Geometric Learning
Final Project

Due: 2/3/2021

Project Requirements

Choose **one** project definition from two options supplied below. Complete all sections as detailed, and collect the results into a well explained and organized report in English. Submit your source code implementation and your report to the relevant Moodle submission box. Bonus points will be awarded for **exceptional analysis and/or interesting extensions** to the project's definition.

(Option 1) Geometric Moments & Neural Shape Analysis

1. Read the paper: Joseph-Rivlin et al, "Momen(e)t: Flavor the Moments in Learning to Classify Shapes", *ICCV Workshops*, 2019. The paper is available on Moodle. Furthermore, revise the lecture notes on PointNet and Momenet.

2. Review the article's implementation via Tensorflow: here. Seeing the dataset is small, you can use a CPU for the computation, or a GPU if you have one accessible. You may or may not use the existing implementation, as you prefer.

3. Rerun the experiments for **shape classification** with the **ModelNet40** dataset with varying inputs (where the shapes themselves are represented as point clouds):

   (a) Classic PointNet (as presented in the article) (input consists of 1st order moments, i.e., the x,y and z coordinates.).

   (b) Classic Moment (as presented in the article) - (input consists of 9 input channels - 1st & 2nd order moments).

   (c) 1st, 2nd, and 3rd order moments.

   (d) Repeat the first 3, while also adding consistently oriented vertex normals, as computed in HW1 (an additional three input channels).

   Note: This requires direct access to the mesh topology, and is not "fairly" applicable for our point cloud input. Approximations that do not use the topology exist (e.g. normals are estimated by locally fitting a plane to each vertex, and computing its normal).

(e) Repeat all above test cases with at least one other type of geometric pre-lifting - e.g. harmonics defined in the embedding space such as

$$\sin(\pi x), \cos(\pi x), \sin(2\pi x), \sin(\pi y), \sin(\pi xy), ...,$$

where $x, y, z \in [-1, 1]$.

- You may need to rescale your point cloud input to fit inside the unit sphere in $\mathbb{R}^3$, promising a well-behaved dynamic range in the harmonic case.
- Limit your pre-lifting space to less than 300 dimensions.

(f) Provide detailed analysis and conclusions on the effect of the different input coordinates w.r.t. the shape classification downstream task.

(g) **Possible Bonus Extension**: Link your results and analysis to SIREN.

(Option 2) The Dynamic Laplacian

1. Read the paper: Banisch and Koltai, "Understanding the geometry of transport: Diffusion maps for Lagrangian trajectory data unravel coherent sets", *Chaos*, 2017. The paper is available on Moodle.

   Background: for details on the relation of coherent sets to the classical material of spectral graph theory that was covered in class see the paper by Froyland "Dynamic isoperimetry and the geometry of Lagrangian coherent structures", *Nonlinearity*, 2015, also available on Moodle.

2. Reconstruct all the results reported in Section V part A (Double gyre flow).
   - Generate the data using Equation (25).
   - To understand the data set, it is recommended to view the data as a video clip in time.

3. Reconstruct all the results reported in Section V part B (Bickley jet).
   - The data set is available on Moodle.
   - To understand the data set, it is recommended to view the data as a video clip in time.