Week 5 - Day 2

Magnetic Field and Force

Concept 1: Torque and Force on a Magnetic Dipole

Application: Speaker

Reading:

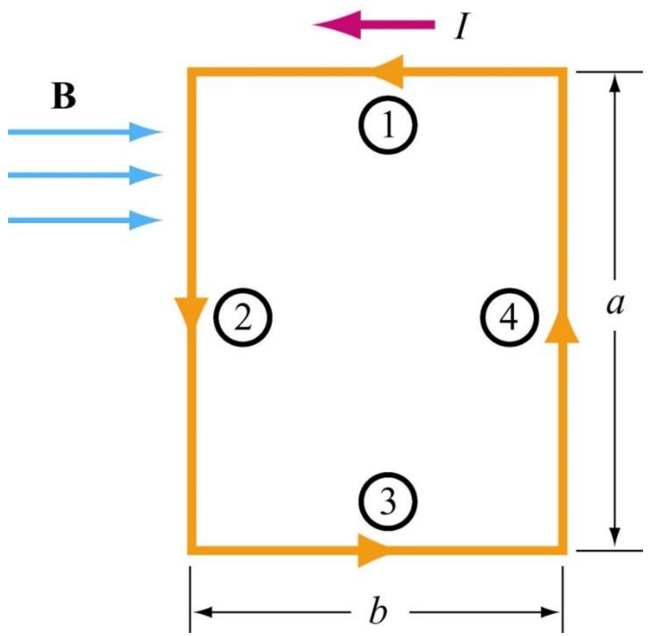
University Physics with Modern Physics – Chapter 27, 28

Introduction to Electricity and Magnetism – Chapter 8,9

Concept 1: Torque and Force on a Magnetic Dipole

Torque on a Current Loop

- Consider the current carrying loop below which is in a uniform external magnetic field $\vec{\boldsymbol{B}}$.
- What are the force and torque on the loop?
- How does the loop move?

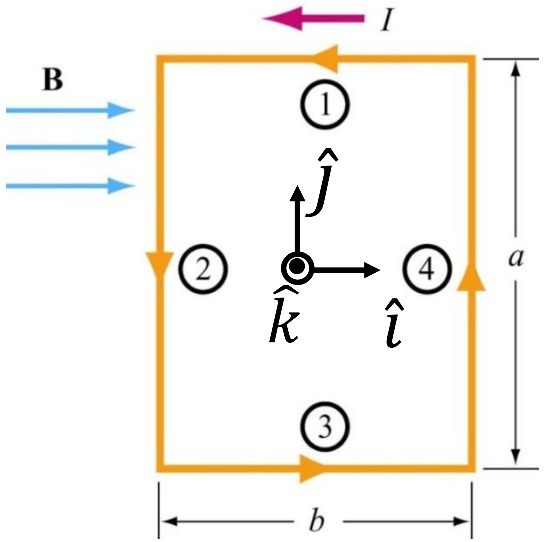


Torque on current Loop

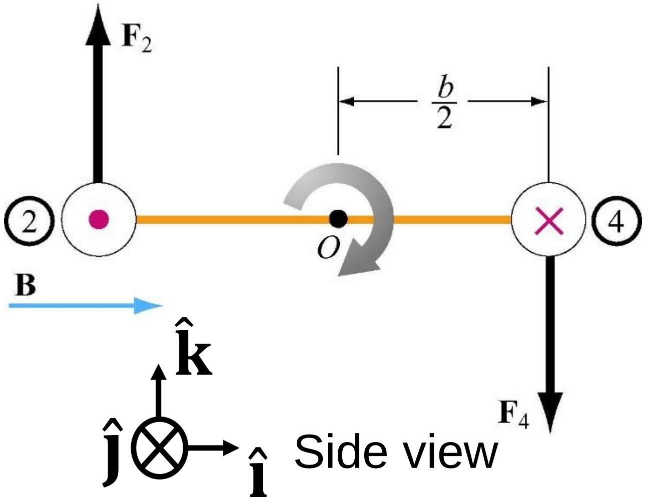
- From $\vec{F}_{mag} = I(\vec{L} \times \vec{B})$, only wire 2 and 4 experience a force.
- From side view, these 2 forces produce a torque around the center, O. $\vec{\tau} = 2F\left(\frac{b}{2}\right)\hat{j} = Fb\hat{j} = (IaB)b\hat{j}$
- Consider area vector, $\vec{A} = A\hat{n} = ab\hat{n}$; $\hat{n} = +\hat{k}$ (in this case)
- And the magnetic field $\vec{B} = B\hat{\imath}$
- Note that this equation $I\vec{A} \times \vec{B}$ can produce a consistent result from the torque equation.
- Thus,

$$\boxed{\vec{\tau} = I\vec{A} \times \vec{B}}$$

- Note: The net force = 0; torque (rotation) $\neq 0$ in a uniform magnetic field.



