# Week 2 - Day 1

## Gauss's Law

Concept 1: Electric Field Lines and Flux

Concept 2: Gauss's Law, Gaussian surface and Charge Enclosed

Concept 3: Application of Gauss's Law – Conductor at Equilibrium

Explanation of Faraday Cages, Screened cables

Reading:

University Physics with Modern Physics – Chapter 22

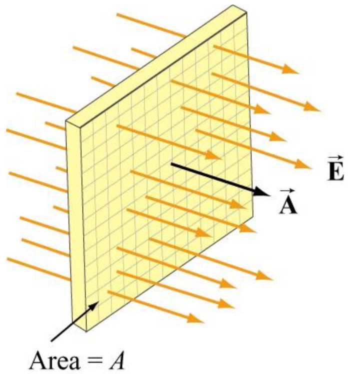
Introduction to Electricity and Magnetism – Chapter 3

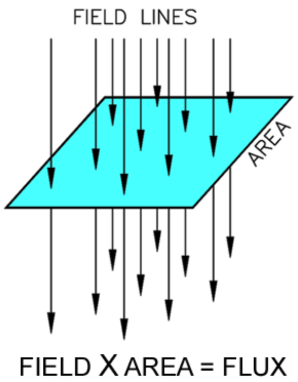
# Concept 1: Electric Field Lines and Flux

To know Gauss's Law, we need to first define and calculate electric flux

# Electric Flux

- You may conceptualize the electric flux, $\Phi_E$ as a measure of the number of electric field lines passing through an area.

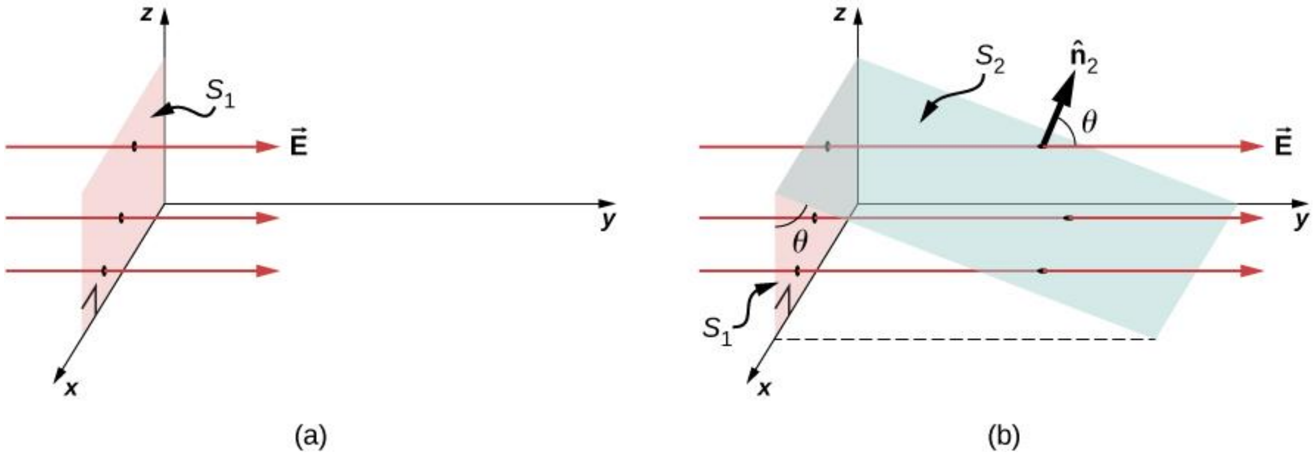



- The larger the area, the more field lines go through it, thus more flux. Similarly, the stronger the electric field is (represented by a greater density of lines), the greater the flux.
- Electric flux, $\Phi_E$ is a scalar quantity. It can be a positive or negative number. The SI unit is newton-meters squared per coulomb ($N \cdot m^2/C$).
- Note: Unlike the general definition of flux, electric flux does not involve the flow rate (time).
- To have an idea, this is somewhat analogous to amount of water flowing across a surface.

# Quantify Electric Flux

- Consider a planar surface $S_1$ of area $A_1$ that is perpendicular to the uniform electric field $\vec{E} = E\hat{\jmath}$, the electric flux through $S_1$ is $\Phi_E = +EA_1$.




- Now consider a surface $S_2$ of area $A_2$ that is inclined at an angle $\theta$ to the xz-plane. Note that because the same number of field lines crosses both $S_1$ and $S_2$, the fluxes through both surfaces must be the same.
- The electric flux through $S_2$ is $\Phi_E = +EA_2\cos(\theta) = +EA_1$.
- We define an area vector pointing along normal direction of a planar surface, i.e. $\vec{A} = A\hat{n}$, then electric flux is given in general:

$$\Phi_E = \vec{E} \cdot \vec{A}$$

# Concept Question 1.1:

- Consider a uniform electric field, $\vec{E}$ that is parallel to a planar surface, $A$.
- Note: We can also say that $\vec{E}$ is perpendicular to surface $\vec{A}$.
- What is the electric flux, $\Phi_E$ that passes through it?

