Week 11 - Day 1

How capacitor can conduct current while there is a dielectric?

↓

Maxwell's Equation

Concept 1: Displacement Current & Complete Maxwell's Equations

Concept 2: Wave Equation

↓

Microwave Oven

Reading:

University Physics with Modern Physics – Chapter 32

Introduction to Electricity and Magnetism – Chapter 13

Concept 1: Displacement Current & Maxwell's Equation

Electromagnetism Review

The Four Fundamental Laws

Faraday's Law

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Gauss' Law

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{\iiint_V \rho \, dV}{\varepsilon_0}$$

$$\begin{tikzcd} \vec{\mathbf{E}} & \rho \arrow[l] \\ \vec{\mathbf{B}} \arrow[u] & \vec{\mathbf{J}} \arrow[l] \end{tikzcd}$$

Gauss' Law

$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Ampere's Law

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint \vec{\mathbf{J}} \cdot \hat{\mathbf{n}} \, da$$

Is there something missing?

What is missing?

Faraday's Law

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Time-varying

Changing magnetic flux induces an electric field

Gauss' Law

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{\iiint_V \rho \, dV}{\varepsilon_0}$$

$\vec{\mathbf{E}}$ $\leftarrow$ ρ

?

$\frac{d}{dt}$

$\vec{\mathbf{B}}$ $\leftarrow$ $\vec{\mathbf{J}}$

?

Changing electric flux induces a magnetic field?

Non Time-varying

Gauss' Law

$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

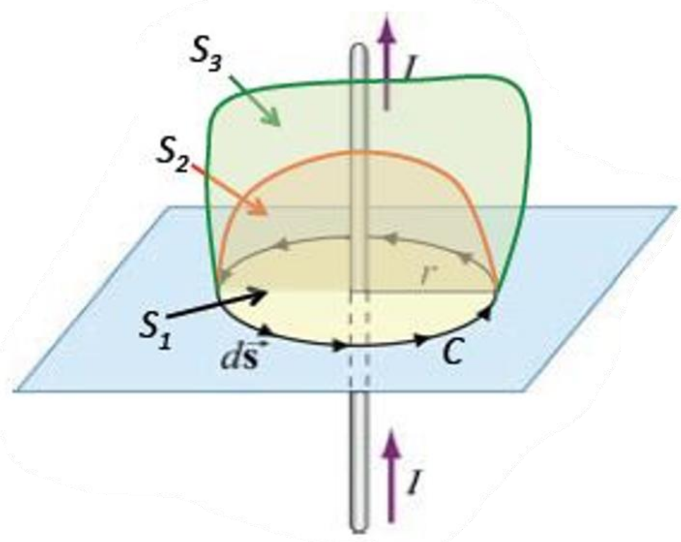
Ampere's Law

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint \vec{\mathbf{J}} \cdot \hat{\mathbf{n}} \, da$$

Re-examine Ampere's law

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o I_{enc}; \quad I_{enc} = \iint_S \vec{J} \cdot d\vec{A}$$

- **Ampere's Law**: the integral of the magnetic field around a closed loop C is proportional to the current I passing through any surface S, whose boundary is loop C itself.
- Infinitely many arbitrary surfaces S can be defined to attach to any loop C – Ampere's law is independent of the choice of S.
- Example: The area S can be formed by
  - *i.* S_1: a plane circular surface bounded by C
  - *ii.* S_2, S_3: any surface (like a paper bag) whose perimeter is the path C.



$$I_{enc} = \iint_{S_1} \vec{J} \cdot d\vec{A} = \iint_{S_2} \vec{J} \cdot d\vec{A} = \iint_{S_3} \vec{J} \cdot d\vec{A}$$

Applying Ampere's Law to Capacitor (at charging-state)

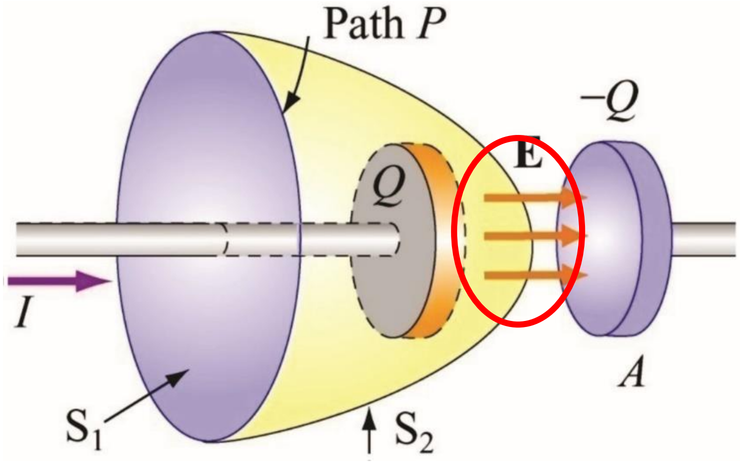
- Consider: A source of emf is connected across a parallel-plate capacitor so that a time-dependent current I develops in the wire. During the transient-state of charging a capacitor:

Use Ampere's Law to calculate the magnetic field around path P,

$$\oint_P \vec{B} \cdot d\vec{s} = \mu_o I_{enc}; \quad I_{enc} = \iint_S \vec{J} \cdot d\vec{A}$$

Version 1: Surface S_1: $I_{enc} = I \ (\neq 0)$

Version 2: Surface S_2: $I_{enc} = 0$ (no conduction current passing through S_2)



Versions 1 and 2 are **inconsistent**… Why?

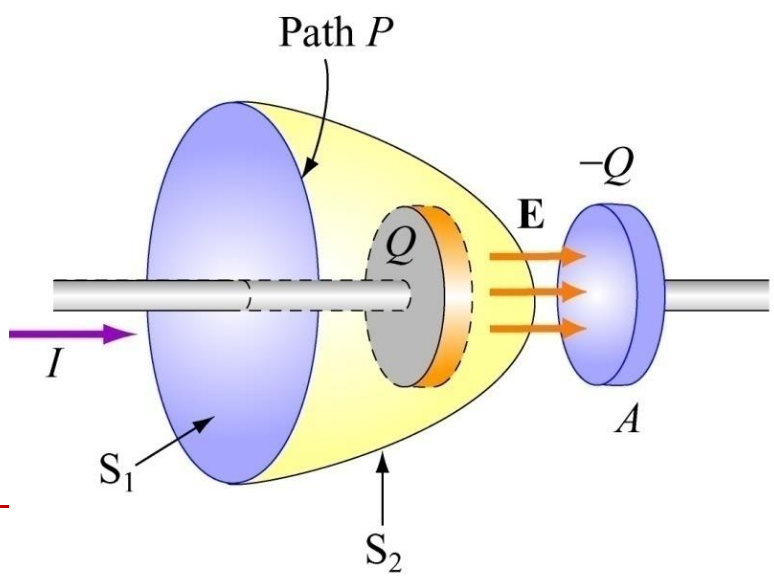
Displacement Current

- We don't have physical current between the capacitor plates, but we do have a changing E field. Can we "make" a current out of that?

$$E = \frac{Q}{\varepsilon_o A} \Rightarrow Q = \varepsilon_o E A = \varepsilon_o \Phi_E$$

$$\frac{dQ}{dt} = \varepsilon_o \frac{d\Phi_E}{dt} \equiv I_{dis}$$

- The quantity I_{dis} is called the **displacement current**.
- I_{dis} **NOT** a flow of charges, but a quantity proportional to changing electric flux
- Same unit as current (i.e. Ampere or C/s)

