Welcome to Technological World

- Look around yourself, think what you have been doing today … you will see that most of the devices we use rely on Electricity and Light

- It turns out the electricity and light, and magnetism too, are closely inter-related.

- In this course we will start from some of the devices you use, show you how they work, their guiding principles, and show how these principles also apply to other technologies.

- We hope that this journey will empower you to use and make new technologies!

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Our Approach

Consider a device

Extract the physical principles that make it work

Extrapolate to other devices/technologies

If you can think quantitatively, many more opportunities will be available!!!

Examples of devices using electricity, magnetism and light

- Telecommunication devices
- Computers and smart devices
- Home Appliances
- Transportation
- Healthcare & Medical Devices

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Many devices: but just a few physical principles

- Fields
  - Electric & magnetic fields.
  - Source, properties of the fields and interaction they involve.

- Lorentz Force Law

$$\vec{\mathrm{F}} = q\left(\vec{\mathrm{E}} + \vec{\mathrm{v}} \times \vec{\mathrm{B}}\right)$$

- Maxwell's Equations

$$\oiint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{Q_{enc}}{\varepsilon_o} \qquad \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{S}} = -\frac{d}{dt}\iint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$$

$$\oiint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0 \qquad \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{S}} = \mu_o I_{enc} + \mu_o \varepsilon_o \frac{d}{dt}\iint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$$

Week 1 – Day 1

How Gecko Crawls on Wall and Ceiling?

↓

Electrical charge

Concept 1: Charges and the Conservation of Charges

Concept 2: Interactions of Point Charges, Coulomb's law

Concept 3: Electric Field of Point Charge and Electric Field Lines

↓

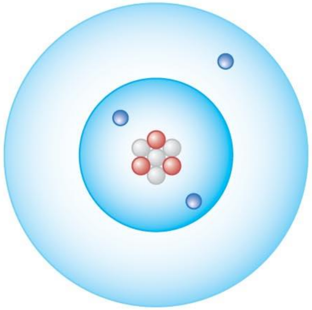
Air purifier, 3-ply face masks

Reading:

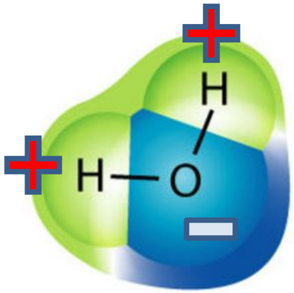
1. University Physics with Modern Physics – Chapter 21
2. Introduction to Electricity and Magnetism – Chapter 1,2

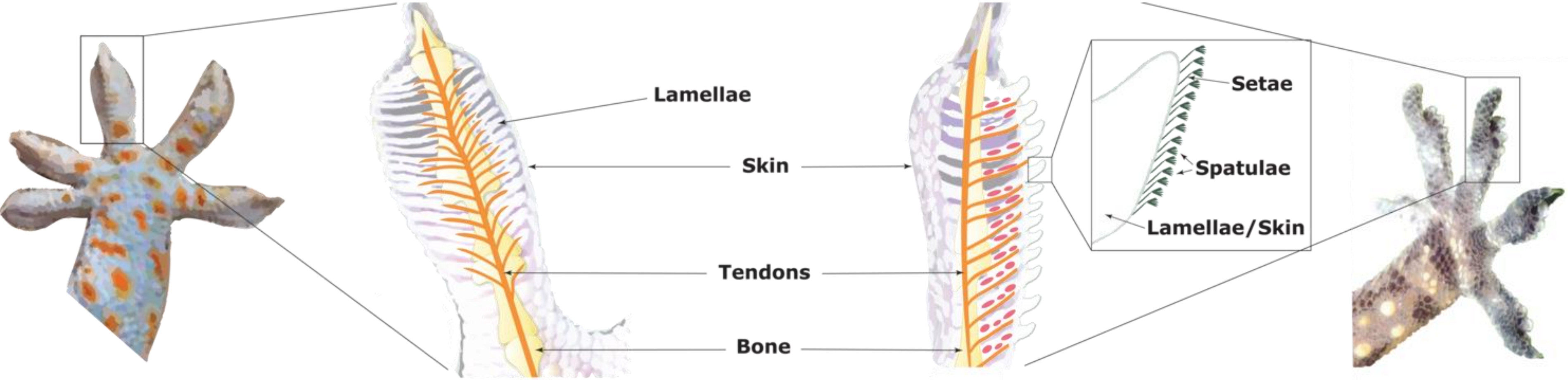
How Gecko Crawls on Wall and Ceiling?

- Gecko seems to live in different world without gravity. It crawls on ceiling and climbs walls as easy as we walk on ground. How does it perform this amazing feat?
- We know there are 2 types of charges : positive charge and negative charge; and like charges repel each other, opposite charges attract one another.
- In an atom, the electrons are the carrier of negative charge while protons are carrier of positive charge. A neutral atom has equal number of electrons and protons.



- However, electrons do not remain stationary but move around, causing uneven charge distribution. The region where more electrons are concentrated is more negatively charged while the remaining part of the atom is more positively charged. This is the formation of electric dipole. Some molecules are permanent dipole because electrons permanently concentrated in certain atom(s), e.g. H_2O.





- Gecko has millions of thin hair bristles called setae at the feet, where the bristle tip ends in structure called spatulae. When the spatulae are pressed against the wall surface, the charges of the molecular dipole induce opposite charges on the wall resulting in electric force of attraction. This is the Van der Waals interaction.

- GeckSkin™ is the technology developed by University of Massachusetts, it was named one of the top five science breakthroughs of 2012 by CNN Money. Adhesive film Geckskin™ is so powerful that an index-card sized piece can hold 320 kg on a smooth surface, such as glass, yet can be easily released, and leaves no residue. Can you imagine the tantalizing possibility of this technology offers?

https://www.youtube.com/watch?v=uC2VShczjrg

Concept 1: Charges and the Conservation of Charges

Let’s jump into the principles!

We got to understand charges better ☺

What you may already know…

- Recall: the gravity is a force between two massive objects. Mass is the charge of the force of gravity, which can act at a distance. Similar phenomena occur with electric charges. We are dealing with forces, hence Newton's laws still apply!

- Electric charge is the physical property of matter (an attribute that is as fundamental as mass). Charge experiences a force when placed in an electromagnetic (EM) field. There exist positive and negative charges, as they behaves differently in EM field.
- Opposite charges attract each other, while same charges repel each other.
- Charges are conserved. They cannot be created but can only be moved around!
- How do we move charges around?
  - Charge separation (e.g. rubbing 2 objects)
  - Transfer charges in or out (e.g. touching a charged object to a neutral object)
  - Induction (e.g. a charge causes an uneven charge distribution of a nearby object without touching)

Let's us go through some demonstrations to gain a deeper understanding, especially relevant for our printer … how to charge objects?

