Week 11 - Day 2

Microwave Oven

Maxwell's Equation

Concept 1: EM Wave Equations

Concept 2: Poynting Vector (Energy in EM waves)

Concept 3: Polarization of Light

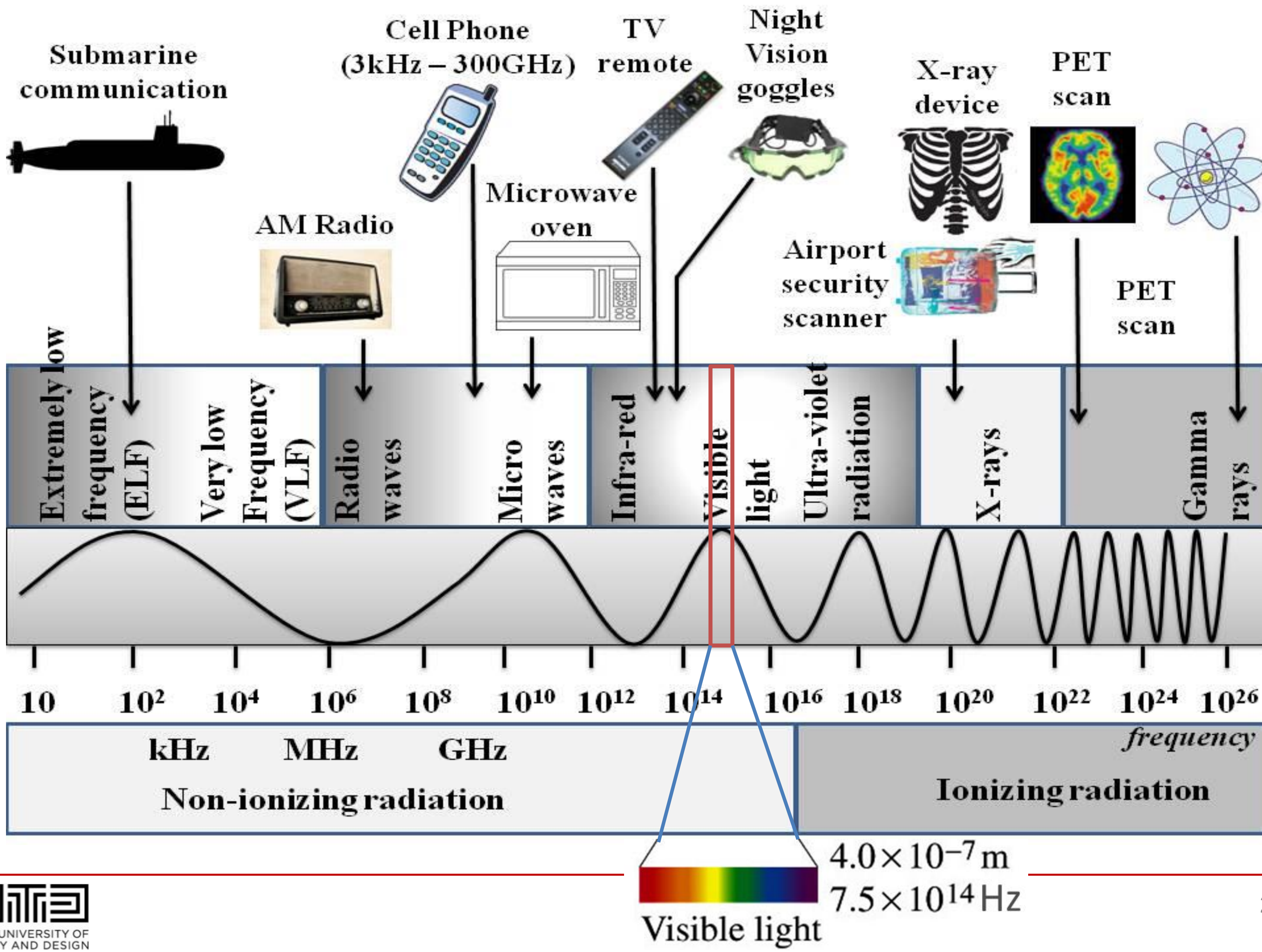
Magnetron (FYI)

Reading:

University Physics with Modern Physics – Chapter 32

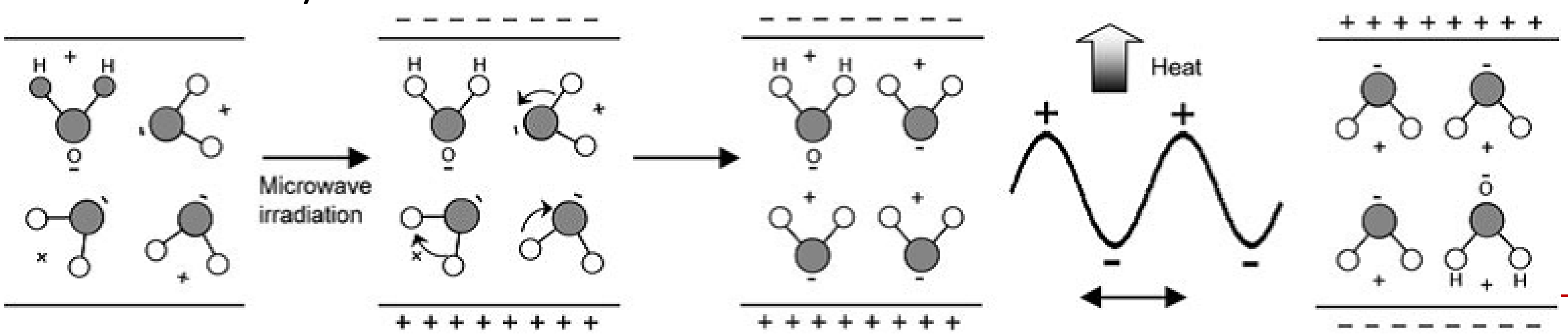
Introduction to Electricity and Magnetism – Chapter 13.3 – 13.6

Electromagnetic Spectrum



Application: Microwave Oven – Interaction of EM Waves with Water Molecules

- Microwave oven uses 2.45 G-Hz EM wave radiation to twist the food's water molecules back and forth billions of times per second. As the water molecules turn, they bump into one another and heat up rapidly, absorbing the microwaves and converts their energy into thermal energy.
- Water molecule can absorb energy from the microwaves because they are electric dipoles (learnt in Week 1!). As the microwaves consist of oscillations of electric field (with magnetic fields), electric dipole molecules tend to align themselves with electric field.
- Glass, ceramics etc. which do not have the freely rotating dipoles are consequently barely heated in a microwave oven.
- 2.45G-Hz is used not because of any resonant effect but simply because it is not in use for communication or radar systems, and this frequency can cook food uniformly.
- The transformer in the oven raises the 220V household voltage to 3,000V or more and delivers it to a magnetron. The magnetron generates microwaves, sending them through a waveguide into the cooking cavity. The stirrer at the end of waveguide distributes the microwaves evenly.



Heating mechanism of water due to microwave field.

We see huge applications of EM wave from its entire spectrum.
But, how does EM wave exist at the first place?

Concept 1: EM Wave Equations

Now, we want to prove that the Four Maxwell's Equations can lead to the EM wave equation!

Recall: 1D Wave Equation

- In general, a wave equation is a partial differential equation, with the following form:

$$\boxed{\frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}}$$

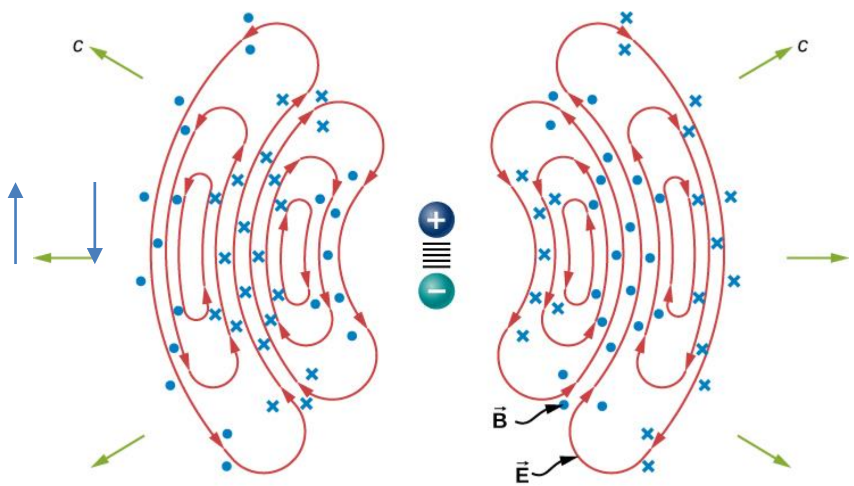
- $y(x,t)$ is a function of position, x and time, t.
  - This function is what we want to find out
  - It is the solution of this wave equation
- v^2 is a constant, where v is the speed of the wave.
- It turns out that the general solution needs to be:

$$y(x,t) = y(x \pm vt)$$

- This function, $y(x \pm vt)$ is called travelling function/wave.
- **Any functions of $\boldsymbol{y(x+vt)}$ or $\boldsymbol{y(x-vt)}$ or a <u>linear combination of both</u> (superposition principle) is a solution of the one-dimensional wave equation.**
- Example:
- $y(x,t) = y_o \sin(kx - \omega t)$ and $y(x,t) = y_o \cos(kx - \omega t)$ are one of the many solutions of the wave differential equation.

