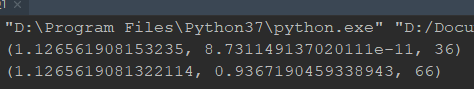
Q1

import math  
  
f = lambda x: math.pow(x, 3) - 1 - math.cos(x)  
  
def bisect (func, range\_min, range\_max, max\_delta):  
 y1 = func(range\_min)  
 y2 = func(range\_max)  
  
 if y1 == 0:  
 return range\_min, 0, 0, 0  
 elif y2 == 0:  
 return range\_max, 0, 0, 0  
  
 # both are positive or negative together  
 if y1 \* y2 > 0:  
 return None  
  
 i = 0  
  
 error = math.fabs(range\_max - range\_min)  
 while error > max\_delta:  
 range\_mid = (range\_min + range\_max) / 2  
 y = func(range\_mid)  
  
  
 if y == 0:  
 return range\_mid, 0, 0, i  
  
 if y2 \* y > 0:  
 y2 = y  
 range\_max = range\_mid  
 else:  
 y1 = y  
 range\_min = range\_mid  
  
 i += 1  
 error = math.fabs(range\_max - range\_min)  
  
 z = (range\_min + range\_max) / 2  
 y = func(z)  
 return z, error, i  
  
def regula\_falsi(func, range\_min, range\_max, max\_delta):  
 y1 = func(range\_min)  
 y2 = func(range\_max)  
  
 if y1 == 0:  
 return range\_min, 0, 0, 0  
 elif y2 == 0:  
 return range\_max, 0, 0, 0  
  
 if y1 \* y2 > 0:  
 return None  
  
 error = math.fabs(range\_max - range\_min)  
 i = 0  
 while error > max\_delta:  
 secant\_zero\_arg = find\_secant\_zero\_arg(func, range\_min, range\_max)  
 y = func(secant\_zero\_arg)  
  
 if y == 0:  
 return secant\_zero\_arg, 0, 0, i  
  
 if y2 \* y > 0:  
 y2 = y  
 range\_max = secant\_zero\_arg  
 else:  
 y1 = y  
 range\_min = secant\_zero\_arg  
  
 i += 1  
 error = math.fabs(y)  
  
 z = secant\_zero\_arg  
 y = func(z)  
 return z, math.fabs((range\_max - range\_min))/2, i  
  
  
def find\_secant\_zero\_arg(func, x1, x2):  
 y1, y2 = func(x1), func(x2)  
 return x2 - y2 \* (x2 - x1)/(y2 - y1)  
  
print(bisect(f, -3, 3, 0.0000000001))  
print(regula\_falsi(f, -3, 3, 0.0000000001))



Bisection took 36 iterations, Regula Falsi 66.  
Meaning that in this case, in order to achieve the same precision – the bisection is more efficient.

Q2

require:  
Therefore g(x) is a contraction mapping for these two ranges.  
  
*We know there is a root at so the interval is the one we need.  
Note that therefore g is decreasing. We’ll find the edges of the interval:  
We’ll find the maximal x where the interval requirement holds:  
So within that interval, g is a contraction mapping where all other convergence requirements hold.*

*Therefore, the convergence occurs at*

*Q3*

*require:  
Therefore:*

*Therefore, g is a contracting mapping.  
  
Converge into:  
true for every x.  
Therefore the convergence is for , which is the domain of Ln (therefore, maximal)*

*Q4*

*from Newton-Raphson’s method:*

*Therefore, for some initial guess , the series is   
We know that the root is zero, and (and therefore , so:*

*So, no iteration will reach the root.  
Also this series has no limit thus it will not converge to 0.*

*Q5*

1. *Negatively assume that there exists s.t .  
   Without loss of generality, assume* .  
    therefore .  
   Contradiction.
2. Let there be some .  
   Note that the function is strictly convex, meaning , thus strictly monotonically increases.   
   if : let A be the tangent at that point. Note that monotonically increases and therefore f(x) will reach 0 before A.  
   Therefore the intersection point of the tangent with the X axis will be greater than the root.  
   Meaning:   
   if : let A be the tangent at that point. Note that monotonically increases, therefore the tangent’s derivative is greater than the derivative at the root. Since the tangent intersects f at the given point and increases faster than f, and the intersection point is at an x greater than 0, than it intersects the X axis at a value greater than f’s root.  
   Meaning:   
     
   Since after , and f is monotonically increasing, .  
   We know that therefore . Thus:
3. Therefore it converges to L.  
   So: For some n large enough:So the limit of is the root of f. We know it is singular, therefore
4. If then:  
   The function monotonically decreases. Therefore, the Tangents have a larger derivative than the function, meaning they will reach the X axis before f.  
   The rest of the proof goes similarly to b.  
   Thus:   
     
   Since after , and f is monotonically decreasing, .  
   We know that therefore . Thus:  
   The proof for c will be equivalent up to symmetrical differences.
5. Let f be a strictly convex monotonous function.  
   If f is monotonically increasing, we have proven in b that it converges to the root.  
   Else, it is monotonically decreasing, and we have shown in d that it also converges to the root.   
   Therefore, in either case it converges to the root for every initial guess.