Video Demonstration on Charges (1)

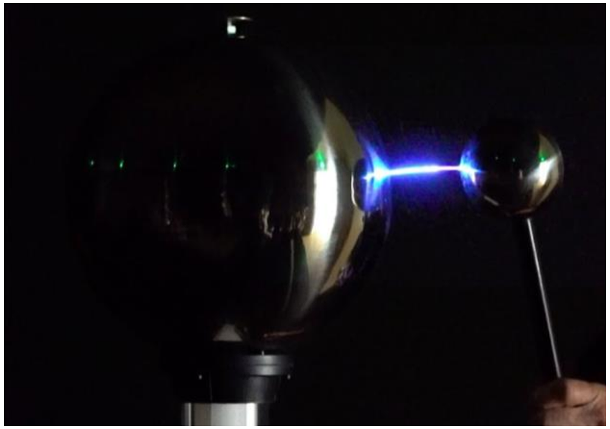
- Bouncing ball with Van de Graaff Generator.
- A ball wrapped in Al foil is placed between the Van de Graaff Generator and a grounded plate wrapped in Al foil.
- When Van de Graaff Generator runs, the ball bounces between the dome and the plate. What happens?
- *The ball is induced with -ve charge at the side near the dome and +ve charge at the side away from the dome. Thus, the ball is attracted to the dome. Upon touching the dome, the -ve charge is neutralized with the +ve charge on the dome, leaving net +ve charge behind which is repelled by the dome towards the plate. Upon touching the plate, the +ve charge is removed (neutralized by the grounded plate). The ball falls back towards the dome, and the cycle begins.*

Inducing Dipoles with a Van de Graaff Generator

https://youtu.be/s_PL9K9c3cg

Summary Demonstration on Charges (2)

- Breakdown of dry air (Van de Graaff generator discharges through ground electrode).
- Place a grounding rod near the Van de Graaff generator dome without touching each other.
- Run the Van de Graaff generator until the arc of spark is observed between the dome and the grounded electrode. What happens?



- *Air is insulator. When the charges are deposited on the dome by the Van de Graaff generator, dipoles are induced in the surrounding air molecules. When more charges are deposited onto the dome, the force of attraction between the opposite charge of the dipoles and the dome, and the repulsive force between the like charge of the dipoles and the dome become so strong, the dipoles charges are being pulled apart and become free to conduct electricity. The air has become conductor causing the discharge.*

Concept Question 1.1

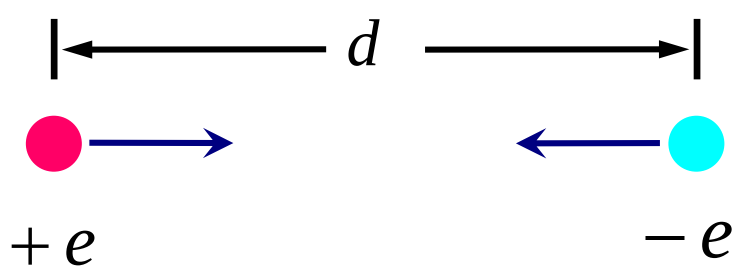
- An electron and a proton, initially separated by a distance d, are released from rest simultaneously. The two particles are free to move. When they collide, they are

A. at the midpoint of their initial separation.

B. closer to the initial position of the proton.

C. closer to the initial position of the electron.

[Hint: mass of proton is $1.67 \times 10^{-27} kg$ and mass of electron is $9.11 \times 10^{-31} kg$]

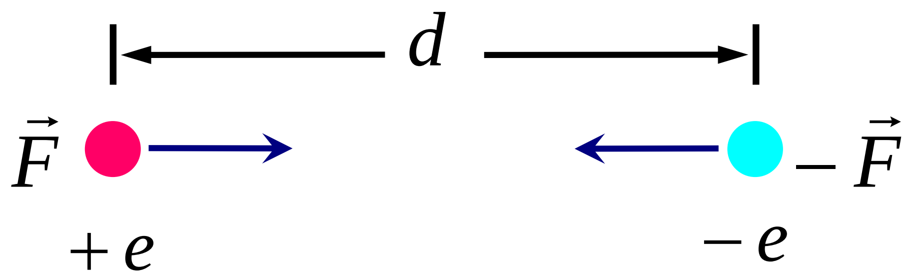


Multiple Choice

Concept Question 1.1: Solution

Answer : B (do you still remember Physical World?).

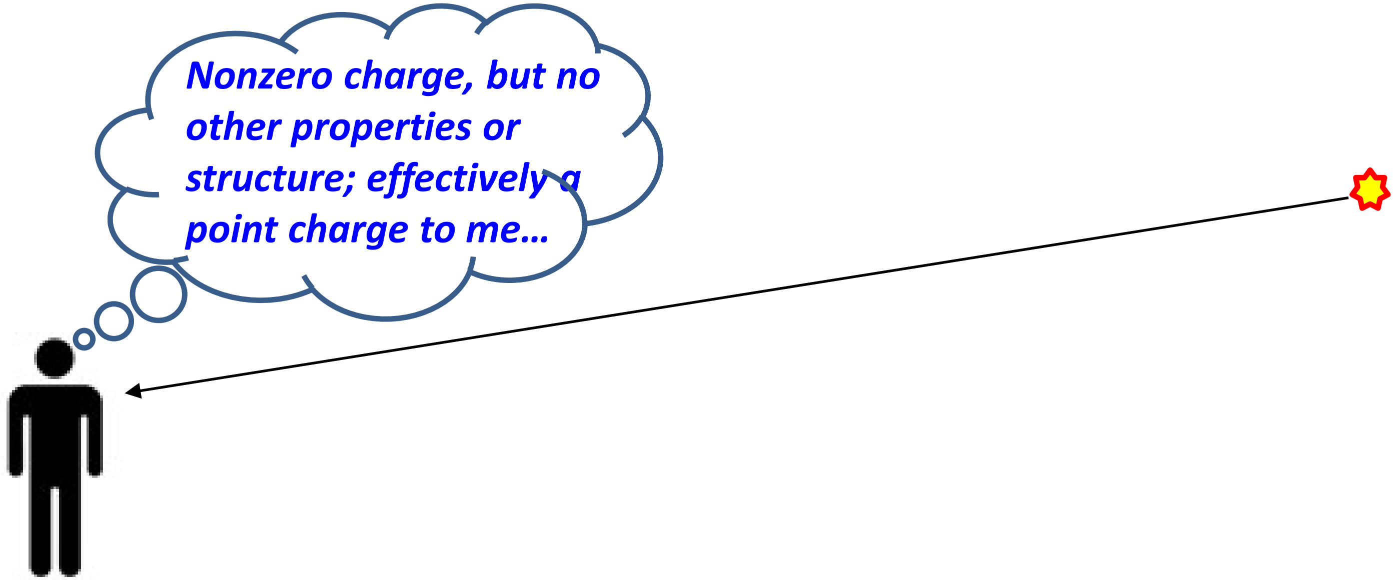
- According to Newton's third law, the forces exerted on the proton and electron are the same in magnitude, but opposite in direction.
- The mass of the proton and electron are different. The mass of the proton is around 2000 times greater than that of the electron, and thus the acceleration of proton is about 2000 times less than that of electron.



