

# Theory of mechanical spin current generation via spin–rotation coupling

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## ABSTRACT

The spin–rotation coupling (SRC) is responsible for angular momentum conversion between the electron spin and rotational deformations of elastic media, whereby spin current generation from mechanical motions is theoretically predicted. Here a surface acoustic wave (SAW) is considered as an example of such elastic deformations. We formulate the spin diffusion equation that is extended to include the influence of the SRC, and from the solution we find that larger spin currents can be obtained in materials with longer spin lifetimes. This distinct feature arises from the fact that the present method is of “purely” mechanical origin, i.e., it relies on neither equilibrium magnetization nor spin–orbit coupling, offering a new route to building the so-called “rare-metal-free” spintronics devices.

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## 1. Introduction

Control of spin current is a key technology in spintronics [1,2], and it relies on basic couplings between an electron spin and other physical quantities such as a magnetic field (Zeeman coupling), magnetization (exchange coupling), and orbital degrees of freedom [spin–orbit coupling (SOC)]. Recently we have shown that the SOC links the spin and mechanical degrees of freedom and enables spin current generation from mechanical rotations and vibrations [3–5]. A work of the series is overviewed in a separate paper of this special issue on Spin Mechanics [6].

In this paper we present a different route for mechanical generation of spin current by considering spin–rotation coupling (SRC), a direct coupling between spin and mechanical rotation [7,8]. The bare SRC for electron spin was rigorously derived from the low energy expansion of the Dirac equation in an accelerated frame of Ref. [9] and is given by

$$H_S = -\mathbf{S} \cdot \mathbf{\Omega}, \quad (1)$$

where  $\mathbf{S} = \hbar\boldsymbol{\sigma}/2$  is the electron spin angular momentum and  $\mathbf{\Omega}$  is the mechanical rotation frequency. One can observe a close similarity between the SRC and the Zeeman coupling (see Table 1), and accordingly, Eq. (1) is renormalized in solid as [7]  $H'_S = -(1 + \delta g)\mathbf{S} \cdot \mathbf{\Omega}$ , where  $\delta g = g - g_0$  denotes the difference

between the renormalized  $g$ -factor of electrons in solids and the bare value  $g_0 = 2$  that appears in the Zeeman coupling.

Historically the SRC has been regarded as a fundamental origin of the Barnett and Einstein–de Haas effects [10–12] and plays a pivotal role in neutron interferometry [13] as well as in tests of general relativity using spin precession in gravitational fields of rotating bodies [14–18]. In the recent context of nanomagnetism, it leads to a variety of studies ranging from the observation of Einstein–de Haas effect in a NiFe film on a microcantilever [19] and mechanical torque due to spin flip on a torsion oscillator [20], to theoretical prediction of rotational doppler shifts in magnetic resonance [21] and effects of mechanical torque in nanostructures [22–29]. However, it has not been used for controlling spin currents.

To induce spin current, it requires the nonequilibrium spin states. Typically, spin accumulation is produced in nonlocal spin valves [30]. In ferromagnets, spin dynamics is excited by ferromagnetic resonance [31], temperature gradient [32], and sound waves in ferromagnets [33,34]. In nonmagnets, the spin Hall effect (SHE) provides an alternative pathway for generating spin current [35], in which a strong SOC is demanded. Essentially, the SOC-mediated mechanical spin current generation (mechanical SHE) discussed in [6] belongs to the latter category.

In contrast to those methods, the SRC-mediated method for generating spin current is of purely mechanical origin, i.e., it is free from equilibrium magnetization and SOC, and hence, is particularly relevant in nonmagnets with longer spin lifetime.

The outline of this paper is as follows. In Section 2, we formulate the spin diffusion equation with the SRC as a source

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term. In Section 3, we solve the spin diffusion equation under lattice rotation driven by a surface acoustic wave. In Section 4, we evaluate the mechanically induced spin current in various non-magnetic materials. In Section 5, we briefly comment on the detection method. Section 6 is devoted to conclusion.

