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***Homework 4***

**Problem 2-1:**

**Demonstrate by means of truth tables the validity of the following identities:**

1. **DeMorgan’s theorem for three variables: (x + y + z)` = x`y`z`, and (xyz)` = x`+y`+z`**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| **(x + y + z)` == NOR**   |  |  |  |  | | --- | --- | --- | --- | | **x** | **y** | **z** | **F** | | 0 | 0 | 0 | **1** | | 0 | 0 | 1 | **0** | | 0 | 1 | 0 | **0** | | 0 | 1 | 1 | **0** | | 1 | 0 | 0 | **0** | | 1 | 0 | 1 | **0** | | 1 | 1 | 0 | **0** | | 1 | 1 | 1 | **0** |   d | **x`y`z`**   |  |  |  |  | | --- | --- | --- | --- | | **x`** | **y`** | **z`** | **F** | | 1 | 1 | 1 | **1** | | 1 | 1 | 0 | **0** | | 1 | 0 | 1 | **0** | | 1 | 0 | 0 | **0** | | 0 | 1 | 1 | **0** | | 0 | 1 | 0 | **0** | | 0 | 0 | 1 | **0** | | 0 | 0 | 0 | **0** |   d |
| **(xyz)` == NAND**   |  |  |  |  | | --- | --- | --- | --- | | **x** | **y** | **z** | **F** | | 0 | 0 | 0 | **1** | | 0 | 0 | 1 | **1** | | 0 | 1 | 0 | **1** | | 0 | 1 | 1 | **1** | | 1 | 0 | 0 | **1** | | 1 | 0 | 1 | **1** | | 1 | 1 | 0 | **1** | | 1 | 1 | 1 | **0** |   d | **x`+y`+z`**   |  |  |  |  | | --- | --- | --- | --- | | **x`** | **y`** | **z`** | **F** | | 1 | 1 | 1 | **1** | | 1 | 1 | 0 | **1** | | 1 | 0 | 1 | **1** | | 1 | 0 | 0 | **1** | | 0 | 1 | 1 | **1** | | 0 | 1 | 0 | **1** | | 0 | 0 | 1 | **1** | | 0 | 0 | 0 | **0** |   d |

1. **The distributive law: x + yz = (x + y)(x + z)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| **x+yz**   |  |  |  |  |  | | --- | --- | --- | --- | --- | | **x** | **y** | **z** | **yz** | **x+yz** | | 0 | 0 | 0 | **0** | **0** | | 0 | 0 | 1 | **0** | **0** | | 0 | 1 | 0 | **0** | **0** | | 0 | 1 | 1 | **1** | **1** | | 1 | 0 | 0 | **0** | **1** | | 1 | 0 | 1 | **0** | **1** | | 1 | 1 | 0 | **0** | **1** | | 1 | 1 | 1 | **1** | **1** |   d | **(x+y)(x+z)**   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **x** | **y** | **z** | **x+y** | **x+z** | **F** | | 0 | 0 | 0 | **0** | **0** | **0** | | 0 | 0 | 1 | **0** | **1** | **0** | | 0 | 1 | 0 | **1** | **0** | **0** | | 0 | 1 | 1 | **1** | **1** | **1** | | 1 | 0 | 0 | **1** | **1** | **1** | | 1 | 0 | 1 | **1** | **1** | **1** | | 1 | 1 | 0 | **1** | **1** | **1** | | 1 | 1 | 1 | **1** | **1** | **1** |   d |

