

SER 232

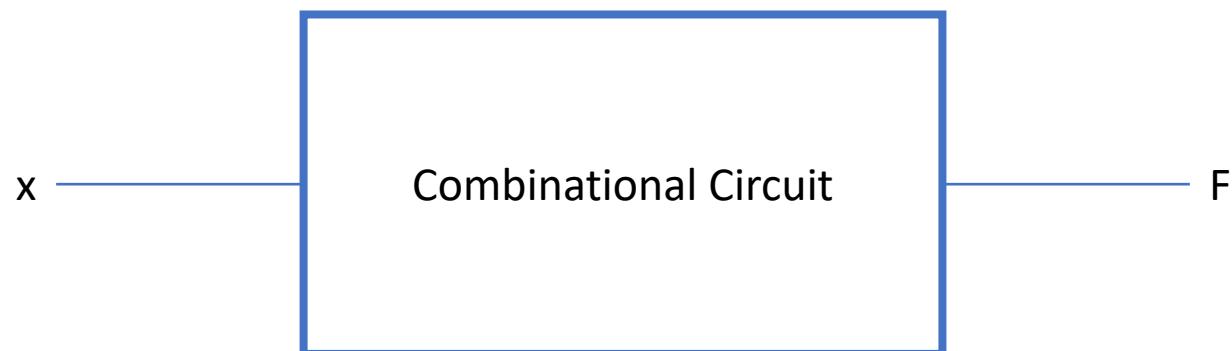
Computer Systems Fundamentals I

Topics

- Combinational Circuit
- Truth Table
- Logic Gates

Combinational Circuit

- A circuit for which the output solely depends on the current input values is called a “combinational circuit”
 - “Outputs are defined by a pure function only based on the current input values”
- Example: Output **F** only depends on current value of input **x**



Truth Table

- Name comes from Boolean values, true and false
- Digital combinational circuits are similar to mathematical functions (or functions in a program)
 - Takes a set of one or more **inputs**
 - For every possible input value there is an **output**
 - All inputs and outputs are digital (1/true or 0/false)

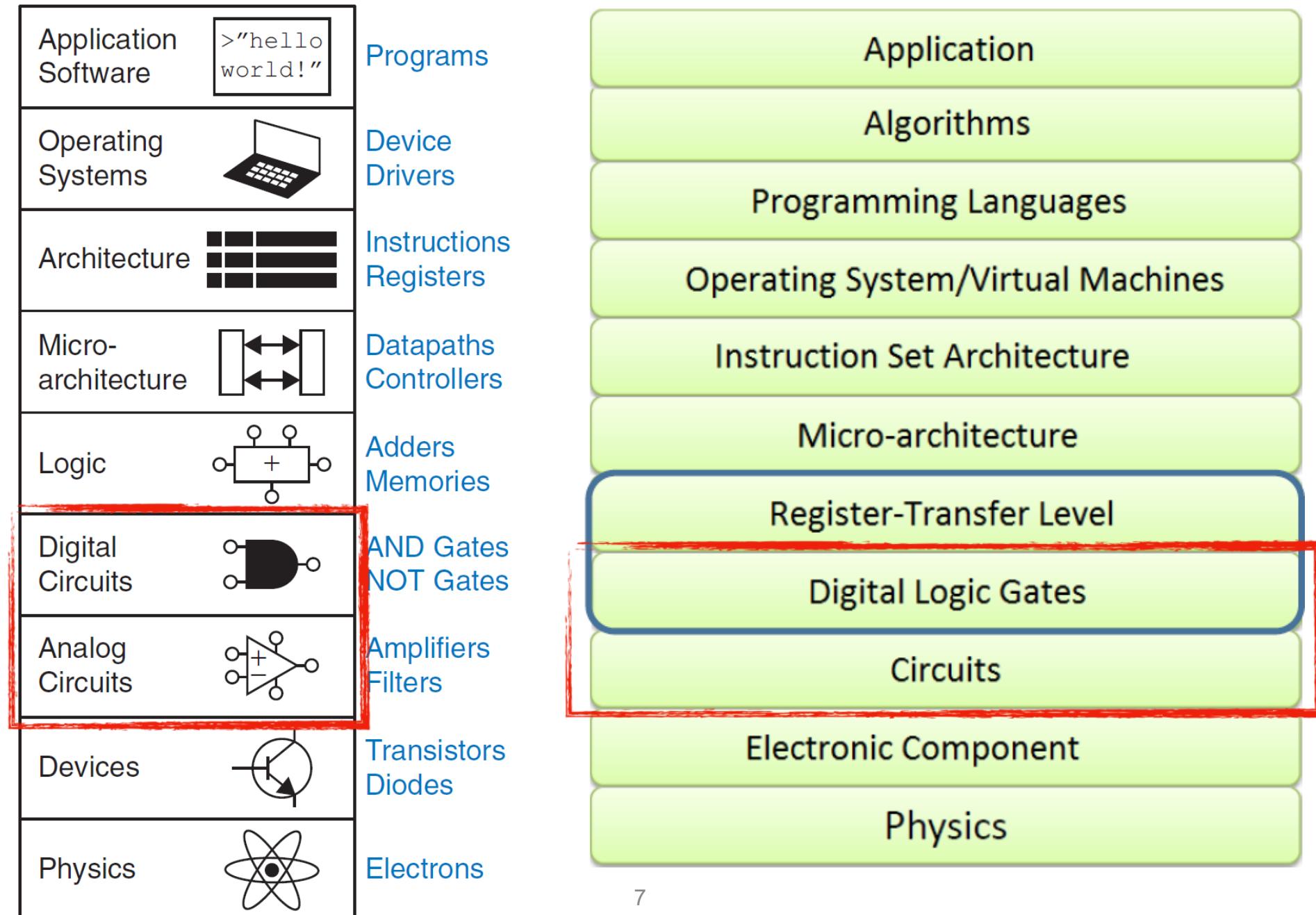
Truth Table

- Table used to describe behavior of circuit
- No knowledge about actual circuit
- Only “what goes in” (input) & “what comes out” (output) visible
 - Also called “blackbox”
- Example: x, y are inputs; F is output

x	y	F

Digital Logic Gates

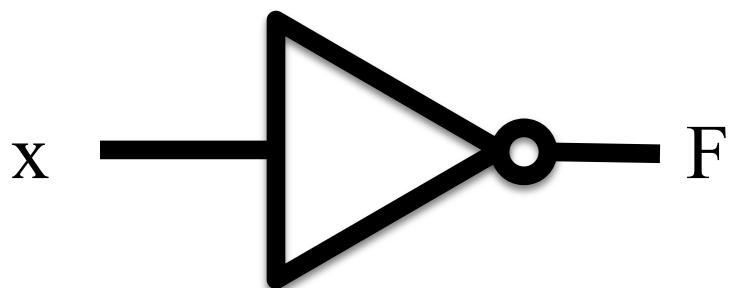
- Abstraction from circuits



Digital Logic Gates

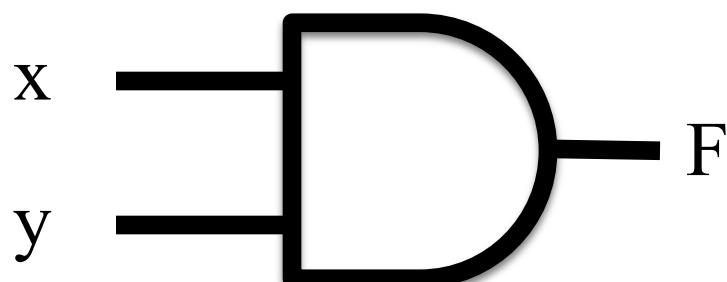
- Abstraction from circuits
 - No need to worry about how analog circuits work
 - Focus on functionality of collection of transistors
 - Logic gates are made out of transistors
 - Wires values are digital (on/1/high or off/0/low)

NOT Gate



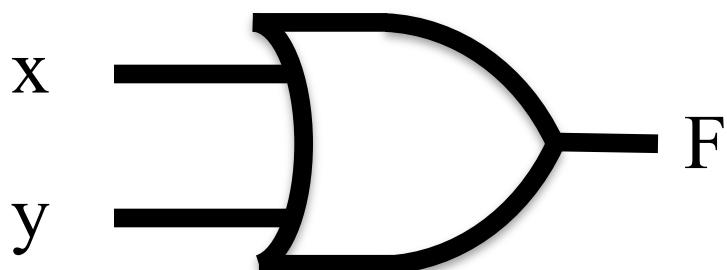
X	F
0	1
1	0

AND Gate



x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

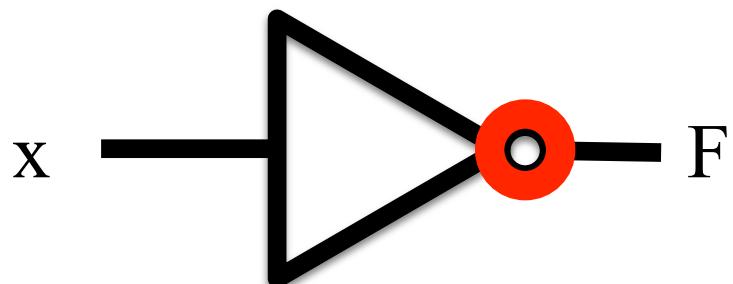
OR Gate



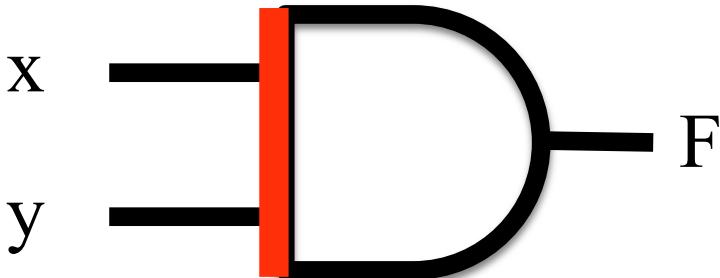
x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

Logic Gates

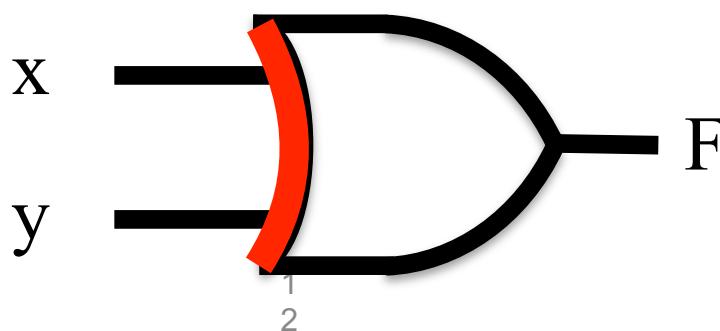
- NOT



- AND



- OR



x	F (NOT)
0	1
1	0

x	y	F (AND)
0	0	0
0	1	0
1	0	0
1	1	1

x	y	F (OR)
0	0	0
0	1	1
1	0	1
1	1	1

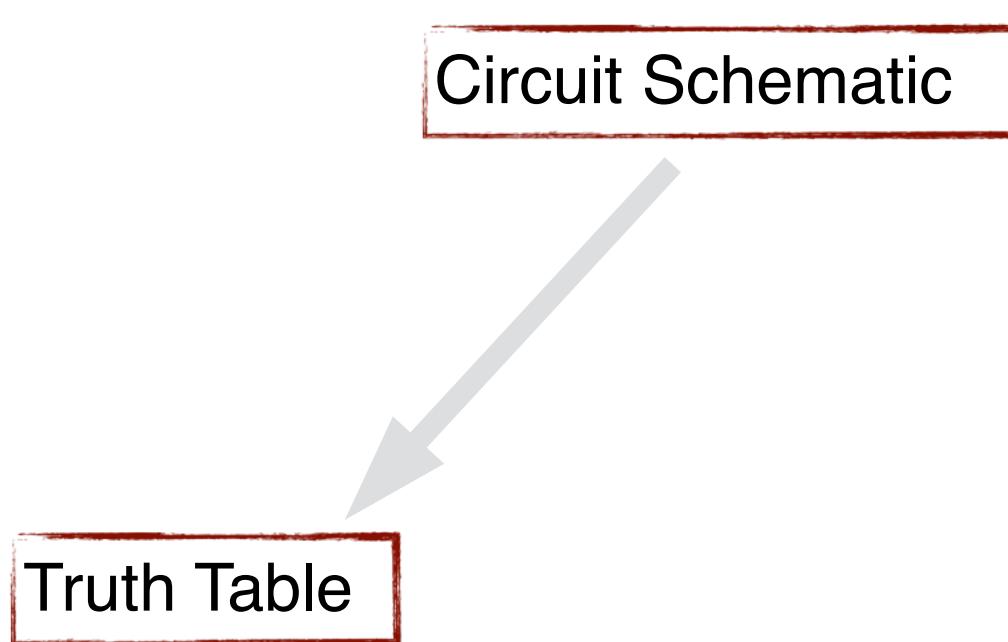
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Computer Systems Fundamentals I

Topics

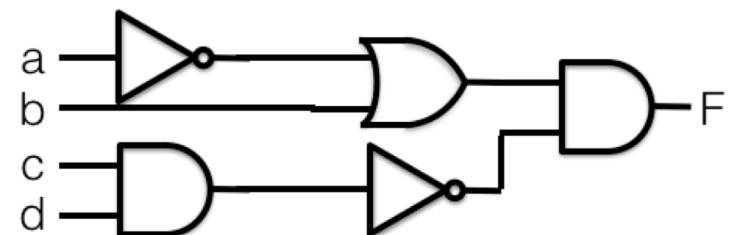
- Circuit Representations
- Circuit Conversion Part 1