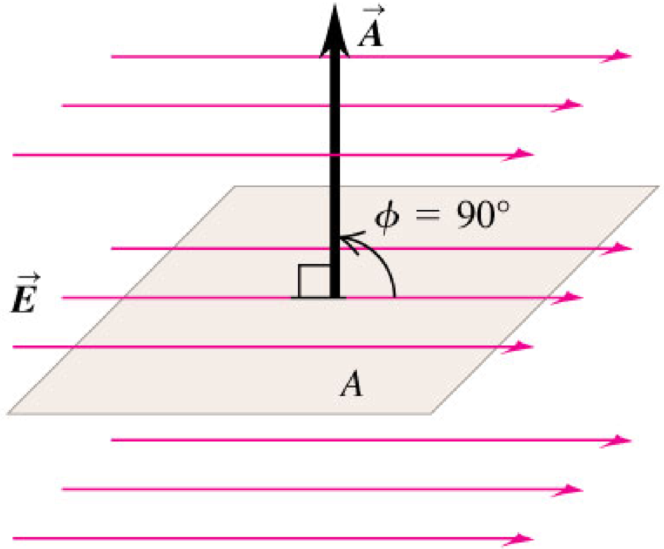
$A.$ $+EA$

$B.$ $-EA$

$C.$ $0$

D. Undefined as this condition is not valid

$E.$ $+EAsin(90^o)$



# Concept Question 1.2: Flux

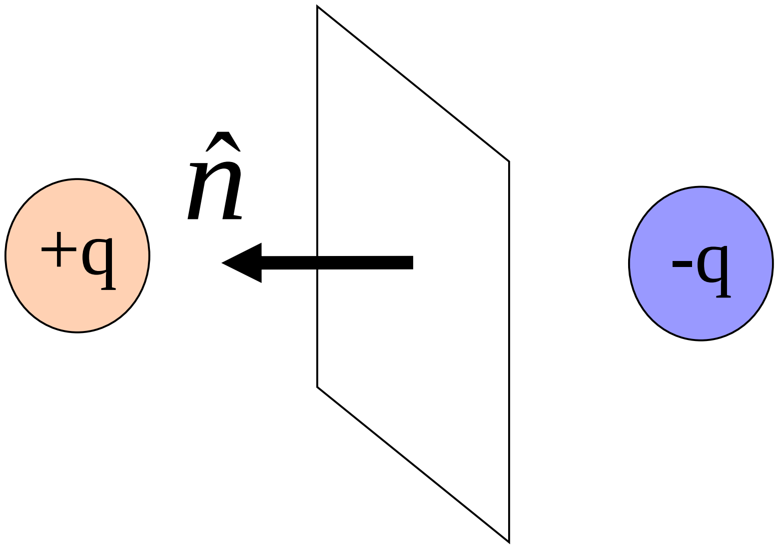
The electric flux through the planar surface below (positive unit normal to left) is:

A. positive.

B. negative.

C. zero.

D. Not well defined.

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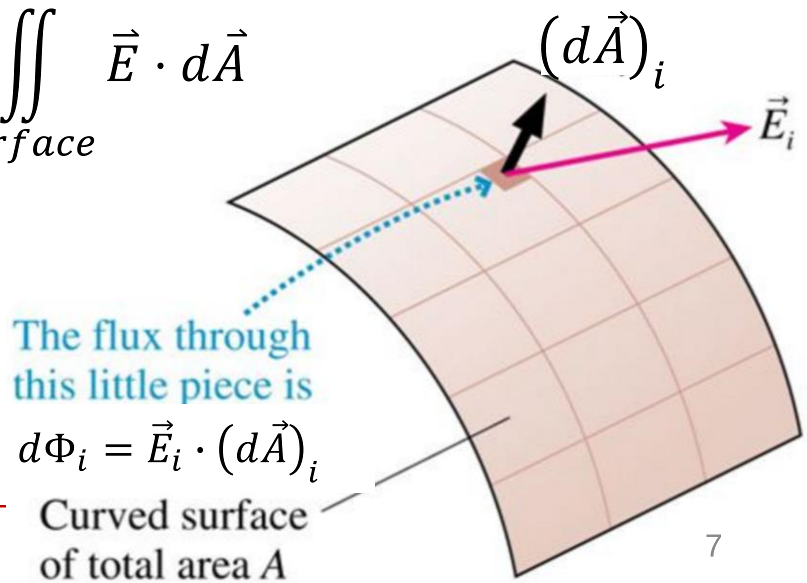
# Quantify Electric Flux: Non-uniform $\vec{E}$ and Curved Surface.

- For non-uniform $\vec{E}$ and a curved surface, we first divide the curved surface into many small pieces of area, $d\vec{A}$.
- At a specific small area, $d\vec{A}_i$, say the electric field is $\vec{E}_i$ (we treat it as constant at that small area), then the electric flux through each small pieces is

$$d\Phi_i = \vec{E}_i \cdot \left(d\vec{A}\right)_i$$

- The total flux that passes through the curved surface would be the sum of many $d\Phi_i$. As the integration form, the electric flux through the whole surface is

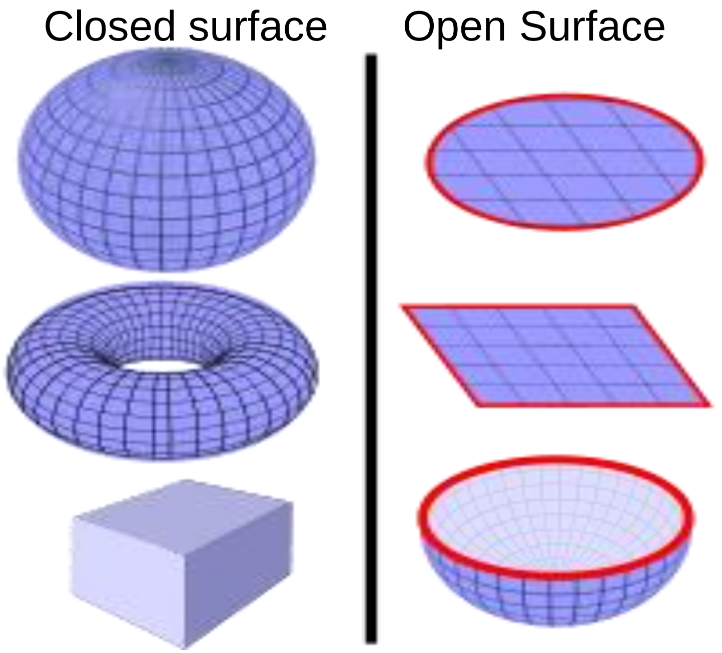
$$\Phi_E = \iint\limits_{surface} d\phi_i = \iint\limits_{surface} \vec{E} \cdot d\vec{A}$$



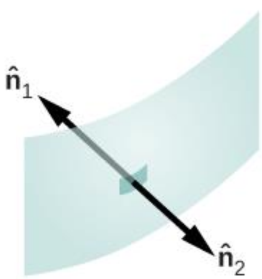

Note: Do not panic about the double integral. It is simply to remind us that we should integrate twice to get a 2D area.

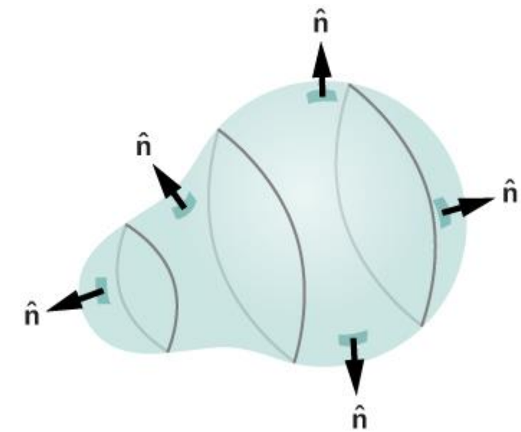
# There are 2 types of Surface: Open and Closed Surfaces

- An open surface
  - Is defined by a closed path boundary.
  - It does NOT contain a volume
  - We can freely define the area vector $\hat{n}$.
- A closed surface
  - contains a volume.
  - It can be made up from many open surfaces.
  - The direction of the normal area vector, $\hat{n}$ always points from inside out.




Note: For an open surface, we can use either direction, as long we are consistent over the entire surface. On a closed surface, $\hat{n}$ is chosen to be the outward normal at every point, to be consistent with the sign convention for electric charge.

