Oscillations, Waves and Optics

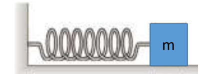
July, 2022, ASTU

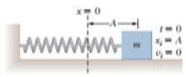
Simple Harmonic Motion

- Periodic or oscillatory motion is motion of an object that regularly returns to a given position after a fixed time interval.

➢ If something is **oscillating** (vibrating) this means that it is moving backwards and forwards, up and down, side to side, and in and out around some central position (equilibrium point).

❖ Simple harmonic motion (SHM) is a special type of periodic motion or oscillation where;
  - ➢ motion is about an equilibrium position at which point no net force acts on the system,
  - ➢ the restoring force is directly proportional to the displacement $\boldsymbol{x}$ from the equilibrium position
  - ➢ acts in the direction opposite to that of displacement.





$$F_s = -kx$$

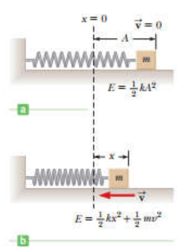
$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

Simple Harmonic Motion

❖ **Velocity as a Function of Position**

➢ Conservation of energy provides a simple method of deriving an expression for the velocity of an object undergoing periodic motion as a function of position.



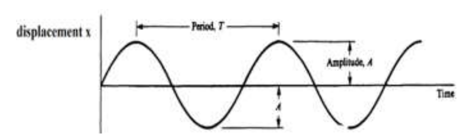
$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

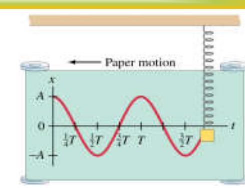
$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

- This expression shows that the object's speed is a maximum at $x = 0$ and is zero at the extreme positions $x = \pm A$.

Simple Harmonic Motion

❖ **Period and Frequency**





$$v_0 = \frac{2\pi A}{T} \qquad T = \frac{2\pi A}{v_0}$$

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}m{v_0}^2 \qquad \frac{A}{v_0} = \sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

❖ The frequency of the periodic motion of a mass on a spring is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

❖ The **angular frequency (ω)** is

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

Simple Harmonic Motion

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \omega^2 = \frac{k}{m}$$

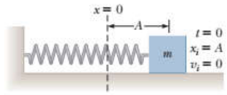
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

❖The solution is

$$x(t) = A\cos(\omega t + \phi)$$

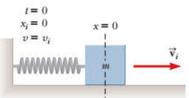
$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \phi) = -\omega A \sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \phi) = -\omega^2 A\cos(\omega t + \phi)$$



✓ First condition

$$x(0) = A\cos\phi = A$$
$$v(0) = -\omega A\sin\phi = 0$$
$$\phi = 0$$



✓ Second condition

$$x(0) = A\cos\phi = 0$$
$$v(0) = -\omega A\sin\phi = v_i$$
$$\phi = \pm\pi/2$$

Simple Harmonic Motion

❖ Example 1

A 0.2kg block connected to a light spring for which the force constant is 5 N/m is free to oscillate on a frictionless, horizontal surface. The block is displaced 5 cm from equilibrium and released from rest. A) Find the period of its motion. B) Determine the maximum speed and acceleration of the block. C) Express the position, velocity, and acceleration as functions of time.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\ \text{N/m}}{200\times10^{-3}\ \text{kg}}} = 5.00\ \text{rad/s}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00\ \text{rad/s}} = 1.26\ \text{s}$$

$$v_{max} = \omega A = (5.00\ \text{rad/s})(5.00\times10^{-2}\ \text{m}) = 0.250\ \text{m/s}$$

$$a_{max} = \omega^2 A = (5.00\ \text{rad/s})^2(5.00\times10^{-2}\ \text{m}) = 1.25\ \text{m/s}^2$$

$$x(0) = A\cos\phi = A \rightarrow \phi = 0$$

$$x = A\cos(\omega t + \phi) = 0.050\,0\cos 5.00t$$
$$v = -\omega A\sin(\omega t + \phi) = -0.250\sin 5.00t$$
$$a = -\omega^2 A\cos(\omega t + \phi) = -1.25\cos 5.00t$$