Converting Representations



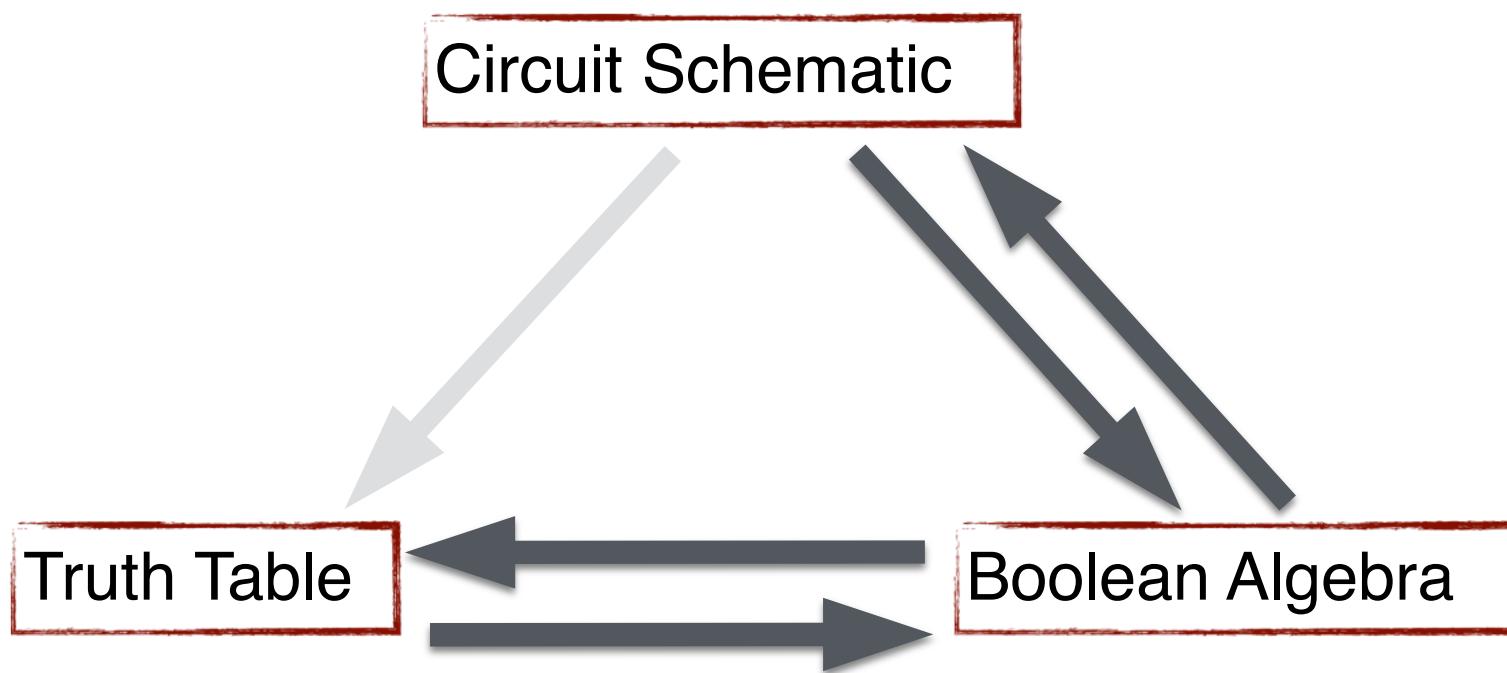
Boolean Algebra

- Ways to represent digital logic gates:
 1. Circuit Schematic
 - Graphical representation
 2. Truth Table
 - Tabular representation
 3. Boolean Algebra
 - Mathematical representation



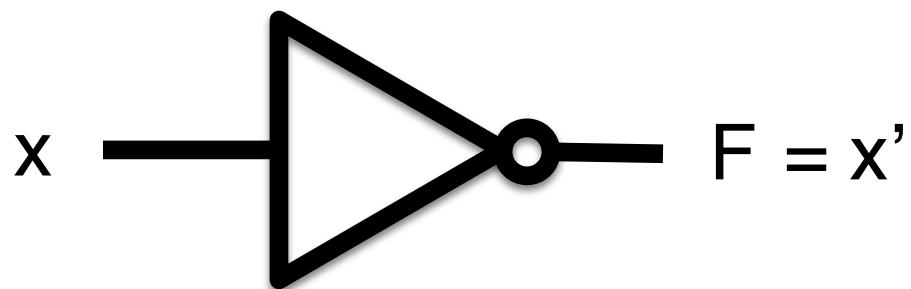
x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

Converting Representations



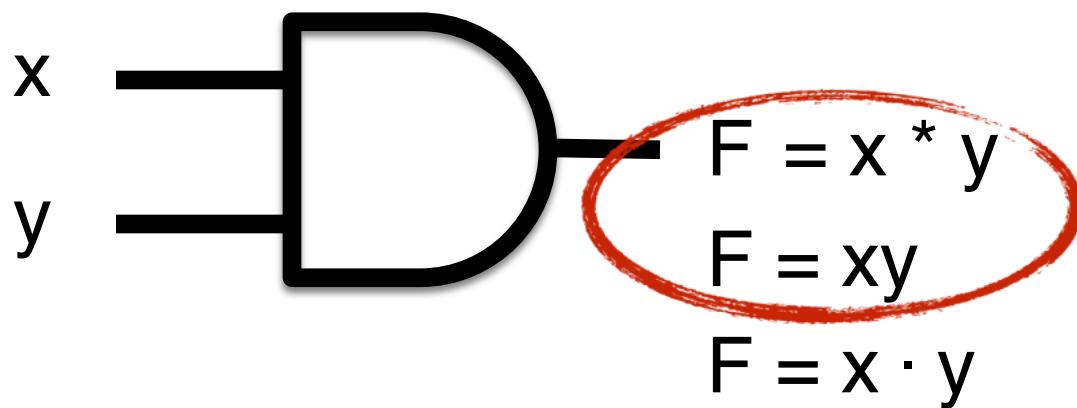
Notation

- NOT Gate



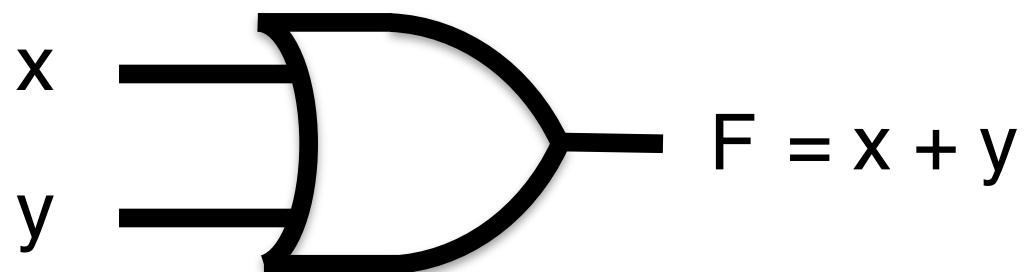
Notation

- AND Gate

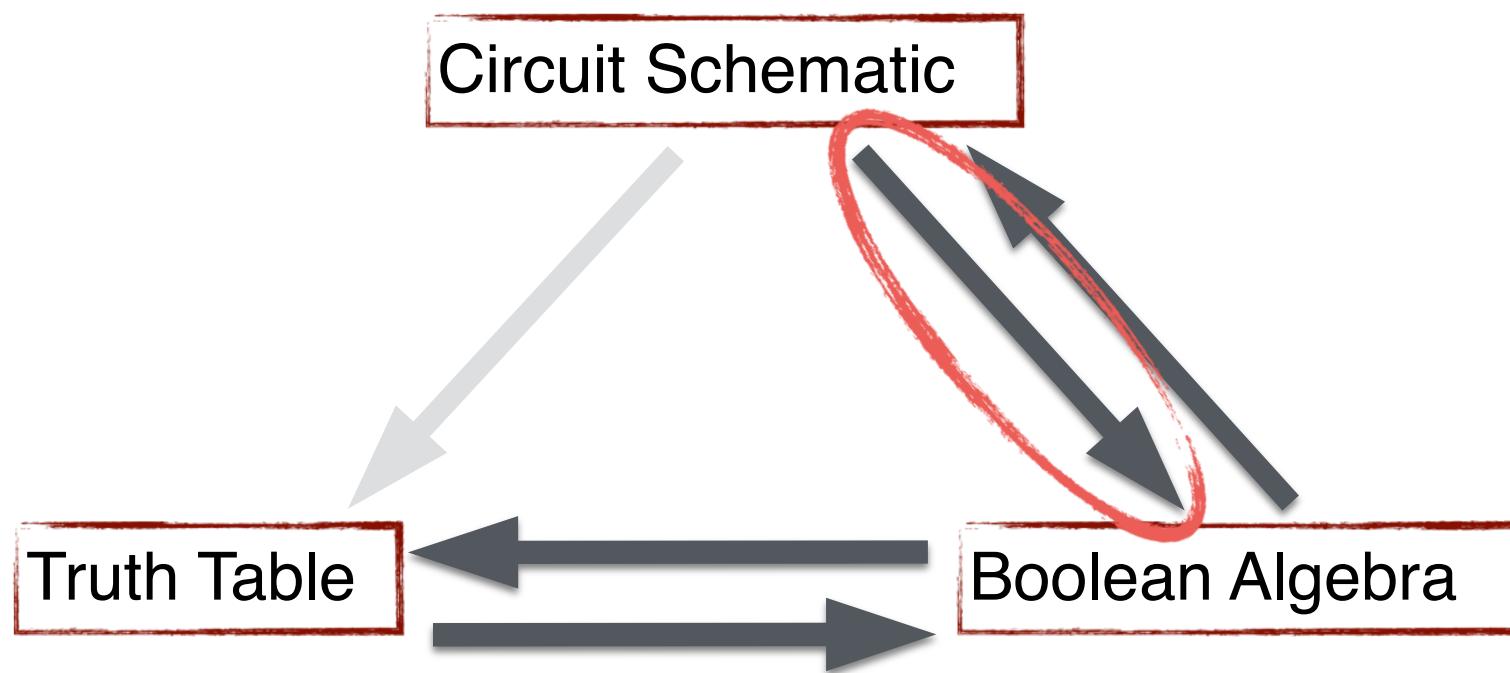


Notation

- OR Gate

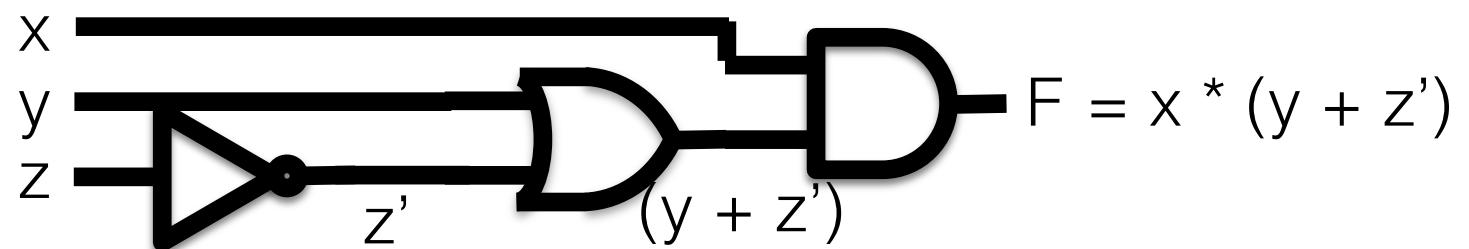


Converting Representations



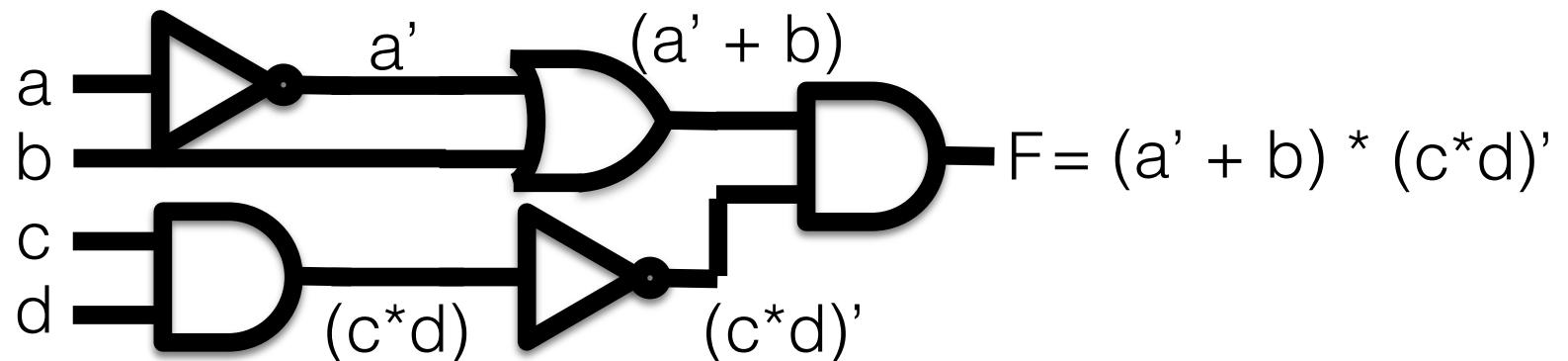
Conversion

- Conversion: Digital Circuit to Boolean Equation

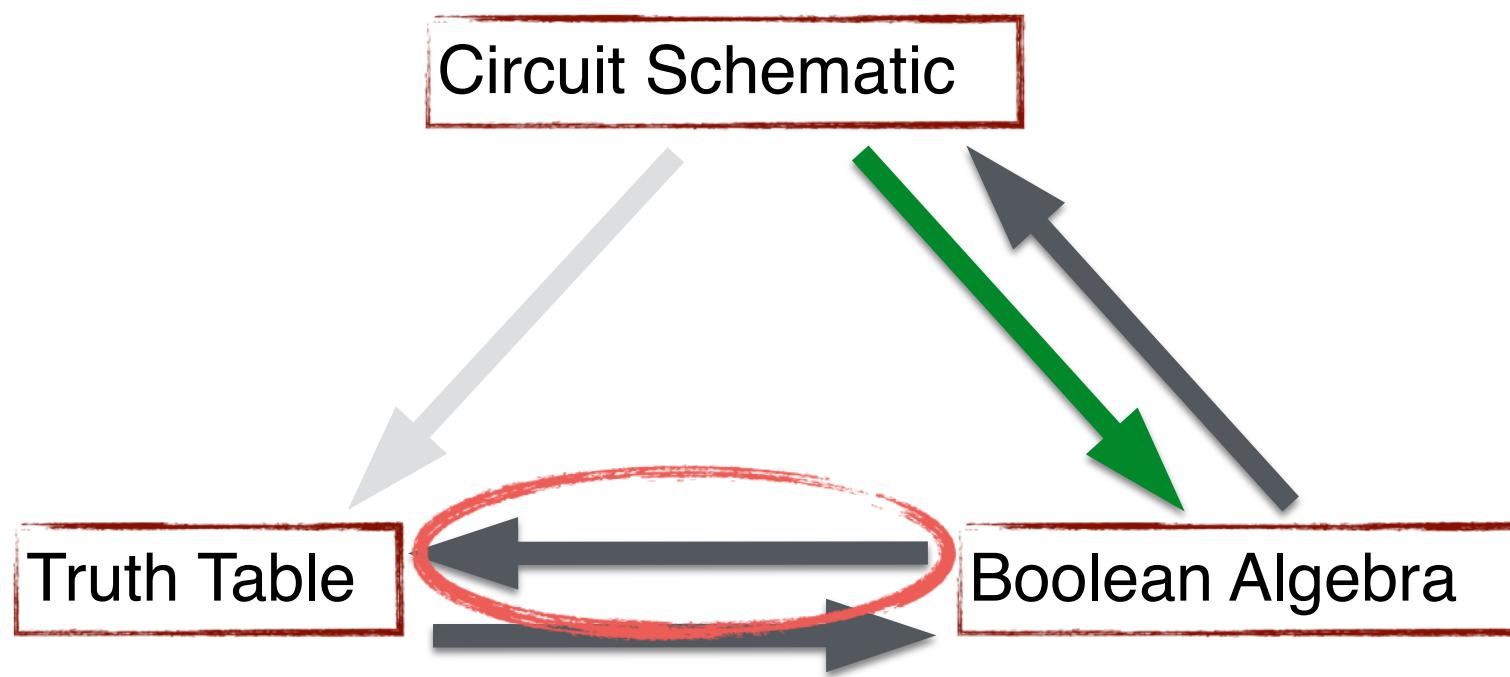


Conversion: Your Turn!

