Week 4 - Day 1

Capacitance

Concept 1: Capacitor and Capacitance

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Supercapacitor vs. Battery, Capacitive Touch Screen/Switch

Reading:

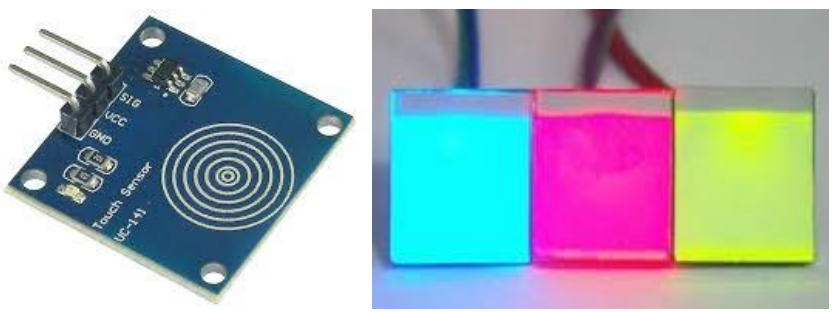
University Physics with Modern Physics – Chapter 24

Introduction to Electricity and Magnetism – Chapter 5

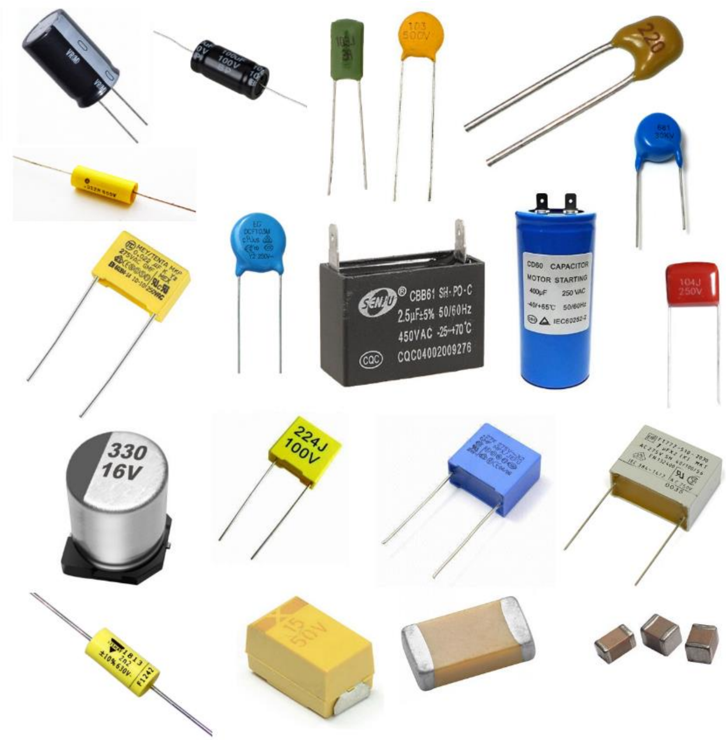
Concept 1: Capacitor and Capacitance

Have you wondered…

- How a capacitor looks like in a circuit?
- The function of a capacitor in a circuit?
- How a capacitive touch screen/touch switch works?



