Week 5 - Day 1

How Northern Light forms?

Magnetic Field and Force

Concept 1: Magnetic field and Gauss Law for Magnetic Field

Concept 2: Magnetic Force for a Moving Charge (Lorentz Law)

Concept 3: Magnetic Force on a Current-Carrying Conductor

Cathode-Ray Tube (CRT), Brushless DC Motor

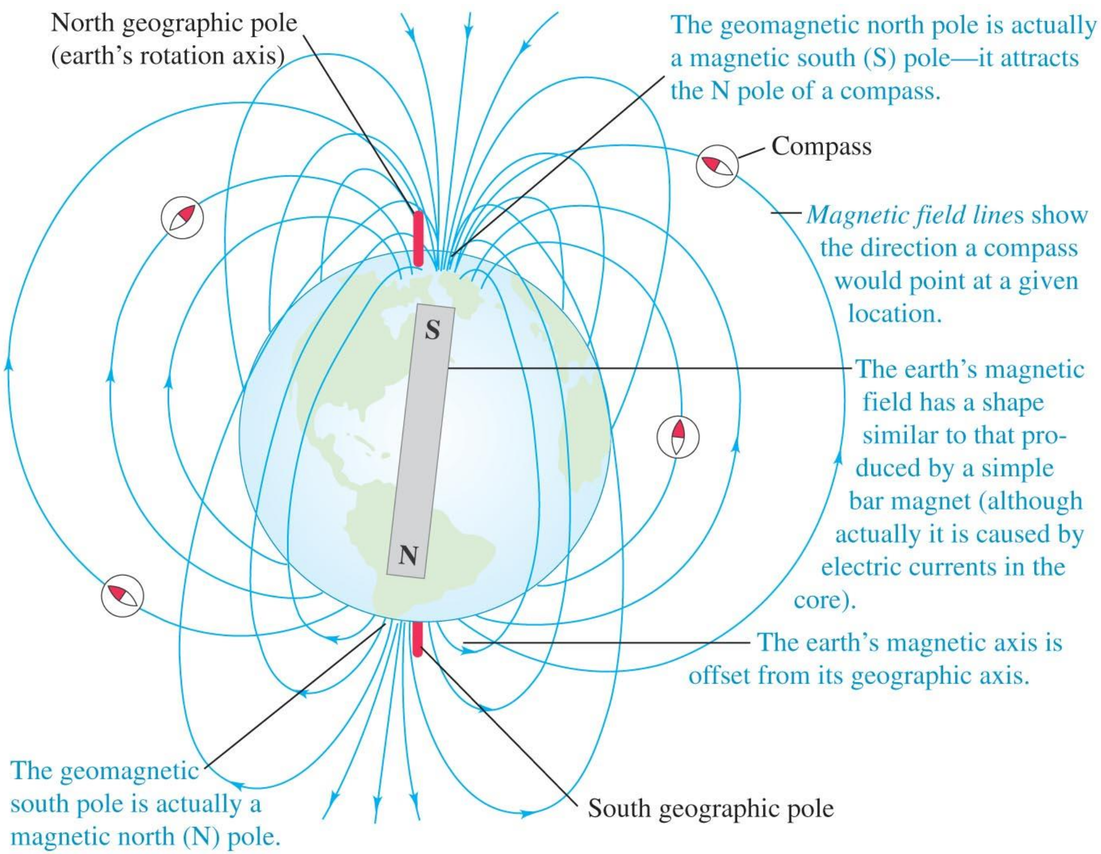
Reading:

University Physics with Modern Physics – Chapter 27

Introduction to Electricity and Magnetism – Chapter 8,9

Why a compass work? Our Earth is a Magnet

- A compass work as our Earth is also a magnet.
- A magnetic bar (dipole) always tends to align with an external magnetic field by a torque produced from the interaction. The north magnetic pole of a magnet is defined as it points to the geographical north (magnetic south) pole of the Earth.
- The Earth's magnetic field is mostly caused by electric currents in the liquid outer core, proposed by the Dynamo Theory.
- It protects the atmosphere and all life on our planet from the attack of solar winds. One of the reasons of life existence on Earth.
- Magnetic Protection: https://youtu.be/AtDAOxaJ4Ms



More information:
http://en.wikipedia.org/wiki/Dynamo_Theory

Application: How Northern Light (Aurora) Forms ?

- Great storms on the sun send gusts of charged solar particles hurtling across space. It is called solar wind.
- The charged particles are accelerated by the Earth's magnetic field due to the interaction of moving charges with external magnetic field (Lorentz Force).
- The Earth magnetic field density is the highest at both north and south poles.
- The charged particles strike atoms and molecules in Earth's atmosphere, they *excite* those atoms, causing them to light up.

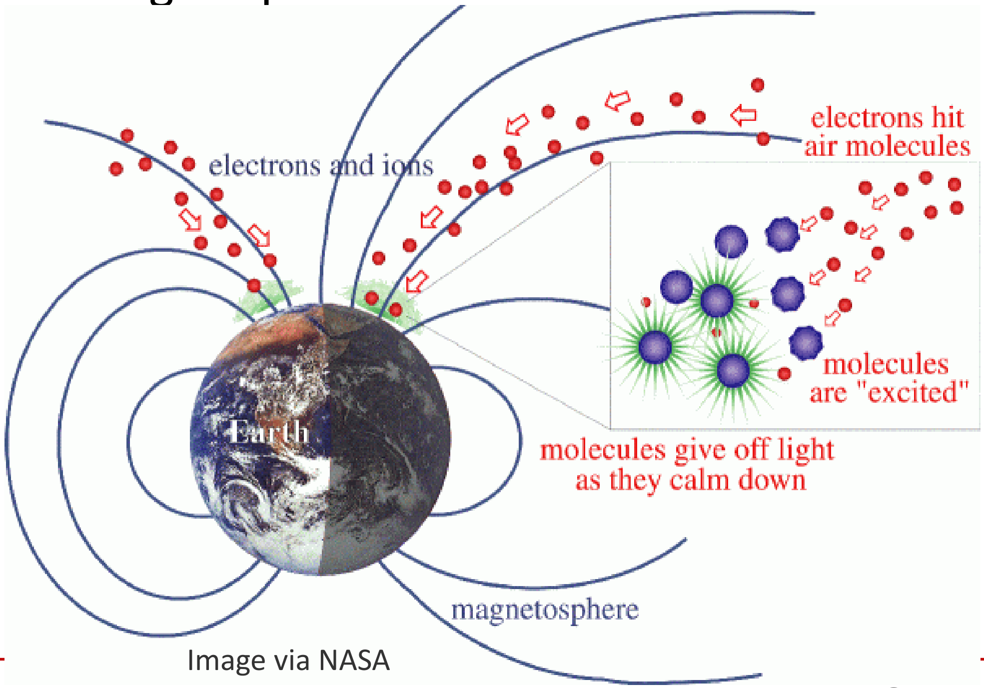


Image via NASA

Concept 1: Magnetic Field and Gauss Law for Magnetic Field

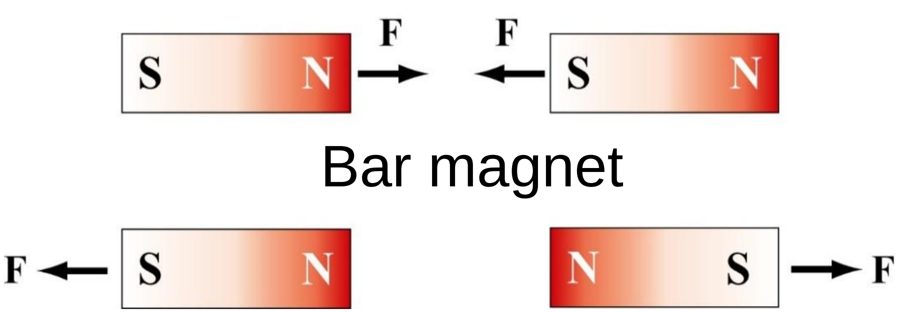
Important Ideas:

