# Linear regression in R

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## Linear regression

In this tutorial we'll learn:

- how to merge datasets
- how to fit linear regression models
- how to split data into test and train sets
- how to tune our models and select features

## Data preparation

We're working with the Capital Bikeshare again this week, so start by reading in usage, weather, stations.

#### Merging data

We have three related datasets to work with, but we can't really get started until we figure out how to combine them. Let's start with *usage* and *weather*. The *usage* dataframe is at the resolution of the hour, while the *weather* data are at the resolution of a day, so we know we're going to have to either duplicate or compress data to merge. I vote compress, let's summarize!

```
head(usage)
##
    bike_id
                     time_start
                                           time_end duration_mins
## 1 W01412 2012-01-01 00:04:00 2012-01-01 00:11:00
## 2 W00524 2012-01-01 00:10:00 2012-01-01 00:29:00
                                                               19
## 3 W00235 2012-01-01 00:10:00 2012-01-01 00:29:00
                                                               19
## 4 W00864 2012-01-01 00:15:00 2012-01-01 00:23:00
                                                                8
## 5 W00995 2012-01-01 00:15:00 2012-01-01 00:23:00
                                                                8
## 6 W00466 2012-01-01 00:17:00 2012-01-01 00:23:00
                                                                6
##
                           station start
                                                     station end cust type
## 1
            7th & R St NW / Shaw Library
                                                   7th & T St NW Registered
## 2
          Georgia & New Hampshire Ave NW
                                            16th & Harvard St NW
                                                                     Casual
## 3
          Georgia & New Hampshire Ave NW 16th & Harvard St NW Registered
## 4
                          14th & V St NW Park Rd & Holmead Pl NW Registered
## 5
                     11th & Kenyon St NW
                                                   7th & T St NW Registered
## 6 Court House Metro / 15th & N Uhle St Lynn & 19th St North Registered
custs_per_day = usage %>%
 group_by(time_start = as.Date(time_start), station_start, cust_type) %%
 summarize(no rentals = n(),
           duration_mins = mean(duration_mins, na.rm = TRUE))
head(custs_per_day)
## Source: local data frame [6 x 5]
## Groups: time_start, station_start
##
##
    time_start
                               station_start cust_type no_rentals
## 1 2012-01-01
                        10th & Monroe St NE Registered
## 2 2012-01-01
                              10th & U St NW
                                                 Casual
                                                                8
```

```
Perfection, now we can merge! What's the key?
```