- Conversion: Digital Circuit to Boolean Equation

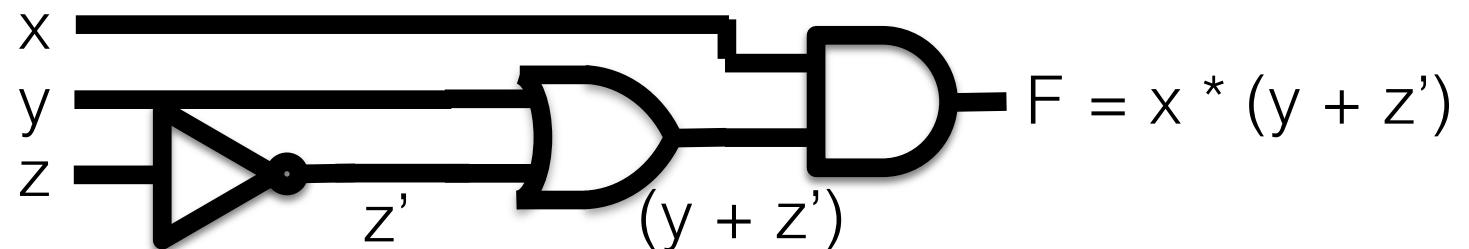


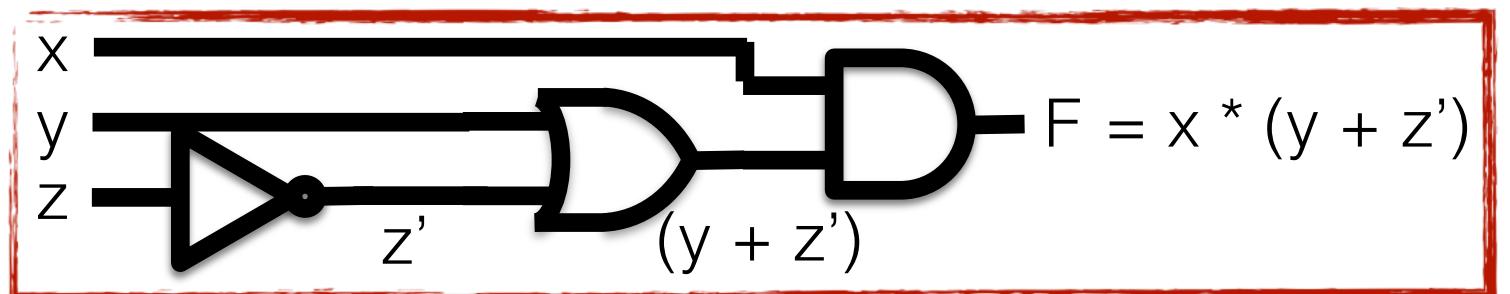
Converting Representations



Conversion

- Conversion: Boolean Equation to Truth Table





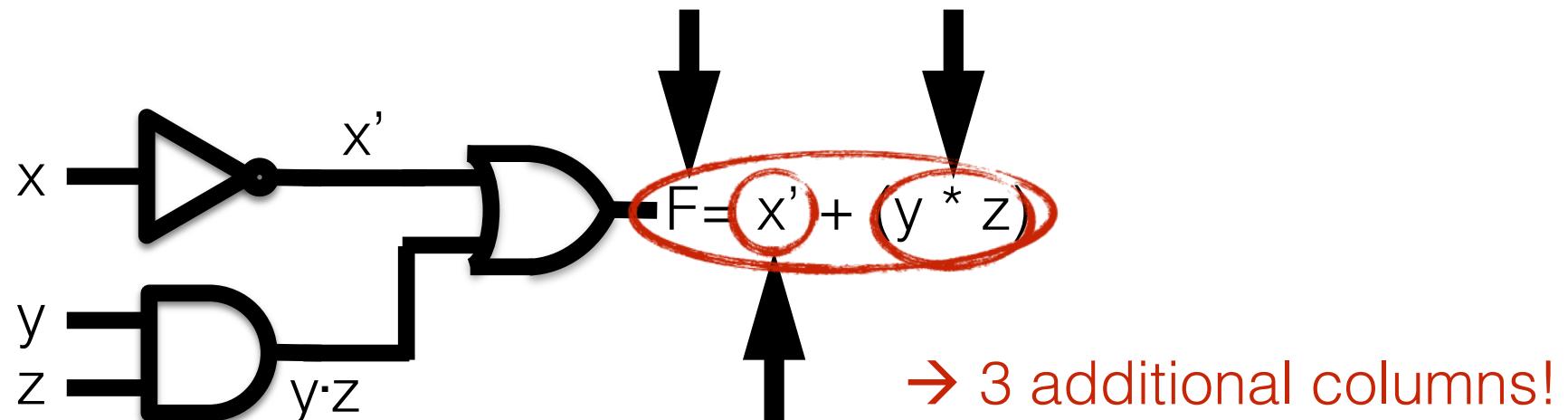
- Conversion: Boolean Equation to Truth Table

x	y	z	z'	$y + z'$	$F = x * (y + z')$
0	0	0	1	1	0
0	0	1	0	0	0
0	1	0	1	1	0
0	1	1	0	1	0
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	1	1

$$F = x * (y + z')$$

Conversion: Your Turn!

- Conversion: Boolean Equation to Truth Table



Conversion: Your Turn!

- Conversion: Boolean Equation to Truth Table

x	y	z	x'	y * z	F = x' + (y * z)
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	0	1	1

$$F = x' + (y * z)$$

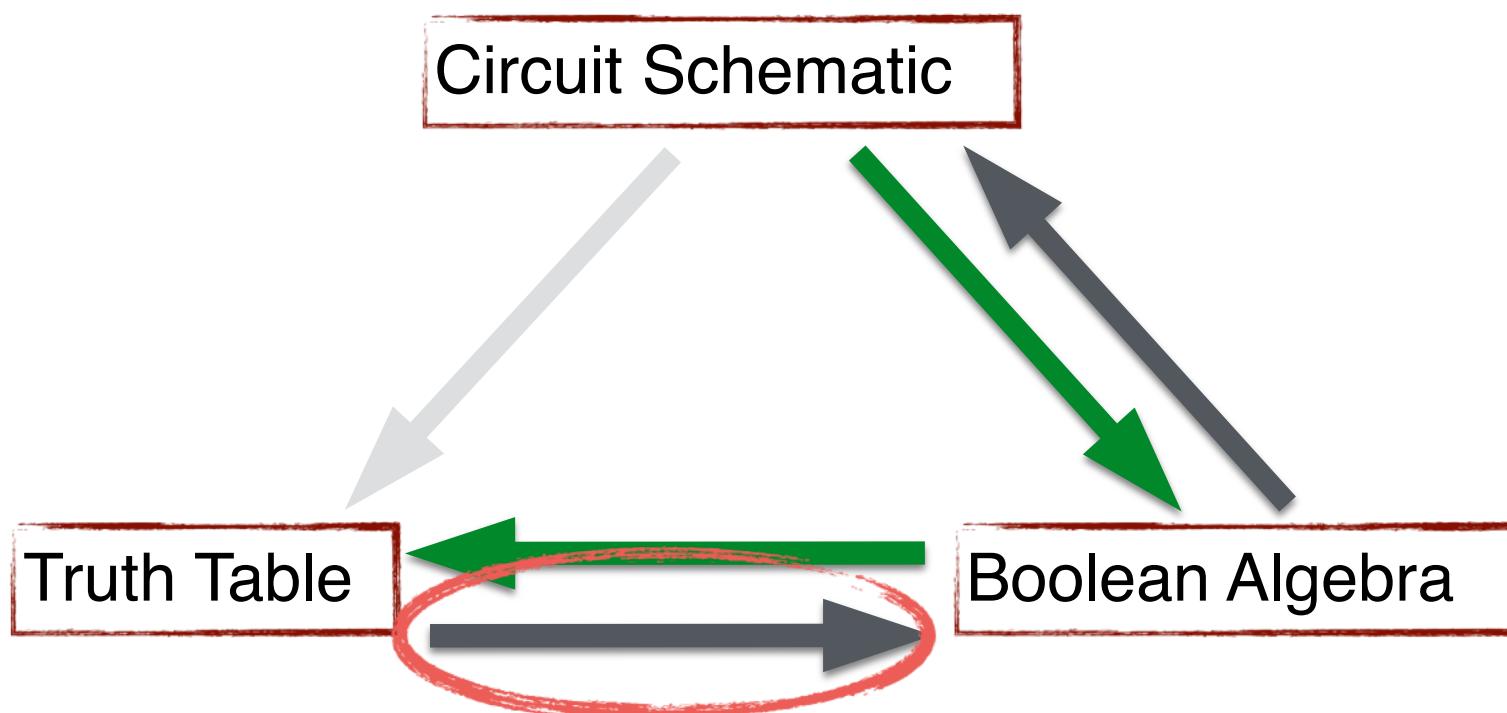
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Computer Systems Fundamentals I

