Week 9 - Day 1

Photoresistor

↓

DC Circuit Analysis

Concept 1: Current and Current Density - How Charges Move in a Conductor

Concept 2: Microscopic and Macroscopic Ohms' Law

Concept 3: Basic DC circuits components

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Potential Divider, Power Rating of an AC Adapter (Day 2)

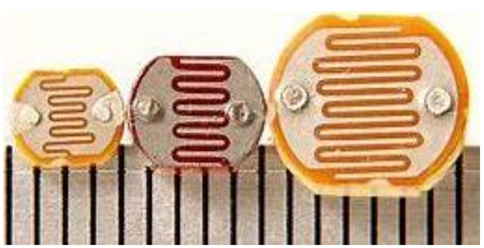
Reading:

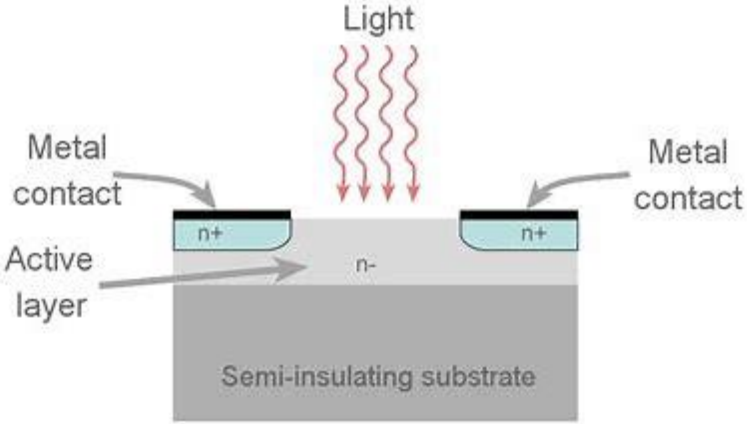
University Physics with Modern Physics – Chapter 25

Introduction to Electricity and Magnetism – Chapter 6,7

Application: Resistance and Light – Photoresistor

- Photoresistors/Light Dependent Resistors (LDR) work based from the principle of photoconductivity. Photoconductivity - phenomenon in which the material's conductivity is increased when light is absorbed by the material.
- An LDR is made by any semiconductor material with a high resistance. It has a high resistance because there are very few electrons that are free to move – they are locked into the crystal lattice (in the valence band) and unable to move.
- When the photons (light) fall on the device, the electrons in the valence band are excited to the conduction band, provided the photons have energy greater than the bandgap of the semiconductor (electrons break free from the crystal lattice so that they can then conduct electricity).
- This results in a lowering of the resistance of the semiconductor and hence the overall LDR resistance.





More Information:

https://www.electronics-notes.com/articles/electronic_components/resistors/light-dependent-resistor-ldr.php

https://www.electrical4u.com/light-dependent-resistor-ldr-working-principle-of-ldr/

Concept 1: Current and Current Density

Current: Flow Of Charge

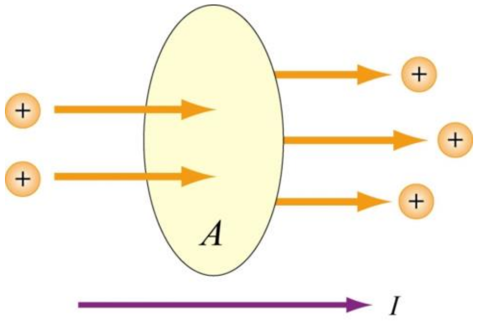
- Definition: Average current I_{av} is the charge ΔQ flowing through area, *A*, in time, Δt.

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

- Instantaneous current: differential limit of I_{av}.

$$I = \frac{dQ}{dt}$$

- Units of Current: Coulomb (*C*) / second (*s*) = Ampere (*A*)

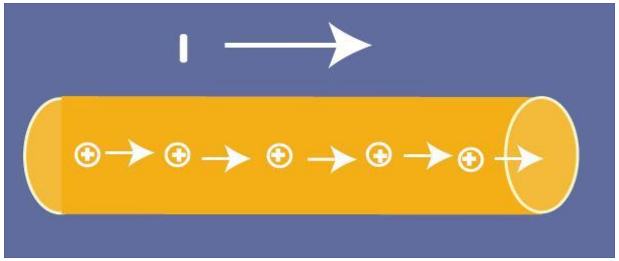


FYI: How Big is One Ampere and What Does it Mean?

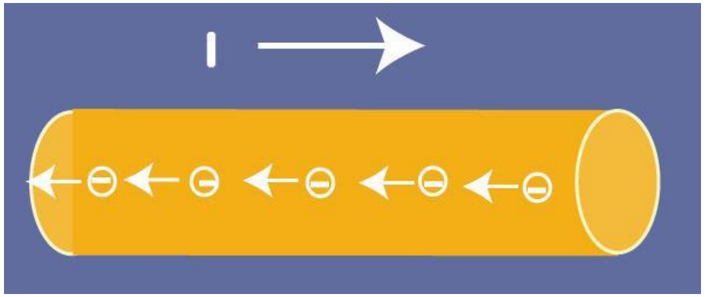
	Range
Household Electronics	~1 A
Battery Powered	~100 mA (1-10 A-Hr)
Household Service	100 A
Lightning Bolt	10 to 100 kA
To hurt	40 (5) mA DC(AC)
To throw	60 (15) mA DC(AC)
To be fatal	0.5 (0.1) A DC(AC)
Fuse/Circuit Breaker	13-30 A

Direction of the Current

- Direction of current is the direction that positive charges flow.



- Physically, it is negative charges that flow in a conductor.
- Due to historical reason, the direction of current is opposite to the direction that negative charges flow.

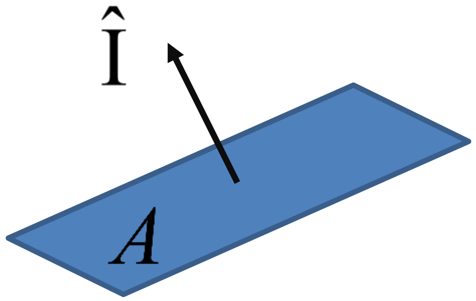


Current Density J

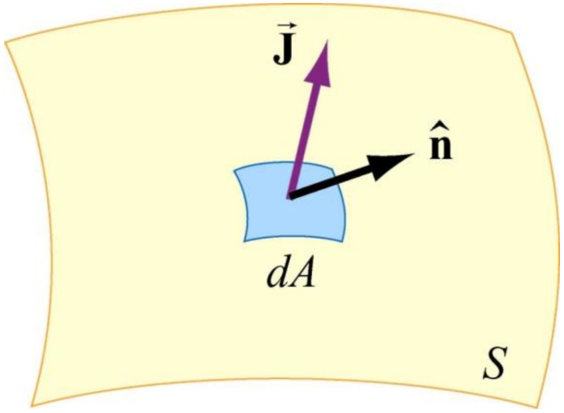
- Current density J: current/area

$$\vec{J} \equiv \frac{I}{A}\hat{I}$$

- $\hat{I}$ points in direction of current



$$I \equiv \int_S \vec{J} \cdot \hat{n} dA = \int_S \vec{J} \cdot d\vec{A}$$



Concept Question 1.1

A current *I* = 200 mA flows in the wire below. What is the magnitude of the current density J?