Concept 2: Interactions of Point Charges, Coulomb's Law

Point Charge

- Electron can be considered a "point" with finite charge but no interior structure.
- Generally, a point charge is a representation of any object with negligible size, shape and structure, compared with distances.
- There is no such thing as point charge (even electron is not exactly a point from quantum mechanic point view). But this concept is a good approximation of a rigid object if it is far away from the observer.



Coulomb's Law

$\vec{F}_{12}$ is defined Force on point charge q_2 due to interaction with point charge q_1

$\vec{F}_{12}$ means force acting on q_2 due to q_1

q_1 and q_2 can be positive or negative values.

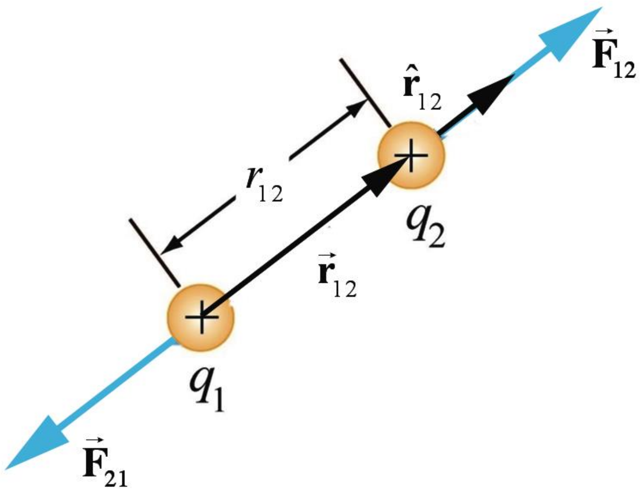
$$\vec{\boldsymbol{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{\boldsymbol{r}}_{12}$$

$k_e = \dfrac{1}{4\pi\varepsilon_o} = 8.9875 \times 10^9 Nm/C^2$

$\hat{r}_{12}$: unit vector pointing from q_1 to q_2

$\hat{r}_{12} = \dfrac{\vec{r}_{12}}{r_{12}}$

r_{12}: distance between the charges



We can also rewrite it in this form.

$$\vec{\boldsymbol{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^3} \vec{\boldsymbol{r}}_{12}$$

Take note of the notation and its meaning.

$\vec{\boldsymbol{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{\boldsymbol{r}}_{12}$ and $\vec{\boldsymbol{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^3} \vec{\boldsymbol{r}}_{12}$ are equivalent.

Note: $\hat{\boldsymbol{r}}_{12}$ may not be the direction of the $\vec{\boldsymbol{F}}_{12}$, as the sign of $\vec{\boldsymbol{F}}_{12}$ can be affected by q_1 and q_2.

The force is a vector!

$$\vec{\boldsymbol{F}}_{12} = \boldsymbol{F}_{12,\,x}\,\hat{\imath} + \boldsymbol{F}_{12,\,y}\,\hat{\jmath} + \boldsymbol{F}_{12,\,z}\,\hat{k}$$

or, equivalently

$$\vec{\boldsymbol{F}}_{12} = \begin{pmatrix} \boldsymbol{F}_{12,\,x} \\ \boldsymbol{F}_{12,\,y} \\ \boldsymbol{F}_{12,\,z} \end{pmatrix}$$

Hence forces from different charges add up as you add up vectors (also referred to as superposition principle).

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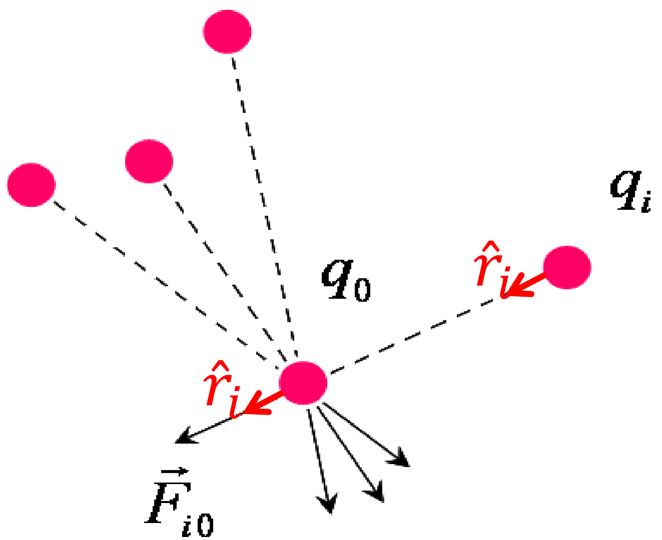
Principle of Superposition

For more than two charges, the net force on any one charge is simply the vector sum of the forces



$$\vec{\mathrm{F}} = \sum_{i=1}^{n} \vec{\mathrm{F}}_{i0} = \sum_{i=1}^{n} k_e \frac{q_i q_0}{r_i^2} \hat{\mathrm{r}}_i$$

$$\vec{\mathrm{F}}_{i0} = k_e \frac{q_i q_0}{r_i^2} \hat{\mathrm{r}}_i$$

$$\vec{\boldsymbol{F}} = \vec{\boldsymbol{F}}_{10} + \vec{\boldsymbol{F}}_{20} + \cdots + \vec{\boldsymbol{F}}_{\mathrm{N}0} = \begin{pmatrix} \boldsymbol{F}_{10,x} + \boldsymbol{F}_{20,x} + \cdots + \boldsymbol{F}_{\mathrm{N}0,x} \\ \boldsymbol{F}_{10,y} + \boldsymbol{F}_{20,y} + \cdots + \boldsymbol{F}_{\mathrm{N}0,y} \\ \boldsymbol{F}_{10,z} + \boldsymbol{F}_{20,z} + \cdots + \boldsymbol{F}_{\mathrm{N}0,z} \end{pmatrix}$$

$$= \left(\boldsymbol{F}_{10,x} + \boldsymbol{F}_{20,x} + \cdots + \boldsymbol{F}_{\mathrm{N}0,x}\right)\hat{\imath}$$
$$+ \left(\boldsymbol{F}_{10,y} + \boldsymbol{F}_{20,y} + \cdots + \boldsymbol{F}_{\mathrm{N}0,y}\right)\hat{\jmath}$$
$$+ \left(\boldsymbol{F}_{10,z} + \boldsymbol{F}_{20,z} + \cdots + \boldsymbol{F}_{\mathrm{N}0,z}\right)\hat{k}$$