## Variables not shown: duration\_mins (dbl)

## 4 2012-01-01 10th St & Constitution Ave NW

## 5 2012-01-01 10th St & Constitution Ave NW Registered

10th & U St NW Registered

11th & H St NE

Casual

Casual

50

34

20

4

## [1] 99356 5

## 3 2012-01-01

## 6 2012-01-01

```
dim(weather)
## [1] 366 15
dim(weather_rentals)
## [1] 99356
                19
head(weather_rentals)
     time_start
##
                                 station start
                                                 cust_type no_rentals
## 1 2012-01-01
                           10th & Monroe St NE Registered
                                                                    10
## 2 2012-01-01
                                10th & U St NW
                                                    Casual
                                                                     8
## 3 2012-01-01
                                10th & U St NW Registered
                                                                    50
## 4 2012-01-01 10th St & Constitution Ave NW
                                                    Casual
                                                                    34
                                                                    20
## 5 2012-01-01 10th St & Constitution Ave NW Registered
## 6 2012-01-01
                                11th & H St NE
                                                    Casual
                                                                     4
##
     duration_mins weekday season_code season_desc is_holiday is_work_day
## 1
          16.40000
                          0
                                       1
                                              Spring
                                                               0
                                                                            0
## 2
                                                                           0
          16.25000
                          0
                                       1
                                              Spring
                                                               0
## 3
          10.00000
                          0
                                       1
                                                               0
                                                                           0
                                              Spring
          20.29412
                                                                           0
## 4
                          0
                                       1
                                              Spring
                                                               0
## 5
          14.20000
                          0
                                       1
                                                               0
                                                                           0
                                              Spring
## 6
          10.00000
                          0
                                       1
                                              Spring
                                                               0
##
     weather_code
                                                       weather_desc temp
## 1
                1 Clear, Few clouds, Partly cloudy, Partly cloudy 0.37
## 2
                1 Clear, Few clouds, Partly cloudy, Partly cloudy 0.37
                1 Clear, Few clouds, Partly cloudy, Partly cloudy 0.37
## 4
                1 Clear, Few clouds, Partly cloudy, Partly cloudy 0.37
## 5
                1 Clear, Few clouds, Partly cloudy, Partly cloudy 0.37
## 6
                1 Clear, Few clouds, Partly cloudy, Partly cloudy 0.37
     subjective_temp humidity windspeed no_casual_riders no_reg_riders
## 1
            0.375621
                        0.6925 0.192167
                                                                     1608
                                                       686
## 2
            0.375621
                        0.6925
                                0.192167
                                                       686
                                                                     1608
                                                                     1608
## 3
            0.375621
                        0.6925 0.192167
                                                       686
            0.375621
                        0.6925 0.192167
                                                       686
                                                                     1608
## 5
            0.375621
                        0.6925
                                0.192167
                                                       686
                                                                     1608
## 6
            0.375621
                        0.6925 0.192167
                                                       686
                                                                     1608
##
     total_riders
## 1
             2294
## 2
             2294
## 3
             2294
## 4
             2294
## 5
             2294
## 6
             2294
```

Great, now we want to merge on the last dataset, stations. What is the key to link weather\_rentals with stations?

```
## [1] 98634 154
```

#### dim(weather\_rentals)

#### ## [1] 99356 19

#### head(final\_data[, 1:30])

```
station_start time_start cust_type no_rentals duration_mins weekday
##
## 1 10th & E St NW 2012-07-25
                                    Casual
                                                     8
                                                             82.37500
                                                                            3
## 2 10th & E St NW 2012-07-25 Registered
                                                                            3
                                                    32
                                                             13.28125
                                                                            2
## 3 10th & E St NW 2012-11-13 Subscriber
                                                    19
                                                             11.73684
## 4 10th & E St NW 2012-09-25 Registered
                                                    41
                                                             12.29268
                                                                            2
                                                                            4
## 5 10th & E St NW 2012-08-09 Registered
                                                    34
                                                             13.61765
## 6 10th & E St NW 2012-11-22 Subscriber
                                                     7
                                                             12.14286
     season_code season_desc is_holiday is_work_day weather_code
## 1
               3
                         Fall
                                       0
                                                    1
                                                                  1
## 2
               3
                         Fall
                                       0
                                                    1
                                                                  1
## 3
               4
                                                                  2
                       Winter
                                       0
                                                    1
## 4
               4
                                       0
                                                    1
                                                                  1
                      Winter
## 5
               3
                         Fall
                                       0
                                                    1
                                                                  1
## 6
               4
                       Winter
                                       1
##
                                                       weather_desc
## 1
                  Clear, Few clouds, Partly cloudy, Partly cloudy 0.724167
## 2
                  Clear, Few clouds, Partly cloudy, Partly cloudy 0.724167
## 3 Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist 0.343333
                  Clear, Few clouds, Partly cloudy, Partly cloudy 0.550000
                  Clear, Few clouds, Partly cloudy, Partly cloudy 0.755833
## 5
## 6
                  Clear, Few clouds, Partly cloudy, Partly cloudy 0.340000
     subjective_temp humidity windspeed no_casual_riders no_reg_riders
            0.654054 0.450000 0.1648000
## 1
                                                      1383
                                                                     6790
## 2
                                                                     6790
            0.654054 0.450000 0.1648000
                                                      1383
## 3
            0.323225 0.662917 0.3420460
                                                       327
                                                                     3767
## 4
            0.544179 0.570000 0.2363210
                                                       845
                                                                     6693
## 5
            0.699508 0.620417 0.1561000
                                                      1196
                                                                     6090
## 6
            0.350371 0.580417 0.0528708
                                                                     1470
                                                       955
     total_riders id terminal_name
                                           lat
                                                    long no bikes
## 1
             8173 199
                               31256 38.89591 -77.02606
## 2
             8173 199
                               31256 38.89591 -77.02606
                                                                 6
## 3
             4094 199
                               31256 38.89591 -77.02606
                                                                 6
## 4
             7538 199
                               31256 38.89591 -77.02606
                                                                 6
## 5
                               31256 38.89591 -77.02606
                                                                 6
             7286 199
             2425 199
                               31256 38.89591 -77.02606
## 6
     no_empty_docks fast_food parking restaurant convenience post_office
## 1
                  8
                             5
                                     2
                                                16
                                                             0
                                                                          1
                                     2
## 2
                  8
                             5
                                                16
                                                             0
                                                                          1
## 3
                  8
                             5
                                     2
                                                             0
                                                16
                                                                          1
                                     2
                  8
                             5
                                                             0
## 4
                                                16
                                                                          1
                                     2
## 5
                  8
                             5
                                                16
                                                             0
                                                                          1
## 6
                  8
                             5
                                     2
                                                16
                                                             0
                                                                          1
```

