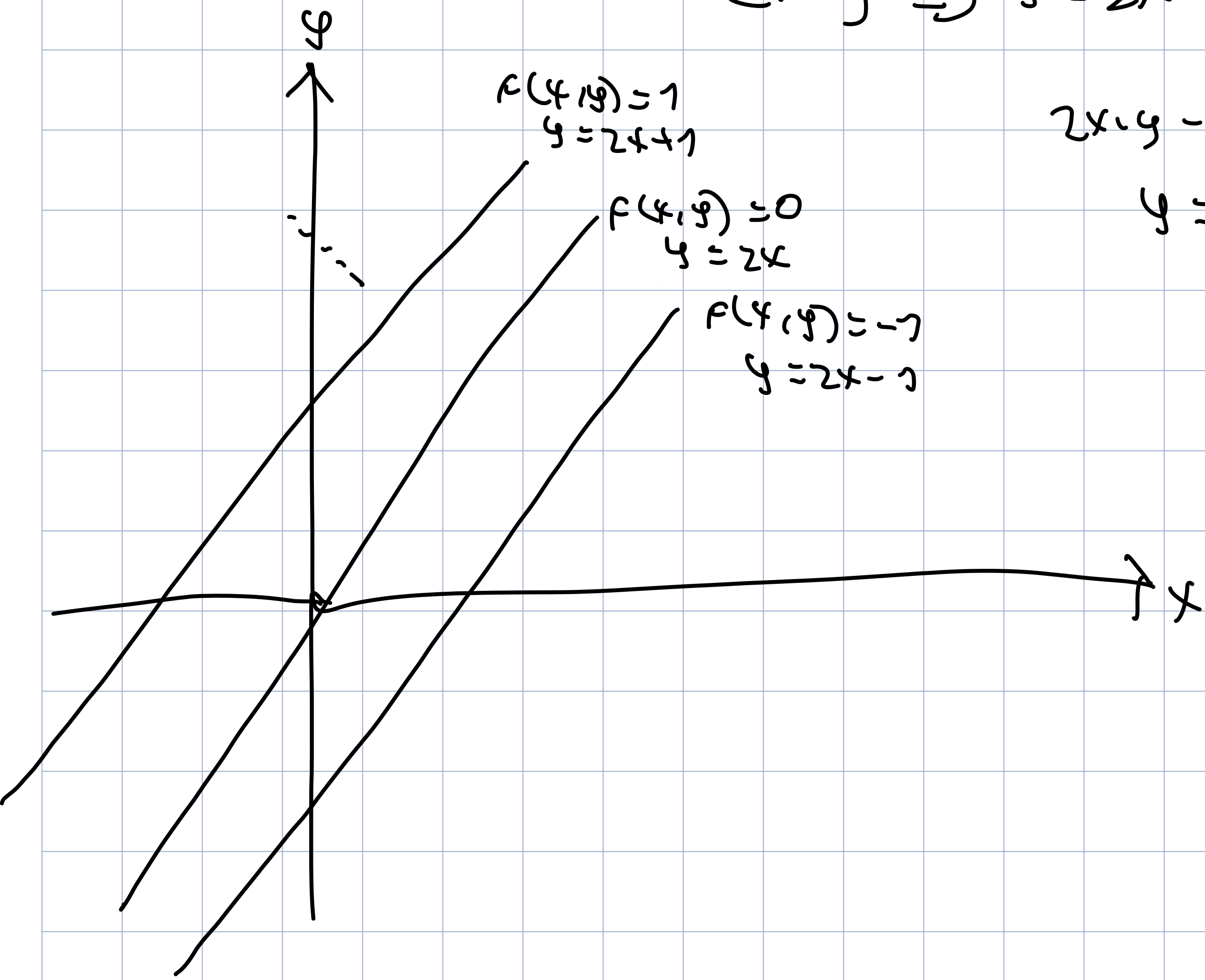
Q2

$2x - y \Rightarrow y = 2x$



$2x - y - 1$

$y = 2x + 1$

$$L(x,y) = 2x - y + \lambda\left(\frac{x^2}{4} + y^2 - 1\right)$$

(1) $\dfrac{\partial L}{\partial x} = 2 + \dfrac{\lambda x}{2} = 0 \Rightarrow \lambda x = -4 \Rightarrow \lambda = \dfrac{-4}{x}$

(2) $\dfrac{\partial L}{\partial y} = -1 + 2\lambda y = 0 \Rightarrow \lambda y = \dfrac{1}{2} \Rightarrow \lambda = \dfrac{1}{2y}$