A. $J = 40\ mA/cm^2$

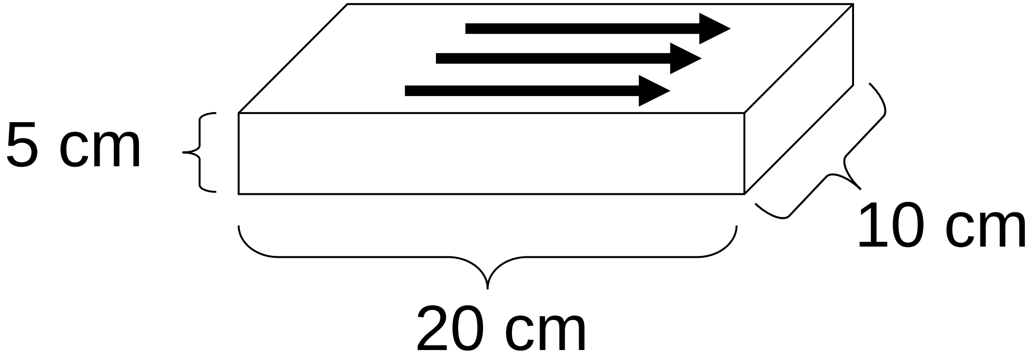
B. $J = 20\ mA/cm^2$

C. $J = 10\ mA/cm^2$

D. $J = 1\ mA/cm^2$

E. $J = 2\ mA/cm^2$

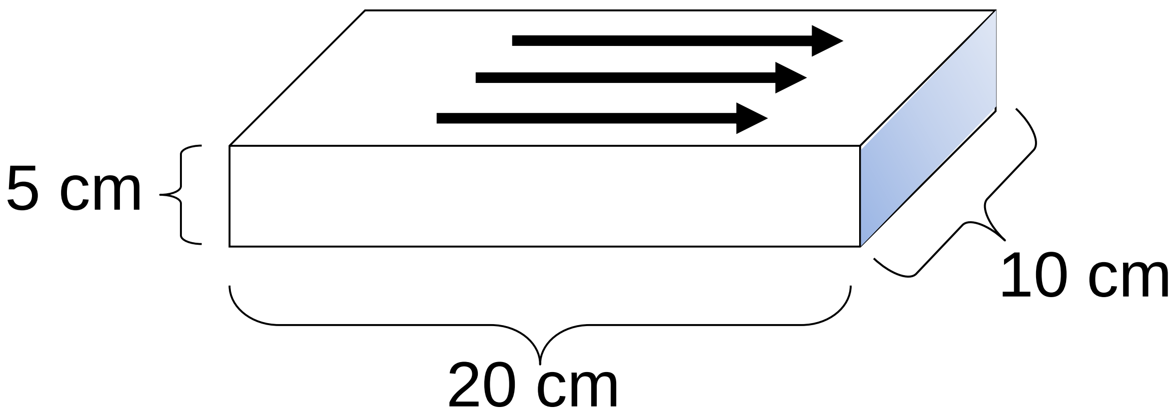
F. $J = 4\ mA/cm^2$



Multiple Choice

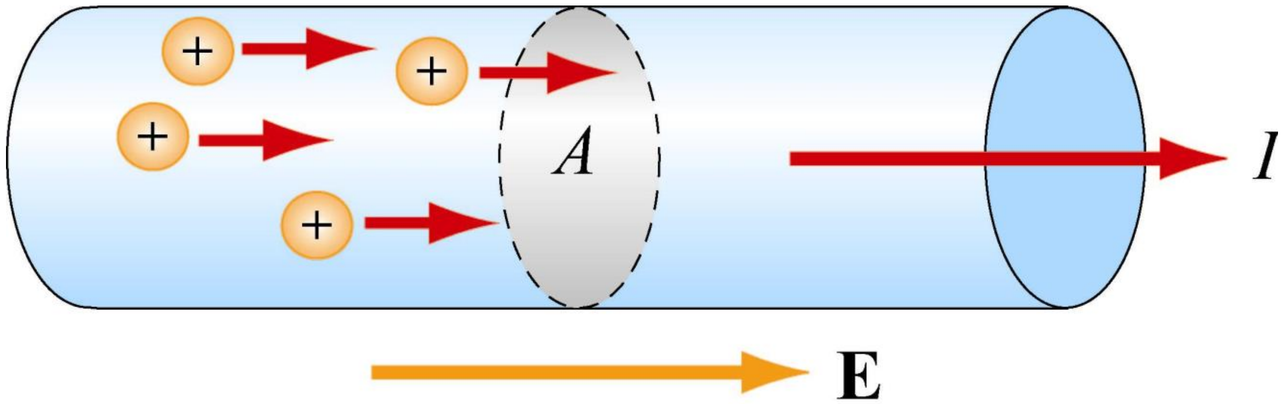
Concept Question 1.1 (Solution)

- The area that matters is the cross-sectional area that the current is punching through – the $50\ cm^2$ area shaded grey.
- So:
- $J = \frac{I}{A} = \frac{200\ mA}{50\ cm^2} = 4\frac{mA}{cm^2}$
- Answer: F. $J = 4\frac{mA}{cm^2}$



Concept 2: Microscopic and Macroscopic Ohms' Law

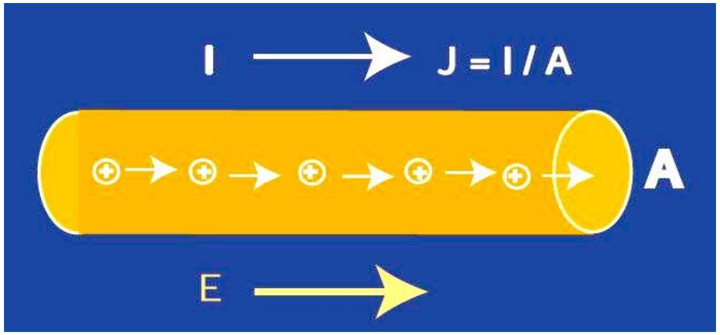
Why Does Current Flow?



- If an electric field is set up in a conductor, charge will move (making a current in direction of $\vec{E}$).
- Note that when current is flowing, the conductor is not an equipotential surface (and $\vec{E}_{inside} \neq 0$)!

Microscopic Ohm's Law

- There is a class of conductor for which the current density $\vec{J}$ that flows through it is proportional to the electric field applied $\vec{E}$. That is $\vec{J} \propto \vec{E}$.
- The proportional constant of that relation, defined as the electrical resistivity ρ is the ratio of the electric field to the current density it creates. $\rho\vec{J} = \vec{E}$.
- We can also set $\vec{J} = \sigma\vec{E}$ where σ is called the electrical conductivity.