#### The lm() function

The function for creating a linear model in R is lm() and the primary arguments are formula and data. Formulas in R are a little funny, instead of an = sign, they are expressed with a  $\sim$ . Let's fit the model we saw in the lecture notes:  $rentals = \beta_0 + \beta_1 * crossing$ . There's a little snag we have to take care of first. Right now we've got repeated measures *i.e.* one measurement per day, so we need to aggregate again. How do we aggregate over date, but still maintain relevant seasonal data?

```
## Source: local data frame [6 x 3]
##
##
                     station_start mean_rentals crossing
## 1
                    10th & E St NW
                                       19.003003
                                                       122
## 2
               10th & Monroe St NE
                                        7.580517
                                                         1
## 3
                    10th & U St NW
                                       37.954876
                                                         5
## 4 10th St & Constitution Ave NW
                                       28.430362
                                                       116
                    11th & H St NE
## 5
                                       20.121875
                                                        73
## 6
               11th & Kenyon St NW
                                       33.718331
                                                        20
```

```
# plot it
ggplot(rentals_crossing, aes(x = crossing, y = mean_rentals)) +
geom_smooth(method = 'lm', size = 2) +
geom_point(size = 4, alpha = 0.60) +
theme_minimal()
```

```
model = lm(mean_rentals ~ crossing, data = rentals_crossing)
```

```
model = lm(mean_rentals ~ crossing, data = rentals_crossing)
# view what is returned in the lm object
attributes(model)
```

```
## $names
## [1] "coefficients" "residuals" "effects" "rank"
## [5] "fitted.values" "assign" "qr" "df.residual"
## [9] "xlevels" "call" "terms" "model"
##
## $class
## [1] "lm"
```

# # get model output summary(model)

```
##
## Call:
## lm(formula = mean_rentals ~ crossing, data = rentals_crossing)
##
## Residuals:
## Min 1Q Median 3Q Max
## -25.735 -10.767 -4.190 6.755 63.079
##
## Coefficients:
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) 15.30402 1.29989 11.773 < 2e-16 ***</pre>
```

```
## crossing
                   0.24127
                                0.03524
                                            6.846 1.11e-10 ***
##
## Signif. codes:
                                0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.8 on 183 degrees of freedom
## Multiple R-squared: 0.2039, Adjusted R-squared: 0.1996
## F-statistic: 46.87 on 1 and 183 DF, p-value: 1.109e-10
# print model diagnostics
par(mfrow = c(2, 2))
plot(model)
                                                      Standardized residuals
                  Residuals vs Fitted
                                                                           Normal Q-Q
                                                            2
                          1470
0105
                                                                                                 1470
Residuals
                                                                                               ഹ2∮05
                      025
      4
                                                            က
      -50
                                                            Ņ
                                                                                               2
                            30
                                                                      -2
                                                                                  0
                                                                                                     3
           15
                20
                      25
                                 35
                                       40
                                             45
                       Fitted values
                                                                        Theoretical Quantiles
Standardized residuals
                                                      Standardized residuals
                    Scale-Location
                                                                     Residuals vs Leverage
      2.0
                          0105
                                                                             0147
                      O25
      1.0
                            0
                                                                            ⊗k's distaneo
      0
                                                            Ņ
           15
                20
                      25
                            30
                                 35
                                             45
                                                                0.00
                                                                          0.02
                                                                                    0.04
                                                                                              0.06
                                       40
```

The attributes() function can be called on just about any object in R and it returns a list of all the things inside. It's a great way to explore objects and see what values are contained inside that could be used in other analysis. For example, extracting the residuals via model\$residuals is useful if we want to print diagnostic plots like those above.