Identify the direction of magnetic field

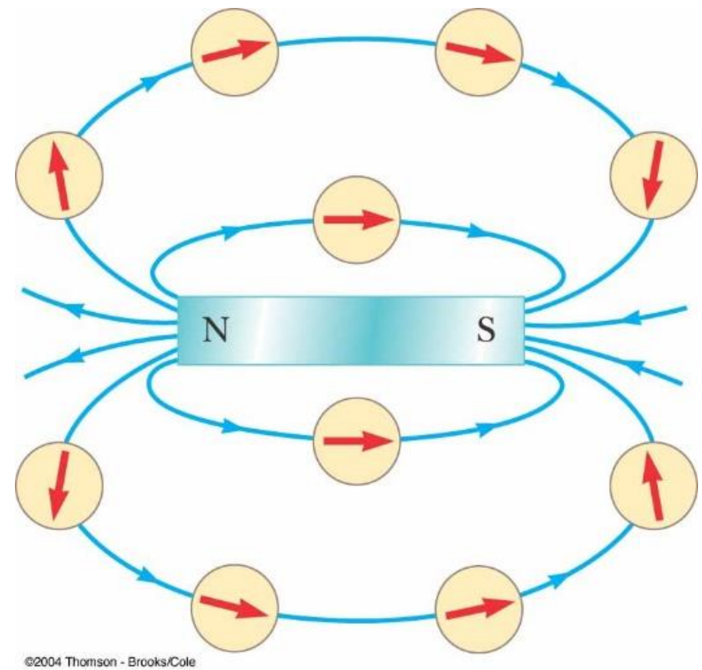
No magnetic monopole exists (so far)

Magnetic Field and Field Lines

- Magnetic field (B) is a vector
- Unit of $\vec{B}$ is 1 Tesla = 10^4 Gauss
- The magnetic field lines outside a magnet point from the North pole to the South pole of the magnet.
- We use a compass to trace the field lines. The north pole of the compass points to the direction of the magnetic field at that location.
- Like poles repel, opposite poles attract.



Like poles repel, opposite poles attract



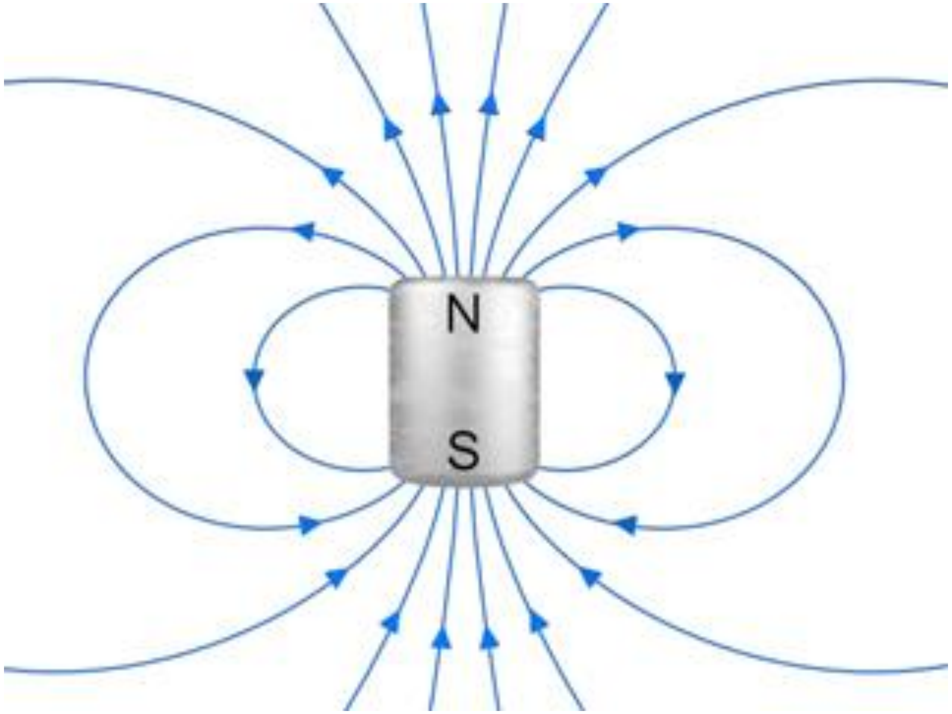
FYI: How Big is a Tesla? (Tesla is a large unit)

	Magnetic field strength
Earth's Field	5×10^{-5} T = 0.5 Gauss
Brain (at scalp)	~1 fT
Refrigerator Magnet	1 mT
Inside MRI	3 T
Good NMR Magnet	18 T
Biggest in Lab	150 T (pulsed)
Biggest in Pulsars	10^8 T

Concept Question 1.1: Magnetic Field Lines

- The picture shows the field lines outside a permanent magnet The field lines inside the magnet point:

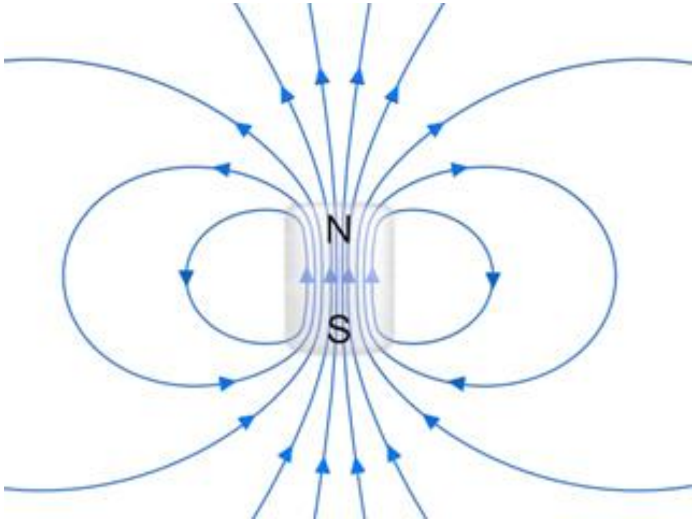
A. Up
B. Down
C. Left to right
D. Right to left
E. The field inside is zero



Multiple Choice

Concept Question 1.1 Solution

- Answer: A. They point up inside the magnet



Magnetic field lines are continuous.

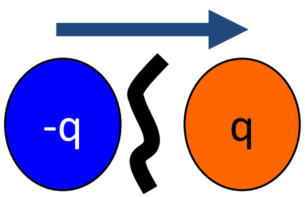
E field lines begin and end on charges.

There are no magnetic charges (monopoles) so **B** field lines ***never*** begin or end

Bar Magnets Are Dipoles!

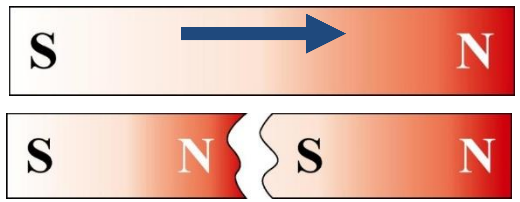
- A magnet (no matter how small it is) always creates a dipole field.
- It always rotates to orient with any external magnetic field.
- It is how a compass works to orient the magnetic field of the Earth, thus north pole of a bar magnet points to the geographical north.
- Unlike the property of mass or charge, which can form a “monopole” (e.g. point mass, positive or negative charge), there is NO such magnetic monopole that can be isolated. In other words, magnetic poles are always found in pairs.

