Week 8 - Day 2

Inductor as vehicle sensors at traffic lights

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Electromagnetic Induction

Concept 1: Self & Mutual Induction

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Transformer, Power Supply/ AC adaptor

Reading:

University Physics with Modern Physics – Chapter 29

Introduction to Electricity and Magnetism – Chapter 10

Application: Inductor as a vehicle sensor at traffic lights

- The pattern in the pavement conceals a large loop of wire with multiple turns, which forms an inductor.
- The metallic frame of a vehicle increases the inductance of the coil.
- There are many possible circuit designs that can be used to convert the change in inductance to a signal that controls the traffic light.
- Example: Using LC oscillating circuit (to be introduced in Circuit Analysis), whose frequency depends upon the L value. The change in frequency can be used to signal other circuitry for processing and actions.
- These inductive detectors are robust, easy to implement and cost effective. It has become very common for managing the flow of traffic.

Similar principle is applied to a metal detector.

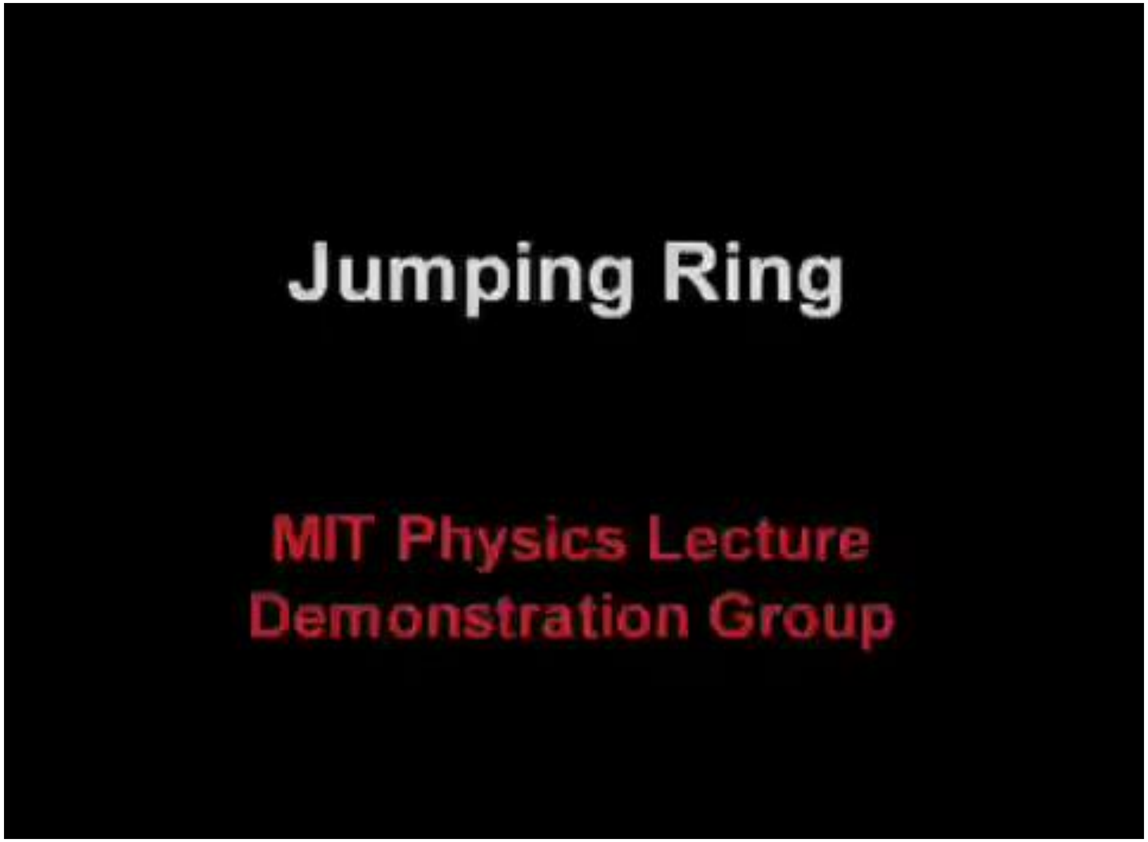
Concept 1: Self and Mutual Induction

WHAT IS INDUCTANCE, L?

Inductance is the property of an electrical conductor by which a change in current flowing through it induces an electromotive force in both the conductor itself (self-inductance) and in any nearby conductors (mutual inductance).

Video Demo : Jumping Ring

- An aluminum ring jumps into the air when the solenoid beneath it is energized.

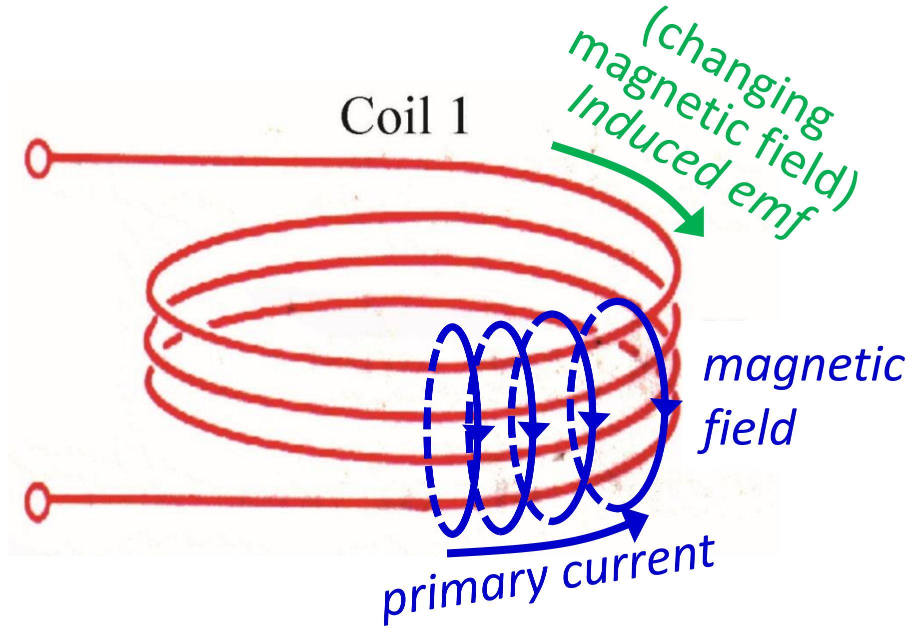


http://techtv.mit.edu/collections/physicsdemos/videos/514-physics-demo-jumping-ring

Video Demo : Jumping Ring

- *So What is Going On?*
- This is a dramatic example of Faraday's Law and Lenz's Law: When current is turned on through the solenoid the created magnetic field tries to permeate the conducting aluminum ring, currents are induced in the ring to try to keep this from happening, and the ring is repelled upwards.