Leverage

When we run summary() on the 1m object, we see the results. The *Call* section just prints back the model specification, and the *Residuals* section contains a summary of the distribution of the errors. The fun stuff is in the *Coefficients* section. In the first row contains the covariate names followed by their estimates, standard errors, t- and p-values. Our model ends up being rentals = 28 + 0.50(crosswalks) which means that the average number of rentals when there are no crosswalks is 28, and the average increases by 1 rental for every two additional crosswalks.

We can fit regressions with multiple covariates the same way:

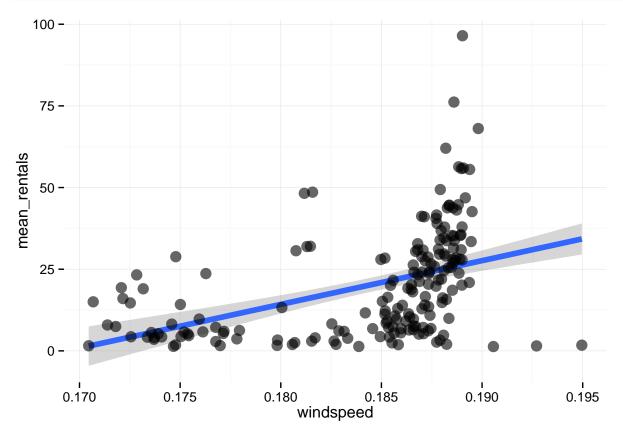
Fitted values

```
# lets include windspeed this time
rentals_multi = data %>%
  group_by(station_start) %>%
  summarize(mean_rentals = mean(no_rentals),
```

```
crossing = mean(crossing),
    windspeed = mean(windspeed))
head(rentals_multi)
```

```
## Source: local data frame [6 x 4]
##
##
                     station_start mean_rentals crossing windspeed
## 1
                                      19.003003
                    10th & E St NW
                                                      122 0.1731664
## 2
               10th & Monroe St NE
                                       7.580517
                                                        1 0.1866016
                    10th & U St NW
## 3
                                       37.954876
                                                        5 0.1890061
## 4 10th St & Constitution Ave NW
                                       28.430362
                                                      116 0.1886993
## 5
                    11th & H St NE
                                                       73 0.1889982
                                       20.121875
## 6
               11th & Kenyon St NW
                                       33.718331
                                                       20 0.1882405
```

```
ggplot(rentals_multi, aes(x = windspeed, y = mean_rentals)) +
  geom_smooth(method = 'lm', size = 2) +
  geom_point(size = 4, alpha = 0.60) +
  theme_minimal()
```



```
model = lm(mean_rentals ~ crossing + windspeed, data = rentals_multi)
summary(model)
```

```
##
## Call:
## lm(formula = mean_rentals ~ crossing + windspeed, data = rentals_multi)
```

