Week 9 - Day 2

DC Circuit Analysis

Concept 1: Power

Concept 2: DC Circuit Analysis – Kirchhoff Voltage and Current Law

Power Rating of an AC Adapter

Reading:

University Physics with Modern Physics – Chapter 25

Introduction to Electricity and Magnetism – Chapter 7

Concept 1: Power

Electrical Power

- Power is change in energy per unit time (rate of change of energy).

$$dU = Pdt = dq\Delta V$$

- So, power to move current through circuit elements:

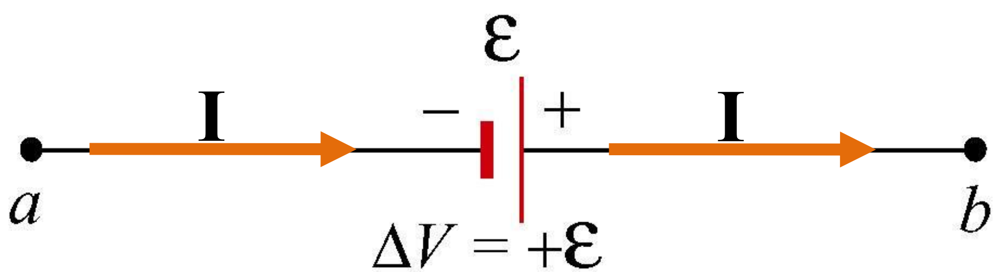
$$\Rightarrow P = \left(\frac{dq}{dt}\right)\Delta V$$

$$P = I\Delta V$$

Power - Battery

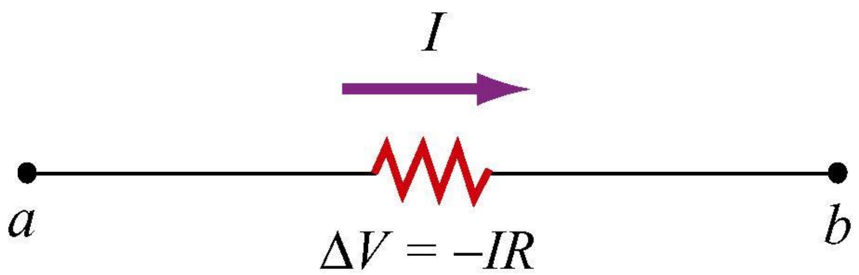
Moving from the negative to positive terminal of a battery **increases** the potential. If current flows in that direction the battery **supplies** power.

$$P_{supplied} = I\Delta V = I\varepsilon$$



Power - Resistor

Moving across a resistor in the direction of current **decreases** the potential. Resistors **always dissipate** power.



$$P_{dissipated} = I|\Delta V| = I^2 R = \frac{\Delta V^2}{R}$$

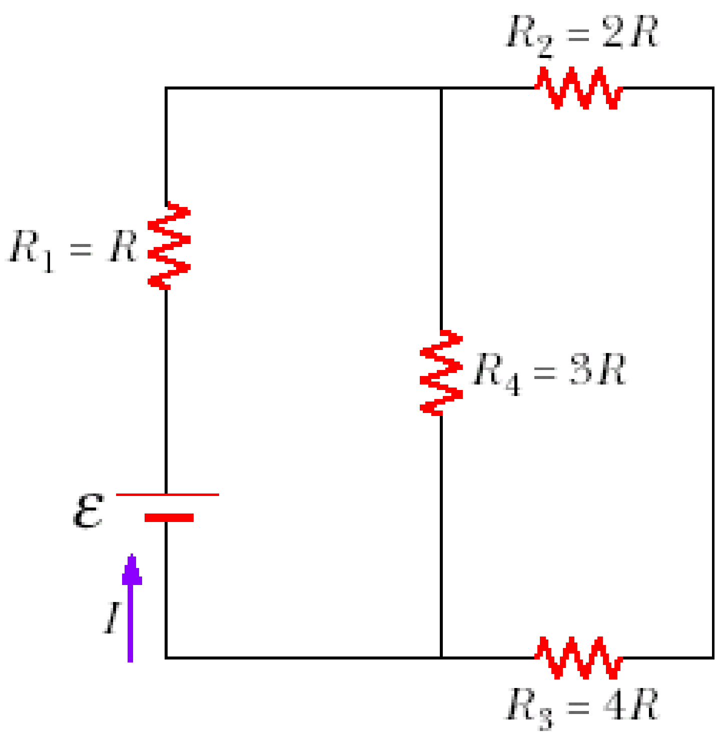
The brightness of an incandescent bulb depends on the temperature of the filament. That in turn depends on the power dissipated of the filament. (We model an incandescent bulb as a resistor).

Case Problem 1.1: Power Dissipated

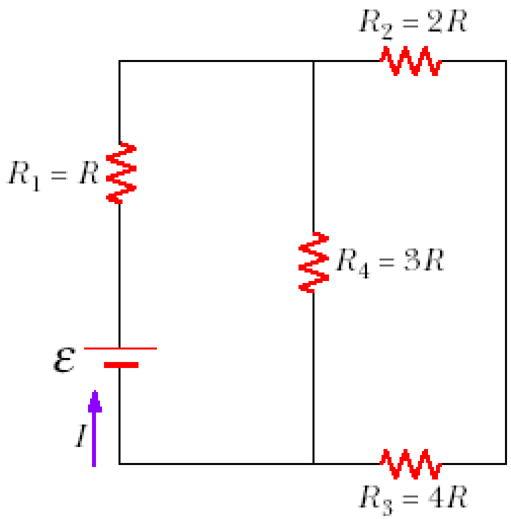
Four resistors are connected to a battery as shown in the figure. The current in the battery is I, the battery emf is ε, and the resistor values are $R_1 = R$, $R_2 = 2R$, $R_3 = 4R$, $R_4 = 3R$.