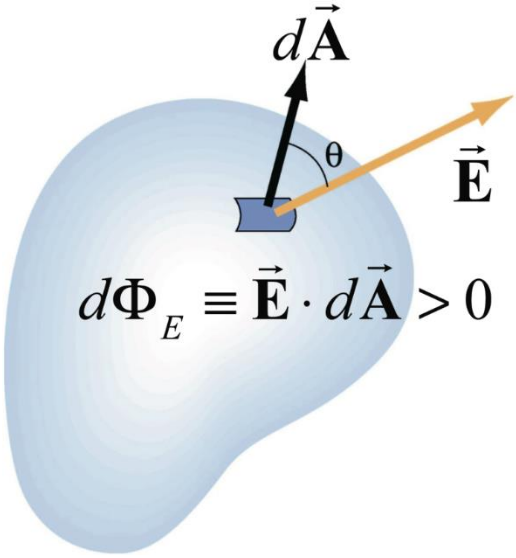


# Quantify Electric Flux: $\vec{E}$ through a closed surface

- Consider a non-uniform $\vec{E}$ passing through a closed (and curved) surface.
  – For closed surface, $d\vec{A}$ is normal to surface and points outward (from inside to outside).
- $d\Phi_E \equiv \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$
- Total flux

$$\Phi_E = \oiint_S d\Phi_E = \oiint_S \vec{E} \cdot d\vec{A}$$




Note:

- $d\Phi_E > 0$ if $\vec{E}$ points outwards
- $d\Phi_E < 0$ if $\vec{E}$ points inwards
- Do not panic about the small little circle in the double integral. It simply means closed surface integral. It is to distinguish between the flux through an open surface and the flux through a closed surface.

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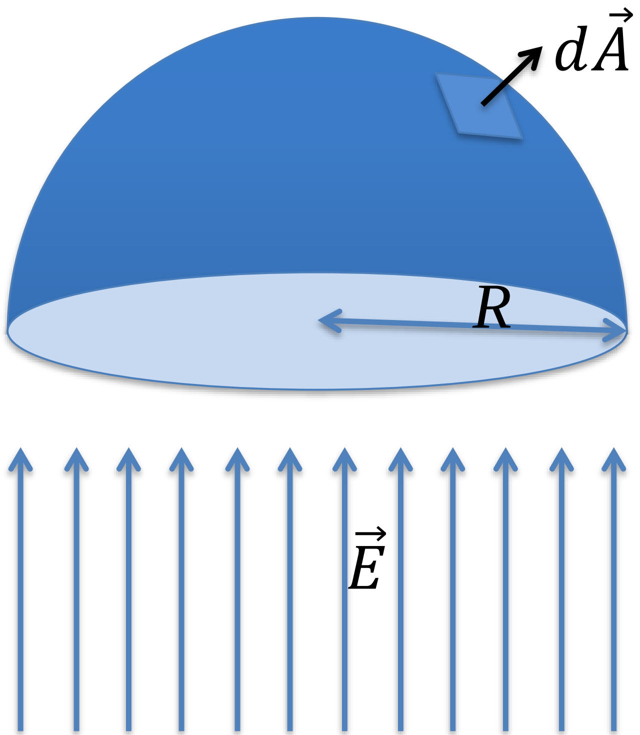
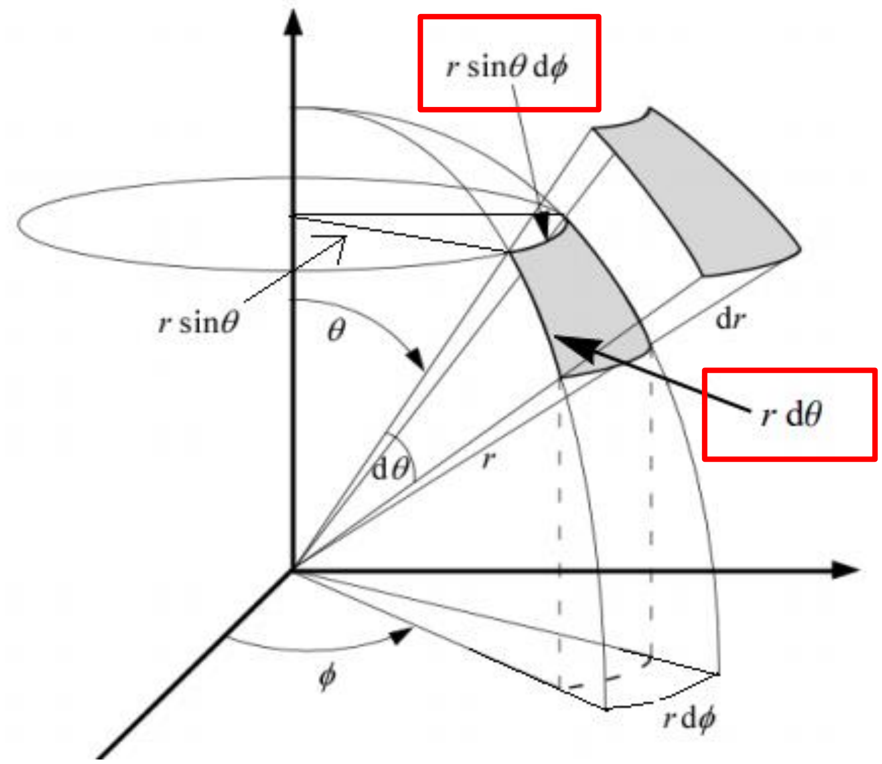
# Case Problem 1.1

- A hemispherical surface with radius R in a region of uniform electric field E is aligned as shown in the figure on the left.
- Based on the definition of electric flux through an open surface, calculate the flux through the hemispherical surface.