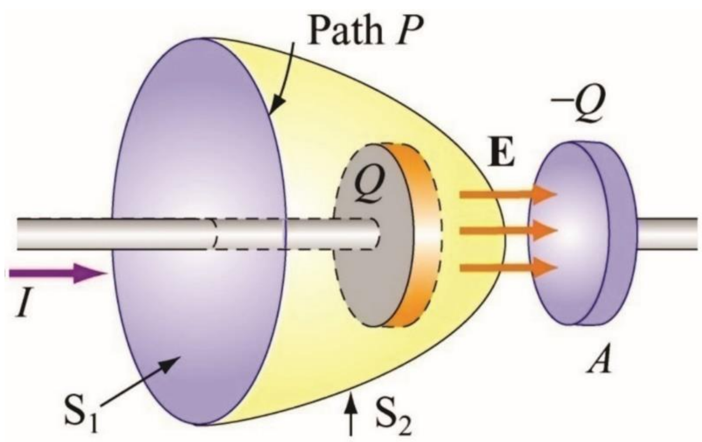


Displacement Current

- $I_{dis} = \varepsilon_o \frac{d\Phi_E}{dt} = \frac{dQ}{dt}$. Since $\Phi_E = \iint_S \vec{E} \cdot d\vec{A}$

$$I_{dis} = \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$

- If surface S_2 encloses all the electric flux due to the charged plate, then $I_{dis} = I_{con}$.
- The conduction current, I_{con}, is carried on by the displacement current I_{dis} in the space between the capacitor.



Maxwell-Ampere's Law (or Generalized Ampere's Law)

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_o \iint_S \vec{J}\cdot d\vec{A} + \mu_o \varepsilon_o \frac{d}{dt}\iint_S \vec{E}\cdot d\vec{A}$$
$$= \mu_o I_{enc} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$
$$= \mu_o (I_{enc} + I_{dis})$$

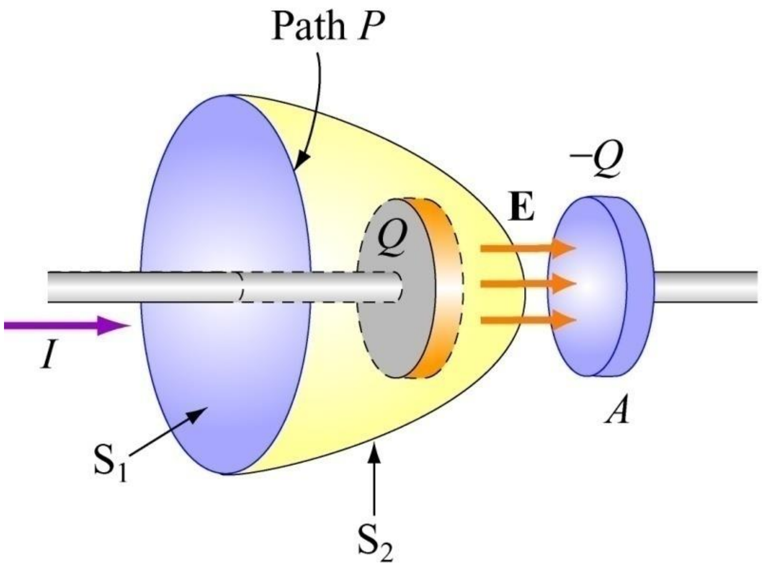
This modified eq. works in all situations!

- "flow of electric charge" -> physical current

$$I_{enc} = \iint_S \vec{J}\cdot d\vec{A}$$

- "changing electric flux" -> displacement current

$$I_{dis} = \varepsilon_o \frac{d}{dt}\iint_S \vec{E}\cdot d\vec{A} = \varepsilon_o \frac{d\Phi_E}{dt}$$



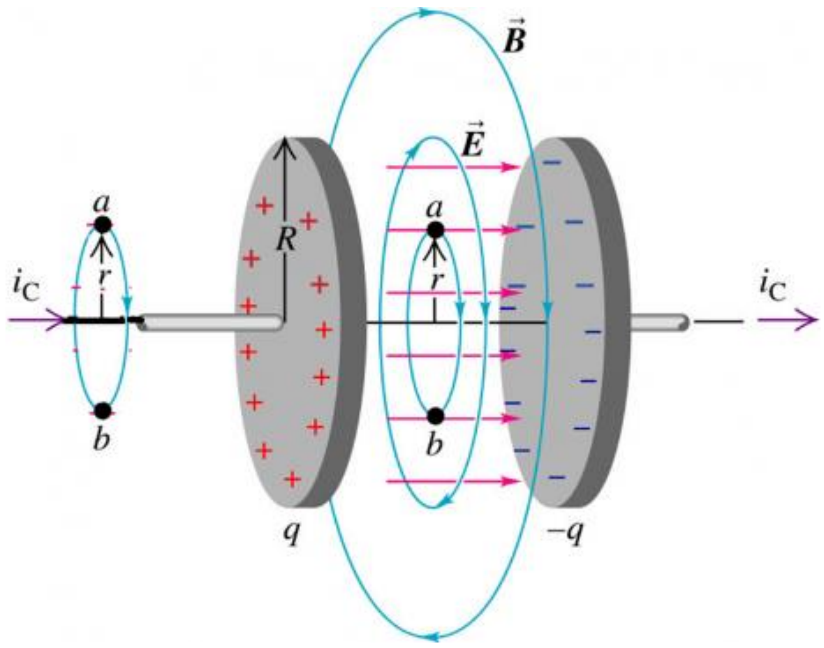
Maxwell-Ampere's Law

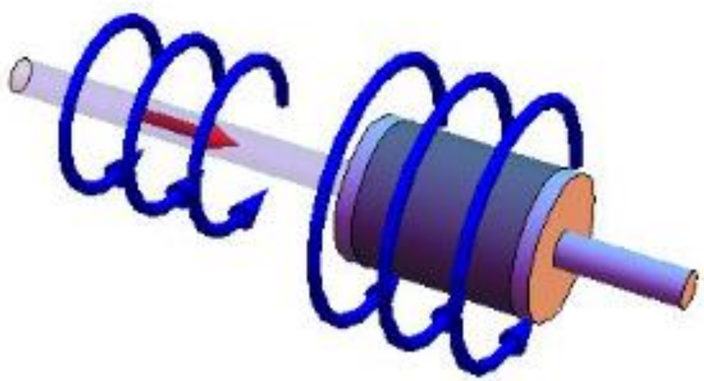
$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o \iint_S \vec{J} \cdot d\vec{A} + \mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$

$$= \mu_o I_{enc} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

$$= \mu_o (I_{enc} + I_{dis})$$

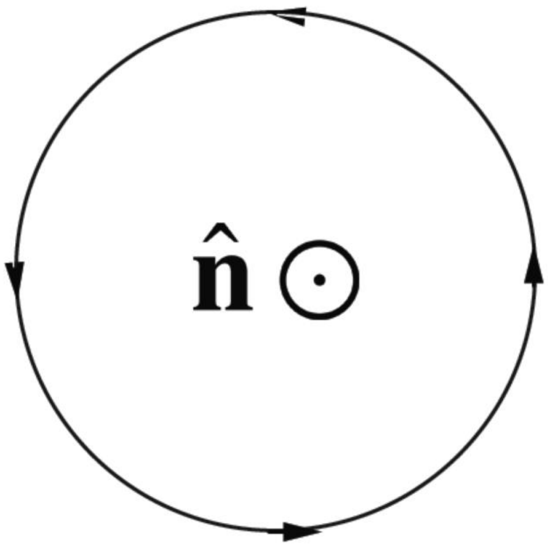
This modified eq. works in all situations!