Self Inductance



- What is the effect of putting current into coil 1?
- There is "self flux": $\Phi_B \equiv LI$ (proportionality constant)
- *L* is the self-inductance (depends on geometry and permeability)
- Faraday's Law

$$\rightarrow \varepsilon_{induced} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

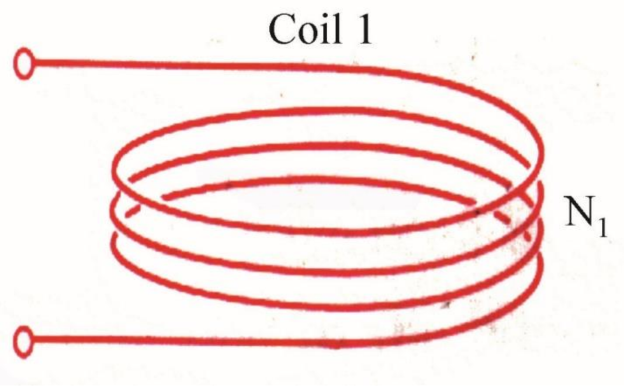
- The induced EMF (thus induced current in the coil) will oppose any change in the primary current $\left(\frac{dI}{dt}\right)$! (has "inertia", so to speak)

Calculating Self Inductance

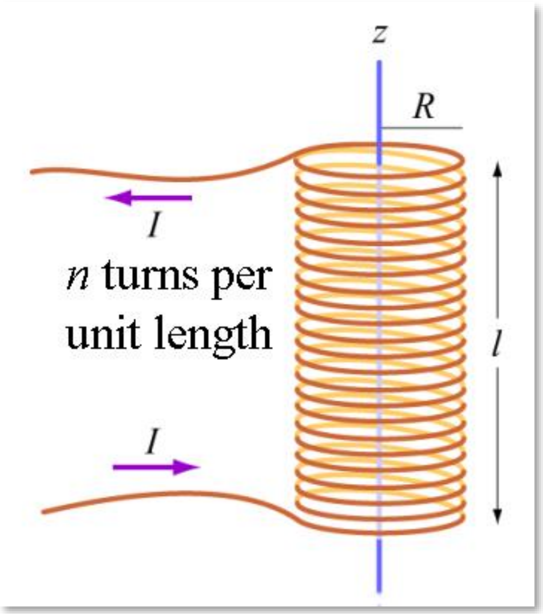
$$L = \frac{\Phi_{B,self}^{total}}{I}$$

- Unit: Henry $1H = 1\frac{V \cdot s}{A}$

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the magnetic flux due to the B field
4. Calculate the self inductance (divide out I)



Worked Example: Solenoid Inductance



Calculate the self-inductance L of a solenoid (n turns per meter, length ℓ, radius R)

From the definition:

$$L = \frac{\Phi_{B,self}^{total}}{\mathrm{I}}$$

We first should find out the total magnetic flux that passes through the solenoid.

Recall:

$$\Phi_B = \iint \vec{B} \cdot d\vec{A}$$

In turn, we need to calculate the magnetic field $\vec{B}$ produced by the solenoid when current I passes through it.

Worked Example: Solenoid Inductance

- As we have done in Week 6, by Ampere's law, we can find the magnetic field of an ideal solenoid.

$$\oint \vec{B} \cdot d\vec{s} = Bl = \mu_o I_{enc} = \mu_o (nl) I$$

$$\Rightarrow B = \mu_o n I$$

- Magnetic flux across each turn

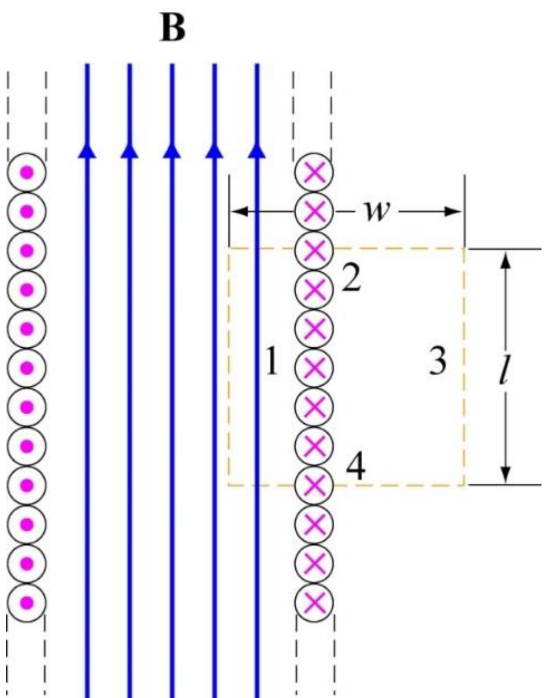
$$\Phi_{B/turn} = \iint \vec{B} \cdot d\vec{A} = BA = \mu_0 n I \pi R^2$$

- Total magnetic flux for N turns

$$\Phi_{B,self}^{total} = N\Phi_{B,turn} = N\mu_0 n I \pi R^2 = \mu_0 n^2 I \pi R^2 l$$

- Inductance over length *l* is

$$L = \frac{\Phi_{B,self}^{total}}{\mathrm{I}} = \mu_0 n^2 \pi R^2 l$$



Number of turns per unit length = *n*

Concept Question 1.1

A very long solenoid consisting of N turns has radius R and length d, (d>>R). Suppose the number of turns is halved keeping all the other parameters fixed. The self inductance