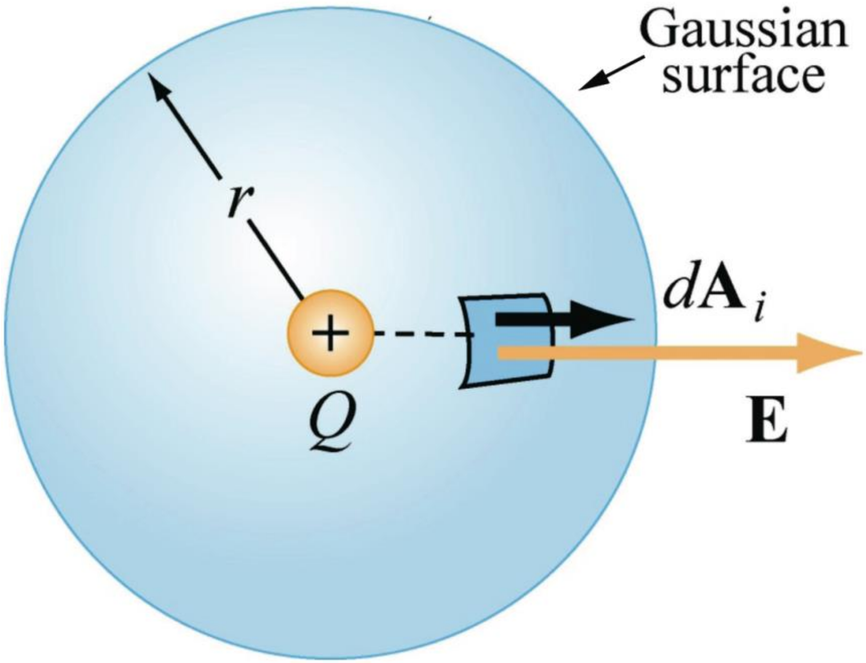


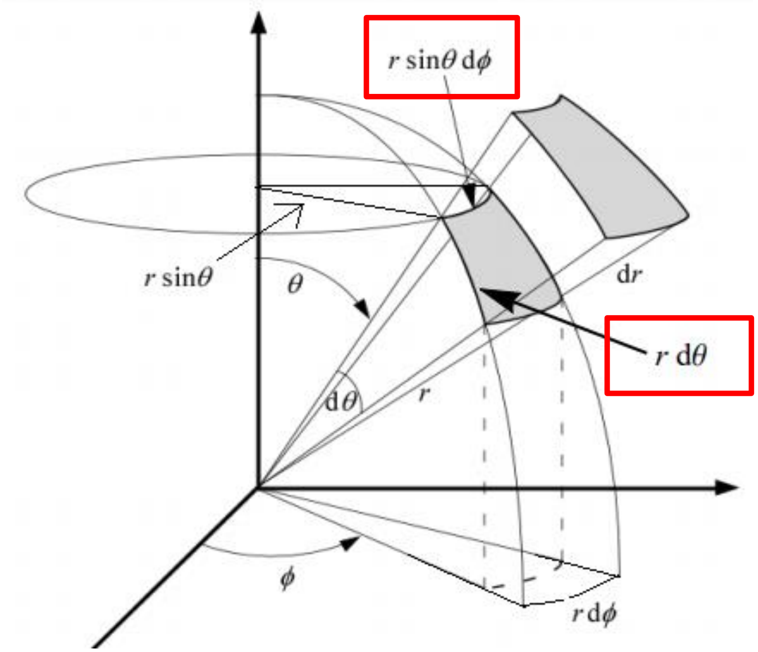




Note: The surface element $d\vec{A}$ in a spherical coordinate is $d\vec{A} = rd\theta\ (rsin\theta d\phi)\hat{r}$

# Case Problem 1.2: Sphere

- Consider a point-like charged object with charge $Q$ located at the origin. What is the electric flux on a spherical surface (Gaussian surface) of radius $r$?

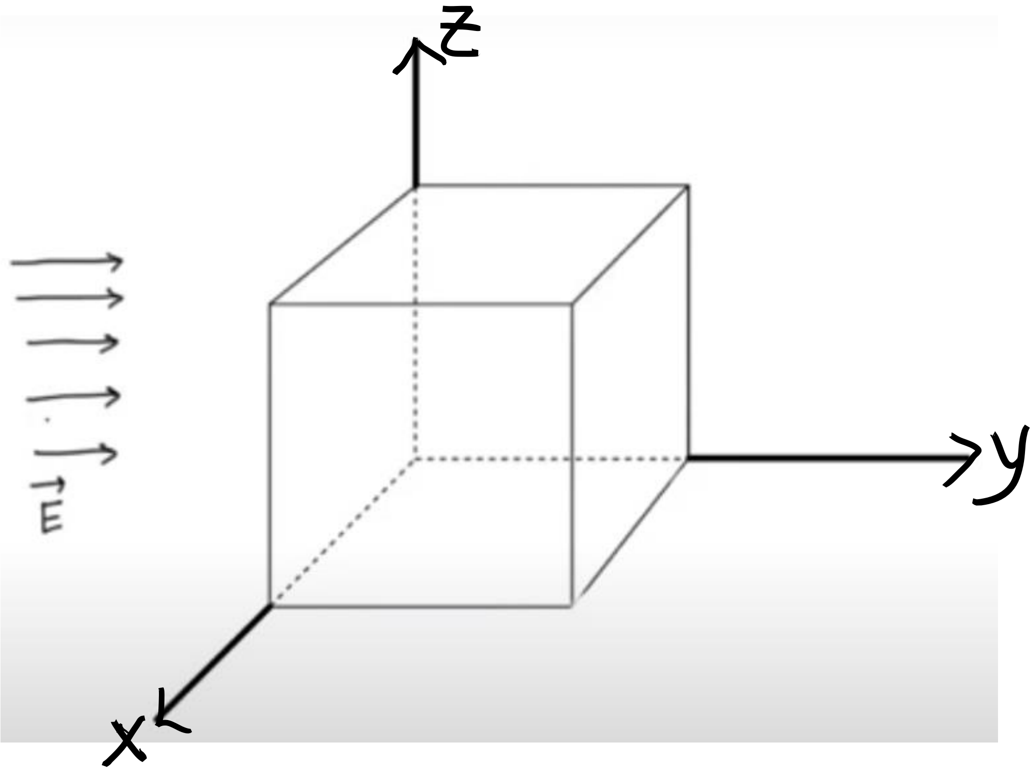



Again, the surface element $d\vec{A}$ in a spherical coordinate is $d\vec{A} = rd\theta\ (rsin\theta d\phi)\hat{r}$

# Case Problem 1.3:

- Consider a uniform $\vec{E}$, as shown in the picture below.
- Calculate the flux passing through a cube box.



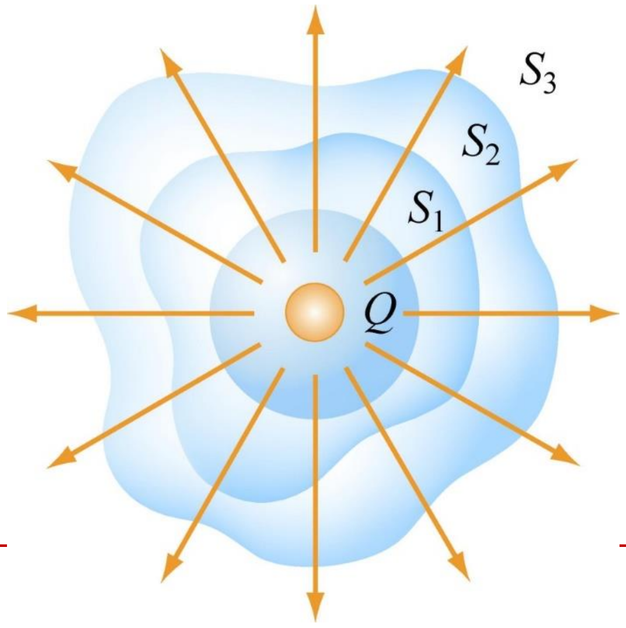
# Concept 2: Gauss's Law, Gaussian Surface and Charge Enclosed

Gaussian surface is just a fictitious surface to help find charge enclosed by the Gaussian surface.

