Week 3 – Day 1

Static wicks on airplane wings

Electric Potential

Concept 1: Work Done by Electrostatic Force

Concept 2: Electrostatic Potential Energy and Configuration Energy

Concept 3: Electric Potential of Electrostatic Field

Lighting rod (Day 2)

Reading:

1. University Physics with Modern Physics – Chapter 23
2. Introduction to Electricity and Magnetism – Chapter 4

Application: Static Wicks/ Static Dischargers/ Static Dissipaters

- Ever notice the mysterious stick on an airplane wing?
- Friction from the aircraft structure and the air/clouds causes an accumulation of static charges (electrons) in its extremities. Static Dischargers are installed to dissipate the charges gradually from the plane.
- Electric charges has the tendency to accumulate at sharp edges and peaky objects. **(WHY? To be found out this week.)**
- Radio and navigation antennas on an airplane are edgy and peaky, meaning that the electrons would accumulate there and create nasty static noise to aircraft communication and jeopardize the navigation.
- Note: Static dischargers are not lightning arrestors and do not affect the likelihood of an aircraft being struck by lightning.

More info: https://youtu.be/N8iQz8Tk3hY

Concept 1: Work Done by Electrostatic Force

Let’s consider a point charge moving along an arbitrary path in an electrostatic field.

What would be the work done by the electrostatic force on the point charge?

Electrical Work

- Electrical force on point charge q_t due to interaction with point charge q_s is

$$\vec{\mathrm{F}}_{st} = k_e \frac{q_s q_t}{{r_{st}}^2} \hat{\mathrm{r}}_{st}$$

- Work done by electrical force moving q_t from point A to B is

Same as in Physical World!

$$W_e = \int_{A_{initial}}^{B^{final}} \vec{F}_{st} \cdot d\vec{s} = \int_A^B k_e \frac{q_s q_t}{{r_{st}}^2} \hat{\mathrm{r}}_{st} \cdot d\vec{s}$$

PATH INTEGRAL

- Note: If $\vec{F}$ is in the same direction with the displacement $d\vec{s}$, W is positive and vice versa.
- The direction of displacement $d\vec{s}$ is determined by the limits of the integral.

Concept Question 1.1: Sign of W

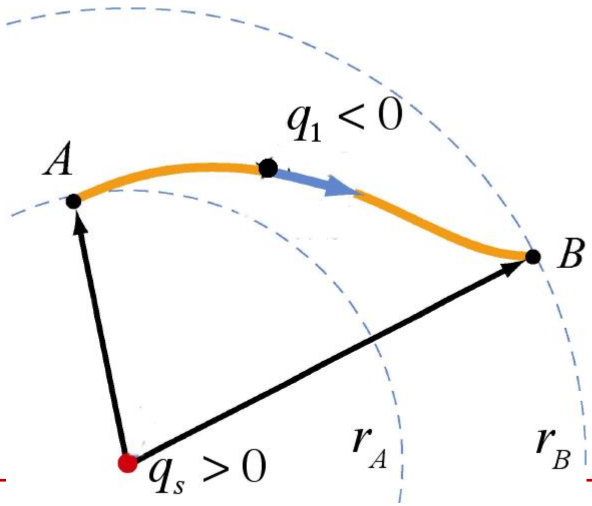
Suppose a fixed positively charged object (charge $q_s > 0$) is at the origin and we move a negatively charged object (charge $q_1 < 0$) from A to B with $r_A < r_B$, where r is the distance from the origin.

A. Work done by the electrostatic force is positive and we do a positive amount of work

B. Work done by the electrostatic force is positive and we do a negative amount of work

C. Work done by the electrostatic force is negative and we do a positive amount of work

D. Work done by the electrostatic force is negative and we do a negative amount of work



Multiple Choice

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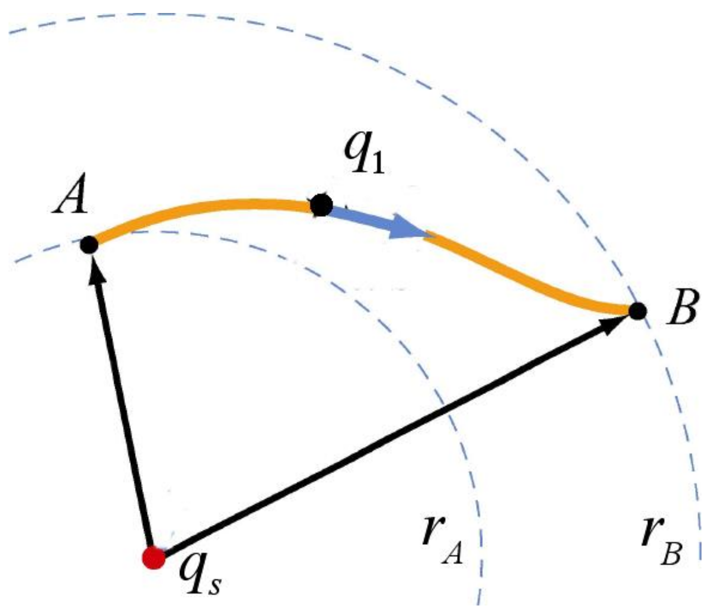
Concept Question 1.1: Solution

Answer C: W is negative, and we do a positive amount of work

W is the work done by the electrical force. This is the opposite of the work that we must do in order to move a charged object in an electric field due to source. The electrical force is attractive, and we are moving the negatively charged object away from the source (opposite the direction of the electric field).