Sign Conventions: Right Hand Rule

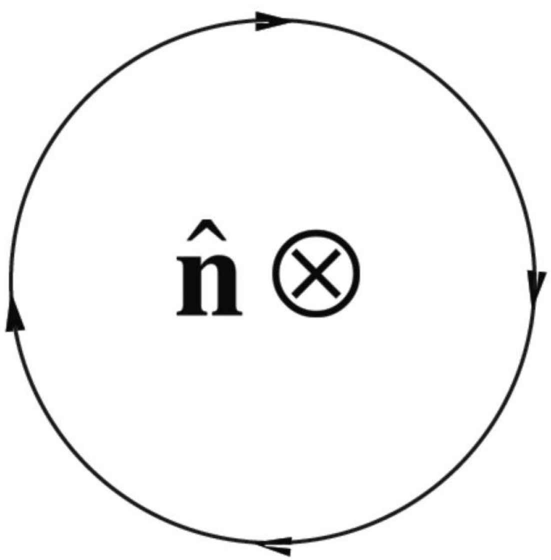
$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o \iint_S \vec{J} \cdot d\vec{A} + \mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$



- Integration direction of B counterclockwise for line integral requires that unit vector n points out of page is positive for surface integral.
- Current and Electric Flux:
- Positive out of page.
- Negative into page.

Sign Conventions: Right Hand Rule

$$\boxed{\oint_C \vec{B} \cdot d\vec{s} = \mu_o \iint_S \vec{J} \cdot d\vec{A} + \mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}}$$



- Integration direction of B clockwise for line integral requires that unit vector n points into page is positive for surface integral.
- Current and Electric flux are
- Positive into the page
- Negative out of page.

Integral Form of Maxwell's Equations

Faraday's Law

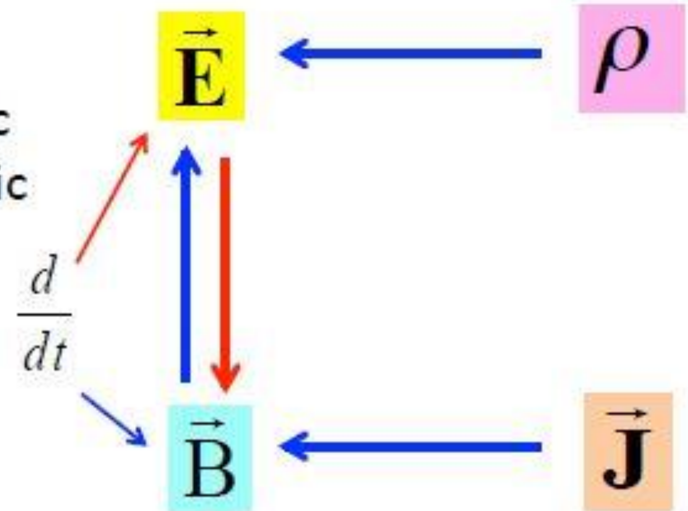
$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt} \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Time-varying

Gauss' Law

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{\iiint_V \rho \, dV}{\varepsilon_0}$$

Changing magnetic flux induces electric field



Changing electric flux induces magnetic field

Time-varying

$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Gauss' Law

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_S \vec{\mathbf{J}} \cdot \hat{\mathbf{n}} \, da + \mu_0 \varepsilon_0 \frac{d}{dt} \iint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} \, da$$

Ampere-Maxwell Law

Maxwell and Displacement Current

The key concept:

- A changing electric field produces a changing magnetic field even when no charges are present and no physical current flows.

- The concept of displacement current allowed James Clerk Maxwell to establish a complete and comprehensive theory of electromagnetism and the prediction of electromagnetic waves which is the foundation for much of 20th century physics.

James Clerk Maxwell
(1831–1879)

James Clerk Maxwell

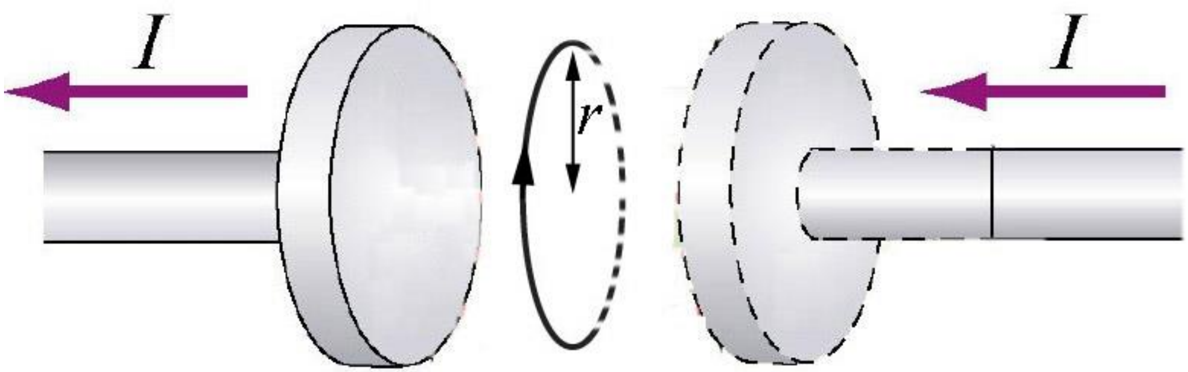
- Born in Scotland, 13 June 1831
- Wrote his first paper "*On the description of oval curves*" in 1846 at the age of 14
- Graduated with a degree in mathematics from Trinity College, Cambridge, in 1854
- Showed the stability of Saturn's ring is achieved by numerous small solid particles (now confirmed by the Voyager spacecraft)
- Whilst at Kings College London, Maxwell calculated the speed of light.
- Worked on the kinetic theory of gasses (Maxwell—Boltzmann theory)
- Became the 1st Cavendish Professor of Physics @ Cambridge in 1871. He designed the Cavendish lab.
- The 1st fully developed form of his *Electricity and Magnetism* equations dates to 1873.
- One of the greatest scientists the world has known died on 5 November 1879. His doctor said: "*No man ever met death more consciously or more calmly.*"

James C Maxwell with his coloured top

Concept Question 1.1: Capacitor

Consider a circular capacitor, with an Amperian circular loop (radius *r*) in the plane midway between the plates. When the capacitor is charging, the line integral of the magnetic field around the circle (in direction shown) is

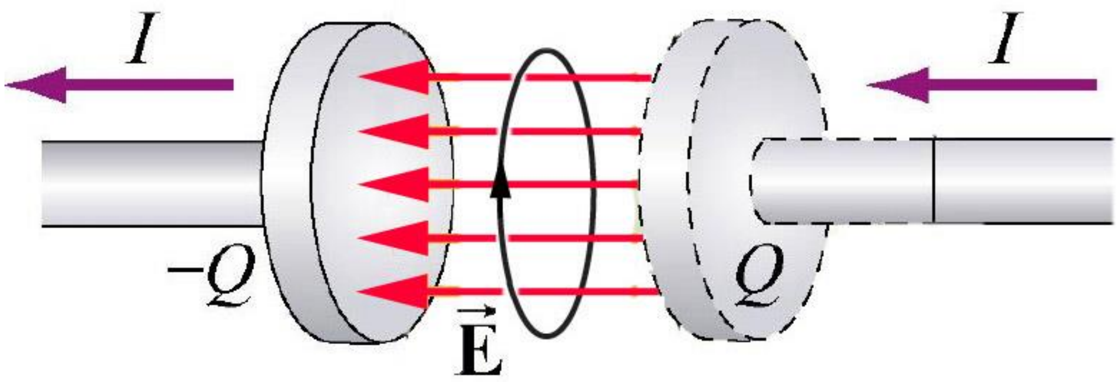
A. Zero
B. Positive
C. Negative
D. Can't tell (need to know direction of E)



Multiple Choice

Concept Question 1.1 Solution: Capacitor

- **Answer B.** The line integral of B is positive.
- There is no enclosed current through the disk. When integrating in the direction shown, the electric flux is positive. Because the plates are charging, the electric flux is increasing. Therefore, the line integral is positive.