Simple Harmonic Motion

❖ Example 2

What if the block were released from the same initial position, $x_i = 5\ cm$, but with an initial velocity of $v_i = -0.100\ m/s$?

$$x(0) = A\cos\phi = x_i$$
$$v(0) = -\omega A\sin\phi = v_i$$

$$\frac{-\omega A\sin\phi}{A\cos\phi} = \frac{v_i}{x_i}$$

$$\tan\phi = -\frac{v_i}{\omega x_i} = -\frac{-0.100\ \text{m/s}}{(5.00\ \text{rad/s})(0.050\,0\ \text{m})} = 0.400$$

$$\phi = \tan^{-1}(0.400) = 0.121\pi$$

$$A = \frac{x_i}{\cos\phi} = \frac{0.050\,0\ \text{m}}{\cos(0.121\pi)} = 0.053\,9\ \text{m}$$

$$v_{max} = \omega A = (5.00\ \text{rad/s})(5.39\times10^{-2}\ \text{m}) = 0.269\ \text{m/s}$$
$$a_{max} = \omega^2 A = (5.00\ \text{rad/s})^2(5.39\times10^{-2}\ \text{m}) = 1.35\ \text{m/s}^2$$

$$x = 0.053\,9\cos(5.00t + 0.121\pi)$$
$$v = -0.269\sin(5.00t + 0.121\pi)$$
$$a = -1.35\cos(5.00t + 0.121\pi)$$

Simple Harmonic Motion

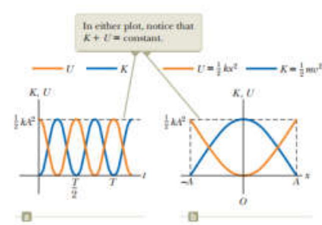
❖ **Energy of the Simple Harmonic Oscillator**

➢ The kinetic energy of the block is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi)$$

➢ The elastic potential energy stored in the spring for any elongation x is given by

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2\cos^2(\omega t + \phi)$$



❖Total mechanical energy

$$E = K + U = \tfrac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$E = \tfrac{1}{2}kA^2$$

Simple Harmonic Motion

❖ Example 3

? A 0.500kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a frictionless, horizontal air track. A) Calculate the maximum speed of the cart if the amplitude of the motion is 3.00 cm. B) What is the velocity of the cart when the position is 2.00 cm? C) Compute the kinetic and potential energies of the system when the position of the cart is 2.00 cm.

$$v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0\ \text{N/m}}{0.500\ \text{kg}}}(0.030\,0\ \text{m}) = 0.190\ \text{m/s}$$

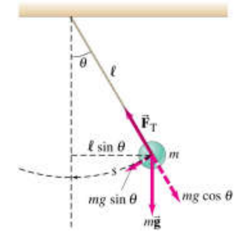
$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.500\ \text{kg})(0.141\ \text{m/s})^2 = 5.00 \times 10^{-3}\ \text{J}$$

$$\begin{aligned} v &= \pm\sqrt{\frac{k}{m}(A^2 - x^2)} \\ &= \pm\sqrt{\frac{20.0\ \text{N/m}}{0.500\ \text{kg}}[(0.030\,0\ \text{m})^2 - (0.020\,0\ \text{m})^2]} \\ &= \pm 0.141\ \text{m/s} \end{aligned}$$

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(20.0\ \text{N/m})(0.0200\ \text{m})^2 = 4.00 \times 10^{-3}\ \text{J}$$