## 2. Spin current dynamics induced by SRC

In this section, we present general treatment of the spin current dynamics induced by the SRC. Here, we consider rotational motion of the lattice

$$\mathbf{\Omega} = \frac{1}{2} \nabla \times \dot{\mathbf{u}}, \quad (2)$$

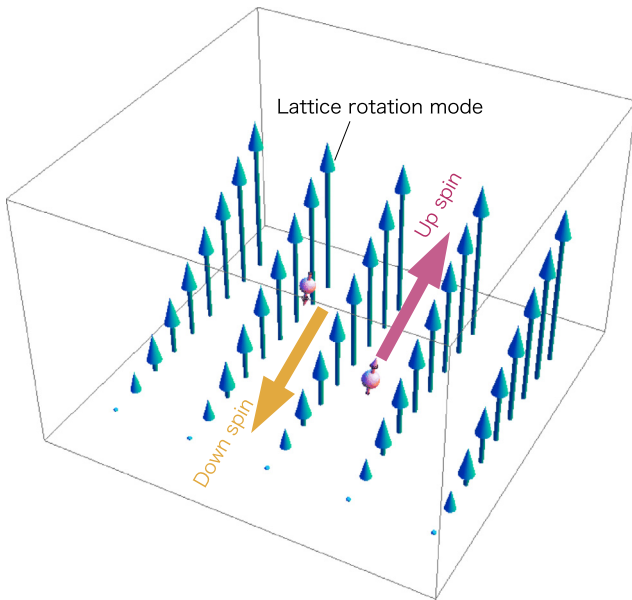
where  $\mathbf{u}$  is the displacement vector of the lattice [36] and the velocity field  $\dot{\mathbf{u}}$  plays the same role as the vector potential in the standard electromagnetism [12]. When the lattice vibration has transverse components, the right hand side of Eq. (2) remains finite. In this case, the mechanical angular momentum of the lattice can couple to spin angular momentum via the SRC,  $H'_S$  [37].

Then, one can expect a mechanical analogue of the Stern–Gerlach effect (SGE): when there is a finite gradient of the mechanical rotation, up and down spins are driven in the opposite direction as shown in Fig. 1. This is the principal driving force for the spin current generation via the SRC. However, a static field gradient in nonmagnetic conductors will be compensated by rearrangement of the electron band after the spin relaxation process and cannot generate spin current steadily.

One means to induce the nonequilibrium spin state is to resort to the lattice dynamics. To quantify the dynamical effect, we

**Table 1**  
Spin-rotation coupling vs. Zeeman coupling.

| Spin-rotation  | Zeeman   |
|--|--|
| $H'_S = -(1+\delta g)\mathbf{S} \cdot \mathbf{\Omega}$         | $H_Z = (2+\delta g)\frac{\mu_B}{h}\mathbf{S} \cdot \mathbf{B}$ |
| $\mathbf{\Omega} = \frac{1}{2} \nabla \times \dot{\mathbf{u}}$ | $\mathbf{B} = \nabla \times \mathbf{A}$                        |
| $\dot{\mathbf{u}}$ (velocity field)                            | $\mathbf{A}$ (vector potential)                                |



**Fig. 1.** (Color online) Mechanical Stern–Gerlach effect: spin current induced by the gradient of mechanical rotation. Blue arrows indicate the direction and magnitude of the mechanical rotation. Up and down spins tend to move oppositely toward the direction for lowering the SRC energy resulting in a pure spin current along the field gradient.