Top view

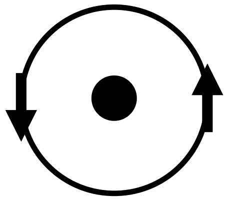


Side view

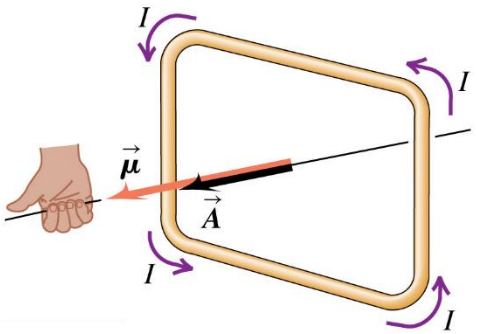
Introduction of Magnetic Dipole Moment, $\vec{\mu}$

- From the result of torque of a current loop in a magnetic field

$$\vec{\tau} = I\vec{A} \times \vec{B}$$



right-hand rule



- We further define:
- Magnetic dipole moment: $\vec{\mu} = I\vec{A}$

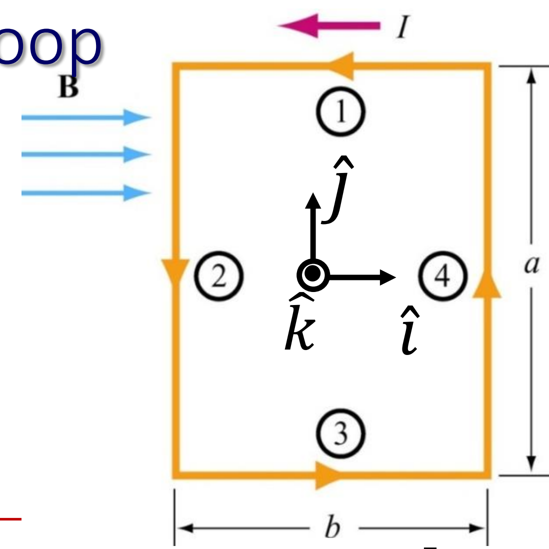
- Magnitude: $|\vec{\mu}| = IA$
- Direction: according to the right-hand rule, the moment is perpendicular to the plane of the loop.

Magnetic moment and Torque on Current Loop

- Place the rectangular current loop in a uniform B field

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$$

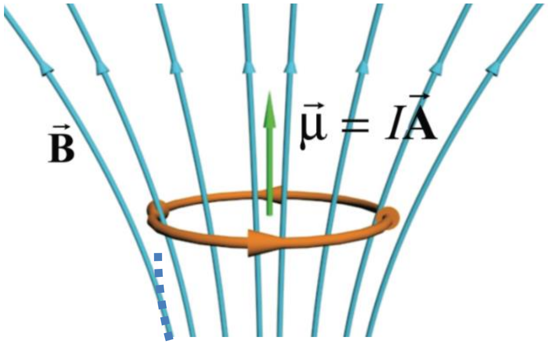
- $\vec{\tau} = \vec{\mu} \times \vec{B} = \mu B\hat{\jmath} = IAB\hat{\jmath}$
- Torque tries to align the magnetic moment vector in the direction of the magnetic field

