Week 2 - Day 2

Gauss's Law

Concept 1: Applying Gauss's Law on Symmetrical Charge Distributions

- Spherical Symmetry
- Cylindrical Symmetry
- Planar Symmetry

A simplified way to find $\vec{E}$ under symmetric conditions.

Reading:

University Physics with Modern Physics – Chapter 22

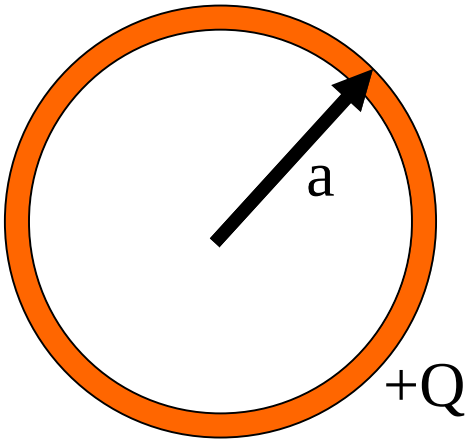
Introduction to Electricity and Magnetism – Chapter 3

Let’s review the concepts of Gauss’s Law

Review Concept Question 1: Spherical Shell

Positive charge Q is distributed uniformly throughout a spherical shell. What is the electric field inside the shell?

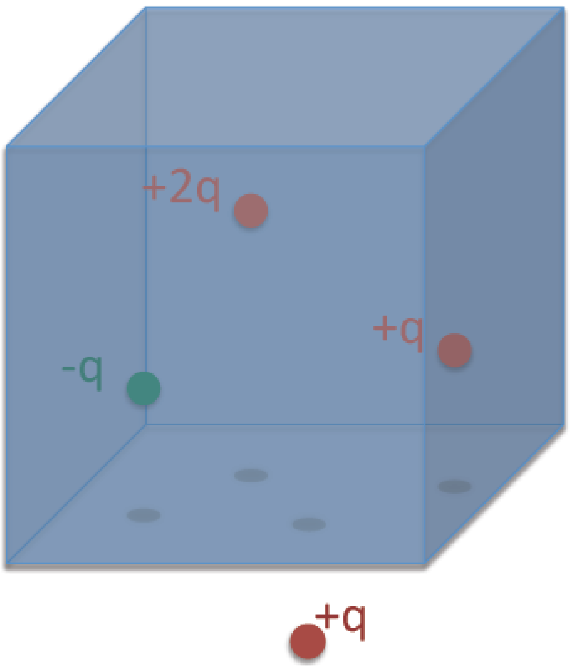
A. Zero
B. Uniform but Non-Zero
C. Still grows linearly
D. Some other functional form (use Gauss' Law)
E. Can't determine with Gauss Law



Review Concept Question 2

- Several charges sit inside or near a cubical object as shown. What is the value of the net flux through the object?

1. $5q/\varepsilon_0$
2. $4q/\varepsilon_0$
3. $2q/\varepsilon_0$
4. Zero
5. $-2q/\varepsilon_0$
6. Impossible to tell