# Arbitrary Gaussian Surfaces

$$\Phi_E = \underset{\substack{closed \\ surface\ S}}{\oiint} \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

- The total flux due to a charge Q, that pass through any enclosed surface (e.g. $S_1$, $S_2$ or $S_3$) are the same (a constant).
- It is true that as area gets bigger $\vec{E}$ gets smaller, but the total electric field lines (or flux) that pass through the enclosed surface are not affected.

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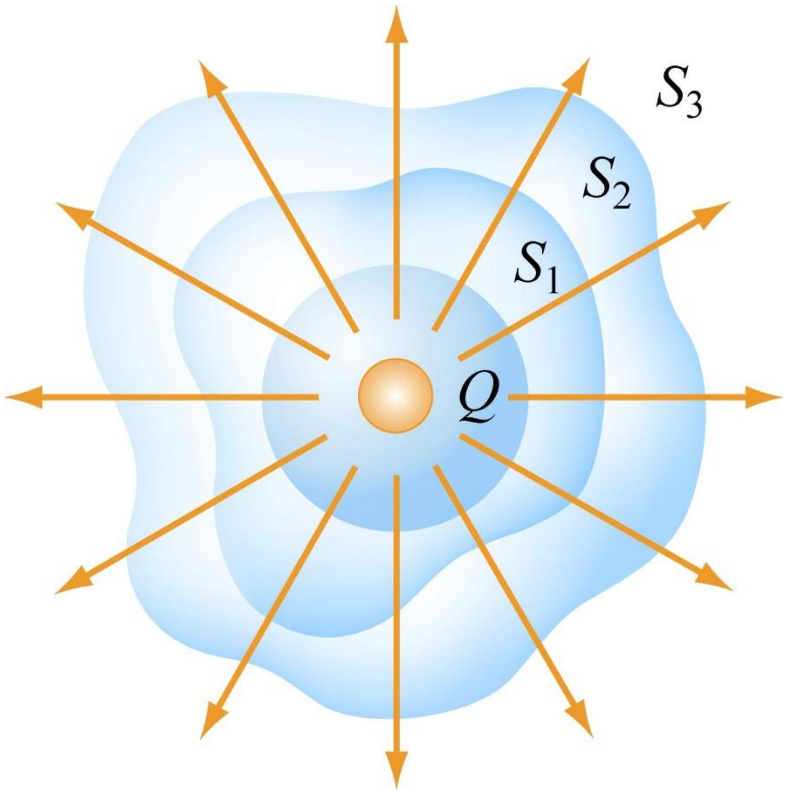
# Gauss’ s Law – The Idea

- The total “flux” of field lines penetrating any of these closed surfaces is the same and depends only on the amount of charge enclosed inside the surface.

$$\Phi_E = \underset{\substack{closed\\ surface\ S}}{\oiint} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

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# Gauss's Law – The 1st Maxwell Equation

- Gauss's law for electric field is the first Maxwell Equation!
- Electric flux $\Phi_E$ for an enclosed surface (the surface integral of electric field $\vec{E}$ over closed surface $S$) is proportional to charge $Q_{enclosed}$ inside the volume of the closed surface.
- A very useful computational technique to find the electric field $\vec{E}$ when the distribution of the charge has 'enough symmetry', such as translational and/or rotational symmetry on symmetrical object.

$$\Phi_E = \oiint_{\substack{closed \\ surface\ S}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

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# Gauss's Law– Carl Friedrich Gauss

- 30 April 1777 – 23 Feb 1855.
- German mathematician and scientist who contributed significantly to many fields, including number theory, statistics, analysis, differential geometry, geodesy, electrostatics, astronomy, and optics.
- Prince of Mathematicians.
- There are > 100 technical terms named after him.
- Child prodigy.

# Concept Question 2.1: Flux thru Sphere

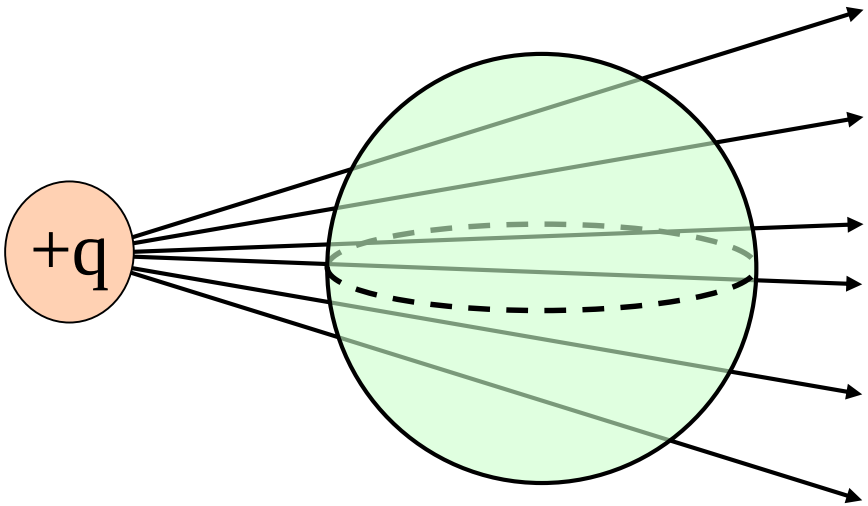
The total flux through the below spherical surface is