Topics

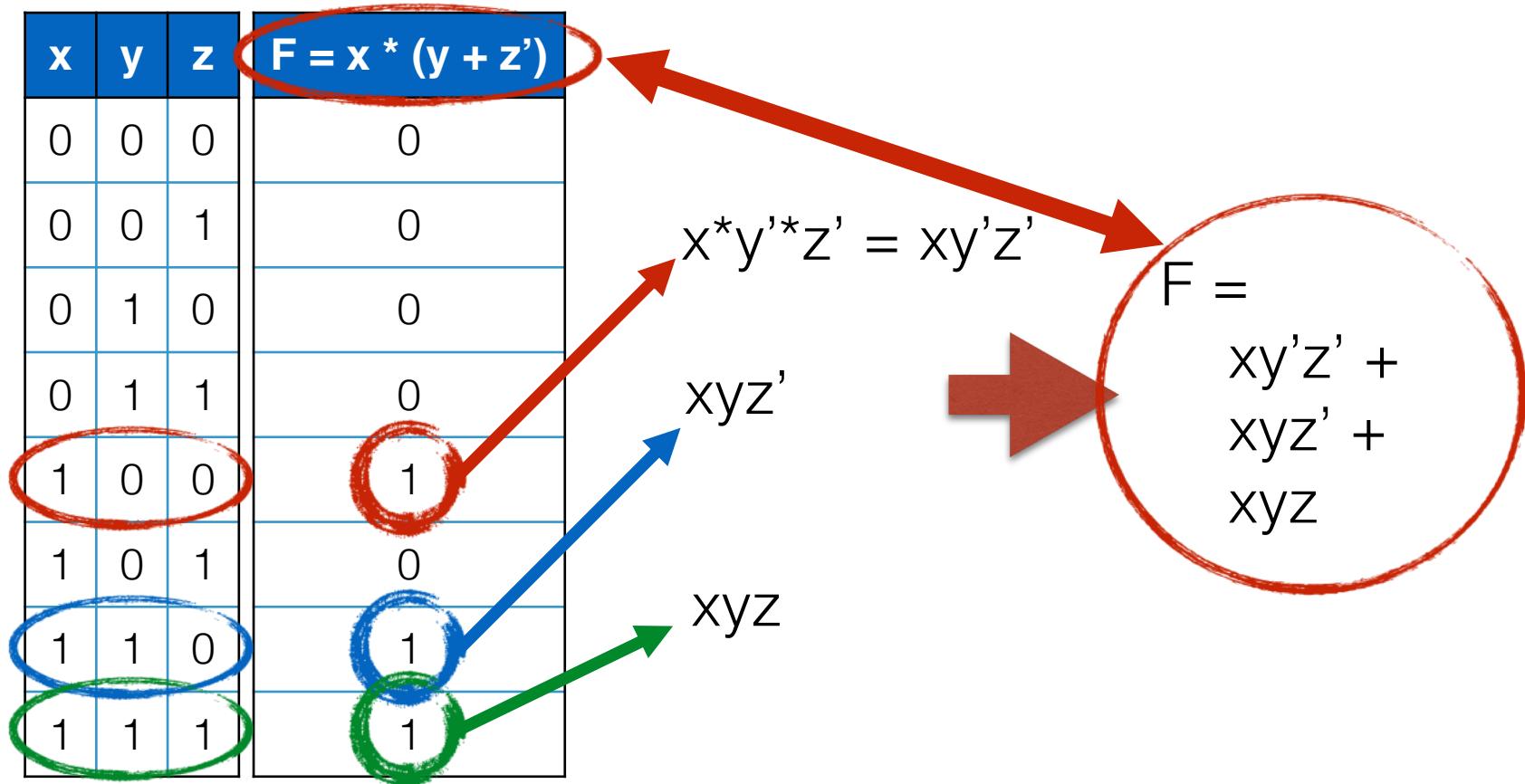
- Circuit Conversion Part 2

Converting Representations



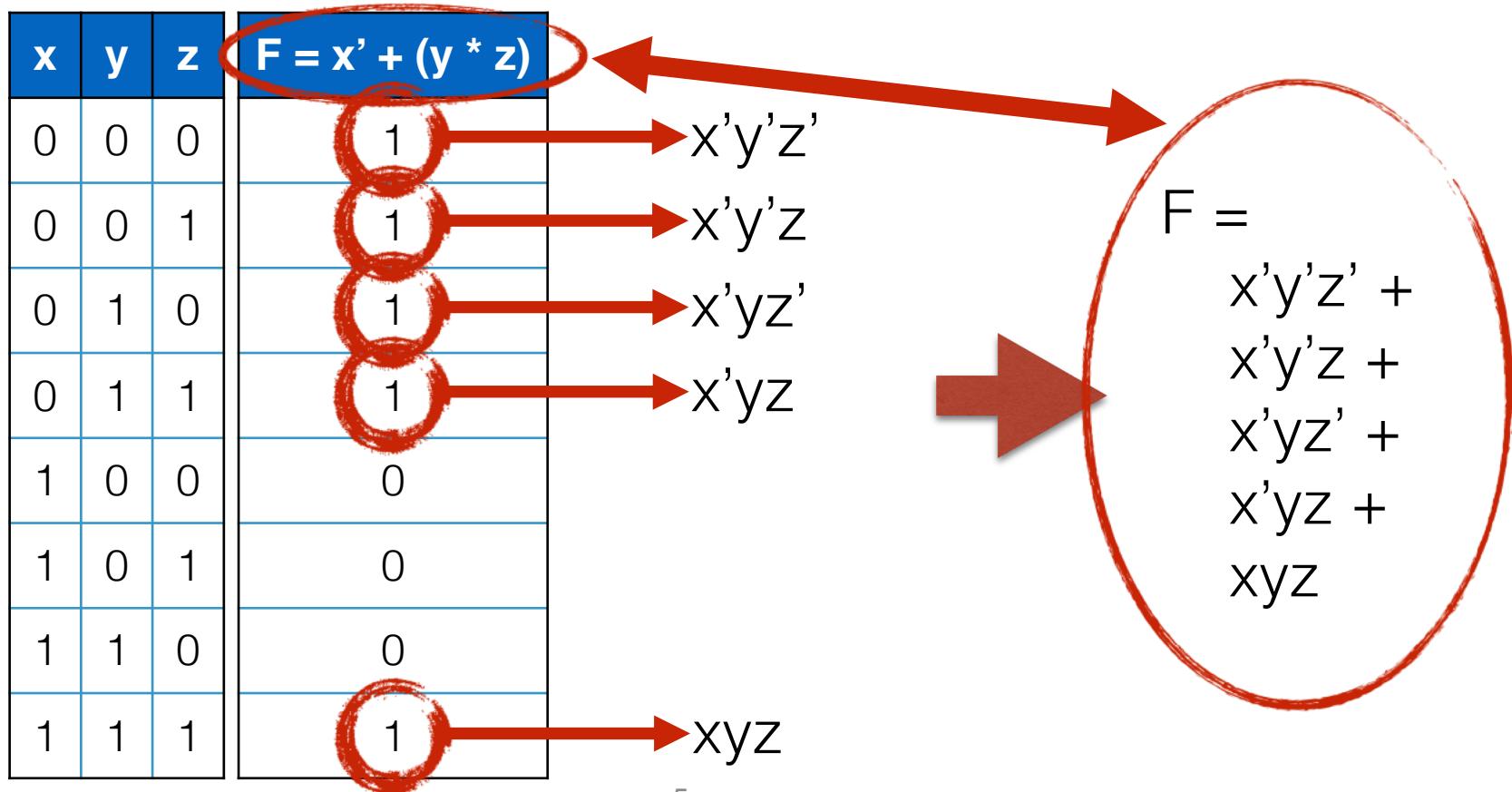
Conversion

- Conversion: Truth Table to Boolean Equation

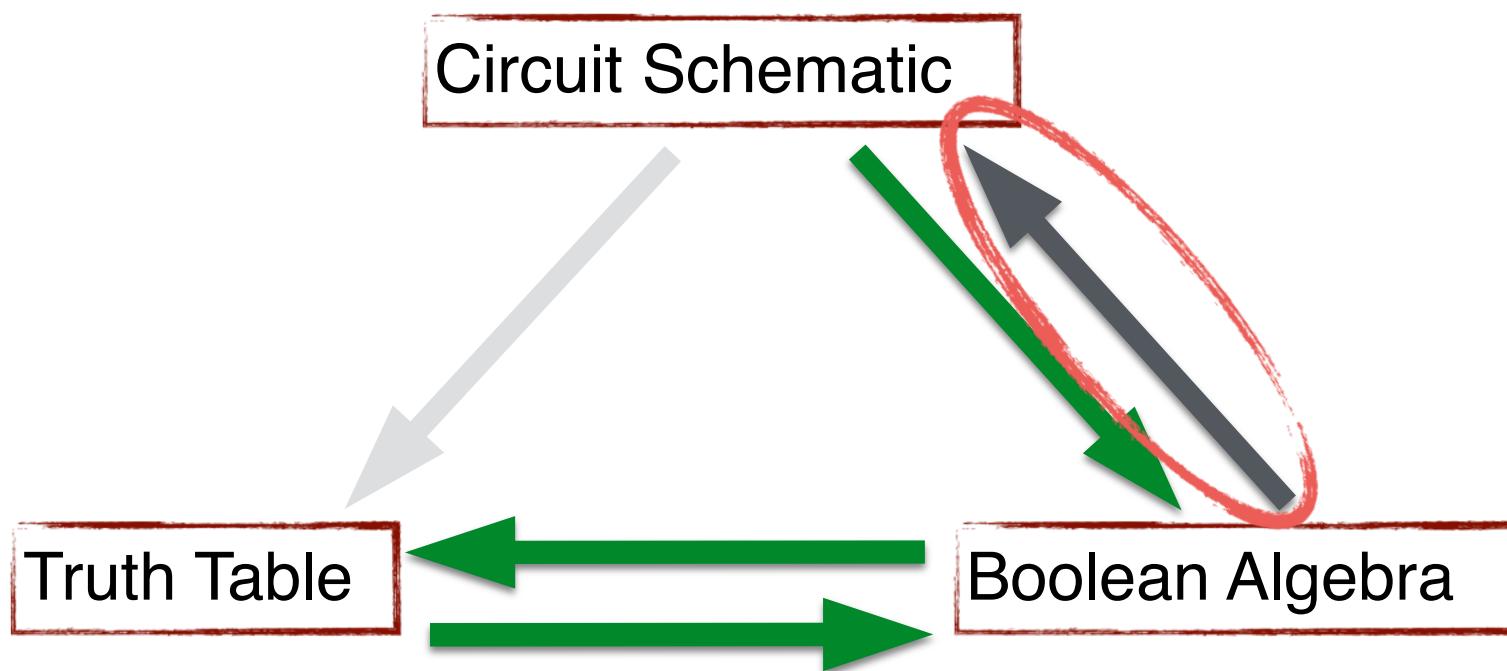


Conversion: Your Turn!

- Conversion: Truth Table to Boolean Equation



Converting Representations



Conversion

- Conversion: Boolean Algebra to Circuit Schematic

$$F = xy'z' + xyz' + xyz$$

Logic Gates

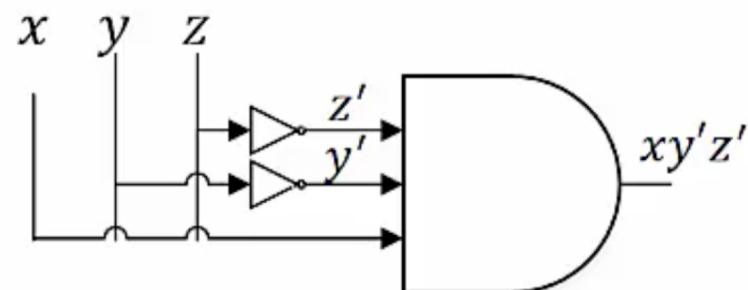
- Logic Gates with more than 2 inputs
 - AND: only 1 if ALL inputs 1
 - OR: only 0 if ALL input 0

x	y	z	F (AND)	x	y	z	F (OR)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	1
0	1	0	0	0	1	0	1
0	1	1	0	0	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	0	1	1	0	1
1	1	1	1	1	1	1	1

Conversion

- Conversion: Boolean Algebra to Circuit Schematic

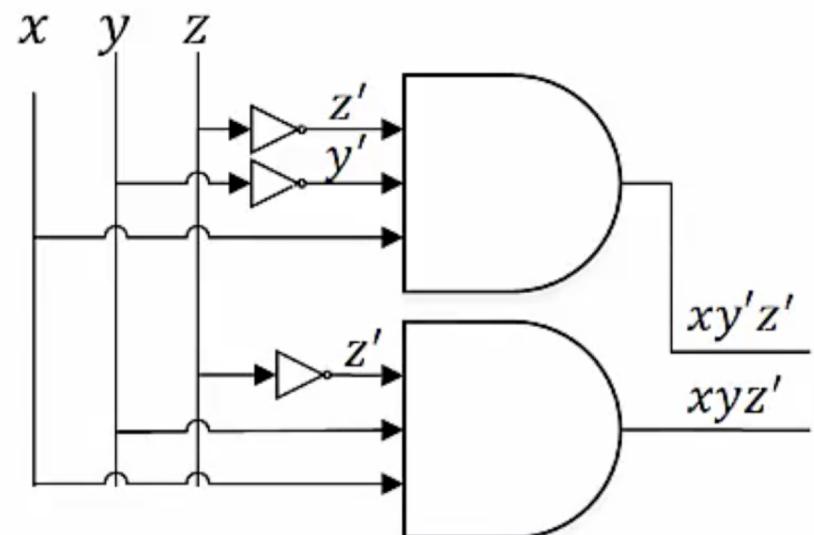
$$F = \mathbf{xy'z'} + xyz' + xyz$$



Conversion

- Conversion: Boolean Algebra to Circuit Schematic

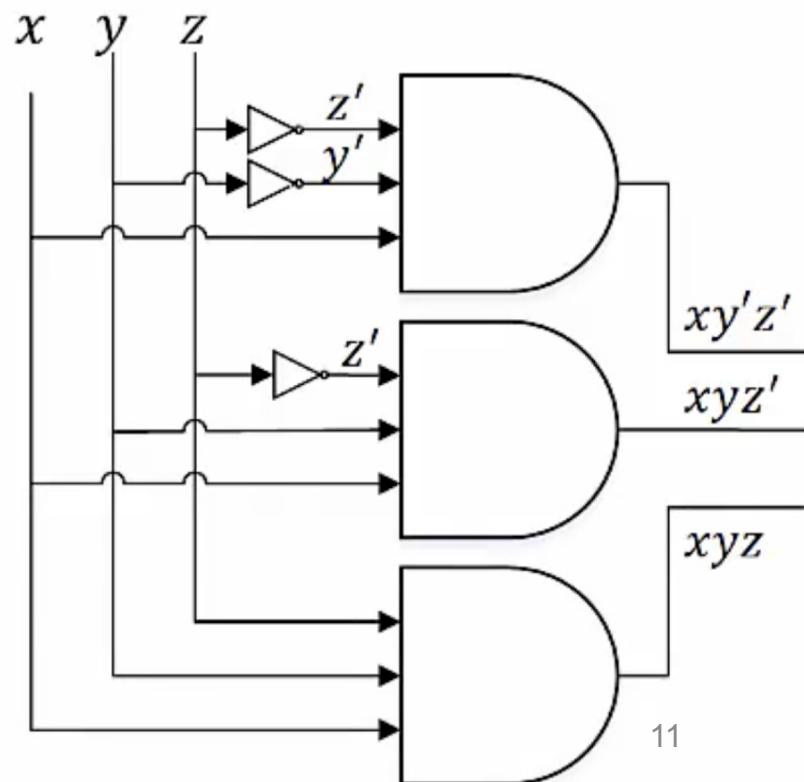
$$F = xy'z' + \mathbf{xyz'} + xyz$$



Conversion

- Conversion: Boolean Algebra to Circuit Schematic

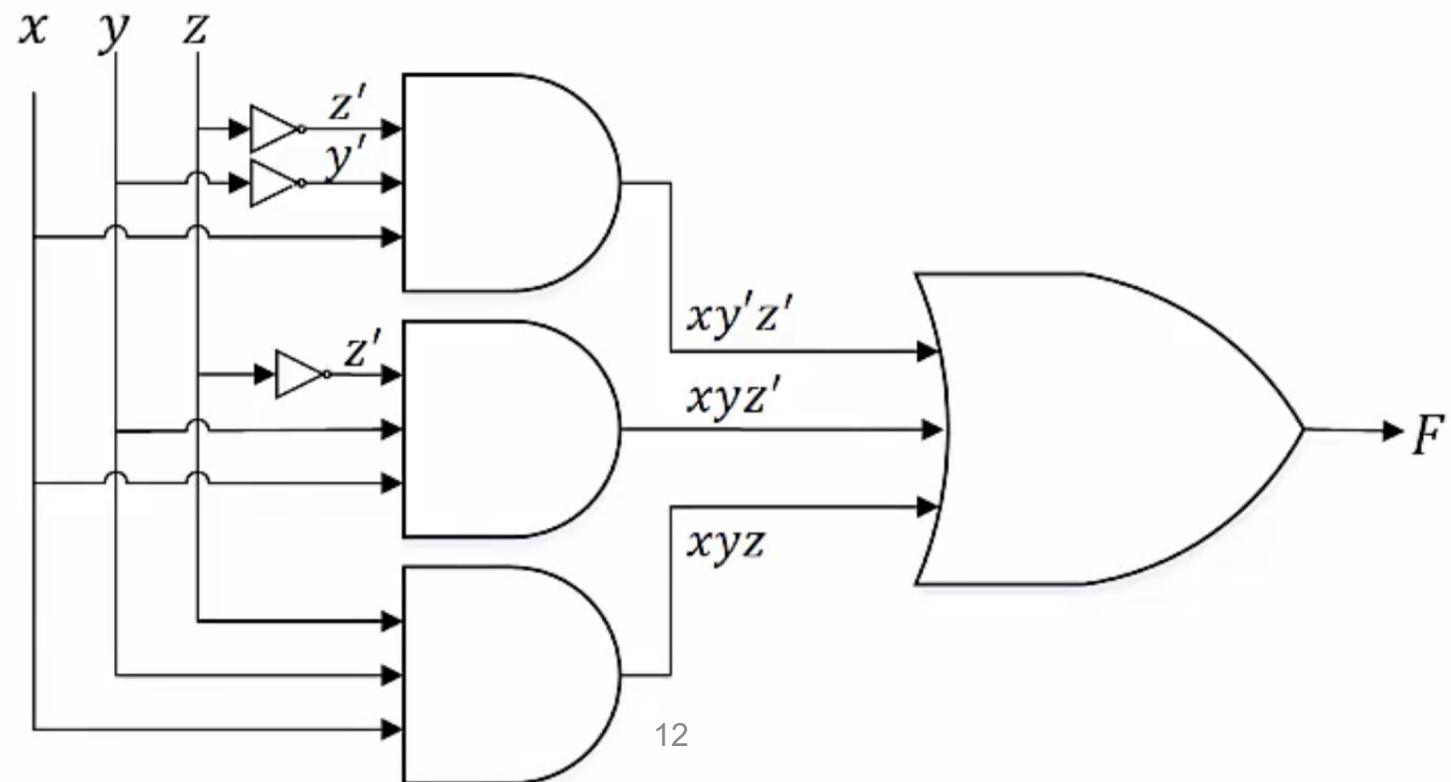
$$F = xy'z' + xyz' + \mathbf{xyz}$$



Conversion

- Conversion: Boolean Algebra to Circuit Schematic

$$F = xy'z' + xyz' + xyz$$



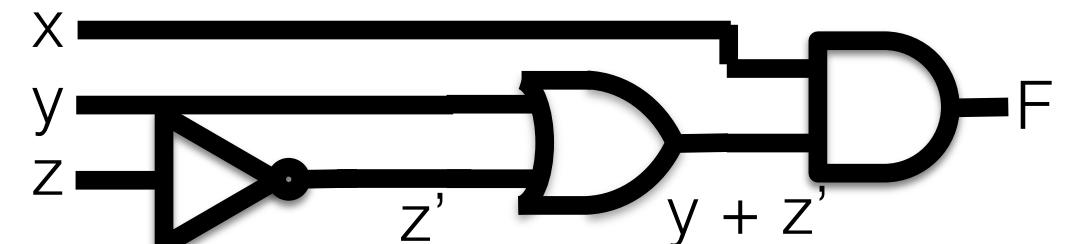
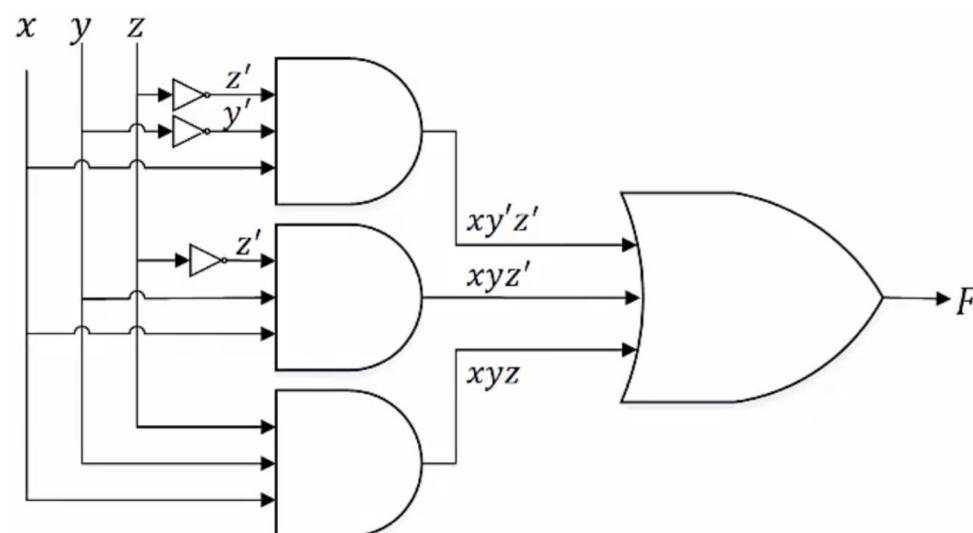
Conversion