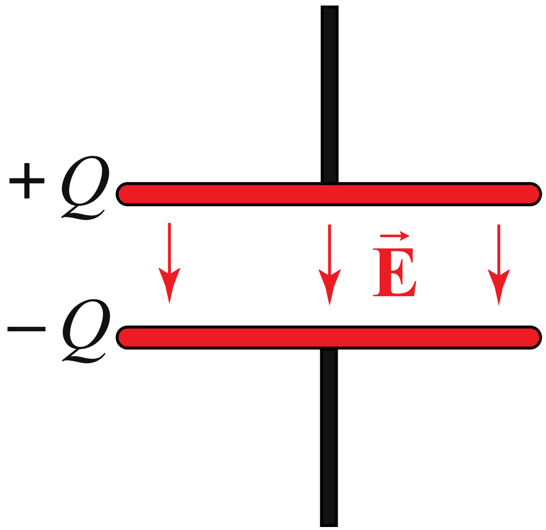


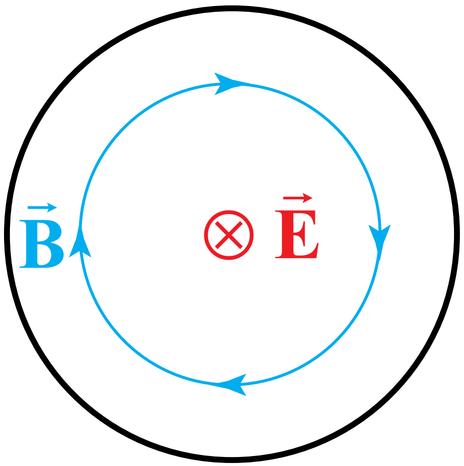
$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$

Concept Question 1.2: Capacitor

- The figures above shows a side and top view of a capacitor with charge *Q* and electric and magnetic fields E and B at time *t*. At this time, the charge *Q* is:

A. Increasing in time
B. Constant in time.
C. Decreasing in time.



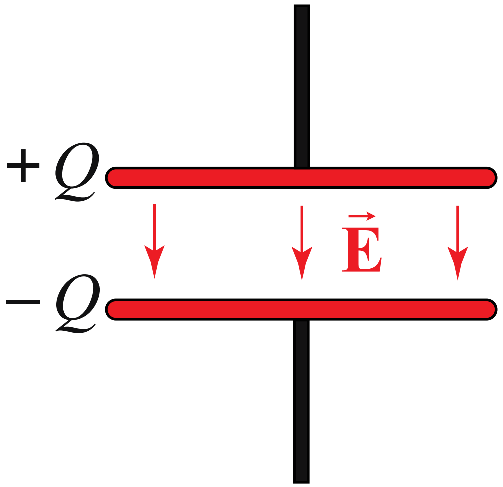


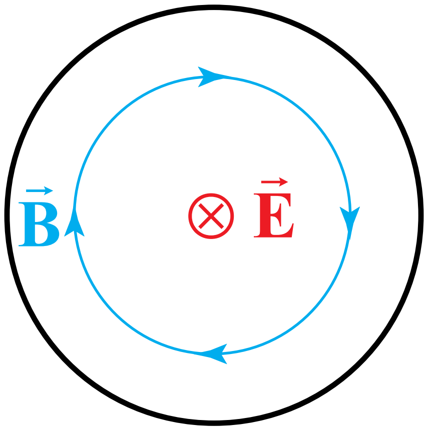
Multiple Choice

Concept Question 1.2 Solution: Capacitor

Answer A. Need the modified Ampere's Law. The B field is clockwise, which means that the if we choose clockwise circulation direction, the electric flux must be increasing in time. So positive charge is increasing on the upper plate.

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o \iint_S \vec{J} \cdot d\vec{A} + \mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$



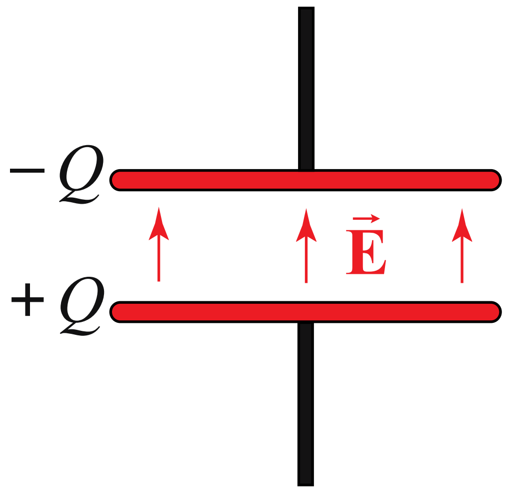


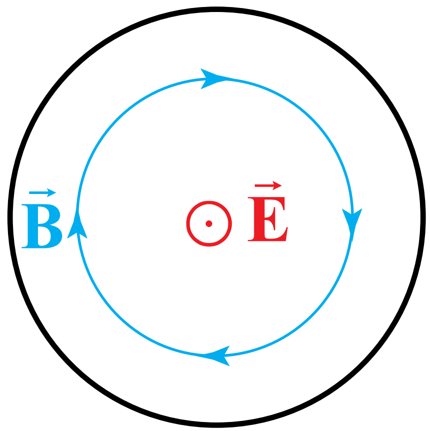
view from above

Concept Question 1.3: Capacitor

- The figures above shows a side and top view of a capacitor with charge *Q* and electric and magnetic fields E and B at time *t*. At this instant the charge *on the lower plate* is

A. Increasing in time
B. Constant in time.
C. Decreasing in time.





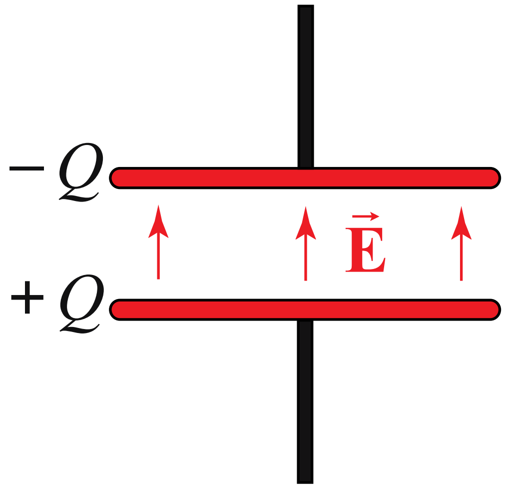
view from above

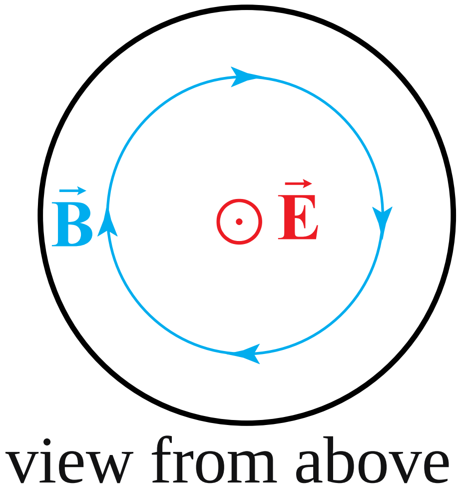
Multiple Choice

Concept Question 1.3 Solution: Capacitor

- **Answer C.** Need the modified Ampere's Law. The B field is clockwise, which means that the if we choose clockwise circulation direction, the electric flux must be decreasing in time. So positive charge on the lower plate is decreasing.