Production of EM waves – A Qualitative View

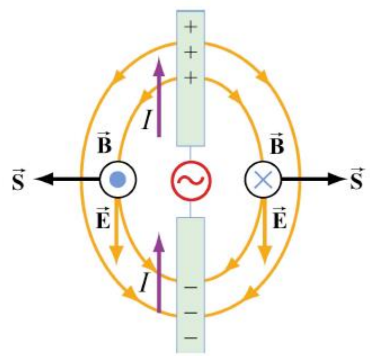
- **Electromagnetic waves** are produced when electric charges are accelerated.
- A common way of producing EM waves is to apply a sinusoidal voltage source or current to an antenna, causing the charges to accumulate near the tips of the antenna. The effect is to produce an **oscillating** electric dipole.

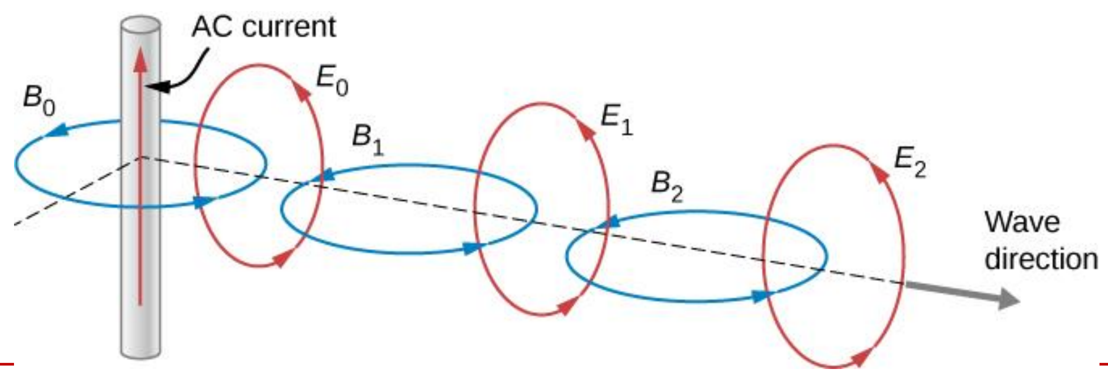


The oscillatory motion of the charges in a dipole antenna produces electromagnetic radiation.

EM waves Production and Propagation: The Concept

- Imagine a time-varying magnetic field $B_o(t)$ produced by the high-frequency alternating current, as shown in the diagram.
- From Faraday's law, the changing magnetic field through a surface induces a time-varying electric field $E_0(t)$ at the boundary of that surface. A field line representation of $E_0(t)$ is shown.
- In turn, the changing electric field $E_0(t)$ creates a magnetic field $B_1(t)$ according to the modified Ampère's law. This changing field induces $E_1(t)$, which induces $B_2(t)$, and so on.
- This self-continuing process leads to the creation of time-varying electric and magnetic fields in regions farther and farther away from the origin. This process may be visualized as the propagation of an EM wave through space.

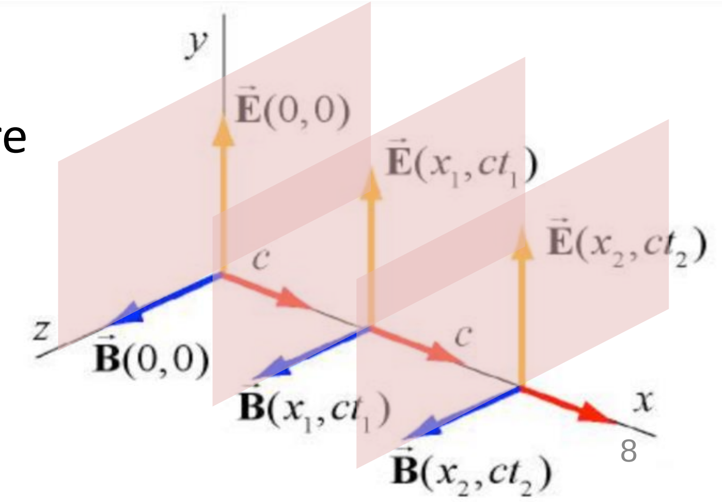




Qualitative visualization of changing $\vec{E}$ and $\vec{B}$ fields propagate through space.

Plane Electromagnetic (EM) Waves in One Direction

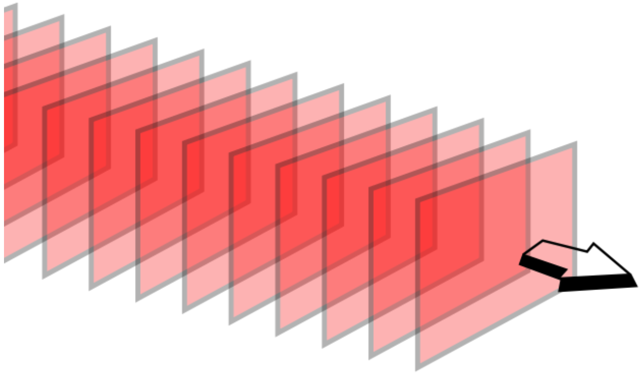
- EM waves consists of an electric field and a magnetic field.
- A general treatment of Maxwell Equations and EM waves is beyond our scope. However, we can investigate a special case: EM wave propagates through free space along one direction, say x-axis → plane EM wave.
- Due to the transverse nature of EM waves, we first assume the electromagnetic field is $\vec{E}(x,t) = E_y(x,y)\,\hat{\jmath}$ and $\vec{B}(x,t) = B_z(x,t)\,\hat{k}$.
- $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of propagation.
- Both expressions say that the EM wave is propagating in the *x*-direction. The electric field $\vec{E}$ of the EM wave is only pointing in the *y*-direction and the magnetic field $\vec{B}$ is only pointing in the *z*-direction.
- It also implies that at any instant both **E** and **B** are **uniform** over the *y*-*z* plane perpendicular to the direction of propagation (*x direction)*.



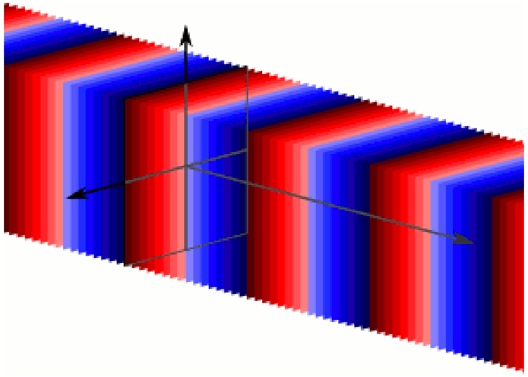
SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

FYI: Plane Wave In General

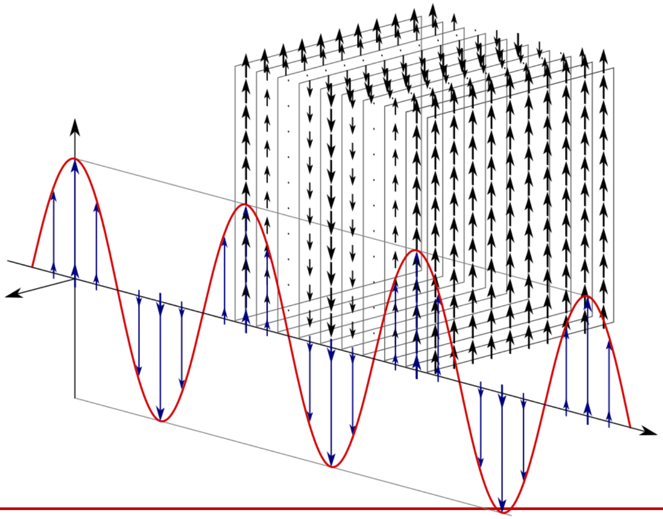
- Consider a special case of wave: Plane Wave



The wavefronts of a plane wave traveling in 3-space



Animation of a 3D plane wave. Each color represents a different phase of the wave.



SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

Maxwell's Equations in Free Space

- We further assume free space, there are no free charge and current. So, in Maxwell's Eq. we have:

1. Gauss's Law
$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{\cancel{Q_{enc}}^{\,0}}{\varepsilon_o}$$

2. Magnetic Gauss's Law
$$\oiint_S \vec{B} \cdot d\vec{A} = 0$$

3. Faraday's Law
$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

4. Ampere-Maxwell Law
$$\oint_C \vec{B} \cdot d\vec{s} = \cancel{\mu_o I_{enc}}^{\,0} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

- Does no free charge and current mean no E and B field in free space? NOT always necessary!!!

Maxwell's Equations in Free Space

- With no free charge and current in free space, we now want to show that the 4 Maxwell's Eq. can lead to the wave equation.

$$\oiint_S \vec{E} \cdot d\vec{A} = 0$$

$$\oiint_S \vec{B} \cdot d\vec{A} = 0$$

$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

$$\frac{\partial^2 B_z(x,t)}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 B_z(x,t)}{\partial t^2}$$

Note: The 1st and 2nd eq. may not be useful here.

- Recall the 1D general wave equation:

$$\frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}$$

- If it is true, the electric and magnetic components of the EM wave obey wave behavior.