A. remains the same.

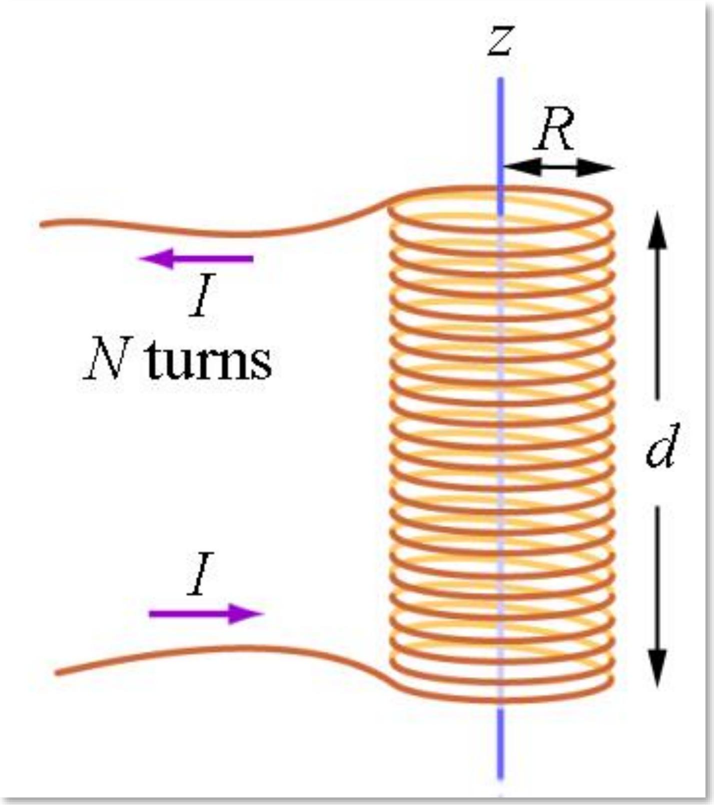
B. doubles.

C. is halved.

D. is four times as large.

E. is four times as small.

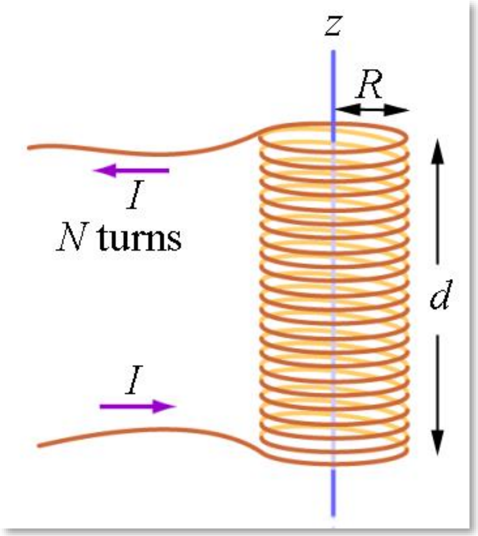
F. None of the above.



Multiple Choice

Concept Question 1.1 (Solution)

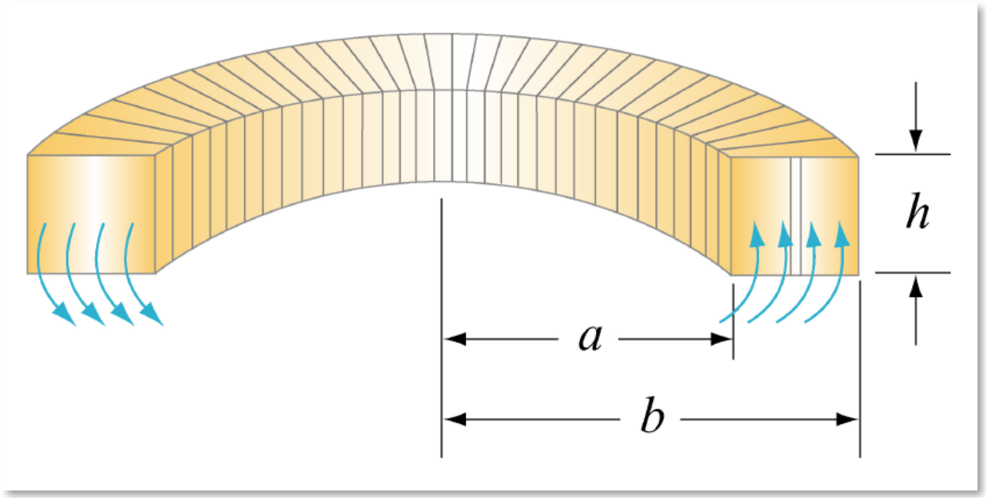
Answer : E. The self-induction of the solenoid is equal to the total flux through the object which is the product of the number of turns time the flux through each turn. The flux through each turn is proportional to the magnitude of magnetic field which is proportional to the number of turns per unit length or hence proportional to the number of turns. Hence the self-induction of the solenoid is proportional to the square of the number of turns. If the number of turns is halved keeping all the other parameters fixed, then the self inductance is four times as small.



Case Problem 1.1 : Toroid

Calculate the self-inductance *L* of a toroid with a square cross section with inner radius *a,* outer radius *b* = *a*+*h,* (height *h*) and *N* square windings.

$$L = \frac{\Phi_{B,self}^{total}}{\mathrm{I}}$$



REMEMBER

1. Assume a current *I* is flowing in your device
2. Calculate the *B* field due to that *I*
3. Calculate the flux due to that *B* field
4. Calculate the self inductance (divide out *I*)

Case Problem 1.1 (Solution)

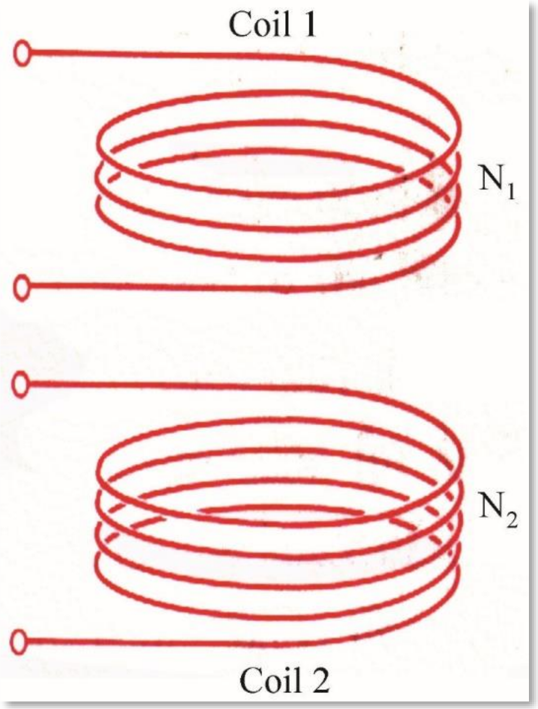
- The magnetic field $B(r)$ is tangential to the circular path and uniform in magnitude along the circular path for a given radius r. By Ampere's law,

$$\int_0^{2\pi} \frac{1}{\mu_0} B(r) r d\theta = NI$$
$$\Rightarrow 2\pi r \frac{B(r)}{\mu_0} = NI \Rightarrow B(r) = \frac{\mu_0 NI}{2\pi r}$$

$$\phi = N \int_a^b B(r) h dr$$
$$\Rightarrow \phi = \int_a^b \frac{\mu_0 N^2 I}{2\pi r} h dr = \frac{\mu_0 N^2 I h}{2\pi} ln \frac{b}{a}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 h}{2\pi} ln \frac{b}{a}$$