Coulomb's Law and Newton's Law of Universal Gravitation

Coulomb's law has the same mathematical form of Newton's law of gravity.

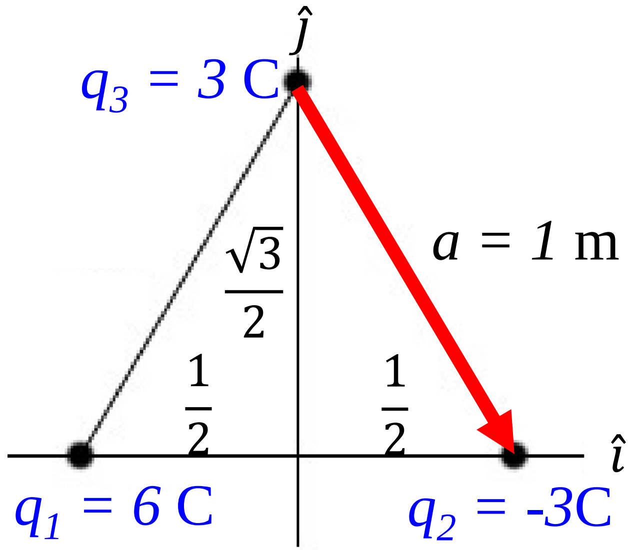
Fantastic!!!

- Both inversely proportional to the square of the distance and same direction.
- Gravity force is always attractive, whereas electric force can be attractive or repulsive depending on the polarity of charges.

$\vec{F}_{12} = k_e \dfrac{q_1 q_2}{r_{12}^{\ 2}} \hat{r}_{12}$	$\vec{F}_{12} = -G \dfrac{m_1 m_2}{r_{12}^{\ 2}} \hat{r}_{12}$
k_e is the Coulomb constant	G is the gravitational constant
$k_e = 8.9875 \times 10^9 \, \mathbf{N \cdot m^2/C^2}$	$G = 6.674 \times 10^{-11} \, \mathbf{N \cdot m^2/kg^2}$

Note: G is much smaller than k_e. This implies that at small scale Coulomb forces dominate over gravitational ones (e.g. electronic systems), while Netwon's force dominates for large masses with small overall charge (e.g. rockets, cars, planets).

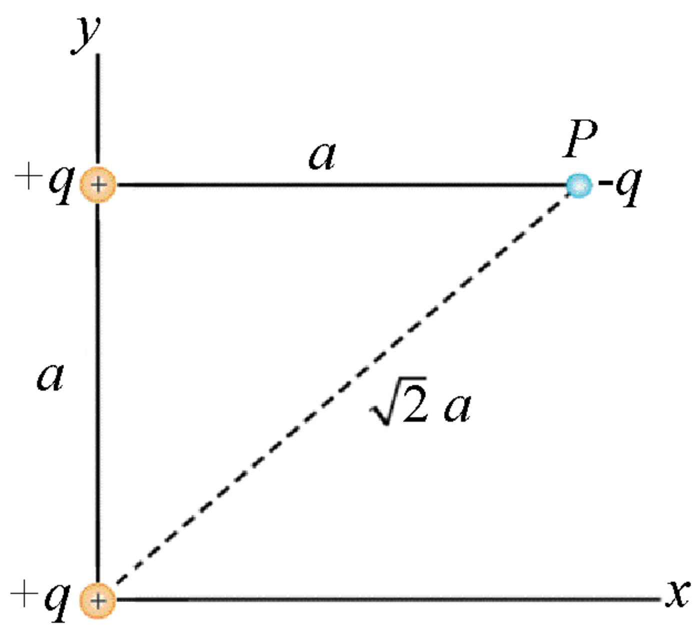
Coulomb's Law: Example



- Consider the following system of charges
- What is the force acting on charge 2 due to 3, $\vec{\mathrm{F}}_{32}$?
- Based on Coulomb's Law, $\vec{\mathrm{F}}_{32} = k_e \frac{q_3 q_2}{r_{32}^3} \vec{\mathrm{r}}_{32}$
- Find $\vec{\mathrm{r}}_{32} = \left(\frac{1}{2}\,\hat{\mathrm{i}} - \frac{\sqrt{3}}{2}\,\hat{\mathrm{j}}\right)$ m
- The magnitude $r_{32} = \left(\sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2}\right) = 1$ m
- Substitute into the equation:
- $\vec{\mathrm{F}}_{32} = k_e \frac{q_3 q_2}{r_{32}^3} \vec{\mathrm{r}}_{32} = (9 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2)(3\ \mathrm{C})(-3\ \mathrm{C}) \frac{\left(\frac{1}{2}\,\hat{\mathrm{i}} - \frac{\sqrt{3}}{2}\,\hat{\mathrm{j}}\right) m}{(1m)^3}$
- $= (4.05 \times 10^{10})\left(-\hat{\mathrm{i}} + \sqrt{3}\,\hat{\mathrm{j}}\right)$ N
- Note: The direction of $\vec{\mathrm{F}}_{32}$ is opposite of $\vec{\mathrm{r}}_{32}$.

Case Problem 2.1: Coulomb force from different charges

Three charged objects are located at the positions shown in the figure. Find a vector expression for the force on the negatively charged object located at the point *P*.