Simple Harmonic Motion

❖ The Simple Pendulum



- The forces acting on the bob are the tension and the weight.
- **T** is the force exerted by the string
- **mg** is the gravitational force
- The tangential component of the gravitational force is the restoring force.
- Recall that the tangential acceleration is

$$F_t = ma_t \;\rightarrow\; -mg\sin\theta = m\frac{d^2s}{dt^2}$$

- This gives another differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta \qquad \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \quad \text{(for small values of } \theta\text{)}$$

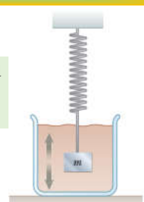
- The equation for θ is the same form as for the spring, with solution

$$\theta(t) = \theta_{max}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{L}} \quad \left(\text{so the period is } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}\right)$$

Simple Harmonic Motion

❖ Damped Oscillations



➢ In many real systems, nonconservative forces such as friction or air resistance also act and retard the motion of the system. Consequently, the mechanical energy of the system diminishes in time, and the motion is said to be *damped*.

$$\sum F_x = -kx - bv_x = ma_x$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \qquad x = Ae^{-(b/2m)t}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \qquad \omega = \sqrt{\omega_0^{\,2} - \left(\frac{b}{2m}\right)^2}$$

where $\omega_0 = \sqrt{k/m}$ represents the angular frequency in the absence of a retarding force (the undamped oscillator) and is called the **natural frequency** of the system.

Simple Harmonic Motion

➢ When the magnitude of the retarding force is small such that ${}^{b}/_{2m} < \omega_0$, the system is said to be **underdamped.**

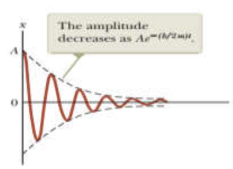
A: underdamping: there are a few small oscillations before the oscillator comes to rest.

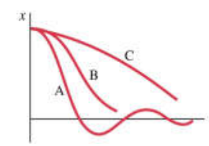
➢ When b reaches a critical value b_c such that ${}^{b_c}/_{2m} = \omega_0$, the system does not oscillate and is said to be **critically damped.**

B: critical damping: this is the fastest way to get to equilibrium.

➢ If the medium is so viscous that the retarding force is large compared with the restoring force that is, if ${}^{b}/_{2m} > \omega_0$, the system is **overdamped.**

C: overdamping: the system is slowed so much that it takes a long time to get to equilibrium.





Simple Harmonic Motion

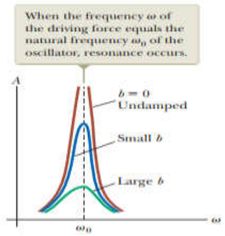
❖Forced Oscillations

- Forced vibrations occur when there is a periodic driving force. This force may or may not have the same period as the natural frequency of the system.
- If the frequency is the same as the natural frequency, the amplitude becomes quite large. This is called resonance.

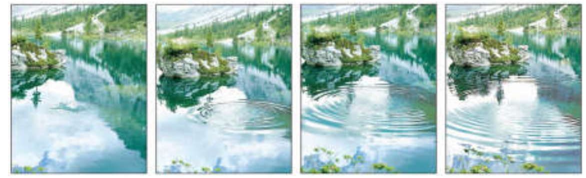
$$\sum F_x = ma_x \rightarrow F_0 \sin \omega t - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

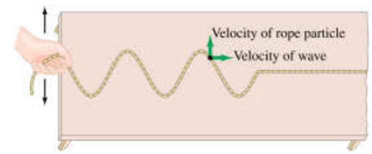
$$x = A\cos(\omega t + \phi)$$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$



Wave Motion

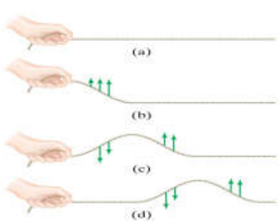


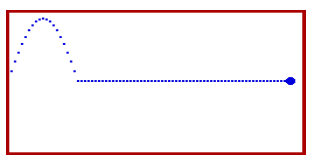


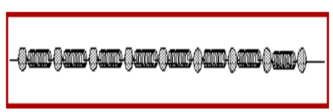
- A wave is a disturbance that carries energy from place to place.
- A wave does NOT carry matter with it! It just moves the matter as it goes through it.