**Problem 2-4:**

**Reduce the following Boolean expressions to the indicated number of literals:**

1. **A`C`+ABC+AC` to three literals**

*from distributive law XY+XZ = X(Y+Z)*

= C`(A`+A)+ABC *from complement law X+X` = 1*

= C`.1+ABC

= C`+ABC *from absorption law XY+X` = Y+X`*

**= C`+AB**

1. **(x`y`+z)`+z+xy+wz to three literals**

*apply DeMorgan’s theorem.*

= ((x`)`+(y`)`)z`+z+xy+wz *from involution law: (A`)` = A*

= (x+y)z`+z+xy+wz

= (x+y)z`+**z+wz**+xy *from absorption law: A+AB = A*

= (x+y)z`+z+xy *from absorption law: A`B+A = B+A*

= **z`(x+y)+z**+xy

= x+y+z+xy *from absorption law: A+AB = A*

= **x+xy**+y+z

**= x+y+z**

1. **A`B(D` + C`D) + B(A + A`CD) to one literal**

*apply absorption law: AB+A` = B+A`*

= A`B(D`+C`)+B(A+A`CD) *apply distribution*

= A`BD`+A`BC`+BA+BA`CD *apply distributive law: AB+AC = A(B+C)*

= B(**A`D`+A**)+A`BC`+BA`CD *apply absorption law: A`B+A = B+A*

= B(D`+A)+A`BC`+BA`CD *apply distributive law: AB+AC = A(B+C)*

= B(D`+A)+B(A`C`+A`CD) *apply distributive law: AB+AC = A(B+C)*

= B(D`+A)+BA`(**CD+C`**) *apply absorption law: AB+A` = B+A`*

= B(D`+A)+BA`(D+C`) *apply distribution*

= BD`+BA+BA`(D+C`) *apply distributive law: AB+AC = A(B+C)*

= BD`+B(A`(D+C`)+A) *apply absorption law: A`B+A = B+A*

= BD`+B(D+C`+A) *apply distribution*

= BD`+BD+BC`+BA *apply distributive law: AB+AC = A(B+C)*

= B(D`+D)+BC`+BA *apply complement law: A+A` = 1*

= B+BC`+BA *apply absorption law: A+AB = A*

= B+BA *apply absorption law: A+AB = A*

**= B**

1. **(A`+C)(A`+C`)(A+B+C`D) to four literals**

*apply distribution*

= (A`+C`)(A+B+C`D)A`+(A`+C`)(A+B+C`D)C *apply distribution*

= (A+B+C`D)A`A`+(A+B+C`D)A`C`+(A`+C`)(A+B+C`D)C

*apply idempotent law: AA=A*

= (A+B+C`D)A`+(A+B+C`D)A`C`+(A`+C`)(A+B+C`D)C

*apply absorption law: A+AB=A*

= (A+B+C`D)A`+(A`+C`)(A+B+C`D)C *apply distribution*

= A`A+A`B+A`C`D+(A`+C`)(A+B+C`D)C *apply complement law: AA`=0*

= 0+A`B+A`C`D+(A`+C`)(A+B+C`D)C *apply identity law: A+0=A*

*=* A`B+A`C`D+(A`+C`)(A+B+C`D)C *apply distribution*

= A`B+A`C`D+(A+B+C`D)CA`+(A+B+C`D)CC` *apply complement law: AA`=0*

= A`B+A`C`D+(A+B+C`D)CA`+0 *apply identity law: A+0=A*

= A`B+A`C`D+(A+B+C`D)CA` *apply distribution*

= A`B+A`C`D+CA`A+CA`B+CA`C`D *apply complement law: AA`=0*

*=* A`B+A`C`D+0+CA`B+0 *apply identity law: A+0=A*

= A`B+A`C`D+CA`B *apply absorption law: A+AB=A*

= A`B+A`C`D *apply distributive law: AB+AC = A(B+C)*

**= A`(B+C`D)**

**Problem 2-5:**

**Find the complement of F = x + yz; then show that FF` = 0 and F + F` = 1**

F` = (x+yz)`

= x`(yz)`

= x`(y`+z`)

= x`y`+x`z`

FF` = (x+yz)(x`y`+x`z`)

= (x+yz).x`.(y`+z`)

= (x`x+x`yz)(y`+z`)

= (0+x`yz)(y`+z`)

= x`yz(y`+z`)

= x`yzy`+x`yzz`

= yy`x`z+x`yzz`

= 0.x`z+x`y.0

= 0+0

= 0

F+F` = x+yz+x`y`+x`z`

= x+yz+x`(y`+z`)

= x+yz+x`.(yz)`

= x+yz+(x+yz)`

= (x+yz)+(x+yz)`

= 1

**Problem 2-8:**

**List the truth table of the function:**

**F = xy + xy` + y`z**

= x(y+y`)+y`z

= x + y`z

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **x** | **y`** | **z** | **y`z** | **x+y`z** |
| 0 | 1 | 0 | **0** | **0** |
| 0 | 1 | 1 | **1** | **1** |
| 0 | 0 | 0 | **0** | **0** |
| 0 | 0 | 1 | **0** | **0** |
| 1 | 1 | 0 | **0** | **1** |
| 1 | 1 | 1 | **1** | **1** |
| 1 | 0 | 0 | **0** | **1** |
| 1 | 0 | 1 | **0** | **1** |