Case Problem 2.1: Solution

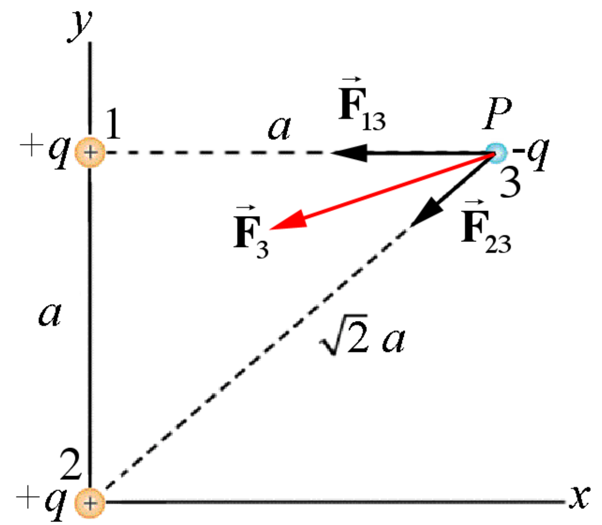
$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \frac{q_2 q_3}{r_{23}^2}\hat{r}_{23}\right)$$

$$q_1 = +q,$$
$$q_2 = +q,$$
$$q_3 = -q$$

$$\vec{r}_{13} = a\,\hat{i}\,;\; r_{13} = a\,;\; \hat{r}_{13} = \hat{i}$$

$$\vec{r}_{23} = a\,\hat{i} + a\,\hat{j};\;\; r_{23} = \sqrt{a^2 + a^2} = \sqrt{2}a$$

$$\hat{r}_{23} = \frac{\sqrt{2}}{2}(\hat{i} + \hat{j})$$



$$\vec{F}_3 = k_e\left(\frac{q_1 q_3}{a^2}\hat{i} + \frac{q_2 q_3}{(\sqrt{2}a)^2}\frac{\sqrt{2}}{2}(\hat{i} + \hat{j})\right)$$

$$= -\frac{k_e q^2}{a^2}\left[\left(1 + \frac{1}{2\sqrt{2}}\right)\hat{i} + \frac{1}{2\sqrt{2}}\hat{j}\right]$$

Concept 3: Electric Field of Point Charge and Electric Field Lines

The force on a charge due to many other charges depends also on the charge itself.

It would be convenient to have a description which is independent of the charge itself, so that we can have an idea of what the force would be if we use a different charge at the same position.

To our rescue comes the concept of **Electric Field** $\vec{E}$

Starting from the force acting on the charge q_0, we can write

$$\vec{\mathrm{F}} = \sum_{i=1}^{N} \vec{\mathrm{F}}_{i0} = \sum_{i=1}^{N} k_e \frac{q_i q_0}{r_i^2} \hat{\mathrm{r}}_i = q_0 \left(\sum_{i=1}^{n} k_e \frac{q_i}{r_i^2} \hat{\mathrm{r}}_i \right) = q_0 \vec{E}$$

Now we can easily evaluate the effect of all the charges on any charge at position 0.

Units of $\vec{E}$ is N/C and it is obviously a vector!

Since $\vec{\mathrm{E}}$ is a vector, the sum of different electric fields follows the rule of addition of vectors.

$$\vec{\mathrm{E}} = \vec{\mathrm{E}}_1 + \vec{\mathrm{E}}_2 + \cdots = \sum_{i=1}^{N} \vec{\mathrm{E}}_i$$

which can be done, for example, component by component.

Note:

- The $\vec{E}$ of a point charge:

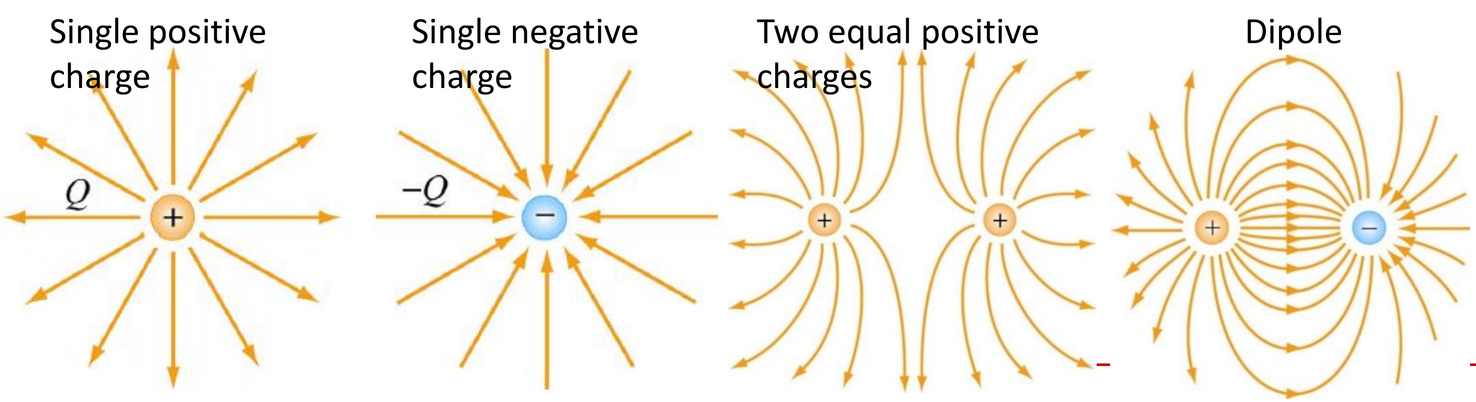
$$\vec{E}_{point\ charge} = \frac{k_e q_s}{r_{sp}^2} \hat{r}_{sp} = \frac{k_e q_s}{r_{sp}^3} \vec{r}_{sp}$$

- The subscript s stands for source, p stands for point of interest.

Electric Field Lines to indicate $\vec{E}$

We use the electric field lines to indicate the direction the electric field is pointing at each point in space. The field line is convenient to understand the direction of force (thus acceleration) on charge particle in the field.

- The direction of $\vec{E}$ vector at any point is tangent to the field lines.
- The field lines must begin on positive charges (or at infinity) and then terminate on negative charges (or at infinity).
- The denser the lines the greater magnitude of $\vec{E}$.
- No two field lines can cross each other.
- Note: The field lines do not indicate the motional trajectory of a charge. It only shows the direction of the acceleration at any point in time.



