

Sheet 5

Q1:  $A = [0, 1, 2]$   $C = ]-1, 3[$   $D = [0, 1, 2]$   $E = \{1, 2\}$  Is  $A = D$  as  $A \cap D$  and  $D \cap A$

Q2: a. Set of integers which equal 1, -1 based on some integer  $K$   $S = \{1, -1\}$

b. set of integers which equal 0, 2 based on some integer  $K$   $T = \{0, 2\}$

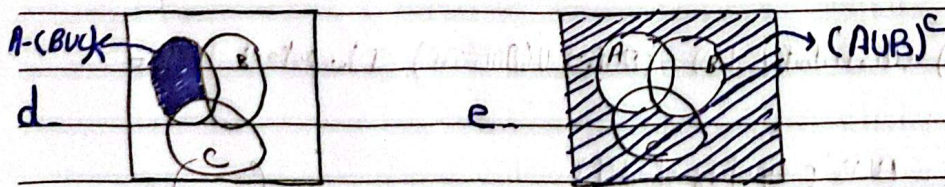
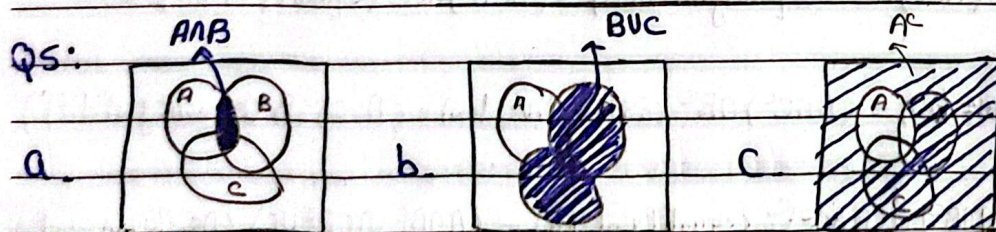
Q3: a yes, as  $f \in A$  and a No, as  $i \notin A$  b yes, as both  $d, g \in A$

c yes, as  $d, g \in C$  d yes, as  $d, g \in A$  but  $C \neq A$

Q4:  $A = ]0, 2]$   $B = [1, 4[$

a  $]0, 4[$  b  $[1, 2]$  c  $] -\infty, 0] \cup ]2, \infty[$  d  $] -\infty, 1[ \cup [4, \infty[$  e  $] -\infty, 0] \cup [4, \infty[$

f  $] -\infty, 1[ \cup ]2, \infty[$  g  $] -\infty, 1[ \cup [2, \infty[$  h  $] -\infty, 0] \cup [4, \infty[$



Q6: a  $(A \cap B) \cup (A \cap B)^c = (A \cap B^c) \cup (A \cap B) = A \cap (B \cap B^c) = A \cap \emptyset = A \neq \text{True}$

b False, let  $A = F, B = T, C = T$   $(A \cap B) \cup C = (F \cap T) \cup T = F \cup T = T$   
 $A \cap (B \cup C) = F \cap (T \cup T) = F \cap T = F \neq T$



c. False, Let  $A = \{3, 4\}$ ,  $B = \{1, 2\}$ ,  $C = \{3, 4, 5, 6\}$  so,  $A \not\subset B$  and  $B \not\subset C$  but  $A \subset C$ .

d. False, Let  $A = \{1, 2\}$ ,  $B = \{1, 2\}$ ,  $C = \{5, 6\}$   $\Rightarrow A \cup C = B \cup C = \{1, 2, 5, 6\}$  but  $A \neq C$

Q7: Let element  $x \in [(A \setminus B) \cap (A \cap B)]$  so,  $x \in (A \setminus B)$  and  $x \in (A \cap B)$

for  $x \in (A \setminus B)$  :  $x \in A$  and  $x \notin B$  for  $x \in (A \cap B)$  :  $x \in A$  and  $x \in B$

so we have  $x \notin B$  and  $x \in B$  contradiction that means  $(A \setminus B) \cap (A \cap B) = \emptyset$

b.  $\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, c, d\}\}$

c. b and d

d. 1  $\{\{1\}, \{2\}, \{3\}\}$  2  $\{\{1, 2\}, \{3\}\}$  3  $\{\{1, 3\}, \{2\}\}$  4  $\{\{1\}, \{2, 3\}\}$  5  $\{\{1, 2, 3\}\}$

Q8: a.  $(A \setminus B) \cap C = (A \setminus B) \cap C^c$  (equality of diff.)  $= (A \cap B^c) \cap C^c$  (equality of diff.)  $=$

$A \cap C^c \cap B^c$  (commutative law)  $= (A \cap C^c) \cap B^c$  (associative law)  $= (A \cap C) \cap B$  (equality of diff.)

b.  $(A \setminus B) \cap (B \setminus C) = (A \cap B^c) \cap (B \cap C^c)$  (equality of diff.)  $= (A \cap B^c) \cap (B^c \cup C)$  (De Morgan law)  $=$

$(A \cap B^c \cap B^c) \cup (A \cap B^c \cap C)$  (distributive law)  $= (A \cap B^c) \cup (A \cap B^c \cap C)$  (Idempotent law)  $=$

$A \cap B^c$  (Absorption law)  $= A \cap B$  (equality of diff.)

c.  $(A \cap B) \cup (B \cap A) = (A \cap B^c) \cup (B \cap A^c)$  (equality of diff.)  $= ((A \cap B^c) \cup B) \cap ((A \cap B^c) \cup A^c)$  (distribution law)  $=$

$((A \cup B) \cap (B^c \cup B)) \cap ((A \cup A^c) \cap (B^c \cup A^c))$  (distribution)  $= ((A \cup B) \cap U) \cap (U \cap (A^c \cup B^c))$  (intersection with U)  $= (A \cup B) \cap (A^c \cup B^c)$

$= (A \cup B) \cap (A \cap B)^c$  (de Morgan)  $= (A \cup B) - (A \cap B)$  (equality of diff.)

d.  $A \cup (B \cap A) = A \cup (B \cap A^c)$  (equality of diff.)  $= (A \cup B) \cap (A \cup A^c)$  (distribution)  $= (A \cup B) \cap U = A \cup B$  (intersection with U)