(3) $\frac{\partial L}{\partial \lambda} = \frac{x^2}{4} + y^2 - 1 = 0$

(1), (2) $\Rightarrow \frac{-4}{x} = \frac{1}{2y} \Rightarrow \boxed{-8y = x}$

$\frac{64y^2}{4} + y^2 - 1 = 0 \Rightarrow 17y^2 = 1 \Rightarrow y^2 = \frac{1}{17}$

$y = \pm \frac{1}{\sqrt{17}}$,

$y = \frac{1}{\sqrt{17}} \Rightarrow x = -\frac{8}{\sqrt{17}} \Rightarrow \left(-\frac{8}{\sqrt{17}}, \frac{1}{\sqrt{17}}\right)$

$y = -\frac{1}{\sqrt{17}} \Rightarrow x = \frac{8}{\sqrt{17}} \Rightarrow \left(\frac{8}{\sqrt{17}}, -\frac{1}{\sqrt{17}}\right)$

$F\left(-\frac{8}{\sqrt{17}}, \frac{1}{\sqrt{17}}\right) = \frac{-16}{\sqrt{17}} - \frac{1}{\sqrt{17}} = \boxed{-\sqrt{17}}$ min

$F\left(\frac{8}{\sqrt{17}}, -\frac{1}{\sqrt{17}}\right) = \frac{16}{\sqrt{17}} + \frac{1}{\sqrt{17}} = \boxed{\sqrt{17}}$ max

Q1

a.

$K(x,y) = (x \cdot y + 1)^3$ is an example of

Inhomogenus polynomial kernel, with $d = 3$.

we know that for $K(x,y)$ to be

a kernel it must satisfy:

$\forall x,y \qquad K(x,y) = \varphi(x) \cdot \varphi(y)$

$K(x,y) = (x \cdot y + 1)^3 = (x \cdot y + 1)^2 (x \cdot y + 1)$

$= ((x \cdot y)^2 + 2 \cdot (x \cdot y) + 1)(x \cdot y + 1)$

$= (x \cdot y)^3 + 2(x \cdot y)^2 + x \cdot y + (x \cdot y)^2 + 2(x \cdot y) + 1$

$= (x \cdot y)^3 + 3(x \cdot y)^2 + 3(x \cdot y) + 1$

$= (x_1 y_1 + x_2 y_2)^3 + 3 \cdot (x_1 y_1 + x_2 y_2)^2$

$+ 3(x_1 y_1 + x_2 y_2) + 1$

$$= \left((x_1y_1)^2 + 2\cdot x_1y_1x_2y_2 + (x_2y_2)^2\right)(x_1y_1 + x_2y_2)$$

$$+3\cdot\left((x_1y_1)^2 + 2x_1y_1x_2y_2 + (x_2y_2)^2\right)$$

$$+3(x_1y_1 + x_2y_2) + 1$$

$$= (x_1y_1)^3 + 2(x_1y_1)^2x_2y_2 + x_1y_1(x_2y_2)^2$$
$$+ (x_2y_2)^3 + 2x_1y_1(x_2y_2)^2 + x_2y_2(x_1y_1)^2$$

$$+ \underbrace{\dots}_{3\cdot()+3\cdot()} + 1$$

$$= (x_1y_1)^3 + 3(x_1y_1)^2x_2y_2 + 3x_1y_1(x_2y_2)^2$$
$$+ (x_2y_2)^3 + 3(x_1y_1)^2 + 6x_1y_1x_2y_2 + 3(x_2y_2)^2$$
$$+3x_1y_1 + 3x_2y_2 + 1 = B$$

Let's Look at:

$$\boxed{\begin{aligned}\phi(x) = (&x_1^3, \sqrt{3}\,x_1^2x_2, \sqrt{3}\,x_1x_2^2, x_2^3,\\ &\sqrt{3}\,x_1^2, \sqrt{6}\,x_1x_2, \sqrt{3}\,x_2^2, \sqrt{3}\,x_1\\ &\sqrt{3}\,x_2, 1)\end{aligned}}$$

we see that

$$\phi(x)\cdot\phi(y) = (x_1^3, \sqrt{3}\, x_1^2 x_2, \sqrt{3}\, x_1 x_2^2, x_2^3, \sqrt{3}\, x_1^2, \sqrt{6}\, x_1 x_2, \sqrt{3}\, x_2^2, \sqrt{3}\, x_1, \sqrt{3}\, x_2, 1) \cdot (y_1^3, \sqrt{3}\, y_1^2 y_2, \sqrt{3}\, y_1 y_2^2, y_2^3, \sqrt{3}\, y_1^2, \sqrt{6}\, y_1 y_2, \sqrt{3}\, y_2^2, \sqrt{3}\, y_1, \sqrt{3}\, y_2, 1) = K$$

b. we called it "kernel for the full rational varieties mapping"

c. if we didn't use the kernel: 10.
with kernel: 2. we save 8.