Week 6 - Day 1

Electromagnetic Lock

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Electromagnet

Concept 1: Biot-Savart Law

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Solenoid, Relay (Day 2)

Reading:

University Physics with Modern Physics – Chapter 28

Introduction to Electricity and Magnetism – Chapter 9

Application: Electromagnetic Lock

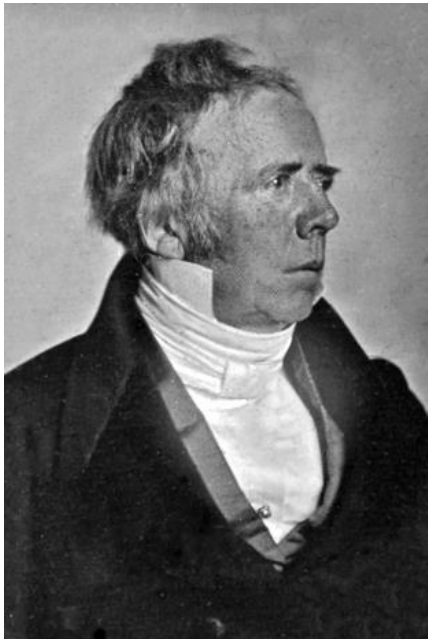
- An electromagnetic lock creates a magnetic field when energized or powered up, causing an electromagnet and armature plate to attract to each other strongly enough to keep a door from opening.
- Advantages: It is controllable (on/off) by signal, can be operated remotely, easy to install and operate, and ideal for security.
- Disadvantages: Requires a constant power source, can disable security in power outage, cost higher in comparison to mechanical locks.

Concept 1: Biot-Savart law

We have a physical law to calculate the magnetic field (magnitude and direction) produced by a current source.

Magnetism without Magnets!

- Hans Christian Ørsted, Danish physicist/chemist, discovered that electric currents create magnetic fields.
- 21 April 1820: during a lecture, Ørsted noticed a compass needle deflected when an electric current was switched on **and** off.
- Initially he thought that magnetic effects (*cf*. light/heat) radiate from all sides of a wire with an electric current.
- Later he concluded instead that an **electric current produces a *circular* magnetic field *along* the wire**.



Hans Christian Ørsted
(1777 – 1851)

Inspiration?

- *Hmmm*: Coulomb's law gives us an elegant relationship between Electric Field and Charge!
- *I wonder*: is there a similar law elegantly governing Magnetic Field and Current?

Charge

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Electric Field

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathrm{r}}$$

Current

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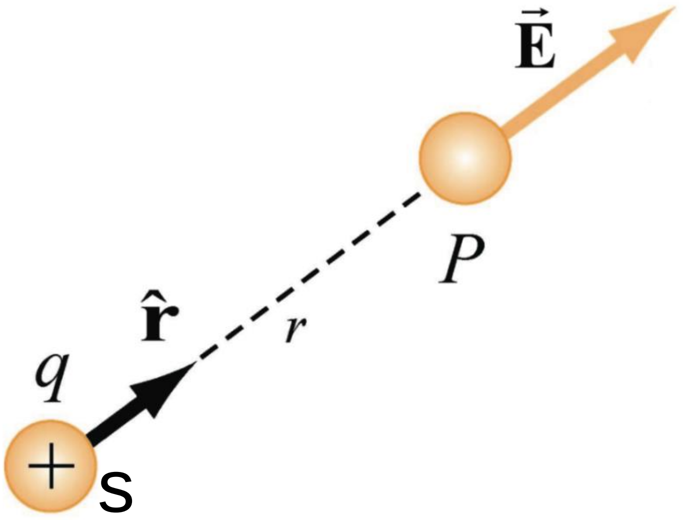
Magnetic Field

$$\vec{\mathbf{B}} = \frac{?}{4\pi}\frac{I}{r^2}\hat{?}$$

Recall: Coulomb's Law

- Named after French physicist Charles Augustin de Coulomb who published the law in 1785.
- Point charge q produces an electric field:

$$\boxed{\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{sp}^2}\hat{\mathbf{r}}_{sp}}$$

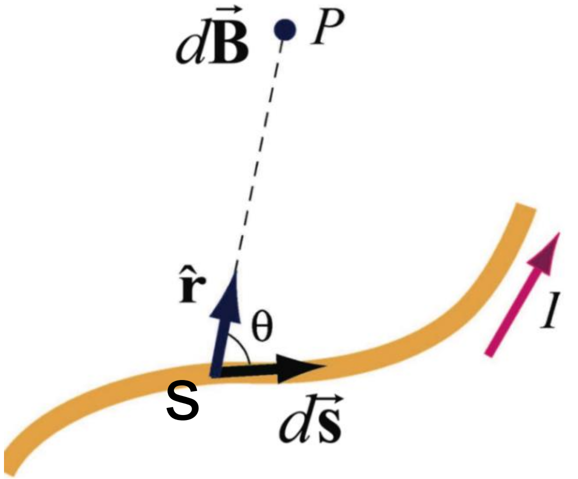


- Note: The direction of $\vec{E}$ is determined by the unit vector, $\hat{r}$ and the sign of q, the source charge.

Biot-Savart Law (pronounced /ˈbiːoʊ səˈvɑr/)

- Named after French physicist, Jean-Baptiste Biot and Felix Savart who discovered this relationship in 1820.
- Current element $d\vec{s}$ of length ds pointing in direction of current I produces a magnetic field.