examine the time evolution of the spin density under the time-dependent SRC. When the mechanical rotation,  $\mathbf{\Omega}$ , whose axis is in the z-direction, is applied, the electron spins align parallel to the axis of rotation. This is known as the Barnett effect [10]. In this case, the bottom of the energy band of the electron is shifted by  $\pm(1+\delta g)\hbar\mathbf{\Omega}/2$  for the up/down spin sub-band as shown in Fig. 2(a). The number density of up (down) spin electrons is then given by

$$n_{\uparrow(\downarrow)} = \int_{\pm(1+\delta g)\hbar\mathbf{\Omega}/2}^{\mu_{\uparrow(\downarrow)}} d\epsilon N_0(\epsilon), \quad (3)$$

where  $N_0$  is the density of states for electrons in non-magnets, and  $\mu_{\uparrow}$  and  $\mu_{\downarrow}$  are the chemical potentials for up and down spin electrons, respectively. Then, spin density is calculated as

$$n_{\uparrow} - n_{\downarrow} \approx N_0[\delta\mu - (1+\delta g)\hbar\mathbf{\Omega}], \quad (4)$$

where

$$\delta\mu = \mu_{\uparrow} - \mu_{\downarrow}, \quad (5)$$

is the spin accumulation. Here, a constant density of state is assumed for simplicity. Spin relaxation occurs in two processes: one is on-site spin flip with the spin lifetime,  $\tau_{sf}$ , and the other is spin diffusion with the diffusion constant,  $D$ , as shown in Fig. 2(b) and (c), respectively. Equating these processes leads to

$$\partial_t(n_{\uparrow} - n_{\downarrow}) = -\frac{N_0\delta\mu}{\tau_{sf}} + D\nabla^2(N_0\delta\mu). \quad (6)$$

Substituting Eq. (4) into Eq. (6), one obtains the extended spin diffusion equation in the presence of the SRC:

$$(\partial_t - D\nabla^2 + \tau_{sf}^{-1})\delta\mu = (1+\delta g)\hbar\partial_t\mathbf{\Omega}. \quad (7)$$

Note that the right-hand-side of Eq. (7) is a source term originating from the SRC and vanishes for constant  $\mathbf{\Omega}$ . In addition, for the uniform rotation the induced spin accumulation is also uniform. The z-polarized spin current can be calculated from the solution of Eq. (7) as

$$\mathbf{J}_s^z = \frac{\sigma_0}{e} \nabla \delta\mu, \quad (8)$$

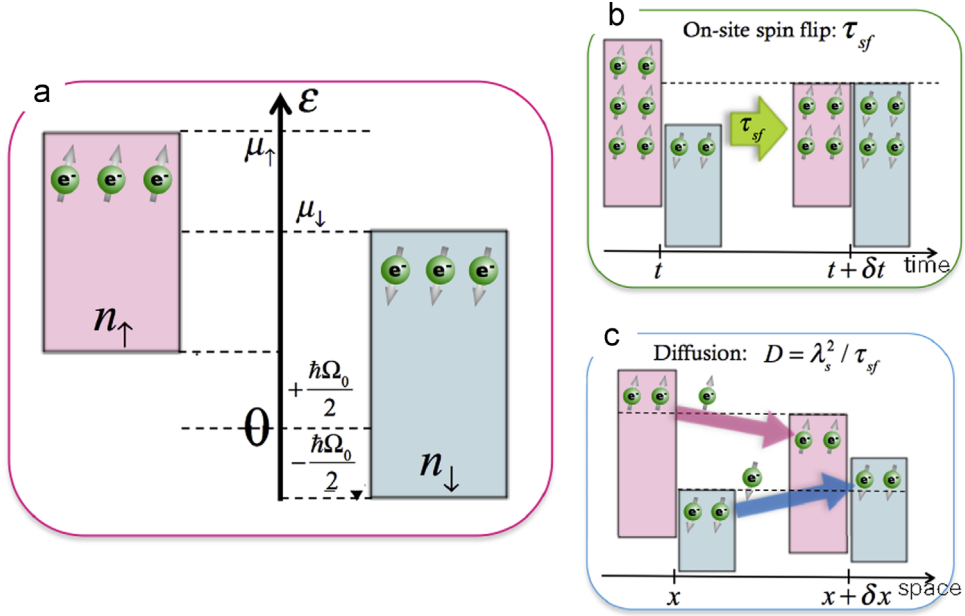
with conductivity  $\sigma_0$ . The condition for generation of spin current is now clear: mechanical rotations have to depend on both time and space. In terms of the lattice vibrations, they should have a transverse component.