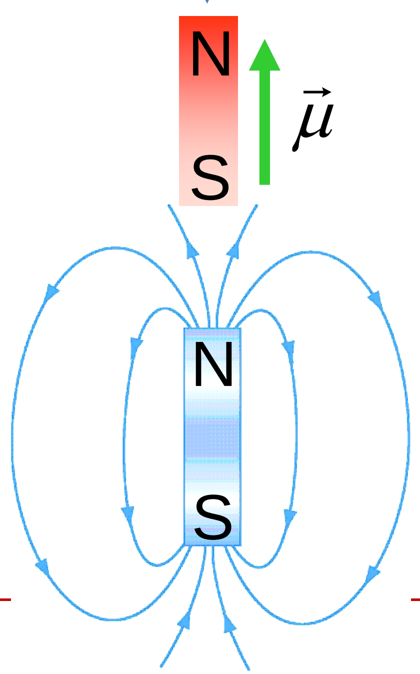


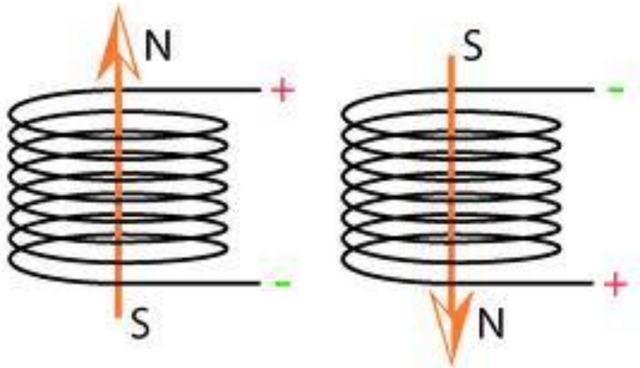
Top view

Force on Magnetic Dipole

- Consider a current loop in a non-uniform magnetic field as shown.
- You can imagine magnetic dipole denoted by $\vec{\mu}$ is like a bar magnetic (red color)
- And the magnetic field is created by the other magnetic bar at the bottom (blue color)
- Both magnetic bars will attract. Thus, the current loop will be attracted downwards.
- This force makes a speaker working. See the application slide.

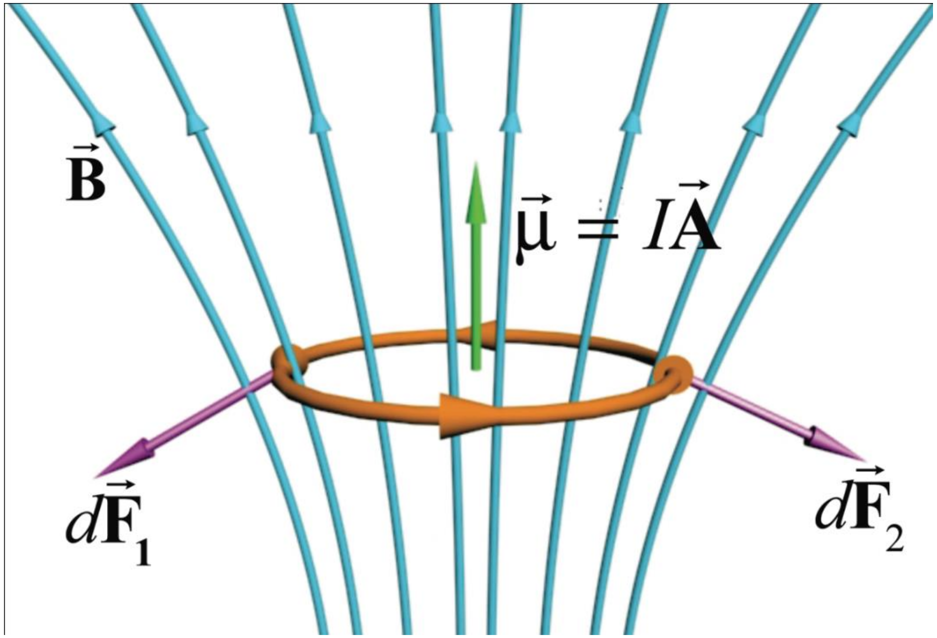






From the view of magnetic force on the loop

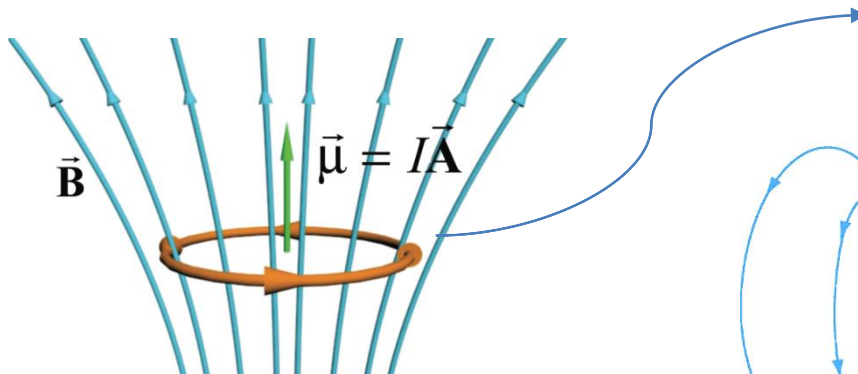
- The net force points downwards (–z)
- The other components of the force cancel out