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

- Consider a rectangular loop which lies in the xy plane, with the left side of the loop at x and the right at x +Δx . The bottom side of the loop is located at y, and the top side of the loop is located at y+Δy.
- Using Faraday's Law

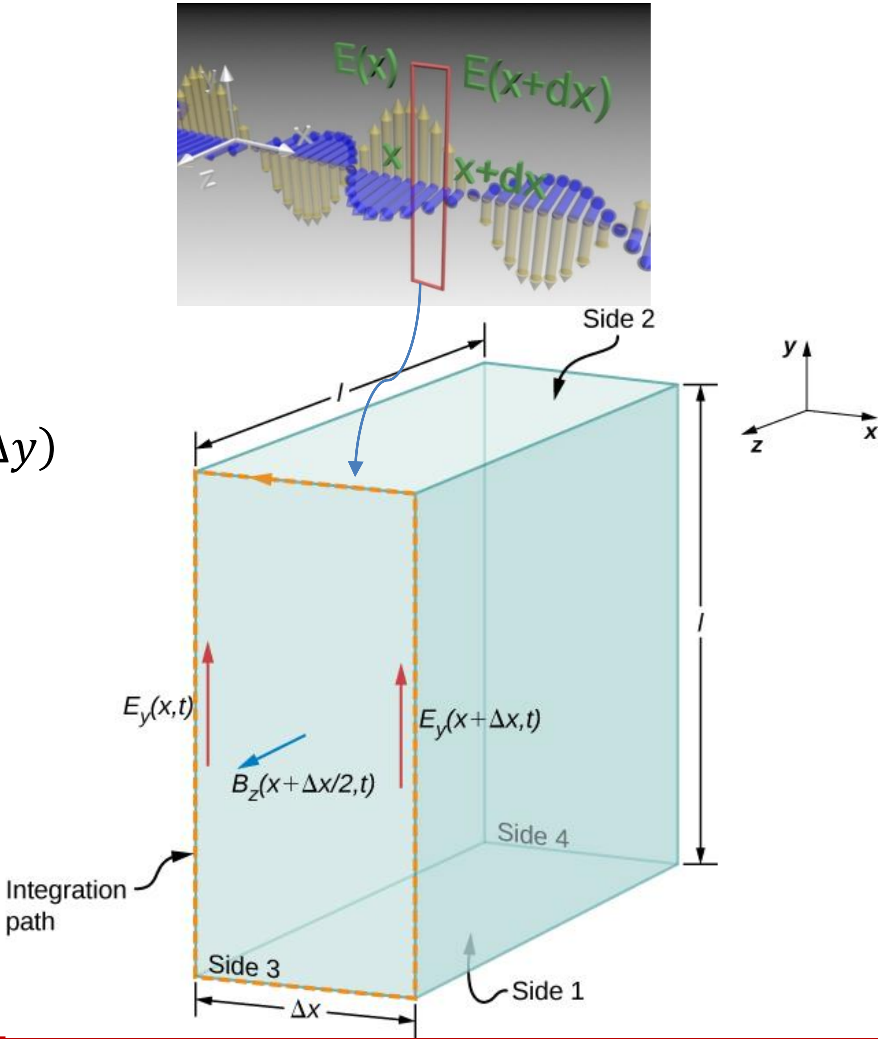
$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_y(x+\Delta x)\Delta y - E_y(x)\Delta y$$

$$-\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{A} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} = -\left(\frac{\partial B_z}{\partial t}\right)(\Delta x \Delta y)$$

$$\therefore \frac{E_y(x+\Delta x) - E_y(x)}{\Delta x} = -\frac{\partial B_z}{\partial t}$$

$$\lim_{\Delta x \to 0} \Rightarrow \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$



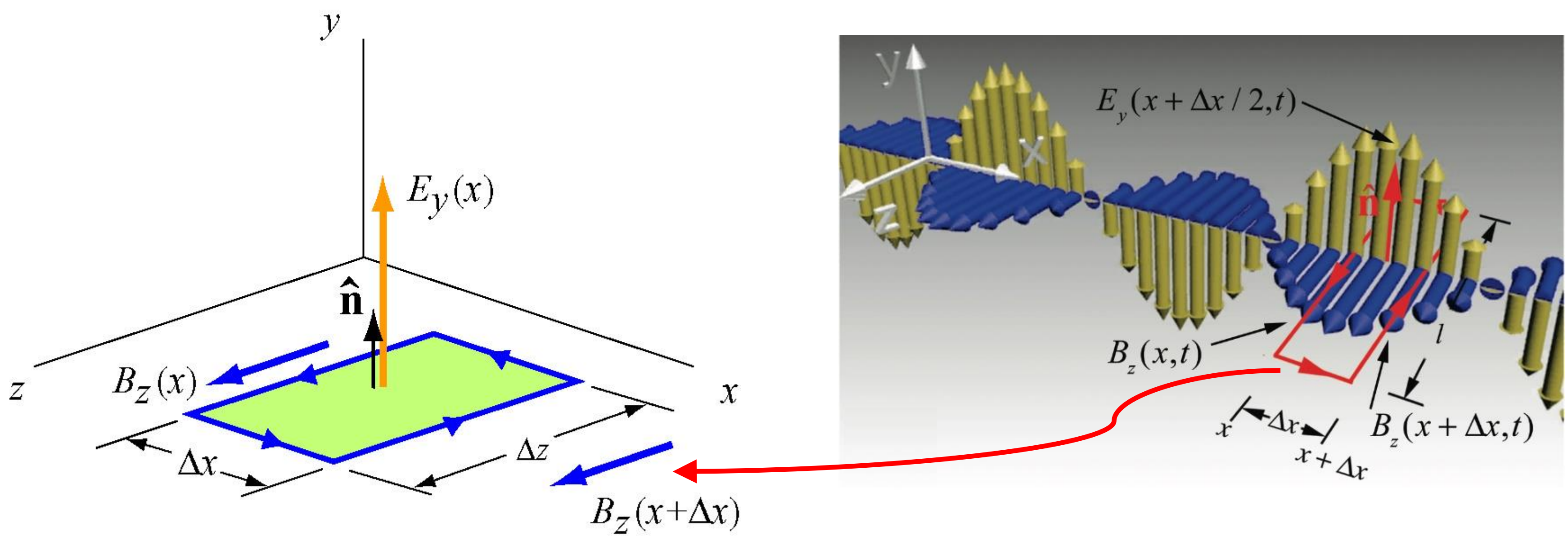
Worked Example

Consider a rectangular loop in the *xz* plane. Show that

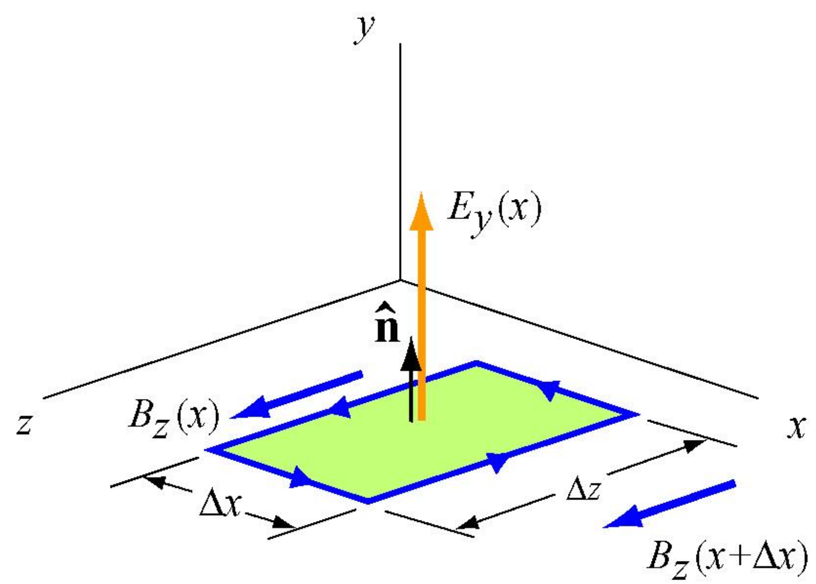
$$-\frac{\partial B_z}{\partial x} = \mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$$

Hint: Start with Ampere-Maxwell Equation and closed loop

$$\oint_C \vec{B} \cdot d\vec{s} = \cancel{\mu_o I_{enc}} + \mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$



- Consider a rectangular loop in the xz plane
- Using Ampere-Maxwell Law



$$\oint_C \vec{B} \cdot d\vec{s} = \cancel{\mu_o I_{enc}} + \mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$

$$\oint_C \vec{B} \cdot d\vec{s} = B_z(x)\Delta z - B_z(x + \Delta x)\Delta z$$

$$\mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A} = \mu_o \varepsilon_o \left(\frac{\partial E_y}{\partial t}\right)(\Delta x \Delta z)$$

$$-\frac{B_z(x + \Delta x) - B_z(x)}{\Delta x} = \mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$$

$$\lim_{\Delta x \to 0} \Rightarrow \quad -\frac{\partial B_z}{\partial x} = \mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$$

Recap:

From Faraday's Law applied to plane wave:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

- This is the equation describing the spatially dependent *E* field produced by the time-dependent *B* field.
- Qualitatively, it says the changing *B* field producing a gradient of *E* field.

- From Ampere-Maxwell's Law applied to plane wave:

$$-\frac{\partial B_z}{\partial x} = \mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$$

- This equation describe the spatially dependent *B* field produced by the time-dependent *E* field.
- Qualitatively, it says the changing *E* field producing a gradient of *B* field.

- Next, we want to combine both equations!

