

# Mechanical generation of spin current by spin-rotation coupling

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Spin-rotation coupling, which is responsible for angular momentum conversion between the electron spin and rotational deformations of elastic media, is exploited for generating spin current. This method requires neither magnetic moments nor spin-orbit interaction. The spin current generated in nonmagnets is calculated in the presence of surface acoustic waves. We solve the spin diffusion equation, extended to include spin-rotation coupling, and find that larger spin currents can be obtained in materials with longer spin lifetimes. Spin accumulation induced on the surface is predicted to be detectable by time-resolved Kerr spectroscopy.

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**Introduction.** Spin current, a flow of spins, is a key concept in the field of spintronics.<sup>1,2</sup> It can be generated from nonequilibrium spin states, i.e., spin accumulation and spin dynamics. The former is routinely produced in nonlocal spin valves.<sup>3</sup> In ferromagnets, the latter is excited by ferromagnetic resonance,<sup>4</sup> temperature gradient,<sup>5</sup> and sound waves in the magnetic insulator.<sup>6</sup> Alternatively, spin currents in nonmagnets have been generated by the spin Hall effect,<sup>7</sup> in which a strong spin-orbit interaction (SOI) is utilized. All these existing methods rely on exchange coupling of spins with local magnetization or on SOI.

In this Rapid Communication, we pursue a new route for generating spin currents by considering spin-rotation coupling:<sup>8</sup>

$$H_S = -\frac{\hbar}{2} \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}, \quad (1)$$

where  $\hbar\boldsymbol{\sigma}/2$  is the electron spin angular momentum and  $\boldsymbol{\Omega}$  is the mechanical rotation frequency. The method requires neither magnetic moments nor SOI. In this sense, the mechanism proposed here is particularly relevant in nonmagnets with longer spin lifetime.

**Nonuniform rotational motion.** Here, we consider rotational motion of the lattice:

$$\boldsymbol{\Omega} = \frac{1}{2} \nabla \cdot \dot{\mathbf{u}}, \quad (2)$$

where  $\mathbf{u}$  is the displacement vector of the lattice.<sup>9</sup> When the lattice vibration has transverse modes, Eq. (2) does not vanish. In such a case, the mechanical angular momentum of the lattice can be converted into spin angular momentum via  $H_S$ .<sup>10</sup> However, as shown later, that  $\boldsymbol{\Omega}$  is finite is insufficient to generate spin currents elastically. Both the time derivative and the gradient of rotational modes are necessary for the generation of spin current. For this purpose, we focus on surface acoustic waves (SAWs), which induce rotational deformations that vary in space and time (Fig. 1).

In the presence of SAW, a gradient of mechanical rotation is induced in the attenuation direction. We extend the spin diffusion equation to include the coupling between elastic rotation and spin. By solving the equation in the presence of SAW, we can evaluate the induced spin current for metals and semiconductors.

**Spin diffusion equation with spin-rotation coupling.** First of all, we examine effects of spin-rotation coupling on spin density. When the mechanical rotation,  $\boldsymbol{\Omega}$ , whose axis is in the  $z$  direction, is applied, the electron spins align parallel to the axis of rotation. This is known as the Barnett effect.<sup>11</sup> In this case, the bottom of the energy band of the electron is shifted by  $\hbar\Omega/2$ . The number density of up (down) spin electrons is then given by

$$n_{\uparrow(\downarrow)} = \int_{\pm\hbar\Omega/2}^{\mu_{\uparrow(\downarrow)}} d\varepsilon N_0(\varepsilon), \quad (3)$$

where  $N_0$  is the density of states for electrons, and  $\mu_{\uparrow}$  and  $\mu_{\downarrow}$  are chemical potentials for up and down spin electrons, respectively. Then, spin density can be estimated as

$$n_{\uparrow} - n_{\downarrow} \approx N_0(\delta\mu - \hbar\Omega), \quad (4)$$

where  $\delta\mu = \mu_{\uparrow} - \mu_{\downarrow}$  is spin accumulation. Here, a constant density of state is assumed for simplicity. Spin relaxation occurs in two processes: one is on-site spin flip with the spin lifetime,  $\tau_{sf}$ , and the other is spin diffusion with the diffusion constant,  $D$ . Equating these processes leads to  $\partial_t(n_{\uparrow} - n_{\downarrow}) = \tau_{sf}^{-1}N_0\delta\mu + D\nabla^2(N_0\delta\mu)$ .