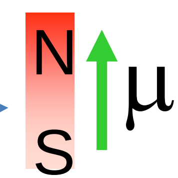


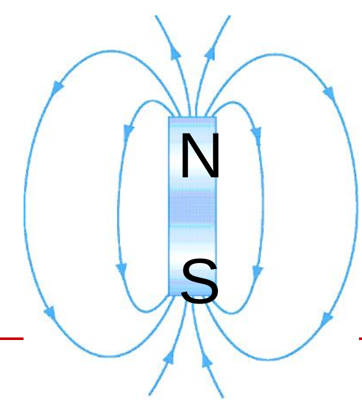
$$d\vec{F}_{mag} = I d\vec{s} \times \vec{B}$$

From the view potential energy

- In general, the magnetic field can be expressed as, $\vec{B} = B_x\vec{\imath} + B_y\vec{\jmath} + B_z\vec{k}$
- Potential energy U of a magnetic moment in magnetic field $\vec{B}$, $U_{dipole} = -\vec{\mu} \cdot \vec{B}$
  - We shall not proof this relation $U_{dipole} = -\vec{\mu} \cdot \vec{B}$ in our syllabus.
- From Physical World, we also learn that $\vec{F} = -\nabla U$. Thus, the force exerted on the closed loop
- $\vec{F} = \vec{\nabla}(\vec{\mu} \cdot \vec{B}) = \mu_z \frac{\partial B_z}{\partial z}\hat{k}$
- Note: $\frac{\partial B_z}{\partial z} < 0$ (magnetic field is weaker at the top), thus $\vec{F}$ pointing downward in z.

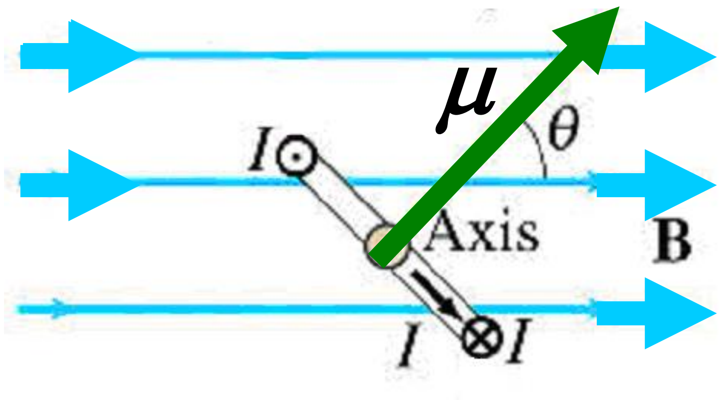






Concept Question 1.1: Dipole in Field

- From rest, the coil above will:



1. rotate clockwise, not move
2. rotate counterclockwise, not move
3. move to the right, not rotate
4. move to the left, not rotate
5. move in another direction, without rotating
6. both move and rotate
7. neither rotate nor move