Capacitive touch switch



Various types of capacitors

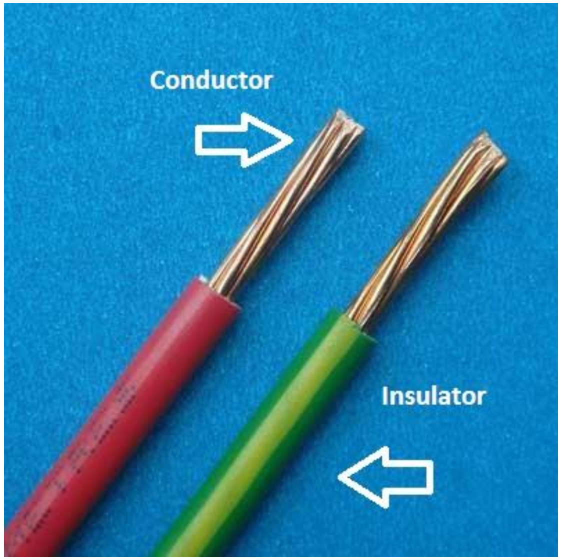
Conductors and Insulators

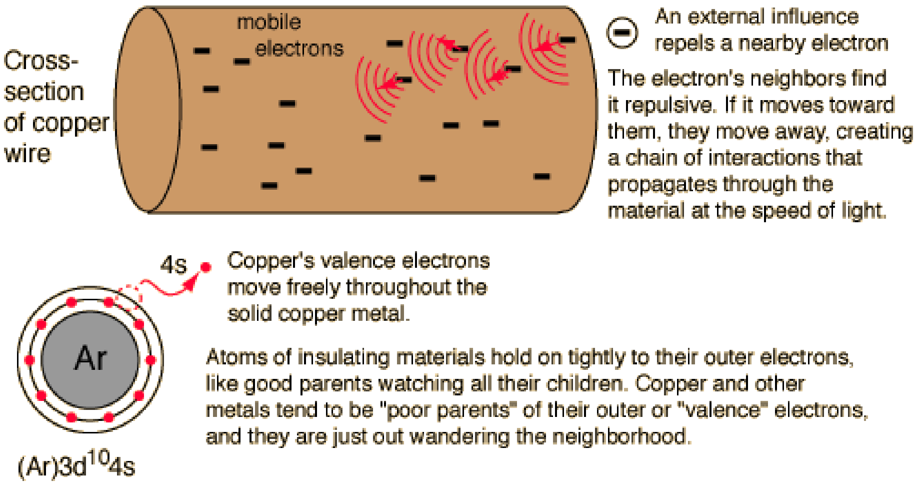
Conductor

- Charges are free to move
- Electrons are weakly bound to the atoms.
- Example: metals, plasma.

Insulator/ Dielectric

- Charges are NOT free to move
- Electrons strongly bound to atoms.
- Examples: plastic, paper, wood, distilled water.





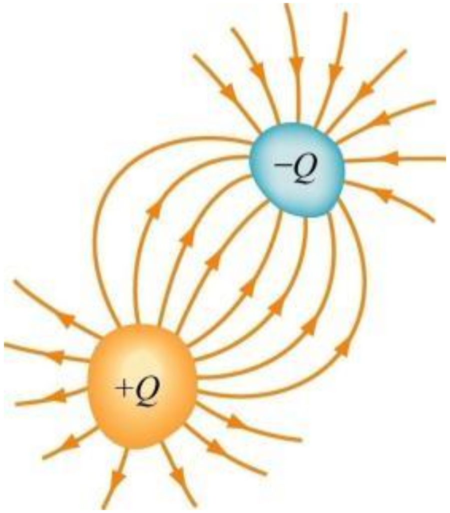
Capacitor: Store Electric Charge

Capacitor:

1. A fundamental electrical component with two isolated conductors and an insulator.
2. The conductors are deposited with equal and opposite charges $\pm Q$.
3. It is used to storage charge and electrical energy

- If the potential difference between the two conductors is ΔV, a constant called the capacitance C is defined as,