1D Wave Equation for Electric Field

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad (1) \qquad -\frac{\partial B_z}{\partial x} = \mu_o \varepsilon_o \frac{\partial E_y}{\partial t} \quad (2)$$

Take *x*-derivative of Eq.(1) and use the Eq. (2)

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

Do the same thing for B:

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 B_z}{\partial t^2}$$

Recall and compare the general wave eq.

$$\frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}$$

Definition of Constants and Wave Speed

- From the EM wave equation, the speed of EM wave/light in vacuum,

$$\boxed{v = c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}}$$

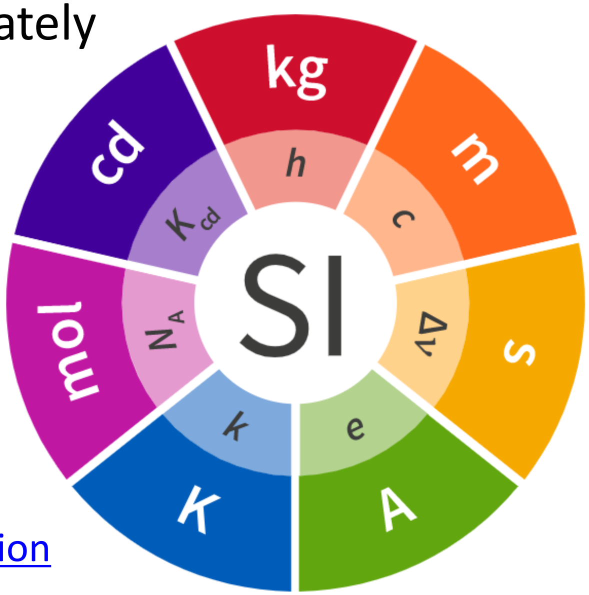
SI Unit Redefinition (2019)

- Speed of light c is precisely defined as $299\ 792\ 458\ ms^{-1}$
  – Usually, we use $c \approx 3 \times 10^8\ ms^{-1}$
- Permeability of free space is approximately

$$\mu_o \approx 4\pi \times 10^{-7}\ Hm^{-1}$$

- Permittivity of free space, $\varepsilon_o = 1/c^2\mu_0$, is approximately

$$\varepsilon_o \approx 8.854 \times 10^{12}\ C^2m^{-1}N^{-1}$$



https://www.nist.gov/content/si-redefinition

A Big Moment: Prediction of Electromagnetic Wave (1864)

- The speed of light has been measured well before the development of EM theory by Maxwell.
- Maxwell himself stated:

  “The velocity is so nearly that of light that it seems we have strong reason to conclude that light itself is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws.”
- In 1887, Heinrich Hertz created EM waves in the laboratory and found that *c* is numerically equal to the speed of light.
- Light is a form of electromagnetic waves!

James Clerk Maxwell

(1831–1879)

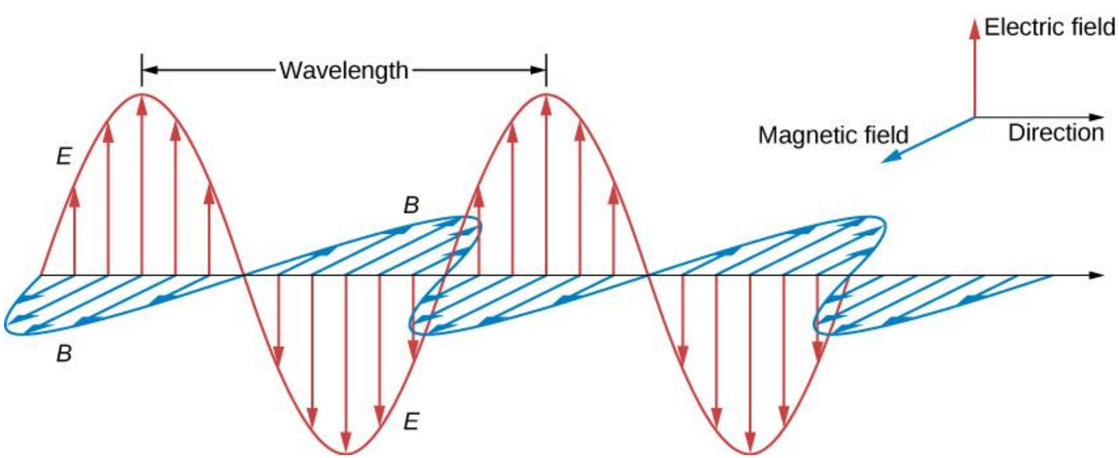
Heinrich Hertz

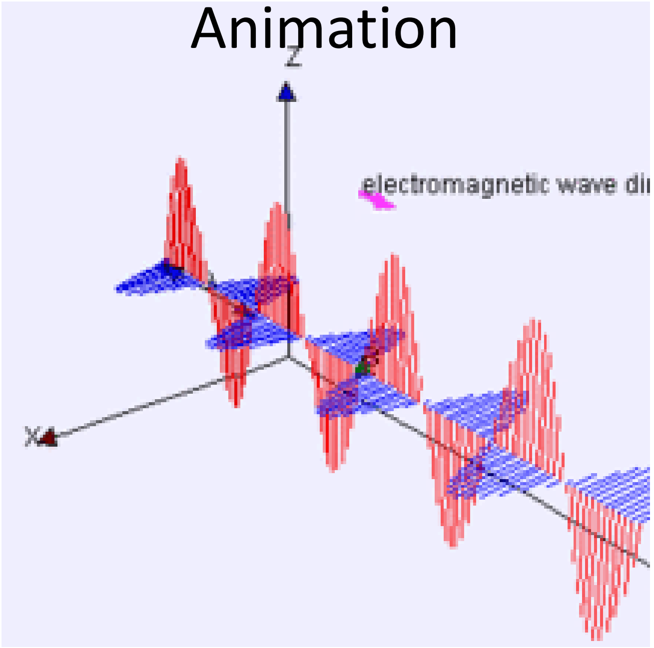
(1857-1894)

Traveling Plane Sinusoidal Electromagnetic Waves

- One useful form of $f(x-ct)$ is a sinusoidal wave function:
- $\vec{E}(x,t) = E_o \sin k(x-ct)\,\hat{j} = E_o \sin(kx-\omega t)\,\hat{j}$
- $\vec{B}(x,t) = B_o \sin k(x-ct)\,\hat{k} = B_o \sin(kx-\omega t)\,\hat{k}$
- Note: $k \equiv \frac{2\pi}{\lambda}$, $\omega \equiv \frac{2\pi}{T} = 2\pi f = kc$. Thus, $c = \frac{\omega}{k} = \frac{\lambda}{T}$
- k is wave number, ω is angular frequency, c is speed of EM wave
- λ is wavelength, f is frequency, T is period

Both $\vec{E}$ and $\vec{B}$ satisfy the wave eq.





Source: https://en.wikipedia.org/wiki/Electromagnetic_radiation#/media/File:Electromagneticwave3D.gif

Case Problem 1.1:

Consider a sinusoidal wave: $\vec{E} = E_o \sin(kx - \omega t)\,\hat{j}$.

Given the relationship between c, k and ω as ;

$$c = \frac{\omega}{k}$$

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

Prove that this function obeys the wave equation:

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$

Case Problem 1.1 Solution:

- Consider a sinusoidal wave: $\vec{E} = E_o \sin(kx - \omega t)\,\hat{j}$.

$$\frac{\partial E_y}{\partial x} = E_o\, k \cos\,(kx - \omega t)$$

$$\frac{\partial^2 E_y}{\partial x^2} = \text{-}E_o\, k^2 \sin(kx - \omega t) \quad \text{............ (1)}$$

$$\frac{\partial E_y}{\partial t} = E_o\,(-w) \cos\,(kx - \omega t)$$

$$\frac{\partial^2 E_y}{\partial t^2} = \text{-}E_o\, w^2 \sin(kx - \omega t) \quad \text{............ (2)}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} \quad \text{............ (3)}$$

Using equations (1) and (2), it can be found that
$\vec{E} = E_o \sin(kx - \omega t)\,\hat{j}$ satisfies the wave equation shown in equation (3) with $\boldsymbol{c} = \frac{\boldsymbol{\omega}}{\boldsymbol{k}}$

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

How the E and B Fields Are Related

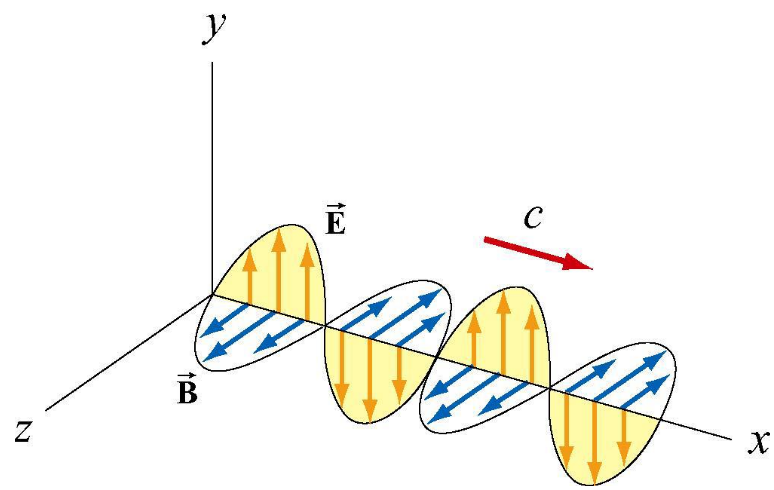
- $\vec{E}(x,t) = E_o \sin(kx - \omega t)\,\hat{j}$
- $\vec{B}(x,t) = B_o \sin(kx - \omega t)\,\hat{k}$
- In Case Problem 1.2, you can show that

$$\frac{E_o}{B_o} = \frac{\omega}{k} = c$$

- Thus, we can also write $\vec{B}(x,t) = \frac{E_o}{c} \sin(kx - \omega t)\,\hat{k}$
- For the transverse nature of EM waves, the E field and the B field is always perpendicular to each other.