Electric Dipole, $\vec{P}$

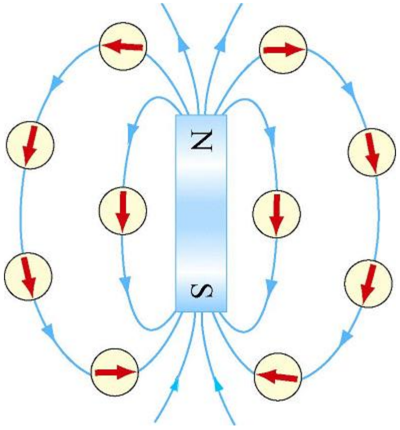


It can be separated to form 2 monopoles (charges)

Magnetic Dipole, $\vec{\mu}$



A dipole is separated and forms 2 dipoles. We cannot separate a dipole into magnetic monopole.



Gauss's Law in Magnetism

- Magnetic monopoles do not exist in isolation.
- This fact leads to another Maxwell's Equation! (the second one).

$$\oiint_S \vec{B} \cdot d\vec{A} = 0 \qquad \text{Magnetic Gauss's Law}$$

- The equation implies magnetic fields do not begin or end at any point.
- The number of B-field lines entering a closed surface equals the number of lines leaving the surface
- The RHS of the equation = 0 implies no magnetic charge or magnetic monopole.
- Unlike Gauss's Law for electrostatic, the application of Gauss's law in magnetism is limited.
- Side Note: Recall the first Maxwell's Equation (about electrostatic field):

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} \qquad \text{Gauss's Law}$$

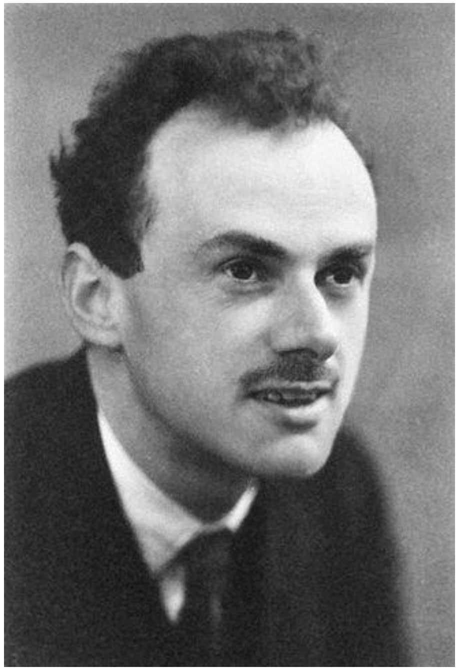
Magnetic monopole?

- Because of the absence of magnetic monopole, thus Maxwell set

$$\oiint_S \vec{B} \cdot d\vec{A} = 0$$

- But there is no explanation on the absence magnetic monopoles

- In 1931, Paul Dirac (Nobel Prize in physics 1933 on Dirac Eq. predicting positron or anti-electron) showed that if *any* magnetic monopoles exist in the universe, then all electric charge in the universe must be quantized. The electric charge *is*, in fact, quantized, then why no magnetic monopole for such elementary particle?



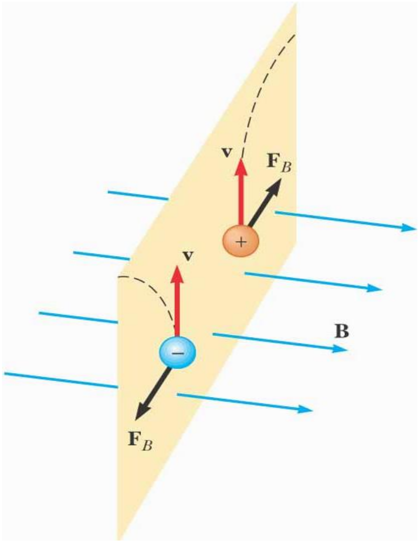
- Magnetic monopole is still an open question, physicists are looking for it, from quantum mechanical point of view.
- As of now, we can safely say that magnetic monopole does not present macroscopically.

Concept 2: Magnetic Force for a Moving Charge Particle

Magnetic Force on a Moving Charge

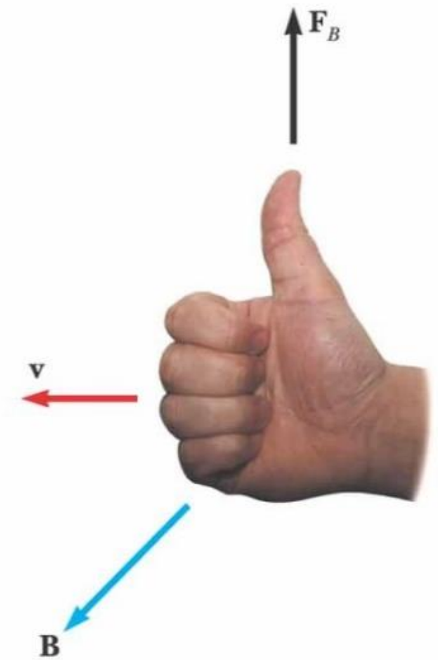
- **Lorentz Force law**

  $\vec{F}_{mag} = q\vec{v} \times \vec{B}$
  - $\vec{F}_{\boldsymbol{mag}}$ is the magnetic force
  - q is the charge
  - $\vec{v}$ is the velocity of the moving charge
  - $\vec{B}$ is the magnetic field
- The magnitude of the magnetic force on a charged particle is
- $\left|\vec{F}_{mag}\right| = q|\vec{v}|\left|\vec{B}\right| \sin\theta$



Recall: Right-Hand Rule (vector cross product)

- The fingers point in the direction of $\vec{v}$
- $\vec{B}$ comes out of your palm
  - Curl your fingers in the direction of $\vec{B}$
- The thumb points in the direction of $\vec{v} \times \vec{B}$ which is the direction of $\vec{F}_B$ if q is positive, but opposite if q is negative.
- $\vec{F}_{Mag} = q\vec{v} \times \vec{B}$



(a)

Concept Question 2.1: SI units - Magnetic Field

- What are the correct SI units for the magnetic field?

A. C/N-m-s

B. N-m-s/C

C. N/C

D. N-s/C-m

E. C-m/N-s

Concept Question 2.1 Solution

- Answer: D
- Since $\vec{F}_B = q\vec{v} \times \vec{B}$

$$\text{B Units} = \frac{\text{newton}}{(\text{coulomb})(\text{meter / second})} = 1\frac{\text{N}}{\text{C}\cdot\text{m/s}}$$