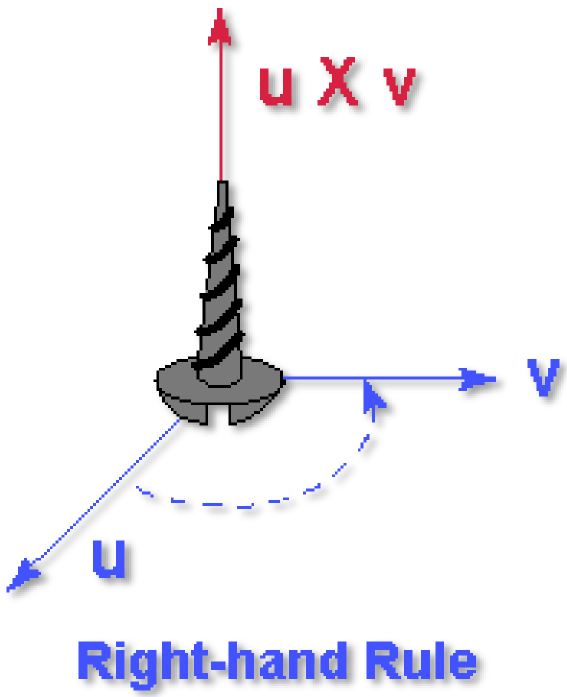
$$\boxed{d\overrightarrow{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{r}_{sp}}{r_{sp}^2}}$$

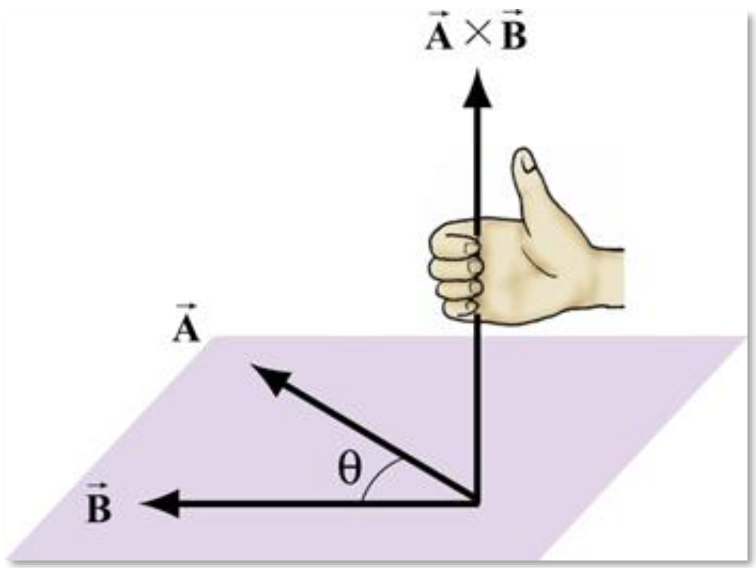


- Note: The direction of the magnetic field is determined by the cross product $d\vec{s} \times \hat{r}$ (right hand rule).

Cross Product Recap…

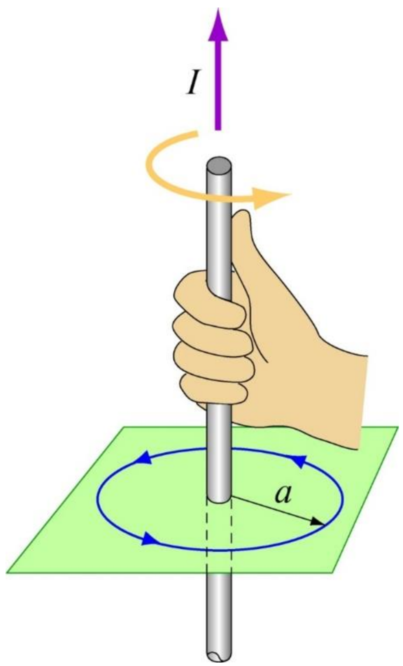
- Direction of Cross Product of Two Vectors is determined using the Right-hand (Curl/Screw) Rule

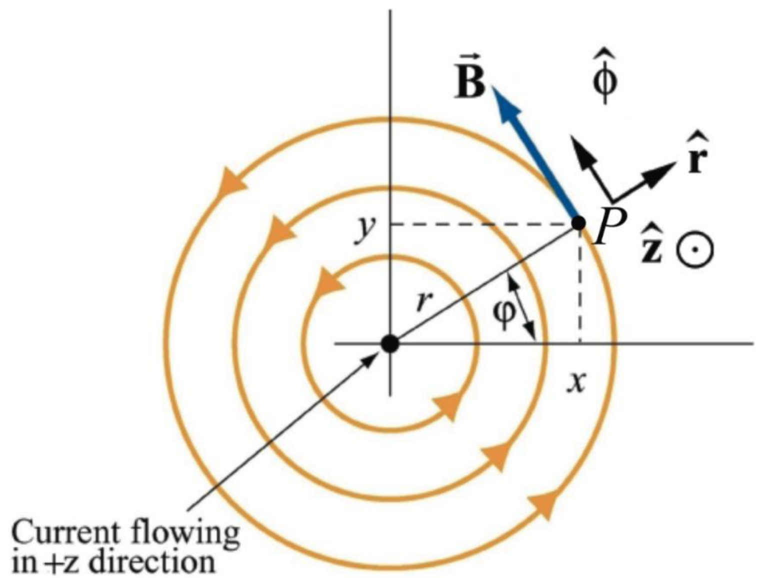




B-field is Circumferential (Right-Hand Rule)

- The direction of current element $Id\vec{s}$ is $\hat{z}$.
- The direction of the magnetic field, $\vec{B}(\vec{r})$ at P is $\hat{z} \times \hat{r} = \hat{\phi}$.