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_o \iint_S \vec{J}\cdot d\vec{A} + \mu_o \varepsilon_o \frac{d}{dt}\iint_S \vec{E}\cdot d\vec{A}$$

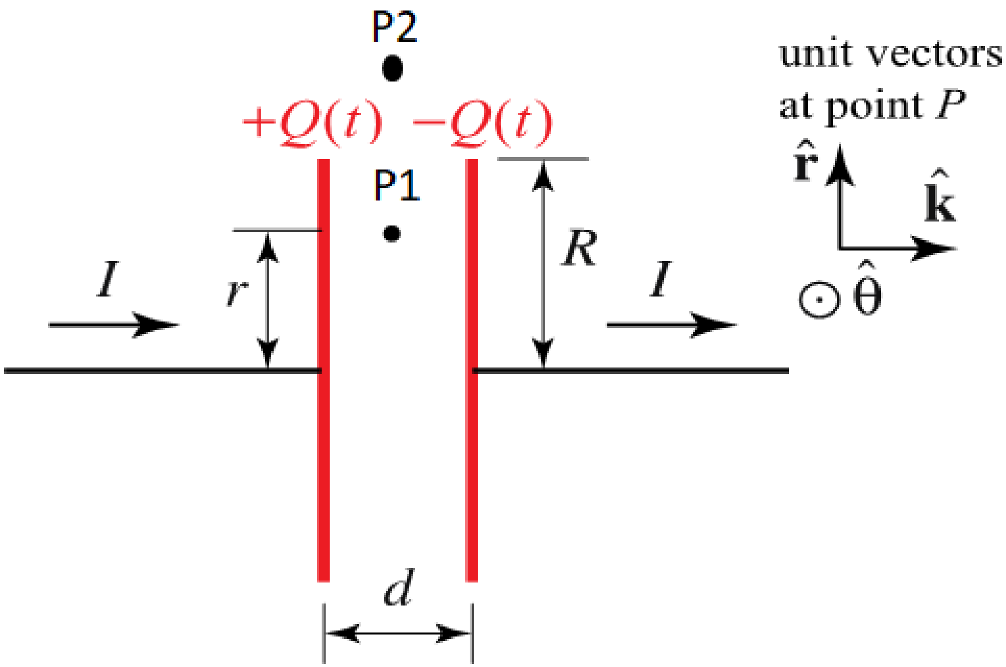




Case Problem 1.1: Capacitor

A circular capacitor of spacing *d* and radius *R* is in a circuit carrying the steady current *I* shown at time *t*. At time *t* = 0, the plates are uncharged.

1. Find the electric field $\vec{E}(t)$ at P1 (mag. & dir.)
2. Find the magnetic field $\vec{B}(\mathrm{t})$ at P1 (r<R)
3. Find the magnetic field $\vec{B}(\mathrm{t})$ at P2 (r>R)



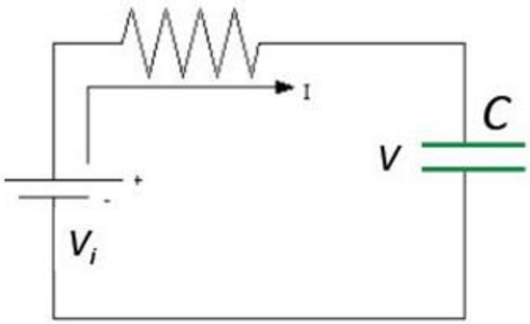
Case Problem 1.1 Solution: Capacitor

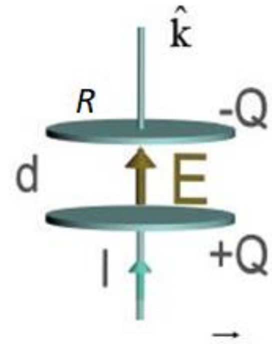
1. Recall from Gauss's Law, the electric field in between 2 parallel plates capacitor, $E = \frac{\sigma}{\varepsilon_o}$. In terms of the coordinate system given in the question, $\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{k} = \frac{Q(t)}{\varepsilon_o \pi R^2}\hat{k}$
2. In between the plates, there is no physical current flow, $I_{enc} = 0$. Ampere-Maxwell Law becomes

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o I_{enc} + \mu_o \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$

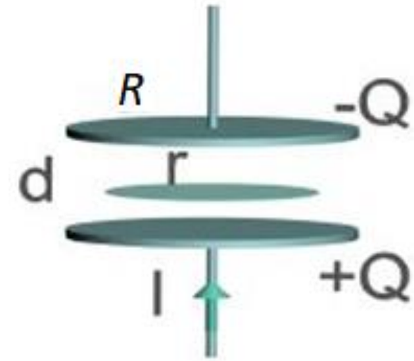
$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o I_{dis} \quad ; I_{dis} = \varepsilon_o \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A}$$

Case Problem 1.1 Solution: (r < R = radius of the disk)





$$I(t) = \frac{dQ(t)}{dt}$$



Consider a flat disk of r<R,

$$\Phi_E = \iint_{\substack{flat \\ disk}} \vec{E} \cdot d\vec{A} = \frac{Q(t)}{\varepsilon_o \pi R^2}(\pi r^2) = \frac{Q(t)}{\varepsilon_o R^2}(r^2)$$

$$I_{dis} = \varepsilon_o \frac{d\Phi_E}{dt} = \varepsilon_o \frac{d}{dt}\left\{\frac{Q(t)}{\varepsilon_o R^2}(r^2)\right\} = \frac{r^2}{R^2}\frac{dQ(t)}{dt} = \frac{r^2}{R^2} I(t)$$

$$\oint_C \vec{B} \cdot d\vec{s} = \mathrm{B}2\pi r = \mu_o \frac{r^2}{R^2} I(t) \rightarrow B = \mu_o \frac{I(t) r}{2\pi R^2}$$

$$\vec{B} = \mu_o \frac{I(t) r}{2\pi R^2} \hat{\theta} \quad \text{for r<R}$$

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Case Problem 1.1 Solution: (R < r)