Mutual Inductance



- Current I_2 in coil 2, induces magnetic flux Φ_{12} in coil 1.
- "Mutual inductance" M_{12}:
- $\Phi_{12} \equiv M_{12} I_2$

$$\varepsilon_{12} \equiv -\frac{d\Phi_{12}}{dt} = -M_{12}\frac{dI_2}{dt}$$

- It tells that change current in coil 2 $\left(\frac{dI_2}{dt}\right)$ induces EMF in coil 1.
- It can be shown that $M_{12} = M_{21} = M$, but the proof is beyond our syllabus.
- It implies that if we passes through the same current change in coil 1, the same amount of EMF can be induced in coil 2.

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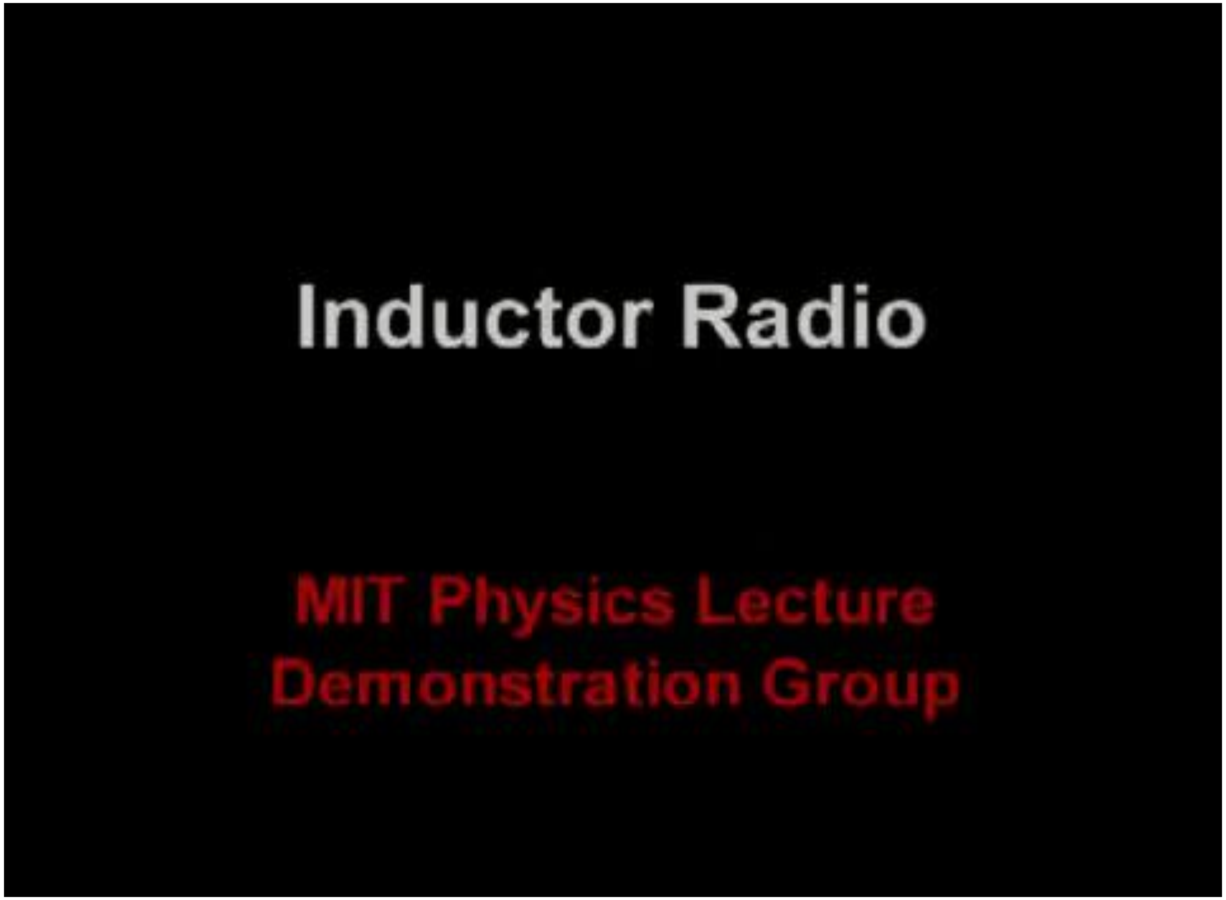
Demonstration: Two Small Coils and Radio



http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H 31&show=0

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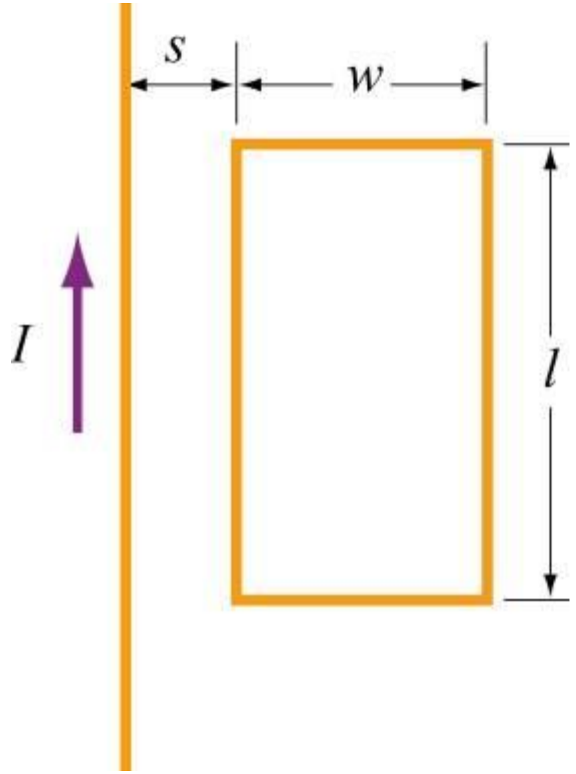
Case Problem 1.2 : Mutual Inductance

- An infinite straight wire carrying current *I* is placed to the left of a rectangular loop of wire with width w and length *l*. What is the mutual inductance of the system?