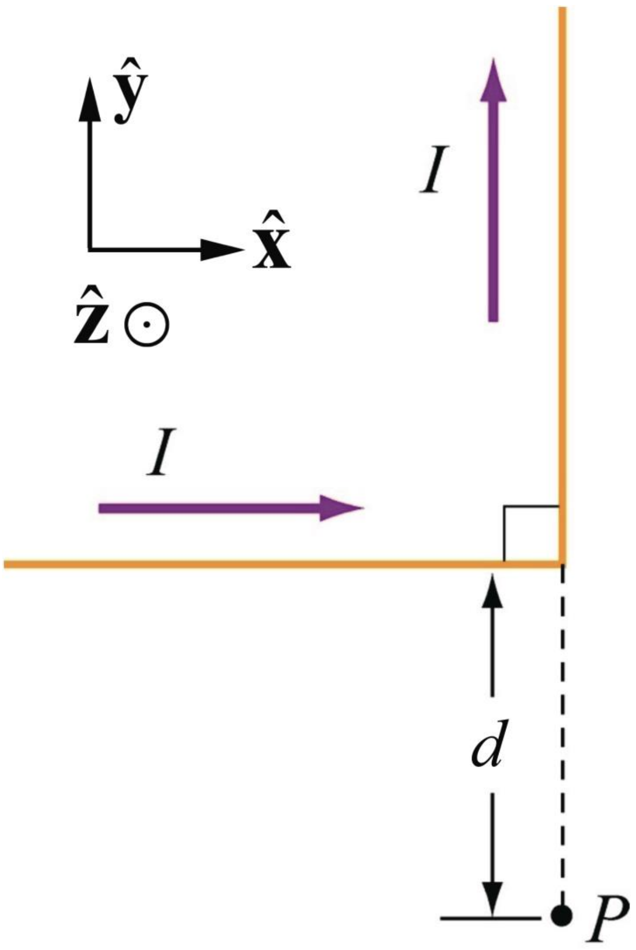




Concept Question 1.1: Direction of Biot-Savart law

The magnetic field at P points towards the

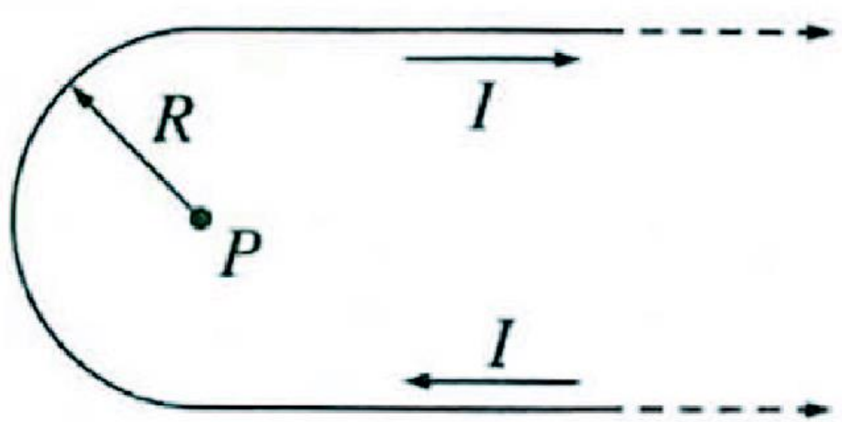
1. +x direction
2. +y direction
3. +z direction
4. -x direction
5. -y direction
6. -z direction
7. Field is zero



Concept Question 1.2: Bent Wire

The magnetic field at *P* is equal to the field of:

1. a semicircle
2. a semicircle plus the field of a long straight wire
3. a semicircle minus the field of a long straight wire
4. none of the above

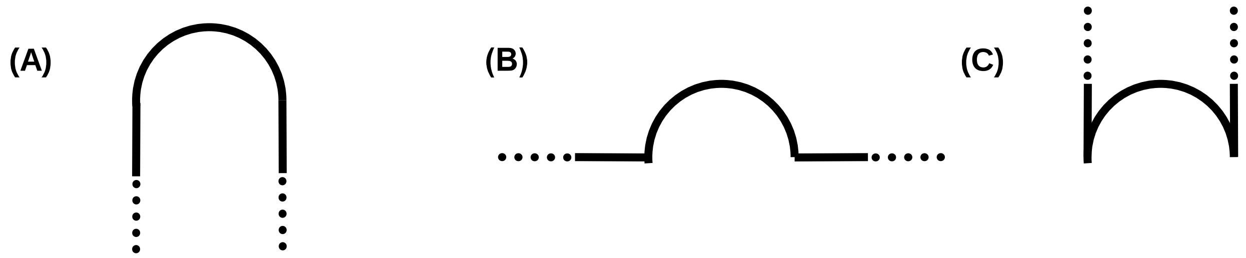


Concept Question 1.3: Shape and Magnetic Field

You are given two long straight pieces of wire and a semicircle of wire and asked to construct a shape that, when a current is passed through it, creates a large magnetic field at the center of the semi-circle. You construct 3 shapes as shown below.

Rank the magnitude of the field generated by these three configurations (assuming the current is the same in each):

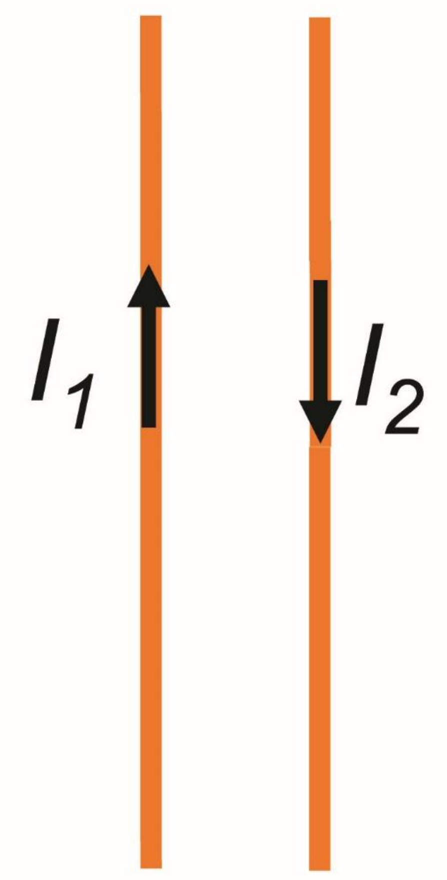
1. A > B > C
2. C > B > A
3. A = C > B
4. B > A = C
5. A > B = C
6. A = B = C



Concept Question 1.4: Parallel Wires

Consider two parallel current carrying wires. With the currents running in the opposite direction, the wires are

1. attracted (opposites attract?)
2. repelled (opposites repel?)
3. pushed another direction
4. not pushed – no net force
5. I don't know



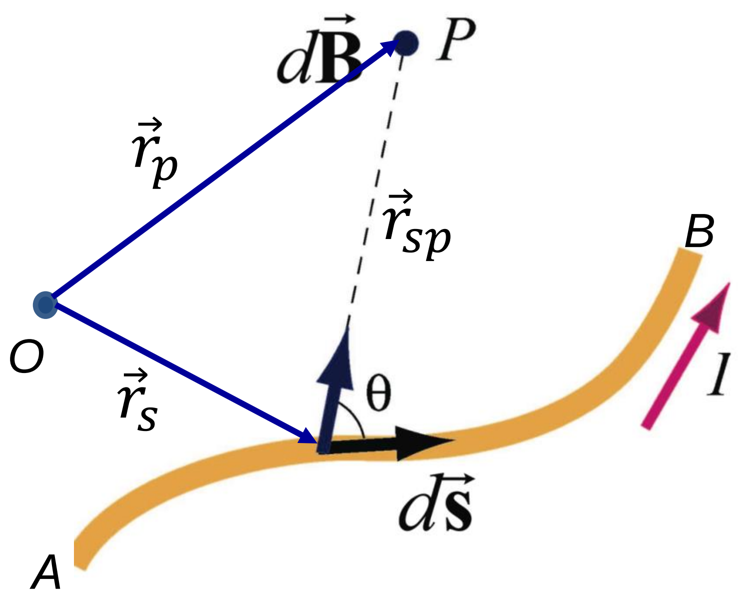
Applying Biot-Savart Law in a Coordinate System