A. Determine the power dissipated by R1, R2, R3 and R4.

B. Determine the power delivered by the emf.



Case Problem 1.1 (solution)

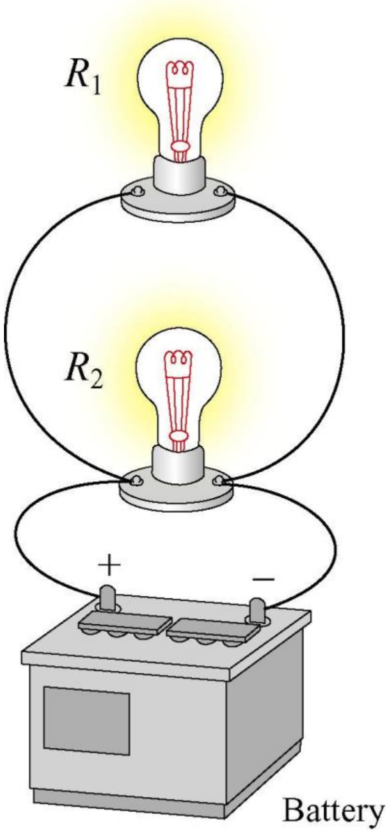


- From previous case problem,
- $I_{R1} = I; I_{R2} = I_{R_3} = \frac{I}{3}; I_{R4} = \frac{2}{3}I$
- $P_{R_1} = I^2R; P_{R_2} = \left(\frac{I}{3}\right)^2 (2R);$
- $P_{R_3} = \left(\frac{I}{3}\right)^2 (4R); P_{R_4} = \left(\frac{2}{3}I\right)^2 (3R)$
- $P_{battery} = \varepsilon I = (3IR)I = 3I^2R$
- Note that the total power dissipated by all resistors, $P_{all\ R} = \frac{9+2+4+12}{9} I^2R = 3I^2R$ are the same as the power supplied by the battery -> conservation of energy.

Concept Question 1.1: Bulbs & Batteries

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in parallel to the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected.

A. Is Higher
B. Is Lower
C. Is The Same
D. Don’t know



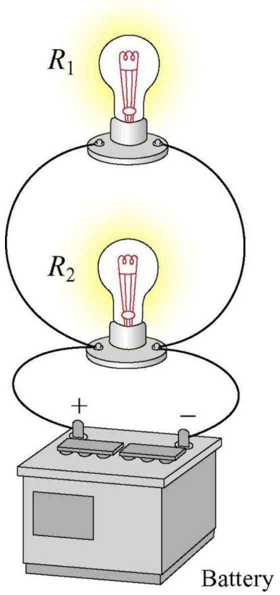
Multiple Choice

Concept Question 1.1 (Solution)

Answer: A. More current flows from the battery

There are several ways to see this:

1. The equivalent resistance of the two light bulbs in parallel is half that of one of the bulbs, and since the resistance is lower the current is higher, for a given voltage.
2. The battery must keep two resistances at the same potential, thus the current flows from the battery must be doubles.



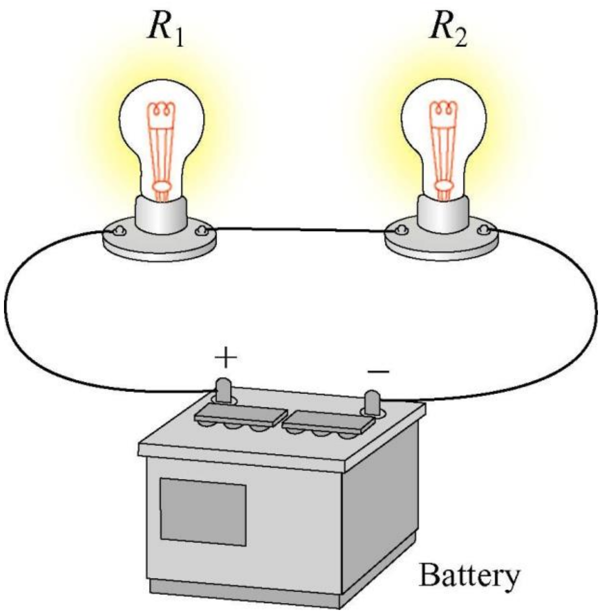
Concept Question 1.2: Bulbs & Batteries

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in series with the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected.

A. Is Higher

B. Is Lower

C. Is The Same

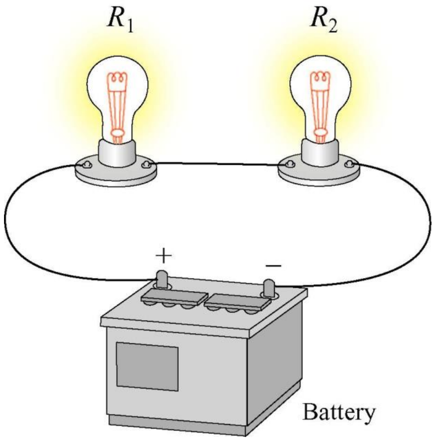


Multiple Choice

Concept Question 1.2 (Solution)

Answer: B. Less current flows from the battery

The equivalent resistance of the two light bulbs in series is twice that of one of the bulbs, and since the resistance is higher the current is lower, for the given voltage.