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Case Problem 1.2 (Solution)

- For any closed circular path of radius r centered around the straight wire, the magnetic field $B(r)$ is tangential to the path and uniform in magnitude along the path. By Ampere's law,

$$\int_0^{2\pi} \frac{1}{\mu_0} B(r) r d\theta = I$$

$$\Rightarrow 2\pi r \frac{B(r)}{\mu_0} = I \Rightarrow B(r) = \frac{\mu_0 I}{2\pi r}$$

- The magnetic flux linkage at the rectangular wire due to the straight wire is

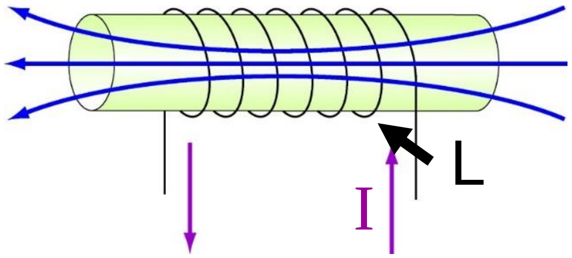
$$\phi = \int_s^{s+w} B(r) l dr$$

$$\Rightarrow \phi = \int_s^{s+w} \frac{\mu_0 I}{2\pi r} l dr = \frac{\mu_0 I l}{2\pi} ln \frac{s+w}{s}$$

- The mutual inductance is given by

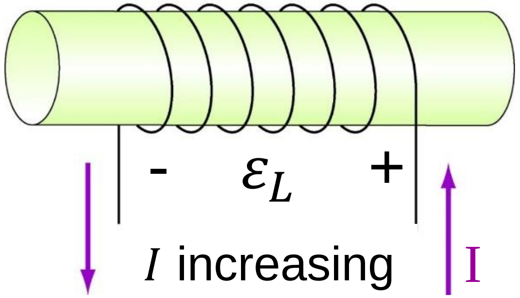
$$M = \frac{\phi}{I} = \frac{\mu_0 l}{2\pi} ln \frac{s+w}{s}$$

Inductor Behavior and Back EMF

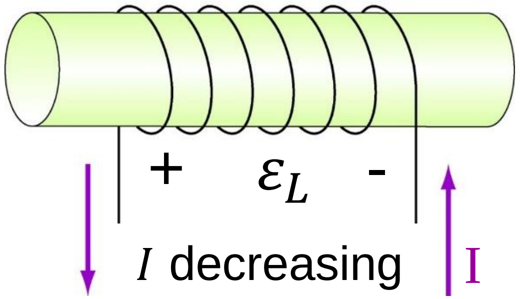


$$\varepsilon_{induced} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

- Inductor with constant current does nothing.

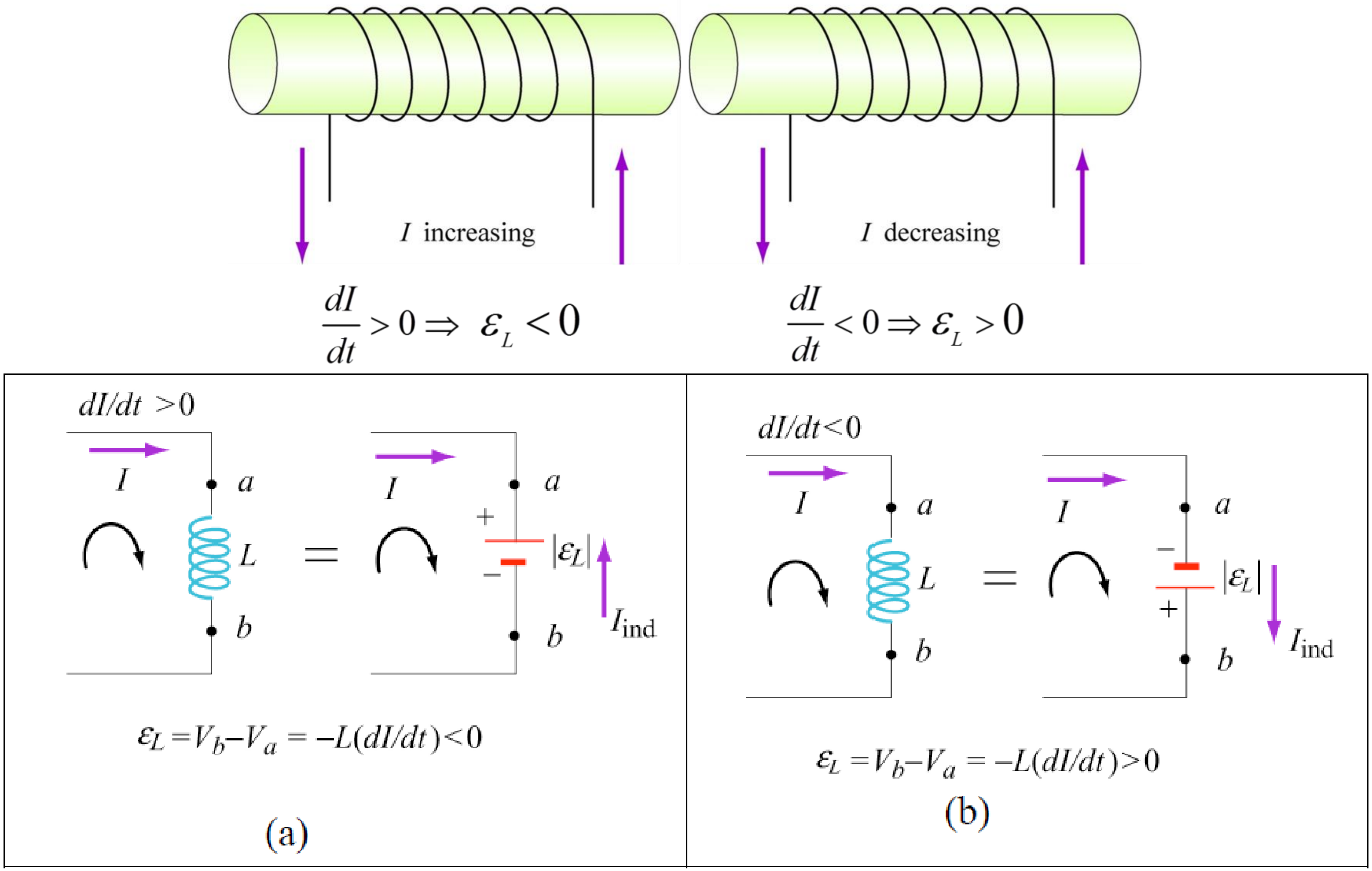


- If current, I increasing, $\frac{dI}{dt} > 0 \Rightarrow \ \varepsilon_L < 0$
- The induced emf, ε_L opposes the change of I, trying to oppose the increasing I.