- Magnetic field due to current element is given by Biot-Savart law.
- The total magnetic field due to a stretch of conductor is the superposition (*i.e.* linear sum) of the magnetic field due to *all current elements* that the conductor is composed of, *i.e.*

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \hat{r}_{sp}}{r_{sp}^2}$$

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \vec{r}_{sp}}{r_{sp}^3}$$

$$\vec{\mathbf{B}} = \int_A^B \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times (\vec{r}_p - \vec{r}_s)}{|\vec{r}_p - \vec{r}_s|^3}$$



(note: $d\vec{\boldsymbol{s}}$ is now crossed with the *relative position vector* $(\vec{\boldsymbol{r}}_p - \vec{\boldsymbol{r}}_s)$ instead of *unit vector* $\hat{\boldsymbol{r}}_{sp}$)

Example: Magnetic Field Due to a Thin Straight Wire

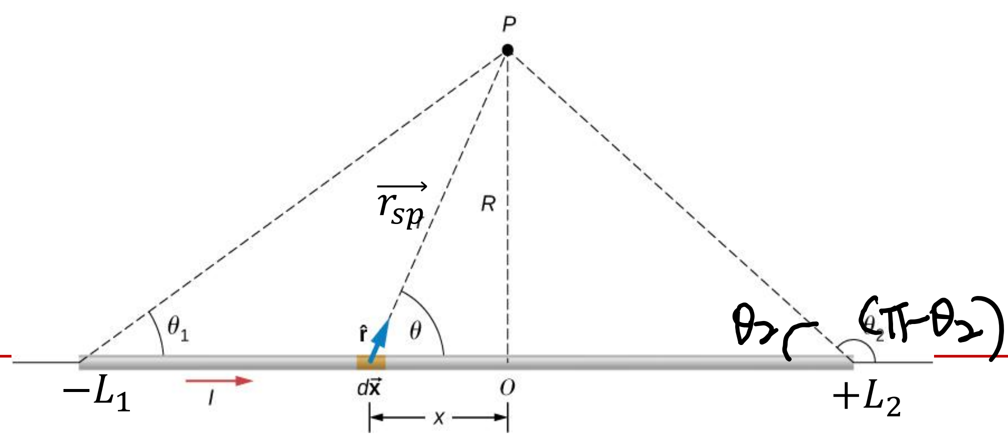
- Consider a finite straight wire that carries a current I. What is the magnetic field at a point P, located a distance R from the wire?
- Magnetic field at point P is contributed by the magnetic field due to each small current element $Id\vec{x}$ along the wire, following the Biot-Savart Law

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times(\vec{r}_p - \vec{r}_s)}{r_{sp}^3}$$

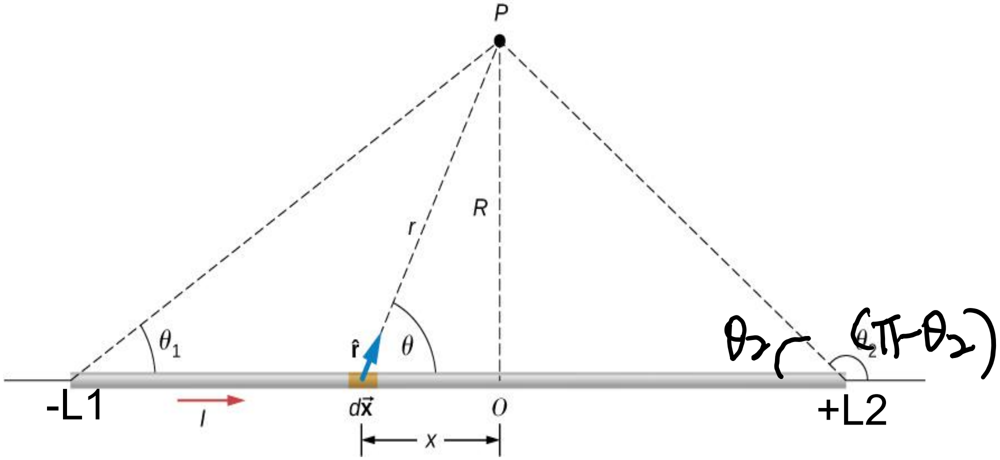
- $d\vec{s}\times(\vec{r}_p - \vec{r}_s) = dx\hat{i}\times(R\hat{j} - x\hat{\imath})$; the direction of $d\vec{B}$ is out of the page $(+\hat{k})$.

$$B = \frac{\mu_0}{4\pi}\int_{wire}\frac{I\left(dx\, R\, \hat{k}\right)}{(x^2+R^2)^{3/2}}$$

Note: θ, x and r are dependent with each others. Our strategy here is to convert θ and r in term of x.



$$B = \frac{\mu_0}{4\pi} \int_{-L_1}^{+L_2} \frac{IR(dx)}{(x^2 + R^2)^{3/2}}$$



Evaluating the integral yields or referring to the integration table,

$$\int \frac{dx}{(x^2 + R^2)^{3/2}} = \frac{1}{R^2} \frac{x}{(x^2 + R^2)^{1/2}}$$

Thus,

$$B = \frac{\mu_0}{4\pi R} \left[\frac{x}{(x^2 + R^2)^{1/2}} \right]_{-L_1}^{+L_2}$$

$$B = \frac{\mu_0}{4\pi R} \left[\frac{L_2}{(L_2^2 + R^2)^{1/2}} - \frac{-L_1}{(L_1^2 + R^2)^{1/2}} \right]$$