3. For $\vec{B}$(t) at P2 (r>R)

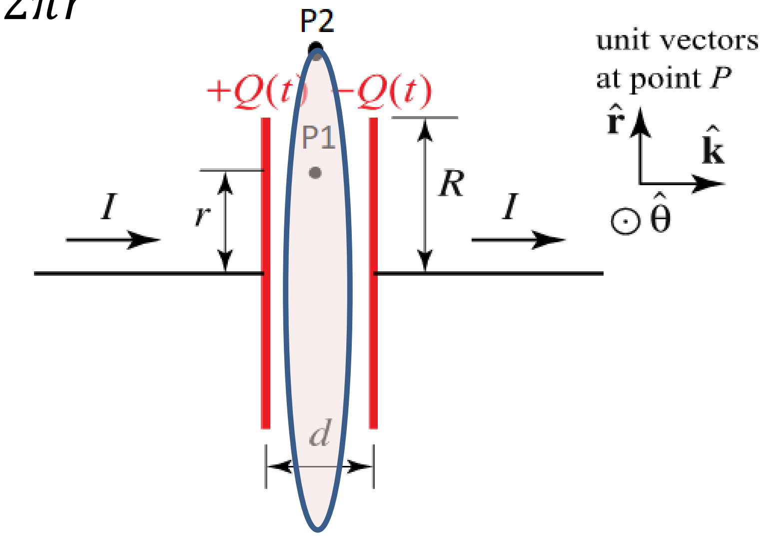
The total electric flux passing through the flat surface,

$$\Phi_E = \iint_{\substack{flat \\ disk}} \vec{E} \cdot d\vec{A} = \frac{Q(t)}{\varepsilon_o \pi R^2}(\pi R^2) = \frac{Q(t)}{\varepsilon_o}$$

The displacement current, $I_{dis} = \varepsilon_o \frac{d\Phi_E}{dt} = \frac{dQ(t)}{dt} = I(t)$.

$$\oint_C \vec{B} \cdot d\vec{s} = \mathrm{B}2\pi r = \mu_o I(t) \rightarrow B = \mu_o \frac{I(t)}{2\pi r}$$

$$\vec{B} = \mu_o \frac{I(t)}{2\pi r} \hat{\theta} \quad \text{for R>r}$$



Case Problem 1.2 : Displacement current in a wire

A long straight, copper wire with a circular cross-section of 2 mm^2 carries a current of 1 A. The current is changing at the rate of 4000 A/s. The resistivity of the copper is $\rho = 2 \times 10^{-8}$ Ω/m.

For copper (metal), the dielectric constant is a complex variable with a real part close to 1, so you may assume $\varepsilon = \varepsilon_o$ for copper.

a) What can be the cause of the current change?

b) What is the magnitude of the magnetic field created at 1 cm from the center of the wire ?

c) Is the displacement current important ?

Case Problem 1.2 Solution

- a) From Ohm's Law, the current change is due to the voltage change, which is also related to the change of the electric field. And this change of E field can induce displacement current!
- b)

From ampere law, $B = \frac{\mu_o}{2\pi r} I(conduction) + I(displacment)$

From Ohm law: $V = IR = \frac{I\rho L}{A}$ and $E = \frac{V}{L} = \frac{I\rho}{A}$

$$I_{displacment} = \varepsilon_o \frac{dE}{dt} \times A = \varepsilon_o \frac{dI}{dt} \times \rho$$

$I_{displacment} \approx 7 \times 10^{-16}$ A << I (conduction) = 1 A

$$B \approx \frac{\mu_o}{2\pi r} I(conduction) = 2 \times 10^{-5}\ T$$

Case Problem 1.3: Displacement current in a dielectric

Suppose that a dielectric material (K = 4.7) of 2.5 mm thick is sandwiched between 2 parallel plates having circular area of 3 cm^2 . At certain time during the charging process, the potential difference between the plates is 120 V and the conduction current I_C = 6 mA.

At this time,

a) What is the charge Q of the plate ?

b) The rate of change of the charge Q ?

Case Problem 1.3 Solution

- Concept: In between the parallel capacitor plate, for both with dielectric and without dielectric, the total displacement current is the same as the conduction current.

(a) From Gauss law: $E = \frac{\sigma}{\varepsilon} = \frac{Q}{A\varepsilon} = \frac{V}{L} \rightarrow Q = \frac{K\varepsilon_O V}{L} \times A$

Q = 0.6 nC

a) During the charging $I_c = I_{displacement}$

$$I_c = \varepsilon \frac{dE}{dt} \times A = \frac{d\sigma}{dt} \times A = \frac{dQ}{dt} = 6 \text{ mA}$$

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Summary: Maxwell's Equations & Electromagnetism

- Gauss's Law

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

- Magnetic Gauss's Law

$$\oiint_S \vec{B} \cdot d\vec{A} = 0$$

- Faraday's Law

$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

- Ampere-Maxwell's Law

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o I_{enc} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

- **E fields are associated with:**
- electric charges (Gauss's Law)
- Time-varying B fields or flux (Faraday's Law)
- **Conservation of magnetic flux**
- No magnetic charge/ monopole (Gauss's Law for Magnetism)
- **B fields are associated with**
- moving electric charges (current) (Ampere's Law)
- Time-varying electric fields or flux (Ampere-Maxwell law)

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Overview of Electromagnetism

$$U_C = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q|\Delta V|$$

$$\vec{E} = k_e \int \frac{dq}{|\vec{r}_p - \vec{r}_s|^3}(\vec{r}_p - \vec{r}_s)$$

$$\vec{F}_{sp} = \frac{k_e q_s q_p}{|\vec{r}_{sp}|^2}\hat{r}_{sp} = \frac{k_e q_s q_p}{|\vec{r}_{sp}|^3}\vec{r}_{sp}$$

3. Faraday's Law

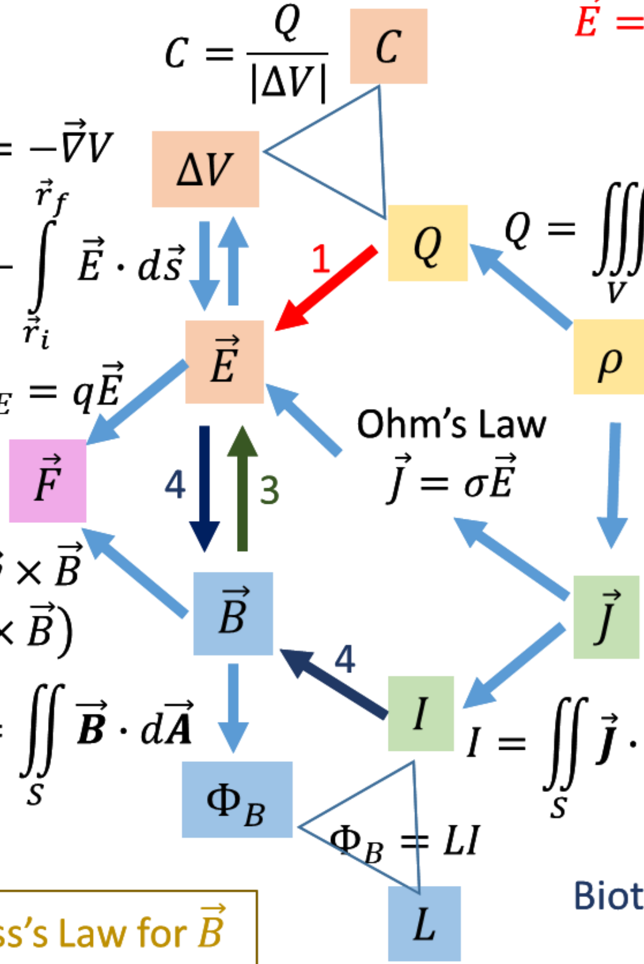
$$\oint_C \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{s}} = -\frac{d\Phi_B}{dt}$$

$$C = \frac{Q}{|\Delta V|}$$

$$\vec{E} = -\vec{\nabla}V$$

$$\Delta V = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{E} \cdot d\vec{s}$$