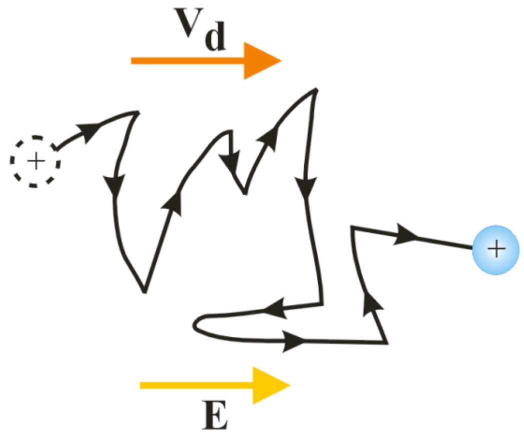
$$\sigma = \frac{J}{E} \qquad \rho = \frac{E}{J} \qquad \sigma = \frac{1}{\rho}$$

- ρ and σ depend only on the microscopic properties of the material, not on its shape.

Conductivity and Resistivity

- Ability of current to flow depends on density of charges and rate of scattering (collision).

$$\sigma = \frac{1}{\rho}$$



- Two quantities summarize this: $S = \Omega^{-1} = \frac{A}{V}$
- σ: conductivity (S/m) (note: this σ is not surface charge density)
  - A measure of a material's ability to conduct an electric current
- ρ: resistivity (Ωm)
  - A measure of the strength that material opposes the flow of current.

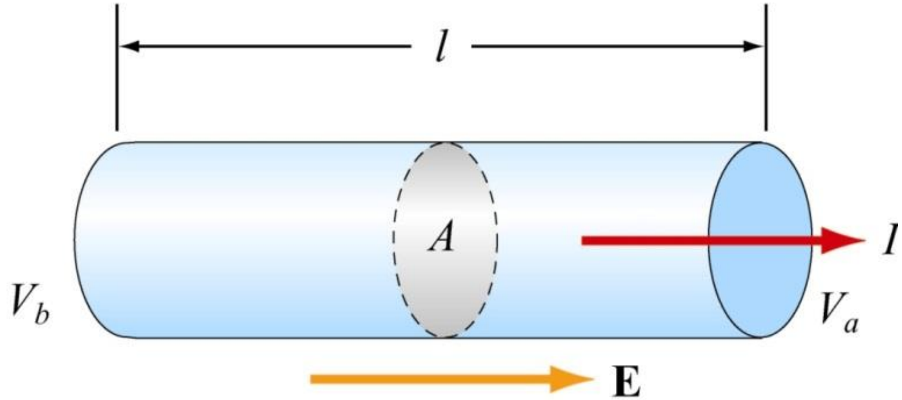
Macroscopic Ohm's Law: $\Delta V = IR$

- What is relationship between electric potential difference and current?
- $\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s} = El$

$$\left.\begin{aligned} J &= \frac{E}{\rho} = \frac{\Delta V/l}{\rho} \\ J &= \frac{I}{A} \end{aligned}\right\} \Rightarrow \Delta V = I\left(\frac{\rho l}{A}\right) \equiv IR$$

$$\Delta V = IR$$

where $R = \frac{\rho l}{A}$



How Big is an Ohm and What does it mean?

Items	Range
Short Copper Wire	milliohms (mΩ)
Notebook paper (thru)	~1 G Ω
Typical resistors	1 Ω to 100 M Ω
You (when dry)	~100 k Ω
You (when wet)	~1 k Ω
Internally (hand to foot)	~500 Ω

Stick your wet fingers in an electrical socket: $I = \frac{\Delta V}{R} = \frac{220}{500} = 0.44A$ Fatal!

Fatal current: >0.1A AC

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Resistance and Temperature

- Over a limited temperature range, the resistivity of a conductor varies approximately linearly with the temperature:
- $\rho = \rho_o \,[1 + \alpha(T - T_o\,)]$
- ρ_o is the resistivity at some reference temperature T_o
- T_o is usually taken to be $20^o C$
- α is the temperature coefficient of resistivity
- SI units of α are $^oC^{-1}$

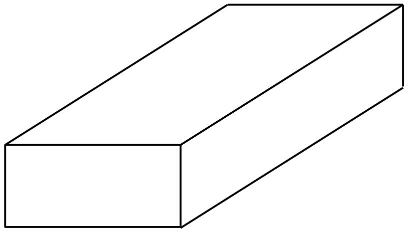
Note:

- Resistance of conductors increases with increasing temperature.
- Not all materials behaves in such a way. As general trend, resistance of intrinsic semiconductor materials decreases with increasing temperature.
- Why is that so?

- Many kitchen appliances produce high temperature by resistive heating: incandescent light bulb, bread toaster, oven, water heater, kettle, etc.
- Engineers always need to consider the effect of temperature on resistance that could affect the performance of the circuit.

Concept Question 2.1:

- A block of carbon, 1.0 cm by 2.0 cm by 5.0 cm, is at one fixed temperature environment. Regarding the resistance and resistivity of the carbon block,
- Which of the following statement is true?



A. The resistance between the two biggest faces is greater than resistance between the two smallest faces.

B. The resistivity between the two biggest faces is greater than resistivity between the two smallest faces.

C. The resistivity between the two biggest faces is smaller than resistivity between the two smallest faces.

D. The resistivity between any two faces is the same.

Multiple Choice

Concept Question 2.1 (solution)

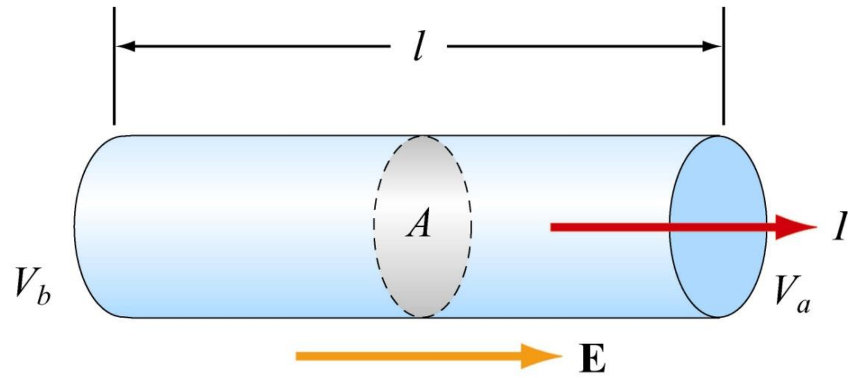
Answer: D. The resistivity between any two faces is the same.

The resistivity of a given material is the same at the same temperature regardless of the shape.

