

Sheet 7

Q1: a. domain = $X = \{1, 3, 5\}$ co-domain = $Y = \{s, t, u, v\}$

b. $g(1) = t, g(3) = t, g(5) = t$ c. Range = $\{t\}$

Q2: a. Not a function; $\{2\}$ has 2 images b. Not a function; $\{5\}$ has no image

c. Not a function; $\{4\}$ has 2 images

$$Q3: g(x) = \frac{2x^3 + 2x}{x^2 + 1} = \frac{2x(x^2 + 1)}{(x^2 + 1)} = 2x - f(x)$$

Q4: Let $m=0, n=4$ so $g(\frac{0}{4}) = g(0) = 0 \neq 4$, let $m=0, n=5$ so $g(\frac{0}{5}) = g(0) = 0 \neq 5$

so g^{-1} has 2 or more images so $g(m/n) = m/n$ is not a function

Q5: a. It is not one-to-one because $\{y\}$ has 2 pre-images, not onto because $\{x\}$ has no pre-images

b. It is one-to-one because each element in range has only one pre-image, not onto because $\{z\}$ has no pre-image

Q6: a) Yes, let $f(n_1) = f(n_2)$ so $2n_1 = 2n_2 \Rightarrow n_1 = n_2$ so it is one-to-one

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b) No, let $f(n) = s$: $2n = s \Rightarrow n = \frac{s}{2} \notin \text{domain}$ so all n can't be obtained from f

So range \neq co-domain

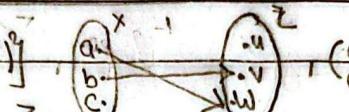
b. Yes, let $m \in \mathbb{Z}$ $m = 2n \Rightarrow n = \frac{m}{2}$ where m is even so $n \in \mathbb{Z}$ so it is onto

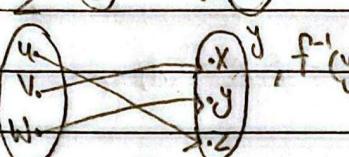
Q7: a. one-to-one, let $f(x_1) = f(x_2)$ $\frac{x_1+1}{x_1} = \frac{x_2+1}{x_2}$ $x_1x_2 + x_2 = x_1x_2 + x_1$ $x_2 - x_1 = 0 \Rightarrow x_1 = x_2$, \therefore one-to-one

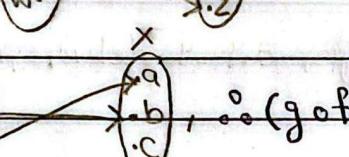
b. one-to-one, let $f(x_1) = f(x_2)$ $\frac{2x_1+1}{x_1} = \frac{2x_2+1}{x_2}$ $2x_1x_2 + x_2 = 2x_1x_2 + x_1$ $x_2 - x_1 = 0 \Rightarrow x_1 = x_2$, \therefore one-to-one

Q8: a. $G \circ F = x^3 - 1$, $F \circ G = (x-1)^3 = x^3 - 3x^2 + 3x - 1$ $\therefore G \circ F \neq F \circ G$

b. $G \circ F = (x^5)^4 - x^{20}$, $F \circ G = (x^4)^5 - x^{20}$ $\therefore G \circ F = F \circ G$

Q9: $g \circ f = g(f(x)) = \{(a, u), (b, v)\}$,  $(g \circ f)^{-1} = \{(u, a), (v, b)\}$

$g^{-1}(z) = y = \{(u, z), (v, x), (w, y)\}$,  $f^{-1}(y) = X = \{(x, b), (y, a)\}$

$f \circ g^{-1} = \{(v, b), (w, a)\}$,  $\therefore (g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Q10: one-to-one: let $f(m, n) = f(m_1, n_2)$ $2^{m_1-1}(2n_1)_1 = 2^{m_2-1}(2n_2)_1$ \therefore Prime factorization is unique

\therefore Power of two are equal $\therefore m_1 = m_2$ $\therefore m_1 - m_2 = 0$ \therefore odd must be the same

$\therefore 2n_1 - 2n_2 = 0$ $\therefore n_1 = n_2$ From 1, 2: $\therefore f(m, n)$ is one-to-one

onto: let $z = 2^a \cdot b$ where $a > 0, b$ is odd

let $a = m-1$ $m = a+1$ $\therefore a$ is an int. $\therefore m$ is an int $\therefore m \geq 1$

let b is odd $b = 2n-1$ $n = \frac{b+1}{2}$ $\therefore b$ is odd $\therefore n$ is even $\therefore n$ is an int.

\therefore for every z , there exists m, n which will satisfy $f(m, n) = z$ $\therefore f(m, n)$ is onto