Galala University, Faculty of Computer Science and Engineering

CSE 131 Logic Design

Dr Gamal Fahmy

Homework 4 sol

Problem: 2-1

Demonstrate by means of truth tables the validity of the following identities:

(a) DeMorgan's theorem for three variables: (x+y+z)′ = x′y′z′ and (xyz)′=x′+y′+z′

(b) The distributive law: x+yz = (x+y)(x+z)

Solution:

(a)

x	y	z	x+y+z	(x+y+z)′
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

x′	y′	z′	x′y′z′
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

x	y	z	xyz	(xyz)′
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

x′	y′	z′	x′+y′+z′
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

(b)

x	y	z	yz	x+yz
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

x+y	x+z	(x+y)(x+z)
0	0	0
0	1	0
1	0	0
1	1	1
1	1	1
1	1	1
1	1	1
1	1	1

Problem: 2-4

Reduce the following Boolean expressions to the indicated number of literals:

(a) $A'C' + ABC + AC'$ to three literals

(b) $(x'y'+z)' + z + xy + wz$ to three literals

(c) $A'B(D'+C'D) + B(A+A'CD)$ to one literal

(d) $(A'+C)(A'+C')(A+B+C'D)$ to four literals

Solution:

(a)
$$
\begin{aligned}
A'C' + ABC + AC' &= A'C' + AC' + ABC \\
&= C'(A'+ A) + ABC \\
&= C'\cdot 1 + ABC \\
&= C' + ABC \\
&= (C'+ AB)(C'+C) && \text{[distributive]} \\
&= AB + C'
\end{aligned}
$$

(b)
$$
\begin{aligned}
(x'y'+z)' + z + xy + wz &= (x'y'+z)' + z + wz + xy \\
&= (x'y'+z)' + z(1+ w) + xy \\
&= (x'y'+z)' + z + xy \\
&= (x + y)z' + z + xy && \text{[DeMorgan]} \\
&= (z + (x + y)) \cdot (z + z') + xy && \text{[distributive]} \\
&= (z + (x + y)) \cdot 1 + xy \\
&= x + y + z + xy \\
&= x + y + z && \text{[absorption]}
\end{aligned}
$$

(c)
$$
\begin{aligned}
A'B(D' + C'D) + B(A+A'CD) &= A'BD' + \underline{A'BC'D} + AB+ \underline{A'BCD} \\
&= A'BD(C+C')+ A'BD'+ AB \\
&= A'BD+ A'BD'+ AB \\
&= A'B(D+D')+ AB \\
&= A'B+ AB \\
&= B(A'+ A) \\
&= B
\end{aligned}
$$

(d)
$$
\begin{aligned}
(A'+C)(A'+C')(A+B+C'D) &= (A'+C)(A'+C')(A+B+C'D) \\
&= (A' + CC')(A + B + C'D) \\
&= A'(A + B + C'D) \\
&= A'A + A'B + A'C'D \\
&= A'B + A'C'D \\
&= A'(B + C'D)
\end{aligned}
$$

Problem 2-5:

Find the complement of $F = x + yz$; then show that $FF' = 0$ and $F + F' = 1$

Solution:

$F = x + yz$

The dual of F is: $x \bullet (y + z)$

Complement each literal: $x' \bullet (y' + z') = F'$

$$FF' = (x + yz) \bullet (x' \bullet (y' + z')) = (xx' + x'yz) \bullet (y' + z') = x'yz \bullet (y' + z') = x'yy'z + x'yzz' = 0$$

$$F + F' = (x + yz) + (x' \bullet (y' + z')) = (x + yz + x') + (x + yz + y' + z') = (1 + yz) + (x + yz + y' + z')$$
$$= 1 + (x + yz + y' + z') = 1$$

Problem 2-8:

List the truth table of the function:

$$F = xy + xy' + y'z$$

Solution:

The truth table is:

x	y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1