Concept Question 2.2: Resistance

When a current flows in a wire of length *L* and cross sectional area *A*, the resistance of the wire is

A. Proportional to *A*; inversely proportional to *L*.
B. Proportional to both *A* and *L*.
C. Proportional to *L*; inversely proportional to *A*.
D. Inversely proportional to both *L* and *A*



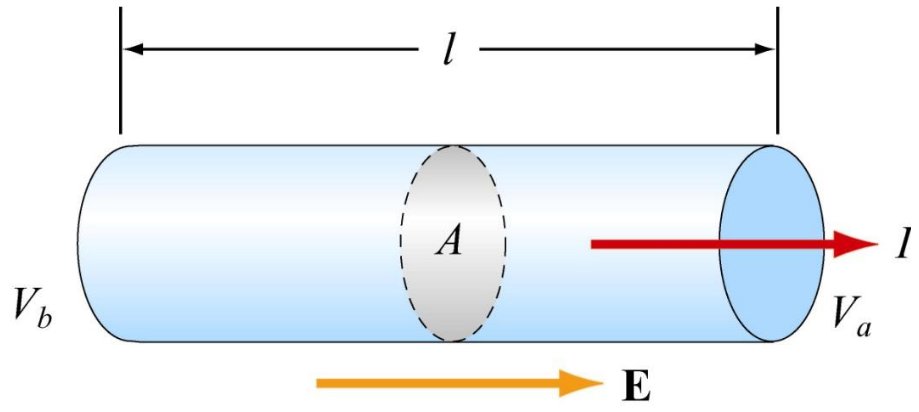
Multiple Choice

Concept Question 2.2 (Solution)

Answer: C. Proportional to *L*; inversely proportional to *A*.

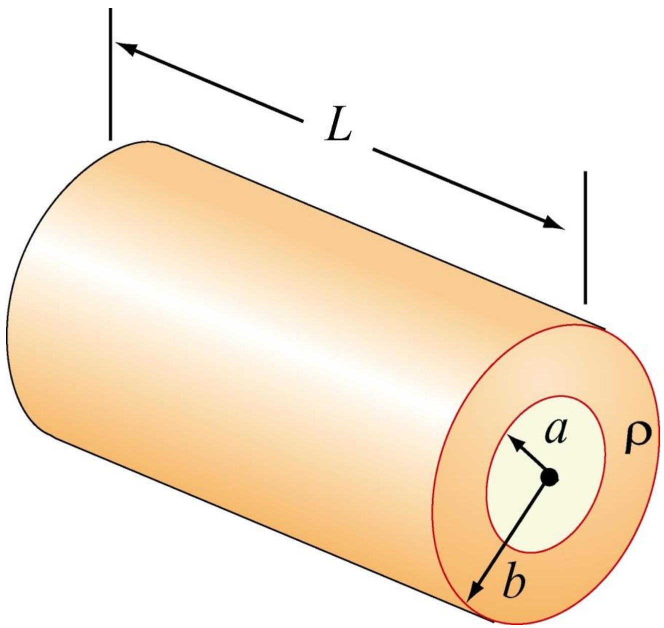
The longer the wire, the higher the resistance. The large the cross-sectional area, the more ways that current can flow through it, so the lower the resistance.

So, if resistivity is ρ_r, then $R = \frac{\rho_r L}{A}$.



Case Problem 2.1: Calculating Resistance

- Consider a hollow cylinder of length L and inner radius a and outer radius b. The material has resistivity ρ.
- Suppose a potential difference is applied between the ends of the cylinder and produces a current flowing parallel to the axis. What is the resistance measured?



Case Problem 2.1: (Solution)

When a potential difference is applied between the ends of the cylinder, current flows parallel to the axis. In this case, the cross-sectional area is

$$A = \pi(b^2 - a^2)$$

and the resistance is given by:

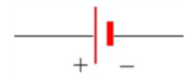
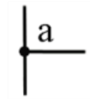
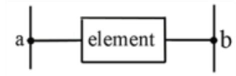
$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi(b^2 - a^2)}$$

Concept 3: Basic DC circuit Components

Voltage supply (battery) and Resistor

Parallel and Series Circuit

Symbols for Circuit Elements

Battery	
Resistor	
Capacitor	
Switch	
Equipotential	
Junction	
Branch	

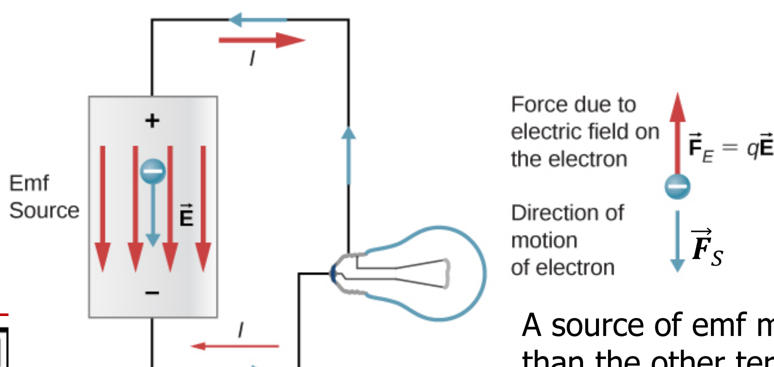
Inductor 

Electromotive Force (EMF)

- Electromotive force (emf) is the potential energy per unit charge supplied by a battery or any electrical energy source. An EMF is needed to move charges around a circuit.
- It is known as the work done per unit charge by an energy source in moving charge q from the negative terminal (lower potential) to the positive terminal (higher potential).
- The symbol of emf is ε. The unit is V.
- EMF is a bad name because it is not a force. EMF is the work done per unit charge.
- Let $\vec{F}_S$ denote the force produced by the energy source, then the EMF is:

$$\varepsilon = \int_{-}^{+} \frac{\vec{F}_S}{q} \cdot d\vec{s} = \int_{-}^{+} \vec{f}_S \cdot d\vec{s} = \int_{-}^{+} \frac{-\vec{F}_E}{q} \cdot d\vec{s}$$

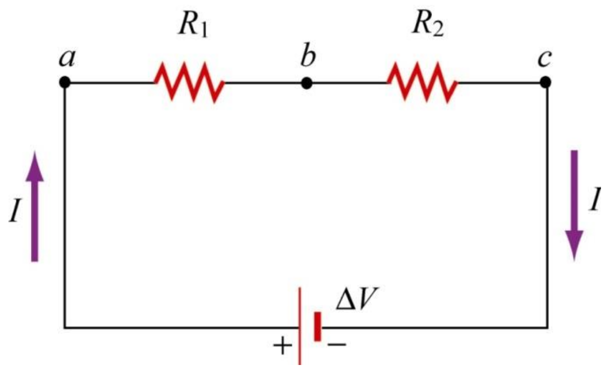
- The energy source needs to produce force/energy to move charges against the internal electric field of the source.
- The origin of this *source force*, in batteries is a chemical force.