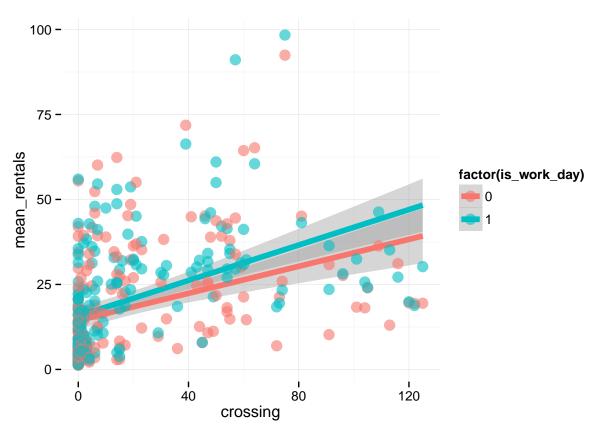
```
##
## Residuals:
##
      Min
               1Q Median
                                      Max
## -26.454 -9.202 -1.752
                            5.080 59.203
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -200.35799
                          34.20198 -5.858 2.15e-08 ***
## crossing
                 0.21373
                            0.03231
                                      6.616 3.99e-10 ***
## windspeed
              1172.33663 185.81081
                                      6.309 2.07e-09 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 13.45 on 182 degrees of freedom
## Multiple R-squared: 0.3468, Adjusted R-squared: 0.3396
## F-statistic: 48.31 on 2 and 182 DF, p-value: < 2.2e-16
```

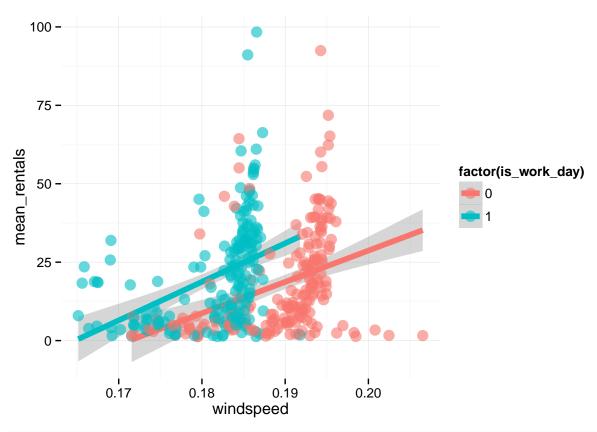
The model coefficients changed quite a lot when we added in windspeed. The intercept is now negative, and the windspeed coefficient is huge! When interpretting coefficients, it's important to keep the scale in mind. Windspeed ranges from 0.05 to 0.44 so when you multiply 2036 by 0.05 for example, you end up with about 102, which is within the range we'd expect.

Let's try one more, this time we'll include a factor variable.

theme\_minimal()

```
rentals_multi = data %>%
  group_by(station_start, is_work_day) %>%
  summarize(mean rentals = mean(no rentals),
            crossing = mean(crossing),
            windspeed = mean(windspeed))
head(rentals multi)
## Source: local data frame [6 x 5]
## Groups: station start
##
##
           station start is work day mean rentals crossing windspeed
## 1
          10th & E St NW
                                                       122 0.1858375
                                   0
                                        19.416667
## 2
          10th & E St NW
                                   1
                                        18.804444
                                                       122 0.1670843
## 3 10th & Monroe St NE
                                   0
                                        5.854054
                                                         1 0.1912622
## 4 10th & Monroe St NE
                                   1
                                        8.584906
                                                         1 0.1838902
## 5
          10th & U St NW
                                   0
                                        41.761062
                                                         5 0.1939839
## 6
          10th & U St NW
                                        36.088937
                                                         5 0.1865657
# plot crossings, colored by is_work_day
ggplot(rentals multi,
       aes(x = crossing, y = mean_rentals, color = factor(is_work_day))) +
  geom_smooth(method = 'lm', size = 2) +
  geom_point(size = 4, alpha = 0.60) +
```





```
##
## Call:
## lm(formula = mean_rentals ~ crossing + windspeed + factor(is_work_day),
##
       data = rentals_multi)
##
##
  Residuals:
##
      Min
                10 Median
                                3Q
                                       Max
   -28.943
           -9.728 -2.500
                             5.734
                                    61.718
##
##
## Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                                               -6.798 4.33e-11 ***
                        -165.77396
                                     24.38634
                                      0.02448
                                                8.316 1.81e-15 ***
## crossing
                           0.20358
## windspeed
                         949.26045
                                    128.75542
                                                7.373 1.13e-12 ***
## factor(is_work_day)1
                          10.05016
                                      1.81045
                                                5.551 5.46e-08 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
\#\# Residual standard error: 14.39 on 366 degrees of freedom
## Multiple R-squared: 0.2868, Adjusted R-squared: 0.281
## F-statistic: 49.06 on 3 and 366 DF, p-value: < 2.2e-16
```

The output looks a little funny now. There's a term called factor(is\_work\_day)1, what does that mean? Factors are category variables and their interpretation is relative to a baseline. Our factor is\_work\_day only

has two levels, 0 and 1, and R sets 0 to the baseline by default. So the interpretation of that term is that we can expect about 17 additional rentals when it is a work day (i.e. is\_work\_day == 0) and the other variables are fixed.