- This is called 1 Tesla (T)
- $1T = 10^4\ Gauss\ (G)$

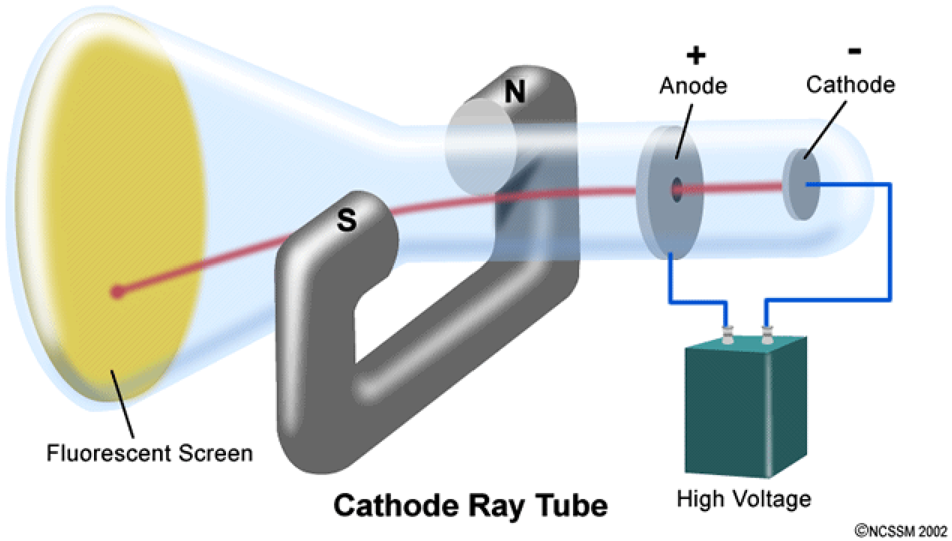
Concept Question 2.2: Force Direction

- Is this picture (deflection direction) correct?
- (Cathode rays are streams of electrons)

A. Yes

B. No

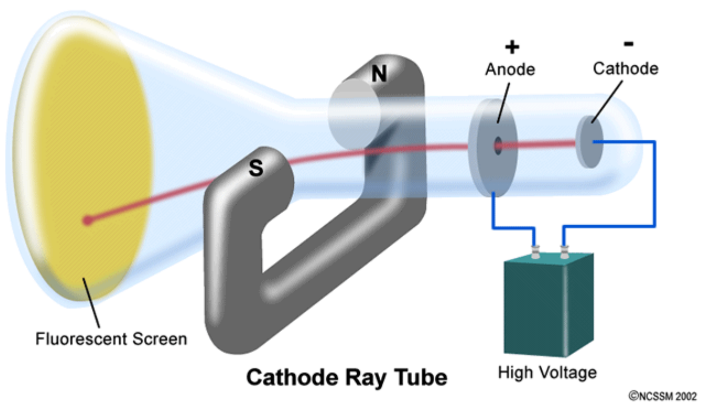
C. I don't know



Multiple Choice

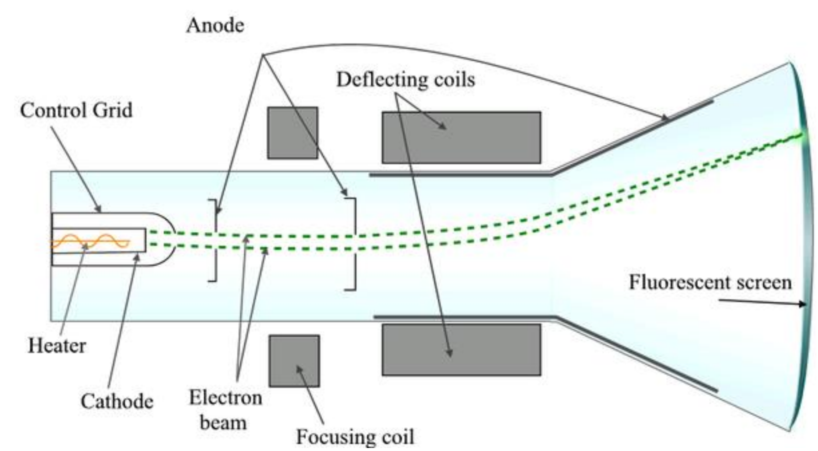
Concept Question 2.2 Solution

- Answer: A. Yes
- Field from N to S, beam velocity right to left, cross product is up. But charges are negative, so force is down, as pictured.



Application: Cathode-ray tube (CRT)

- The cathode-ray tube (CRT) is a vacuum tube that contains one or more electron guns and a phosphorescent screen and is used to display images, such as TV and computer monitor screen.
- Images are produced by the visible light emitted from the fluorescent screen when hit by electron beams.
- Electron beams are bent by magnetic deflection, a varying magnetic field generated by coils and driven by electronic circuits around the neck of the tube
- In 2000s, CRT had been superseded by flat panel display (LCD & OLED). Most production had ceased around 2010.
- CRT are bulky, heavy, high cost, high power consumption, cannot be made in large size.



Case Problem 2.1: Charged Particle in a Magnetic Field

A particle, with mass *m* and charge *q*, is moving in a circular motion inside an external magnetic field with radius *r* and speed *v*.

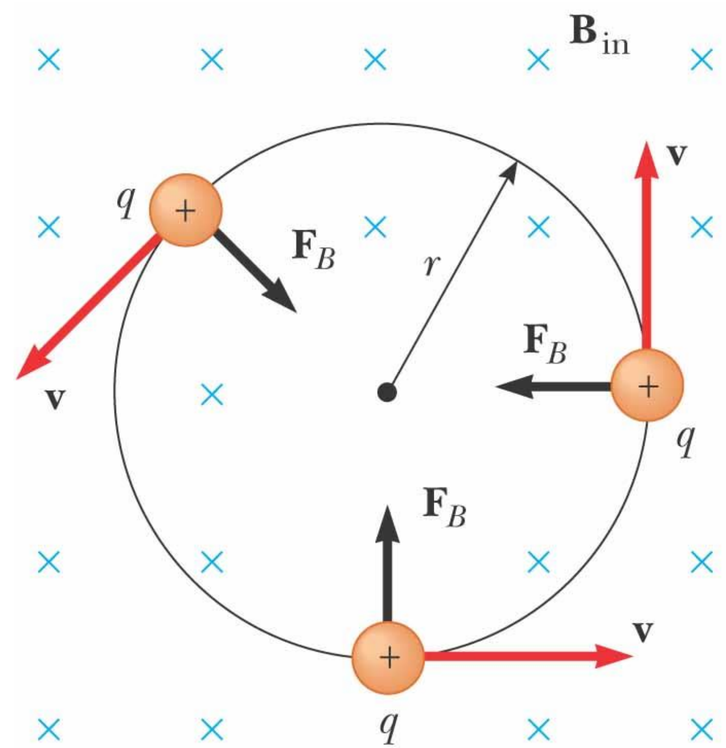
Express

a) radius *r*,

b) angular speed, ω,

c) Period, T

in terms of *q*, v, *B*, and *m*.



Case Problem 2.1 (Solution)

- Since the particle undergoes circular motion, we can equate the magnetic and centripetal forces:

$$F_B = qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB}$$

  - Hence, we can see that *r* is proportional to the momentum of the particle and inversely proportional to the magnetic field

