EX 5 - Theory + SVM

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Q1 - Kernels and mapping functions

$K(x,y) = (x \cdot y + 1)^3$ function over $\mathbb{R}^2 \times \mathbb{R}^2$. Find Ψ for which K is a kernel

A

We'll expand the right hand side argument to find Ψ.

$$(x \cdot y + 1)^3 = (x \cdot y + 1)^2 (x \cdot y + 1) = ((x \cdot y)^2 + 2x \cdot y + 1)(x \cdot y + 1)$$

$$= (x^2 \cdot y^2 + 2x \cdot y + 1)(x \cdot y + 1) = x^3 \cdot y^3 + 2x^2 \cdot y^2 + x \cdot y + x^2 + y^2 + 2x \cdot y + 1$$

$$(x \cdot y)^3 + 3x \cdot y^2 + 3x \cdot y + 1$$

We now will apply $x \in \mathbb{R}^2$ to it and get:

$$(x_1y_1 + x_2y_2)^3 + 3(x_1y_1 + x_2y_2)^2 + 3(x_1y_1 + x_2y_2) + 1$$

And simplify it a bit more:

$$= (x_1y_1 + x_2y_2)^2 (x_1y_1 + x_2y_2) + 3((x_1y_1)^2 + 2x_1y_1x_2y_2 + (x_2y_2)^2) + 3(x_1y_1 + x_2y_2) + 1$$

$$= ((x_1y_1)^2 + 2x_1y_1x_2y_2 + (x_2y_2)^2)(x_1y_1 + x_2y_2) + 3((x_1y_1)^2 + 2x_1y_1x_2y_2 + (x_2y_2)^2)) + 3(x_1y_1 + x_2y_2) + 1$$

$$= (x_1y_1)^3 + 2(x_1y_1)^2 x_2y_2 + (x_2y_2)^2 (x_1y_1) + (x_1y_1)^2 (x_2y_2) + 2(x_2y_2)^2 x_1y_1 + ...$$

$$...(x_2y_2)^3 + 3((x_1y_1)^2 + 2x_1y_1x_2y_2 + (x_2y_2)^2)) + 3(x_1y_1 + x_2y_2) + 1$$

$$= (x_1y_1)^3 + 3(x_1y_1)^2(x_2y_2) + 3(x_2y_2)^2(x_1y_1) + (x_2y_2)^3 ...$$

$$... + 3(x_1y_1)^2 + 6x_1y_1x_2y_2 + 3(x_2y_2)^2 + 3x_1y_1 + 3x_2y_2 + 1$$

Meaning, for $\Psi(x)$ to afford the kernel$K$ it means:

$$\Psi(x) = x_1^3 + \sqrt{3}x_1^2x_2 + \sqrt{3}x_2^2x_1 + x_2^3 + \sqrt{3}x_1^2 + \sqrt{6}x_1x_2 + \sqrt{3}x_2^2 + \sqrt{3}x_1 + \sqrt{3}x_2 + 1$$

$$\Psi(y) = y_1^3 + \sqrt{3}y_1^2y_2 + \sqrt{3}y_2^2y_1 + y_2^3 + \sqrt{3}y_1^2 + \sqrt{6}y_1y_2 + \sqrt{3}y_2^2 + \sqrt{3}y_1 + \sqrt{3}y_2 + 1$$

B

In class we called it "kernel for the full rational variaties mapping".

C

By using $K(x, y)$ instead of $\Psi(x) \cdot \Psi(y)$, we're moving the calculations to be done in $\mathbb{R}^2$ instead of in $\mathbb{R}^{10}$, so we'll save 8 multiplications per instance.

Q2 - Lagrange multipliers

A

$f(x, y) = 2x - y$. Find the min and max point for f under $g(x, y) = \frac{x^2}{4} + y^2 = 1$.

We can write the constraint as

$$\frac{x^2}{4} + y^2 - 1 = 0$$

And write the Lagrangian:

$$L(x, y) = 2x - y + \lambda(\frac{x^2}{4} + y^2 - 1)$$

Next, we'll derive by x, y and λ:

$$(1)\frac{\partial L}{\partial x} = 2 + \frac{2\lambda x}{4} = 0$$
$$\lambda = -4x$$

$$(2)\frac{\partial L}{\partial y} = -1 + 2\lambda y = 0$$

$$\lambda = \frac{1}{2y}$$

We'll put both (1) and (2) togehter and:

$$(3) - 4x = \frac{1}{2y}$$

$$x = -\frac{1}{8y}$$

$$y = -8x$$

$$\frac{\partial L}{\partial \lambda} = \frac{x^2}{4} + y^2 - 1 = 0$$

Plug in (3)

$$\frac{64y^2}{4} + y^2 - 1 = 0$$

$$17y^2 = 1$$

$$y = \pm\frac{1}{\sqrt{17}}$$

$$x = \mp\frac{8}{\sqrt{17}}$$

Plug it back in f:

$$f(-\frac{8}{\sqrt{17}}, \frac{1}{\sqrt{17}}) = -\sqrt{17}$$

$$f(\frac{8}{\sqrt{17}}, -\frac{1}{\sqrt{17}}) = \sqrt{17}$$

It seems that $\sqrt{17}$ is the max point and that $-\sqrt{17}$is the min point

Q3- PAC Learning

The three vectors $u = (\frac{\sqrt{3}}{2}, \frac{1}{2})$, $w = (\frac{\sqrt{3}}{2}, -\frac{1}{2})$, $v = (0, -1)$ define a origin-centered upright equilateral triangle, which serves as the *true* C. We'll define a cricle around it that bounds the triangle and we'll call the radius of that circle r^*.