Note that it can also be expressed as

$$\boxed{B = \frac{\mu_0}{4\pi R} [\cos(\theta_2) + \cos(\theta_1)]}$$

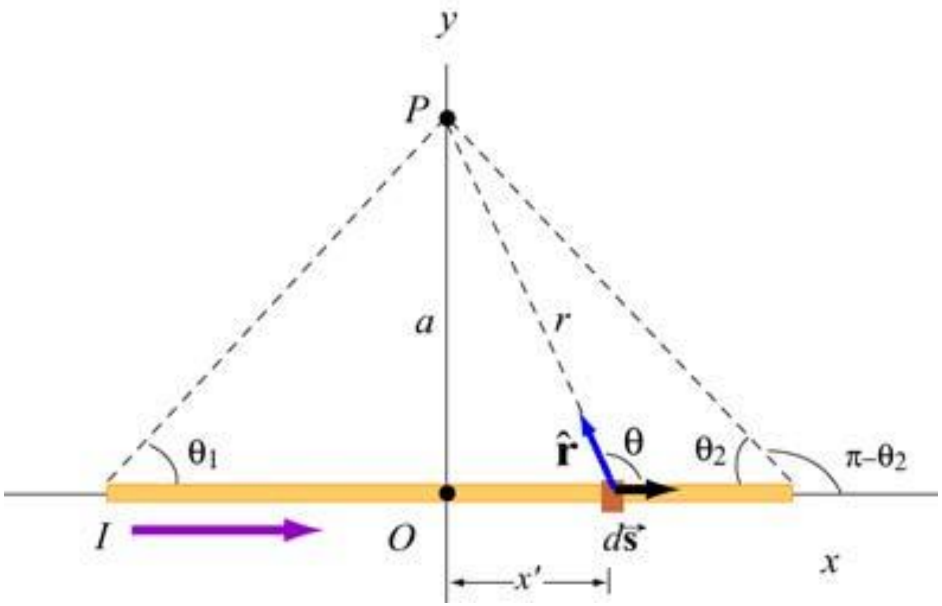
If the wire is infinitely long, the angles $\theta_1, \theta_2 \rightarrow 0$

Then, $|\vec{\mathbf{B}}| = \mathrm{B} = \frac{\mu_0 I}{2\pi a}$

The magnetic field at P is pointing out of the page (+z direction).

Alternative: Magnetic Field Due to a Thin Straight Wire

- $d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$
- $|d\vec{s}\times\hat{r}| = dx' sin\theta$
- $|d\vec{\mathbf{B}}| = \frac{\mu_0}{4\pi}\frac{I(dx' sin\theta)}{r^2}$



- Note: θ, x' and r are dependent with each other
- $r = \frac{a}{sin(\pi-\theta)} = a\csc\theta$
- $x' = a\, cot(\pi-\theta) = -a\, cot\theta \rightarrow dx' = a csc^2\theta d\theta$
- $|d\vec{\mathbf{B}}| = \frac{\mu_0}{4\pi}\frac{I(dx' sin\theta)}{r^2} = \frac{\mu_0}{4\pi}\frac{(a\, csc^2\theta d\theta\, sin\theta)}{(a\, csc\theta)^2} = \frac{\mu_0 I sin\theta d\theta}{4\pi a}$

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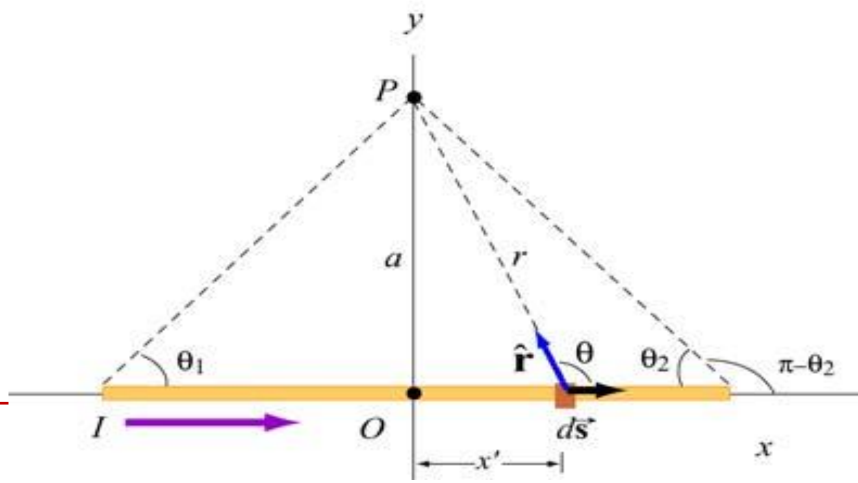
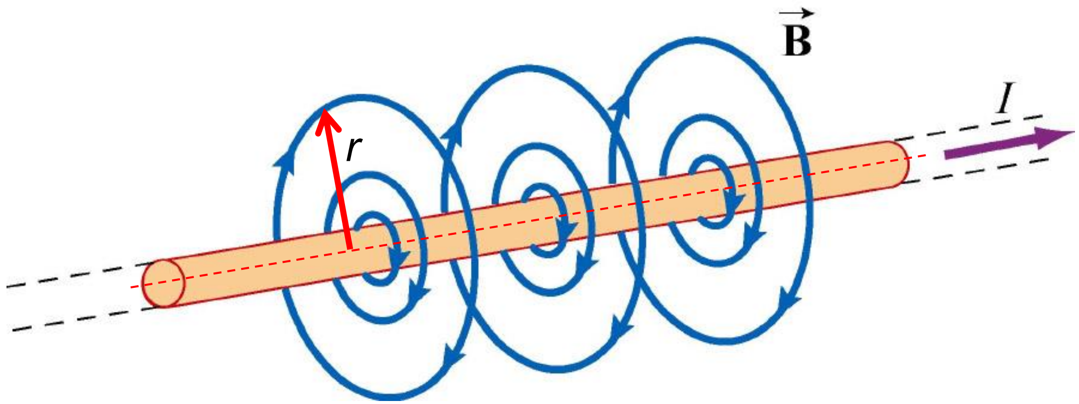
Alternative: Magnetic Field Due to a Thin Straight Wire

- $\left|d\vec{\mathbf{B}}\right| = \frac{\mu_0 I sin\theta d\theta}{4\pi a}$
- $\Rightarrow \left|\vec{\mathbf{B}}\right| = \int \left|d\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\pi-\theta_2} sin\theta d\theta$
- The direction of magnetic field due to all current elements is in the +z axis.
- $\Rightarrow \left|\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{4\pi a}(-cos\theta)_{\theta_1}^{\pi-\theta_2}\Big| = \frac{\mu_0 I}{4\pi a}(cos\theta_2 + cos\theta_1)$
- If the wire is infinitely long, the angles $\theta_1, \theta_2 \rightarrow 0$
- Then, $\left|\vec{\mathbf{B}}\right| = \mathrm{B} = \frac{\mu_0 I}{2\pi a}$
- The magnetic field at P is pointing out of the plane(+z direction)