- The angular speed (cyclotron frequency) and period is

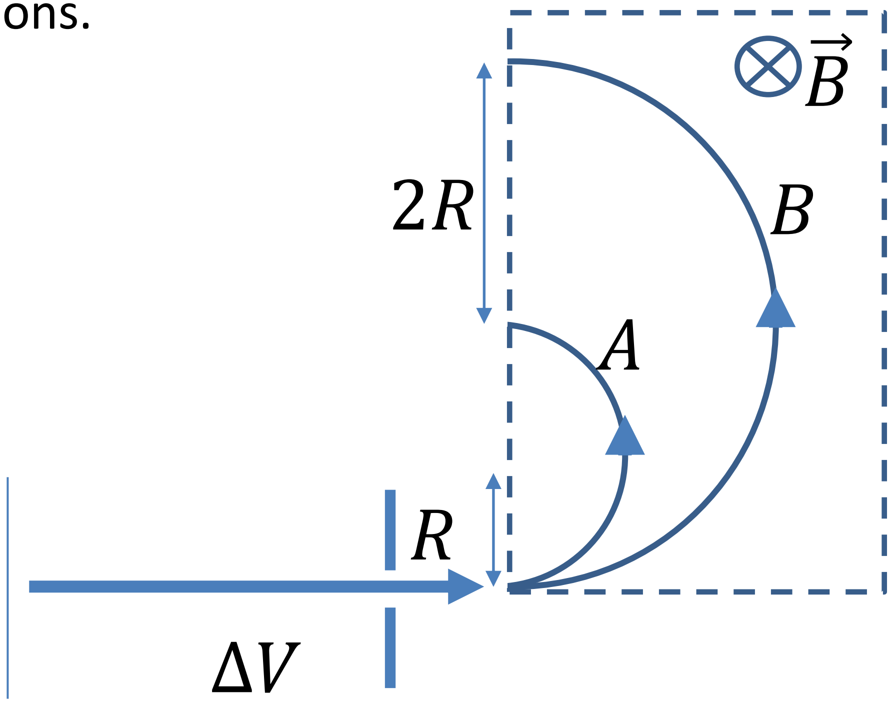
$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

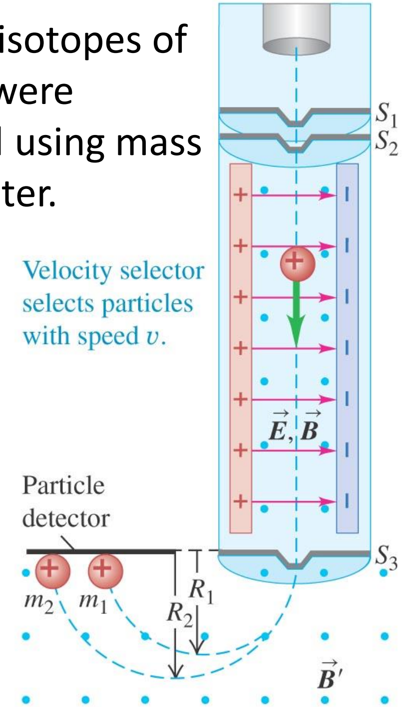
Case Problem 2.2: Charged Particles in Magnetic Field

Particle A with charge q and mass m_A and particle B with charge $2q$ and mass m_B, are accelerated from rest by a potential difference ΔV, and subsequently deflected by a uniform magnetic field into semicircular paths. The radii of the trajectories by particle A and B are R and $2R$, respectively. The direction of the magnetic field is perpendicular to the velocity of the particle. What is their mass ratio?

Note: This is the working principle of mass spectrometer. It measures the masses of ions.



FYI: Many isotopes of elements were discovered using mass spectrometer.



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

Case Problem 2.2 (Solution)

- Kinetic energy gained due to the potential difference:

$$\frac{1}{2}mv^2 = q\Delta V \quad \rightarrow \quad v = \sqrt{\frac{2q\Delta V}{m}}$$

- The magnetic force is pointing radially inward (centripetal force) and hence, the particles are able to subsequently move in a semicircular path:

$$\frac{mv^2}{r} = qvB \quad \rightarrow r = \frac{mv}{qB}$$

- Substitute v, $r = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$

$$\frac{r_A}{r_B} = \frac{1}{2} = \frac{\sqrt{\frac{m_A}{q_A}}}{\sqrt{\frac{m_B}{q_B}}} \Rightarrow \frac{1}{2} = \sqrt{\frac{\frac{m_A}{q}}{\frac{m_B}{2q}}} \Rightarrow \frac{1}{8} = \frac{m_A}{m_B}$$

Putting it Together: Lorentz Force

- Electric force on charged particles in electric fields

$$\vec{F}_{elec} = q\vec{E}$$

- Magnetic force on charged particles in electric and magnetic fields

$$\vec{F}_{mag} = q\vec{v} \times \vec{B}$$

- Thus, the total force on charged particles, namely Lorentz Force

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

Differences Between E-force and B-force

Direction of force

- The electric force acts along the direction of the electric field
- The magnetic force acts perpendicular to the magnetic field

Motion

- The electric force acts on a charged particle regardless of whether the particle is moving
- The magnetic force acts on a charged particle only when the particle is in motion

Differences Between E-force and B-force

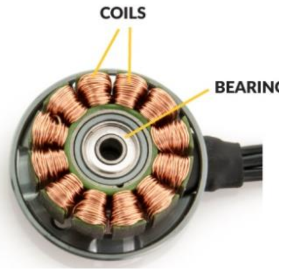
- Work done by the force
  - The electric force does work in displacing a charged particle
  - The magnetic force associated with **a steady-state magnetic field** does **no work when a particle is displaced**. WHY?
- This is because the force due to B is perpendicular to the displacement.

- When a charged particle moves with a velocity v through a magnetic field, the field can **alter the direction of the velocity**, but not the speed or the kinetic energy.

- The kinetic energy of a charged particle moving through a B field cannot be altered by the B field alone.

Application of Magnetic Force: DC Brushless Motor

- Common types of motor:
  - DC brush motor
  - DC brushless motor
  - Stepper motor
  - Servo motor
  - AC motors
- Example: Drones use DC brushless motor to fly, PC cooling fan, etc.
- How a DC brushless motor work?

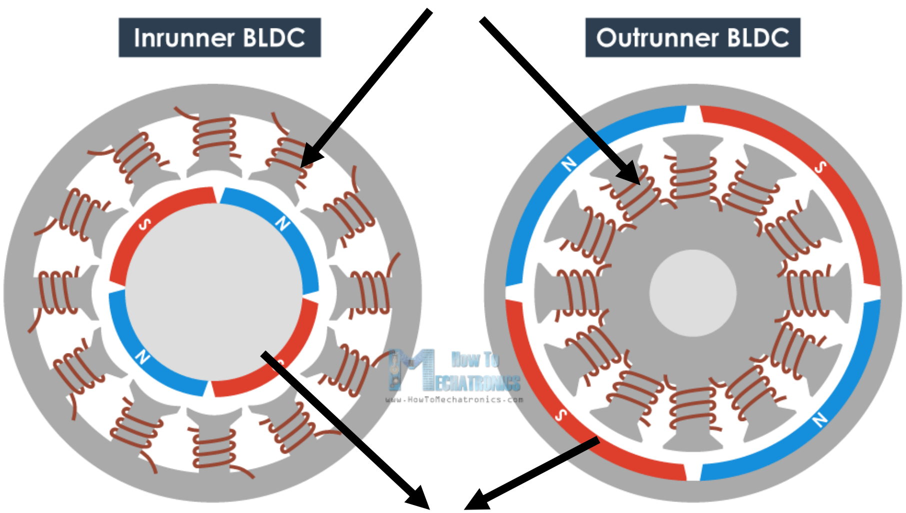


DC brushless motor

Application: Working Principle of a Brushless Motor

- A brushless DC motor (BLDC motor/ BL motor) is an electronically commuted DC motor which does not have brushes.
- The controller provides sequential pulses of current to the motor windings, in which the frequency of current controls the speed and torque of the synchronous motor.
- 2 main parts: Rotor (moving part with magnets) & Stator (non-moving part with electromagnetic coils)
- 2 main designs: Inner Rotor (In-runner) & Outer Rotor (Out-runner)

Stator – electromagnet coils do not move



Rotor – permanent magnets that rotate

Application: Advantages & Disadvantages of a Brushless Motor

The advantages of a BLDC motor:

- BLDC motors do not have brushes, the mechanical energy loss due to friction is less which enhanced efficiency, make it more reliable, high life expectancies, and maintenance free operation.
- Its velocity is determined by the frequency at which current is supplied, not the voltage. It can operate at high-speed under any condition.
- There is no spark from the commutator and much less noise during operation, electromagnetic interference is also reduced.
- More electromagnets could be used on the stator for more precise control.
- BLDC motors accelerate and decelerate easily as they are having low rotor inertia.
- It is high performance motor that can provide large torque.

