Bottom-Up Parsing

- A **bottom-up parser** creates the parse tree of the given input starting from leaves towards the root.
- A bottom-up parser tries to find the right-most derivation of the given input in the reverse order.

  $S \Rightarrow \ldots \Rightarrow \omega$ (the right-most derivation of ω)

  $\leftarrow$ (the bottom-up parser finds the right-most derivation in the reverse order)
- Bottom-up parsing is also known as **shift-reduce parsing** because its two main actions are shift and reduce.
  - At each shift action, the current symbol in the input string is pushed to a stack.
  - At each reduction step, the symbols at the top of the stack (this symbol sequence is the right side of a production) will replaced by the non-terminal at the left side of that production.
  - There are also two more actions: accept and error.

Shift-Reduce Parsing

- A shift-reduce parser tries to reduce the given input string into the starting symbol.

  a string $\xrightarrow[\text{reduced to}]{}$ the starting symbol

- At each reduction step, a substring of the input matching to the right side of a production rule is replaced by the non-terminal at the left side of that production rule.
- If the substring is chosen correctly, the right most derivation of that string is created in the reverse order.

Rightmost Derivation: $S \overset{*}{\underset{rm}{\Rightarrow}} \omega$

Shift-Reduce Parser finds: $\omega \underset{rm}{\Leftarrow} \ldots \underset{rm}{\Leftarrow} S$

Shift-Reduce Parsing -- Example

$S \to aABb$

$A \to aA \mid a$

$B \to bB \mid b$

input string: `aaabb`

`aaAbb`

`aAbb` $\Downarrow$ reduction

`aABb`

`S`

$$S \underset{rm}{\Rightarrow} aABb \underset{rm}{\Rightarrow} aAbb \underset{rm}{\Rightarrow} aaAbb \underset{rm}{\Rightarrow} aaabb$$

Right Sentential Forms

- How do we know which substring to be replaced at each reduction step?

A Shift-Reduce Parser

$E \to E+T \mid T$

$T \to T*F \mid F$

$F \to (E) \mid id$

Right-Most Derivation of `id+id*id`

$E \Rightarrow E+T \Rightarrow E+T*F \Rightarrow E+T*id \Rightarrow E+F*id$

$\Rightarrow E+id*id \Rightarrow T+id*id \Rightarrow F+id*id \Rightarrow id+id*id$

Right-Most Sentential Form	Reducing Production
$\underline{id}+id*id$	$F \to id$
$\underline{F}+id*id$	$T \to F$
$\underline{T}+id*id$	$E \to T$
$E+\underline{id}*id$	$F \to id$
$E+\underline{F}*id$	$T \to F$
$E+T*\underline{id}$	$F \to id$
$E+\underline{T*F}$	$T \to T*F$
$\underline{E+T}$	$E \to E+T$
E	

a **handle** of a string is a substring that matches the right side of a production rule

Handles are red and underlined in the right-sentential forms.

A Stack Implementation of A Shift-Reduce Parser

- There are four possible actions of a shift-parser action:

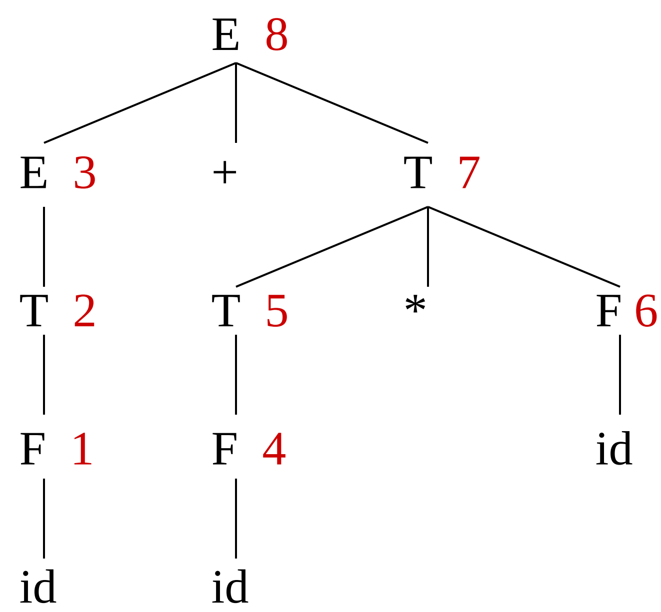
  1. **Shift** : The next input symbol is shifted onto the top of the stack.
  2. **Reduce**: Replace the handle on the top of the stack by the non-terminal.
  3. **Accept**: Successful completion of parsing.
  4. **Error**: Parser discovers a syntax error, and calls an error recovery routine.