Now, we obtain the extended spin diffusion equation in the presence of spin-rotation coupling:

$$(\partial_t - D\nabla^2 + \tau_{sf}^{-1})\delta\mu = \hbar\partial_t\Omega, \quad (5)$$

The right-hand side of Eq. (5) is a source term originating from spin-rotation coupling. Z-polarized spin current can be calculated from the solution of Eq. (5) as

$$\mathbf{J}_s^z = \frac{\sigma_0}{e} \nabla \delta\mu, \quad (6)$$

with conductivity  $\sigma_0$ . If the mechanical rotation is constant with time, the source term vanishes. Moreover, even if mechanical rotation depends on time, the uniform rotation in space cannot generate spin currents because spin accumulation is independent of position.

**Spin accumulation induced by SAW.** Let us consider generation of spin current due to spin-rotation coupling of SAWs in nonmagnetic metals or semiconductors. Our setup is shown in Fig. 1(a). SAWs are generated in the  $xz$  plane and penetrate a nonmagnetic material along the  $y$  direction.

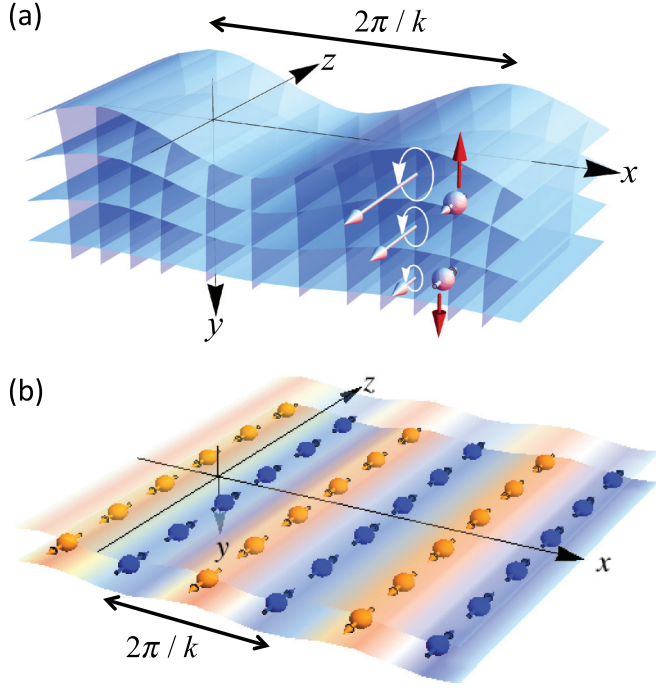


FIG. 1. (Color online) Snapshot of mechanical generation of spin current induced by SAW. (a) In the presence of a SAW propagating in the  $x$  direction, a gradient of mechanical rotation around the  $z$  axis is induced. The rotation couples to electron spins, and then the  $z$ -polarized spin current flows in the  $y$  direction. (b) Spin accumulation induced on the surface. Because spins are polarized parallel to the rotation axis ( $\pm z$ ), the striped pattern of spin accumulation arises at the surface.

They then induce mechanical rotation around the  $z$  axis, whose frequency  $\Omega = (0, 0, \Omega)$  is given by<sup>9</sup>

$$\Omega(x, y, t) = \frac{\omega^2 u_0}{2c_t} \exp\{-k_t y + i(kx - \omega t)\}, \quad (7)$$

where  $\omega$  and  $u_0$  are the frequency and amplitude of the mechanical resonator,  $k$  is the wave number,  $c_t$  is the transverse sound velocity, and  $k_t$  is the transverse wave number. The frequency  $\omega$  is related to the wave number as  $\omega = c_t k \xi$  and the transverse wave number as  $k_t = k\sqrt{1 - \xi^2}$ , where  $\xi$  satisfies the equation  $\xi^6 - 8\xi^4 + 8\xi^3(3 - 2c_t^2/c_l^2) - 16(1 - c_t^2/c_l^2) = 0$  and  $c_l$  is the longitudinal sound velocity. The Poisson ratio,  $\nu$ , is related to the ratio of velocities as  $(c_t/c_l)^2 = (1 - 2\nu)/(2(1 + \nu))$ , and  $\nu$  and  $\xi$  are related as  $\xi \approx (0.875 + 1.12\nu)/(1 + \nu)$ .