The disadvantages of a BLDC motor:

- BLDC motor cost more than a brushed DC motor.
- Limited high power could be supplied to BLDC motor, otherwise, too much heat weakens the magnets, and the insulation of winding may get damaged.

References:

- Brushless DC Motor, How it works ?
- What is a BRUSHLESS MOTOR and how it works - Torque - Hall effect - 3D animation
- How Brushless Motor and ESC Work and How To Control them using Arduino

Concept 3: Magnetic Force on a Current-Carrying Conductor

Current: Flow Of Charge

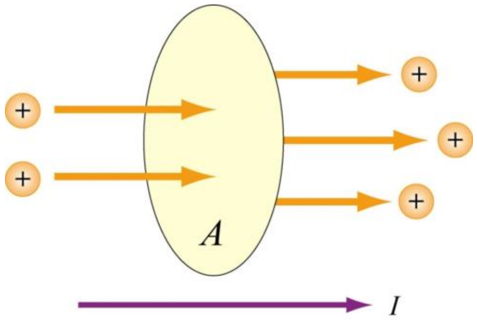
- Definition: Average current I_{av} is the charge ΔQ flowing through area, *A,* in time, Δt.

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

- Instantaneous current: differential limit of I_{av}.

$$I = \frac{dQ}{dt}$$

- Units of Current: Coulomb (*C*) / second (*s*) = Ampere (*A*)



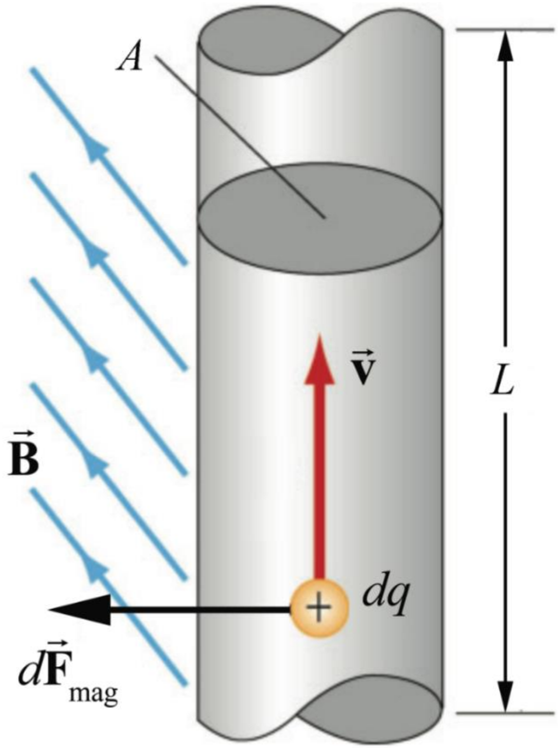
Magnetic Force on Current-Carrying Wire

Magnetic force acting on a charge dq

- $d\vec{F}_{mag} = dq(\vec{v} \times \vec{B})$
- Note: $dq\vec{v} = dq\frac{d\vec{s}}{dt} = \frac{dq}{dt}d\vec{s} = Id\vec{s}$
- $d\vec{F}_{mag} = Id\vec{s} \times \vec{B}$

Thus, magnetic force acting on a current-carrying wire in a magnetic field, $d\vec{B}$,

$$\vec{F}_{mag} = \int Id\vec{s} \times \vec{B}$$



- If the wire is inside a uniform magnetic field, then

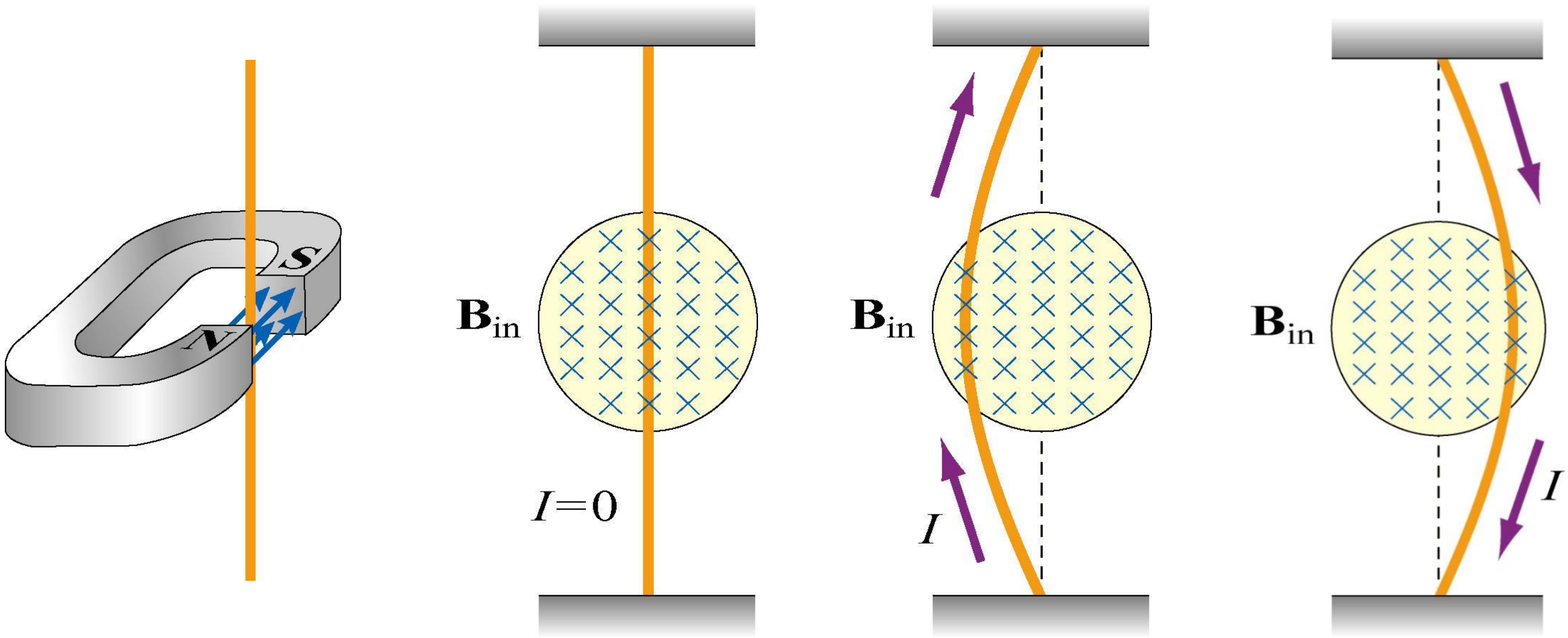
$$\vec{F}_{mag} = \left(\int_{wire} Id\vec{s}\right) \times \vec{B}$$

- If the wire is straight, the cross product of $\vec{B}$ and the unit vector along the wire is constant, hence we obtain

$$\vec{F}_{mag} = I(\vec{L} \times \vec{B})$$

Magnetic Force on Current-Carrying Wire

- Current is made of moving charges, and we know that charges moving in a magnetic field experience a force.



