

Sheet 6

Q1: a. domain = $X = \{1, 3, 5\}$ Co domain = $Y = \{s, t, u, v\}$

b. $g(1) = t, g(3) = t, g(5) = t$ c. Range = $\{t\}$

Q2: a. Not a function, $\{2\}$ has 2 images b. Not a function, $\{5\}$ has no image

c. Not a function, $\{4\}$ has 2 images

$$Q3: g(x) = \frac{2x^3 + 2x}{x^2 + 1} = \frac{2x(x^2 + 1)}{(x^2 + 1)} = 2x = f(x)$$

Q4: Let $m=0, n=4$ so $g(\frac{0}{4}) = g(0) = 0 \cdot 4 = 4$, let $m=0, n=5$ so $g(\frac{0}{5}) = g(0) = 0 \cdot 5 = 5$

So g has 2 or more images so $g(\frac{m}{n}) = m \cdot n$ is not a function

Q5: a. It is not one to one because $\{y\}$ has 2 pre-images, not onto because $\{x, z\}$ has no pre-images

b. It is one-to-one because each element in range has only one pre image, not onto because $\{z\}$ has no pre image

Q6: a. yes, let $f(n_1) = f(n_2)$ so $2n_1 = 2n_2$ so $n_1 = n_2$ so f is one-to-one

b. No, let $f(n) = 5$ so $2n = 5$ so $n = \frac{5}{2} = 1.5 \notin \text{domain}$ so all m can't be obtained from f

So range \neq co-domain

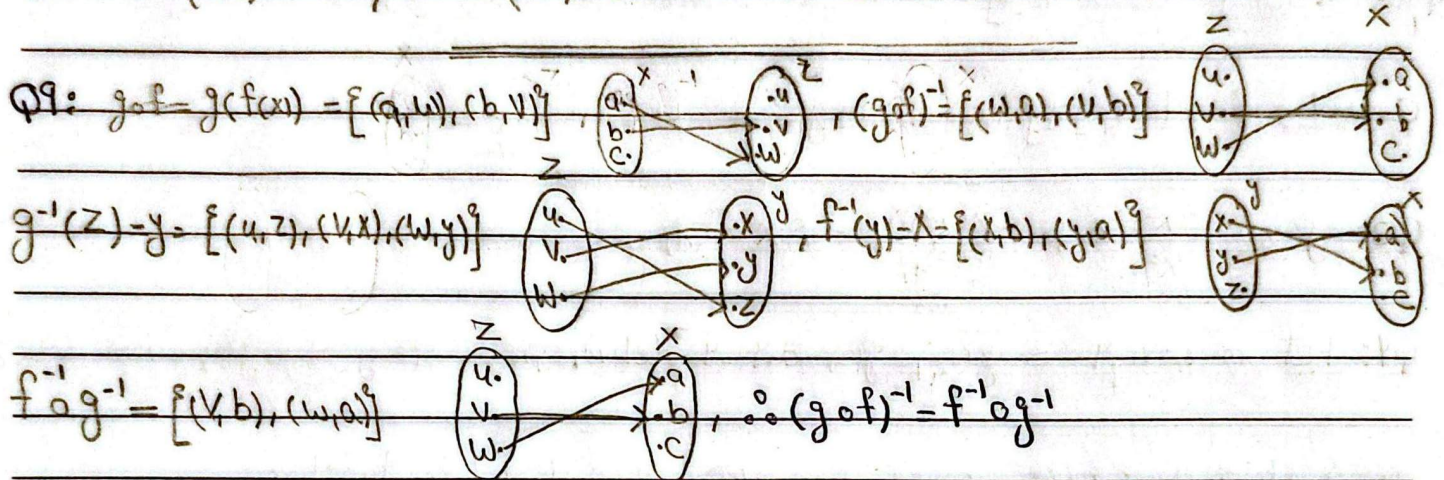
b. Yes, let $m \in 2\mathbb{Z}$ so $m = 2n$ so $n = \frac{m}{2}$ where n is even so $n \in \mathbb{Z}$ so f is onto

Q7: a. one to one, let $f(x_1) = f(x_2)$ $\frac{x_1+1}{x_1} = \frac{x_2+1}{x_2}$ $\cancel{x_2} \cancel{x_1} + x_2 = \cancel{x_2} \cancel{x_1} + x_1$ $x_2 = x_1$ \therefore one to one

b. one to one, let $f(x_1) = f(x_2)$ $\frac{2x_1+1}{x_1} = \frac{2x_2+1}{x_2}$ $\cancel{2} \cancel{x_2} + x_2 = \cancel{2} \cancel{x_1} + x_1$ $x_2 = x_1$ \therefore one to one

Q8: a. $G \circ F = x^3 - 1$, $F \circ G = (x-1)^3 = x^3 - 3x^2 + 3x - 1$ So $G \circ F \neq F \circ G$

b. $G \circ F = (x^5)^4 = x^{20}$, $F \circ G = (x^4)^5 = x^{20}$ So $G \circ F = F \circ G$



Q10: one to one: let $f(m, n) = f(m_2, n_2)$ $2^{m_1-1}(2n_1-1) = 2^{m_2-1}(2n_2-1)$ \therefore Prime factorization is unique

\therefore Power of two are equal $\therefore m_1 - 1 = m_2 - 1$ $\therefore m_1 = m_2$ ① \therefore odd must be the same

$\therefore 2n_1 - 1 = 2n_2 - 1$ $\therefore n_1 = n_2$ ② From 1, 2: $\therefore f(m, n)$ is one-to-one

onto: let $z = 2^a \cdot b$ where $a \geq 0$, b is odd

let $a = m - 1$ $m = a + 1$ $\therefore a$ is an int. $\therefore m$ is an int $\therefore m \geq 1$

let b is odd $b = 2n - 1$ $n = \frac{b+1}{2}$ $\therefore b$ is odd $\therefore b+1$ is even $\therefore n$ is an int.

\therefore for every z , there exists m, n which will satisfy $f(m, n) = z$ $\therefore f(m, n)$ is onto