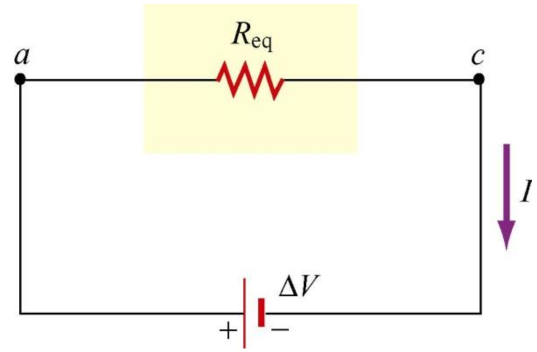


A source of emf maintains one terminal at a higher electric potential than the other terminal, acting as a source of current in a circuit.

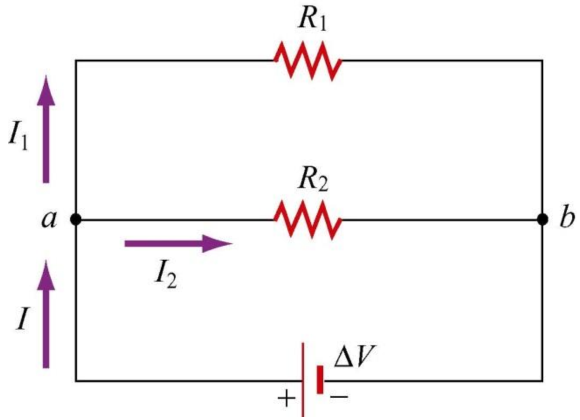
Series and Parallel Resistive Circuit

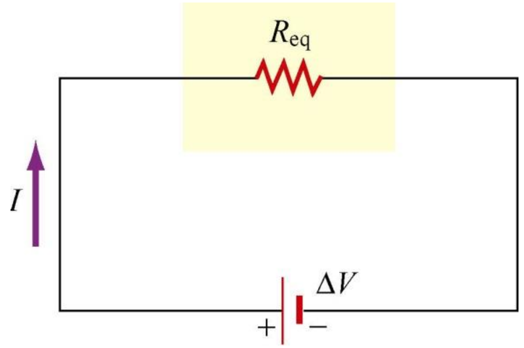
- What is the equivalent resistance? Apply Ohm's law to prove it.





$$R_{eq} = ?$$

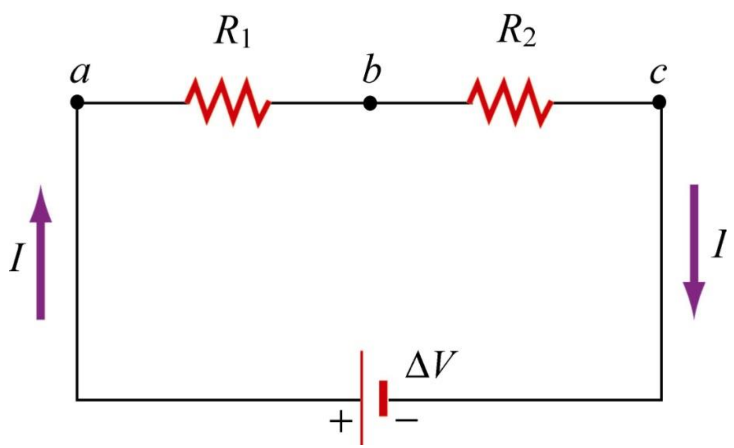


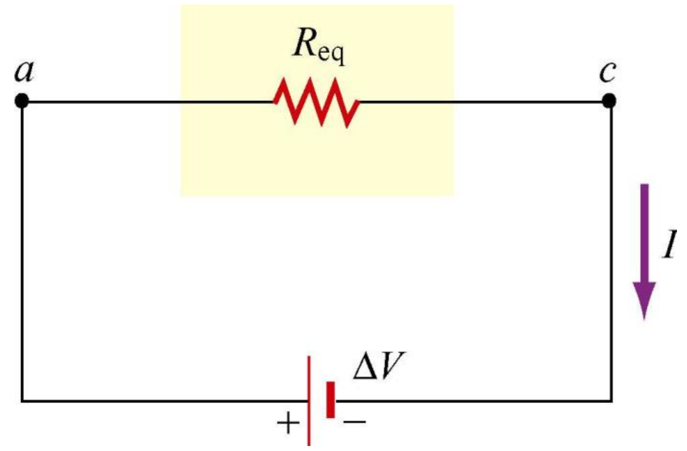


$$R_{eq} = ?$$

Resistors In Series

- Key concept: The same current *I* must flow through both resistors.
- And, $\Delta V = \Delta V_1 + \Delta V_2$

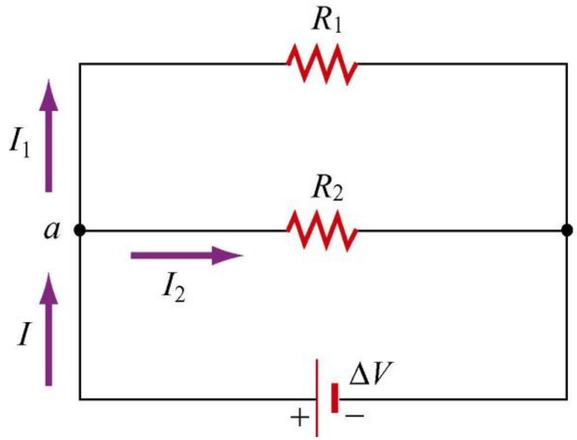


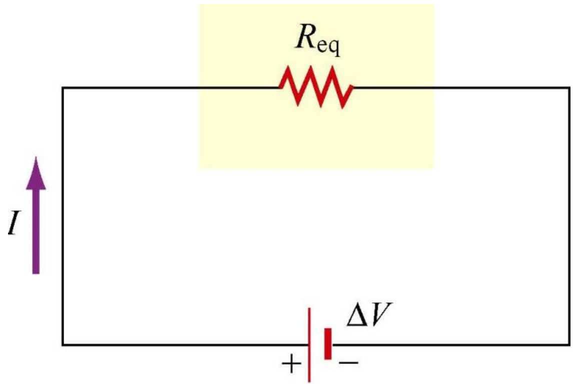


$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2) = IR_{eq}$$
$$R_{eq} = R_1 + R_2$$

Resistors In Parallel

- Key concept: Potential difference across R_1 and R_2 are the same.
- And, $I = I_1 + I_2$