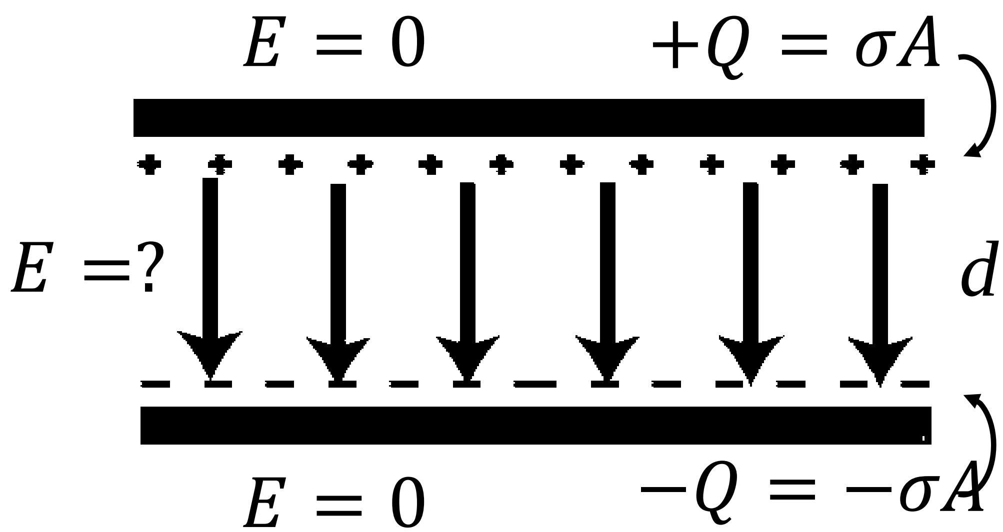
$$C = \frac{Q}{|\Delta V|}$$



- Units: C/V (Coulombs per Volt) or F (Farads).
- C is always Positive (we take positive Q and $|\Delta V|$)
- Capacitance: The ability to store charge

Parallel Plate Capacitor

- The simplest form of a capacitor is the ideal parallel plate capacitor.
- It is usually assumed that the dimensions of the plates are very long compared to the distance d between the plates.



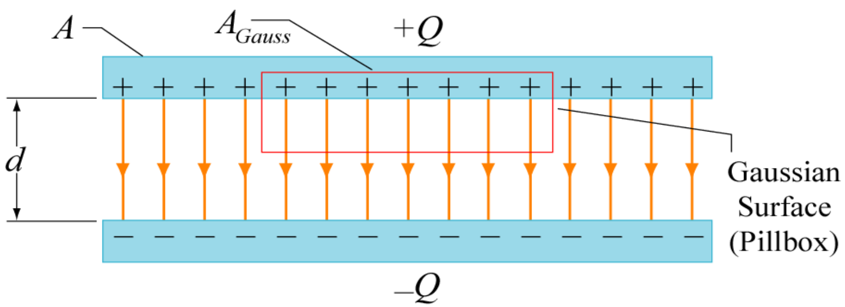
Demonstration: How a Capacitor really looks like?

Parallel Plate Capacitor: Calculating E with Gauss's Law then ΔV

- Note: Recall Week 2 (Gauss's Law) to find the $\vec{E}$ field in between 2 parallel plates with opposite charges.

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_o} \Rightarrow E(A_{Gauss}) = \frac{\sigma A_{Gauss}}{\varepsilon_o}$$

$$E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{A\varepsilon_o}$$



The potential difference between 2 plates:

$$\Delta V = -\int_{bottom}^{top} \vec{E} \cdot d\vec{s} = Ed = \frac{Q}{A\varepsilon_o} d$$

Thus,

$$C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_o A}{d}$$

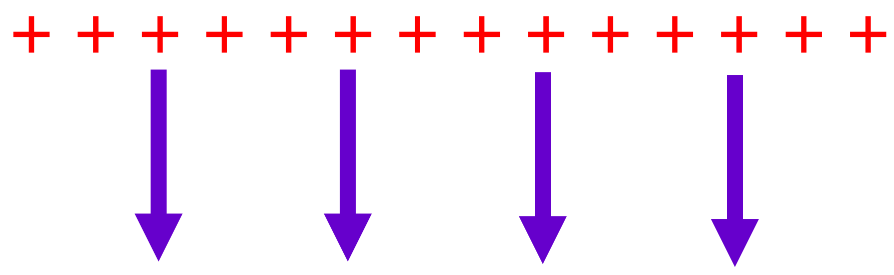
Note: Capacitance C depends only on the geometric factors (in this case, A and d for a parallel plate capacitor).

Recall: Alternative Way to calculate $\vec{E}$ between 2 plates

- Using superposition method.