Spin accumulation generated by the SAW can be evaluated by solving Eq. (5). By inserting Eq. (7) and  $\delta\mu = \delta\mu_y(y, t)e^{ikx}$  into Eq. (5), the spin diffusion equation can be rewritten as

$$(\partial_t - D\partial_y^2 + \tilde{\tau}_{sf}^{-1})\delta\mu_y(y, t) = -i\omega\hbar\Omega_0 e^{-k_t y - i\omega t}, \quad (8)$$

where  $\tilde{\tau}_{sf} = \tau_{sf}(1 + \lambda_s^2 k^2)^{-1}$  with the spin diffusion length  $\lambda_s = \sqrt{D\tau_{sf}}$  and  $\Omega_0 = \omega^2 u_0 / 2c_t$ . With the boundary condition  $\partial_y \delta\mu = 0$  on the surface  $y = 0$ , the solution is given by

$$\delta\mu_y(y, t) = -i\omega\hbar\Omega_0 \int_0^\infty dt' \int_0^\infty dy' \frac{\theta(t - t')e^{-(t-t')/\tilde{\tau}_{sf}}}{\sqrt{4\pi D(t-t')}} \times \left[ e^{-\frac{(y-y')^2}{4D(t-t')}} + e^{-\frac{(y+y')^2}{4D(t-t')}} \right] e^{-k_t y' - i\omega t'}. \quad (9)$$

Here, let us consider the time evolution of spin accumulation at the surface,  $y = 0$ . Because each spin aligns parallel to the rotation axis, i.e., the  $\pm z$  axis, a striped pattern of spin accumulation [shown in Fig. 1(b)] arises at the surface. The period of spatial pattern is the same as the wavelength of SAW,  $2\pi/k$ .

Recently, spin precession controlled by SAW was observed by using the time-resolved polar magneto-optic Kerr effect (MOKE),<sup>12,13</sup> in which the spatial and time resolution reaches the order of 10  $\mu\text{m}$  and 10 ns, respectively. In our case, in-plane spin polarization is induced. Therefore, transversal or longitudinal MOKE can be used to observe patterns shown in Fig. 1(b).

*Spin current from SAW.* From Eqs. (6) and (9) we obtain  $z$ -polarized spin current in the  $y$  direction. In Fig. 2, time evolution of the SAW-induced spin current is shown. The spin current,  $J_s^z$ , is plotted as a function of  $k_t y$  and  $\omega t$  in Fig. 2(a). The spin current oscillates with the same frequency as that of the mechanical resonator,  $\omega$ . The maximum amplitude,  $J_s^{\text{Max}}$ ,

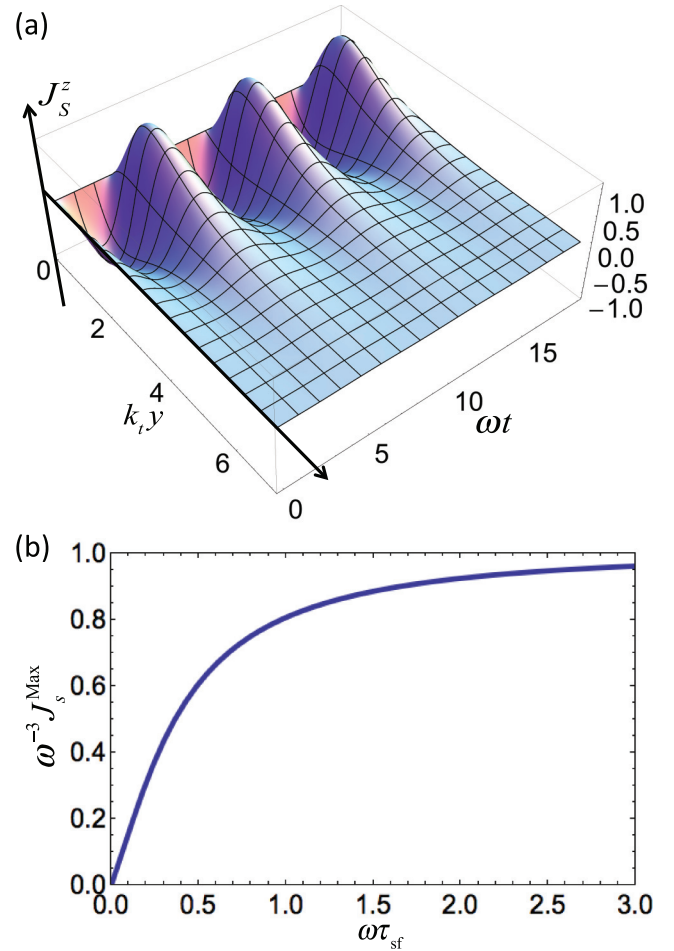


FIG. 2. (Color online) (a) Spin current induced by SAW  $J_s^z$  plotted as a function of  $k_t y$  and  $\omega t$  for fixed  $x$  and  $z$ . The spin current oscillates with time. Maximum amplitude is located near the surface,  $k_t y \approx 1$ . (b) Maximum amplitude of the spin current scaled by  $\omega^3$  plotted as a function of  $\omega\tau_{sf}$ . When  $\omega\tau_{sf} \ll 1$ , the scaled amplitude,  $J_s^{\text{Max}}\omega^{-3}$ , increases linearly. On the other hand, it saturates when  $\omega\tau_{sf} \gg 1$ . Accordingly, the maximum amplitude,  $J_s^{\text{Max}}$ , is proportional to  $\omega^4$  in the former case, whereas  $J_s^{\text{Max}} \propto \omega^3$  in the latter case.