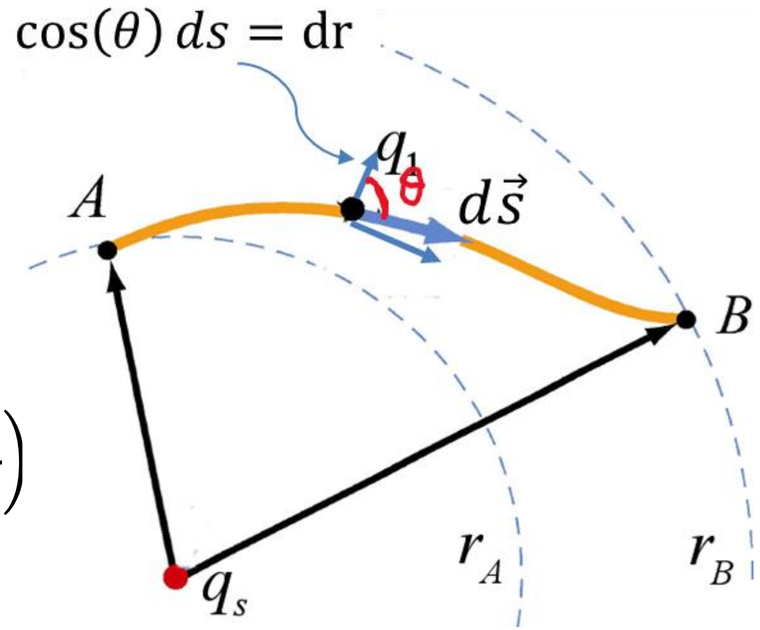
Case Problem 1.1: Work Done by Electrical Force

- A point-like charged source object (charge q_s) is held fixed. A second point-like charged object (charge q_1)is initially at a distance r_A from the fixed source and moves to a final distance r_B from the fixed source. What is the work done by the electrical force on the moving object?
- Hint: What coordinate system is best suited for this problem?



Case Problem 1.1: Solution

- For point charge of q_s, $\vec{\mathrm{E}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_s}{r^2}\right)\hat{r}$
- $dW = \vec{F}\cdot d\vec{s} = q_1\vec{E}\cdot d\vec{s} = q_1 E\cos\theta\, ds$
- $\cos\theta\, ds = dr$
- $dW = q_1 E dr$
- $W = \int_{r_A}^{r_B} q_1 E dr = \frac{q_s q_1}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{q_s q_1}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$



- Or alternatively,
- In general, $d\vec{s}$ can be decomposed into polar coordinate as $d\vec{s} = dr\,\hat{r} + rd\theta\,\hat{\theta}$
- $dW = \vec{F}\cdot d\vec{s} = q_1\vec{E}\cdot d\vec{s} = \mathrm{q}_1\left\{\frac{1}{4\pi\varepsilon_0}\left(\frac{q_s}{r^2}\right)\hat{r}\right\}\cdot\left\{dr\,\hat{r} + rd\theta\,\hat{\theta}\right\} = \frac{q_1 q_s}{4\pi\varepsilon_0}\left(\frac{1}{r^2}\right)dr$

Sign of W: Negative Work

- $W = \frac{q_s q_1}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$ is the work done by electrostatic field of q_s to move q_1 from point A (starting point) to point B (final point).
- Suppose a fixed positively charged source (charge $q_s > 0$) is at the origin and a positively charged object (charge $q_1 > 0$) moves from A to B with $r_A > r_B$, where r is the distance from the origin, then $W < 0$.