$$Q = \iiint_V \rho \, dV$$



1. Gauss's Law for $\vec{E}$

$$\oiint_S \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{Q_{enc}}{\varepsilon_o}$$

$$\vec{\tau}_E = \vec{\mu}_E \times \vec{E}$$

$$\vec{F}_E = q\vec{E}$$

$$\vec{\tau}_B = \vec{\mu}_B \times \vec{B}$$

Ohm's Law

$$\vec{J} = \sigma\vec{E}$$

4. Ampere-Maxwell's Law

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = \mu_o I_{enc} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} = I\left(\vec{L} \times \vec{B}\right)$$

$$u_E = \frac{1}{2}\varepsilon_o E^2$$

$$u_B = \frac{B^2}{2\mu_o}$$

$$\Phi_B = \iint_S \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$$

$$I = \iint_S \vec{\boldsymbol{J}} \cdot d\vec{\boldsymbol{A}}$$

$$\Phi_E = \iint_S \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$$

$$\Phi_B = LI$$

Biot-Savarts' Law

$$\vec{B} = \frac{\mu_o I}{4\pi}\int \frac{d\vec{l} \times (\vec{r}_p - \vec{r}_s)}{|\vec{r}_p - \vec{r}_s|^3}$$

2. Gauss's Law for $\vec{B}$

$$\oiint_S \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$$

$$U_L = \frac{1}{2}LI^2$$

Concept 2: Wave Equation

Recognize the wave equation, a (partial) differential equation that describes wave behaviors.

Recognize the traveling function/wave is the solution of the wave equation.

Define parameters that describes a sinusoidal wave:

- Wavelength, wave number
- Frequency, angular frequency, period
- Speed
- The relation between wavenumber, angular frequency and speed.
- Amplitude

1D Wave Equation

- In general, a wave equation is a partial differential equation, with the following form:

$$\boxed{\frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}}$$

- $y(x,t)$ is a function of position, x and time, t.
  - This function is what we want to find out
  - It is the solution of this wave equation
- v^2 is a constant, where v is the speed of the wave.
- It turns out that the general solution needs to be:

$$y(x,t) = y(x \pm vt)$$

- This function, $y(x \pm vt)$ is called travelling function/wave.
- **Any functions of $\boldsymbol{y(x+vt)}$ or $\boldsymbol{y(x-vt)}$ or a linear combination of both (superposition principle) is a solution of the one-dimensional wave equation.**

Traveling Function: $f(x \pm vt)$

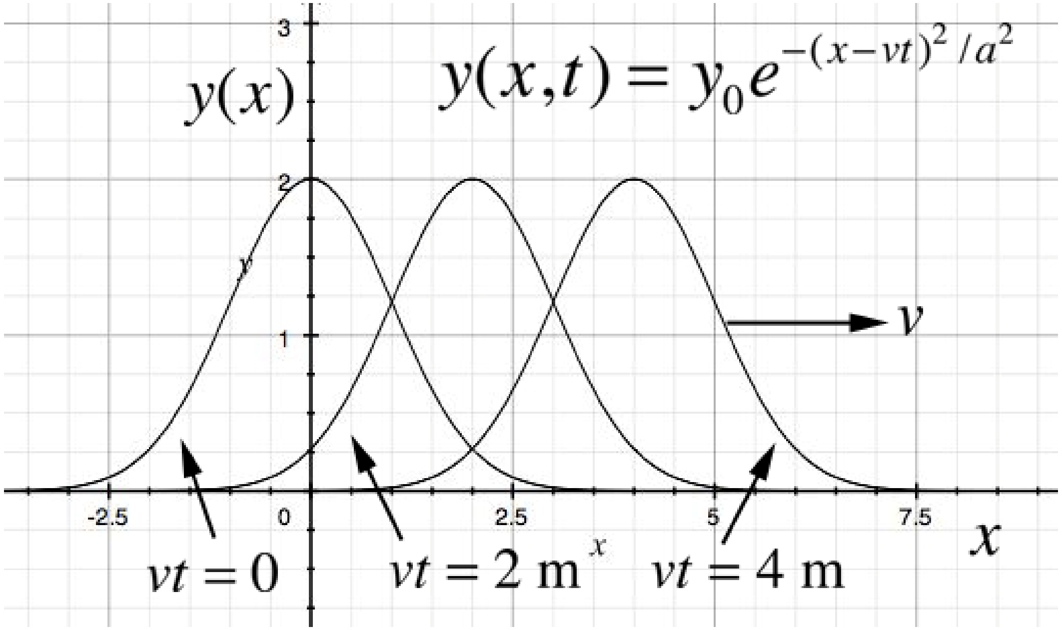
- $f(x - vt)$ is a right traveling function
  - corresponds to a wave traveling in the positive *x*-direction with speed *v*
- $f(x + vt)$ is a left traveling function
  - corresponds to a wave traveling in the negative *x*-direction with speed *v*
- "Traveling" means that the shape of these individual arbitrary functions with respect to *x* stays constant, however the functions are translated left and right with time at the speed *v*.

https://www.geogebra.org/m/YKuG3zNZ

https://www.desmos.com/calculator/uuhphomccb

Understanding $f(x-vt)$: Traveling Wave

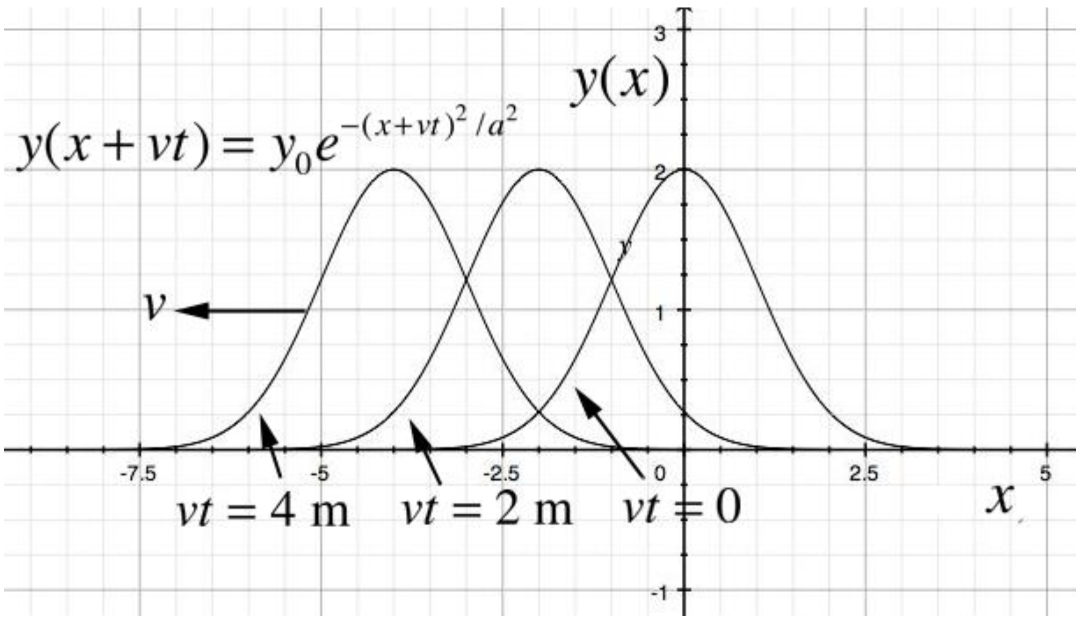
- Consider $y(x-vt) = y_o e^{-(x-vt)^2/a^2}$
- At $t = 0$:
- $y(x-vt) = y_o e^{-(x)^2/a^2}$
- At $vt = 2m$:
- $y(x-vt) = y_o e^{-(x-2m)^2/a^2}$
- At $vt = 4m$:
- $y(x-vt) = y_o e^{-(x-4m)^2/a^2}$



- Note: $y(x-vt)$ is travelling in the positive x-direction with velocity v.