is found near the surface,  $k_t y \approx 1$ . In Fig. 2(b), maximum amplitude scaled by  $\omega^3$  is plotted as a function of  $\omega\tau_{sf}$ . The scaled amplitude increases linearly when  $\omega\tau_{sf} \ll 1$ , whereas it saturates when  $\tau_{sf} \gg \omega^{-1}$ . In other words,  $J_s^{\text{Max}} \propto \omega^4$  for  $\omega\tau_{sf} \ll 1$ , whereas  $J_s^{\text{Max}} \propto \omega^3$  for  $\omega\tau_{sf} \gg 1$ .

To clarify material dependence of spin current, we use an asymptotic solution of Eq. (5) for  $k_t y \gg 1$ :

$$\delta\mu \approx \frac{i\omega\tilde{\tau}_{sf}}{i\omega\tilde{\tau}_{sf} + \lambda_s^2 k_t^2 - 1} \hbar\Omega_0 e^{-k_t y + i(kx - \omega t)}, \quad (10)$$

which leads to

$$J_s^z \approx \frac{-i\omega\tilde{\tau}_{sf}}{i\omega\tilde{\tau}_{sf} + \lambda_s^2 k_t^2 - 1} \frac{\hbar\sigma_0}{2e} \frac{\omega^3 u_0}{c_t^2} \frac{\sqrt{1 - \xi^2}}{\xi} e^{-k_t y + i(kx - \omega t)}, \quad (11)$$

where  $\Omega_0 = \omega^2 u_0 / 2c_t$ .

When spin relaxation is absent,  $\omega\tilde{\tau}_{sf} \gg 1$ , one obtains

$$\delta\mu \approx \hbar\Omega_0 e^{-k_t y + i(kx - \omega t)} \quad (12)$$

and

$$J_s^z \approx -(\sigma_0/e)k_t \hbar\Omega_0 e^{-k_t y + i(kx - \omega t)}. \quad (13)$$

When  $\omega\tilde{\tau}_{sf} \ll 1$  and  $\lambda_s k_t \ll 1$ , the spin current becomes

$$J_s^z \approx \omega\tau_{sf} \frac{\hbar\sigma_0}{2e} \frac{\omega^3 u_0}{c_t^2} \frac{\sqrt{1 - \xi^2}}{\xi} e^{-k_t y + i(kx - \omega t + \pi/2)}. \quad (14)$$

As seen in Eq. (14), the larger spin current can be obtained from materials with the longer spin lifetime, namely, weaker SOI.

Let us examine the SAW-induced spin current in typical nonmagnetic materials. Using Eqs. (6) and (9), the maximum value of the spin current for Al, Cu, Ag, Au, and  $n$ -doped GaAs normalized by that of Pt,  $J_s^{\text{Max,Pt}}$ , is computed as listed in Table I. The ratio of the maximum amplitude of the spin current to that of Pt,  $\bar{J}_s$ , is defined as  $\bar{J}_s = J_s^{\text{Max}} / J_s^{\text{Max,Pt}}$ . The ratio depends mainly on the conductivity,  $\sigma_0$ , and spin lifetime,  $\tau_{sf}$ . The order,  $\bar{J}_s^{\text{Cu}} > \bar{J}_s^{\text{Al}} > \bar{J}_s^{\text{Ag}} > \bar{J}_s^{\text{Au}} > \bar{J}_s^{\text{Pt}} = 1$ , is unchanged, since these materials well satisfy  $\omega\tau_{sf} \ll 1$ . For GaAs, the ratio,  $\bar{J}_s^{\text{GaAs}}$ , is greater than 1 for  $\omega < 1$  GHz, whereas it becomes smaller than 1 for  $\omega > 1$  GHz. This happens because the spin lifetime of GaAs is much longer than that of Pt; i.e., the dimensionless parameter,  $\omega\tau_{sf}$ , becomes much greater than 1 when  $\omega > 1$  GHz. In such a case,  $J_s^{\text{Max}}/\omega^3$  for GaAs saturates, whereas that for Pt linearly increases, as shown in Fig. 2(b).