Magnetic Field of an Infinitely Long Wire

Thus, for an infinitely long wire, the magnetic field at a distance *r* away from the axis is given by



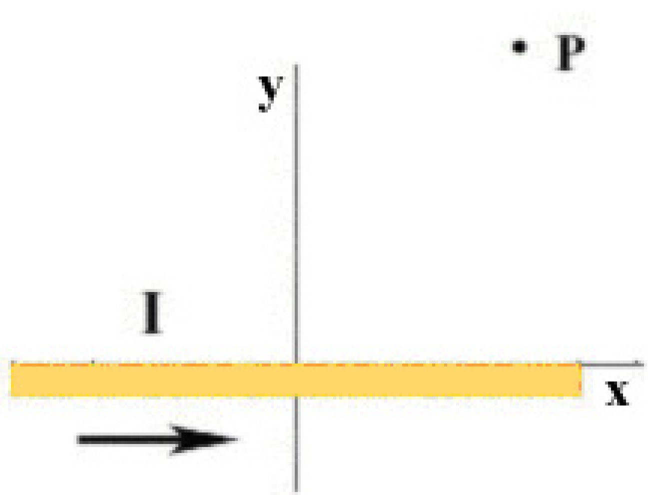
$$\left|\vec{B}\right| = \mathrm{B} = \frac{\mu_0 I}{2\pi r} \text{ or } \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$$

- The direction is circumferential.
- This is simply Ampere's Law that you will learn in next session

Case Problem 1.1

Consider a straight wire of length L with a current $\mathbf{I}$ flowing in the wire (here, we will neither worry about the return path of the current nor the source for the current).

Find the integral vector expression for the magnetic field due to the straight wire at point P, which does not lie on the perpendicular bisector of the wire.



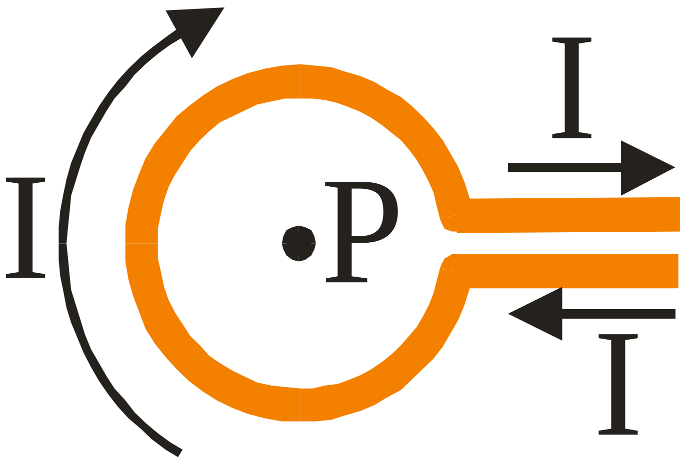
Integration table:

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{(x^2+a^2)}}$$

Case Problem 1.2

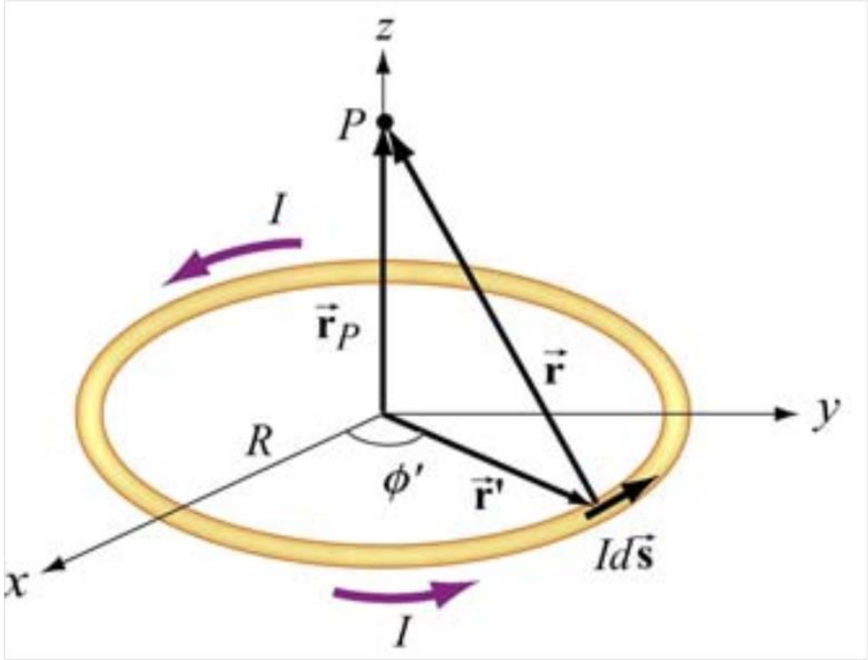
Consider a coil with radius R and current I. What is the magnetic field at point P?

1. Think about it:
   - Legs contribute nothing
   - Ring makes field into page
2. Choose a suitable ds
3. Pick your coordinates
4. Use Biot-Savart Law



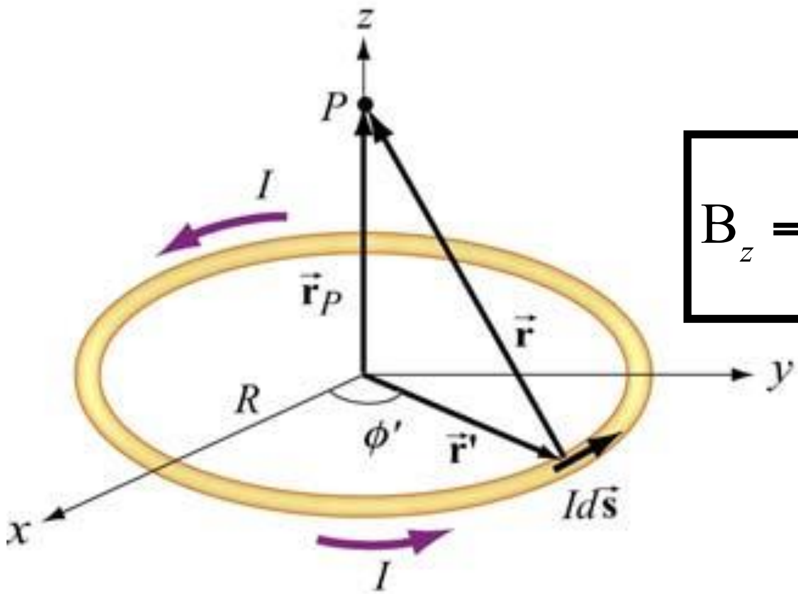
Case Problem 1.3

Consider a coil with radius R, carrying a current I. What is $\vec{B}$ at point P?