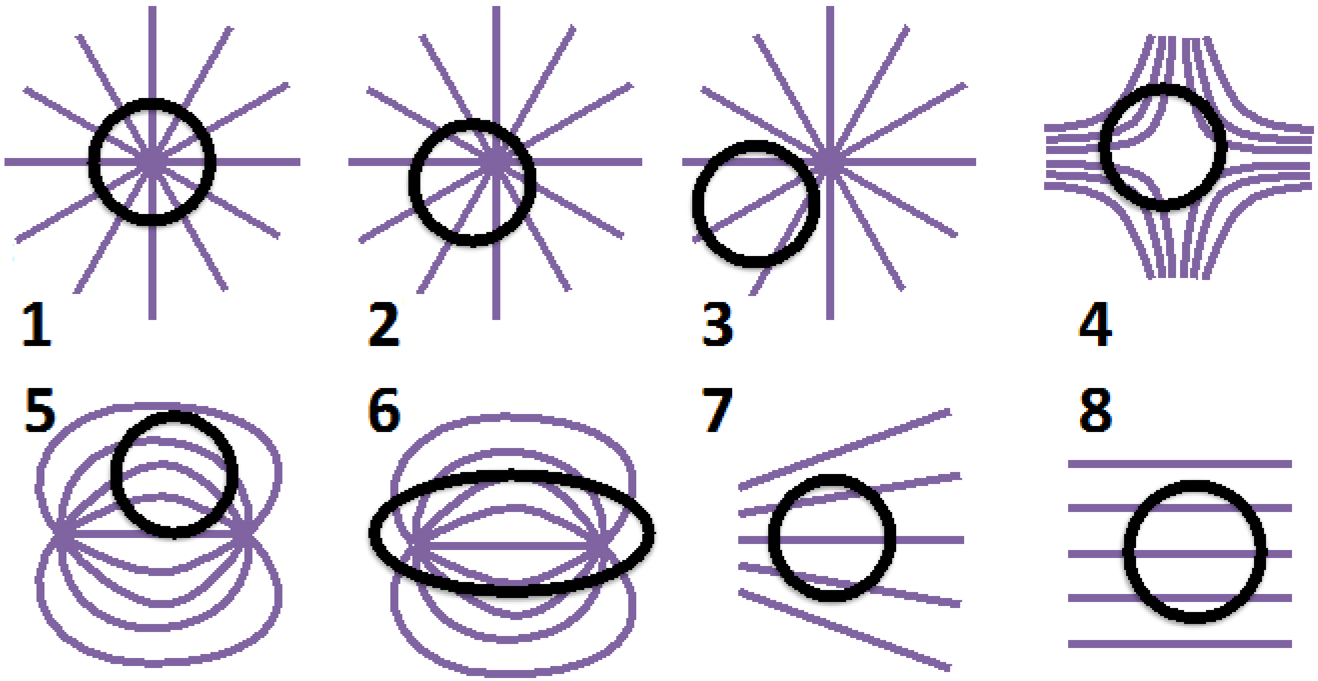


Review Concept Question 2: Answer

- Answer: 3. $\frac{+2q}{\varepsilon_0}$
- There are 3 charges enclosed by the cubical box, +2q, +q and –q. The total enclosed charge is +2q.
- Gauss's Law tells us that the flux that passing through the close surface is always equal to the enclosed charge by the surface/ε_o.
- $\Phi_E = \frac{+2q}{\varepsilon_0}$

Review Concept Question 3

- The black shapes in the picture below are closed surfaces. The colored lines are electric field lines. For which cases is the flux through the surface non zero?



9: Impossible to tell.

Concept 1: Applying Gauss's Law on Symmetrical Charge Distribution

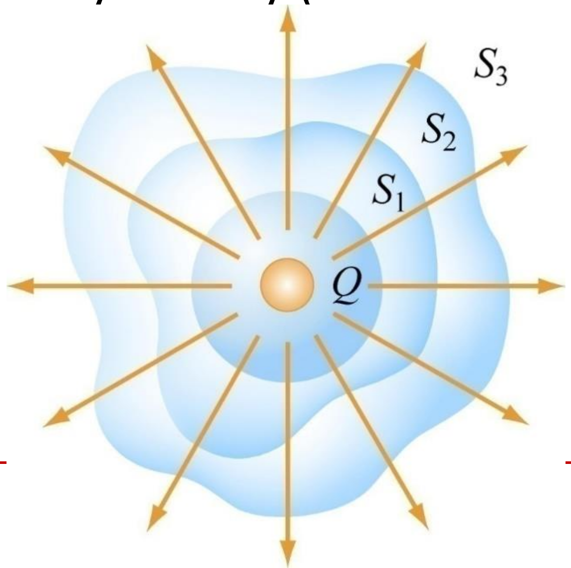
The simplified way to find $\vec{E}$ (under symmetric conditions) helps to understand and analyze other concepts in the subsequent weeks.

Choosing Gaussian Surface

- Desired **E field:** Perpendicular to surface and uniform on surface A. Flux is EA or –EA on these surfaces.
- Other **E field:** Parallel to surface. Flux is zero on these surfaces.

$$\oiint_{\substack{closed\\ surface\ S}} \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

- Gauss's Law is
  – **True** for all closed surfaces, but only **useful** to calculate electric field for problems with plenty of symmetry (based on charge distribution).



Symmetry & Gaussian Surfaces

- Desired **E field**: perpendicular to surface and constant on surface. So Gauss' s Law useful to calculate electric field from highly symmetric sources.

Source (charge distribution) Symmetry	Gaussian Surface
Spherical	Concentric Sphere
Cylindrical	Coaxial Cylinder
Planar	Gaussian "Pillbox"

Skills to develop:

Describe and picture the electric field pattern and electric field lines for highly symmetrical charge distribution.

Construct an appropriate Gaussian surface and apply Gauss's Law on highly symmetrical charge distribution to calculate the electric field.