A. positive (net outward flux).

B. negative (net inward flux).

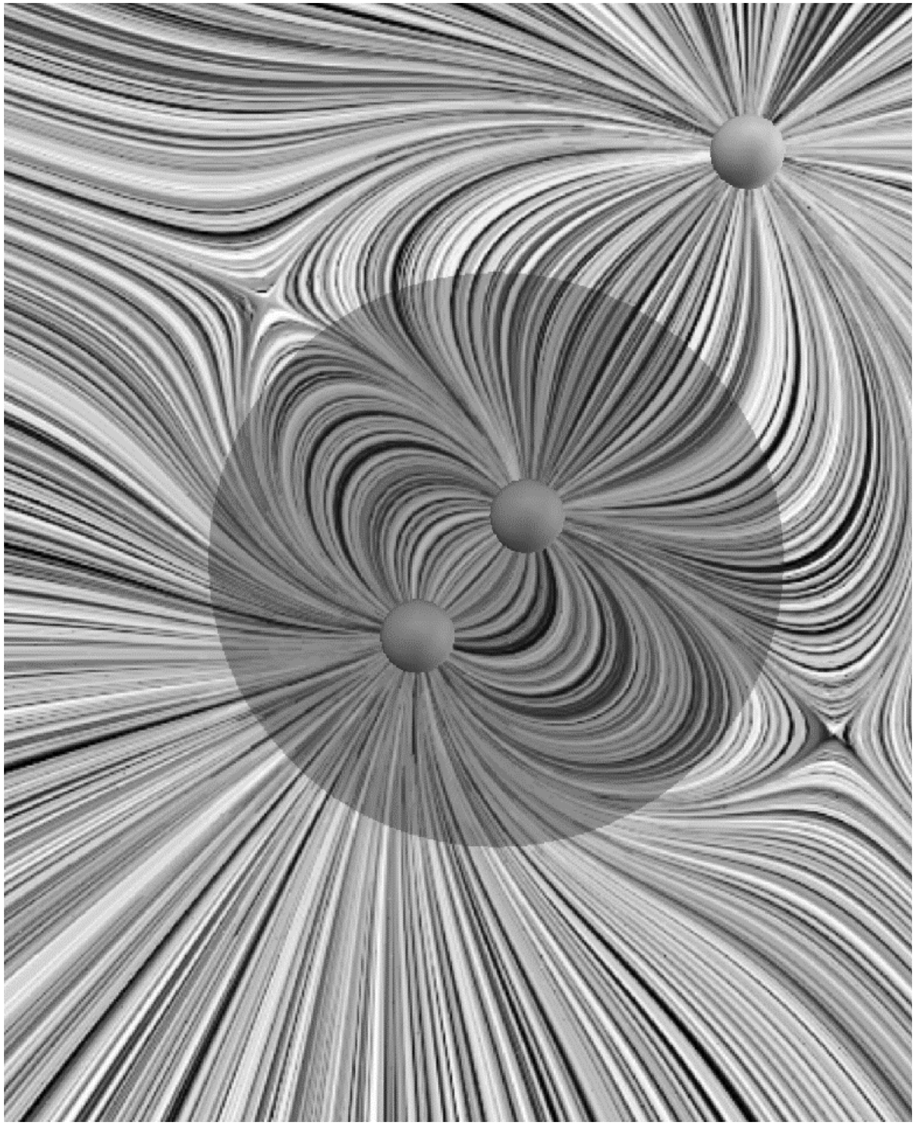
C. zero.

D. Not well defined.



# Concept Question 2.2: Gauss's Law

- The grass seeds figure shows the electric field of three charges with charges +1, +1, and -1, The Gaussian surface in the figure is a sphere containing two of the charges. The electric flux through the spherical Gaussian surface is



A. Positive
B. Negative
C. Zero
D. Impossible to determine without more information.

# Concept Question 2.3: Gaussian Surface

- Two point charges, $+q$ (in red) and $-q$ (in blue), are arranged as shown.
- Through which closed surface(s) is the net electric flux equal to zero?

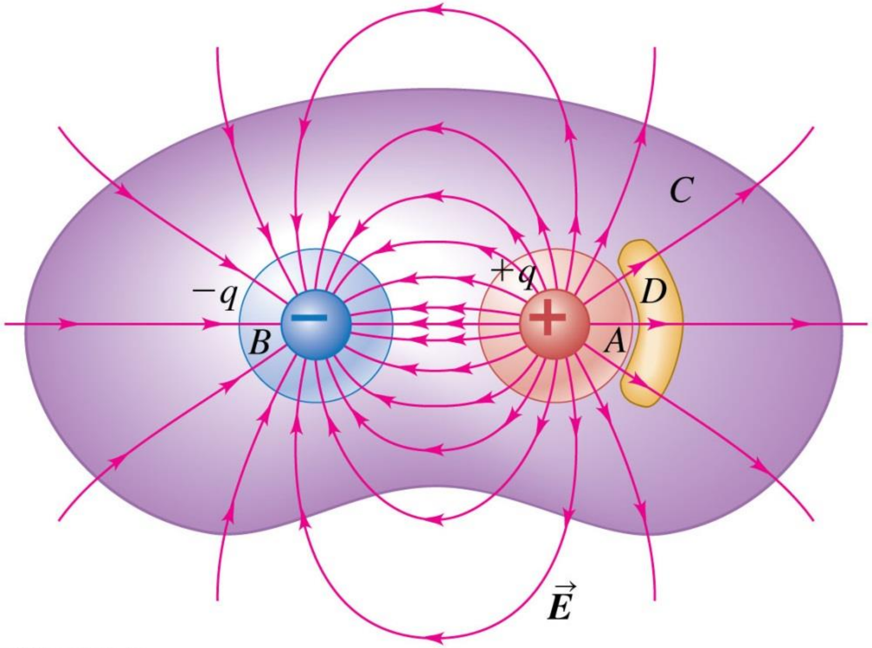
A. surface *A*

B. surface *B*

C. surface *C*

D. surface *D*

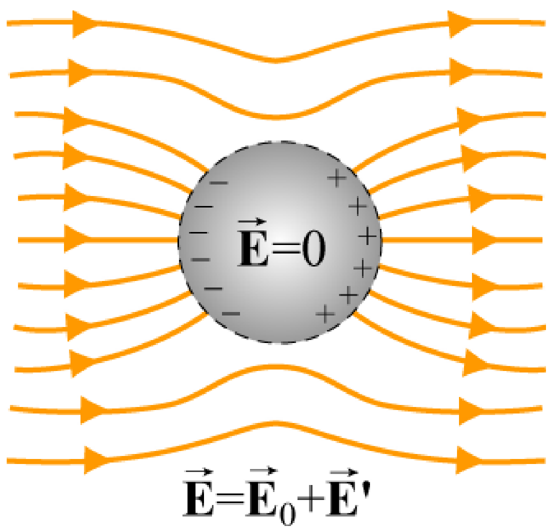
E. both surface *C* and surface *D*



# Concept 3: Application of Gauss’s Law – Conductor at Equilibrium

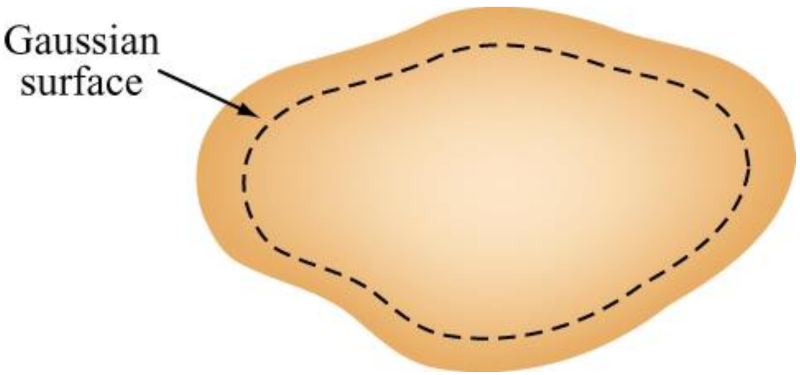
# Why so special about conductors?