Top plate: $E = \dfrac{\sigma}{2\varepsilon_o}$

Bottom plate: $E = \dfrac{\sigma}{2\varepsilon_o}$



- E in between the 2 plates with opposite charge,

$$E_{total} = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o} = \frac{Q}{A\varepsilon_o}$$

Example: Large Plane Capacitor

- A 1 Farad capacitor has its plates separated by 1 mm of air. What is the area of its plates?
- $C = \frac{\varepsilon_o A}{d} \Rightarrow A = \frac{Cd}{\varepsilon_o}$
- Put the numbers in:
- $A = (1F \times 1 \times 10^3 m) \times 8.85 \times 10^{-12} C^2\, N^{-1} m^{-2} = 1.12 \times 10^8 m^2$
- This would give us a capacitor with plates 10 km × 10 km, which is rather impractical. A farad is a very big unit, and we are much more likely to use microfarads (μF ~10^{-6} F) or nanofarads (nF ~10^{-9} F) or picofarads (pF ~10^{-12} F).
- 1F capacitor is usually called supercapacitor. The size can be small, but it does not hold high voltage (5.5V in this example).

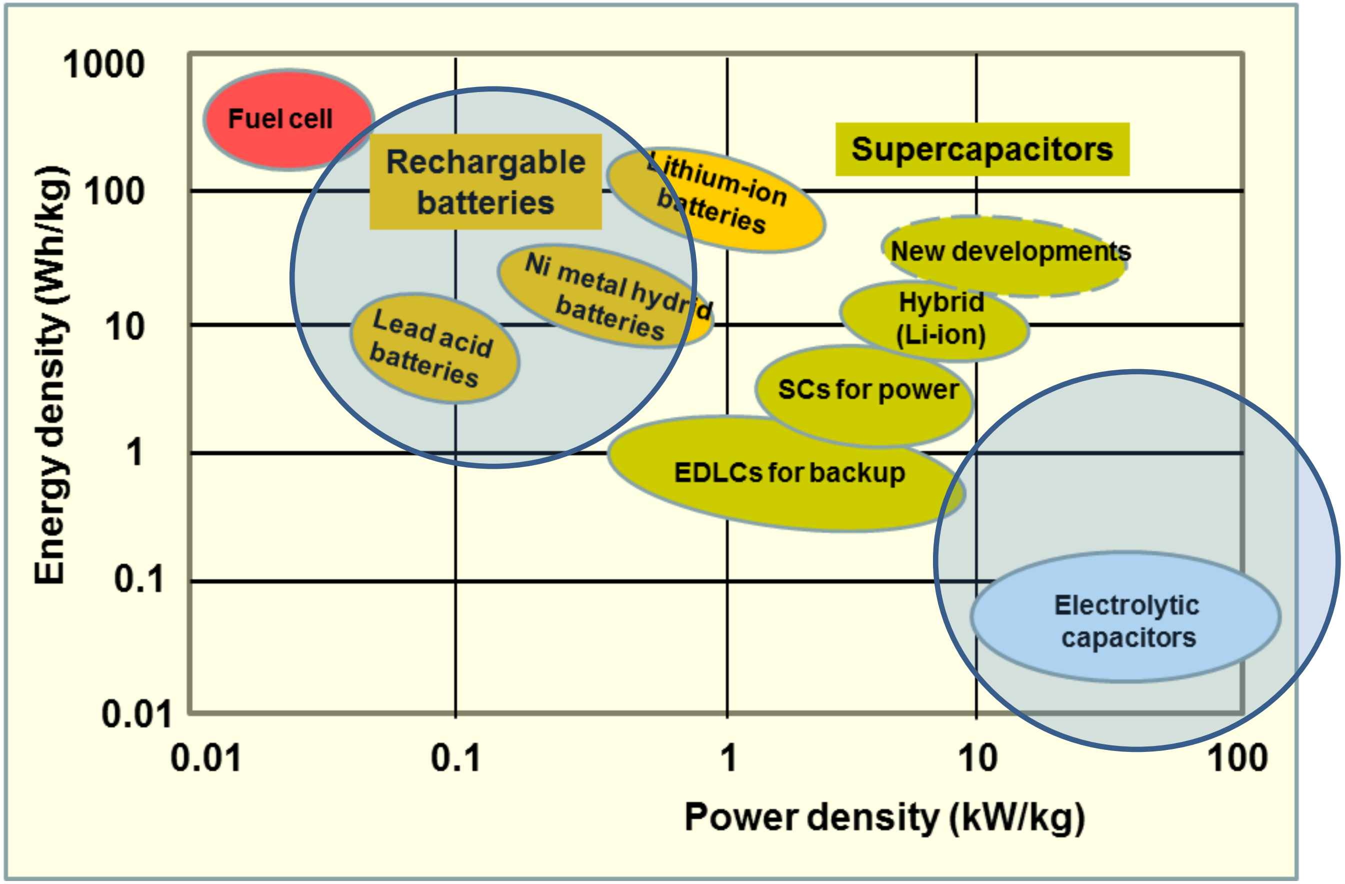
Demonstration: Charging and Discharging a 1F Capacitor