We'll define r^T, a second cricle that bounds another origin-centered upright equilateral triangle, that is defined by drawing the tightest circle around the new data that we have recieved. This triangle represents our hypothesis $L(D)$.

Furthermore, we'll define a third circle, r'^{ϵ}, which is defined by $argminf_r\pi[x_1, x_1 \in A_r] \leq \varepsilon$, where A_r is the annulus, which bounds a third origin-centered upright equilateral triangle. Meaning, the probability of new data landing in the area between r^T and r^* is $\leq \epsilon$.

We'll note that the both new triangles (bounded by r^ϵ and r^T) are neccasarily inside the origin, true concepts triangle (bounded by r^*).

We want to find

$$P(\{D \in X^m : Err(L(D), c)\} > \epsilon) < \delta$$

In the good case, a new instance of the X^m lands in between the the triangle that is bounded by r^ϵ and the triangle that is bounded by r. The probability of error is $\leq \epsilon$. Meaning, we don't have any issue.

In the bad case, the new instances will miss both the traingles that are bounded by r^* and r^ϵ.

The probability of that happening is $(1-\epsilon)^m \leq e^{-\epsilon m}$. We can define $m \geq \frac{ln(\frac{1}{\delta})}{\epsilon}$ and get:

$$e^{m\epsilon} \leq e^{-ln(\frac{1}{\delta})} = \delta$$

Which is small enough.

Meaning, the sample complexity here is

$$m(\epsilon, \delta) = \frac{1}{\epsilon} ln \frac{1}{\delta}$$

For example, if $\epsilon = \delta = 0.05$, then $m = \frac{1}{0.05} ln \frac{1}{0.05} \approx 60$.

The time complexity of finding the bounded origin-centered upright equilateral around the origin by each of the circles is $O(1)$ as it is not dependent on the sample size. The time complexity of the generating the circle itself is the $O(m)$. Meaning, the time complexity of the solution is $O(m) + O(1) = O(m)$.

Q4 - Confidence Intervals

We'll compute the confidence interval, according to the data provided. We'll take the upper bound of the interval, to find the maximum percentage that we can commit to in 95% confidence.

$$p \in [\hat{p} + 2se, \hat{p} - 2se]$$

$$se = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

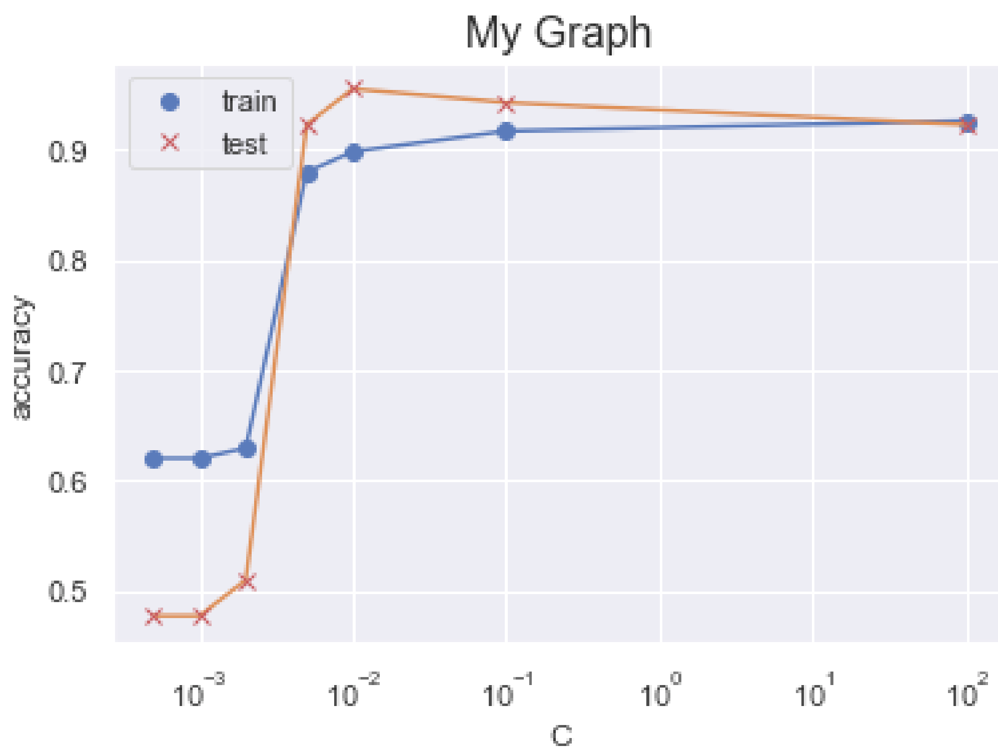
We know that $\hat{p} = 0.2$, $n = 1000$. We'll plug it in and get:

$$se = \sqrt{\frac{0.2(1-0.2)}{1000}} = 0.0126$$

$$p \in [0.175, 0.225]$$

The maximum error, with 95% confidnce, is 22.5%.

Q5 - SVM



My Graph
train
test
0.9
0.8
0.7
0.6
0.5
accuracy
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
C