Case Problem 1.2: How the E and B Fields Are Related

- $\vec{E}(x,t) = E_o \sin(kx - \omega t)\,\hat{j}$
- $\vec{B}(x,t) = B_o \sin(kx - \omega t)\,\hat{k}$



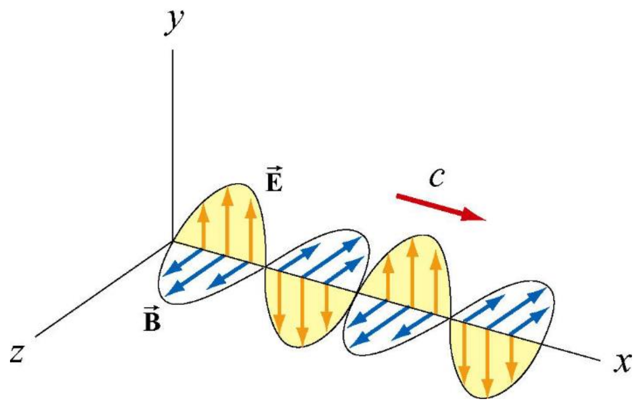
- Prove the following relation:

$$\frac{E_o}{B_o} = \frac{\omega}{k} = c$$

- Do you have to worry the magnetic field of the EM waves emitted by your hand phone or others ?

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

Case Problem 1.2 Solution



- $\vec{E}(x,t) = E_o \sin(kx - \omega t)\,\hat{j}$
- $\vec{B}(x,t) = B_o \sin(kx - \omega t)\,\hat{k}$
- Obtain the below expressions
- $\frac{\partial B_z}{\partial t} = -\omega B_o \cos(kx - \omega t)$; $\frac{\partial E_y}{\partial x} = kE_o \cos(kx - \omega t)$
- $\frac{\partial B_z}{\partial x} = kB_o \cos(kx - \omega t)$; $\frac{\partial E_y}{\partial t} = -\omega E_o \cos(kx - \omega t)$
- We can either use the relation either (1) $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$ or (2) $-\frac{\partial B_z}{\partial x} = \mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad (1)$$

$$kE_o \cos(kx - \omega t) = -(-\omega B_o \cos(kx - \omega t))$$

$$kE_o = \omega B_o$$

$$\frac{E_o}{B_o} = \frac{\omega}{k} = c$$

$$-\frac{\partial B_z}{\partial x} = \mu_o \varepsilon_o \frac{\partial E_y}{\partial t} \quad (2)$$

$$-kB_o \cos(kx - \omega t) = \mu_o \varepsilon_o(-\omega E_o \cos(kx - \omega t))$$

$$\frac{E_o}{B_o} = \frac{k}{\mu_o \varepsilon_o \omega} = c^2\left(\frac{k}{\omega}\right) = \frac{\omega}{k} = c$$

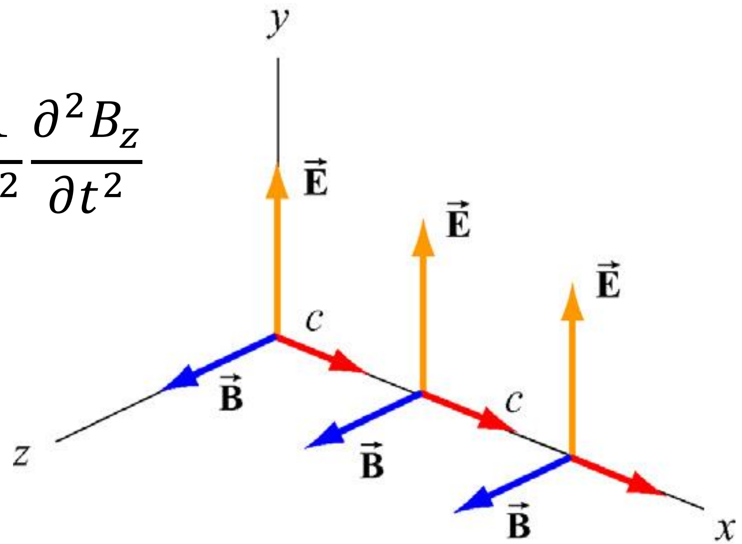
- In application, the electric field is much higher than the magnetic field by a factor of c.

Summary

- In the EM Wave eq., both electric & magnetic fields travel like waves: $f(x \pm vt)$ satisfying:

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} \quad ; \quad \frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}$$

With speed $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$



- Electric and magnetic fields are related by:

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

$$-\frac{\partial B_z}{\partial x} = \mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$$

Summary (cont'd)

- For transverse EM waves, the E and B fields are always perpendicular to each other.
- Direction of $\vec{E} \times \vec{B}$ gives the direction of propagation of the EM wave.
- It is important to express the E and B field direction and the propagation direction correctly, e.g.

$$\vec{E} = E_o \cos(kx - \omega t)\,\hat{j}$$

$$\vec{B} = \frac{E_o}{c} \cos(kx - \omega t)\,\hat{k}$$

- $\vec{E} = E_o \cos(kx - \omega t)\,\hat{j}$ itself describes fully all the properties of this EM wave.
  - Amplitude (E_o), wavelength $\left(\lambda = \frac{2\pi}{k}\right)$, frequency $\left(f = \frac{\omega}{2\pi}\right)$ and speed $\left(c = \frac{\omega}{k}\right)$.
  - E field oscillation direction $(\hat{j})$ and the wave propagation direction $(+x)$.
  - Therefore, we also know the B field oscillation direction (even though not explicitly written out)

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

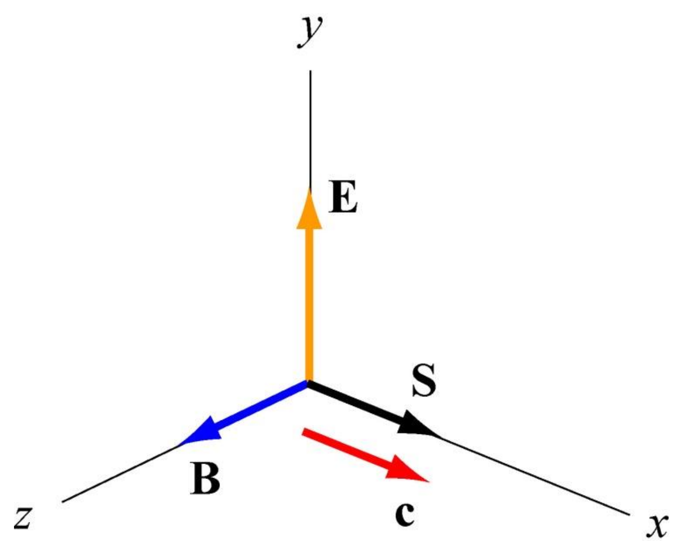
Concept 2: Poynting Vector - Energy in EM waves

Poynting Vector and Intensity

- Direction of energy flow = direction of wave propagation

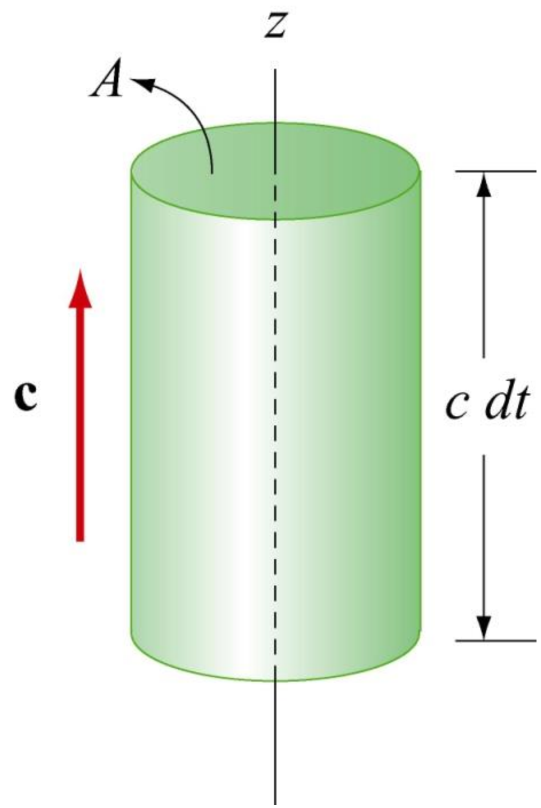
$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_o}$$

- $\vec{S}$ is known as Poynthing Vector.
- Unit: Joules per square meter per sec or Watts per square meter



Proof: Energy in EM Waves

- Energy densities: $u_E = \frac{1}{2}\varepsilon_o E^2$, $u_B = \frac{1}{2\mu_o} B^2$
- Consider cylinder:
- $dU = (u_E + u_B)Adz = \frac{1}{2}\left(\varepsilon_o E^2 + \frac{B^2}{\mu_o}\right)Acdt$
- What is rate of energy flow per unit area?
- $S = \frac{1}{A}\frac{dU}{dt} = \frac{c}{2}\left(\varepsilon_o E^2 + \frac{B^2}{\mu_o}\right) = \frac{c}{2}\left(\varepsilon_o cEB + \frac{EB}{c\mu_o}\right)$
- $= \frac{EB}{2\mu_0}(\varepsilon_0\mu_0 c^2 + 1)$
- $= \frac{EB}{\mu_o} \rightarrow \vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_o}$

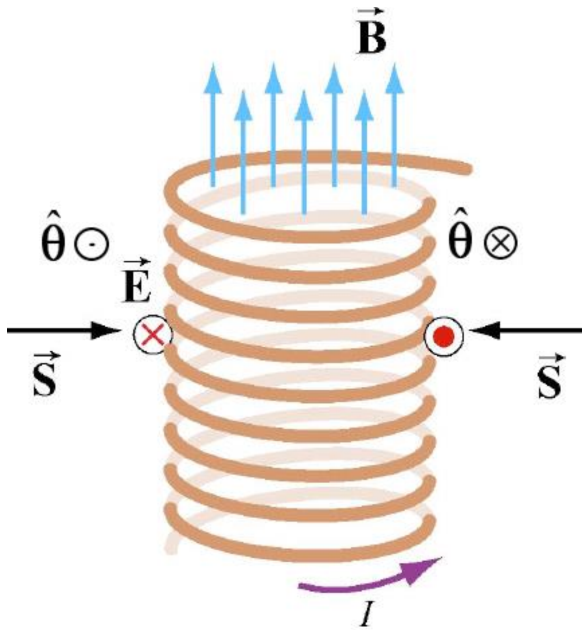


Note:
$$E = cB$$
$$c = \frac{1}{\sqrt{\mu_o\varepsilon_o}}$$

Case Problem 2.1

(a) Consider a very long solenoid (see figure), using Ampere law to calculate **B** and Faraday law to calculate **E** as indicated in the figure. The current *I* is a function of time.