- Conversion: Boolean Algebra to Circuit Schematic
 - Equivalence can be shown with Boolean Algebra
 - Alternative: Create and compare Truth Tables

$$F = xy'z' + xyz' + xyz$$

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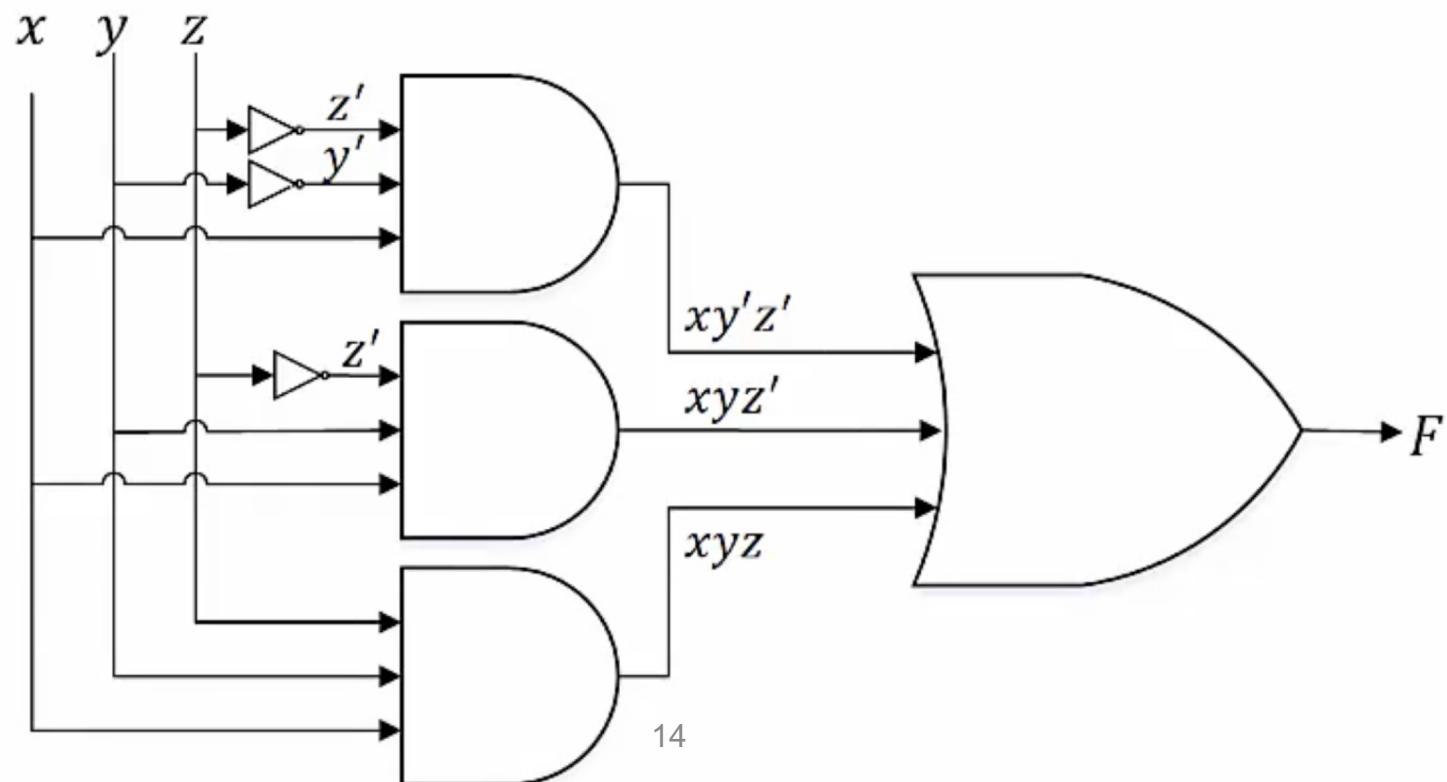
$$F = x(y + z')$$



Conversion: Your Turn!

- Create Truth Table for this circuit schematic

$$F = xy'z' + xyz' + xyz$$



Conversion: Your Turn!

- Create Truth Table for this circuit schematic

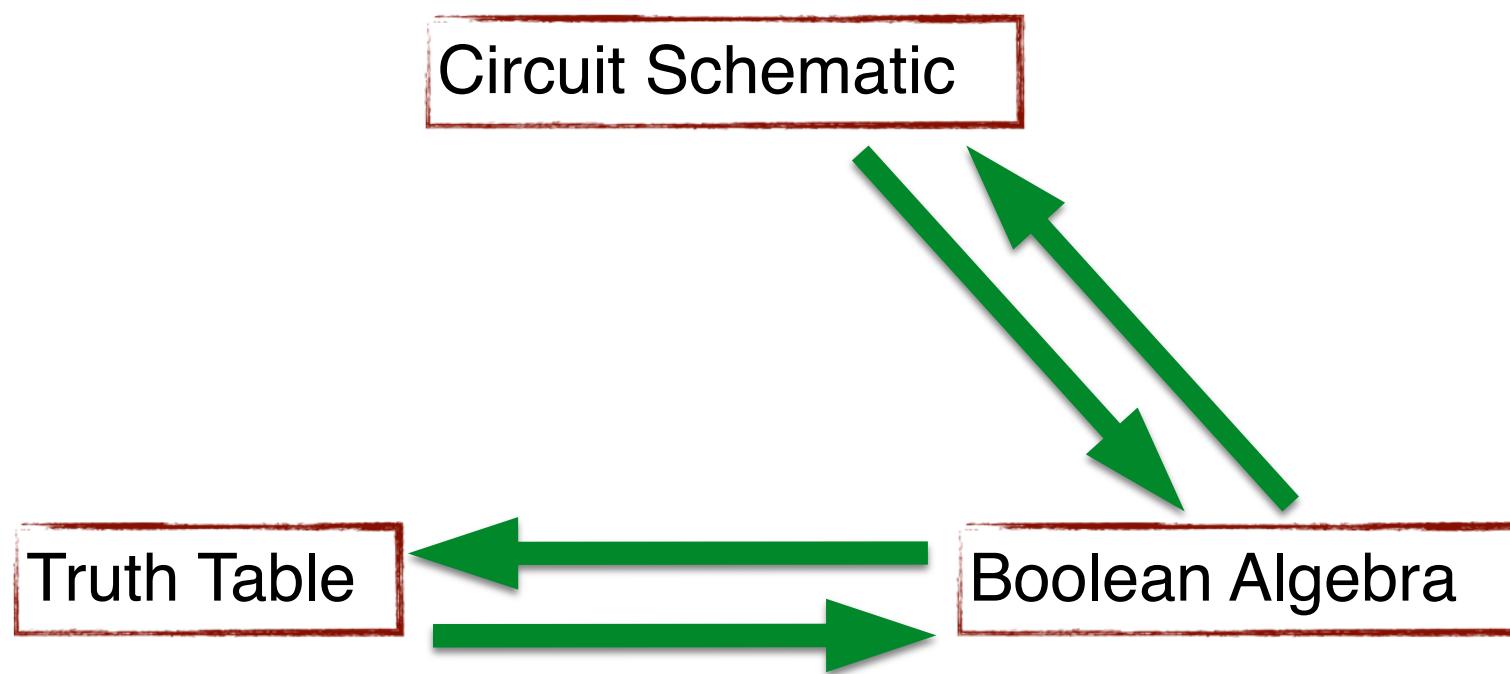
$$F = xy'z' + xyz' + xyz$$

x	y	z	y'	z'	xy'z'	xyz'	xyz	F
0	0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0
0	1	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0	0
1	0	0	1	1	1	0	0	1
1	0	1	1	0	0	0	0	0
1	1	0	0	1	0	1	0	1
1	1	1	0	0	0	0	1	1

=

$F = x^* (y + z')$
0
0
0
0
1
0
1
1

Converting Representations



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Computer Systems Fundamentals I

Topics

- Boolean Algebra Part 1

Boolean Algebra

- Can be used to:
 - Verify that two different equations describe same behavior
 - Simplify Boolean equations
 - Make it easier to create circuits and truth tables from it

Prove Equivalence

- Converting circuit to truth table & back two different Boolean equations resulted:
 - $F = x * (y + z')$
 - $F = x*y'*z' + x*y*z' + x*y*z$
- Proving equivalence with Boolean algebra:

$$F = x*y'*z' + x*y*z' + x*y*z$$

$$F = x * (y'*z' + y*z' + y*z) \quad (distributive\ property)$$

Distributivity

- Distributive property (of AND over OR)

$$\rightarrow a * (b + c) = a * b + a * c$$

- Distributive property (of OR over AND)

$$\rightarrow a + (b * c) = (a + b) * (a + c)$$

Prove Equivalence

- Prove equation below is equal to: $F = x * (y + z')$

$$F = x^*y'^*z' + x^*y^*z' + x^*y^*z$$

$$F = x^* (y'^*z' + y^*z' + y^*z) \quad (distributive\ property)$$

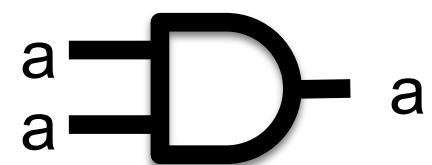
$$F = x^* (y'^*z' + y^*z' + y^*z' + y^*z) \quad (idempotence\ prop.)$$

Idempotence

- Idempotence property

$$\rightarrow a + a = a$$

$$\rightarrow a * a = a$$



Prove Equivalence

- Prove equation below is equal to: $F = x * (y + z')$

$$F = x^*y'^*z' + x^*y^*z' + x^*y^*z$$

$$F = x^* (y'^*z' + y^*z' + y^*z) \quad (\textit{distributive property})$$

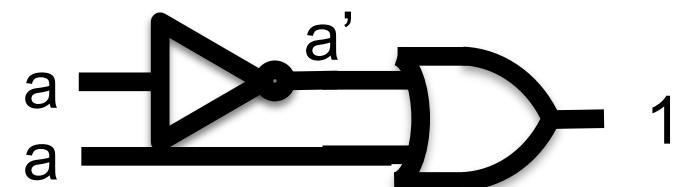
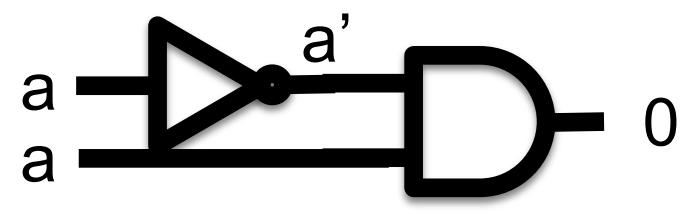
$$F = x^* (y'^*z' + y^*z' + y^*z' + y^*z) \quad (\textit{idempotence prop.})$$

$$F = x^* (z'^*(y' + y) + y^*(z' + z)) \quad (\textit{distributive property})$$

$$F = x^* (z'^*(1) + y^*(1)) \quad (\textit{complement prop.})$$

Complementation

- Complement property 1
 $\rightarrow a * a' = 0$
- Complement property 2
 $\rightarrow a + a' = 1$



Prove Equivalence

- Prove equation below is equal to: $F = x * (y + z')$

$$F = x^*y'^*z' + x^*y^*z' + x^*y^*z$$

$$F = x^* (y'^*z' + y^*z' + y^*z) \quad (\textit{distributive property})$$

$$F = x^* (y'^*z' + y^*z' + y^*z' + y^*z) \quad (\textit{idempotence prop.})$$

$$F = x^* (z'^*(y' + y) + y^*(z' + z)) \quad (\textit{distributive property})$$

$$F = x^* (z'^*(1) + y^*(1)) \quad (\textit{complement prop.})$$

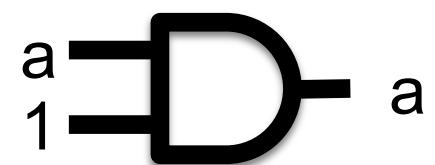
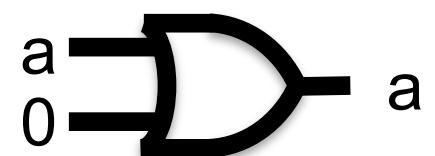
$$F = x^* (z' + y) \quad (\textit{identity property})$$