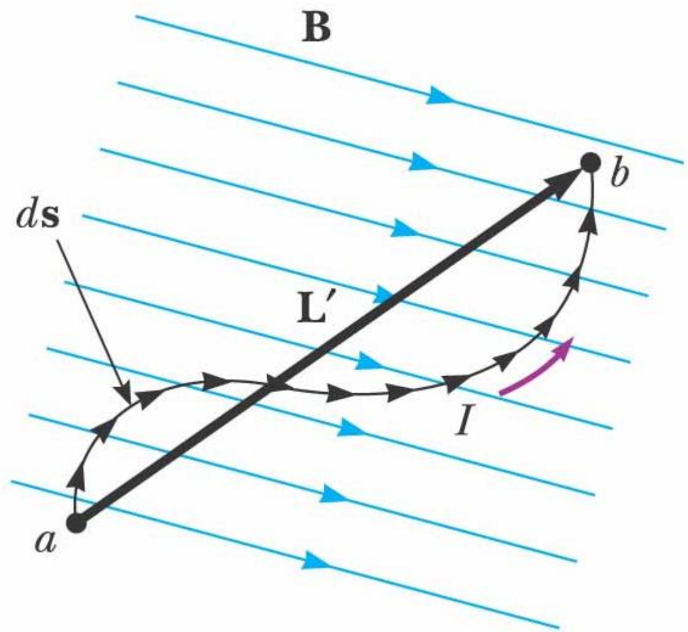
Concept question 3.1 (for a curved wire)

- Which of the following two wires has the largest magnetic force acting on it assuming that both carry the same current, and start (and end) at the same location?

A. curved segment

B. straight segment

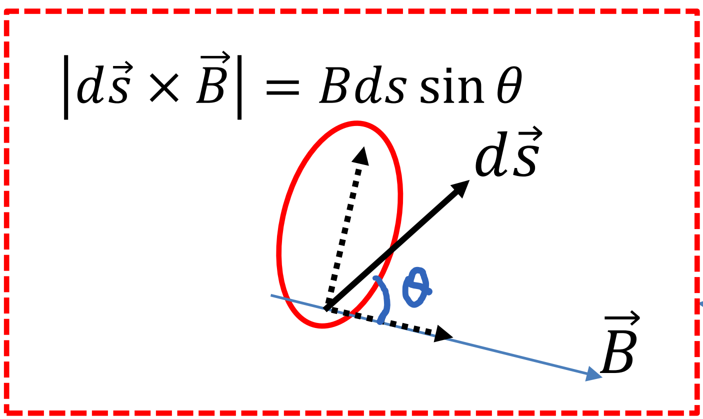
C. Same

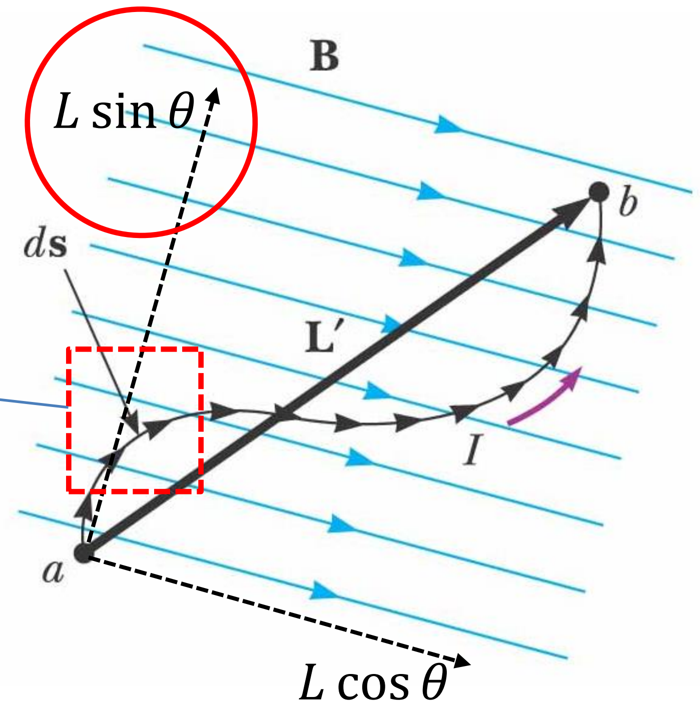


Multiple Choice

Concept Question 3.1 Solution

- **Answer: C The same.**
- **L'** is the displacement vector from a to b.
- The magnetic force on a curved current-carrying wire in a uniform field <u>is equal to that on a straight wire connecting the end points and carrying the same current</u>





By cross product, no matter which direction $d\vec{s}$ is pointing, the effective component is always the one that is perpendicular to the $\vec{B}$

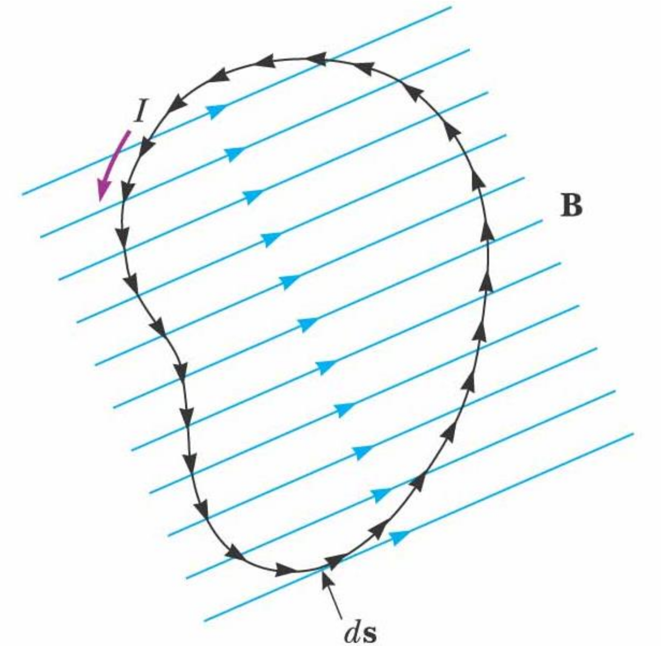
Concept Question 3.2 – for a closed loop wire

- What is the net magnetic force acting on any closed current loop in a uniform magnetic field?

A. ZERO.

B. NONZERO depending on the shape of the closed loop.

C. NONZERO irrespective of the shape of the closed loop.



Multiple Choice

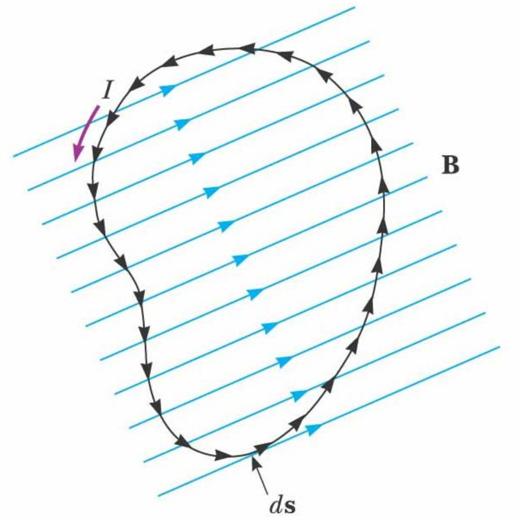
Concept Question 3.2 Solution

The ANSWER is A - The net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

We have an arbitrarily shaped closed loop carrying current I in a uniform magnetic field.

The displacement vector of closed loop, where the start and end point are the same is ZERO, so the vector sum of the force is also ZERO.

