

1. Find MGUs for the following pairs of type expressions (if exists):

a) $[T1 * [T1 \rightarrow T2] \rightarrow N]$, $[[T3 \rightarrow T4] * [T5 \rightarrow \text{Number}] \rightarrow N]$

$\{ T1 = [T3 \rightarrow T4], T5 = [T3 \rightarrow T4], T2 = \text{Number} \}$

b) $[T1 * [T1 \rightarrow T2] \rightarrow N]$, $[\text{Number} * [\text{Symbol} \rightarrow T3] \rightarrow N]$

$T1 = \text{number} \ \& \ T1 = \text{symbol}$

$\text{number} \neq \text{symbol}$

Therefore no unifier.

c) $T1, T2$

$\{T1=T2\}$

d) $\text{Number}, \text{Number}$

$\{\}$

2. Explain why we can typecheck `letrec` expressions without specific problems related to recursion and without the need for a recursive environment like we had in the interpreter.

The interpreter goes over the body of all the expressions multiple times (can do it one time or infinite times). The reason for that is the fact that the interpreter's environment boxes and values to their corresponding identifiers. The typechecker only goes over the body of the expression one time because the environment of the type binds the types to their corresponding identifiers. In `letrec` the typechecker only needs to check constraints.

3. In the type equation implementation - we represent Type Variables (TVar) with a content field (which is a box which contains a Type Expression value or `#f` when empty). In this representation, we can have a TVar refer in its content to another TVar - repeatedly, leading to a chain of TVars. Design a program which, when we pass it to the type inference algorithm, creates a chain of length 4 of $\text{Tvar1} \rightarrow \text{Tvar2} \rightarrow \text{Tvar3} \rightarrow \text{Tvar4}$. Write a test to demonstrate this configuration.

```
(let ((a 6))
```

```
  (let ((b a))
```

```
    (let ((c b))
```

```
      (let ((d c)) d)
```

```
    )))
```

>6

$T_d \rightarrow T_c \rightarrow T_b \rightarrow T_a$. T_a is number.