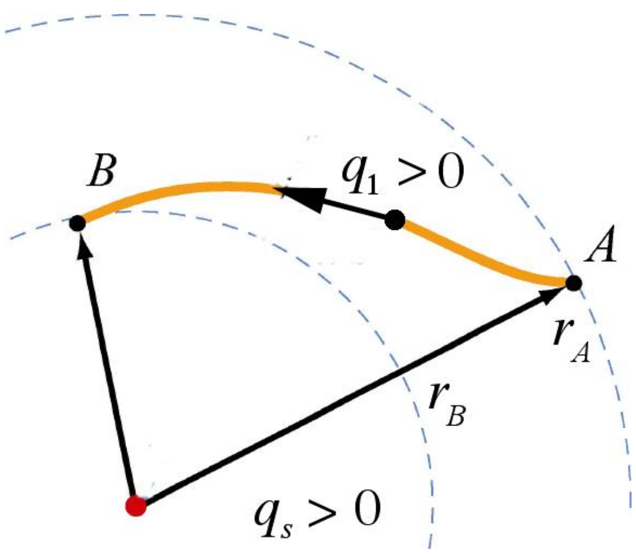
$$r_A > r_B \Rightarrow \frac{1}{r_B} - \frac{1}{r_A} > 0 \text{ and } q_s q_1 > 0 \Rightarrow W = -k_e q_s q_1 \left(\frac{1}{r_B} - \frac{1}{r_A}\right) < 0$$

Note:

Intuitively, both like charges repel.

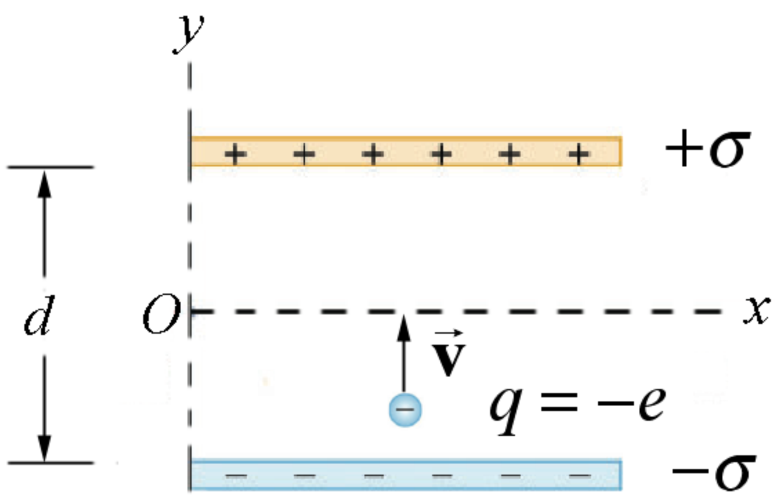
If they move apart due to repulsion, electrostatic force/field does positive work.

If they move closer due to external force, electrostatic force/field does negative work.



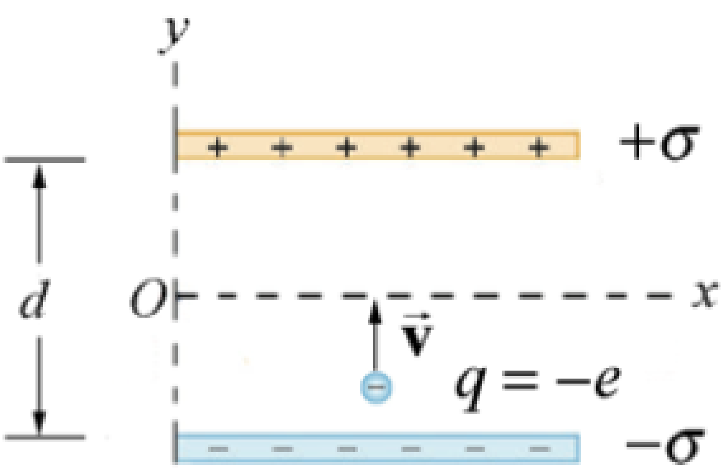
Case Problem 1.2: Work-Kinetic Energy in a Uniform Electric field

- Consider two thin oppositely uniform charged thin plates separated by a distance d. The surface charge densities on the plates are uniform and equal in magnitude. An electron with charge $-e$ and mass m is released from rest at the negative plate and moves to the positive plate. What is the speed of the electron when it reaches the positive plate?



Case Problem 1.2: Solution

- According to the work-energy theorem, the change in kinetic energy during any displacement is equal to the work done on the system.
- Recall electric field between 2 parallel plates with opposite charge: $E = \frac{\sigma}{\varepsilon_0}$



$\Delta K = W$

We have

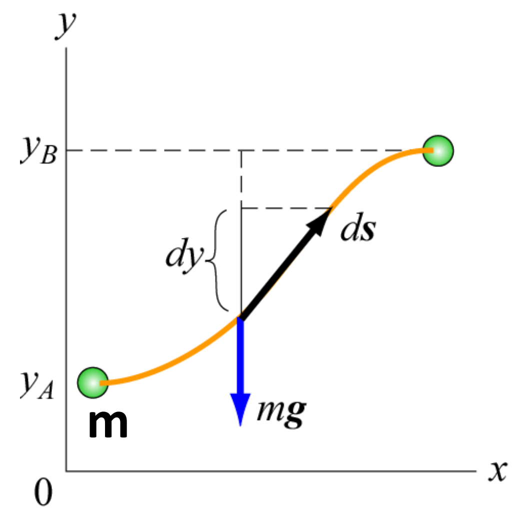
$$\vec{F} = -e\vec{E} = \frac{e\sigma}{\varepsilon_0}\hat{\jmath}$$

$$W = \int_0^d \vec{F} \cdot d\vec{s} = \int_0^d \frac{e\sigma}{\varepsilon_0} dy = \frac{e\sigma d}{\varepsilon_0}$$

$$\frac{1}{2}mv^2 = \frac{e\sigma d}{\varepsilon_0} \Rightarrow v = \sqrt{\frac{2e\sigma d}{m\varepsilon_0}}$$