- If current, I decreasing, $\frac{dI}{dt} < 0 \Rightarrow \ \varepsilon_L > 0$
- The induced emf, ε_L opposes the change of I, trying to sustain the decreasing I.

2 scenarios

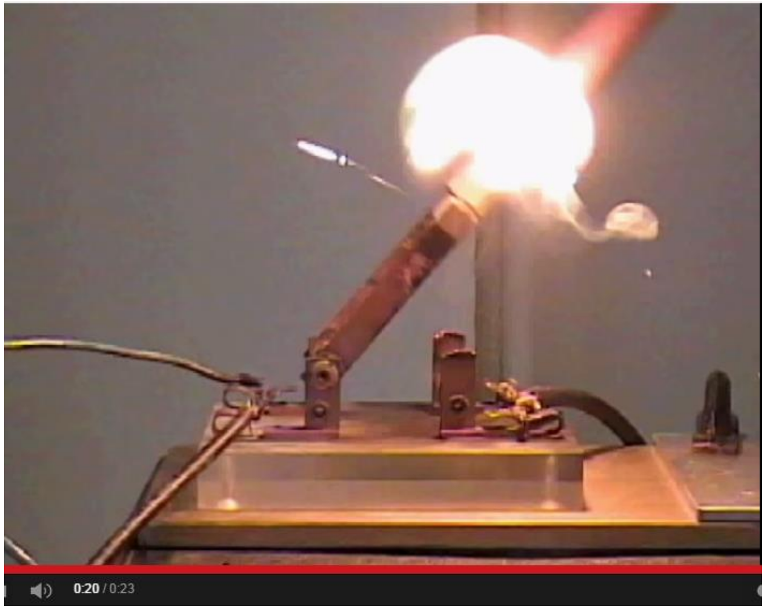


An inductor acts like a battery in the opposite direction of current to preserve I from increasing.

An inductor acts like a battery in the same direction of current to preserve I from decreasing.

Notice the spark when the solenoid is switched off

- The spark is due to the back emf (energy stored in the solenoid)
- http://youtu.be/aSmMFog10D0?list=PL860B6886A47E5490
- http://youtu.be/Pl7KyVIJ1iE



For big EMF to happen,

$$\varepsilon_L = -L\frac{dI}{dt}$$

3 conditions contribute to huge EMF:

1. Big L
2. Big dI
3. Small dt

Energy to "Charge" Inductor = Energy Stored in Inductor

1. Start with "uncharged" inductor

2. Gradually increase current. Must do work:

$$dW = Pdt = \varepsilon I\, dt = L\frac{dI}{dt}Idt = LI\, dI$$

3. Integrate up to find total work done:

$$W = \int dW = \int_{I=0}^{I} LI\, dI = \frac{1}{2}LI^2$$

Note: Work done to "charge" inductor transforms to energy stored by the inductor, U_L. Thus,

$$\boxed{U_L = \frac{1}{2}LI^2}$$

But, where is energy stored?

Worked Example : Solenoid

- Consider an ideal solenoid: length *l*, radius *R*, *n* turns/length and current *I*

$$B = \mu_o n I$$
$$L = \mu_o n^2 \pi R^2 l$$

$$U_B = \frac{1}{2} L I^2 = \frac{1}{2} (\mu_o n^2 \pi R^2 l) I^2$$

$$U_B = \underbrace{\left(\frac{B^2}{2\mu_o}\right)}_{\text{Energy Density}} \underbrace{\pi R^2 l}_{\text{Volume}}$$

This workout shows that indeed the energy in a solenoid/ inductor is stored in the form of magnetic field produced, $\vec{B}$.

It also implies that in order to create a field in space, energy is needed.

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Summary: Energy Density of $\vec{B}$ and Energy Density of $\vec{E}$

- Energy is stored in the magnetic field
- *Magnetic Energy Density:*

$$u_B = \frac{B^2}{2\mu_o}$$

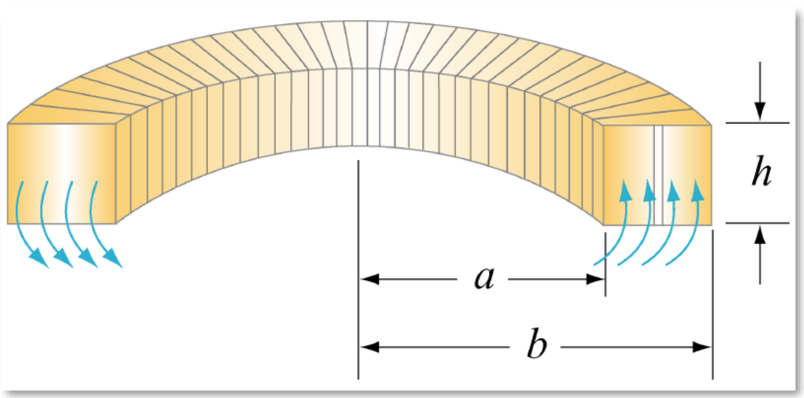
- Energy is stored in the electric field
- *Electric Energy Density:*

$$u_E = \frac{\varepsilon_o E^2}{2}$$

Worked Example : Energy Stored in Toroid

Consider a toroid with a square cross section with inner radius a, outer radius b = a+h, (height h) and N square windings with current I. Calculate

1. the energy stored in the magnetic field of the toroid.
2. The self-inductance of the toroid.