Multiple Choice

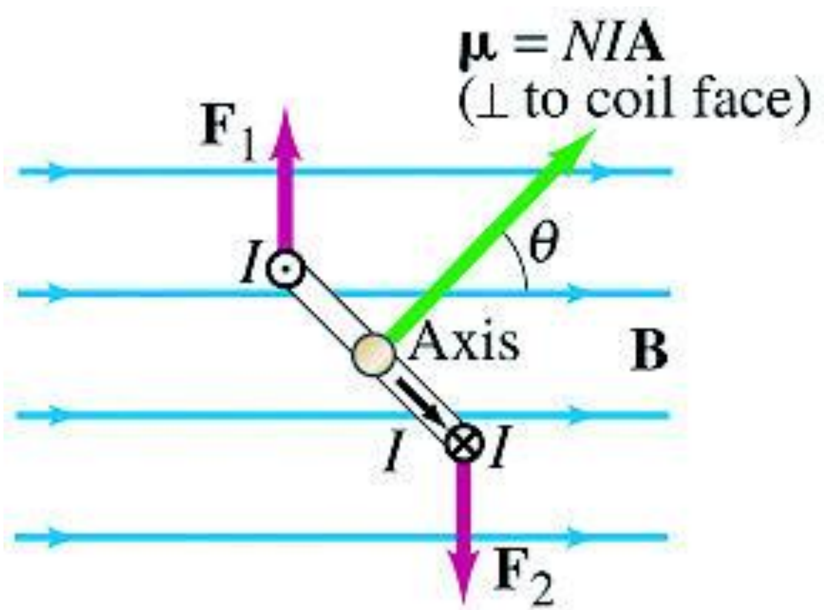
Concept Question 1.1 (Solution)

Answer: 1. Coil will rotate clockwise (not move)

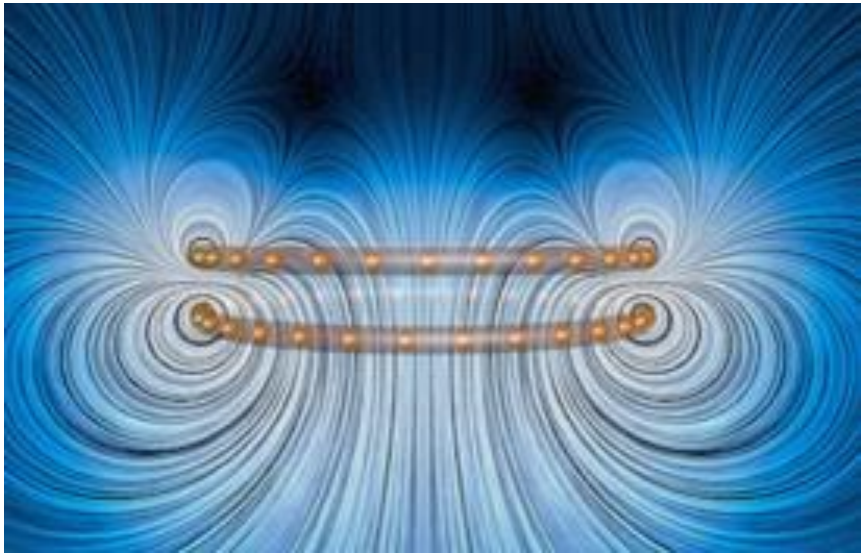
No net force so no center of mass motion.

BUT Magnetic dipoles rotate to align with external field. This is the working principle of a simple brushed DC motor and a compass.

We can always represent a current loop as a magnetic dipole, which also represents a small magnetic bar.



Concept Question 1.2: Current Carrying Coils



Which of the following options is true ? The above coils carry

1. parallel currents that attract
2. parallel currents that repel
3. opposite currents that attract
4. opposite currents that repel