Concept 2: Electrostatic Potential Energy and Configuration Energy

Recall: Work by Gravity Near Earth's Surface



- Work done by gravity moving m from *A* to *B*:
- $W_g = \int_A^B \vec{F}_g \cdot d\vec{s}$ where $\vec{F}_g = -mg\hat{\jmath}$ and $d\vec{s} = dx\hat{\imath} + dy\hat{\jmath}$
- $= \int_{y_A}^{y_B} mg\, dy = -mg(y_B - y_A)$

- **W_g depends only on endpoints (initial and final points) but not on path taken – Conservative Force**
- Examples of conservative force: Gravitational force, Spring force and Electrostatic force.
- Examples of non-conservative force: Friction and normal force.

The Idea: Gravity vs Electrostatics

Mass M

$$\vec{g} = -G\frac{M}{r^2}\hat{r}$$

$$\vec{F}_g = m\vec{g}$$

Charge q ($\pm$)

$$\vec{E} = k_e\frac{q_s}{r^2}\hat{r}$$

$$\vec{F}_E = q\vec{E}$$

Both forces are conservative, so…

$$\Delta U_g = -\int_A^B \vec{F}_g \cdot d\vec{s}$$

$$\Delta V_g = -\int_A^B \vec{g} \cdot d\vec{s}$$

$$\Delta U = -\int_A^B \vec{F}_E \cdot d\vec{s}$$

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

Work done, Kinetic and Potential Energy

- The work done by the electric field is

$$W = q\int_A^B \vec{\mathrm{E}} \cdot d\vec{\mathrm{s}}$$

- As this work is done by the field, the change of kinetic energy must be equal to W.

$$W = \Delta K$$

- From energy conservation $\Delta U + \Delta K = 0$, thus we have the change of potential energy equal to

$$\Delta U = -\Delta K = -W$$

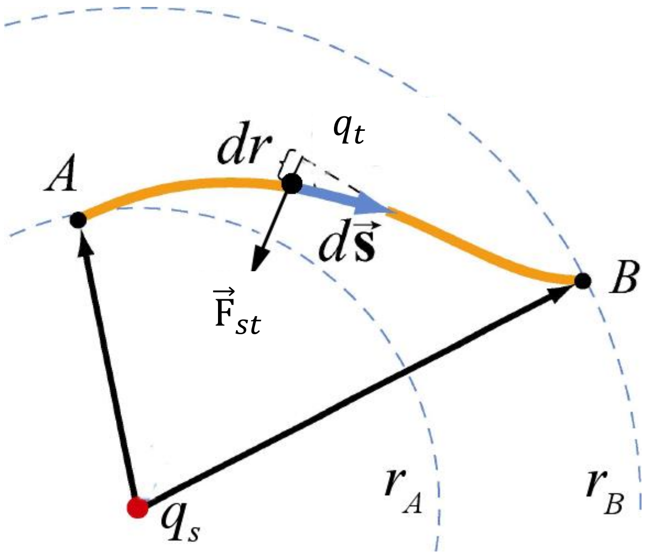
$$\Delta U = -q\int_A^B \vec{\mathrm{E}} \cdot d\vec{\mathrm{s}}$$

Potential Energy Difference, ΔU

- Suppose point charge q_s is fixed and located at the origin and point charge q_t moves from an initial position A at distance r_A from the origin to a final position B at distance r_B from the origin.
- The potential energy difference due to the interaction is defined to be the negative of the work done on q_t in moving from A to B:

$$\Delta U \equiv U_B - U_A = -W = -\int_A^B \vec{\mathrm{F}}_{st} \cdot d\vec{\mathrm{s}} = k_e q_s q_t \left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

- It is the potential energy difference of charge q_t between point B and A.
- Or it is the potential energy of charge q_t at point B wrt to A.



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Potential Energy: Reference Point

- Choose the reference point for the potential energy at infinity, i.e.

$$U(\infty) \equiv 0$$

- Then set $r_A = \infty$ and $r_B = r$.

- The potential energy difference of point charge q_t between ∞ and any point on a circle of radius r from point charge q_s is

$$U(r) - U(\infty) = U(r) = \frac{k_e q_s q_t}{r}$$

Concept Question 2.1: Motion of Charged Objects

Two oppositely charged are released from rest in an electric field.