(b) From the calculate **E** and **B**, calculate the Polynting vector **S**



Case Problem 2.1 Solution

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o I_{enc} \quad \rightarrow \quad Bl = \mu_o (NI)$$

- By Ampere's Law, the magnetic field inside a solenoid, $\vec{B} = \mu_o nI\, \hat{k}$
- Thus, the rate of increase of the magnitude of the magnetic field is

$$\frac{dB}{dt} = \mu_o n \frac{dI}{dt}$$

- According to Faraday's Law, changing magnetic flux results in an induced electric field

$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

- Calculating and equating the two sides of Faraday's Law yields

$$E 2\pi r = -\mu_o n \frac{dI}{dt} \pi r^2$$

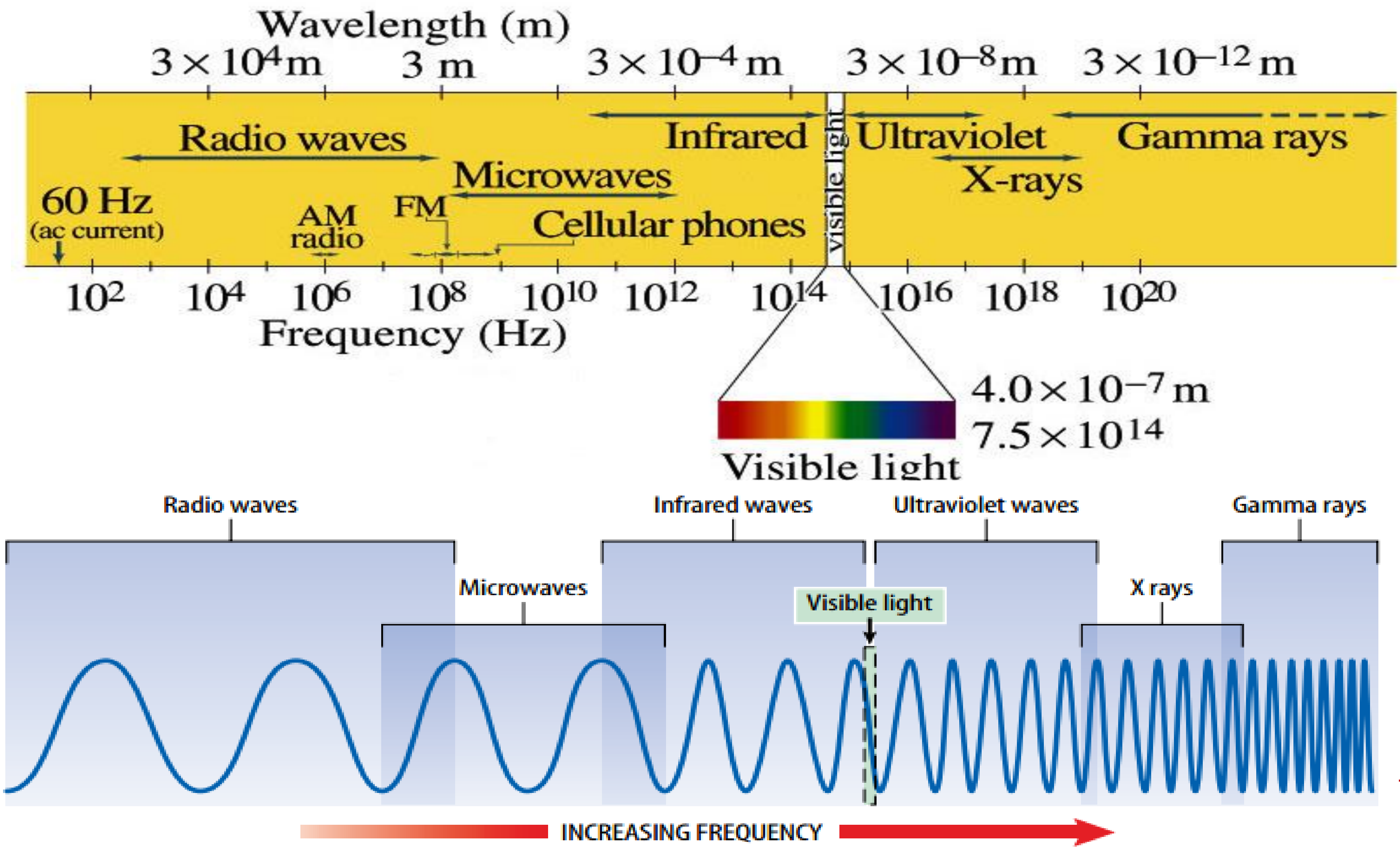
- The electric field is therefore

$$\vec{E} = -\frac{\mu_o n r}{2} \frac{dI}{dt}\, \hat{\theta}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_o} = \frac{1}{\mu_o}\left(-\frac{\mu_o n r}{2}\frac{dI}{dt}\,\hat{\theta}\right) \times \mu_o n I\, \hat{k} = -\frac{\mu_o n^2 r I}{2}\frac{dI}{dt}\,\hat{r}$$

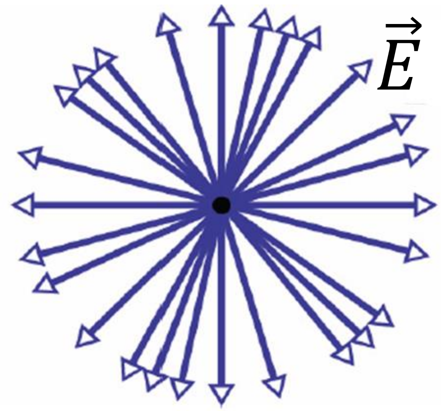
Electromagnetic Spectrum

- Wavelength and frequency are related by: $c = f\lambda$



Concept 3: Polarization of Light

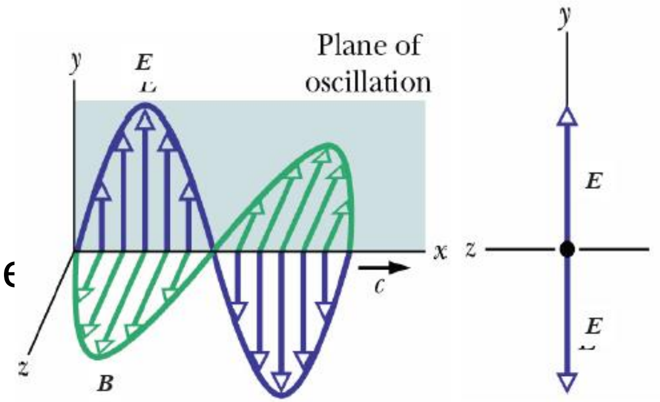
Polarization of Light



- **Electromagnetic** waves are usually generated by oscillating electric charges and the direction of oscillation determines the orientation of the waves electric field.
- In most common light sources, the oscillating atoms are randomly orientated E.
- Although electric field of each wave lies in a single plane, the overall beam contains electric fields oscillating in all planes ⇒ known as unpolarized waves.

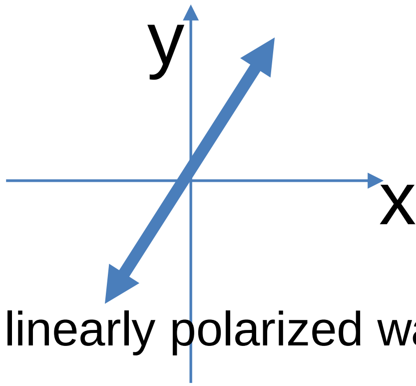
Linear Polarization

- If oscillating charges are confined to move in only one plane ⇒electric field also oscillates in that plane ⇒ wave is polarized (see picture in y-direction).
- A wave where the $\vec{E}$ field vibrates in the same direction at all times is said to be linearly polarized and the plane of oscillation is called the plane of polarization.

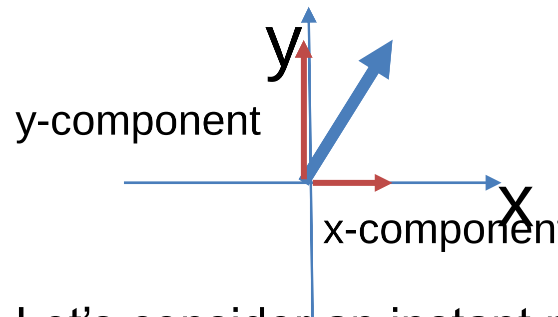


We only draw E (ignore B), simply because B is always associated to E.

