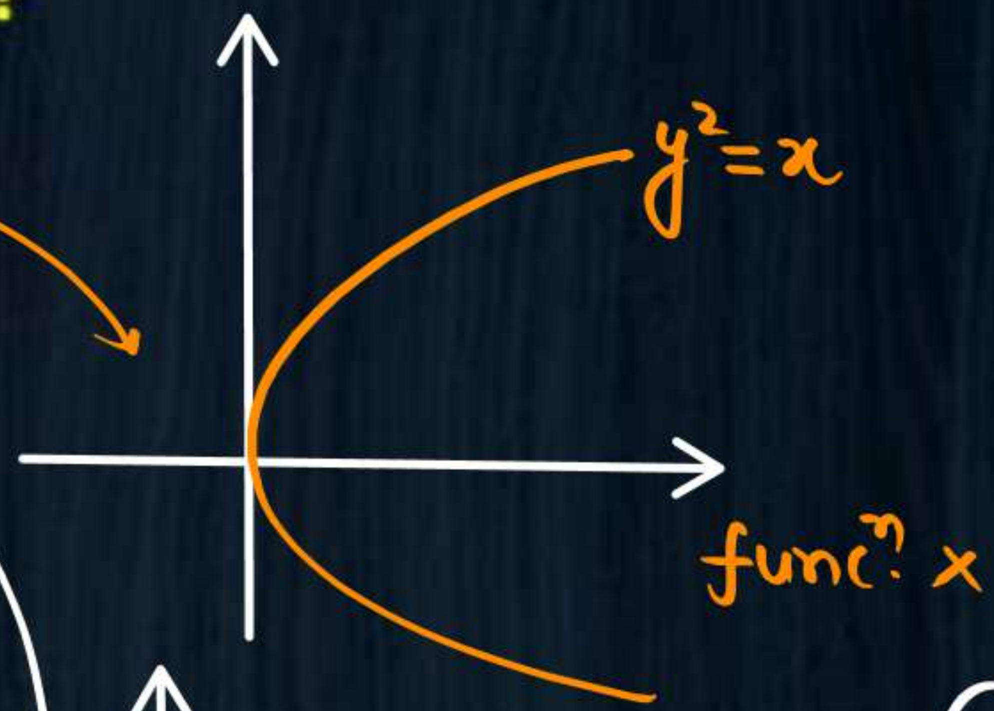
(a) $\{(x, y) : \boxed{y^2 = x}, x, y \in R\}$