Solution:

- The magnetic field in the torus is given by

$$B = \frac{\mu_o NI}{2\pi r}$$

- The stored energy is then

$$U_B = \frac{1}{2\mu_0} \int_{allspace} B^2 \, dV_{vol}$$

$$= \frac{1}{2\mu_0} \int_a^b B^2 h 2\pi r dr = \frac{h\pi}{\mu_0} \int_a^b \left(\frac{\mu_0 NI}{2\pi r}\right)^2 r dr$$

$$= \frac{h\mu_0 N^2 I^2}{4\pi} \int_a^b \frac{dr}{r} \rightarrow \quad U_B = \frac{h\mu_0 N^2 I^2}{4\pi} \ln \frac{b}{a}$$

- The self-inductance is

$$L = \frac{2U_B}{I^2} = \frac{h\mu_0 N^2}{2\pi} \ln \frac{b}{a}$$

Case Problem 1.3: Self Inductance, Energy, Induced Electric Fields, and Faraday's Law

A wire is wrapped N = 100 times around a cylinder of non-magnetized material of radius a = 3.0 cm and length l = 25 cm. At t = 0, the current through the wire is increased according to

$$I(t) = bt, 0 < t < 2\text{ s}$$

where $b = 2\text{x}10^{-1}\text{ As}^{-1}$. Assume the direction of the current through each turn is clockwise when seen from above. After t >= 2 s the current remains constant. A small wire loop of radius r = 1 cm is placed on the solenoid's axis at the mid point of the solenoid with the normal to the plane of the loop pointing along the solenoid's axis. The loop has resistance R = 5 Ω .

a. After t >= 2 s, use Ampere's Law to find the direction and magnitude of the magnetic field within the solenoid. (Ignore edge effects).

b. Sketch the magnetic field lines everywhere including edge effects.

c. What is the self-inductance of the solenoid?

d. The rate of doing work against the back emf is $dW/dt = -\varepsilon I$, where the emf is given by $\varepsilon = -L(dI/dt)$. How much work is done against the back emf in order to reach a steady current after t >= 2 s in the solenoid?

e. The stored energy in a magnetic field is equal to $U = \frac{1}{2\mu_0}\int_{all\ space} B^2 dV$. How much energy is stored in the magnetic field of the solenoid? Do you expect your result to agree or disagree with your answer in part e)? Briefly explain your reasoning.

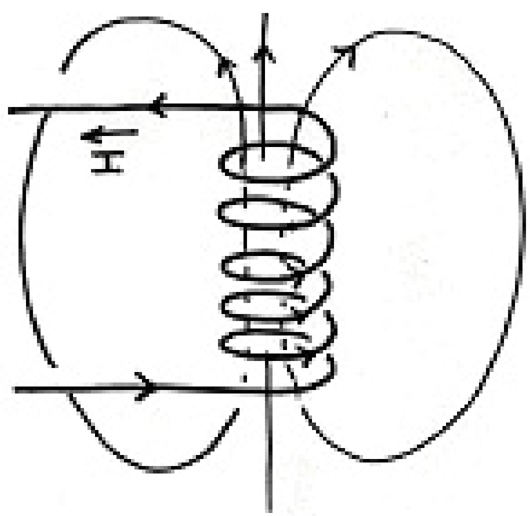
f. The changing magnetic flux inside the solenoid produces an induced tangential electric field. During the interval, $0 < t < 2$ s, find an expression for the induced electric field as a function of distance from the central axis of the solenoid.

Case Problem 1.3 (Solution)

a. If edge effect is ignored, the magnetic field *B* can be assumed to be constant over the length of the solenoid. By Ampere's law, the magnetic field strength is given by

$$Bl = \mu_0 NI \Rightarrow B = \frac{\mu_0 NI}{l} = 2 \times 10^{-4}\ \mathrm{T}$$

b.



Case Problem 1.3 (Solution)

c. The self inductance *L* is given by total magnetic flux linkage *Φ* divided by the current input *I*. i.e.

$$\Phi = NBA = N\frac{\mu_0 NI}{l}\pi a^2 = \frac{\mu_0 N^2 I\pi a^2}{l}$$

$$L = \frac{\Phi}{I} = \frac{\mu_0 N^2 \pi a^2}{l} = 1.4 \times 10^{-4}\ \mathrm{H}$$

d. The total work done is given by

$$W = \int_0^{2\ \mathrm{s}} \varepsilon I dt = \int_0^{2\ \mathrm{s}} L\frac{dI}{dt} I dt$$

$$\Rightarrow W = \frac{\mu_0 N^2 \pi a^2}{l}\int_0^{2\ \mathrm{s}} b^2 t dt = \frac{\mu_0 N^2 \pi a^2}{l} b^2 \frac{t^2}{2} \quad = 1.12 \times 10^{-5}\ \mathrm{J}$$

Case Problem 1.3 (Solution)

e. The total volume of the solenoid is $l\pi a^2$
   - If the magnetic field is taken to be constant within the solenoid by ignoring the edge effect, then total energy in the magnetic field is

$$U = \int_{vol} \frac{1}{2\mu_0} B^2\, dV = \frac{1}{2\mu_0} \left(\frac{\mu_0 NI}{l}\right)^2 l\pi a^2 = \frac{\pi\mu_0 N^2 I^2 a^2}{2l} = \frac{1}{2} LI^2$$

e. By Faraday's law,

$$E(2\pi r) = -\frac{d(B\pi r^2)}{dt}$$

$$\Rightarrow E = -\frac{\mu_0 N r}{2l}\frac{dI}{dt} = -\frac{\mu_0 N r b}{2l}$$

Transformer

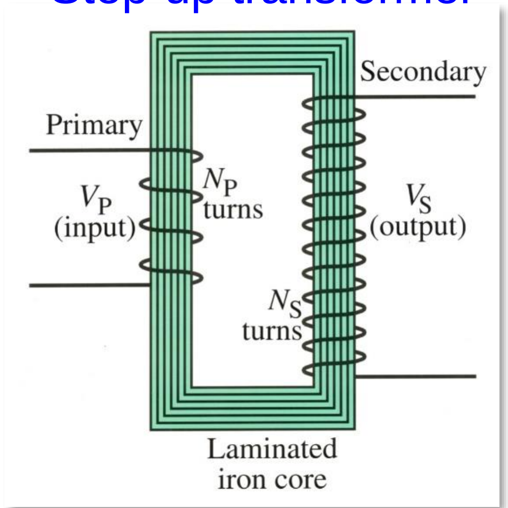
- An electrical device that changes (steps up or steps down) an AC voltage.
- Transformers have become essential for the transmission, distribution, and utilization of AC electric power.