It is worth noting that the spin current generated in metals with weak spin-orbit interaction such as Al and Cu is much

larger than that in Pt. In addition, the spin current in  $n$ -doped GaAs is comparable to that in Pt. Although conductivities of semiconductors are much smaller than those of metals, the spin lifetime is much longer. Hence, the amplitude of the induced spin current in GaAs is comparable to that in Pt for  $\omega\tau_{sf}^{\text{GaAs}} < 1$ .

Recently, SAWs in the GHz frequency range have been used for spin manipulation.<sup>13,14</sup> Here, we evaluate spin current at such high frequencies. In the case of  $u_0 = 10^{-9}$  m,  $\omega/2\pi = 10$  GHz, Pt has the maximum amplitude,  $J_s^{z,\text{Pt}} \approx 4 \times 10^6$  A/m<sup>2</sup>.

*Enhancement of the SAW-induced spin current.* Very recently, it has been predicted that spin-rotation coupling can be enhanced by an interband mixing of solids.<sup>20</sup>

$$H'_S = -(1 + \delta g) \frac{\hbar}{2} \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}. \quad (15)$$

Here,  $\delta g$  is given by  $\delta g = g - g_0$ , where  $g_0 = 2$  and  $g$  are electron  $g$  factors in vacuum and solids, respectively. Considering this enhancement, the mechanical rotation,  $\boldsymbol{\Omega}$ , inserted into the extended spin diffusion equation, Eq. (5), is replaced by  $(1 + \delta g)\boldsymbol{\Omega}$ . Consequently, the spin accumulation,  $\delta\mu$ , is modified as  $\delta\mu \rightarrow (1 + \delta g)\delta\mu$ , and accordingly, the induced spin current as  $J_s^z \rightarrow (1 + \delta g)J_s^z$ . For lightly doped  $n$ -InSb at low temperature,  $g \approx -49$  has been employed in a recent experiment.<sup>21</sup> In this case, one obtains  $\delta g \approx -51$ . Therefore, the amplitude of the spin current can be 50 times larger.

*Discussion and conclusion.* The method of spin-current generation using spin-rotation coupling is purely of mechanical origin; i.e., it is independent of exchange coupling and SOI. Lattice dynamics directly excites the nonequilibrium state of electron spins, and consequently, spin current can be generated in nonmagnets.

Conventionally, generation of spin current in nonmagnetic materials has required strong SOI because the spin Hall effect has been utilized. In other words, nonmagnetic materials with short spin lifetimes have been used. On the contrary, the mechanism proposed here requires longer spin lifetimes to generate larger spin currents. This means that Al and Cu, which have been considered as good materials for a spin conducting channel, can be favorable for generating spin current. Therefore, more options are available for spin-current generation in nonmagnets than ever before. For instance, this method can provide a different route to excite the spin-torque ferromagnetic resonance<sup>22</sup> without relying on the spin Hall effects.

These results can be generalized for other lattice dynamics. SAW discussed above is the Rayleigh wave, which induces rotation with the axis parallel to the surface. For instance,

TABLE I. SAW-induced spin current for Pt, Al, Cu, Ag, Au, and GaAs. The ratio is given by  $\bar{J}_s = J_s^{\text{Max}} / J_s^{\text{Max,Pt}}$ , where  $J_s^{\text{Max,Pt}}$  is the maximum amplitude of the spin current for Pt. The ratio  $\bar{J}_s$  depends on the Poisson's ratio,  $\nu$ ; the transverse velocity,  $c_t$ ; conductivity,  $\sigma_0$ ; and spin lifetime,  $\tau_{sf}$ .

	$\nu$	$c_t$ [m/s]	$\sigma_0$ [ $10^7(\Omega\text{m})^{-1}$ ]	$\tau_{sf}$ [ps]	$\bar{J}_s(0.1\text{GHz})$	$\bar{J}_s(1\text{GHz})$	$\bar{J}_s(2.5\text{GHz})$	$\bar{J}_s(10\text{GHz})$	Ref.
Pt	0.377	1730	0.96	0.3	1	1	1	1	15
Al	0.345	3040	1.7	100	390	290	210	62	16
Cu	0.343	2270	7.0	42	950	700	650	330	3
Ag	0.367	1660	2.9	3.5	44	38	34	32	17
Au	0.44	1220	2.5	2.8	42	35	33	30	18
GaAs	0.31	2486	$3.3 \times 10^{-4}$	$10^5$	1.6	0.13	0.050	0.013	19

the Love wave,<sup>23</sup> a horizontally polarized shear wave, can be utilized to generate spin currents whose spin polarization is perpendicular to the surface. The use of spin rotation coupling, argued here, opens up a new pathway for creating spin currents by elastic waves.

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