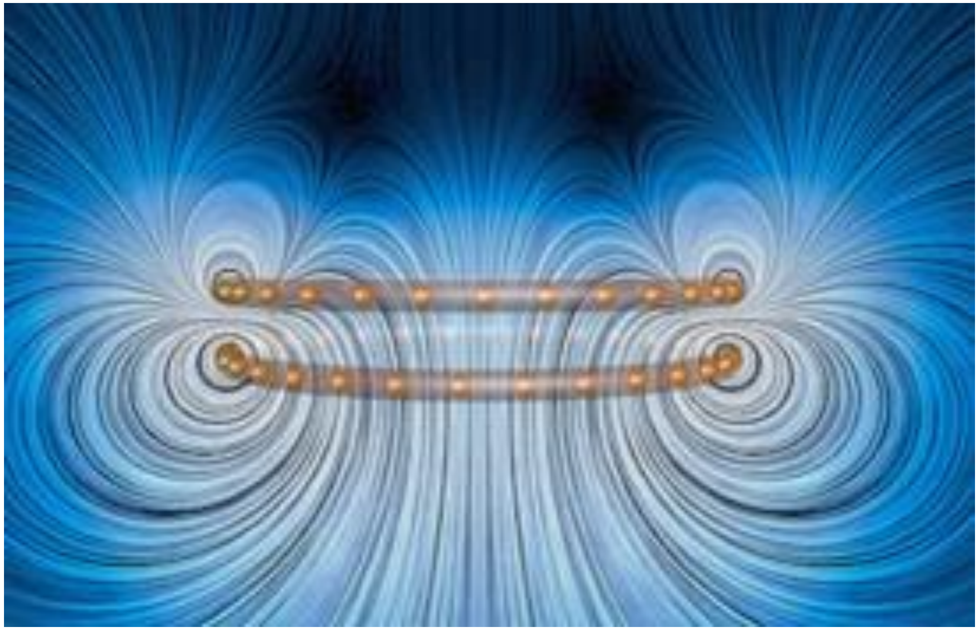
Multiple Choice

Concept Question 1.2 (Solution)

Answer: 4. Opposite currents, repelling.

Look at the field lines at the edge between the coils.

They are jammed in, want to push out. Also, must be in opposite directions.

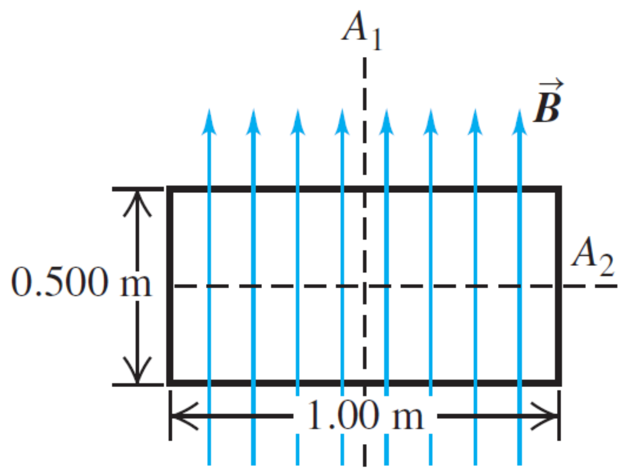


Case Problem 1.1: Torque on a Current Loop

A uniform rectangular coil of total mass 212 g and dimensions 0.500 m by 1.00 m is oriented with its plane parallel to a uniform 3.00 T magnetic filed as illustrated in the figure. A current of 2.00 A is suddenly started in the coil.

a) About which axis (A_1 or A_2) will the coil begin to rotate? Why?

b) Assuming moment of inertia of the coil about the rotation axis is 0.0102 kg.m^2, find the magnitude of the initial angular acceleration of the coil just after the current is started.



Case Problem 1.1 (Solution)

a) A_2, because only the top and bottom sides of the coil experience a force.

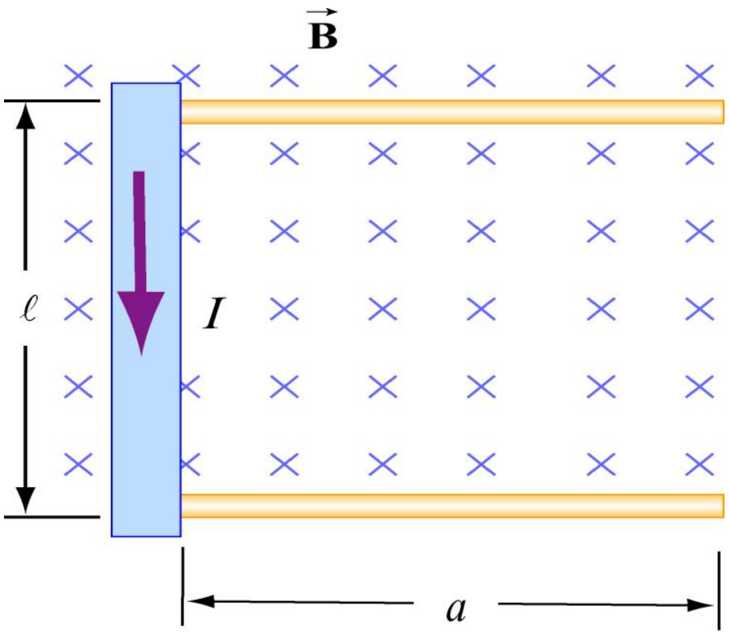
b) The magnetic field exerts a torque on the current-carrying coil, which causes it to turn. We can use the rotational form of Newton's second law to find the angular acceleration of the coil.