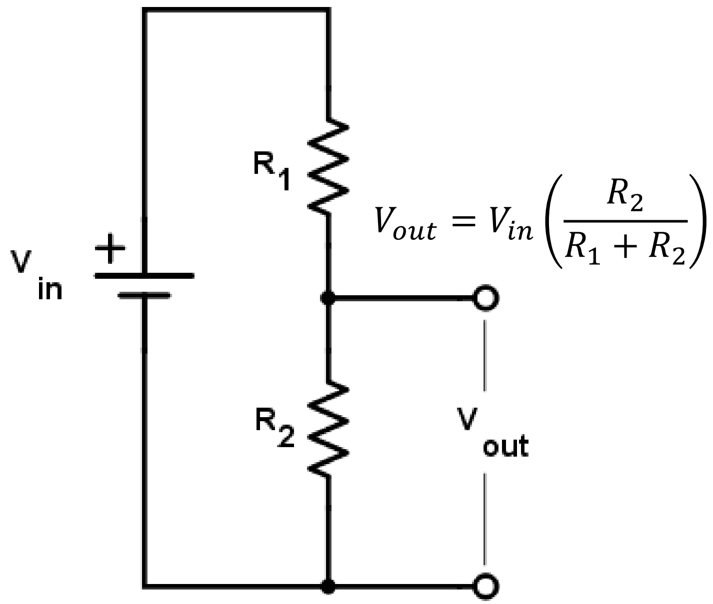
$$\Delta V = \Delta V_1 = \Delta V_2 = I_1 R_1 = I_2 R_2 = I R_{eq}$$

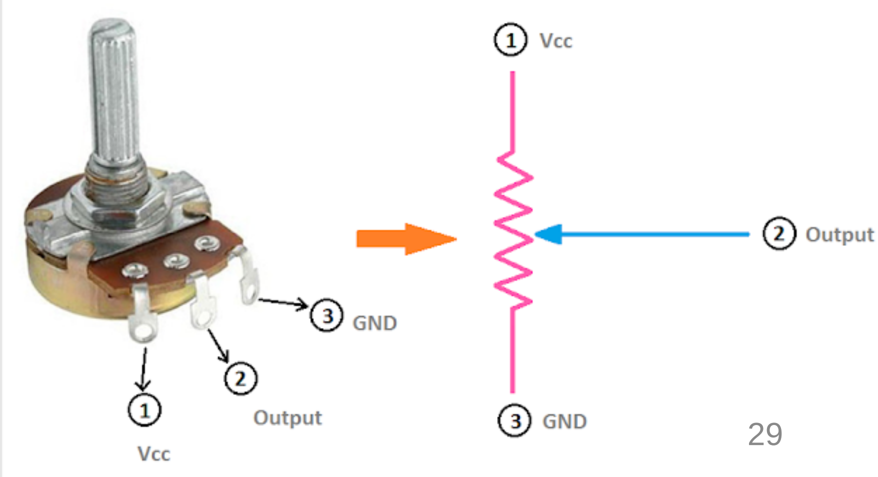
$$I = I_1 + I_2 = \frac{\Delta V}{R_1} = \frac{\Delta V}{R_2} = \frac{\Delta V}{R_{eq}}$$

$$\Rightarrow \frac{1}{\mathrm{R}_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Application of a Series Circuit: Potential Divider

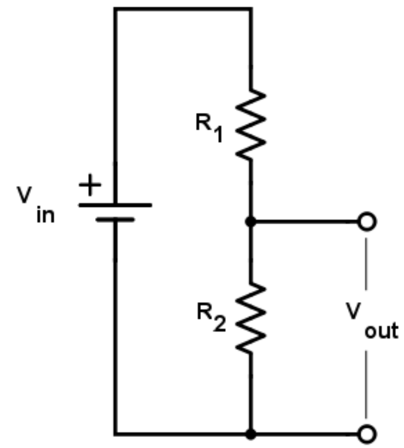
- Consider 2 resistors in series. One can proof that $V_{out} = V_{in}\left(\frac{R_2}{R_1+R_2}\right)$.
- V_{out} is a fraction of V_{in}. It is called a potential divider circuit.
- If R_2 is replaced by a variable resistor that can be controlled by turning a dial, then V_{out} can also be controlled.
- This potential divider circuit is applied in a device, called **potentiometer.** It is used for tuning or volume control in many circuits.

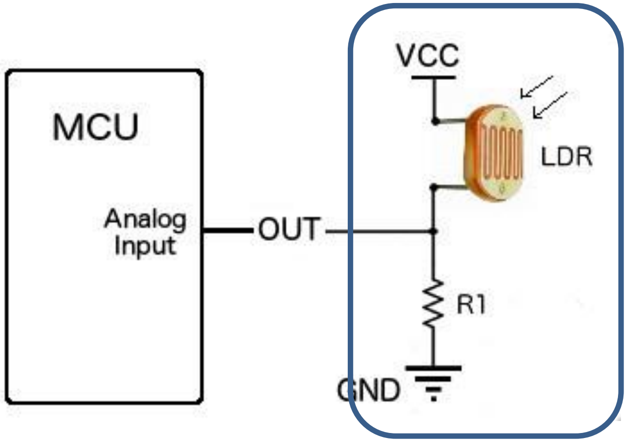




Application of a Series Circuit: Potential Divider

- Another common use of the potential divider is to replace R_1 or R_2 with a sensor such as an LDR. As the resistance of the sensor changes, V_{out} changes as well. This change can then be used to trigger a transistor or can be fed into the input of a microcontroller.
- It can also be used for:
  - Sensor measurement (e.g. LDR).
  - Adjusting the signal level/ signal attenuator.
  - Reducing the magnitude of a voltage so it can be measured (such as high voltage measurement).
  - Creating a reference voltage (for active devices, e.g. amplifier).
  - Etc.