Applying Gauss's Law

1. Based on the charge distribution, perceive the electric field pattern.
2. Identify regions in which to calculate the electric field.
3. Choose a Gaussian surface S, so that the surface is either perpendicular or parallel to the $\vec{E}$.
4. Calculate $\Phi_E = \oiint_S \vec{E} \cdot d\vec{A}$ (left hand side of Gauss's Law)
5. Calculate q_{enc}, charge enclosed by surface S (right hand side of Gauss's Law)
6. Apply Gauss's Law (equal LFS to RHS) to calculate the unknown electric field.

$$\oiint_{\substack{closed \\ surface\ S}} \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

Applying Gauss's Law

- To apply Gauss's Law to find the electric field, the charge distribution needs to be:
  - Spherical Symmetry
  - Cylindrical Symmetry
  - Planar Symmetry
- Out of these symmetry, Gauss's Law is not useful to calculate electric field (by hand).

- Gauss's Law is always true, but not always useful to calculate electric field! It is only useful for finding electric field in highly symmetric cases!

Concept Question 1.1: Application of Gauss's law

- For which of the following charge distributions would Gauss' s law *not* be useful for calculating the electric field?

A. a uniformly charged sphere of radius *R*

B. a spherical shell of radius *R* with charge uniformly distributed over its surface

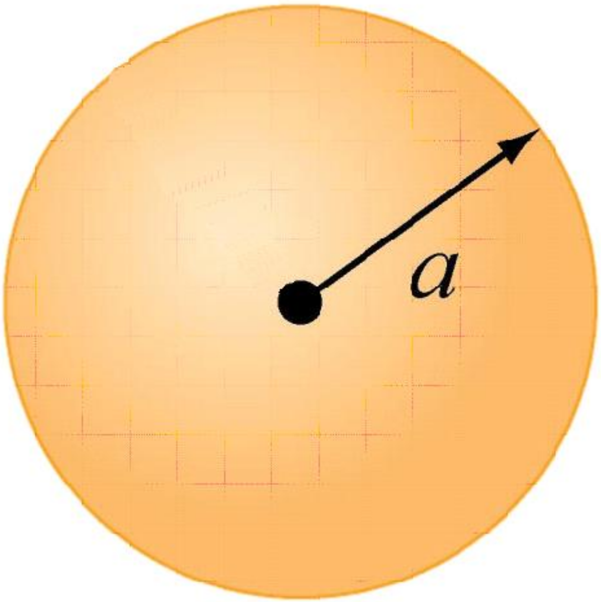
C. a right circular cylinder of radius *R* and height *h* with charge uniformly distributed over its surface

D. an infinitely long circular cylinder of radius *R* with charge uniformly distributed over its surface

E. Gauss' s law would be useful for finding the electric field in all of these cases.

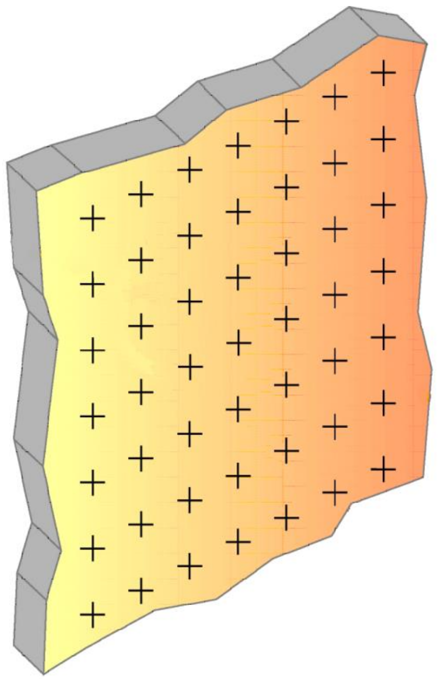
Case Problem 1: Spherical Symmetry

- $+Q$ uniformly distributed throughout non-conducting solid sphere of radius a. Find $\vec{E}$ everywhere.

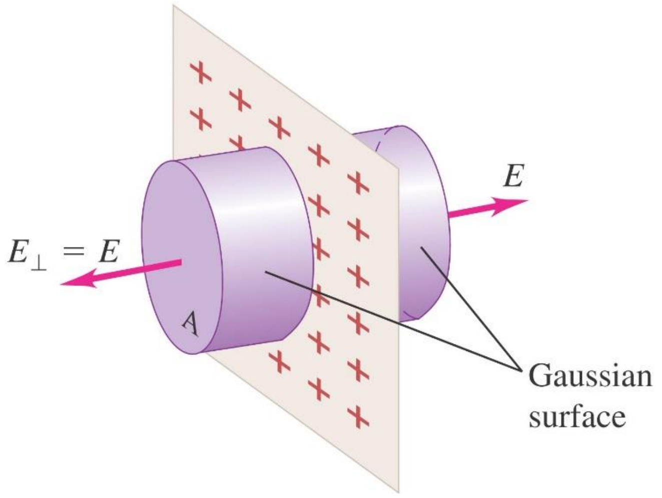


Case Problem 2: Planar Symmetry

- Consider an infinite thin slab with uniform positive charge density σ. Find a vector expression for the direction and magnitude of the electric field outside the slab. Make sure you show your Gaussian closed surface.