- We charge up a 1F capacitor with a 9V battery.
- Once it is fully charged, by waiting couple seconds, we connect the capacitor directly to a LED.
- We can observe how long it can light up a LED, compared to a normal capacitor, like $220\mu F$.

vs.

Application: Supercapacitors vs Batteries

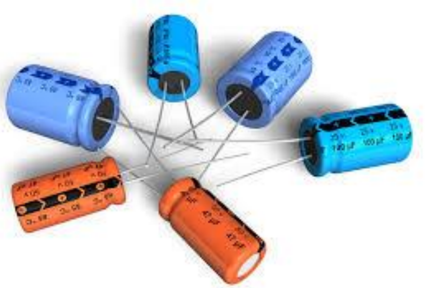
- Both devices store and release electrical energy. But what are the differences between the two?



Application: Capacitor vs. Rechargeable Battery

Capacitor/Supercapacitor

- The potential energy in a capacitor is stored in an electric field.
- A capacitor can discharge and charge faster than a battery (Greater power density).
- Capacitors store the electrical energy directly on the conducting plates, so discharging rate is directly related to the conduction capabilities of the plates.)



Rechargeable Battery (e.g. Li ion)

- A battery stores its potential energy in a chemical form.
- The technology for chemical storage currently yields greater energy densities (capable of storing more energy per weight).
- A battery discharges slower than a capacitor (there is a latency associated with the chemical reaction to transfer the chemical energy into electrical energy.)

Concept Question 1.1:

A parallel-plate capacitor is charged until the plates have equal and opposite charges $\pm$Q, separated by a distance *d*, and **then disconnected from the charging source (battery).** The plates are pulled apart to a distance $D > d$. What happens to the magnitude of the potential difference V and charge Q?

1. V, Q increases.
2. V increases, Q is the same.
3. V increases, Q decreases.
4. V is the same, Q increases.
5. V is the same, Q is the same.
6. V is the same, Q decreases.
7. V decreases, Q increases.
8. V decreases, Q is the same.
9. V decreases, Q decreases.

Multiple Choice

Concept Question 1.1: Solution

Answer: 2. V increases, Q is the same

With no battery connected to the plates the charge on them has no possibility of changing.

In this situation, the electric field doesn't change when you change the distance between the plates, so:

$$V = E\, d$$

As *d* increases, *V* increases.

Concept Question 1.2:

A parallel-plate capacitor is charged until the plates have equal and opposite charges $\pm Q$, separated by a distance d. While still **connected to the charging source,** the plates are pulled apart to a distance $D > d$. What happens to the magnitude of the potential difference V and charge Q?

1. V, Q increases.
2. V increases, Q is the same.
3. V increases, Q decreases.
4. V is the same, Q increases.
5. V is the same, Q is the same.
6. V is the same, Q decreases.
7. V decreases, Q increases.
8. V decreases, Q is the same.
9. V decreases, Q decreases.

Multiple Choice

Concept Question 1.2: Solution

Answer: 6. V is the same, Q decreases

With a charging source (battery) connected to the plates the potential V between them is held constant

In this situation, since

$$V = E\,d$$

As d increases, E must decrease.

Since the electric field is proportional to the charge on the plates, Q must decrease as well.

Concept Question 1.3

- A parallel-plate capacitor, disconnected from a battery, has plates with equal and opposite charges, separated by a distance d.
- Suppose the plates are pulled apart until separated by a distance $D > d$.
- How does the final electrostatic energy stored in the capacitor compare to the initial energy?

1. The final stored energy is smaller
2. The final stored energy is larger
3. Stored energy does not change.

We shall investigate the quantitative analysis of energy stored in a capacitor in Week 4 Day 2.