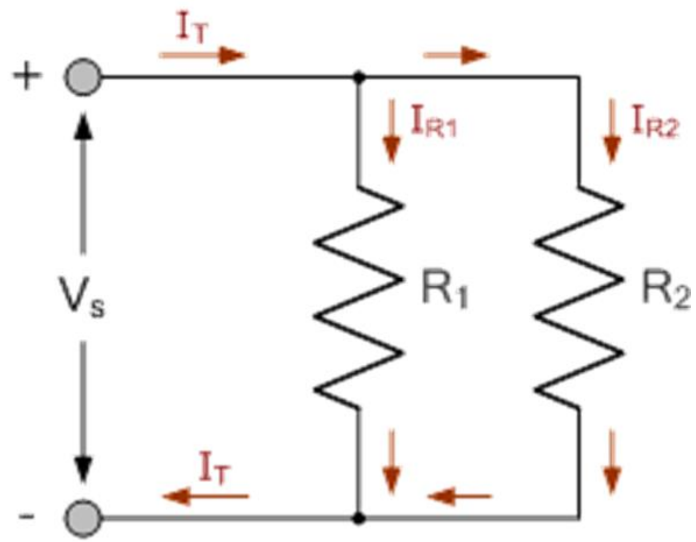




A potential divider circuit

Case Problem 3.1: Current Divider

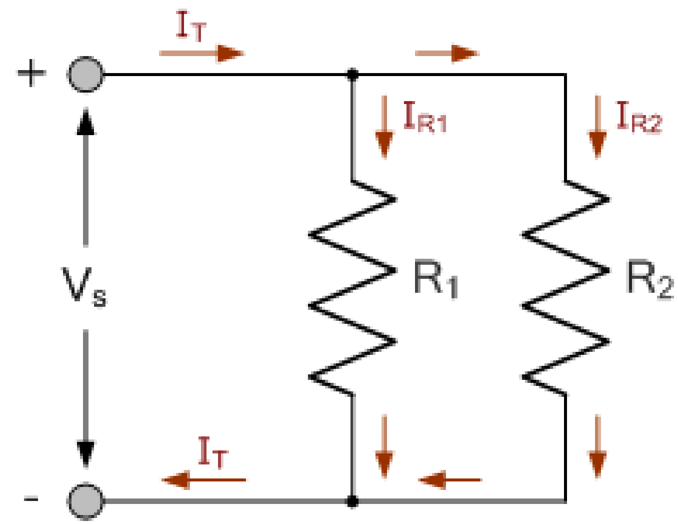
- Current divider are parallel circuits in which the supply current divides into a number of parallel paths.
- Find I_{R1} and I_{R2} as a function of $I_{\mathrm{T}}, \mathrm{R}_1$ and R_2.



- Note: I_{R1} and I_{R_2} are fraction of the total I_T. It is called a current divider circuit.
- Application: Some electronic components has a limit of how much current can flow through. By using a current divider, the current flowing through the component can be minimized and smaller component size can be used.

Case Problem 3.1: (Solution)

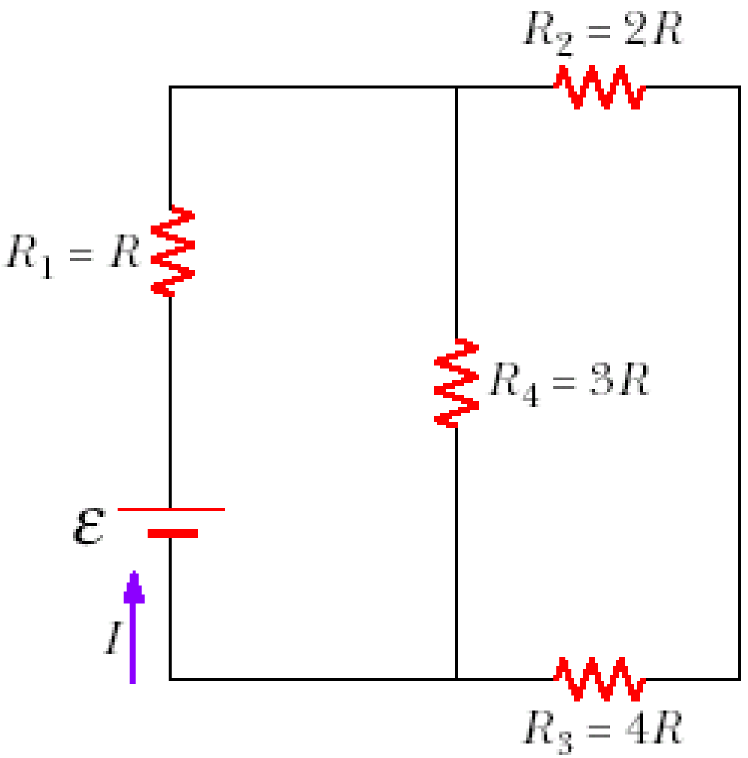
- $I_T = I_{R1} + I_{R2}$ (KCL)
- $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$ (parallel circuit)
- $V_s = I_T R_{eq} = I_T \left(\frac{R_1 R_2}{R_1 + R_2}\right)$
- $I_{R1} = \frac{V_s}{R_1} = I_T \left(\frac{R_2}{R_1 + R_2}\right)$
- $I_{R_2} = \frac{V_s}{R_2} = I_T \left(\frac{R_1}{R_1 + R_2}\right)$



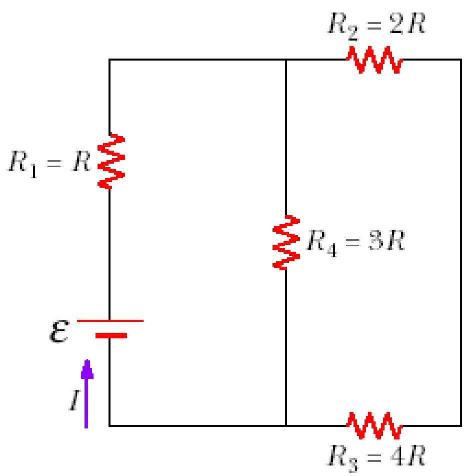
Case Problem 3.2: Four Resistors

Four resistors are connected to a battery as shown in the figure. The current supplied by the battery is I, the battery emf is ε, and the resistor values are $R_1 = R$, $R_2 = 2R$, $R_3 = 4R$, $R_4 = 3R$.

Determine the current in each resistor in terms of I.



Case Problem 3.2 (Solution)



- R_{eq} = R_1 in series with (R_4 in parallel with (R_2+R_3))
- R_4 in parallel with (R_2 + R_3) = $\frac{1}{\frac{1}{3}+\frac{1}{6}} = 2\Omega$
- R_{eq} = R + 2R = 3R.
- Total current supplied by the battery, $I = \frac{\varepsilon}{3}$.
- Note: The brunch of 6Ω is twice that of $R_4 = 3\Omega$. Thus, twice as much current goes through R_4 as through R_2 *and* R_3.

Design Case Problem 1:

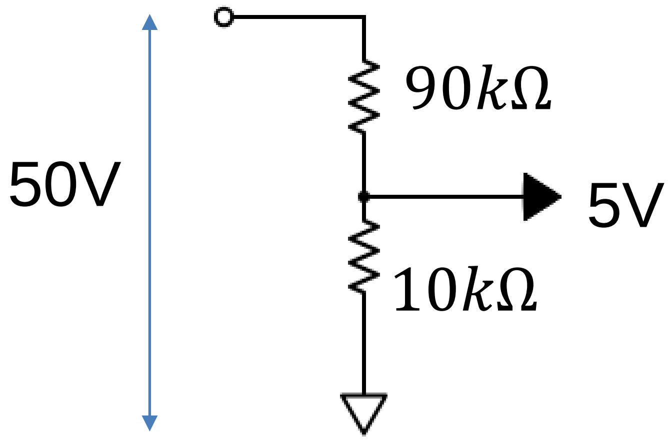
- You want to measure a voltage signal of 50V with a voltmeter that can only handle 5V.
- Suggest a simple way for this measurement. Label clearly the value of the components you have chosen.