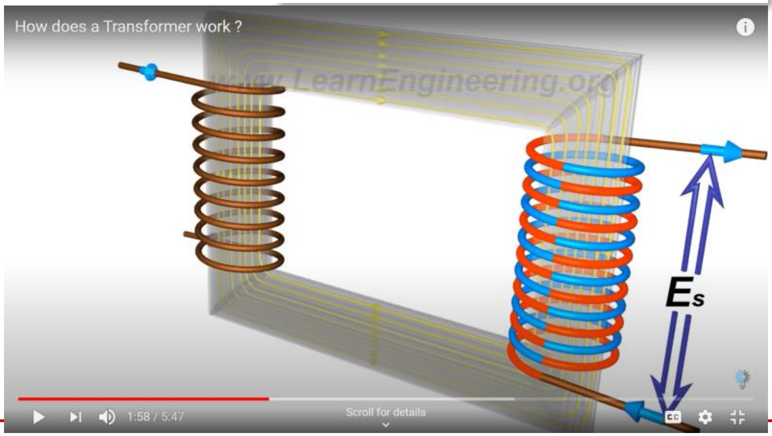
- Ideally, the flux Φ through each turn at primary and secondary is the same, so does $\frac{d\Phi}{dt}$:

$$\varepsilon_p = N_p \frac{d\Phi}{dt}; \quad \varepsilon_s = N_s \frac{d\Phi}{dt}$$
$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$$

- $N_s > N_p$: step-up transformer
- $N_s < N_p$: step-down transformer

Step-up transformer





https://youtu.be/vh_aCAHThTQ

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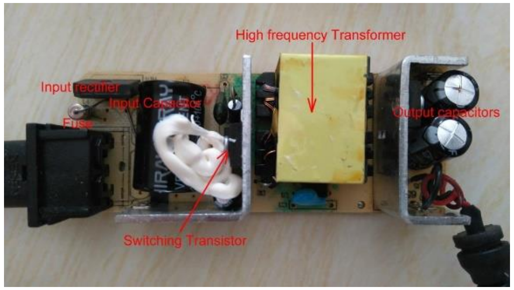
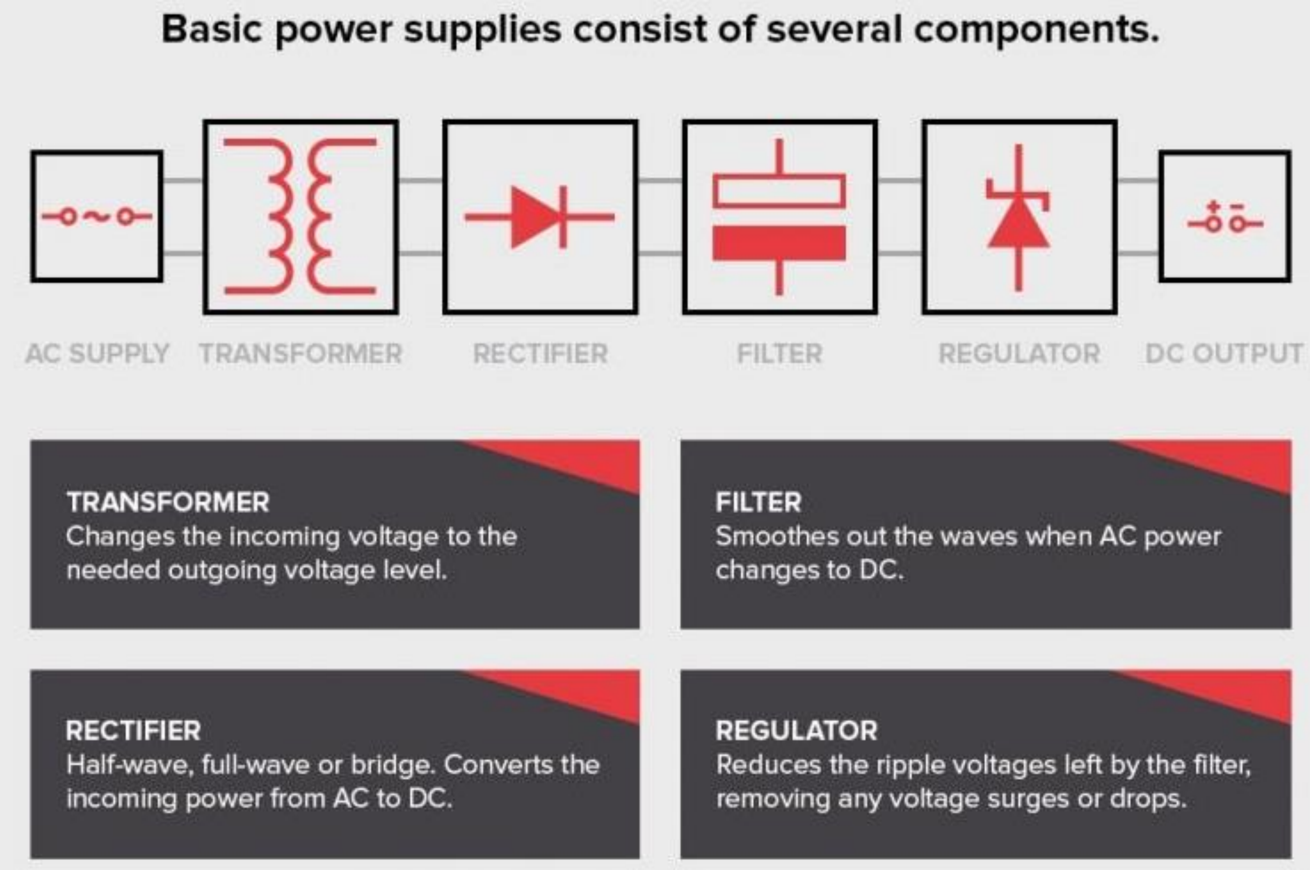
Inductors and Transformers are Everywhere.



Application: Power Supply/ AC adapter

- Basic power supplies consist of several parts. These components help the unit to step up or down voltage, convert power and reduces ripple voltages, which are residual variations in the voltage and results in wasted power and overheating.



A disassembled AC adapter reveals a simple, unregulated linear DC supply circuit: a transformer, four diodes in a bridge rectifier, and an electrolytic capacitor to smooth the waveform.

Summary : Self and Mutual Inductance

- Self inductance is defined as

$$L = \frac{\Phi_{B,self}^{total}}{I}$$

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the magnetic flux due to the B field
4. Calculate the self inductance (divide out I)

- Another way to calculate self inductance is to deduce the value from the energy stored by the magnetic field,

$$U_B = \frac{1}{2}LI^2 \quad \rightarrow \quad L = \frac{2U_B}{I^2}$$

- Mutual inductance is defined as

$$M_{12} = \frac{\Phi_{12}}{I_2} = M_{21} = \frac{\Phi_{21}}{I_1}$$

In-Class Worksheet

FYI: How Special Relativity Makes Magnets Work

- http://youtu.be/1TKSfAkWWN0

How Special Relativity Makes Magnets Work