- If an electric field is present inside a conductor, free charges will move around (not an electrostatic condition).
- The free charges in a conductor will redistribute themselves and very quickly reach electrostatic equilibrium (all charges do not move anymore).
- It implies that at electrostatic equilibrium, the electric field inside a conductor is ZERO!




Placing a conductor in a uniform electric field $\vec{E}_0$

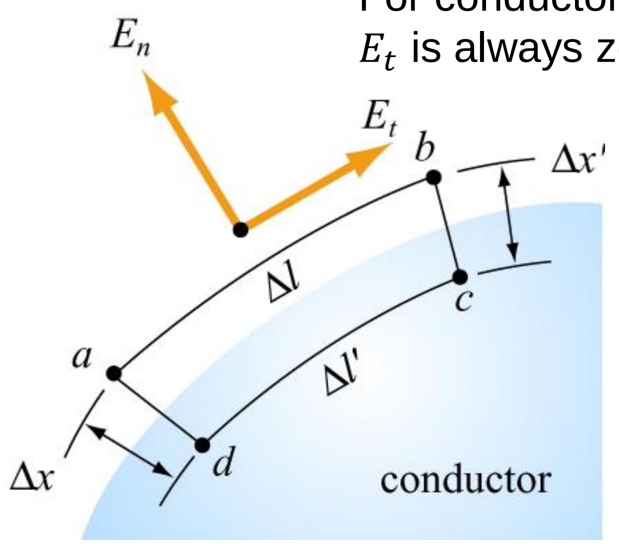
You can have external electric field outside a conductor, but inside the conductor, the $\vec{E}$ is always zero at equilibrium condition.



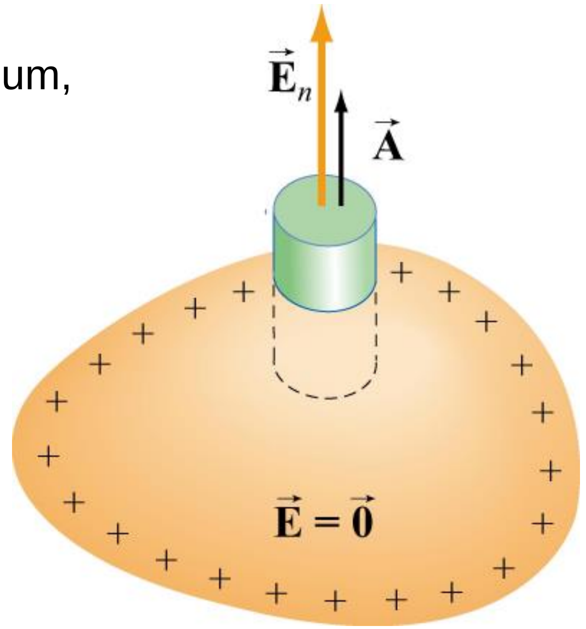

We draw a Gaussian surface inside a conductor. The enclosed charge is zero.

- Now we know E field inside a conductor is always 0.
- We rightfully draw a Gaussian surface just underneath a conductor surface (or inside a conductor). No electric field inside a conductor means no flux passing through the Gaussian surface.
- According to Gauss's law, the enclosed net charges has to be ZERO! It implies that any excessive charges can only stay on the surface of a conductor in equilibrium, regardless of the shape of the conductor.
- The distribution of the charges on surface may not be necessary uniform. It depends on the shape/ geometry of the conductor.

- In electrostatic equilibrium, the tangential component of $\vec{E}$, $E_t$ is zero on the surface of a conductor. (Otherwise, charges will move tangentially along the surface – a non-static condition.)
- $\vec{E}$ can only be normal to the surface just outside the conductor.



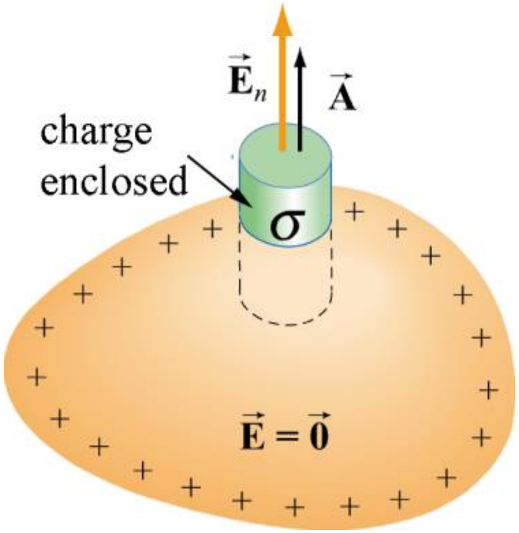

For conductor at equilibrium,
$E_t$ is always zero



# Electric Field on Surface of Conductor

- We can calculate the $\vec{E}$ perpendicular to the surface of the conductor when there are excessive charges on surface.
- Applying Gauss' s Law using a closed cylinder/pill box:

$$E_{surface}A = \frac{\sigma A}{\varepsilon_o} \Rightarrow E_{surface} = \frac{\sigma}{\varepsilon_o}$$




Note: The surface charge density, $\sigma$ may not be uniform along the surface.

# Summary - Conductors in Electrostatic Equilibrium

- The electric field inside a conductor vanishes.
- Any excess charge placed on a conductor resides entirely on the surface of the conductor.
- The electric field is perpendicular to the surface of a conductor everywhere on that surface.
- The magnitude of the electric field just above the surface of a conductor is given by $E_{surface} = \frac{\sigma}{\varepsilon_o}$. Note that $\sigma$ is the surface charge density at the particular spot; $\sigma$ may not be uniform throughout the whole surface.