- We know magnetic torque is given by: $\vec{\tau} = \vec{\mu} \times \vec{B}$
- And also, rotational form of Newton's second law is: $\sum \tau = I\alpha$
- Hence,

$$\left|\vec{\mu} \times \vec{B}\right| = I\alpha \ \rightarrow \ \alpha = 290\, rad/s^2$$

Case Problem 1.2: Rolling Rod

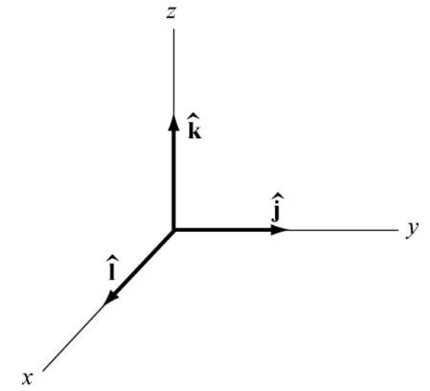
- A rod with a mass *m* and a radius *R* is mounted on two parallel rails of length *a* separated by a distance *l*, as shown in the Figure. The rod carries a current *I* and rolls without slipping along the rails which are placed in a uniform magnetic field $\vec{\mathbf{B}}$ directed into the page. If the rod is initially at rest, what is its speed as it leaves the rails?
- During this rolling, the moment of inertia, $I_{rod} = \frac{mR^2}{2}$



Case Problem 1.2 (Solution)

Using the coordinate system shown on the right, the magnetic force acting on the rod is given by:

$$\vec{F}_{mag} = I\vec{l} \times \vec{B} = Il\hat{\imath} \times B(-\hat{k}) = IlB\hat{\jmath}$$



The total work done by the magnetic force on the rod as it moves through the region is

$$W = \int \vec{F}_B \cdot d\vec{s} = F_B a = (IlB)a$$

By the work-energy theorem, *W* must be equal to the change in kinetic energy:

$$\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Case Problem 1.2 (Solution)

Since the moment of inertia of the rod is given by $\frac{mR^2}{2}$, we have

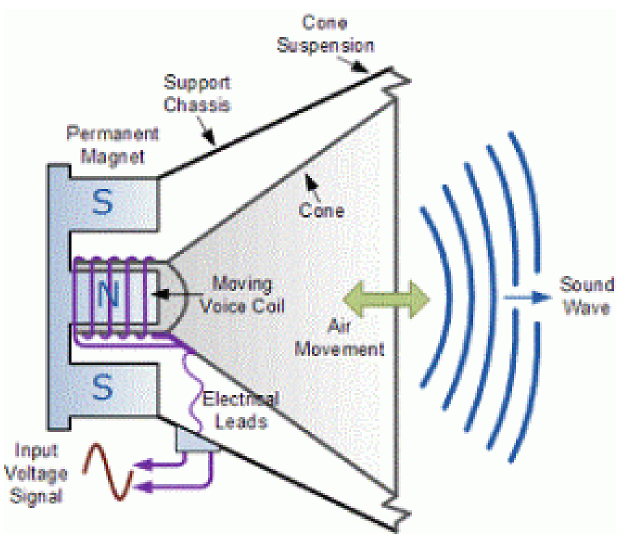
$$IlBa = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

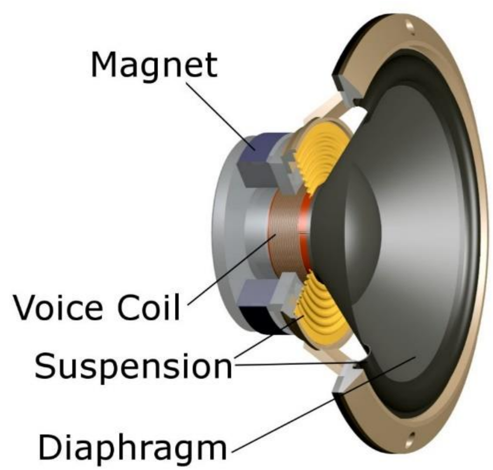
Thus, the speed of the rod as it leaves the rails is