Wave Motion

- Some waves do not need matter (called a "medium") to be able to move (for example, through space). These are called electromagnetic waves (or EM waves).
- Some waves MUST have a medium in order to move. These are called mechanical waves.



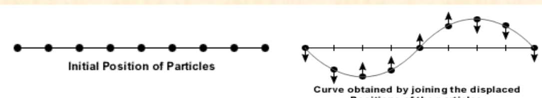




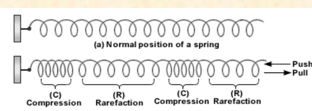
Wave Motion

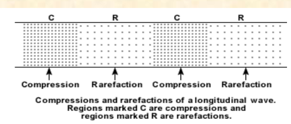
❖ Type of wave

1. Transverse waves: Waves in which the medium moves at right angles to the direction of the wave.



2. Compressional (or longitudinal) waves: Waves in which the medium moves back and forth in the same direction as the wave.





Wave Motion

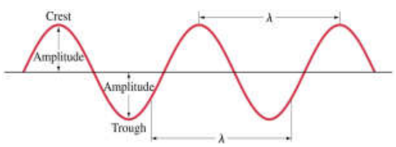
- Crest: the highest point of the wave.
- Trough: the lowest point of the wave.
- Compression: where the particles are close together
- Rarefaction: where the particles are spread apart

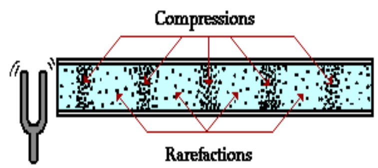
✓ Frequency (f): how many waves go past a point in one second; unit of measurement is hertz (Hz).

✓ Wavelength (λ): The distance between one point on a wave and the exact same place on the next wave.

✓ Amplitude (A): how far the medium moves from rest position (where it is when not moving).

✓ Period (T): is the time it takes for one cycle to complete.

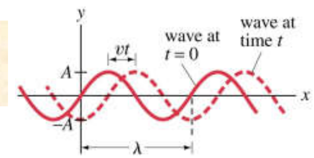




Wave Motion

❖ Traveling wave

➢ The function describing the positions of the elements of the medium through which the sinusoidal wave is traveling can be written



$$y(x, 0) = A \sin\left(\frac{2\pi}{\lambda} x\right)$$

$$y(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

$$k \equiv \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T}$$

$$v = \lambda f.$$

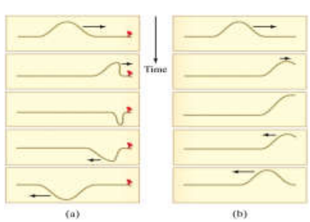
$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

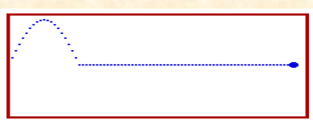
$$y = A \sin(kx - \omega t)$$

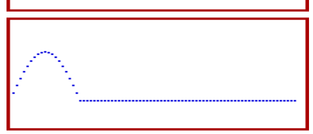
Wave Motion

❖ **Reflection and Transmission of Waves**

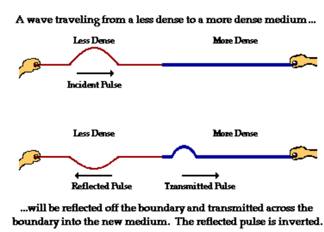
- A wave reaching the end of its medium, but where the medium is still free to move, will be reflected (b), and its reflection will be upright.
- A wave hitting an obstacle will be reflected (a), and its reflection will be inverted.