- Initial stack just contains only the end-marker $.
- The end of the input string is marked by the end-marker $.

A Stack Implementation of A Shift-Reduce Parser

Stack	**Input**	**Action**
\$	id+id*id \$	shift
\$id	+id*id\$	reduce by F $\rightarrow$ id
\$F	+id*id\$	reduce by T $\rightarrow$ F
\$T	+id*id\$	reduce by E $\rightarrow$ T
\$E	+id*id\$	shift
\$E+	id*id\$	shift
\$E+id	*id\$	reduce by F $\rightarrow$ id
\$E+F	*id\$	reduce by T $\rightarrow$ F
\$E+T	*id\$	shift
\$E+T*	id\$	shift
\$E+T*id	\$	reduce by F $\rightarrow$ id
\$E+T*F	\$	reduce by T $\rightarrow$ T*F
\$E+T	\$	reduce by E $\rightarrow$ E+T
\$E	\$	accept

Parse Tree



Conflicts During Shift-Reduce Parsing

- There are context-free grammars for which shift-reduce parsers cannot be used.
- Stack contents and the next input symbol may not decide action:
  - **shift/reduce conflict**: Whether make a shift operation or a reduction.
  - **reduce/reduce conflict**: The parser cannot decide which of several reductions to make.
- If a shift-reduce parser cannot be used for a grammar, that grammar is called as non-LR(k) grammar.

left to right scanning

right-most derivation

k lookhead

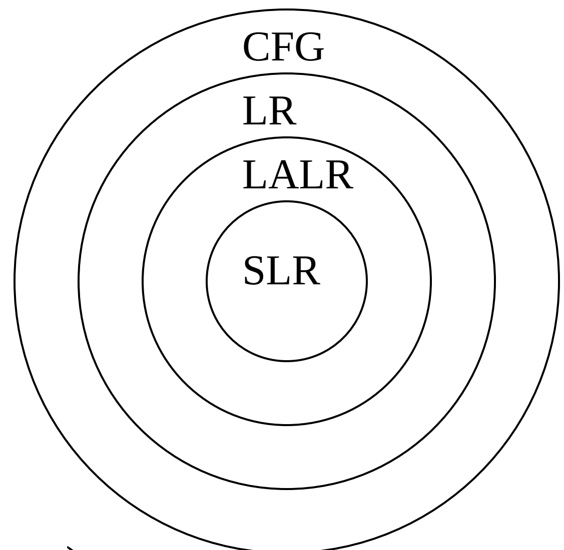
- An ambiguous grammar can never be a LR grammar.

Shift-Reduce Parsers

- There are two main categories of shift-reduce parsers

1. Operator-Precedence Parser

– simple, but only a small class of grammars.



2. LR-Parsers

– covers wide range of grammars.

- SLR – simple LR parser
- LR – most general LR parser
- LALR – intermediate LR parser (lookhead LR parser)

– SLR, LR and LALR work same, only their parsing tables are different.