## 3. Spin accumulation induced by SAW

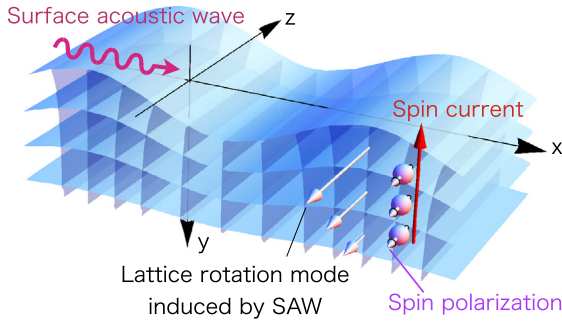
In the previous section, we observed that the spatio-temporal lattice deformation can generate spin current. To realize this situation, we consider the Rayleigh type surface acoustic waves (SAWs) where rotational deformations vary in space and time as illustrated in Fig. 3. In the presence of SAW, a gradient of mechanical rotation is induced in the attenuation direction in addition to the SAW propagation direction. In this section, by solving Eq. (7) in the presence of SAW, we evaluate the induced spin accumulation at the surface of a nonmagnetic film.

Let us consider generation of spin current due to the SRC of SAWs in nonmagnetic metals or semiconductors. Our setup is shown in Fig. 3. SAWs propagate along the x-direction and penetrate a nonmagnetic material along the y-direction. The viscous damping of the lattice vibrations is small and neglected. Mechanical rotation around the z-axis is then produced and the angular frequency  $\mathbf{\Omega} = (0, 0, \Omega)$  of the rotational motion is given by [36]

$$\Omega(x, y, t) = \frac{\omega^2 u_0}{2c_t} \exp\{-k_t y + i(kx - \omega t)\}, \quad (9)$$



**Fig. 2.** (Color online) Spin dynamics under the influence of the SRC. (a) Shifts of the spin dependent energy bands due to the onset of the SRC,  $\pm \hbar\Omega_0/2$ . The resultant spin accumulation,  $\delta\mu = \mu_\uparrow - \mu_\downarrow$ , relaxes in two processes: (b) spin flip in time and (c) spin diffusion in space.



**Fig. 3.** (Color online) Schematic illustration of mechanical spin current generation in the presence of a SAW propagating in the  $x$ -direction. Mechanical rotation induced around the  $z$ -axis couples to electron spins as an effective magnetic field, and its gradient causes the  $z$ -polarized spin current flows in the  $y$ -direction in addition to the  $x$ -direction.

where  $\omega$  and  $u_0$  are the frequency and amplitude of the mechanical rotation,  $k$  is the wave number,  $c_t$  is the transverse sound velocity, and  $k_t$  is the transverse wave number. The frequency  $\omega$  is related to the wave number as

$$\omega = c_t k \xi, \quad (10)$$

and the transverse wave number is

$$k_t = k \sqrt{1 - \xi^2}, \quad (11)$$

where  $\xi$  satisfies the equation

$$\xi^6 - 8\xi^4 + 8\xi^3(3 - 2c_t^2/c_l^2) - 16(1 - c_t^2/c_l^2) = 0, \quad (12)$$

and  $c_l$  is the longitudinal sound velocity. The Poisson ratio,  $\nu$ , is related to the ratio of velocities as