Identity

- Identity property

$$\rightarrow a + 0 = a$$

$$\rightarrow a * 1 = a$$



Prove Equivalence

- Prove equation below is equal to: $F = x * (y + z')$

$$F = x^*y'^*z' + x^*y^*z' + x^*y^*z$$

$$F = x^* (y'^*z' + y^*z' + y^*z) \quad (\textit{distributive property})$$

$$F = x^* (y'^*z' + y^*z' + y^*z' + y^*z) \quad (\textit{idempotence prop.})$$

$$F = x^* (z'^*(y' + y) + y^*(z' + z)) \quad (\textit{distributive property})$$

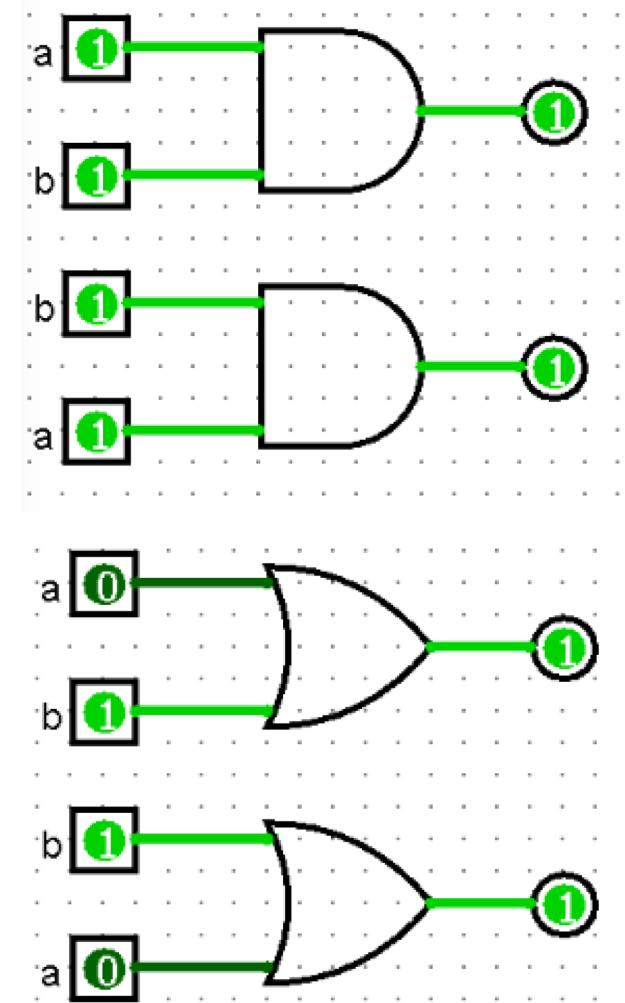
$$F = x^* (z'^*(1) + y^*(1)) \quad (\textit{complement prop.})$$

$$F = x^* (z' + y) \quad (\textit{identity property})$$

$$F = x^* (y + z') \quad (\textit{commutativity prop.})$$

Commutativity

- Commutative property
→ $a + b = b + a$
- $a * b = b * a$



Prove Equivalence

- Prove equation below is equal to: $F = x * (y + z')$

Equivalent!

$$F = x^*y'^*z' + x^*y^*z' + x^*y^*z$$

$$F = x^*(y'^*z' + y^*z' + y^*z) \quad (\textit{distributive property})$$

$$F = x^*(y'^*z' + y^*z' + y^*z' + y^*z) \quad (\textit{idempotence prop.})$$

$$F = x^*(z'^*(y' + y) + y^*(z' + z)) \quad (\textit{distributive property})$$

$$F = x^*(z'^*(1) + y^*(1)) \quad (\textit{complement prop.})$$

$$F = x^*(z' + y) \quad (\textit{identity property})$$

$$F = x^*(y + z') \quad (\textit{commutativity prop.})$$

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Computer Systems Fundamentals I

Topics

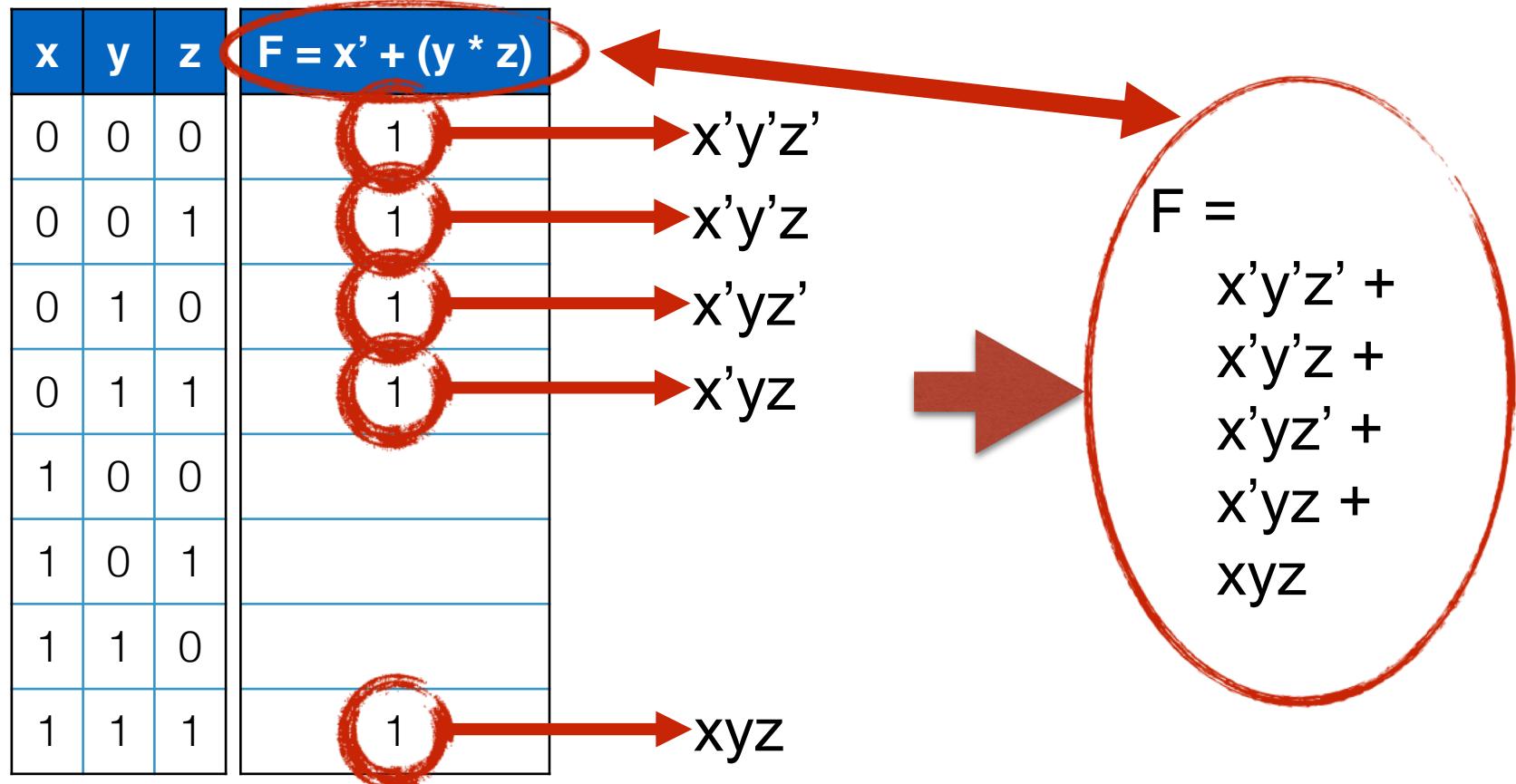
- Boolean Algebra Part 2

Boolean Algebra

- Covered laws so far:

- Distributivity
- Idempotence
- Complementation
- Identity
- Commutativity

Boolean Algebra



Boolean Algebra

- Making sure equation is equal to: $F = x' + (y^* z)$

$$F = x''^* z' + x''^* y^* z + x''^* y^* z' + x''^* y^* z + x^* y^* z$$

$$F = x'^*(y''^* z' + y''^* z + y^* z' + y^* z) + x^* y^* z \quad (\textit{distributive property})$$

$$F = x''^* (y'^*(z' + z) + y^*(z' + z)) + x^* y^* z \quad (\textit{distributive property})$$

$$F = x''^* (y'^*(1) + y^*(1)) + x^* y^* z \quad (\textit{complement property})$$

$$F = x''^* (y' + y) + x^* y^* z \quad (\textit{identity property})$$

$$F = x''^* (1) + x^* y^* z \quad (\textit{complement property})$$

$$F = x' + x^* y^* z \quad (\textit{identity property})$$

$$F = (x' + x)^* (x' + y)^* (x' + z) \quad (\textit{distributivity property})$$

$$F = (1)^* (x' + y)^* (x' + z) \quad (\textit{complement property})$$

$$F = (x' + y)^* (x' + z) \quad (\textit{identity property})$$

$$F = (x' + y)^* x' + (x' + y)^* z \quad (\textit{distributivity property})$$

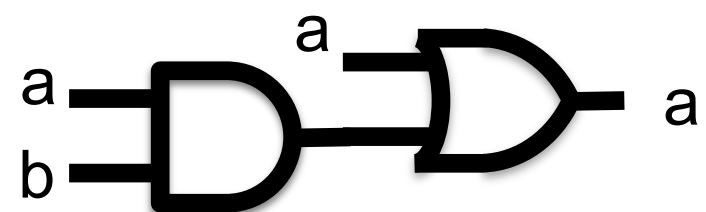
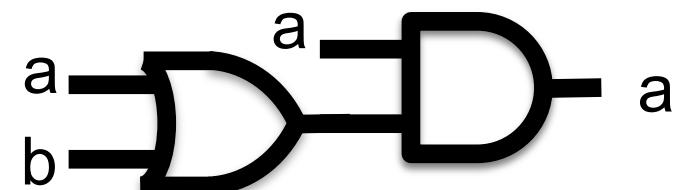
$$F = x' + (x' + y)^* z \quad (\textit{absorption property})$$

Absorption

- Absorption property

$$\rightarrow a * (a + b) = a$$

$$\rightarrow a + (a * b) = a$$



Boolean Algebra

- Making sure equation is equal to: $F = x' + (y^* z)$

$$F = x''^*z' + x''^*y^*z + x''^*y^*z' + x''^*y^*z + x^*y^*z$$

$$F = \cancel{x}''^*(y''^*z' + y''^*z + y^*z' + y^*z) + x^*y^*z$$

$$F = x''^*(\cancel{y}''^*(z' + z) + \cancel{y}^*(z' + z)) + x^*y^*z$$

$$F = x''^*(y''^*(1) + y^*(1)) + x^*y^*z$$

$$F = x''^*(\cancel{y}^* + \cancel{y}) + x^*y^*z$$

$$F = x''^*(1) + x^*y^*z$$

$$F = \cancel{x}' + x^*y^*z$$

$$F = (\cancel{x}' + x)^* (\cancel{x}' + y)^* (\cancel{x}' + z)$$

$$F = (1)^* (x' + y)^* (x' + z)$$

$$F = (\cancel{x}' + y)^* (x' + z)$$

$$F = (\cancel{x}' + y)^* x' + (\cancel{x}' + y)^* z$$

$$F = \cancel{x}' + (x' + y)^* z$$

$$F = x' + x^*\cancel{z} + y^*\cancel{z}$$

$$F = \cancel{x}' + (y^* z)$$

(distributive property)

(distributive property)

(complement property)

(identity property)

(complement property)

(identity property)

(distributivity property)

(complement property)

(identity property)

(distributivity property)

(absorption property)

(distributivity property)

(absorption property)

Boolean Algebra Laws

- Covered
 - Distributivity
 - Idempotence
 - Complementation
 - Identity
 - Commutativity
 - Absorption
- Remaining:
 - De Morgan
 - Associativity
 - Double Negation
 - Annihilator

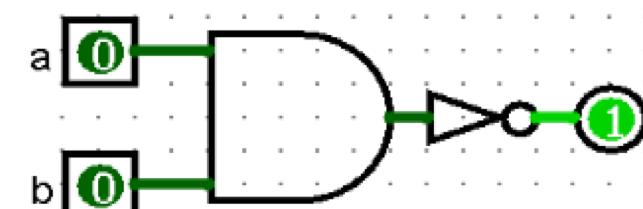
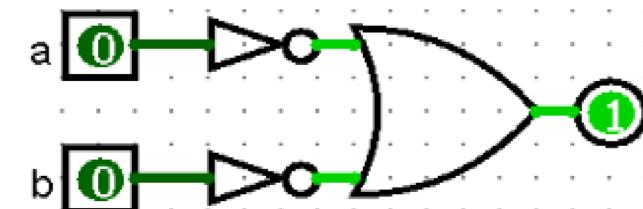
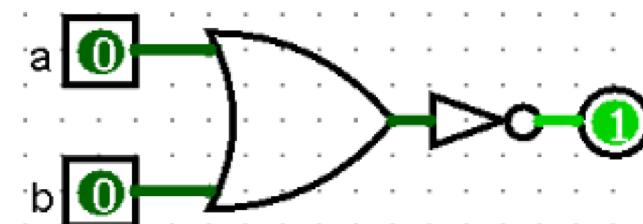
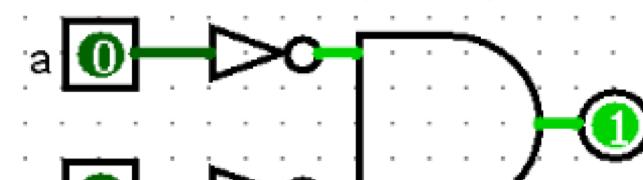
De Morgan

- De Morgan 1:

$$\rightarrow a' * b' = (a + b)'$$

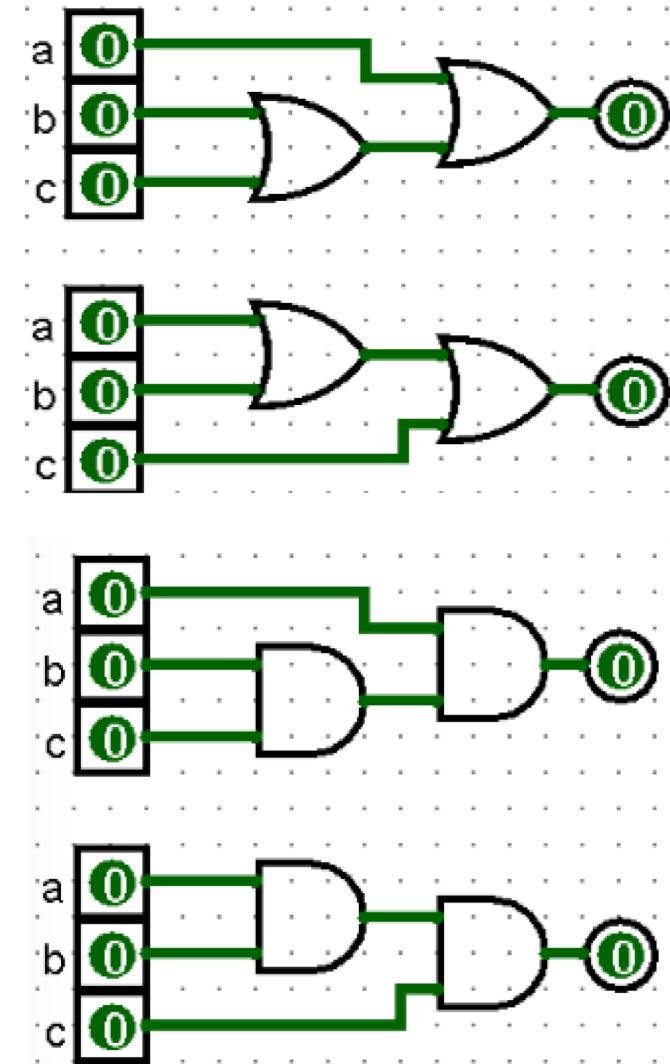
- De Morgan 2:

$$\rightarrow a' + b' = (a * b)'$$



Associativity

- Associativity of OR:
→ $a + (b+c) = (a+b)+c$
- Associativity of AND:
→ $a * (b*c) = (a*b)*c$