A. Both charged objects will move from lower to higher potential energy.
B. Both charged objects will move from higher to lower potential energy.
C. The positively charged object will move from higher to lower potential energy; the negatively charged object will move from lower to higher potential energy.
D. The positively charged object will move from lower to higher potential energy; the negatively charged object will move from higher to lower potential energy.

Multiple Choice

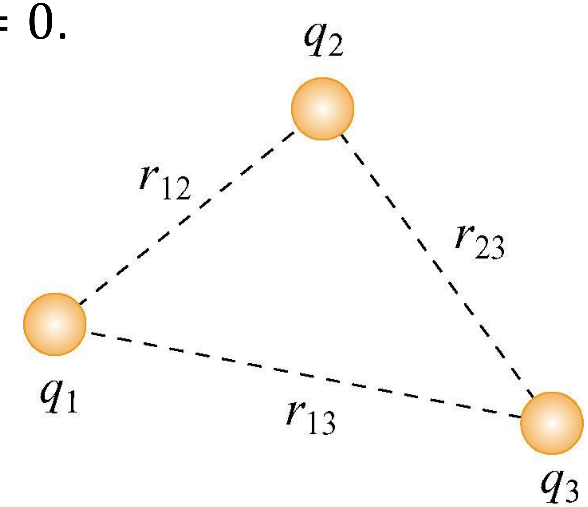
Concept Question 2.1: Solution

- Answer: B. Both charged objects will move from higher to lower potential energy so that

$$\Delta U < 0$$

Configuration Energy

- What is the potential energy stored in a configuration of charged objects? Start with all the charged objects at infinity. Choose $U(\infty) = 0$.



1. Bring in the first charged object.

$$\Delta U_1 = 0$$

2. Bring in the second charged object

$$\Delta U_2 = U_{12} = \frac{k_e q_1 q_2}{r_{12}}$$

3. Bring in the third charged object

$$\Delta U_3 = U_{23} + U_{13} = \frac{k_e q_2 q_3}{r_{23}} + \frac{k_e q_1 q_3}{r_{13}}$$

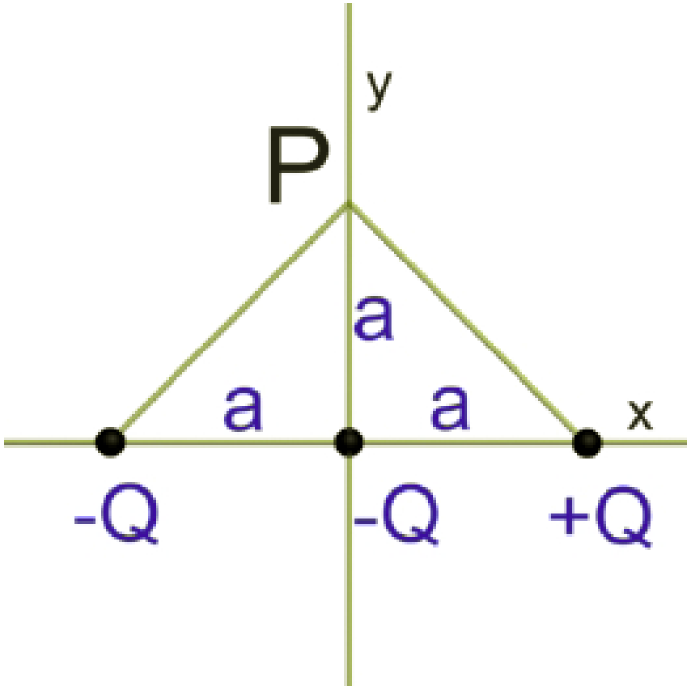
4. Configuration energy

$$\Delta U = U_{12} + U_{23} + U_{13} = \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_2 q_3}{r_{23}} + \frac{k_e q_1 q_3}{r_{13}}$$

Note: Configuration energy the potential energy stored in a configuration of charged object. It is also the energy needed to form such configuration.

Case Problem 2.1: Build Up Charge Config

- How much energy does it take you to assemble the charges into the configuration at left? Assuming they all started out an infinite distance apart.