Concept Question 1.3: Power

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in series with the first light bulb. After the second light bulb is connected, the light (power) from the first bulb (compared to when only one bulb was connected)

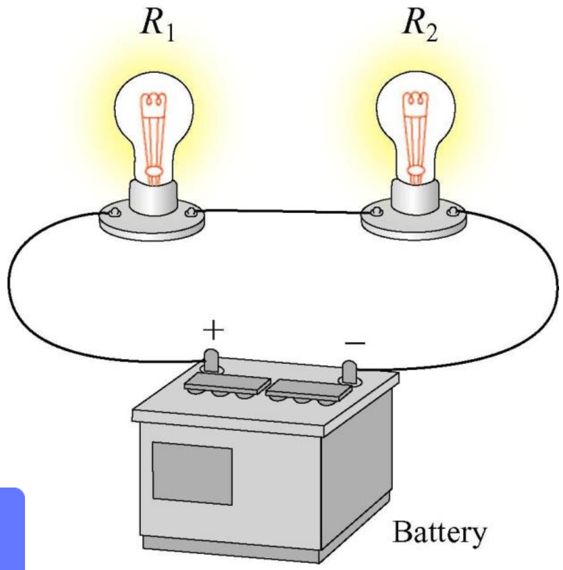
A. Is four times higher

B. Is twice as high

C. Is the same

D. Is half as much

E. Is $\frac{1}{4}$ as much

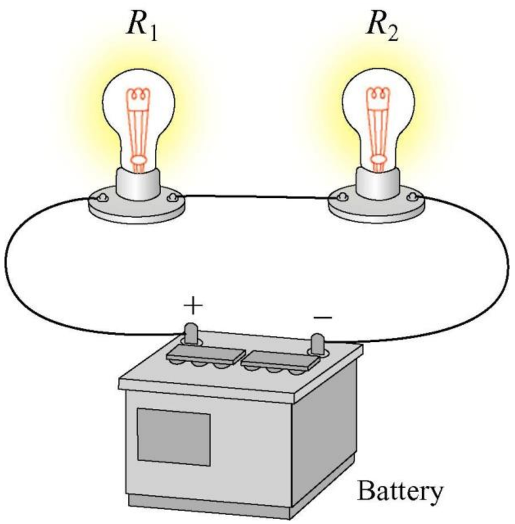


Concept Question 1.3 (Solution)

- Answer: E. Is 1/4 as bright

- In the series circuit, R doubles, the current that flow through the circuit is cut in half. So power delivered by the battery is half what it was. But that power is further divided between two bulbs now.

- $P = I^2R$

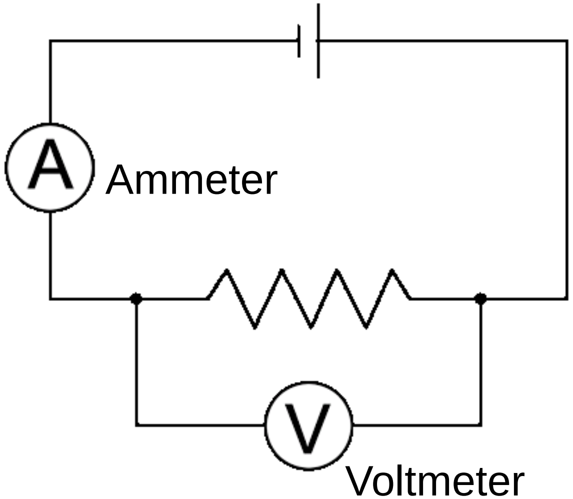


Application: Power Rating of an AC Adapter

- A power supply for electronic devices. It plugs into a wall outlet and convert AC to a single DC voltage.
- It comes with the output voltage and the rated output current. The output power rating = output voltage x rated output current.
- The rated output current is the maximum load current that a power supply can provide at a specified ambient temperature. A power supply can never provide more current than that of the rated current.
- The current drawn from a power supply depends on the power rating of the load circuit.
- A load requiring more current than rated output current of the power supply will not get enough power and may not operate as expected. If the power supply (without protection) provides the excess current beyond its rated amount for a prolonged time, it may damage the components.
- We always select a higher rating power supply than the load.

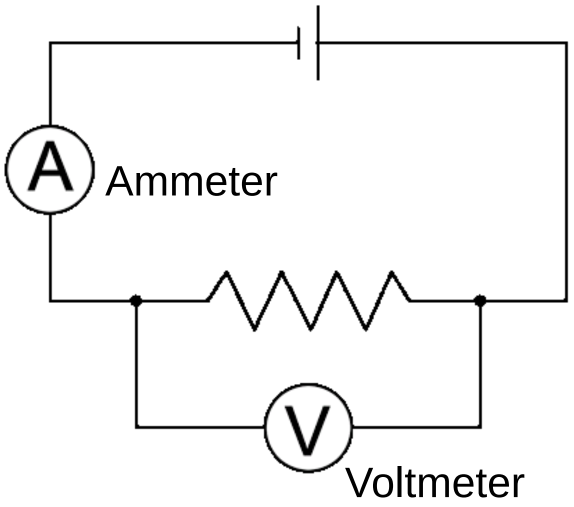
How to use an Ammeter and Voltmeter

- To measure voltage, the leads of the voltmeters are placed in parallel with the two terminals.
- Voltage difference is the difference in potential energy per unit of charge between two points in a circuit. Thus, the voltmeter must be placed between these two points.
- A voltmeter should have a large resistance so that it does not divert much current from the component whose voltage is being measured.
- To measure current, we need to insert the meter in series with other components.
- An ammeter should have a small resistance so that its effect on the overall resistance (and thus the current) is small.
- Convention:
  – Red: positive terminal
  – Black: negative terminal



A digital multimeter

Demo: Series and Parallel Circuit



- Estimating the temperature of the light bulb filament.
- Using ohmmeter in the multimeter to measure the resistance of a light bulb, R_o
- Build a simple circuit to measure the current passing through the light bulb and the voltage across the bulb.
- Use Ohm's Law to deduce the resistance of the bulb while it is on, R. From the temperature-resistance relation for conductor:
- $\rho = \rho_o\,[1 + \alpha(T - T_o\,)]$ or $R = R_o\,[1 + \alpha(T - T_o\,)]$
- Temperature coefficient of tungsten, $\alpha = 0.0045\ ^{\mathrm{o}}\mathrm{C}^{-1}$
- Take T_o as room temperature 25 ^{o}C
- With R, R_o and T_o, we can find out T of the filament.