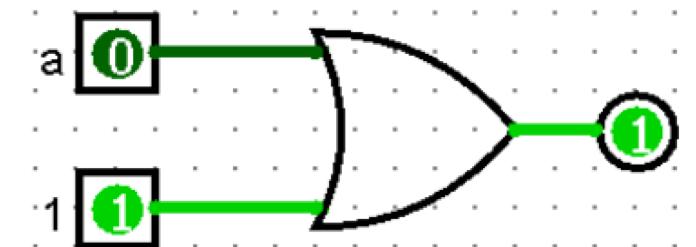
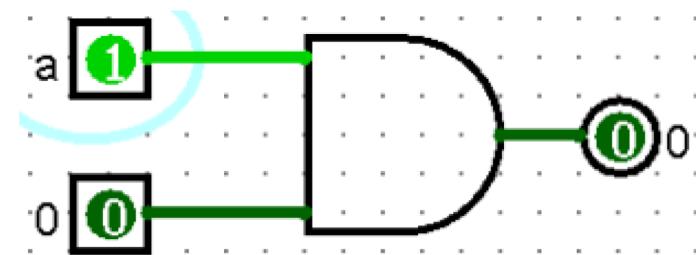
Double Negation

$$\rightarrow (a')' = a$$



Annihilator

- Annihilator for AND:
→ $a^*0 = 0$
- Annihilator for OR:
→ $a+1 = 1$



Boolean Algebra

- Making sure equation is equal to: $F = x' + (y * z)$

$$F = x''y''z' + x''y''z + x''y'z' + x''y'z + x'y'z$$

$$F = x''(y''z' + y''z + y'z' + y'z) + x'y'z$$

$$F = x''(y''(z' + z) + y'(z' + z)) + x'y'z$$

$$F = x''(y''(1) + y'(1)) + x'y'z$$

$$F = x''(y' + y) + x'y'z$$

$$F = x''(1) + x'y'z$$

$$F = x' + x'y'z$$

$$F = (x' + x) * (x' + y) * (x' + z)$$

$$F = (1) * (x' + y) * (x' + z)$$

$$F = (x' + y) * (x' + z)$$

$$F = (x' + y) * x' + (x' + y) * z$$

$$F = x' + (x' + y) * z$$

$$F = x' + x'*z + y*z$$

$$F = x' + (y * z)$$

(distributive property)

(distributive property)

(complement property)

(identity property)

(complement property)

(identity property)

(distributivity property)

(complement property)

(identity property)

(distributivity property)

(absorption property)

(distributivity property)

(absorption property)

13 Steps!

Try to do it
in fewer
steps

Boolean Algebra

- Making sure equation is equal to: $F = x' + (y * z)$

Steps	$F = x'*y'*z' + x'*y'*z + x'*y*z' + x*y*z + x*y*z$	Original
1	$F = x'*(y'*(z'+z) + y*(z'+z)) + x*y*z$	Distributive
2	$F = x'*(1) + x*y*z$	Compliment
3	$F = x' + x*y*z$	Identity
4	$F = (x' + x) * (x' + y) * (x' + z)$	Distributive
5	$F = 1 * (x' + y) * (x' + z)$	Compliment
6	$F = ((x' + y)*(x' + z))$	Identity
7	$F = x' + (y*z)$	Distributive

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Computer Systems Fundamentals I

Topics

- Merging Truth Tables

Truth Table: “Don’t Care” Symbol

- In some situations there will be a row in a truth table where an input (or output) doesn’t matter
 - In these instances it is common to use a don’t care symbol
 - Often an “x”, sometimes “-“
 - Best practice: define symbol you use
 - We will use “x”

Truth Table: “Don’t Care” Symbol

- Suppose we want a circuit where the output is 1 if at least one input is 1
- Question:** Truth table complete? Does it cover all possible input combinations?

x	y	z	F
x	x	1	1
x	1	x	1
1	x	x	1
0	0	0	0

This row represents the following input combinations for inputs xyz:

- 001
- 011
- 101
- 111

Truth Table: “Don’t Care” Symbol

- Find two rows which differ at one input
- Replace this input with *don’t care* symbol in one row
- Remove other row

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
1	0	1	1
...			

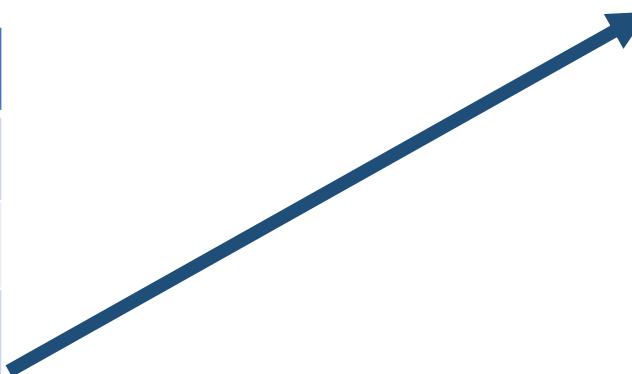
Diagram illustrating the simplification of a truth table using a don't care symbol:

- The original truth table has four rows.
- Rows 1 and 3 are highlighted in red, and their second column (b) is circled in red. This indicates they differ from Row 2 at input b.
- Row 2 is highlighted in green, and its first column (a) is circled in green. This indicates it differs from Row 4 at input a.
- Arrows point from the circled inputs (b in Row 1/3 and a in Row 2) to the simplified truth table.
- The simplified truth table shows:
 - Row 1: a=0, b=x, c=0, F=0
 - Row 2: a=x, b=0, c=1, F=1
 - Rows 3 and 4 are omitted.

Truth Table: “Don’t Care” Symbol

- Find two rows which differ at one input
- Replace this input with *don’t care* symbol in one row
- Remove other row

a	b	c	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



a	b	c	F
0	0	x	1
0	1	x	0
1	0	x	1
1	1	0	0
1	1	1	1

a	b	c	F
x	0	x	1
0	1	x	0
1	1	0	0
1	1	1	1

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Computer Systems Fundamentals I

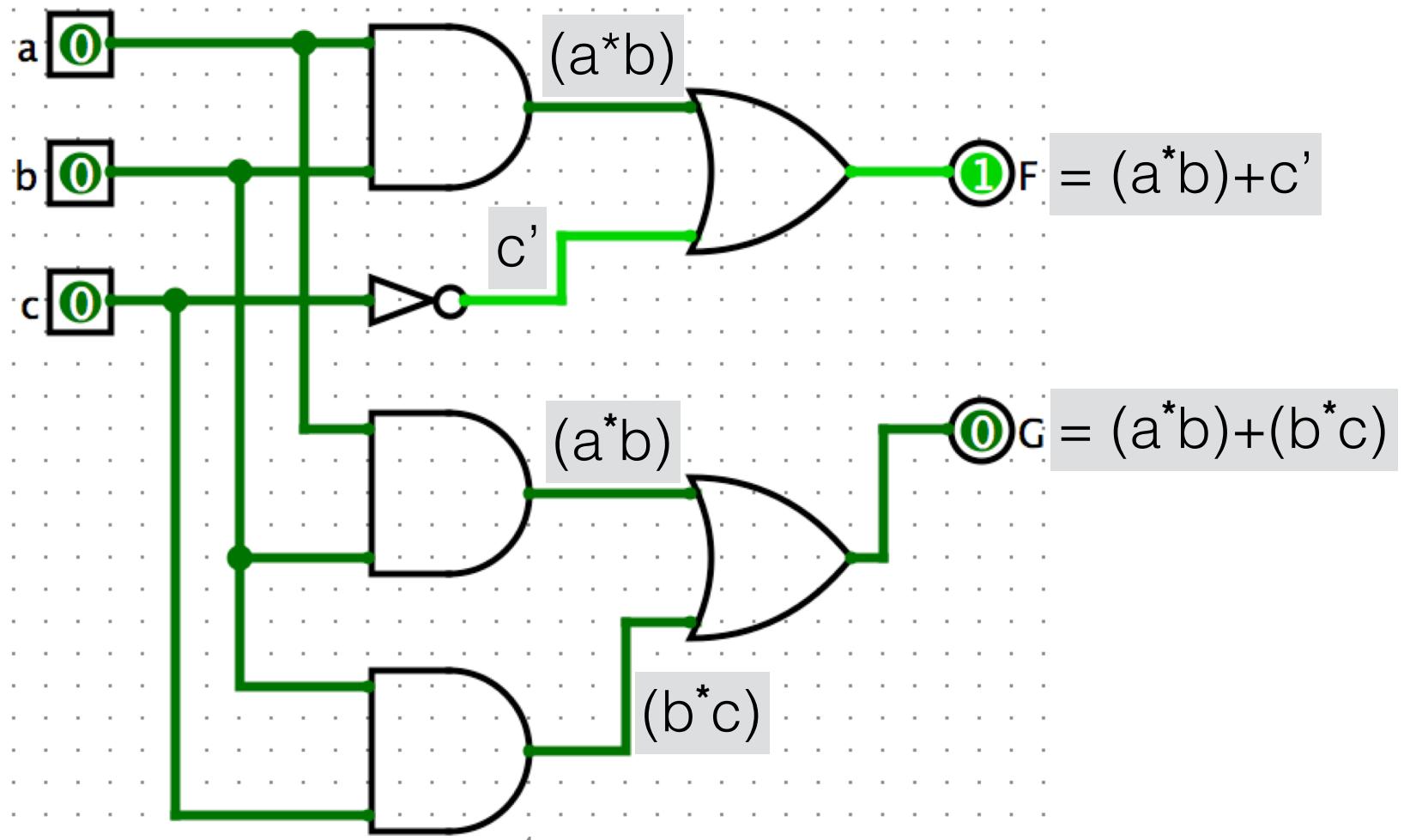
Topics

- Multiple Output Circuits
- Boolean Equations Terminology
- Minterm

Multiple Outputs

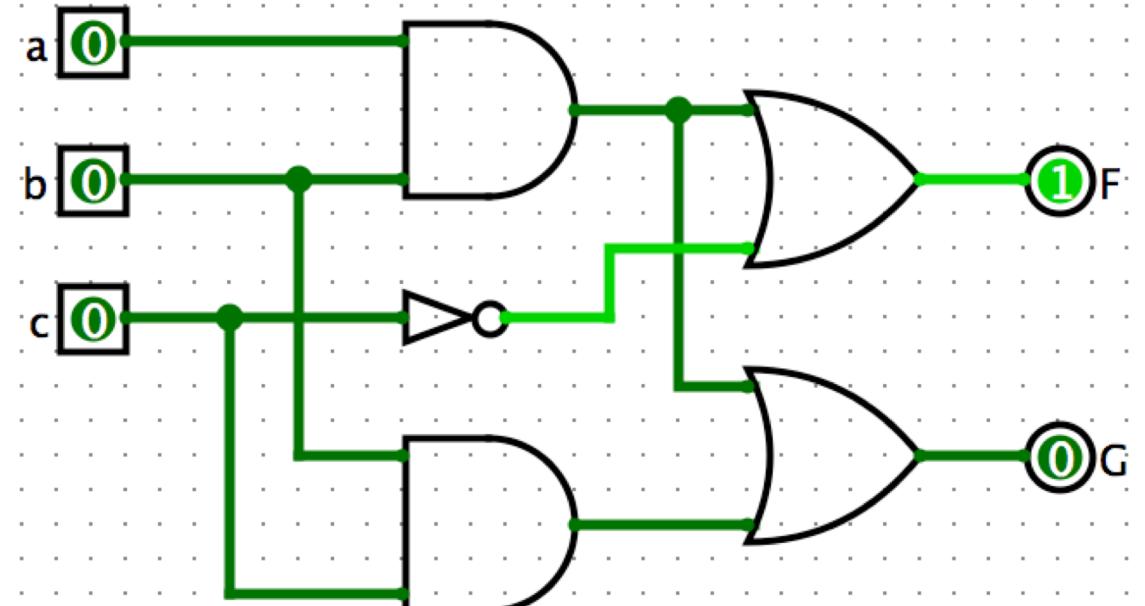
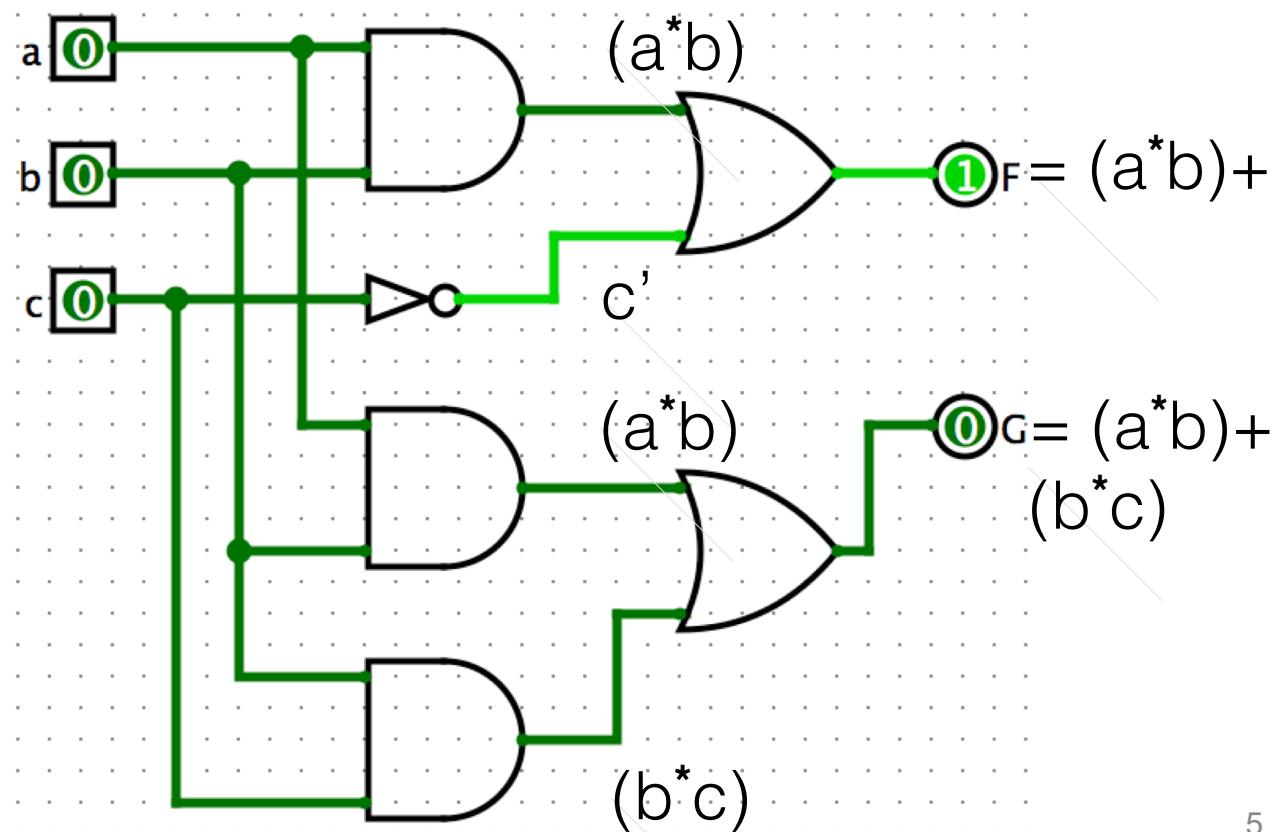
- Circuits can have more than one output
 - Multiple Boolean equations can describe behavior of circuits with multiple outputs
 - Each output can be described with one equation