Design Case Problem 1: (solution)

Possible solution:

- We can use a simple potential divider to reduce the signal level.
- From the example of the circuit, the total current flow in the circuit:

$$\frac{50V}{100k\Omega} = 0.5mA$$

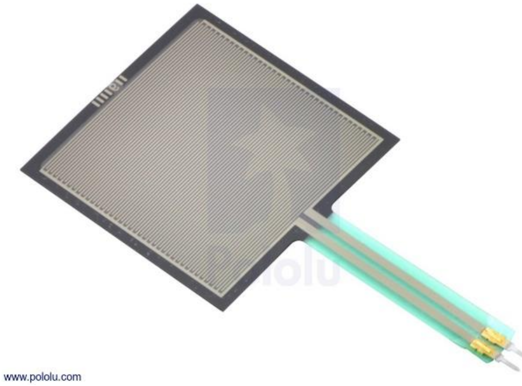


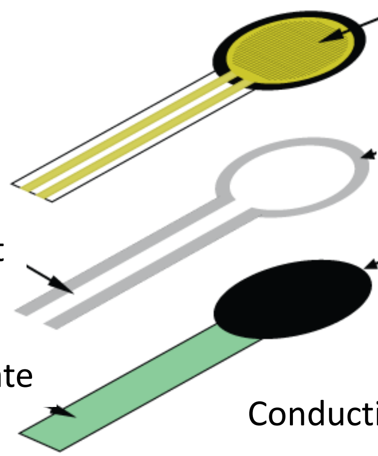
Note:

- Low value of resistors (e.g. $9\Omega\ and\ 1\Omega$) should be avoided as high current can flow through the whole circuit. (wasting energy and can cause fire!)
- It should not be in $M\Omega$ range. The common amount of the input resistance of a modern digital voltmeter is 10 MΩ. With a choice of MΩ resistors, it creates a parallel circuit of the load circuit, causing less accuracy of the measurement.

Application: Working Principle of Force Sensing Resistor

- Force Sensing Resistors (FSR) are a polymer thick film device which exhibits a decrease in resistance with an increase in the force/ pressure applied to the active surface.
- The main structure:
  - An active area consisting of two sets of interdigitated traces that are electronically isolated from one another.
  - A plastic spacer, which includes an air vent through the tail, to separate the conductive film with the active area.
  - A flexible substrate coated with a thick polymer conductive film, aligned with the active area.





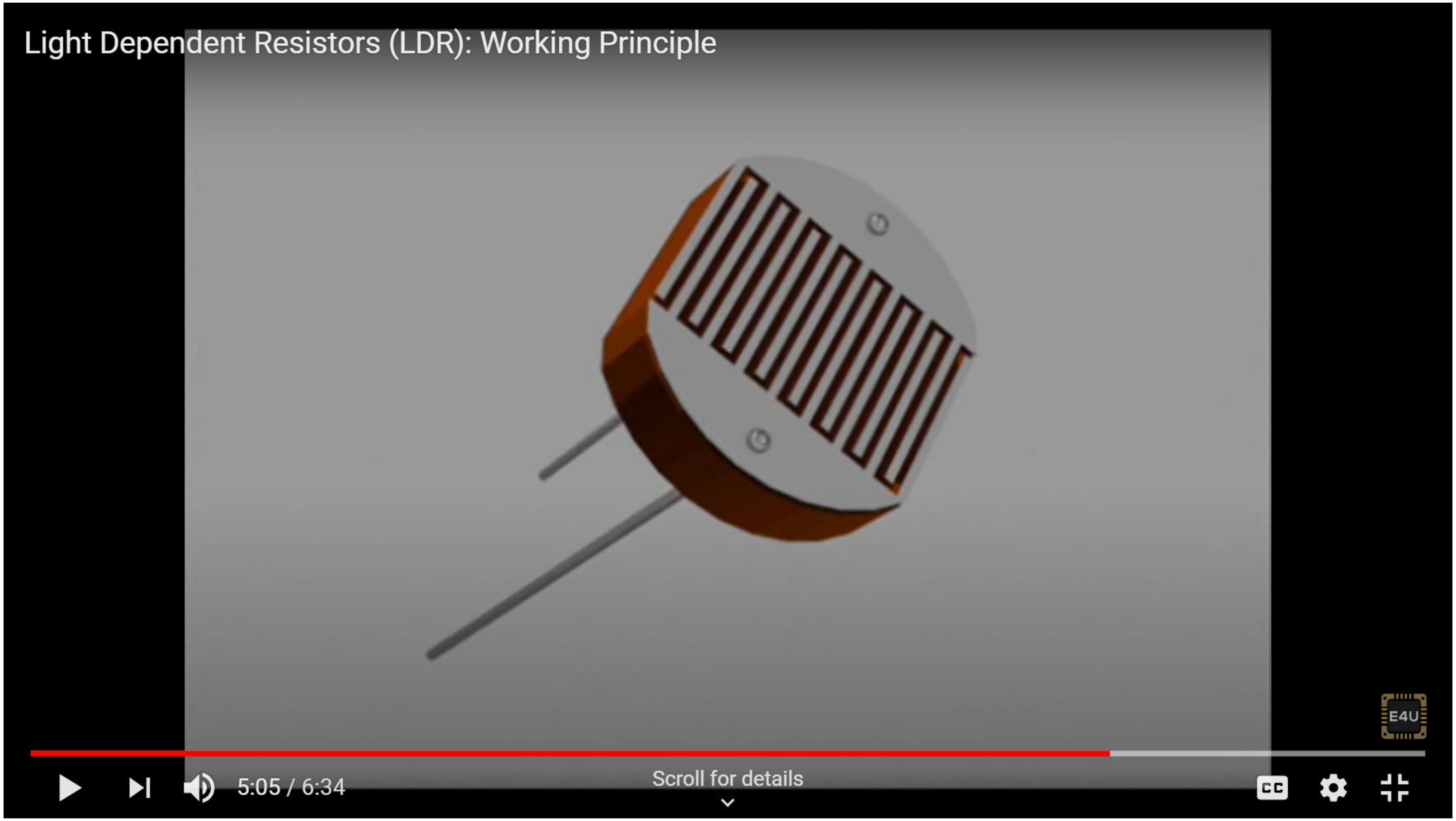
- When external force is applied to the sensor, air from the spacer opening is pushed through the air vent in the tail, and the conductive material on the substrate comes into contact with parts of the active area. The more of the active area that touches the conductive element, the lower the resistance.
- Low cost, simple design, but **not suitable for precision measurements.**

Demo: Force Sensing Resistor (optional)

Application: Working Principle of Strain Gauge

- A strain gauge is a sensor to measure strain on an object when deformation causes change to its electrical resistance.
- Electrical conductance (or resistance) depends on a conductor's geometry, as suggested by $R = \frac{\rho l}{A}$.
- Whenever a conductor is stretched within the limits of its elasticity, it doesn't break but, gets narrower and longer. Conversely, when it is compressed, it gets shorter and broader, ultimately changing its resistance.
- This resistance change is usually measured using a Wheatstone bridge.
- Applications: Strain gauges are extensively used in the field of geotechnical monitoring to keep a constant check on structures, dams, tunnels, buildings, bridges, stress and strain on rails, airplane, etc. so that the mishaps can be avoided well on time.

FYI: More About LDR?



https://youtu.be/ilN8XIK77dc