Understanding $f(x+vt)$: Traveling Wave

- Consider
- $y(x+vt) = y_o e^{-(x+vt)^2/a^2}$
- At $t = 0$:
- $y(x+vt) = y_o e^{-(x)^2/a^2}$



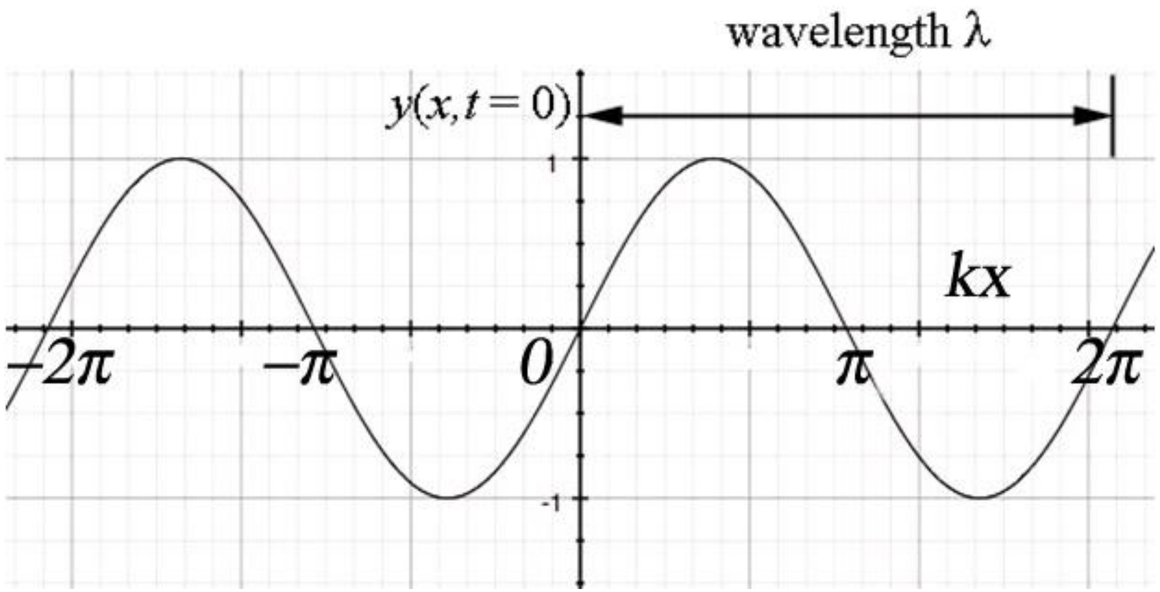
- At $vt = 2m$:
- $y(x+vt) = y_o e^{-(x+2m)^2/a^2}$
- At $vt = 4m$:
- $y(x+vt) = y_o e^{-(x+4m)^2/a^2}$
- Note: $y(x+vt)$ is travelling in the negative x-direction with velocity v.

Traveling Sinusoidal Wave: Wavelength, λ and Wave Number, k

- Consider $y(x,t) = y_o \sin(kx - \omega t)$
- Look at $t = 0$: $\mathrm{y(x,0) = y_o sin(kx)}$

This sine function has a form of this kind $f(x - vt)$

- λ is called the wavelength, k is called the wave number
- When $x = \lambda$
- $\Rightarrow k\lambda = 2\pi$
- $\Rightarrow k = \frac{2\pi}{\lambda}$

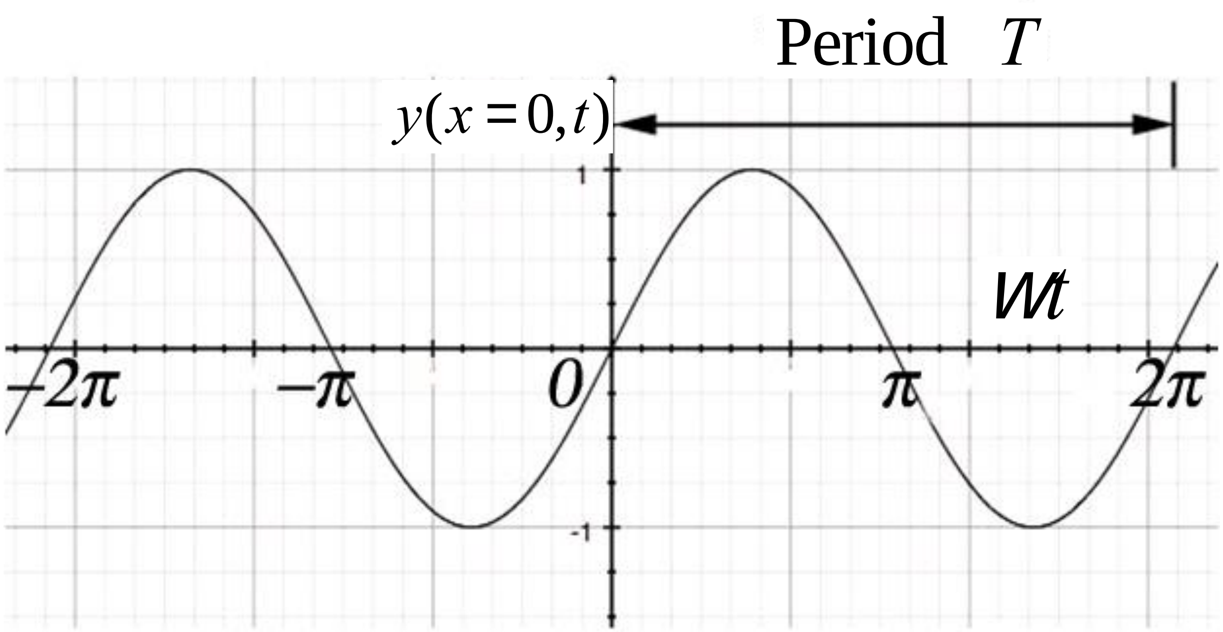


Note: Don't be confused with Simple Harmonic Motion/Oscillation!!

Traveling Sinusoidal Wave: Period, T and Angular Frequency, ω

- Consider $y(x,t) = y_o \sin(kx - \omega t)$
- Look at x= 0: $y(0,t) = -y_o \sin(\omega t)$

- T is called the period, ω is called the angular frequency.
- When $t = T$
- $\Rightarrow \omega T = 2\pi$
- $\Rightarrow \omega = \frac{2\pi}{T}$



Traveling Sinusoidal Wave: Summary

$$y(x,t) = y_o \sin[k(x - ct)] = y_o \sin(kx - \omega t)$$

- Two periodicities:
  - Spatial period: wavelength λ
  - Temporal period: period T

- **Wavenumber** $k = \frac{2\pi}{\lambda}$
- **Angular frequency** $\omega = 2\pi f = \frac{2\pi}{T}$
- **Dispersion relation** $\lambda = cT \Rightarrow \omega = kc$
- **Frequency** $f = \frac{1}{T}$

Note: The speed of the wave, $v = f\lambda$