Alexandria University
Faculty of Engineering
Computer and Systems Engineering Department

Numerical Computing

Assigned: October 16, 2023
Due: October 23, 2023

Sheet 1

1. Show that the binary equivalent to 1/7 = $0.\overline{001}_2$ is equivalent to
$$\frac{1}{7} = \frac{1}{8} + \frac{1}{64} + \frac{1}{512} + \cdots$$
[Use geometric series]

2. a. Evaluate the polynomial $y = x^3 - 7x^2 + 8x - 0.35$ at x=1.37. Use 3-digit arithmetic with chopping. Evaluate the percent relative error.
   b. Repeat (a) but express y as $= \big((x-7)x+8\big)x - 0.35$. Evaluate the error and compare with part (a).

3. The Maclaurin series for $cosx$ is
$$cosx = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$
Starting with simplest version, $cosx = 1$, add terms one at a time to estimate $\cos\left(\frac{\pi}{4}\right)$. After each new term is added, compute the true and approximate percent relative errors. Use your calculator to determine the true value. Add terms until the absolute value of the approximate error estimate below an error criterion confirming to two significant figures.

4. Use zero-order through fourth-order Taylor series expansions to predict $f(2)$ for $f(x) = \ln(x)$ using a base point at $x = 1$. Compare the true percent relative error ε_t for each approximation. Discuss the meaning of the results.

5. Show that the relative error incurred by using $x^2 + \frac{x^3}{6}$ to approximate $e^x - \cos x - x$ is roughly 10^{-24} for $|x| \le 5 \times 10^{-8}$.

6. **Verify** that: f(x) and g(x) are identical functions.
   f(x) = $1 - \sin x$ g(x) = $\frac{cos^2 x}{1+\sin x}$
   a. Which function should be used for computations when x is near $\frac{\pi}{2}$? Why?
   b. Which function should be used for computations when x is near $\frac{3\pi}{2}$? Why?

7. The derivative of $f(x) = \frac{1}{(1-3x^2)}$ is given by $\frac{6x}{(1-3x^2)^2}$. Do you expect to have difficulties evaluating this function at x=0.577? Try it using 3- and 4-digit arithmetic with chopping.

8. Given $f(x) = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+1}}$ assuming 3-decimal mantissa with rounding:
   a. Evaluate $f(1000)$ directly.
   b. Evaluate $f(1000)$ as accurate as possible using an alternative approach.
   c. Find the relative error of $f(1000)$ in part (a) and (b).

9. Construct an algorithm which computes the roots of the quadratic equation $ax^2 + bx + c = 0$ and which avoids as many round-off error problems as possible. Test your algorithm by computing the roots of the quadratic equations: $0.2\,x^2 - 47.91x + 6 = 0$ and $0.025\,x^2 + 7x - 0.1 = 0$. Use 4 decimal digit rounding arithmetic in your calculations.

10. Bonus:
    a. Make a MATLAB Program for binary and decimal base number conversion.
    b. Deliver: The source code snippets and sample runs.