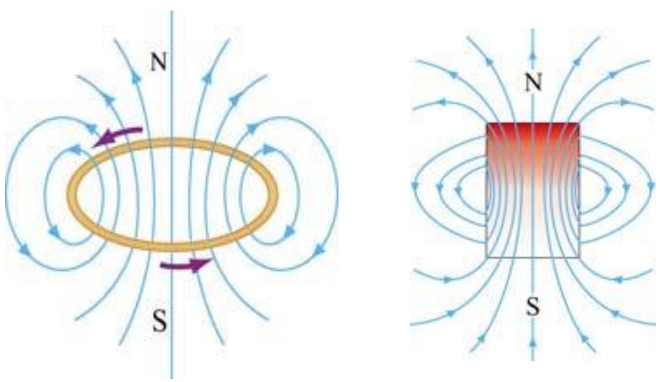
Magnetic Field on the Axis of a Circular Current Loop

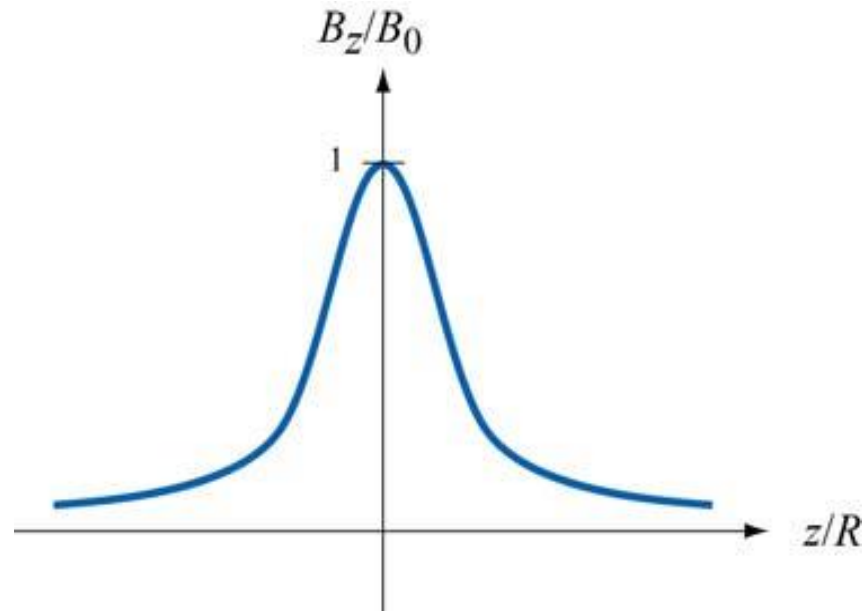
- Note: a circular current loop can be viewed as magnetic dipole.



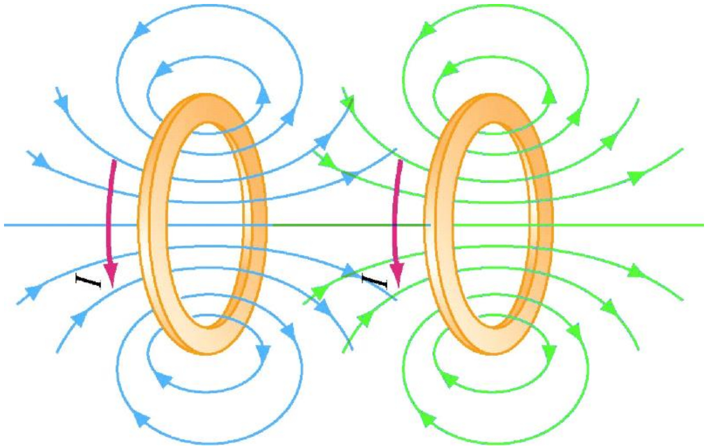
$$\boxed{B_z = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}}$$

$$\boxed{B_0 = B_z(z=0) = \frac{\mu_0 I}{2R}}$$

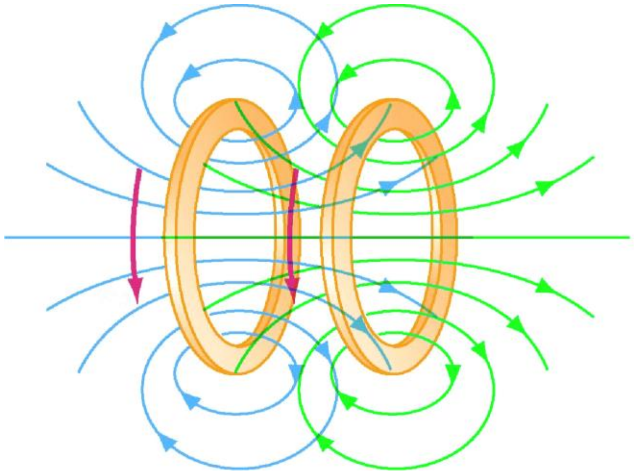


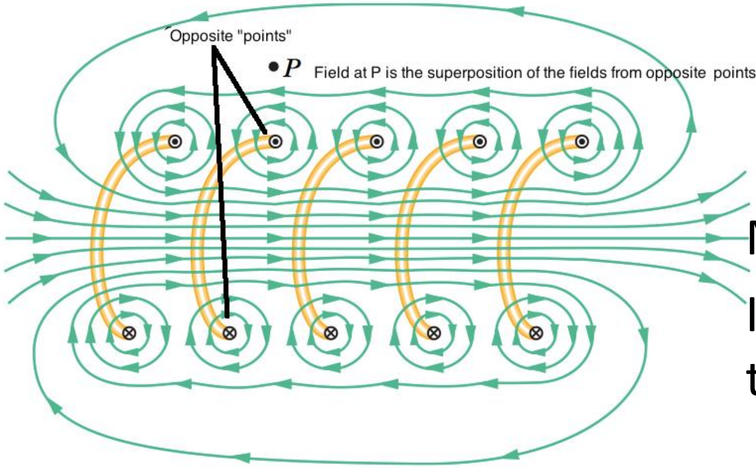


- 2 current loops align along the axis.
- Magnetic fields due to each loops superimpose in the space.