Examples of Electric Field Line (Figure from MIT notes)

Concept Question 3.1: 5 Equal Charges

- Six equal positive charges *q* sit at the vertices of a regular hexagon with sides of length R. We remove the bottom charge. The electric field at the center of the hexagon (point P) is:

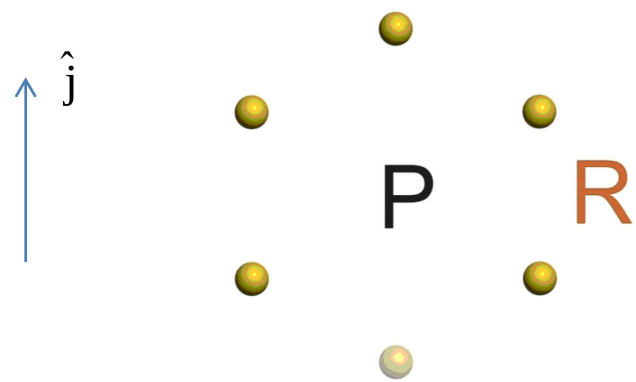
A. $\vec{\mathrm{E}} = \dfrac{2k_e q}{R^2}\hat{\mathrm{j}}$

B. $\vec{\mathrm{E}} = -\dfrac{2k_e q}{R^2}\hat{\mathrm{j}}$

C. $\vec{\mathrm{E}} = \dfrac{k_e q}{R^2}\hat{\mathrm{j}}$

D. $\vec{\mathrm{E}} = -\dfrac{k_e q}{R^2}\hat{\mathrm{j}}$

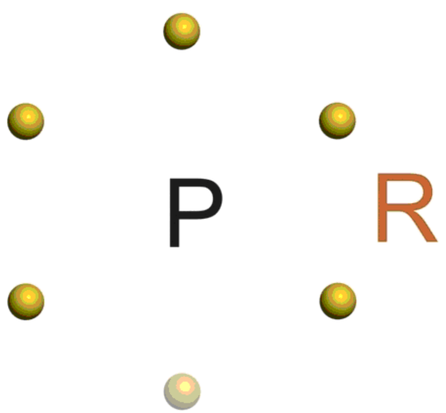
E. $\vec{\mathrm{E}} = 0$



Multiple Choice

Concept Question 3.1: Solution

- Answer: $\text{D. } \vec{\mathrm{E}} = -\dfrac{k_e q}{R^2}\,\hat{\mathrm{j}}$
- E fields of the side pairs cancel (symmetry)
- E at center due only to top charge (R away)
- Field points downward



Concept Question 3.2: Field Lines

Electric field lines show:

A. Directions of the forces that always exist in space.

B. Directions in which positive charges on those lines will accelerate.

C. Paths that charges will follow.

D. A and B.

E. A, B and C.

Multiple Choice

Concept Question 3.2: Solution

- Answer: B. Directions positive charges accelerate.

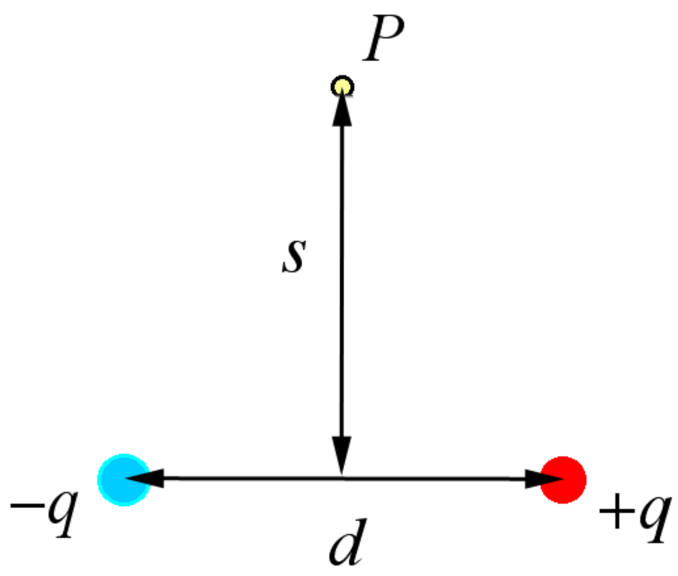
- Remarks:
- A is not correct because we do not know the sign of charge, so we do not know direction of force
- C is not correct. Charged particles do NOT move along field lines because they may have a non-zero initial velocity and also be affected by other forces like gravity.

Case Problem 3.1: Electric Field of a Dipole

- Consider two-point charges of equal magnitude but opposite signs, separated by a distance d. Point P lies at a distance s above the midpoint between the charges.

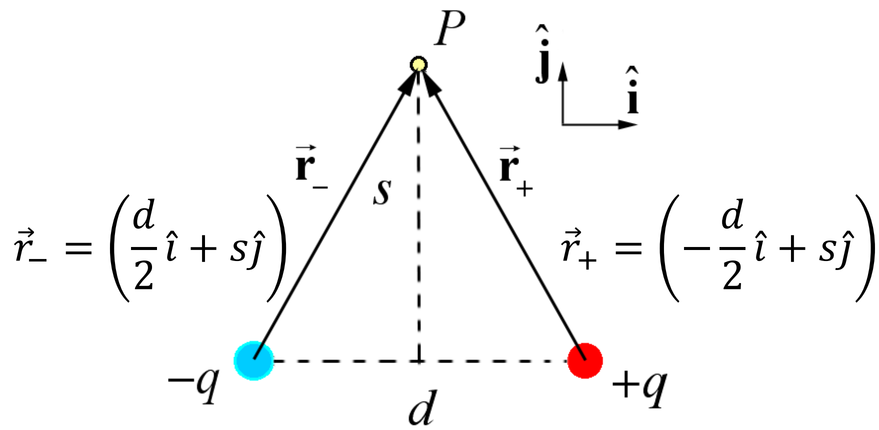
a) What is the E field at P?

b) What is the E field where s is much larger than d ($s \gg d$)?

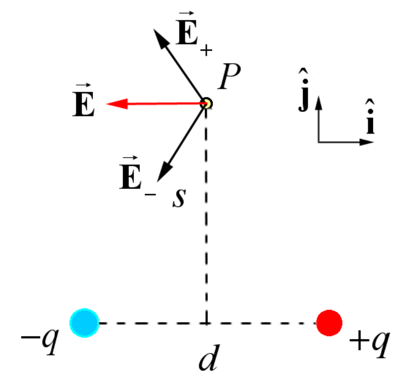


Case Problem 3.1 Solution

Part a) Setup the coordinate system and the origin as shown.