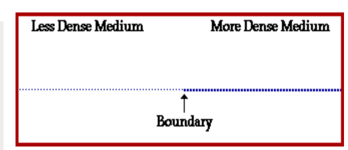


Wave Motion



- In this situation part of the wave is reflected, and part of the wave is transmitted.
- Part of the wave energy is transferred to the more dense medium, and part is reflected.
- The transmitted pulse is upright, while the reflected pulse is inverted.

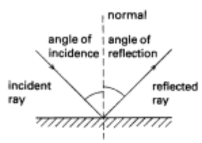
- The speed and wavelength of the reflected wave remain the same, but the amplitude decreases.
- The speed, wavelength, and amplitude of the transmitted pulse are all smaller than in the incident pulse.

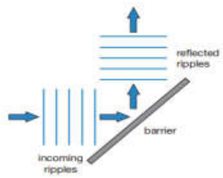


Wave Motion

- Reflection;

✓ *Bounce off of wave.*
✓ *Occurs when a waves reaches an obstacle/barrier.*
✓ *Traveling in the opposite direction.*
✓ *Its frequency does not change.*
✓ *Obey first law of reflection of light.*
  ❖ *The angle of incident = the angle of reflection*

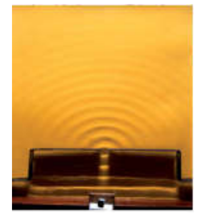




Wave Motion

- Diffraction;

✓ *A spreading of wave after passing the edge of an obstacle or gap/slit.*
✓ *Narrow gap has more effect.*
✓ *Wide gap has less effect.*



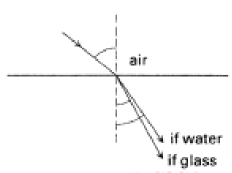
➢ Large obstacle, small wavelength = low diffraction

➢ Small obstacle, large wavelength = large diffraction

Wave Motion

- Refraction;

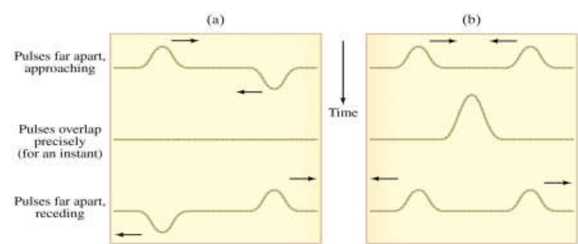
✓ *breaking of wave.*
✓ *Is a measure of the extent to which a medium reflects light.*
✓ *Change speed and direction as they move from one material to another.*
✓ *The wavelength may decrease/increase.*
✓ *It use in lenses, camera, telescope.*



Wave Motion

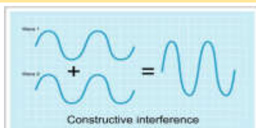
➢ The superposition principle says that when two waves pass through the same point, the displacement is the arithmetic sum of the individual displacements.

➢ In the figure below, (a) exhibits destructive interference and (b) exhibits constructive interference.

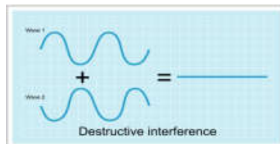


Wave Motion

- Interference;
  - ✓ *Add up or cancel out.*
  - ✓ *There are two types: constructive and destructive.*
- Constructive;
  - ✓ *If two waves are in phase with each other they combine to make a bigger wave.*
  - ✓ *Constructive interference occurs when the crests of one wave are over the crests of another wave*

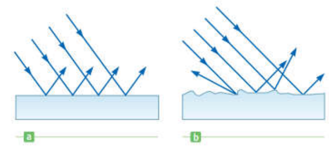


- Destructive;
  - ✓ *If two waves are out of phase with each other they cancel each other, so that there is no wave.*



Light and Optics

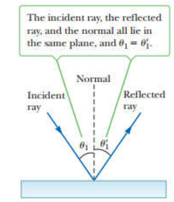
➢ light travels in a straight-line path in a homogeneous medium, until it encounters a boundary between two different materials.



