

Homework 1

Due: Thursday 18 January

1. Find all solutions of the system of linear equations

$$\begin{aligned}x_1 + x_2 + x_3 &= 1 \\2x_1 - x_2 + 3x_3 &= 3 \\x_1 - 2x_2 + 2x_3 &= 2.\end{aligned}$$

2. Determine the values of k for which the following augmented matrices give a consistent system of linear equations:

(a) $\left[\begin{array}{cc|c} 1 & -3 & k \\ -2 & 6 & -5 \end{array} \right]$

(b) $\left[\begin{array}{cc|c} 1 & k & 4 \\ 3 & 6 & 8 \end{array} \right]$

3. Show that all of the following three planes always intersect regardless of the values of a and b .

$$\begin{aligned}x_1 - x_2 - x_3 &= 1 \\x_1 + x_2 + ax_3 &= -1 \\x_1 - x_2 + bx_3 &= 1\end{aligned}$$

If the three planes intersect at infinitely many points, what can you say about the values of a and b ? If the three planes intersect at a single point only, what can you say about the values of a and b ?

4. Find conditions on a, b so that the following system has: no solution, one solution, infinitely solutions.

$$\begin{aligned}x - 2y &= 1 \\ax + by &= 5\end{aligned}$$

5. Find the conditions on a and b so that the following system has: no solution, one solution and infinitely many solutions.

$$\begin{aligned}ax + y &= 1 \\2x + y &= b\end{aligned}$$