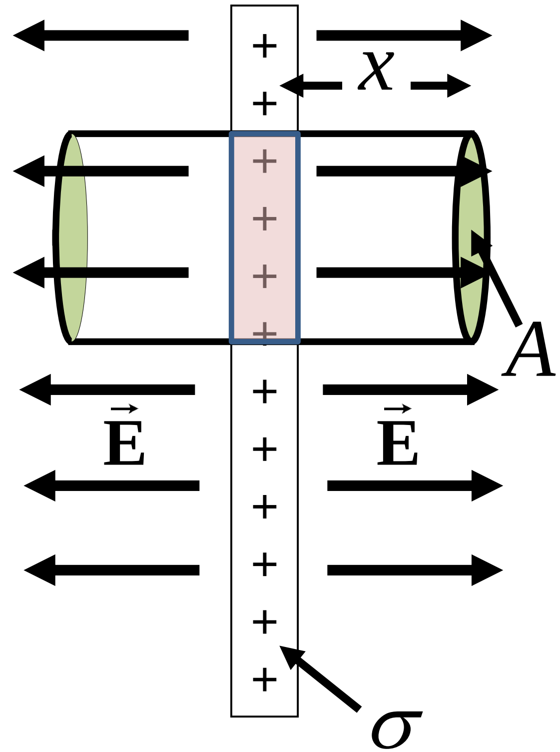


Hint: You may consider the Gaussian surface as shown.



- Total charge enclosed: $q_{enclosed} = \sigma A$
- NOTE: No flux through side of cylinder, only endcaps surface

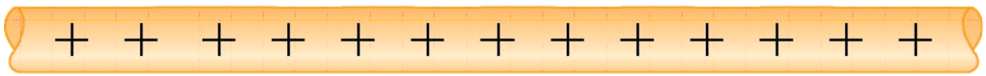
$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = E \oiint_S dA = EA_{Endcaps} = E(2A)$$

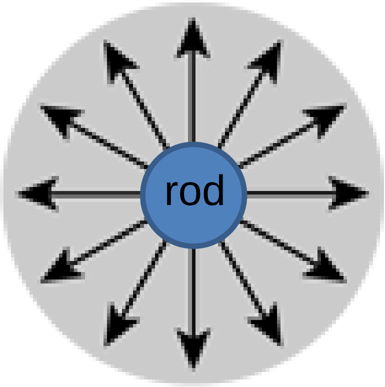


- Thus, using Gauss's Law
- $\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$
- $\mathrm{E(2A)} = \frac{\sigma A}{\varepsilon_0}$
- $\Rightarrow E = \frac{\sigma}{2\varepsilon_o}$
- Thus, $\vec{E} = +\frac{\sigma}{2\varepsilon_o}\hat{\imath}$ (for x>0)
- $\vec{E} = -\frac{\sigma}{2\varepsilon_o}\hat{\imath}$ (for x<0)
- Note: For infinite large charged slab, E field is uniform and independent of the distance x away from the slab.

Case Problem 3: Cylindrical Symmetry

- An infinitely long rod has a uniform positive linear charge density λ. Find the direction and magnitude of the electric field outside the rod. Clearly show your choice of Gaussian closed surface.
- Note that the resultant E field is pointing radial outward.