$$\left(\frac{c_t}{c_l}\right)^2 = \frac{1 - 2\nu}{2(1 + \nu)}, \quad (13)$$

and  $\nu$  and  $\xi$  are related as

$$\xi \approx \frac{0.875 + 1.12\nu}{1 + \nu}. \quad (14)$$

Spin accumulation generated by the SAW can be evaluated by solving Eq. (7). By separating the variables as

$$\delta\mu = \delta\mu_y(y, t)e^{ikx}, \quad (15)$$

and inserting it with Eq. (9) into Eq. (7), the spin diffusion equation can be rewritten as

$$(\partial_t - D\partial_y^2 + \tilde{\tau}_{sf}^{-1})\delta\mu_y(y, t) = -i\omega\hbar\Omega_0 e^{-k_t y - i\omega t}, \quad (16)$$

where the effective relaxation time is given by

$$\tilde{\tau}_{sf}^{-1} = \tau_{sf}^{-1}(1 + \lambda_s^2 k^2), \quad (17)$$

with the spin diffusion length

$$\lambda_s = \sqrt{D\tau_{sf}}, \quad (18)$$

and

$$\Omega_0 = (1 + \delta g) \frac{\omega^2 u_0}{2c_t}. \quad (19)$$

With the boundary condition

$$\partial_y \delta\mu = 0, \quad (20)$$

on the surface  $y=0$ , the solution is calculated by Green's function and the method of image as

$$\frac{\delta\mu_y(y, t)}{\hbar\Omega_0} = -i(\omega\tilde{\tau}_{sf}) \int_0^\infty dT' \int_0^\infty dY' \times K(Y, Y'; T - T') e^{-(k_t \tilde{\lambda}_s) Y'} e^{-i(\omega\tilde{\tau}_{sf}) T'}, \quad (21)$$

where  $T = t/\tilde{\tau}_{sf}$ ,  $Y = y/\tilde{\lambda}_s$  with  $\tilde{\lambda}_s = \sqrt{D\tilde{\tau}_{sf}}$ , and

$$K(Y, Y'; T) = \frac{\theta(T)e^{-T}}{\sqrt{4\pi T}} (e^{-(Y - Y')^2/4T} + e^{-(Y + Y')^2/4T}). \quad (22)$$

Let us consider the time evolution of spin accumulation at the surface,  $y=0$ . Because each spin aligns parallel to the rotation axis, i.e., the  $\pm z$ -axis, a striped pattern of spin accumulation with the in-plane polarization arises at the surface with the same wavelength of SAW,  $2\pi/k$ .

#### 4. Spin current from SAW

Next, we examine the spin current (8) from the spin accumulation obtained in the previous section. In the  $x$  direction, the  $z$ -polarized spin current is

$$J_{sx}^z(x, y, t) = J_{sx0}^z e^{ik_x x} \int_0^\infty dT' \int_0^\infty dY' \times K(Y, Y'; T - T') e^{-(k_t \tilde{\lambda}_s)Y' - i(\omega \tilde{\tau}_{sf})T'}, \quad (23)$$

while in the  $y$  direction

$$J_{sy}^z(x, y, t) = iJ_{sy0}^z e^{ik_x x} \int_0^\infty dT' \int_0^\infty dY' \times F(Y, Y'; T - T') e^{-(k_t \tilde{\lambda}_s)Y' - i(\omega \tilde{\tau}_{sf})T'}, \quad (24)$$

where

$$F(Y, Y'; T) = \frac{\theta(T)e^{-T}}{\sqrt{16\pi T^3}} (e^{-(Y-Y')^2/4T} (Y-Y') + e^{-(Y+Y')^2/4T} (Y+Y')), \quad (25)$$

and

$$J_{sx0}^z = (\omega \tilde{\tau}_{sf}) \frac{\sigma_0}{e} k \hbar \Omega_0, \quad J_{sy0}^z = (\omega \tilde{\tau}_{sf}) \frac{\sigma_0}{e} \frac{\hbar \Omega_0}{\tilde{\lambda}_s}. \quad (26)$$