- The reflection of light from such a smooth surface is called **specular reflection**.
- The reflection from any rough surface is known as **diffuse reflection**.

❖ *The angle of reflection equals the angle of incidence:*

$$\theta_1' = \theta_1$$



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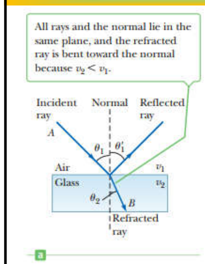
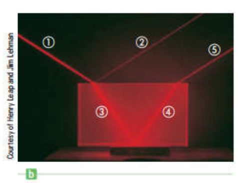
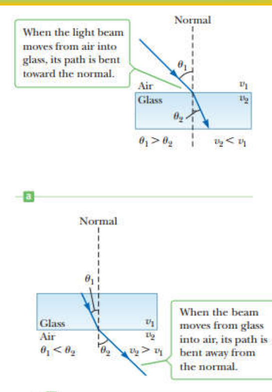




$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \text{constant}$$



Light and Optics

❖ The Law of Refraction

➢ The **index of refraction**, n, of a medium is defined as:

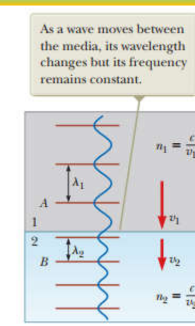
$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}$$

❖ As light travels from one medium to another, its frequency doesn't change.

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Snell's law of refraction.

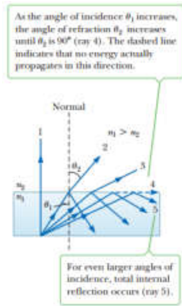


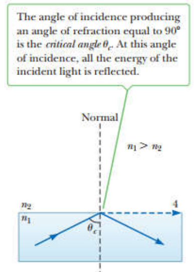
Light and Optics

- At some particular angle of incidence θ_c, called the **critical angle,** the refracted light ray moves parallel to the boundary so that $\theta_2 = 90^o$. For angles of incidence greater than θ_c, the ray is entirely reflected at the boundary.

$$n_1 \sin\theta_c = n_2 \sin 90° = n_2$$

$$\sin\theta_c = \frac{n_2}{n_1} \quad \text{for } n_1 > n_2$$





Light and Optics

- Example 4

- A light ray of wavelength 589 nm (produced by a sodium lamp) traveling through air is incident on a smooth, flat slab of crown glass at an angle $\theta_1 = 30^o$ to the normal. **(a)** Find the angle of refraction, θ_2. **(b)** At what angle θ_3 does the ray leave the glass as it re-enters the air?

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

$$\sin\theta_2 = \left(\frac{1.00}{1.52}\right)(\sin 30.0°) = 0.329$$

$$\theta_2 = \sin^{-1}(0.329) = 19.2°$$

$$\sin\theta_3 = \frac{n_2}{n_3}\sin\theta_2 = \frac{1.52}{1.00}\sin(19.2°) = 0.500$$

$$\theta_3 = \sin^{-1}(0.500) = 30.0°$$

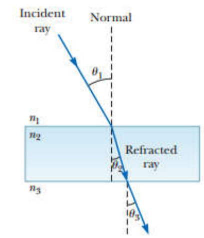


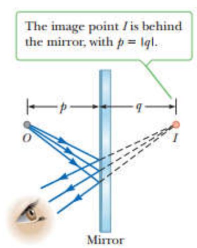
Image Formation

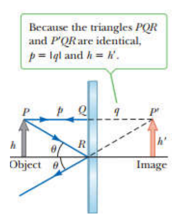
- Images are formed at the point where rays of light actually intersect or where they appear to originate.

- The image formed by an object placed in front of a flat mirror is as far behind the mirror as the object is in front of the mirror.

- The **lateral magnification** M is defined as:

$$M = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h}$$





End

The End