Case Problem 2.1: Solution

- To build the system we bring the charges in from left to right, one at a time
- $U = \sum_i U_i$
- $U_1 = 0$ First one is free
- $U_2 = k_e \frac{q_1 q_2}{r} = k_e \frac{(-Q)(-Q)}{a} = k_e \frac{Q^2}{a}$
- $U_3 = k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23}} = k_e \frac{(-Q)(Q)}{2a} + k_e \frac{(-Q)(Q)}{a} = -\frac{3}{2} k_e \frac{Q^2}{a}$
- Sum to find the configuration energy
- $\sum U_i = 0 + k_e \frac{Q^2}{a} + \left(-\frac{3}{2} k_e \frac{Q^2}{a}\right) = -\frac{k_e}{2} \frac{Q^2}{a}$

Concept 3: Electric Potential of Electrostatic Field

Electric Potential Difference, ΔV

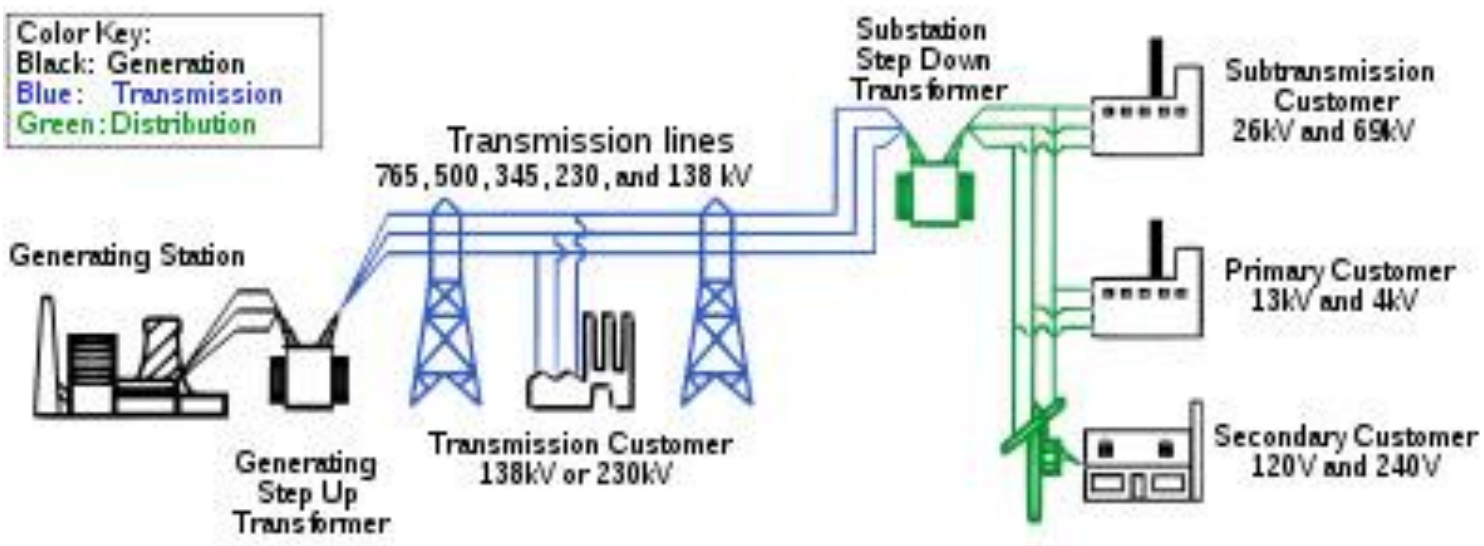
- Change in potential energy per unit test charge (small point charge q_t) moving from point A to B:

$$\Delta V \equiv \frac{\Delta U}{q_t} = -\int_A^B \frac{\vec{F}}{q_t} \cdot d\vec{s} = -\int_A^B \vec{E} \cdot d\vec{s}$$

- Units: Joules/Coulomb = Volts
- Note:
- The negative sign is important. It exists from the definition, $\Delta U = -W_e$.
- Potential difference is a scalar quantity. There is no direction associated with it.
- It tells you the potential energy difference of a charge q_t between 2 points.
- In Electrical Engineering, we call potential difference as voltage.

How Big is a Volt?

AA Batteries	1.5 V	High Voltage Transmission Lines	100 kV-700 kV
Car Batteries	12 V	Van der Graaf	300 kV
US Outlet (AC) Singapore (AC)	120 V 230 V	Tesla Coil	500 kV
Distribution Power Lines	120 V- 70 kV	Lightning	10-1000 MV



FYI: Tesla Coil

- A Tesla coil is an electrical resonant transformer circuit designed by inventor Nikola Tesla in 1891. It is used to produce high-voltage, low-current, high frequency alternating-current electricity.

http://www.youtube.com/watch?v=FY-AS13fl30

E Field and Potential: Effects

- If a charged particle q is placed in an electric field $\vec{E}$, there will be a force acting on the particle given by $\vec{F} = q\vec{E}$
- If q is moved over a displacement of electric potential ΔV, then the potential energy change is $\Delta U = q\Delta V$.
- If there is no external force acting on the particle over the displacement, kinetic energy change $\Delta K = -\Delta U$.
- If the kinetic energy is to remain constant over the displacement, an external work done by an external force must be applied, i.e.

$$W_{ext} = -\Delta K = \Delta U = q\Delta V$$

Concept Question 3.1: Motion of Charged Objects

Two oppositely charged are released from rest in an electric field.