- Depends on the coordinate system that we set, a linearly polarized wave(light) can be decomposed into x and y components. E field is a vector.

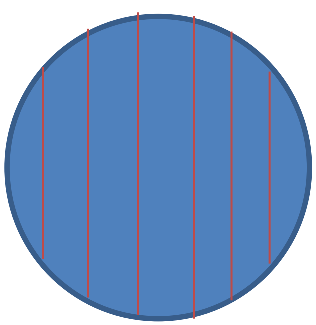


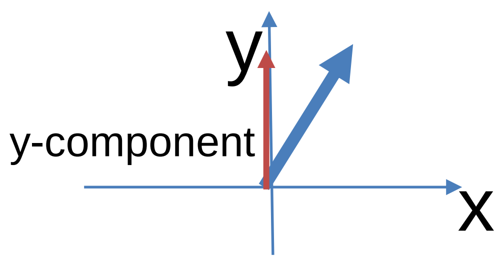
A linearly polarized wave

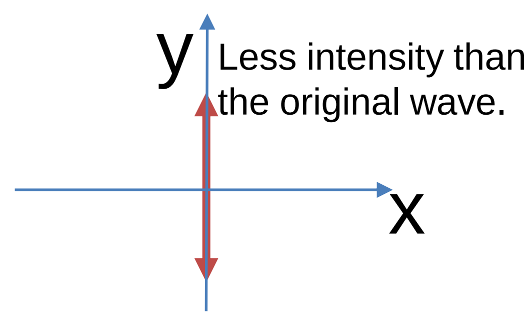


Let’s consider an instant moment when the E field is positive.

- If the polarizer is set to be in vertical direction, only the y-component of the E field can go through.



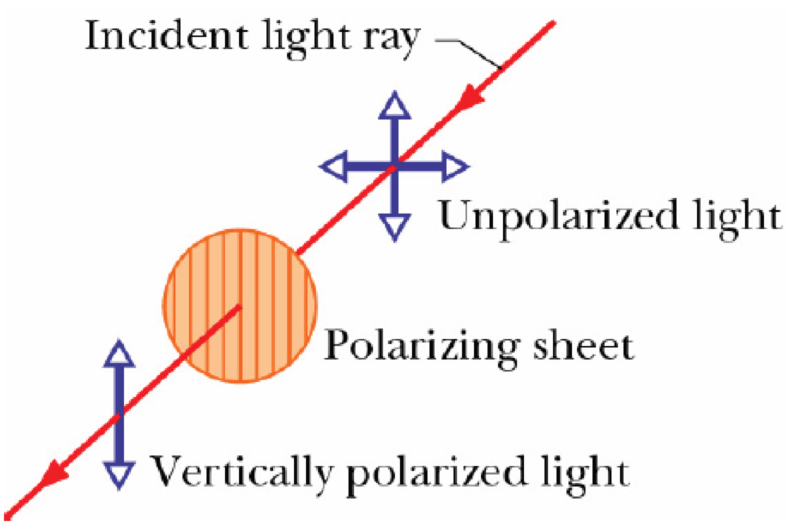


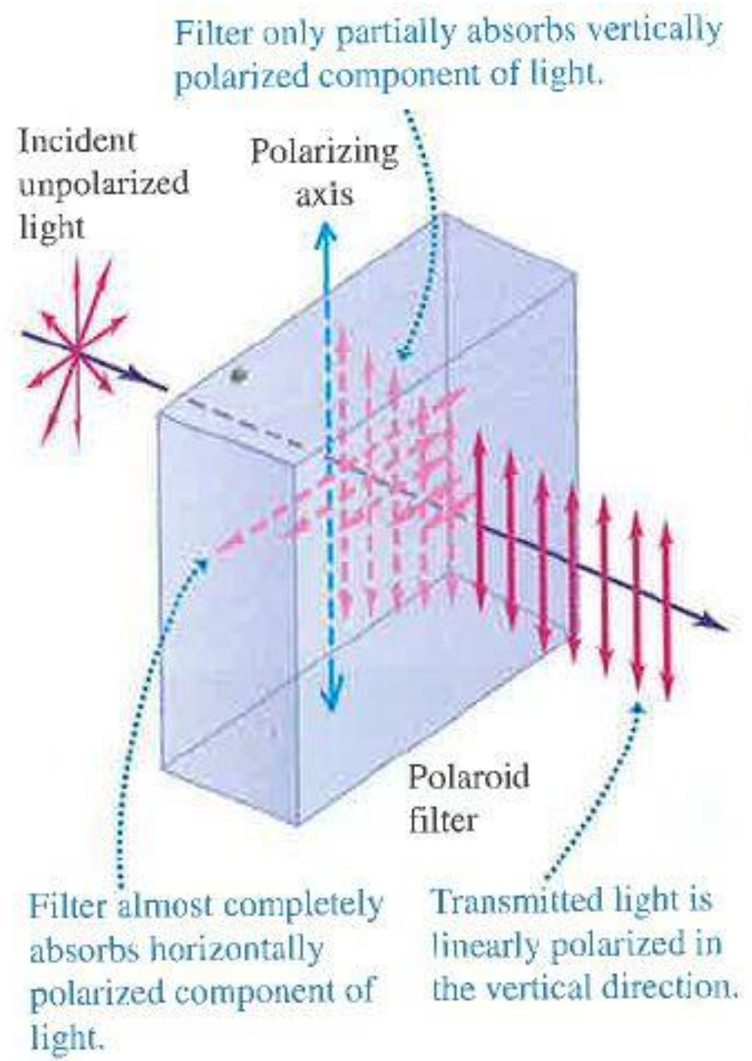


SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

Polarization by Selective Absorption

- Transform unpolarized light into polarized light by sending it through a polarizing sheet or filters
- The transmitted intensity is 50%
- It was invented in 1932 by Edwind Land while he was an undergraduate student.