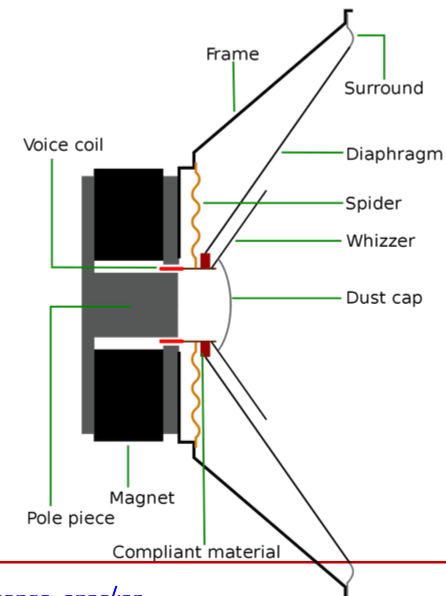
$$\mathrm{v} = \sqrt{\frac{4IlBa}{3m}}$$

Application: Speaker

- The speaker cone vibrates, pushing and pulling the air to create sound waves.
- 3 main parts: An electromagnet (voice coil), a permanent magnet and a cone.
- As the electrical current (i.e. your music) flows back and forth through the coil, the electromagnet either attracts or repels the permanent magnet. This moves the coil back and forward, pulling and pushing the loudspeaker cone.
- Speakers (a current-driven device) require much more power to push and pull the air to make sound. An audio amplifier is always needed to boost a low power electrical music signal into a higher power state to drive the speakers.

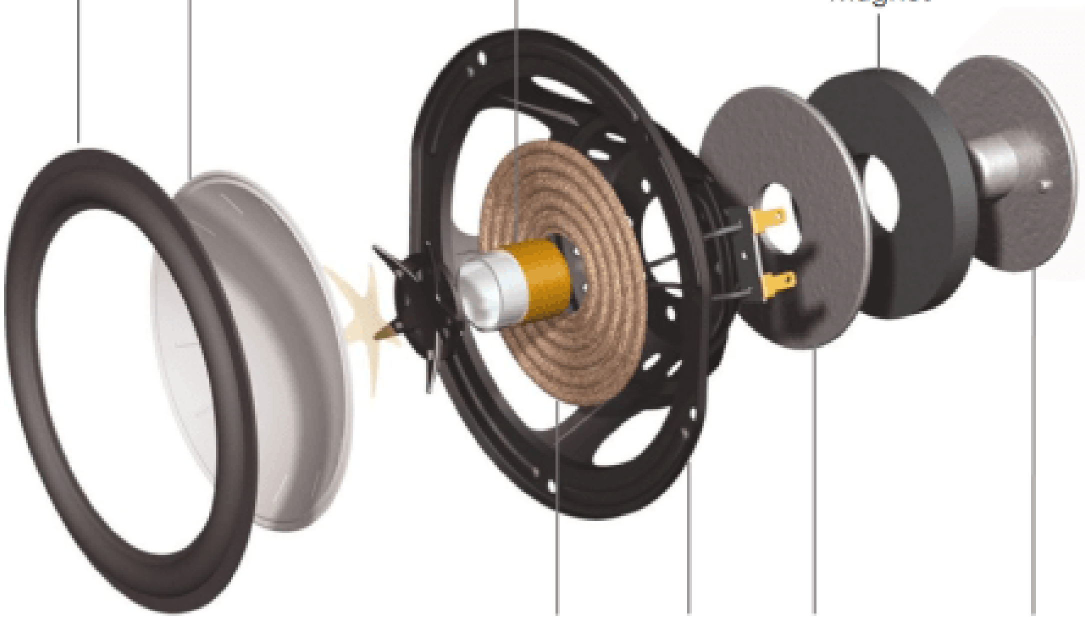






Picture source: https://en.wikipedia.org/wiki/Full-range_speaker

FYI: More Detail Structure



Picture source: http://www.audioaffair.co.uk/blog/

Surround: A piece of elastic rubber, foam, or textile that flexibly fastens the diaphragm to the basket (outer frame)

Cone (Diaphragm): Moves in and out to push air and make sound.

Voice Coil: The coil that moves the diaphragm back and forth.

Magnet: Typically made from ferrite or powerful neodymium

Basket: The sturdy metal framework around which the speaker is built.

Spider (suspension): A flexible, corrugated support that holds the voice coil in place, while allowing it to move freely.

More Info:
https://www.explainthatstuff.com/loudspeakers.html

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

1D Pre-analysis Worksheet