- The electric field at point P is a superposition of both E by +q, $\vec{\mathrm{E}}_+$ and E by –q, $\vec{\mathrm{E}}_-$. Thus, $\vec{\mathrm{E}}_p = \vec{\mathrm{E}}_+ + \vec{\mathrm{E}}_-$.
- From point charge, $\vec{E} = \frac{k_e q_s}{r_{sp}^3}\vec{r}_{sp}$ $\vec{\mathrm{E}}_+ = \frac{k_e q}{r_+^3}\vec{\mathrm{r}}_+$; $\vec{\mathrm{E}}_- = \frac{k_e(-q)}{r_-^3}\vec{\mathrm{r}}_-$
- $\vec{\mathrm{r}}_+ = -\frac{d}{2}\hat{\imath} + s\hat{\jmath}$; $\vec{\mathrm{r}}_- = \frac{d}{2}\hat{\imath} + s\,\hat{\jmath}$; $r_+ = r_- = \sqrt{s^2 + \left(\frac{d}{2}\right)^2}$;
- $\vec{\mathrm{E}}_+ = k_e \frac{(+q)}{\left(s^2+\left(\frac{d}{2}\right)^2\right)^{3/2}}\left(-\frac{d}{2}\hat{\imath} + s\,\hat{\jmath}\right)$; $\vec{\mathrm{E}}_- = k_e \frac{(-q)}{\left(s^2+\left(\frac{d}{2}\right)^2\right)^{3/2}}\left(\frac{d}{2}\hat{\imath} + s\,\hat{\jmath}\right)$
- $\vec{\mathrm{E}}_p = k_e \frac{q}{\left[\left(\frac{d}{2}\right)^2+s^2\right]^{3/2}}\left[\left(-\frac{d}{2} - \frac{\mathrm{d}}{2}\right)\hat{\imath} + (\mathrm{s} - s)\hat{\jmath}\right]$
- $\vec{\mathrm{E}}_p = k_e \frac{qd}{\left[\left(\frac{d}{2}\right)^2+s^2\right]^{3/2}}(-\hat{\imath})$



Note that y-components are cancelled with each other.

Can you figure out the $\vec{E}$ if both are $+q$?

Case Problem 3.1 Solution

Part b) Consider $z \gg d$.

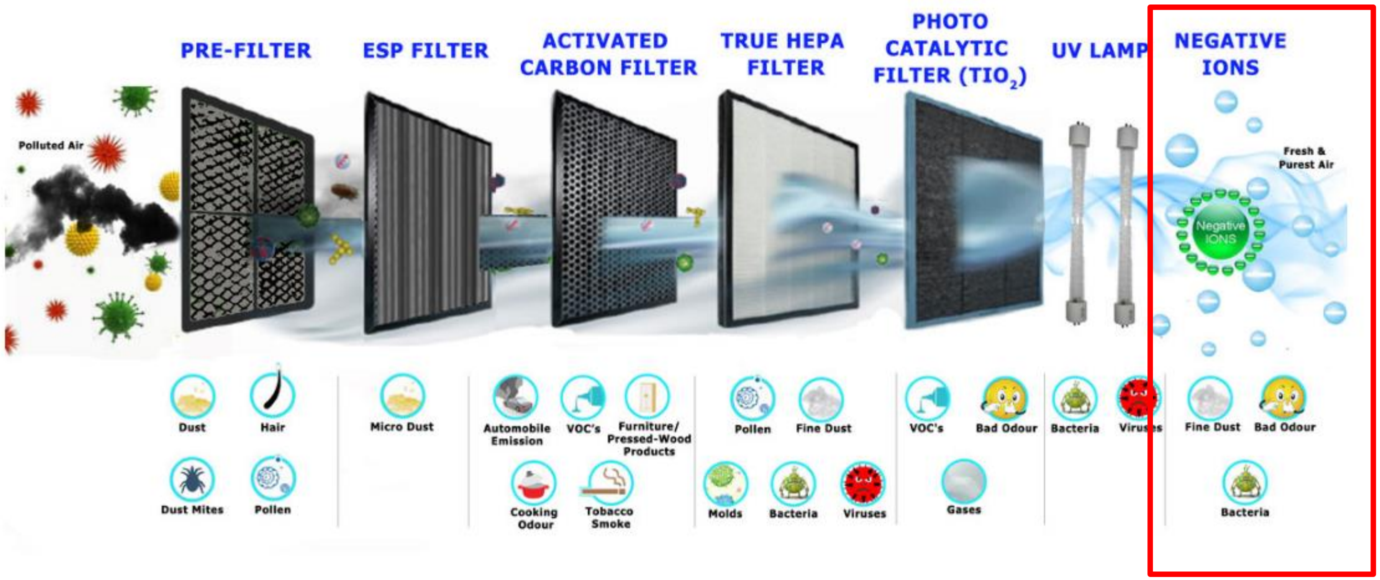
From part a) result, $\vec{\mathrm{E}}_p = \frac{k_e q d}{\left[\left(\frac{d}{2}\right)^2 + s^2\right]^{3/2}} \hat{\imath}$

- Recall the Taylor expansion that $(1+x)^n \approx 1 + n\,x$ for small x
- $\left[s^2 + \left(\frac{d}{2}\right)^2\right]^{-3/2} = \frac{1}{s^3}\left[1 + \left(\frac{d}{2s}\right)^2\right]^{-3/2} \approx \frac{1}{s^3}\left[1 - \frac{3}{2}\left(\frac{d}{2s}\right)^2\right] \approx \frac{1}{s^3}$
- We ignore $\left(\frac{d}{s}\right)^2$ and any higher order terms for they are small.
- Thus, for s $\gg d$

$$\vec{\mathrm{E}}_p = \frac{k_e q d}{s^3} \hat{\imath}$$

- This is the dipole electric field. It drops faster than $1/r^2$.
- This is quite remarkable. The fact that the charges are slightly separated one from the other, and that their sum is 0, results in a very different dependence of the electric field, $1/r^3$ instead of $1/r^2$. Cool!!!