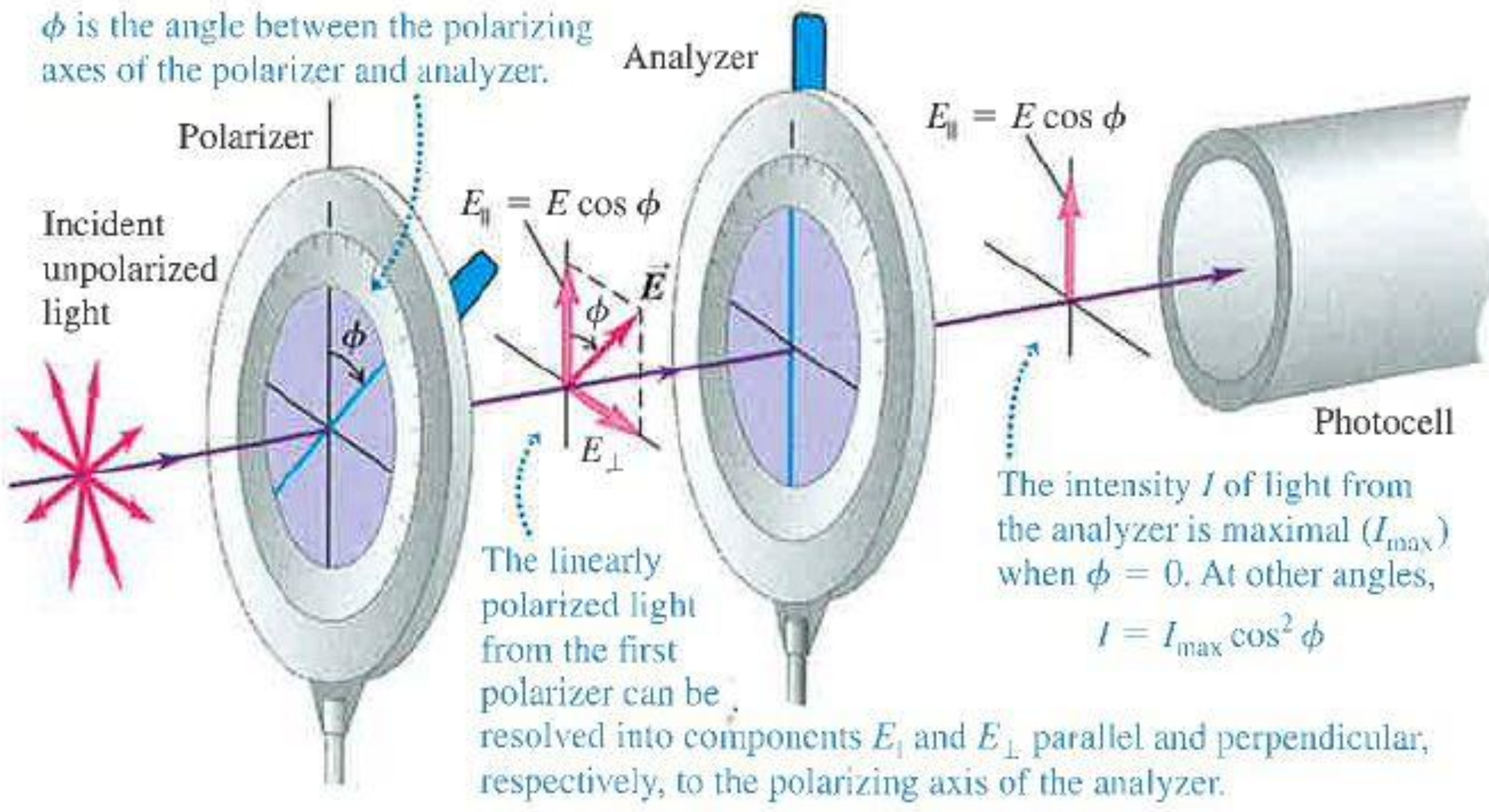




Intensity of the polarized light

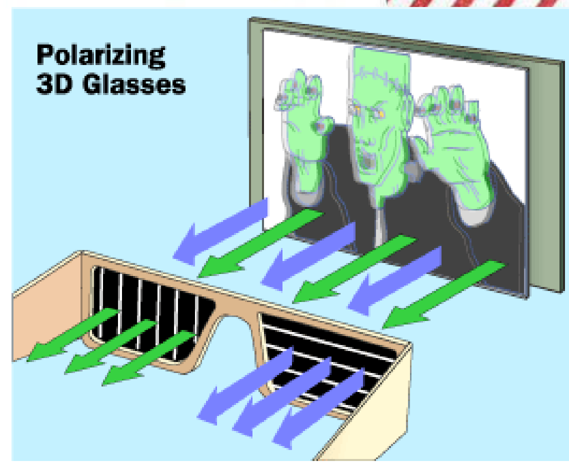
- Malus's law, polarized light passing through an analyzer.

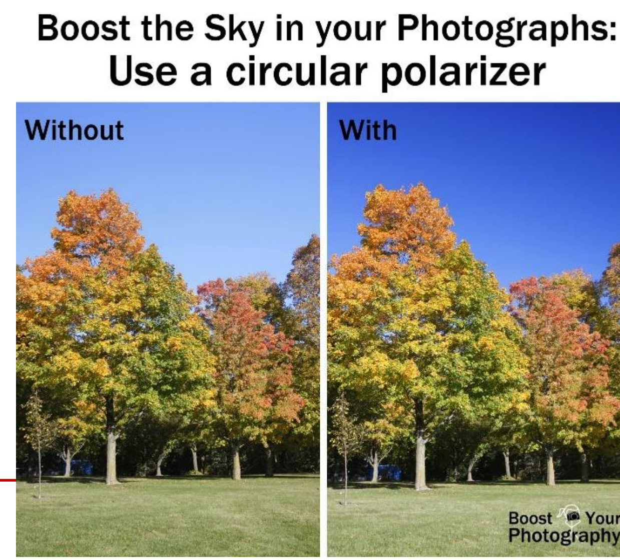
$$I = I_{max} \cos^2 \phi$$



Examples and Demo

- These photos show the view through Polaroid sunglasses whose polarizing axes are (left) aligned ($\phi = 0$) and (right) perpendicular ($\phi = 90^o$). The transmitted intensity is greatest when the axes are aligned; it is zero when the axes are perpendicular.
- 3D movie is based on the polarization effect. 3D glasses are polarizers.
- A polarizer prevents polarized light from entering the lens without affecting the color temperature. When rotated to the appropriate angle, the filter can largely eliminate reflections off glass or water. It can reduce glare and excess skylight, thereby increasing the sharpness, contrast, and color saturation of a captured image.





Additional Materials

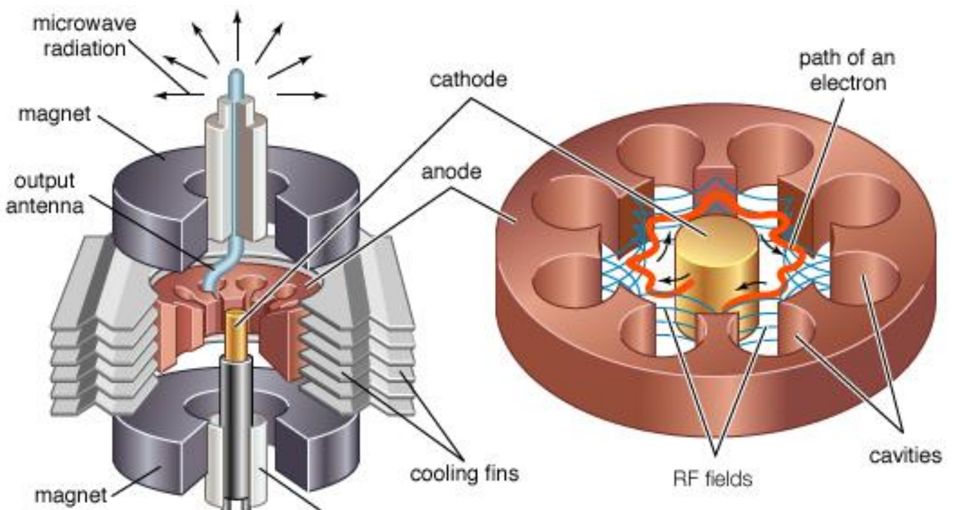
Euler's Formula: $e^{ix} = \cos x + i \sin x$

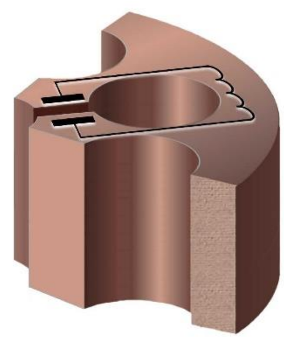
- Euler's formula is ubiquitous in engineering, such as signal processing, wave analysis, etc.
- $e^{ix} = \cos x + i \sin x$ is a complex number. $\cos x$ is the real part, $\sin x$ is the imaginary part. Note that we can write $\cos x = Re\{e^{ix}\}$; $\sin x = Im\{e^{ix}\}$.
- e^{ix} is usually used to describe a sinusoidal function ($\cos x$ or $\sin x$).
- It can be used to describe our plane wave too. Example:
- $\vec{E} = E_o \cos(kx - \omega t)\,\hat{j} = \mathrm{Re}\{E_o e^{i(kx-\omega t)}\}\,\hat{j}$
- It helps us to transform a trigonometry or a sinusoidal function into an exponential function. We can deal with e function with easier math.
- Example: we know $\frac{d}{dx}\cos x = -\sin x$. Using Euler's formula, we can write, $\frac{d}{dx}\cos x = \frac{\mathrm{d}}{\mathrm{dx}} Re\{e^{ix}\} = \mathrm{Re}\left\{\frac{d}{dx} e^{ix}\right\} = Re\{ie^{ix}\} = Re\{\, i(\cos x + i \sin x)\} = Re\{-\sin x + i \cos x\} = -\sin x$

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

FYI: Application - Magnetron

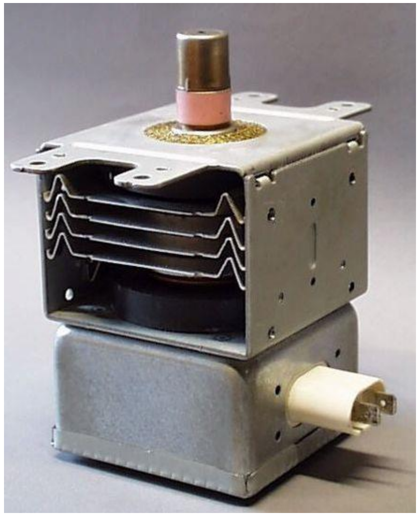
- The device that generates microwave is called magnetron.
- A magnetron consists of two parts: the tube core and permanent magnets.
  In the tube core, the filament (cathode of the tube) which is carefully sealed into the tube is placed at the centre of the magnetron. The anode is a hollow cylinder of copper block which surrounds the cathode and the tube core.
- A magnetic field (by the permanent magnets) is in parallel to the filament. When the electrons are accelerated from cathode to anode, they spiral outward in a curve rather than moving directly to the anode (by Lorentz Force!).
- The anode block of the tube includes many cylindrical cavities, which each serve as a resonant LC circuit, causing electrons to oscillate at the resonant frequency. Microwaves are emitted by this interaction of the electrons with the magnetic fields and the arrangement cavities. An antenna will transmit a part of the RF energy.
- The resonant frequency of the LC oscillator is defined entirely by the cavity dimensions.



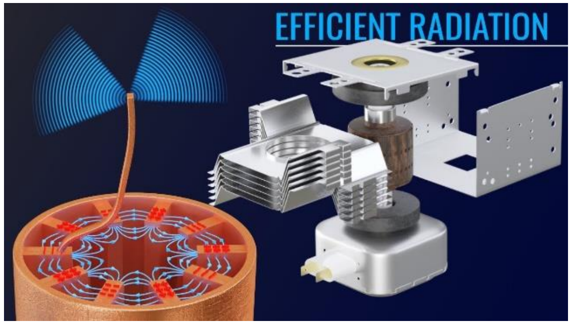


A resonant cavity in the anode block forms a parallel LC resonant circuit: The opposite anode walls of a slot are the capacitor, the detour around the hole is the inductance (with only one turn).

FYI: Magnetron (Extra Information)



Magnetron from a microwave oven with magnet in its mounting box. The horizontal plates form a heat sink, cooled by airflow from a fan. The magnetic field is produced by two powerful ring magnets, the lower of which is just visible. Almost all modern oven magnetrons are of similar layout and appearance. (from Wikipedia)



Magnetron, How does it work?

https://youtu.be/bUsS5KUMLvw

What is a MAGNETRON - How Does it Work

https://youtu.be/5DpYlnHT-0s

FYI: About 5G Network

- https://youtu.be/g-gGeAe-PJA