In Fig. 4, spatial profiles along the  $y$  direction of the SAW-induced spin accumulation and spin current for a certain time are shown. Because of the interplay between the initially excited spin current and the back-flow near the surface,  $\delta\mu_y$  has the maximum near the surface,  $y \approx \tilde{\lambda}_s$ , and the  $z$ -polarized spin currents flow both to the surface and inward to the sample. While spin current also flows in the  $x$  direction due to the periodic modulation of the rotation amplitude it might be difficult to observe such an in-plane spin current and is irrelevant to spin injection into attached contacts. These profiles oscillate with the same frequency and wavelength along the  $x$  direction as those of the SAW.

Let us examine the mechanically induced spin current in typical nonmagnetic materials. As seen in Eq. (26),  $J_{sy0}^z \propto \tau_{sf}^{1/2}$  and the larger spin current can be obtained from materials with the longer spin lifetime, i.e., weaker SOC. Using Eqs. (8) and (21), the maximum values of the spin current for Cu ( $\tau_{sf} = 42$  ps) and Pt ( $\tau_{sf} = 0.3$  ps) are computed in comparison with the SOC-mediated generation of spin current from mechanical rotation and vibration as listed in Table 2. It is worth noting that the spin current generated in metals with weak spin–orbit interaction such as Al and Cu is much larger than that in Pt.

#### 5. Detection of SAW-induced spin signals

Experimentally, spin precession controlled by SAW was observed by using the time-resolved polar magneto-optic Kerr effect (MOKE) [38,39] in which the spatial and time resolution of the order of 10  $\mu\text{m}$  and 10 ns, respectively, have been achieved. MOKE measurements for metallic multilayers are also investigated [40]. Since in-plane spin polarization is induced in our setup, transversal or longitudinal MOKE measurements are relevant to the SAW-induced spin accumulation [41]. Conventionally, generation of spin current in nonmagnetic materials has required strong SOI because the spin Hall effect has been utilized. In other words, nonmagnetic materials with short spin lifetimes have been used. On the contrary, the mechanism proposed here requires longer spin lifetimes to generate larger spin currents. This means that Al and Cu, which have been considered as good materials for a spin conducting channel, can be favorable for generating spin current. Therefore, more options are available for spin-current generation in nonmagnets than ever before.

The AC nature of the proposed spin current has several advantages: one of the valuable applications would be for the spin-torque ferromagnetic resonance (ST-FMR) [42], in which AC spin currents excite magnetization dynamics of nanomagnets in the several GHz range. The predicted AC spin current can provide an alternative route for the ST-FMR: conventionally an AC charge current is converted into an AC spin current via the SHE which requires materials with the large spin-Hall angle such as Pt and Ta whereas our method does not rely on the SHE and Cu or Al can be used. This indicates that our finding has a significant impact on the development of “rare-metal-free” spintronic devices and promotes the further inquiry of the ST-FMR technique.

Moreover, the interaction between SAW and spin current revealed here has potentials to equip new functionalities with conventional (charge-current based) SAW devices which are built in mobile phones for RF bandpass filtering. From an application point of view, we note that the SAW-induced spin current in nonmagnets does not rely on magnetic properties. This feature is of great advantage that the spin current can be utilized in a magnetically shielded space.

When top or bottom contacts are attached for detection they would modify the properties of the SAW. Such modification of the SAW due to putting contacts on the sample does not essentially change our results. Technically, one can easily handle the modified SAWs and estimate resulting spin currents by solving our equations with certain boundary conditions. In addition, SAW devices with contacts are well-controlled in a lot of experiments or industrial applications. In the context of spintronics, SAW-induced spin