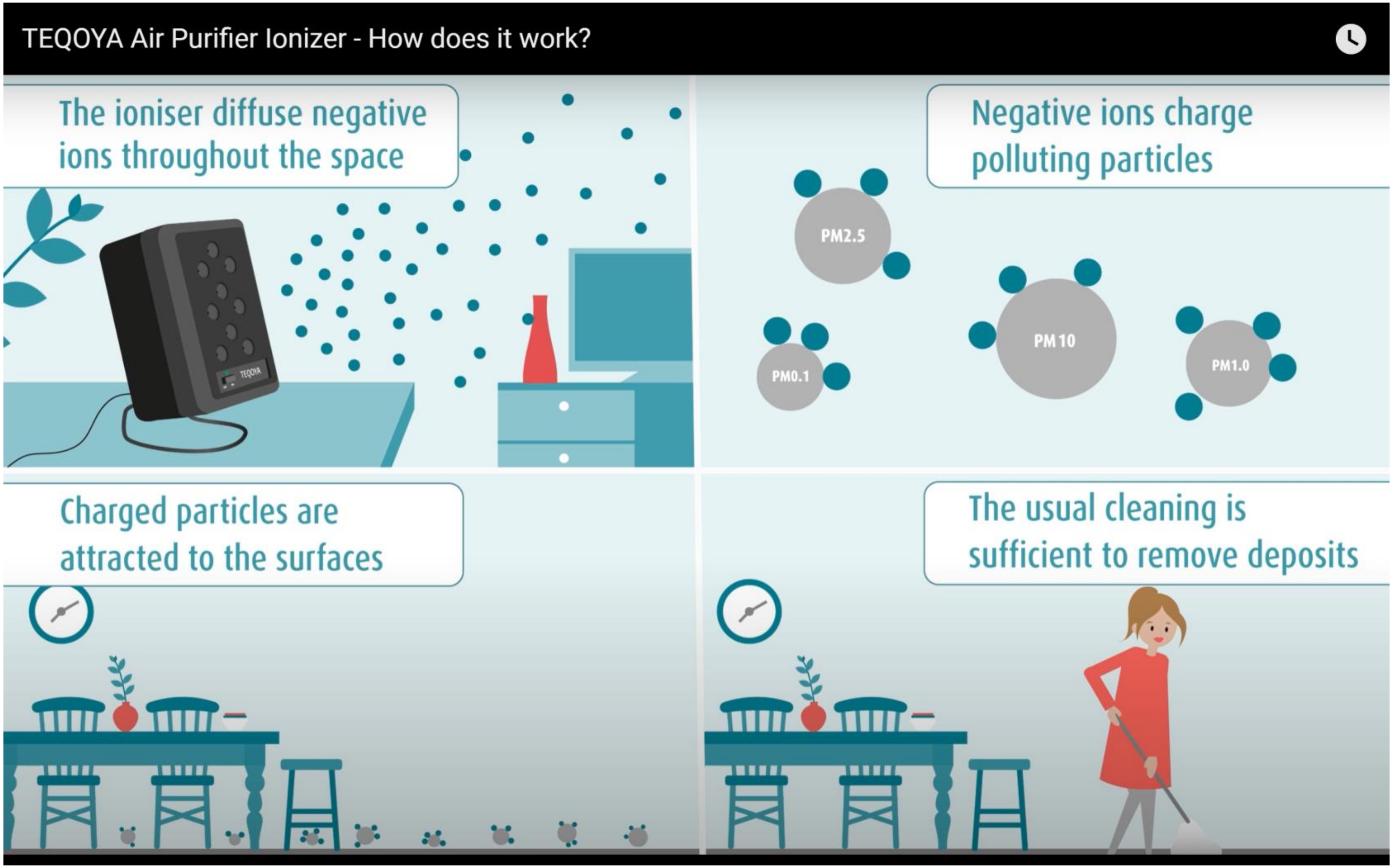
Application: The Working Principle of an Air Purifier



Several main components:

- Pre-filter: filters bigger particles and dust
- Activated Carbon: filters smells, odors, poisonous gases, VOCs (Volatile organic compounds)
- HEPA filter (High Efficiency Particulate Air) filter: Stops small particles (>0.3 micron) including bacteria and some viruses.
- HEPA definition by US Department of Energy: remove at least 99.97% of airborne particles 0.3 micrometers (µm) in diameter.
- UV light: kills bacteria and viruses.
- Ionizer: produces and diffuses negative ions to space.

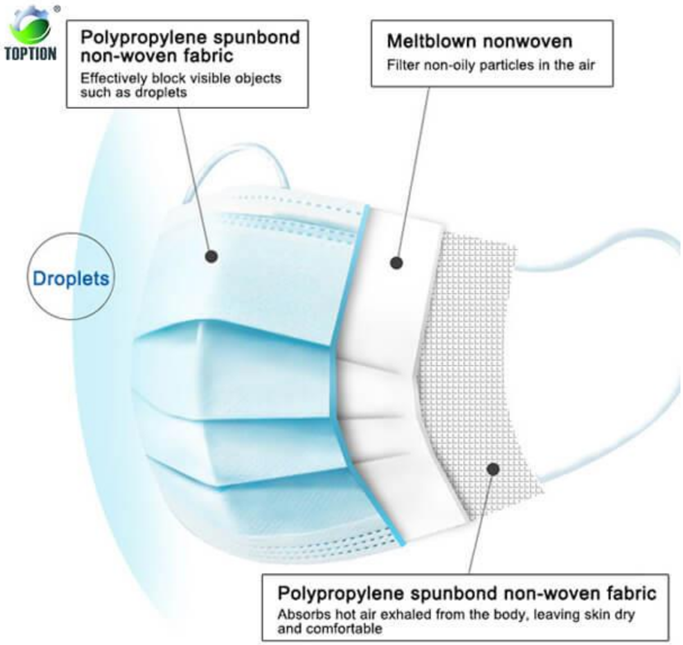
Application: How negative ions help to purify air?

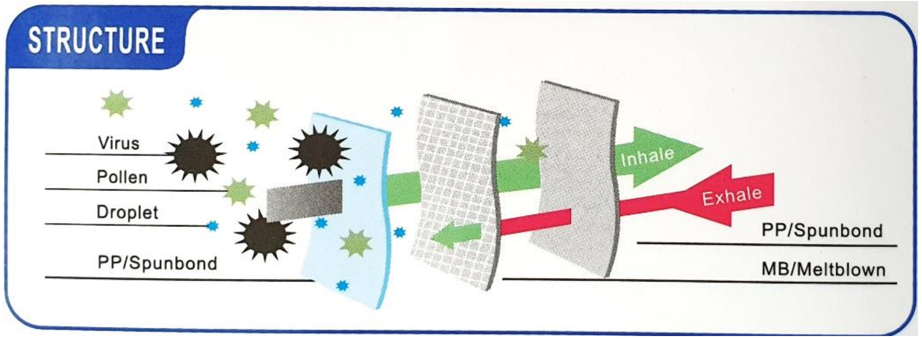


Captured from Youtube

Application: How a 3-ply face mask works?

- Outer layer: made from polypropylene (PP), having a water repellent (hydrophobic) property to block droplets and spray.
- Middle layer: melt-blown nonwoven fabric to block small particles in the air.
- Inner layer: absorb damp from breathing to keep face dry and comfort for long hours of wearing.
- Electrostatic charge can be induced on these polymer fabrics. Charges induces polarization of small particles to be captured in the fabric by the electrostatic force.





Captured from a product packaging in Sg market