Multiple Outputs

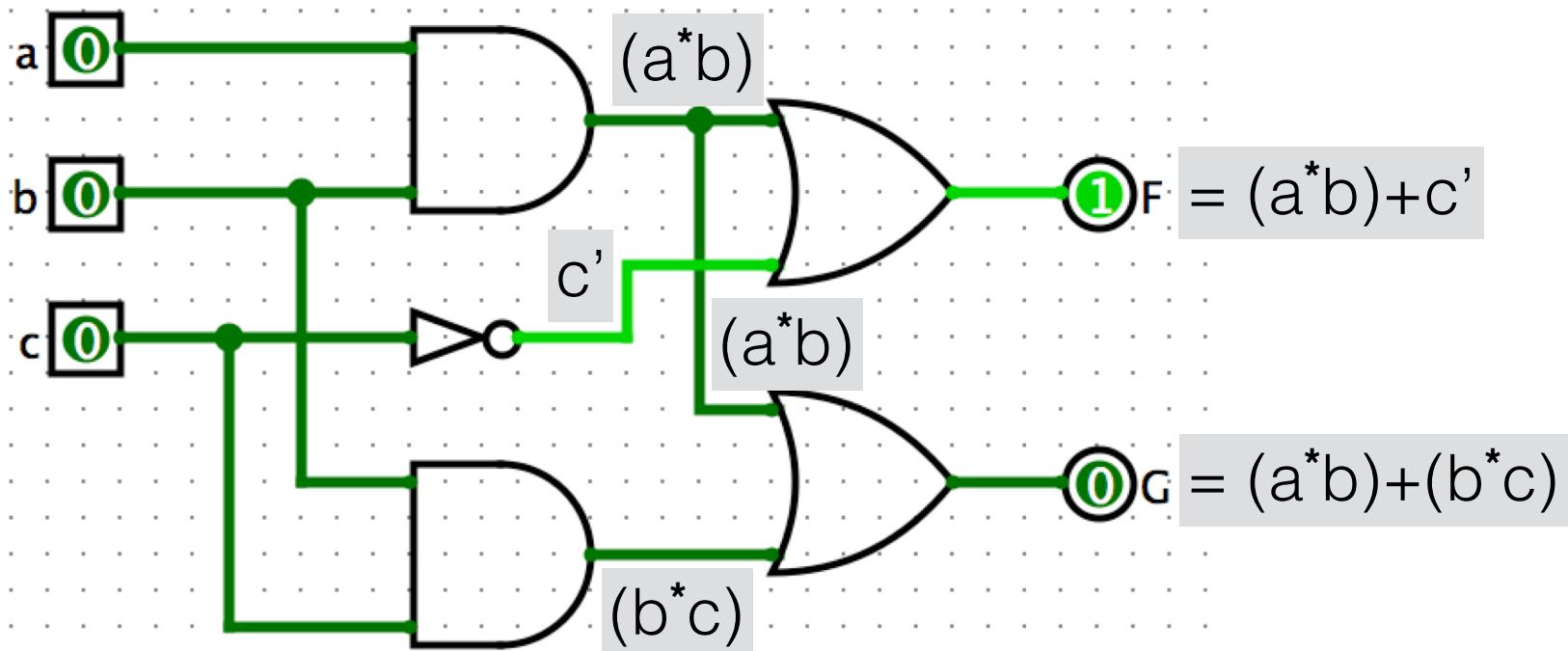


Multiple Outputs

- Do these two circuits have equal behavior?
 - Check if equal: Compare Boolean equations or truth tables



Multiple Outputs



Identical to Boolean equations of first circuit, thus same behavior

Boolean Equations - Terminology

- Example equation: $F(a,b,c) = a'bc + abc' + ab + c$
- **Variable**
 - Represents a value (0 or 1)
 - Three variables: a, b, and c
- **Literal**
 - Appearance of a variable, in true or complemented form
 - Nine literals: a' , b , c , a , b , c' , a , b , and c
- **Product term**
 - Product of literals (AND gate)
 - Four product terms: $a'bc$, abc' , ab , c
- **Sum-of-products**
 - Equation written as OR of product terms only
 - Above equation is in sum-of-products form. “ $F = (a+b)c + d$ ” is not.

Checking Equivalency of Boolean Equations

Sum of Minterms/Canonical Form:

- **Minterm**: product term with every function literal appearing exactly once, in true or complemented form (*Vahid – Digital Design Slides*)

Checking Equivalency

- Simplify to Sum of Products
- Expand to Sum of Minterms

Example

- Are these boolean equations equivalent?
 $F(a,b) = ab' + a'$ vs $F(a,b) = ab' + a'b + a'b'$
 1. $ab' + a' * 1$ Identity
 2. $ab' + a'(b + b')$ Complement
 3. $ab' + a'b + a'b'$ Distribute

Minterm

- If each of a circuit's input variables are combined with a logic AND in a single term, it is called a ***minterm***
- A ***minterm*** only includes AND and NOT gates
- If the whole equation consists exclusively of minterms that are combined with logic OR, this form is called ***canonical sum of minterms***

Minterm

- Conversion: Truth Table to Boolean Equation

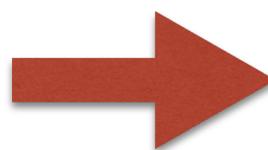
x	y	z	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

F
0
0
0
0
1
0
1
1

$xy'z'$

xyz'

xyz



$$\begin{aligned}F(x,y,z) = \\ xy'z' + \\ xyz' + \\ xyz\end{aligned}$$

Minterm

- $F(x,y,z) = \textcircled{xy'z'} + \textcircled{xyz'} + \textcircled{xyz}$
→ 3 *minterms*
 - Each term includes all inputs/function literal: x, y, z
 - Each term is a product-term (only includes AND and NOT gates)
- F is in *canonical sum of minterms* form
 - All *minterms* are combined with logical OR (sum-of-products)

Minterm

- Are the following equations in *canonical sum of minterms* form?

$$1. F(a,b,c) = abc + a'b'c + a'bc$$

$$2. F(a,b,c) = a (bc + bc')$$

$$\rightarrow F(a,b,c) = abc + abc' \text{ (distributive property)}$$

$$3. F(a,b,c) = a'b'c' + a'b + abc'$$

$$4. F(a,b,c) = a$$

$$5. F(a,b,c) = ab + bc$$

Minterm

- Can we turn them into canonical sum of minterm forms?

3. $F(a,b,c) = a'b'c' + a'b + abc'$

- $a'b$ stands for $a'bc$ and $a'bc'$ (equal to having a *don't care* for c in truth table)

- Replace non-minterms with all possible minterms

$$\rightarrow F(a,b,c) = a'b'c' + \underline{a'bc} + \underline{a'bc'} + abc'$$

4. $F(a,b,c) = a$

$$\rightarrow F(a,b,c) = abc + abc' + ab'c + ab'c'$$

5. $F(a,b,c) = ab + bc$

$$\rightarrow F(a,b,c) = abc + abc' + a'bc + abc \text{ (next: remove redundant terms)}$$

$$\rightarrow F(a,b,c) = abc + abc' + a'bc$$

SER 232

Computer Systems Fundamentals I

Topics

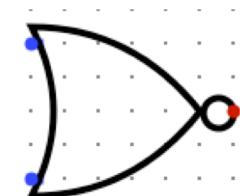
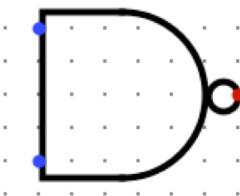
- More Logic Gates & Applications

Logic Gates

- More than the 3 basic logic gates exist
- Every other gate can be created with those 3 basic gates: AND, OR, NOT

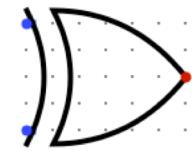
More Logic Gates

- NAND
 - Combination of AND and NOT gate: $(x \cdot y)'$
- NOR
 - Combination of OR and NOT gate: $(x + y)'$



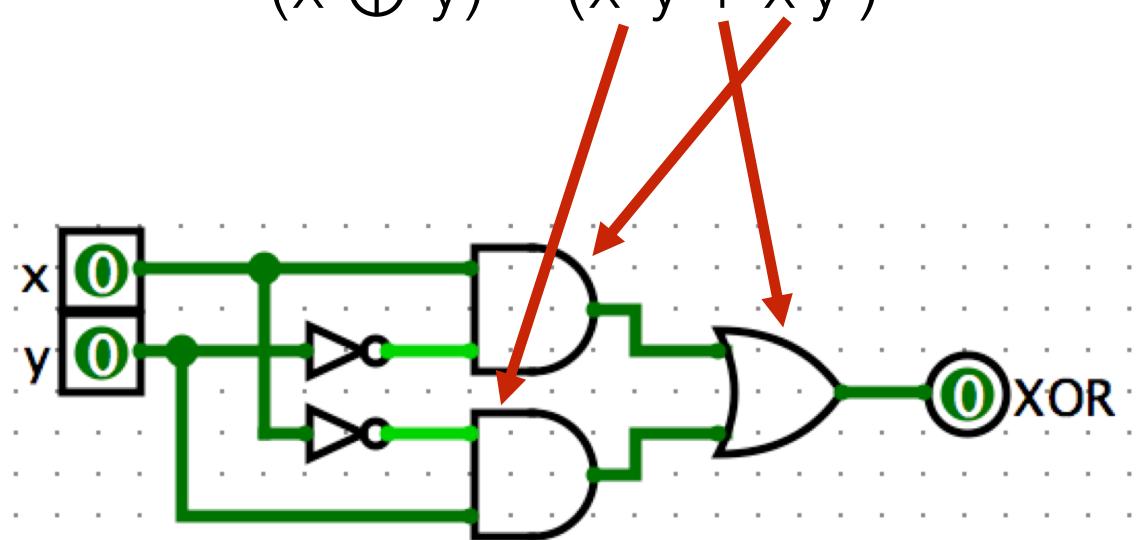
Logic Gates: XOR

- XOR
 - Symbol for boolean algebra: \oplus



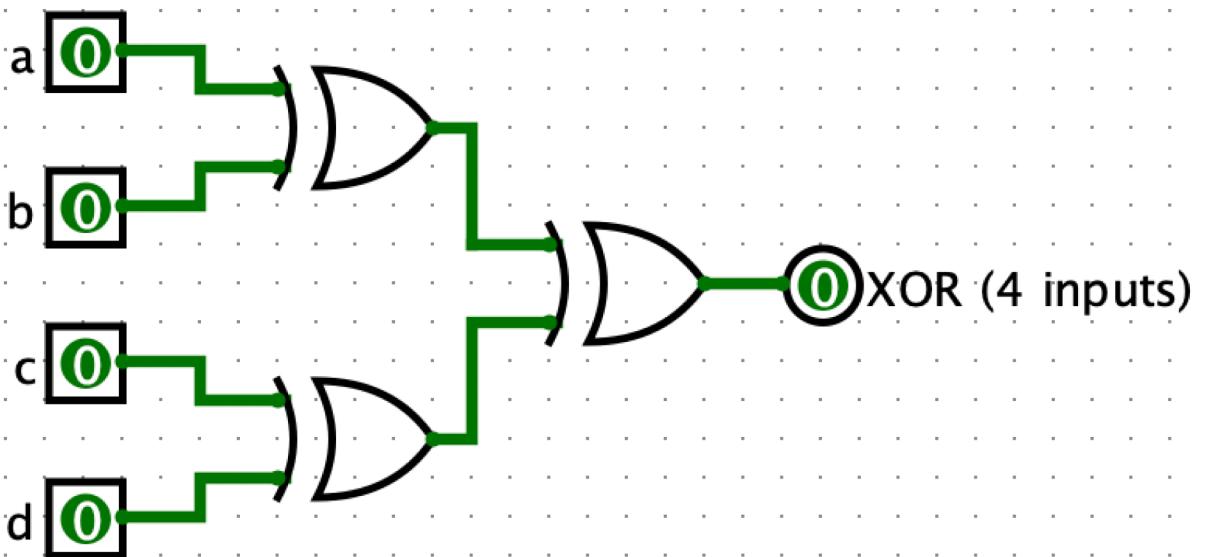
x	y	XOR
0	0	0
0	1	1
1	0	1
1	1	0

$$(x \oplus y) = (x'y + xy')$$



Logic Gates: XOR

- XOR with multiple inputs
 - XOR outputs
 - 1 if there is an odd number of 1s on the inputs
 - 0 if there is an even number of 1s on the inputs



0 XOR 1 =

a	b	c	d	XOR
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Logic Gates: XOR Applied

- A so called *parity bit* indicates if binary value has even or odd amount of 1s
 - XOR generates exactly that information
 - Can be used to:
 - ... efficiently create data redundancy to prevent data-loss during hard drive failures
 - ... simple check if a communication was erroneous (does not catch all error cases)

Logic Gates: XOR Applied

- Parity HDD combines data from two other drives with *bitwise* XOR:
 - <content HDD 1> **XOR** <content HDD2> = <content parity drive>
- Why does that help in case of HDD failures?
 - Loosing one of the three HDDs, two remaining can be used to reconstruct the data of the lost HDD
 - HDD 1 and 2 sizes must be equal
 - Parity HDD must be equal or larger than HDD 1 & 2
 - Parity HDD can secure twice the amount of data than its own size
 - If HDD 1 & 2 each have size of 6 TB, parity HDD secures 12 TB of data with only 6 TB of space

Logic Gates: XOR Applied

- Parity HDD:
 - $\text{<content HDD 1>} \text{ XOR } \text{<content HDD2>} = \text{<content parity HDD>}$
 - Failure example:
 1. $01001111 \text{ XOR } 11100001 = 10101110$
 2. HDD 2 fails and data is lost
 3. $01001111 \text{ XOR } ???????? = 10101110$
 4. Reconstruct: HDD1 **XOR** parity HDD = 11100001

Logic Gates: XOR Applied

- Check if communication was erroneous:
 - Transfer data (e.g. 8 bit) + parity bit of that data
 - Parity bit is XOR of all transferred bits (data: 11001101; parity bit: 1)
 - Receiver calculates parity bit of data and compares to sent parity bit
 - Equal: Communication most likely correct
 - Not equal: Communication was erroneous (e.g. bit “flipped” during transfer)
 - Example: Sent **11001101** + 1 but received **11001100** + 1
 - Receiver calculates parity bit of received data: parity bit = 0
 - Received parity bit (1) not equal to calculated parity bit (0) = error in communication!

More Logic Gates

- XNOR
 - Combination of XOR and NOT gate: $(x \oplus y)'$