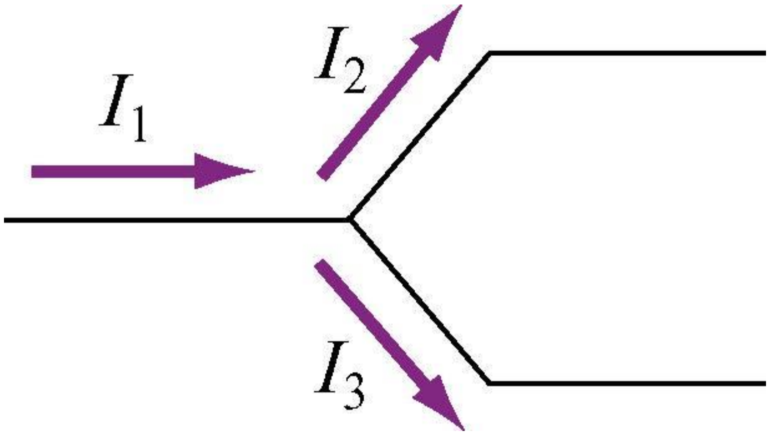
Concept 2: DC Circuit Analysis – Kirchhoff Voltage and Current Law

Kirchhoff's Current Law (KCL) or Kirchhoff's Junction Rule

- By current conservation, the sum of the currents entering any junction in a circuit must be equal the sum of currents leaving that junction.
- $\sum I_{in} = \sum I_{out}$

Example:

- $I_1 = I_2 + I_3$

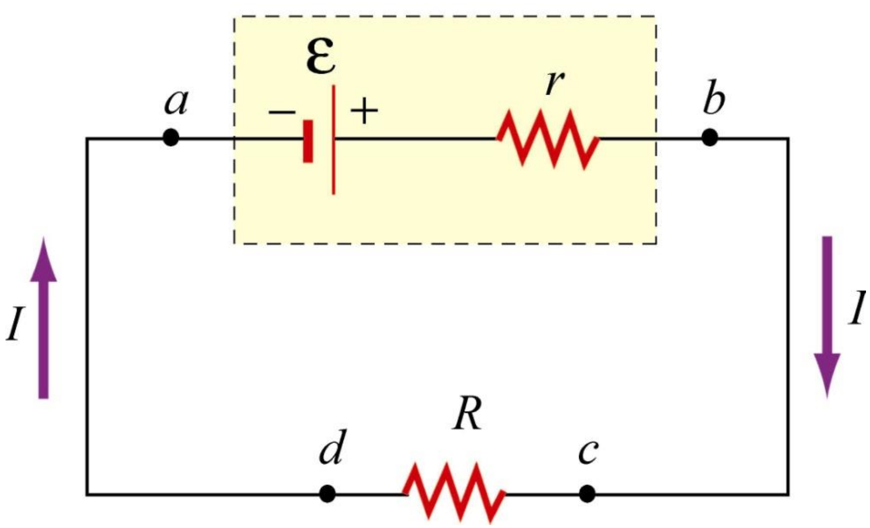


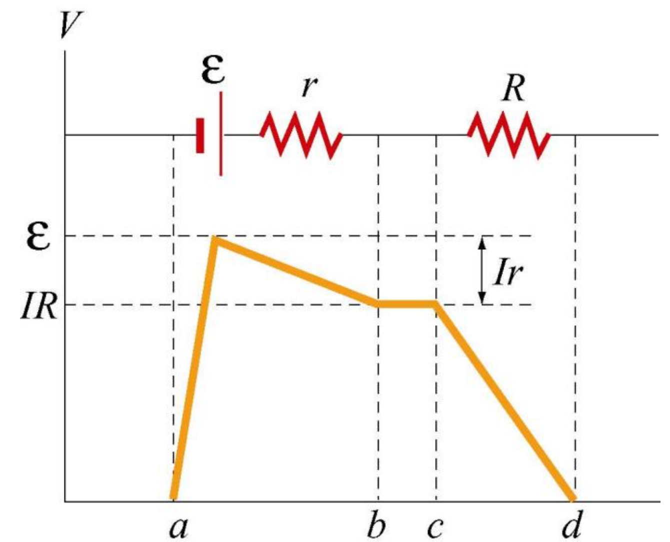
Kirchhoff's Voltage Law (KVL) Kirchhoff's Loop Rule

- The sum of the potential differences, ΔV across all elements around any closed-circuit loop must be zero.

