Q2

require:  
Therefore g(x) is a contraction mapping for these two ranges.  
  
*We know there is a root at so the interval is the one we need.  
Note that therefore g is decreasing. We’ll find the edges of the interval:  
We’ll find the maximal x where the interval requirement holds:  
So within that interval, g is a contraction mapping where all other convergence requirements hold.*

*Therefore, the convergence occurs at*

*Q3*

*require:  
Therefore:*

*Therefore, g is a contracting mapping.  
  
Converge into:  
true for every x.  
Therefore the convergence is for , which is the domain of Ln (therefore, maximal)*

*Q4*

*from Newton-Raphson’s method:*

*Therefore, for some initial guess , the series is   
We know that the root is zero, and (and therefore , so:*

*So, no iteration will reach the root.  
Also this series has no limit thus it will not converge to 0.*

*Q5*

1. *Negatively assume that there exists s.t .  
   Without loss of generality, assume* .  
    therefore .  
   Contradiction.
2. Let there be some .  
   Note that the function is strictly convex, meaning , thus strictly monotonically increases.   
   if : let A be the tangent at that point. Note that monotonically increases and therefore f(x) will reach 0 before A.  
   Therefore the intersection point of the tangent with the X axis will be greater than the root.  
   Meaning:   
   if : let A be the tangent at that point. Note that monotonically increases, therefore the tangent’s derivative is greater than the derivative at the root. Since the tangent intersects f at the given point and increases faster than f, and the intersection point is at an x greater than 0, than it intersects the X axis at a value greater than f’s root.  
   Meaning:   
     
   Since after , and f is monotonically increasing, .  
   We know that therefore . Thus:
3. Therefore it converges to L.  
   So: For some n large enough:So the limit of is the root of f. We know it is singular, therefore
4. If then:  
   The function monotonically decreases. Therefore, the Tangents have a larger derivative than the function, meaning they will reach the X axis before f.  
   The rest of the proof goes similarly to b.  
   Thus:   
     
   Since after , and f is monotonically decreasing, .  
   We know that therefore . Thus:  
   The proof for c will be equivalent up to symmetrical differences.
5. Let f be a strictly convex monotonous function.  
   If f is monotonically increasing, we have proven in b that it converges to the root.  
   Else, it is monotonically decreasing, and we have shown in d that it also converges to the root.   
   Therefore, in either case it converges to the root for every initial guess.