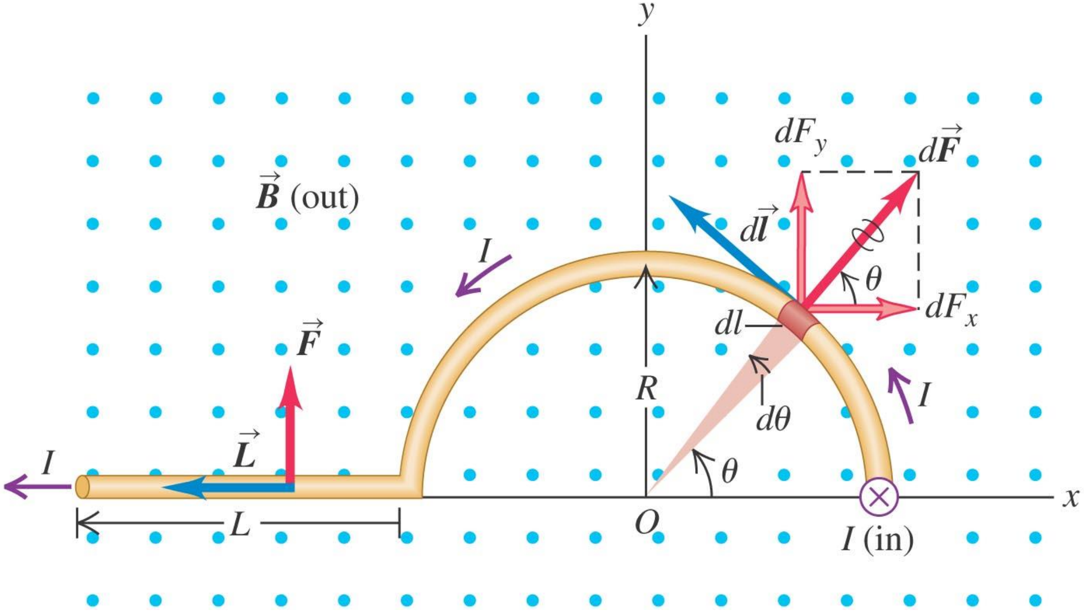
Note: However, the loop can experience a torque!



Case Problem 3.1

The magnetic field $\vec{B}$ is uniform and points out of the page. The conductor, carrying current I, has three segments:

1) a straight segment with length L going into the page,
2) a semicircle with radius R, and
3) another straight segment with length L parallel to the x-axis. Find the total magnetic force on this conductor.

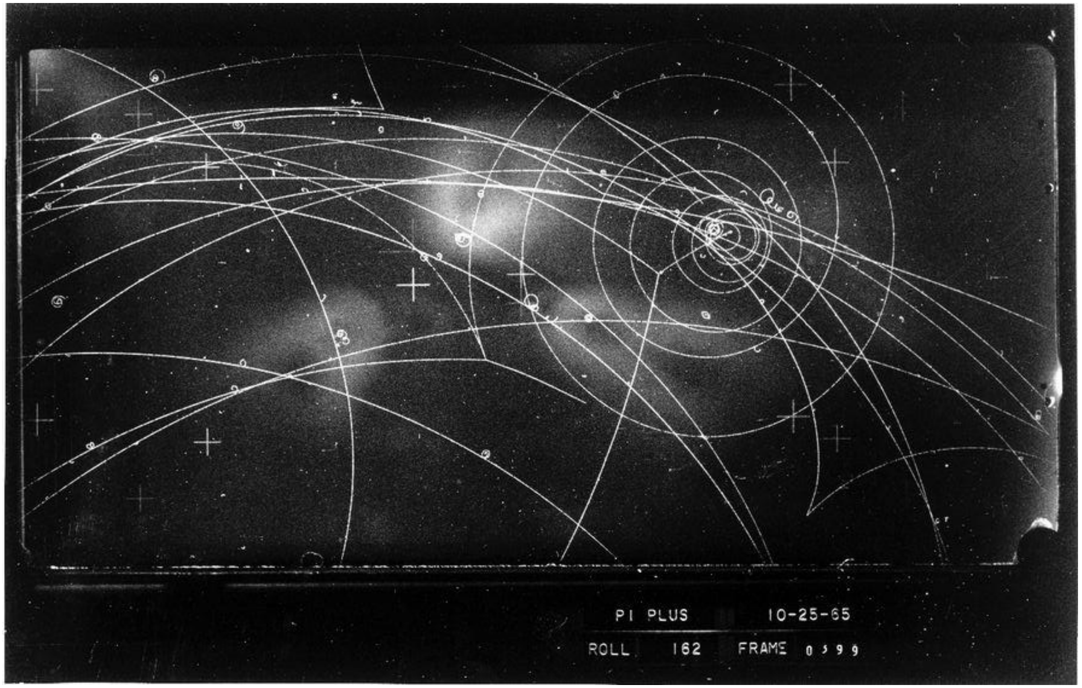


Case Problem 3.1 Solution

- For segment 1, $\vec{L} = -L\hat{k}$. $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$. The wire is antiparallel with the magnetic field. Note: $\sin(180^o) = 0$.
- For segment 3, $\vec{L} = -L\,\hat{\imath}$. $\vec{F}_3 = I\left(-L\,\hat{\imath} \times B\,\hat{k}\right) = -ILB\,(-\hat{\jmath}) = ILB\hat{\jmath}$.
- For segment 2, $dl = Rd\theta = Rd\theta$. The direction of $d\vec{l} \times \vec{B}$ is radially outward from the center. The magnitude $dF_2 = IBdl = IB(Rd\theta)$. The components of the force in the Cartesian coordinate, $d\vec{F}_2 = IBRd\theta\,(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$
- $F_{2x} = IRB\int_0^{\pi}\cos\theta\,d\theta = 0$; $F_{2y} = IRB\int_0^{\pi}\sin\theta\,d\theta = 2IRB$
- $\vec{F}_2 = 2IRB\,\hat{\jmath}$
- $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 + 2IRB\,\hat{\jmath} + ILB\,\hat{\jmath} = IB\,(2R + L)\,\hat{\jmath}$
- Note: We can see from the picture that by symmetry, the x-component of $\vec{F}_2$ would be zero. The result of $\vec{F}_2$ is the force that would be exerted if we replaced the semicircle with a straight wire of length $2R$ along the x-axis.
- The result reinforces the concept of Concept Question 3.1.

FYI: Detection of Charge Element Particles

- An example of the film photo from Fermilab Bubble Chamber, indicating charged particles' trajectory in a magnetic field.



- Why is the trajectory helical, instead of a circular one?

FYI: Application of magnetic force: Magnetic Levitation Train (MAGLEV)

- Typical speed 350 to 430 km/hour
- Two different approaches
  - Japan – repulsion
  - Germany – attraction (on and off) using electromagnet
- Cost - $50M Euro per mile
- In future – if we put a vacuum tunnel to reduce the air resistance, 3500 km / hour (London to Tokyo in 3.5 hours)
- How it works? https://www.youtube.com/watch?v=aIwbrZ4knpg

News: World's fastest train (Jan 21 2021)

https://www.traveller.com.au/worlds-fastest-train-china-unveils-620-kmh-maglev-prototype-h1thzl

FYI: Future applications - Speedy travel in a vacuum tube

Evacuated (or Vacuum) tube transport technology that would allow travel from Toronto to Beijing in two hours, travelling at 6500 km per hour, for a cost of $100 round trip. Travel would be “clean, green, fast, comfortable and affordable for all,” and safe, not vulnerable to environmental conditions.

https://www.youtube.com/watch?v=03kVU2FYl6U

Hyperloop Explained

https://youtu.be/zcikLQZI5wQ