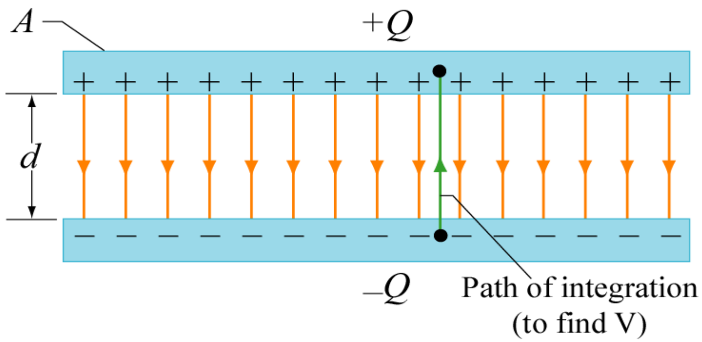
Multiple Choice

Concept Question 1.3: Solution

Answer: 2. The stored energy increases

As you pull apart the capacitor plates you increase the amount of space in which the E field is non-zero and hence increase the stored energy. Where does the extra energy come from? From the work you do pulling the plates apart.

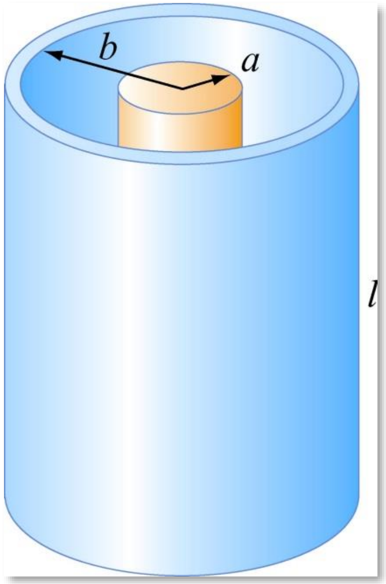
Review: Parallel Plate Capacitor



- We have seen that for a plate capacitor the capacitance can be calculated using $C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_o A}{d}$
- *Capacitance* depends on geometrical factors, so we want to explore other geometries and calculate the respective capacitance.
- *Note: The key is to calculate ΔV for different geometry.*

Case Problem 1.1: Cylindrical Shell

- Two concentric cylinders have radii a and b and height l, with b > a. The inner cylinder carries total charge -Q, and the outer cylinder carries total charge Q. We ignore end effects. Express capacitance in terms of the radii a and b, the height l, and any other constants which you may find necessary.



Case Problem 1.1: Solution

- By symmetry, the electrostatic field in between in the inner and outer shells must be radially inwards and possesses the same magnitude at the same distance *r* away from the center. Thus, by Gauss law with cylindrical Gaussian surface that neglect the edge effect,

$$\oiint \vec{E}\cdot d\vec{A} = -\frac{Q}{\varepsilon_o} \Rightarrow \oiint E dA = -\frac{Q}{\varepsilon_o} \Rightarrow E(2\pi r l) = -\frac{Q}{\varepsilon_o} \Rightarrow E = -\frac{Q}{2\pi r l \varepsilon_o}$$

- The potential difference of the outer shell with respect to inner shell is the negative of the work done by the electrostatic field from the inner to the outer shell, i.e.

$$V = -\int_a^b \vec{E}\cdot d\vec{s} = -\int_a^b -\frac{Q}{2\pi l \varepsilon_o r} dr = \frac{Q}{2\pi l \varepsilon_o} \ln\left(\frac{b}{a}\right)$$

- Capacitance is

$$C = \frac{Q}{V} = \frac{2\pi l \varepsilon_o}{lnb - lna} = \frac{2\pi l \varepsilon_o}{\ln\left(\frac{b}{a}\right)}$$

Video Demonstration: Capacitor explosion from excessive voltage

Not for you to try at home!

http://youtu.be/_WheLp0RdLQ

- Every single capacitor has a voltage rating, which is the maximum voltage you can apply to it before it malfunctions.
- If the capacitor experiences an excessive voltage between its terminals higher than its rated voltage, the dielectric may break down and electrons will flow between the thin metal layers inside of the capacitor, creating a short and heat.

In Class Worksheet