# Concept Question 3.1: A Conductor with Cavity

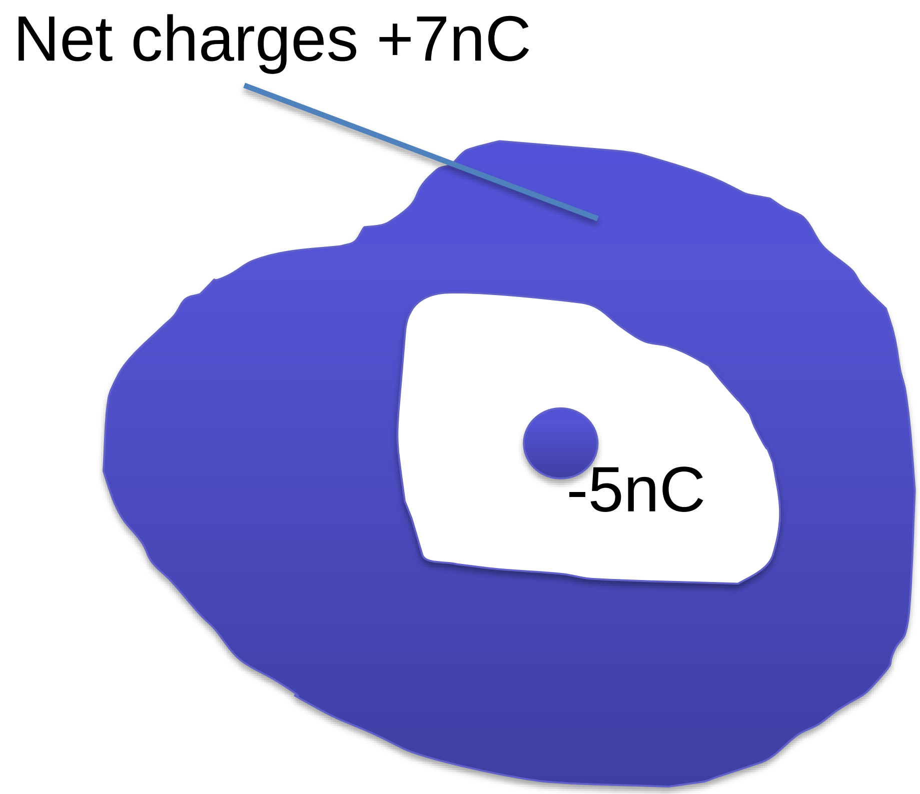
- A conductor with a cavity carries a total charge of +7nC. A point charge of -5nC is inside the conductor.
- How is the charge distributed on the inner surface of cavity wall and outside surface?

A. Inner cavity wall 0C, outside surface +7nC

B. Inner cavity wall +5nC, outside +2nC

C. Not enough information: exact geometry need to be provided

D. Inner cavity wall -5nC, outside +12nC



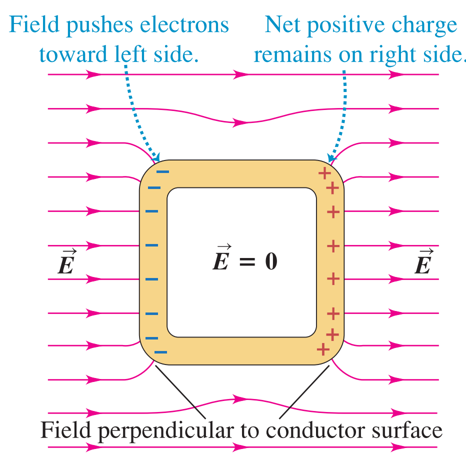
# Demo – Faraday Cage

- Turn on the Van de Graaff generator and observe what happen to  the tinsel and electroscope. Now cover the tinsel and the electroscope with a metallic cage (Faraday cage). What happen to the tinsel and electroscope then? Why?
- Turn on a radio. Put the Faraday cage over the radio and observe what happen to the sound. Why?
- Put a hand phone in the metallic container, try calling the number of the hand phone. Does the hand phone ring at all? Why?

Now you know how the metallic shield works in all signal cables!

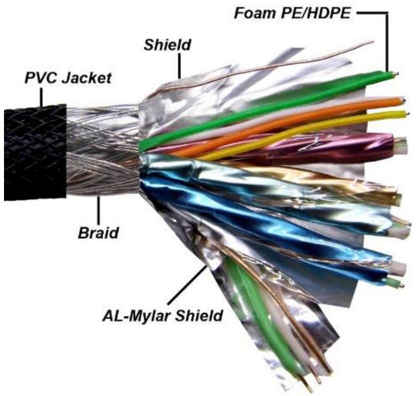
# Application: Faraday Cage

- Even there is a cavity inside a conductor, $\vec{E}$ inside is also zero. It implies external $\vec{E}$ cannot penetrate in.
- Faraday cage works because the external electrical field causes the electrical charges within the cages conducting material to be disturbed such that they cancel the fields effect inside the cage.
- That is the reason you may lose phone or radio signal in elevators or buildings with metallic conducting frames and walls, which simulate a Faraday cage effect.
- You are safe in cars, trains or even airplane during lighting as the metallic compartments are essentially Faraday cages, protecting passengers from electric charges.



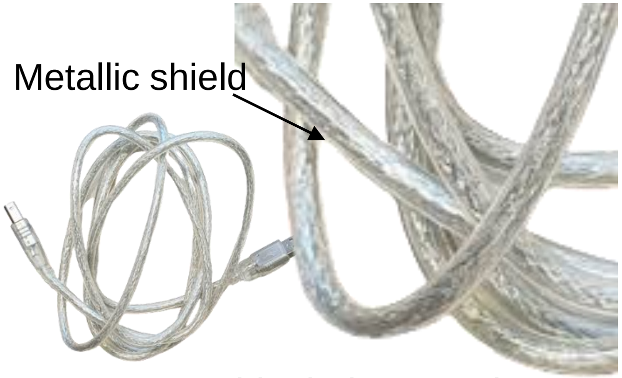

- Microwave ovens are one common product that use a Faraday cage. Instead of keeping microwaves out, the door seals and outer case contain microwaves into a small cooking chamber that cook your food.
- Space Launch Complex (SLC-40) with SpaceX Falcon 9 launch infrastructure has four towers surrounding the rocket as lightning arresters, acting like a giant Faraday cage (from Wikipedia).
- Electronic components in automobiles and aircraft utilize Faraday cages to protect signals from interference.



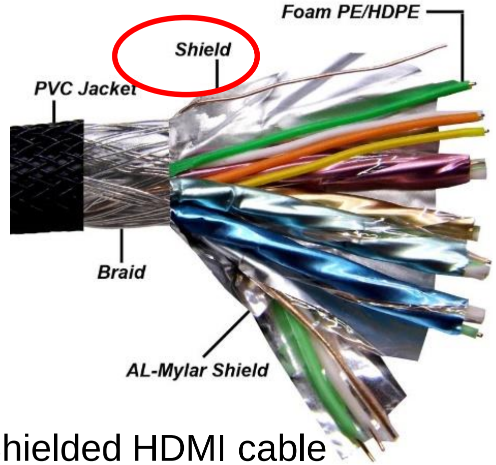


# Application: Metallic Shielding in Screened Cables

- You may notice any signal cable (also called screened cable), such as USB, VGA, HDMI, network, coaxial cables for cable television, always have a metallic shielding wrapping around those signal wires.
- Not only it helps to strengthen the mechanical structure, more importantly, the metallic shield protects the internal conductors from external electrical noise and prevents the RF signals from leaking out.
- A conductor has such a property to eliminate external electric field from penetrating into it.
- Gauss's Law explains it!




Shielded USB cable




Shielded HDMI cable