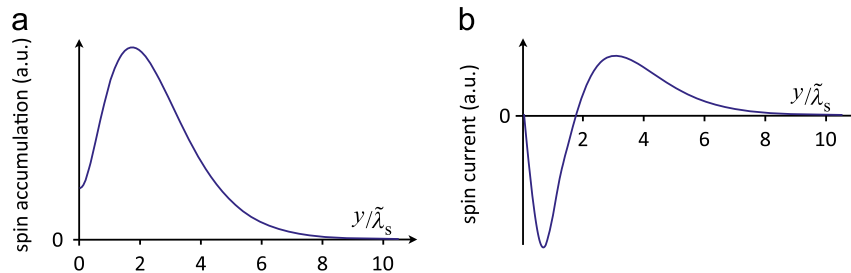


Fig. 4. (Color online) Snapshot of the SAW-induced spin accumulation (a) and spin current (b). Parameters used here are  $k_t \tilde{\lambda}_s = 1$ ,  $\omega \tilde{\tau}_{sf} = 1$ .

Table 2  
Mechanical spin current generation in nonmagnetic materials.

| Method         | Mechanical motion   | Input parameters                                | $J_s(\text{Pt})$ (A/m <sup>2</sup> ) | $J_s(\text{Cu})$ (A/m <sup>2</sup> ) | Ref.  |
|----------------|---------------------|---|--------------------------------------|--------------------------------------|-------|
| Mechanical SHE | Rigid rotation      | $B = 1$ T, $r = 1$ mm, $\Omega/(2\pi) = 10$ kHz | $10^6$                               | –                                    | [3,4] |
| Mechanical SHE | Rigid vibration     | $u = 1$ nm, $\omega/(2\pi) = 2.5$ GHz           | $10^6$                               | –                                    | [5]   |
| Mechanical SGE | Elastic deformation | $u_0 = 1$ nm, $\omega/(2\pi) = 2.5$ GHz         | $10^4$                               | $10^7$                               | [8]   |



current in ferromagnets is indeed observed by Pt contacts attached to the sample using the inverse spin Hall effect [33,34].

The acoustic wave may affect spin relaxation via the SOC. If ones use materials with strong SOC, the strain SOC will be modified by the SAW, which can affect the spin relaxation. However, in the present paper, we seek the opposite limit: in the case of Al or Cu, which have negligible SOC, the SAW cannot cause the spin relaxation whereas the magnitude of the generated spin current becomes larger. This is one of the novel properties of our proposal which upsets the conventional wisdom of spin current generation using nonmagnetic materials.

The present results are not restricted to the specific modes of lattices and can be generalized for other lattice dynamics. SAW discussed above is the Rayleigh wave, which induces rotation with the axis parallel to the surface. For instance, the Love wave [43], a horizontally polarized shear wave, can be utilized to generate spin currents whose spin polarization is perpendicular to the surface. More generally, the *transverse* phonons in bulk can excite the spin dynamics, in which the AC spin current flows along the phonon propagation direction.

## 6. Conclusion

We have shown new spin current physics arising from the spin–rotation coupling, a fundamental interaction originating from the general relativity, but which has not been used for generating spin current. The proposed method is of purely mechanical origin, i.e., it is independent of exchange coupling and spin–orbit coupling. Lattice dynamics directly excites the nonequilibrium state of electron spins, and consequently, spin current can be generated in nonmagnets.

To include effects of spin–rotation coupling the spin diffusion equation has been extended and the solution of the equation has revealed that the spin current is generated parallel to the gradient of the rotation. As an example, we have theoretically demonstrated that SAW, a situation in which rotational motion of lattice couples with electron spins, can be exploited for spin current generation, illustrating that larger spin current can be generated in nonmagnetic materials with longer spin lifetimes. These findings imply that the proposed method would have the merits in building the so-called “rare-metal free” spin devices and can be used to excite ferromagnetic resonance via spin-transfer-torque effects without spin Hall effects.

The use of spin–rotation coupling, argued here, opens up a new pathway for creating spin currents by elastic waves and would be one of the principal concepts in the field of novel technology and science *Spin Mechanics*.

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