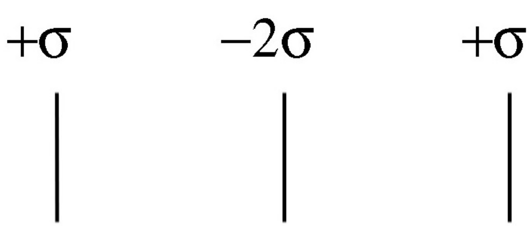
Top view

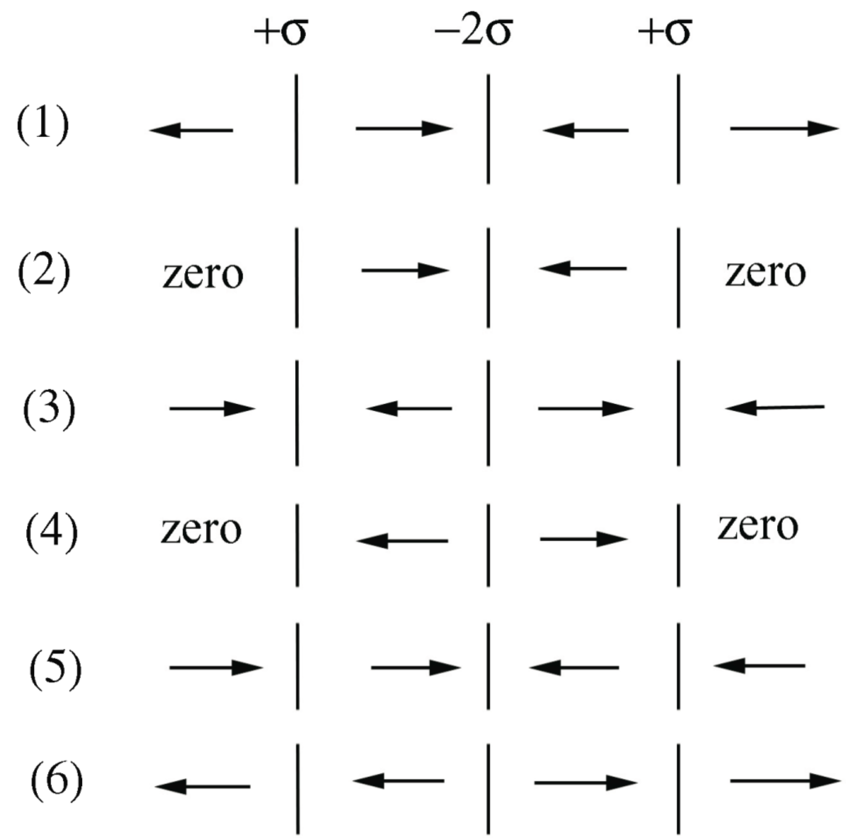
Summary of $\vec{E}$ for Basic Charged Structures

Charge Distribution	E Field (Outside) Strength
Dipole	Falls off by $1/r^3$
Spherical	$\vec{E} = k_e \frac{Q}{r^2} \hat{r}$ (Falls off by $1/r^2$)
Cylindrical or Line (infinite)	$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r} \hat{r}$ (Falls off by $1/r$)
Plane (infinite)	$\vec{E} = \pm \frac{\sigma}{2\varepsilon_o} \hat{\imath}$ (Uniform on both sides (constant))

Concept Question 1.2: Superposition

Three infinite sheets of charge are shown above. The sheet in the middle is negatively charged with charge per unit area -2σ, and the other two sheets are positively charged with charge per unit area $+\sigma$. Which set of arrows (and zeros) best describes the electric field?



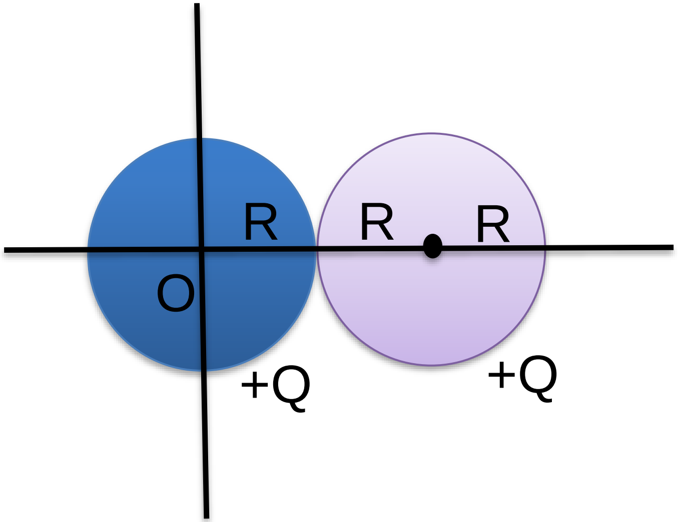


Extra Case Problem 1

Positive charge is distributed uniformly over each of two spherical volumes with radius R.

Find the magnitude and direction of Electric field at

a) x=0
b) x=R/2
c) x=R
d) x=3R



Hint: E inside an uniformly distributed charged sphere $= k_e \frac{Qr}{R^3}\hat{r}$.

E outside an uniformly distributed sphere $= k_e \frac{Q}{r^2}\hat{r}$

In Class Worksheet