A. Both charged objects will move from lower to higher electric potential.

B. Both charged objects will move from higher to lower electric potential.

C. The positively charged object will move from higher to lower electric potential; the negatively charged object will move from lower to higher electric potential.

D. The positively charged object will move from lower to higher electric potential; the negatively charged object will move from higher to lower electric potential.

Multiple Choice

Concept Question 3.1: Solution

Answer:

C. The positively charged object will move from higher to lower electric potential; the negatively charged object will move from lower to higher electric potential.

For the positively charged object:

$$\Delta V < 0 \Rightarrow \Delta U = q\Delta V < 0$$

For the negatively charged object:

$$\Delta V > 0 \Rightarrow \Delta U = q\Delta V < 0$$

Potential Created by Point Charge

- $\vec{E} = kQ\frac{\hat{r}}{r^2}$; $d\vec{s} = dr\,\hat{r} + rd\phi\,\widehat{\phi}$

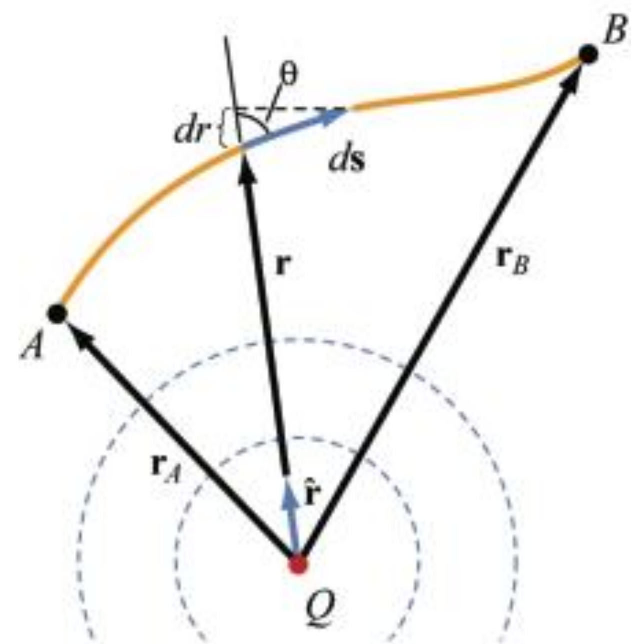
$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$= -\int_A^B kQ\frac{\hat{r}}{r^2} \cdot (dr\,\hat{r} + rd\theta\,\hat{\theta}) = -kQ\int_A^B \frac{dr}{r^2}$$

$$= kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

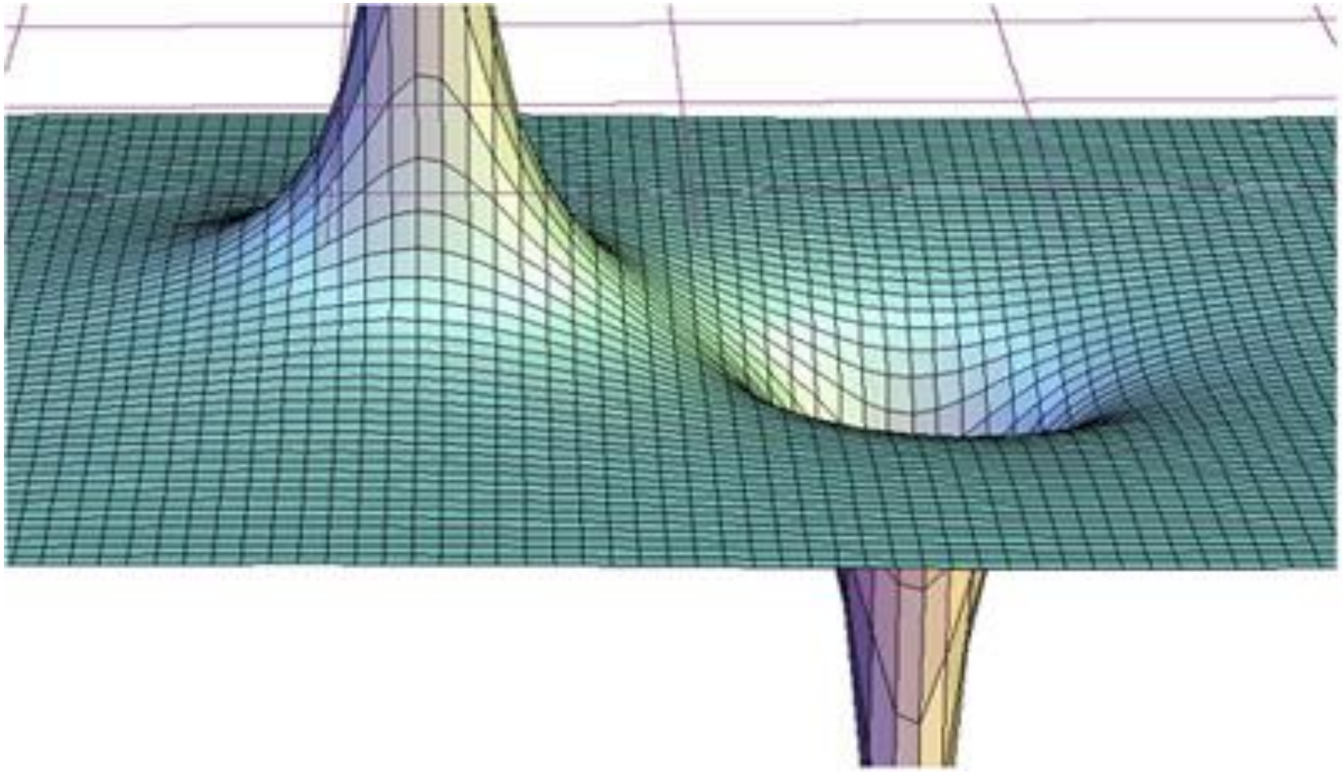
- Take reference at $\mathrm{r_A} = \infty$:

$$V_\mathrm{Q}(r) = \frac{kQ}{r}$$



Potential Landscape

Positive Charge



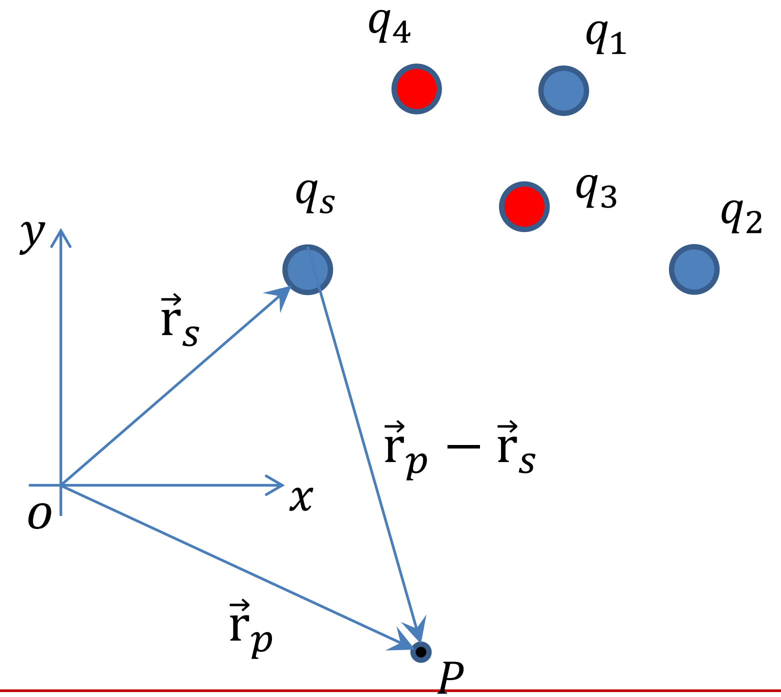
Negative Charge

Potential by Multiple Point-Charge

- The net electric field at point P is the superposition of all the electric fields contributed by individual point charges.
- Thus, electric potential at *P* due to N discrete point charges is

$$V(\vec{r}) - V(\infty) = k_e \sum_{i=1}^{N} \frac{q_i}{|\vec{r} - \vec{r}_i|}$$

- where $V(\infty) \equiv 0$



Concept Question 3.2: Two Point Charges

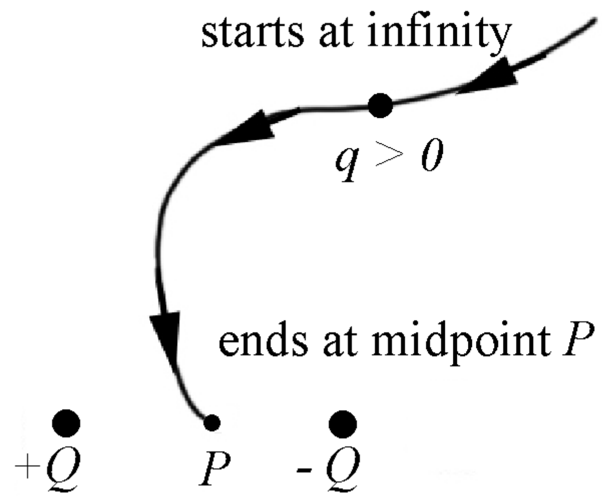
The work done in moving a positively charged object that starts from rest at infinity and ends at rest at the point P midway between two charges of magnitude $+Q$ and $-Q$

A. is positive.

B. is negative.

C. is zero.

D. can not be determined – not enough info is given.



Multiple Choice

Concept Question 3.2: Solution

Answer : C. Work from ∞ to P is zero.

The potential at ∞ is zero.

The potential at P is zero because equal and opposite potentials are superimposed from the two point charges (remember: V is a scalar, not a vector)