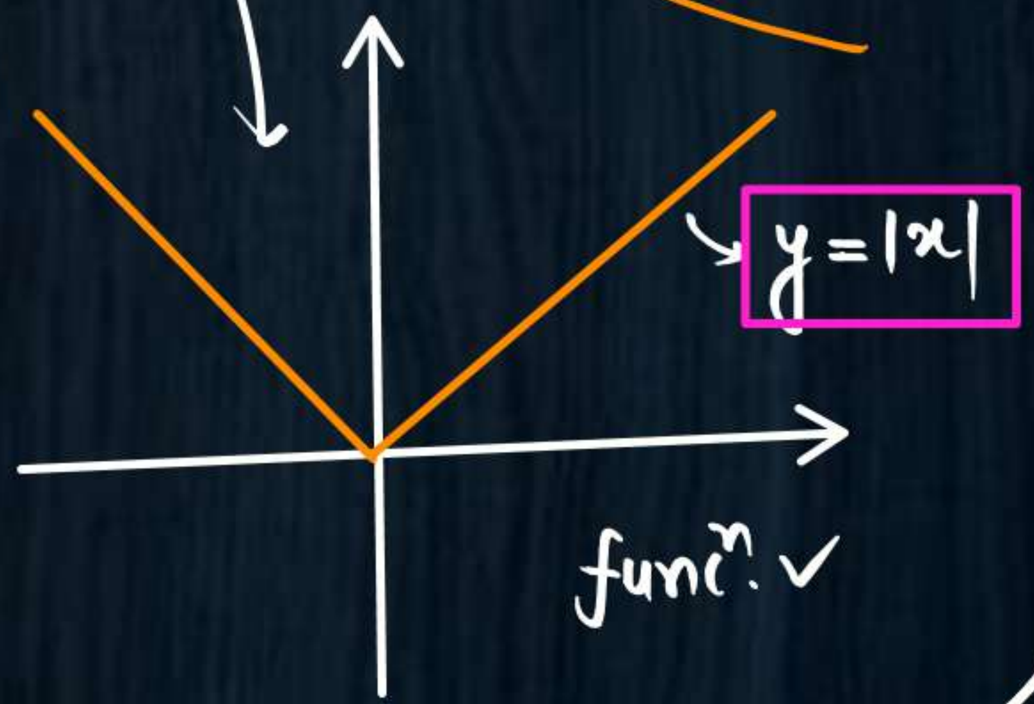
(b) $\{(x, y) : y = |x|, x, y \in R\}$

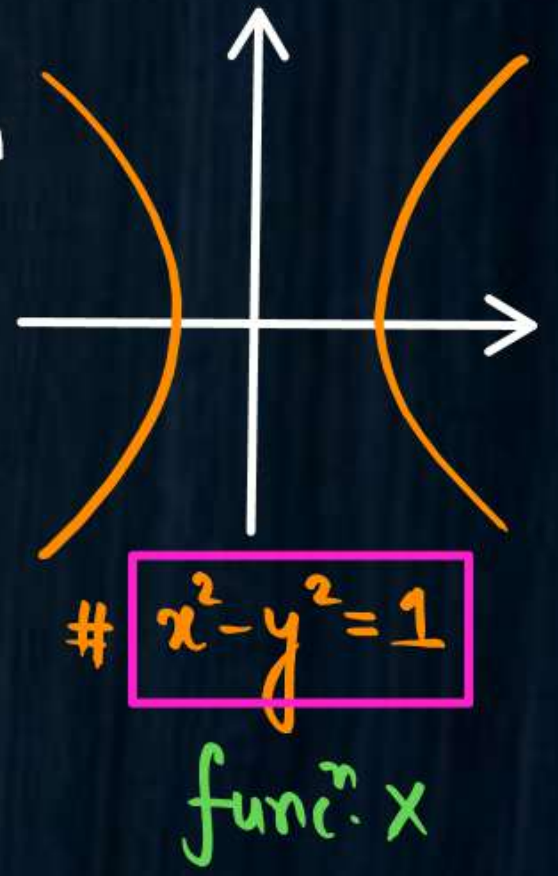
(c) $\{(x, y) : x^2 + y^2 = 1, x, y \in R\}$

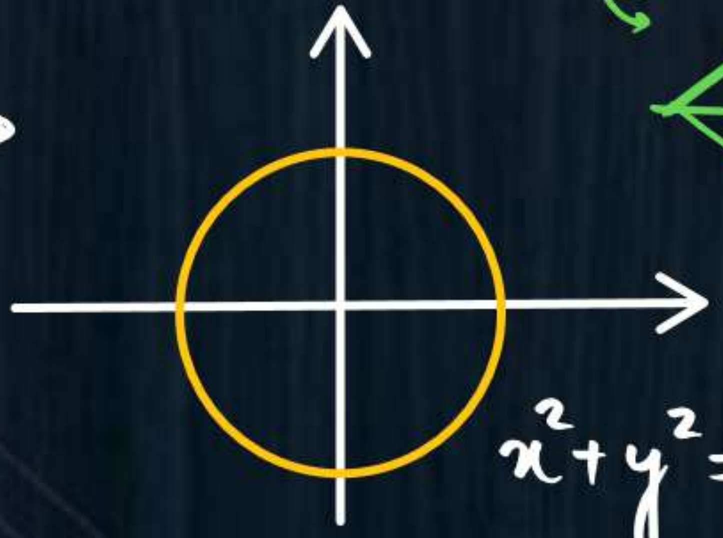
(d) $\{(x, y) : x^2 - y^2 = 1, x, y \in R\}$

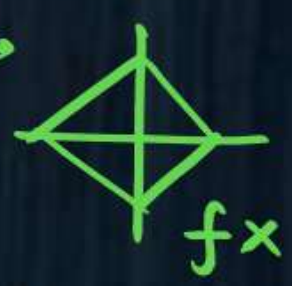
(e) $\{(x, y) \in R^2, |x| + |y| = 1\}$











DOMAIN, RANGE & CO-DOMAIN

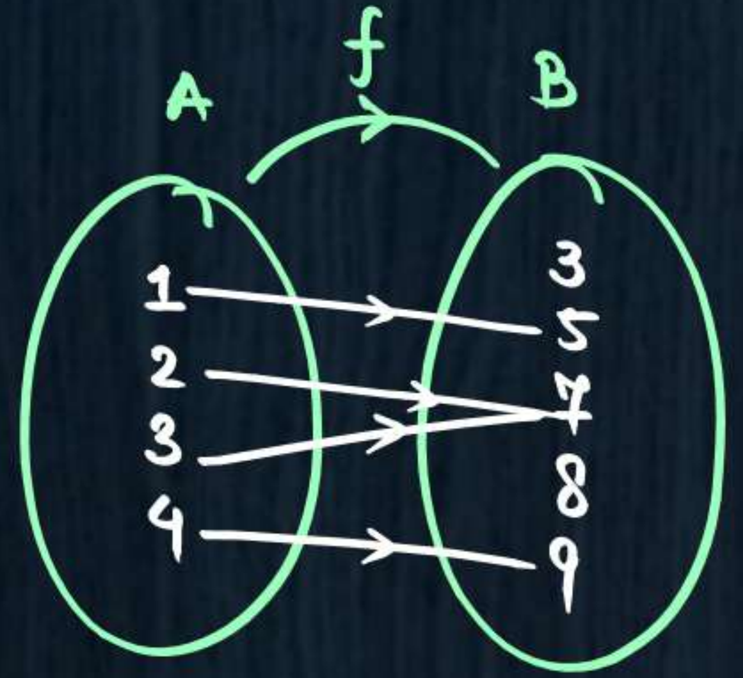
\# If function 'f' is defined from set A to set B ($f: A \to B$) then:

(i) Domain of f = complete set A.

(ii) Range of f = elements of set B related to elements of set A.

(iii) Co-domain of f = complete set-B

Ex:



Domain = A = {1, 2, 3, 4}

Range = {5, 7, 9}

Co-domain = B

$f(x) =$

Ex: $f : [-1, 3] \to [5, 11)$

then :

number of integers in domain = 5

maximum no. of integers in Range = 6

Domain $\equiv$ $\boxed{x \in [-1, 3]}$

Ex: 'f' defined on Real no. set $\to$

$f : R \to R$

or

$f : (-\infty, \infty) \to (-\infty, \infty)$

Note :

Domain $\equiv$ Set of pre-image | values of x | input | X-axis covered by graph

Range $\equiv$ Set of functional image | values of y | output | Y-axis covered by graph

FIRST TEST

01 MAY 2022 (SUNDAY)

SYLLABUS:

(All about Relations)

OR

(All till Relations)

keep doing hard work & keep smiling

THANK YOU

to all FUTURE IITIANS