#### Train and test data

For all analyses in this class we'll need to divide our data into train and test sets. We'll do this using a package called *caret*. Check out this nice overview for more details.

## The caret package

The *caret* package in R contains helper functions that provide a unified framework for data cleaning/splitting, model training, and comparison. I highly recommend the optional reading this week which provides a great overview of the *caret* package.

```
install.packages('caret', dependencies = TRUE)
library(caret)
set.seed(1234) # set a seed
```

Setting a seed in R insures that you get identical results each time you run your code. Since resampling methods are inherently probabilistic, every time we rerun them we'll get slightly different answers. Setting the seed to the same number insures that we get identical randomness each time the code is run, and that's helpful for debugging.

#### Splitting data into test and train sets

In data mining we're interested in creating models for prediction, and we'll assess the quality of our models by quantifying their prediction accuracy. To measure prediction quality, we hold out a portion of our data called the *test* set. The *training* data is used to build the model.

```
## Resample1
## [1,] 14
## [2,] 25
## [3,] 27
## [4,] 61
## [5,] 71
## [6,] 83
```

```
training_set = data[in_train, ]
testing_set = data[-in_train, ]
dim(training_set)
```

```
## [1] 73977 154
```

```
dim(testing_set)
```

```
## [1] 24657 154
```

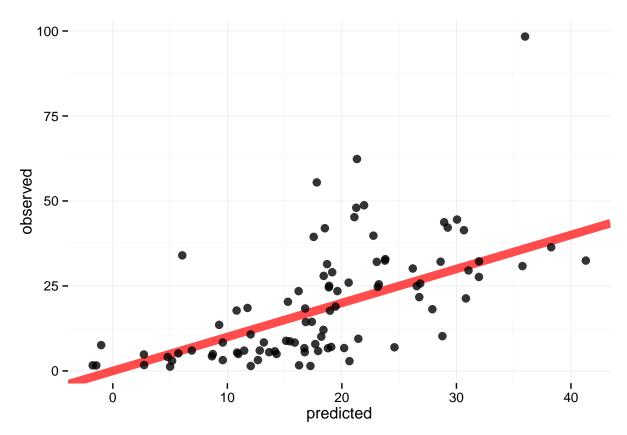
Note: I recommend doing all data processing and aggregation steps before splitting out your train/test sets.

## Fitting / Training

A workhorse function in the *caret* package in the train function. This function can be used to evaluate performance parameters, choose optimal models based on the values of those parameters, and estimate model performance. For regression we can use it in place of the lm() function. Here's our last regression model using the train function.

```
## Resample1
## [1,] 3
## [2,] 17
## [3,] 18
## [4,] 41
## [5,] 44
## [6,] 47
```

```
## Linear Regression
##
## 278 samples
##
     4 predictor
## No pre-processing
## Resampling: Bootstrapped (25 reps)
##
## Summary of sample sizes: 278, 278, 278, 278, 278, 278, ...
##
## Resampling results
##
##
    RMSE
              Rsquared
                         RMSE SD
                                   Rsquared SD
     14.8344 0.2514065 1.143292 0.05124707
##
##
##
```



Our prediction accuracy is not so great for this model. The RMSE is about 31 which means that on average the predictions are off by about 31 rentals.

## Feature Selection

Next time!

## Project tips

We saw this issue before when we constructed our SLR.

 $\label{eq:model_data} $$ model_data = data \%>\% \ group\_by(station\_start, weekday, season\_code, is\_holiday, is\_work\_day, weather\_code) \%>\% \ summarize(no\_rentals = mean(no\_rentals)) $$ head(model_data)$