Q1

μ_1 = true mean of pop. model 1 stoves (time to boil water)
μ_2 = true mean of pop. model 2 stoves (time to boil water)
$\bar{X}_1$ = sample mean model 1 $\quad$ $\bar{X}_2$ = sample mean model 2

$\bar{X}_1 = 11.2 \quad S_1 = 2.6 \quad n_1 = 12$
$\bar{X}_2 = 9.8 \quad S_2 = 3.2 \quad n_2 = 14$

a) $H_0: \mu_1 = \mu_2 \rightarrow \mu_1 - \mu_2 = 0$
$H_a: \mu_1 \neq \mu_2 \rightarrow \mu_1 - \mu_2 \neq 0$

b) Assumptions:
- Populations of Model 1 & 2 stoves are normal
- Independent random samples

$$T = \frac{\bar{x} - \bar{y} - \mu_{x-y}}{\hat{\sigma}_{x-y}} \text{ with } \gamma \text{ df} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1}{n_1} + \frac{S_2}{n_2}}}$$

$$= \frac{1.4 - 0}{\sqrt{\frac{2.6}{12} + \frac{3.2}{14}}} = \frac{1.4}{.67} = 2.10 \text{ w/ } \gamma \text{ df}$$

c) $$\gamma = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{1.68}{.03 + .04} = \frac{1.68}{.07} = 24$$

Pval $= 2P(T > |t_{cal}, 24) = 2(.023) = .046$

d) At $\alpha = .05$ $P < .05$ $(.046 < .05)$ so reject H_0

e) At 95% confidence level, there is enough evidence to conclude there is a diff in avg time to boil 2 quarts of water at 50°C between Model 1 & 2 stoves

Q2

a) $\bar{X}_C = 80$ $S_c = 9.24$ where C is control group
$\bar{X}_P = 72$ $S_P = 9.79$ where P is Pet group
$\bar{X}_F = 82$ $S_F = 8.73$ where F is friend group

μ_x = true avg heart rate of subjects in x group's population
$H_0: \mu_C = \mu_P = \mu_F$
H_a: At least one of the population means is diff from others

b) Assumptions:

- samples/groups are independent of each other
- within groups observations are ind
- samples are taken from pops w/ normal dist
- Pops have equal variances

$F_{0.001}(2,42) = 8.25$

c) $\bar{X} = \frac{\bar{X}_C + \bar{X}_P + \bar{X}_F}{3} = \frac{80+72+82}{3} = 74.67$

$SST_r = \sum_{i=1}^{n} n_i(\bar{x}_i - \bar{x})^2$

$= 15(80-74.67)^2 + 15(72-74.67)^2 + 15(82-74.67)^2 = 1339$

$MST_r = \frac{SST_r}{k-1} = \frac{1339}{2} = 669.50$

$SSE = (n_1-1)s_1^2 + (n_2-1)s_2^2 + (n_3-1)s_3^2 = 14(9.24)^2 + 14(9.79)^2 + 14(8.73)^2$

$= 3604.08$

$MSE = \frac{SSE}{n-k} = \frac{3604.08}{45-3} = 85.81$ $F = \frac{MST_r}{MSE} = \frac{669.5}{85.81} = 7.80$

d) At 0.001 significance $7.80 < 8.25$ $F_{calc} < F_{crit}$ so fail to reject null hypo

e) We conclude at .001 significance level there is not enough evidence to reject H_0, that population mean heart rate of subjects in the 3 groups is diff from each other

Q3

a) $\bar{X}_1 = 23.4 \quad n_1 = 16 \quad S_1 = 10.1 \qquad CI = 99.9\%$

$\bar{X}_2 = 13.1 \quad n_2 = 24 \quad S_2 = 8.5$

$df = n_1 + n_2 - 2 = 16 + 24 - 2 = 38$

Since pop vars are equal sample variances should be pooled

$$S_p^2 = \frac{(n_1-1)\cdot s_1^2 + (n_2-1)\cdot s_2^2}{n_1+n_2-2} = \frac{(15)(10.1)^2 + (23)(8.5)^2}{38} = 84$$

$CI = (x_1 - x_2) \pm t^* \sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ where $t^* = 3.5660$ @ $t_{.0005,38}$

$$(23.4 - 13.1) \pm 3.566 \sqrt{84\left(\frac{1}{16} + \frac{1}{24}\right)} = 10.3 \pm 10.518$$

CI @ $99.9\% = [-0.218, 20.818]$

b) Type II error $= 1 - \beta = \phi(z - z_{1-\alpha/2}) + \phi(-z - z_{1-\alpha/2})$ where

$$z = \frac{\mu_A - \mu_B}{\sigma\sqrt{\frac{1}{n_A} + \frac{1}{n_B}}} \to \phi($$

Q4

a) $m = n = \dfrac{\left(z_{\alpha/2}\sqrt{\frac{(p_1+p_2)(q_1+q_2)}{2}} + z_\beta\sqrt{p_1q_1+p_2q_2}\right)^2}{(p_1-p_2)^2}$

$z_{\alpha/2} = z_{.005} = 2.575$

$z_\beta = z_{.12} = -1.175$

$\dfrac{\left(2.575\sqrt{\frac{(.1+.05)(.9+.95)}{2}} + 1.175\sqrt{(.1)(.9)+(.05)(.95)}\right)^2}{(.05)^2}$

$\dfrac{(.9591+.4357)^2}{(.05)^2} = \dfrac{1.9455}{.0025} \doteq 778.19 \to \boxed{779}$

b) error

c) interval estimate

d) narrower

e) True

f) $SE = \sqrt{\dfrac{\hat{p}_1\hat{q}_1}{m} + \dfrac{\hat{p}_2\hat{q}_2}{n}} = \sqrt{\dfrac{(.3)(.7)}{200} + \dfrac{(.167)(.833)}{300}} = \boxed{.0389}$

$60/200 = .3 = \hat{p}_1 \quad \hat{q}_1 = .7$

$50/300 = .167 = \hat{p}_2 \quad \hat{q}_2 = 1 - .167 = .833$

$m = 200 \quad n = 300$

g) $df = n-1 = 4$

$x_1 - x_2$: $-6 \; 2 \; 4 \; 5 \; 2$

$S = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}} =$

$\bar{x} = \dfrac{7}{5} = 1.4$

$S = \sqrt{\dfrac{75.2}{4}} = \boxed{4.34}$

$\sum_{i=1}^{n}(x_i-\bar{x})^2 = (-6-1.4)^2 + (2-1.4)^2 + (4-1.4)^2 + (5-1.4)^2 + (2-1.4)^2 = 75$

h) $\bar{x}_A = 7 \quad \bar{x}_B = 11 \quad \bar{x}_C = 6 \quad \bar{x}_0 = (7+11+6)/3 = 8$

$SSTr = \sum_{i=1}^{n} m\cdot(\bar{x}_i - \bar{x}_0)^2 = 4\cdot(7-8)^2 + 4(11-8)^2 + 4(6-8)^2 = 4+36+16 = \boxed{56}$

$SST = (5-8)^2 + (9-8)^2 + (6-8)^2 + (9-8)^2 + (11-8)^2 + (14-8)^2 + (10-8)^2 + (6-8)^2 + (3-8)^2 + (7-8)^2 =$
$9+1+4+1+9+36+4+4+25+1 = \boxed{94}$