$$\sum_i \Delta V_i = -\oint \vec{E}_{static} \cdot d\vec{s} = 0$$

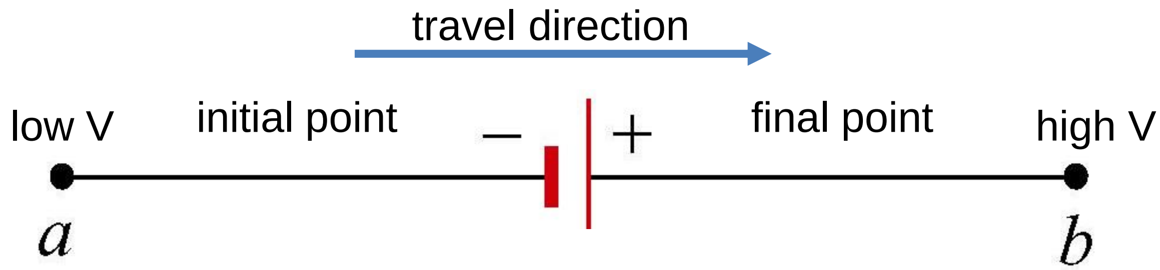
- The rules for determining ΔV across a battery and a resistor (with a designated travel direction) are based on the sign conventions in the next two slides.





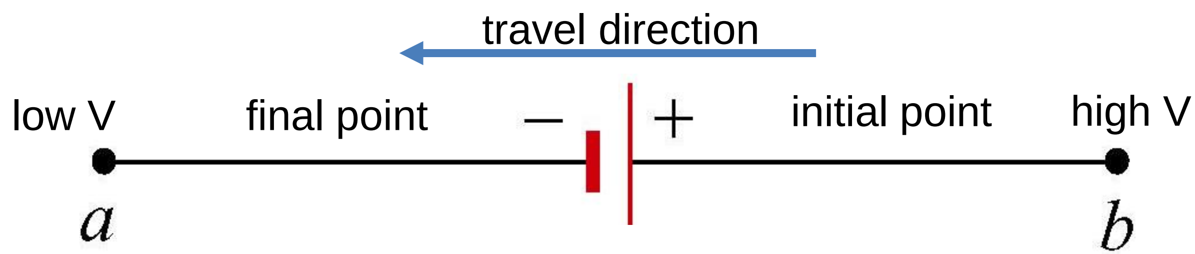
Sign Conventions - Battery

- When moving from the negative to positive terminal of a battery, the change in potential **increases**, as shown below:



$$\Delta V = V_b - V_a = +\varepsilon$$

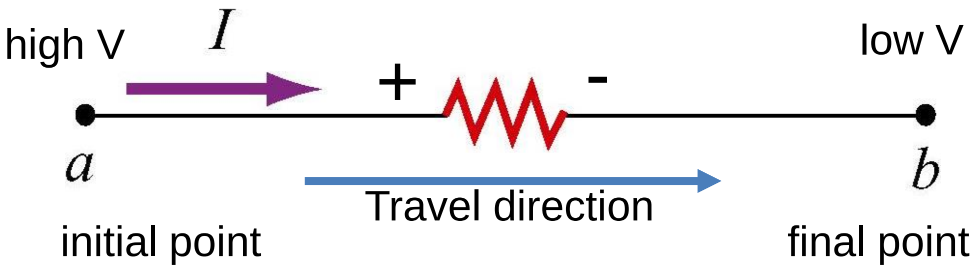
- Likewise, moving from positive to negative terminal of a battery, the change in potential decreases.



$$\Delta V = V_a - V_b = -\varepsilon$$

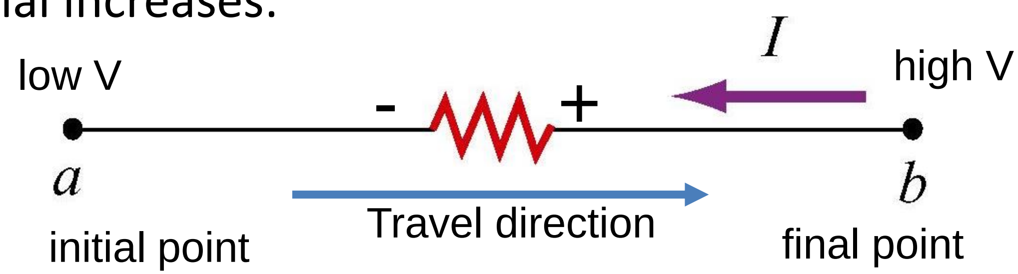
Sign Conventions - Resistor

- As convention, the entrance of the current through a resistor has higher potential. Thus, when moving across a resistor in the current I direction, the change in potential decreases:



$$\Delta V = V_b - V_a = -\mathbf{IR}$$

- Likewise, if moving across a resistor in the **opposite** current direction, the change in potential increases:



$$\Delta V = V_b - V_a = +\mathbf{IR}$$

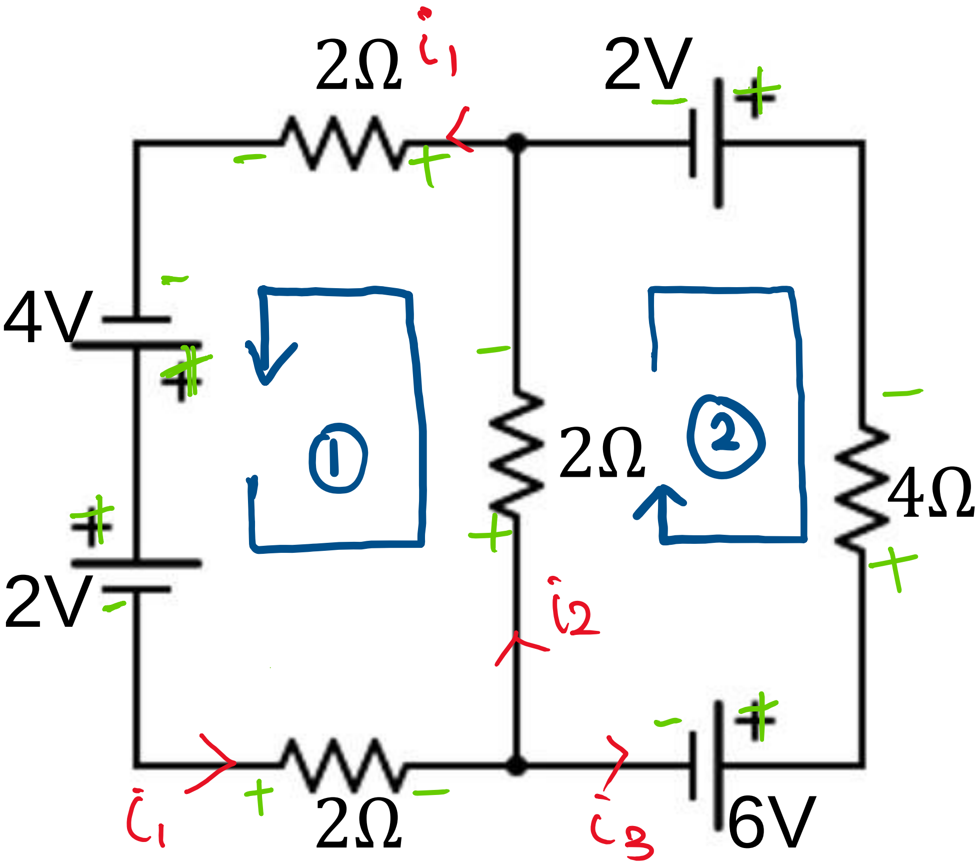
Circuit Analysis using KVL and KCL

1. Assign current direction in the circuit (can be arbitrary)
2. Determine sign of potential based on sign convention for each components
3. Assign an analysis loop direction (can be arbitrary)
4. Based on KCL, write down junction equations (1 per junction)
5. Based on KVL, write down loop equations (1 per loop)
6. Solve simultaneous equations for the unknowns.

Tips:

1. Junction equations needed if there is a junction in the circuit.
2. You need N equations to solve N unknowns.

Work Example: Applying KVL and KCL



- Step 1: Assign current direction
- Step 2: Potential sign
- Step 3: Loop direction
- Step 4: KCL
  - $\boldsymbol{i_1 = i_2 + i_3} \quad \mathbf{(1)}$
- Step 5: KVL

Loop 1:

$$4V - 2V - 2i_1 - 2i_2 - 2i_1 = 0$$

$$\boldsymbol{2V - 4i_1 - 2i_2 = 0} \quad \mathbf{(2)}$$

Loop 2:

$$2V + 4i_3 - 6V - 2i_2 = 0$$

$$\boldsymbol{4i_3 - 4V - 2i_2 = 0} \quad \mathbf{(3)}$$

- With the 3 equations, we can solve the 3 unknowns.

Work Example: Applying KVL and KCL

- Step 6: Solving the simultaneous equations

$$i_1 = i_2 + i_3 \quad (1)$$
$$2V - 4i_1 - 2i_2 = 0 \quad (2)$$
$$4i_3 - 4V - 2i_2 = 0 \quad (3)$$

- Sub (1) into (3):

$$4(i_1 - i_2) - 4V - 2i_2 = 0 \quad \rightarrow \quad 4i_1 - 6i_2 - 4V = 0 \quad (4)$$

- (2) + (4):

$$-2V - 8i_2 = 0 \rightarrow i_2 = -\frac{1}{4}A \qquad (5)$$

- Sub the result (5) into (2)

$$4i_1 = 2V - 2\left(-\frac{1}{4}\right) \rightarrow i_1 = \frac{5}{8}A \quad (6)$$

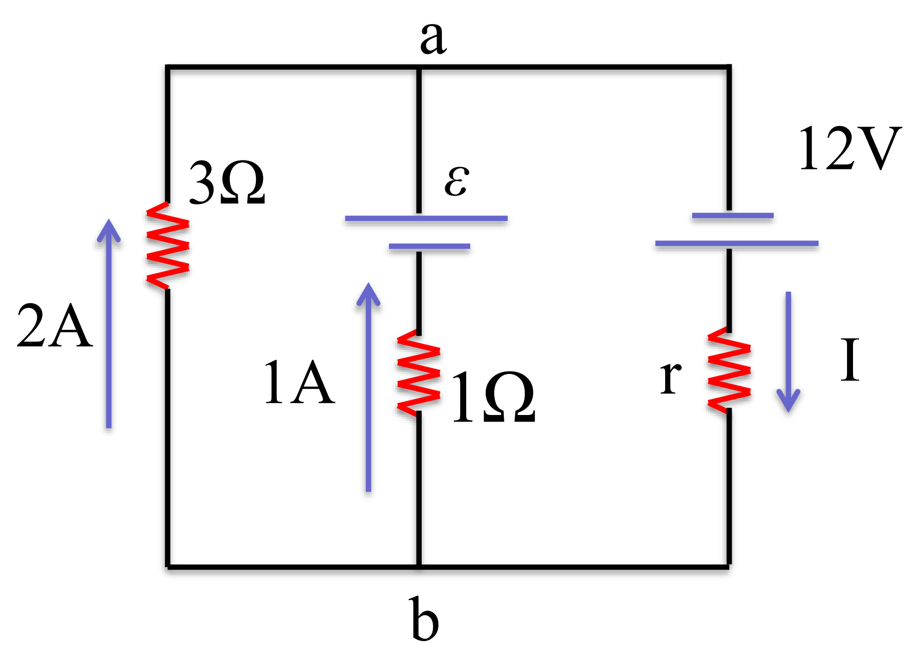
- From (1): $i_3 = i_1 - i_2$

$$i_3 = \frac{5}{8} - \left(-\frac{1}{4}\right) = \frac{7}{8}A$$

- Note: i_2 is a negative value, which indicates i_2 flows in the opposite to that shown in the diagram.