- Move the 2 loops closer and magnetic fields reinforce strongly in the space between 2 loops.





Multiple current loops form the basis of a solenoid.

Ideally, it produces a uniform magnetic field inside the solenoid while zero field outside the solenoid.

http://web.mit.edu/viz/EM/visualizations/magnetostatics/MagneticFieldConfigurations/tworings/tworings.htm

Summary

- Current carrying element $Id\vec{s}$ gives rise to magnetic field $d\vec{\mathbf{B}}$ at point P (position vector $\vec{r}$ with respect to current element) following Biot-Savart Law, i.e.

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \hat{r}_{sp}}{r_{sp}^2}$$

- The total magnetic field due to a piece of current carrying conductor is given by the integration of the contribution of all current carrying elements, i.e.

$$\vec{\mathbf{B}} = \int_A^B \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \left(\vec{r}_p - \vec{r}_s\right)}{|\vec{r}_p - \vec{r}_s|^3}$$

where $\vec{r}_p$ is the position vector of point P, $\vec{r}_s$ is the position of current element $Id\vec{s}$. Integration carried along the wire from point A to B.

Problem Solving Strategy

The law states that the magnetic field at a point P due to a length element $d\vec{\mathbf{s}}$ carrying a steady current I located at $\vec{\mathbf{r}}$ away is given by

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times\vec{\mathbf{r}}}{r^3}$$

The calculation of the magnetic field may be carried out as follows:

(1) Source point: Choose an appropriate coordinate system and write down an expression for the differential current element $I\,d\vec{\mathbf{s}}$, and the vector $\vec{\mathbf{r}}\,'$ describing the position of $I\,d\vec{\mathbf{s}}$. The magnitude $r' = |\vec{\mathbf{r}}\,'|$ is the distance between $I\,d\vec{\mathbf{s}}$ and the origin. Variables with a "prime" are used for the source point.

(2) Field point: The field point P is the point in space where the magnetic field due to the current distribution is to be calculated. Using the same coordinate system, write down the position vector $\vec{\mathbf{r}}_P$ for the field point P. The quantity $r_P = |\vec{\mathbf{r}}_P|$ is the distance between the origin and P.

Problem Solving Strategy

(3) Relative position vector: The relative position between the source point and the field point is characterized by the relative position vector $\vec{\mathbf{r}} = \vec{\mathbf{r}}_P - \vec{\mathbf{r}}\,'$. The corresponding unit vector is

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} = \frac{\vec{\mathbf{r}}_P - \vec{\mathbf{r}}\,'}{|\vec{\mathbf{r}}_P - \vec{\mathbf{r}}\,'|}$$

where $r = |\vec{\mathbf{r}}| = |\vec{\mathbf{r}}_P - \vec{\mathbf{r}}\,'|$ is the distance between the source and the field point P.

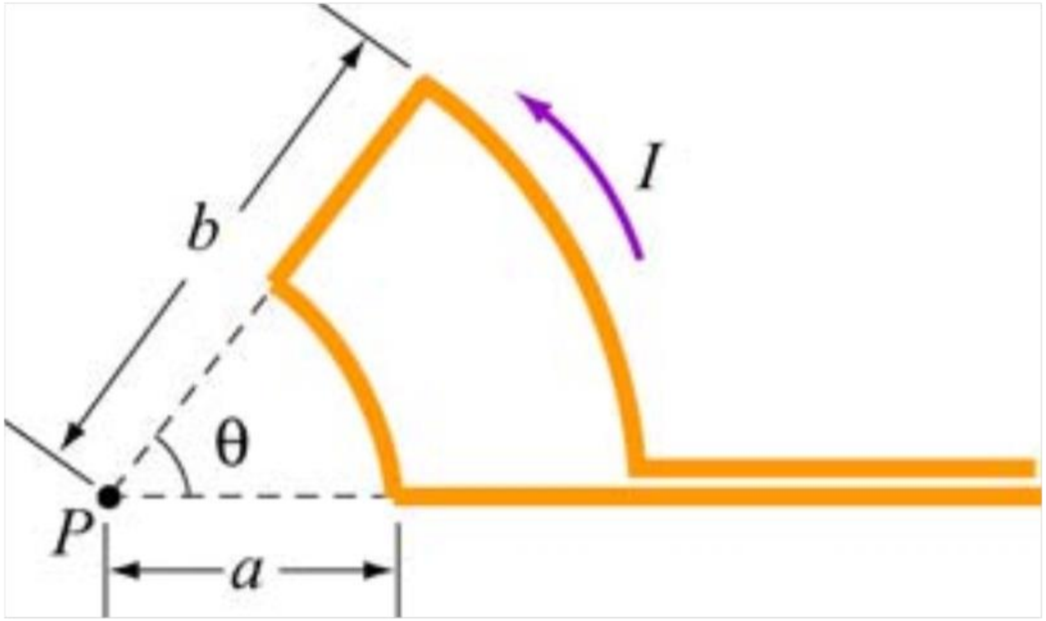
(4) Calculate the cross product $d\vec{\mathbf{s}} \times \hat{\mathbf{r}}$ or $d\vec{\mathbf{s}} \times \vec{\mathbf{r}}$. The resultant vector gives the direction of the magnetic field $\vec{\mathbf{B}}$, according to the Biot-Savart law.

(5) Substitute the expressions obtained to $d\vec{\mathbf{B}}$ and simplify as much as possible.

(6) Complete the integration to obtain $\vec{\mathbf{B}}$ if possible. The size or the geometry of the system is reflected in the integration limits. Change of variables sometimes may help to complete the integration.

Extra Case Problem 1

Consider the current-carrying loop formed of radial lines and segments of circles whose centers are at point P as shown below. Find the magnetic field $\vec{\mathbf{B}}$ at P.



Extra Case Problem 2

An infinitely long current-carrying wire is bent into a hairpin. Find the magnetic field at point P which is located at the center of the semi-circle.