Case Problem 1: Charging a Battery

- In the circuit below, a 12V power supply with unknown internal resistance r is connected to a battery with emf ε and internal resistance 1Ω and to a light bulb of resistance 3Ω carrying a current of 2A. The current through the battery is
- 1A in the direction shown.
- Find r, ε and the current I through the power supply.



Case Problem 1 (Solution)

Apply the junction rule, to point *a:* - *I* + 1 A + 2 A = 0 so *I* = 3 A

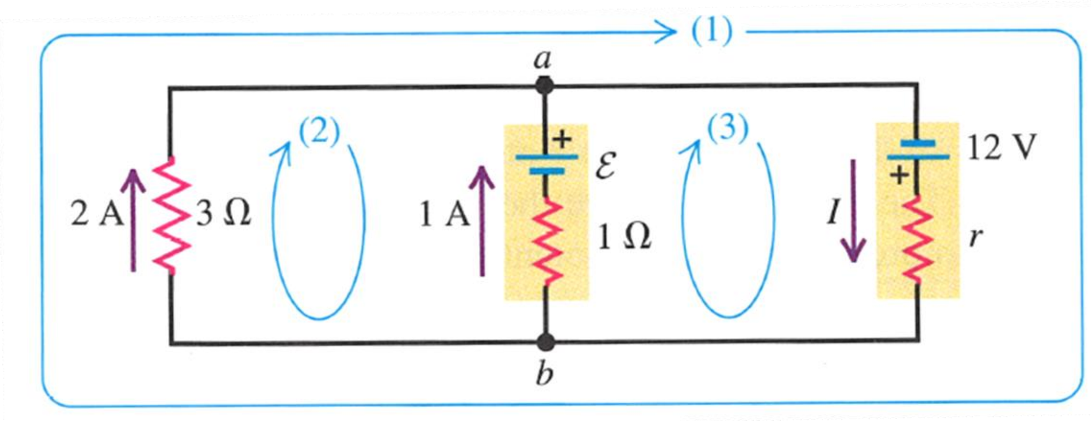
To determine *r*, we apply the loop rule to loop (1):

$$12V - (3A)r - (2A)(3\Omega) = 0$$
$$r = 2\Omega$$

To determine , we apply the loop rule to the left-hand loop (2):

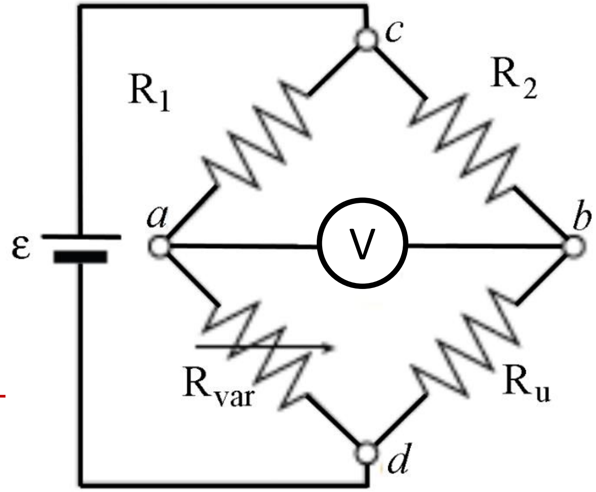
$$-\varepsilon + (1A)(1\Omega) - (2A)(3\Omega) = 0$$
$$\varepsilon = -5V$$

The negative value for *ε* shows that the actual polarity of this emf is opposite to that shown in the figure. Current goes into the positive terminal of a battery means the battery is being recharged.



Application of Wheatstone Bridge

- A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance.
- This circuit provides very high accurate measurements by detecting zero current with a galvanometer.
- Many sensors change its resistance by force, temperature, pressure, etc. which thereby allows the use of Wheatstone bridge in measuring those elements indirectly.
- Examples: strain gauge, resistance thermometer (thermistor), photoresistor (LDR-Light Dependent Resistor), etc.
- Modification of this circuit can be used to measure capacitance, inductance and impedance.

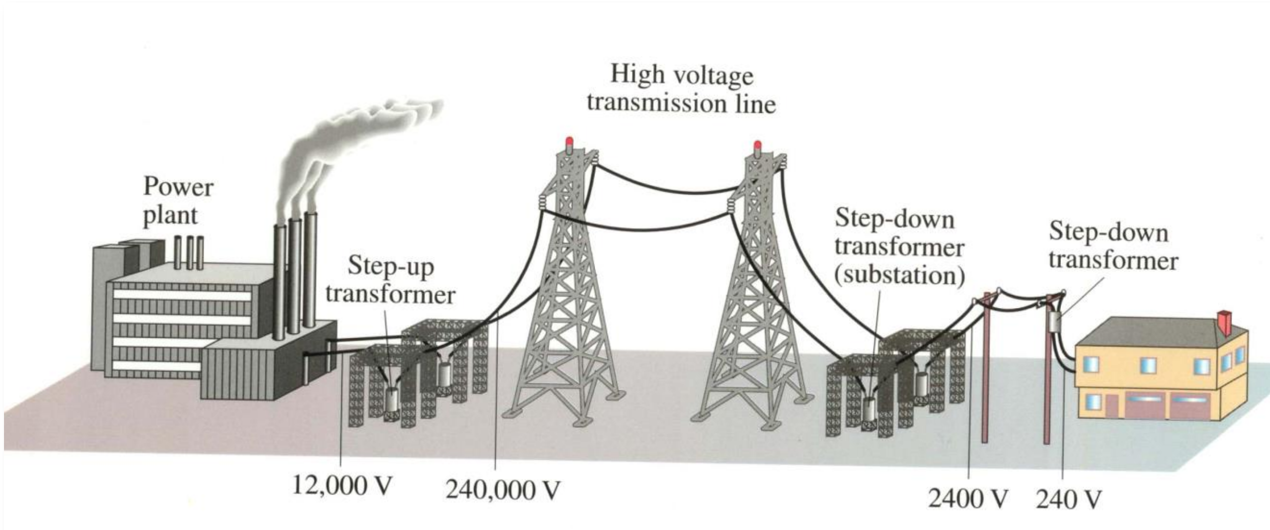


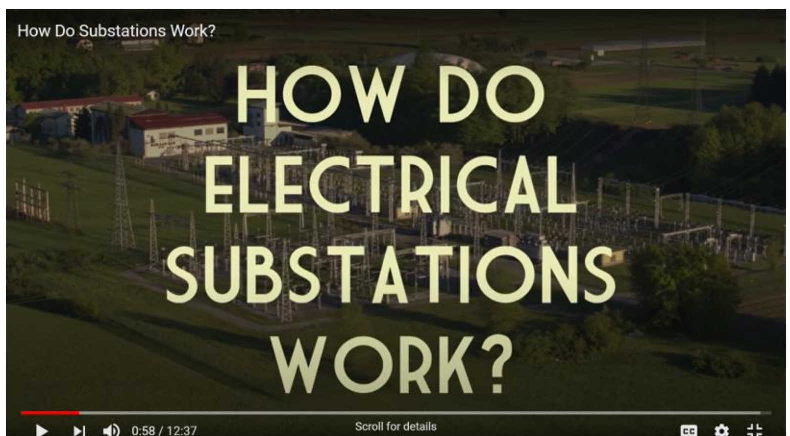
In Class Worksheet

FYI: Special Notes about Power Transmission

Transmission of Electrical Power

- Power loss can be greatly reduced if transmitted at high voltage





FYI: https://youtu.be/7Q-aVBv7PWM

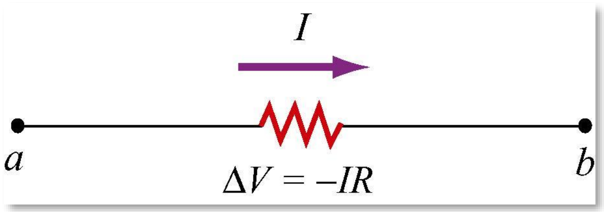
Recall: Electrical Power

- Power is change in energy per unit time
- So power to move current through circuit elements:

$$P = \frac{d}{dt}U = \frac{d}{dt}(q\Delta V) = \frac{dq}{dt}\Delta V$$

$$P = I\Delta V$$

- Power – Resistor
- Moving across a resistor in the direction of current **decreases** your potential. Resistors **always dissipate** power



$$P_{\text{dissipated}} = I|\Delta V| = I^2R = \frac{\Delta V^2}{R}$$

Worked Example : Transmission lines

An average of 120 kW of electric power is sent from a power plant. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is sent at (a) 240 V, and (b) 24,000 V.

(a) $$I = \frac{P}{V} = \frac{1.2 \times 10^5 W}{2.4 \times 10^2 V} = 500\,A$$

83% loss!!

$$P_L = I^2 R = (500\,A)^2 (0.40\Omega) = 100kW$$

(b) $$I = \frac{P}{V} = \frac{1.2 \times 10^5 W}{2.4 \times 10^4 V} = 5.0\,A$$

0.0083% loss

$$P_L = I^2 R = (5.0\,A)^2 (0.40\Omega) = 10W$$

Transmission lines

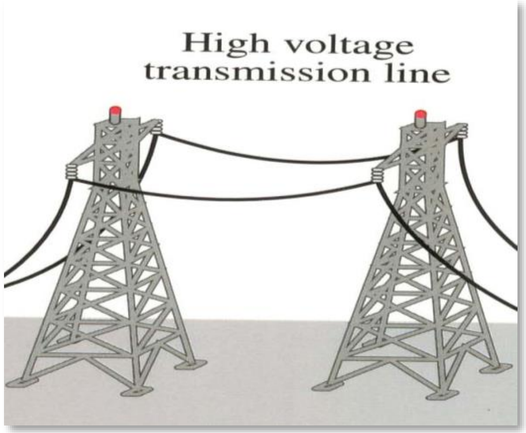
- We just calculated that I^2R is smaller for bigger voltages.
- What about $\frac{\Delta V^2}{R}$? Isn't that bigger? Which equation to be used?

120 kW

24,000 V

$0.40\ \Omega$

0 V



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Transmission lines (cont.)

- What about $\frac{\Delta V^2}{R}$? Isn't that bigger? Which equation to be used?
  - Note that ΔV here refers to the voltage drop across the resistor (in this case is the resistance of the transmission line), which is $\Delta V = IR = 5(0.4) = 2.0V$
  - Thus, $\frac{\Delta V^2}{R} = \frac{2^2}{0.4} = 10W$ (consistent with previous analysis)
- We need to be clear about the meaning of each notation in an equation! Do not blindly use an equation!