LR Parsers

- The most powerful shift-reduce parsing (yet efficient) is:

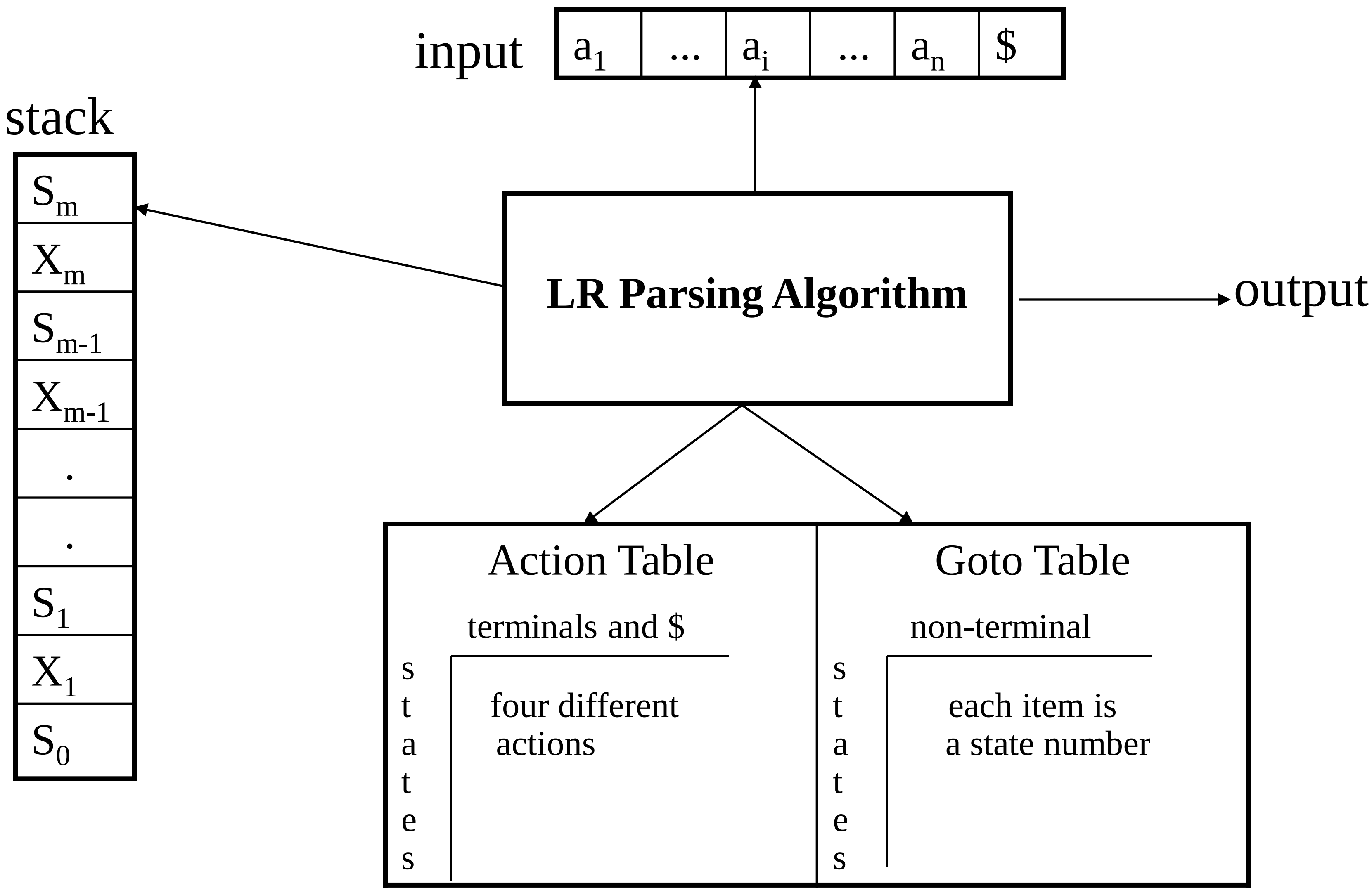
LR(k) parsing.

left to right scanning | right-most derivation | k lookhead (k is omitted ➔ it is 1)

- LR parsing is attractive because:
  - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient.
  - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers.

    LL(1)-Grammars $\subset$ LR(1)-Grammars
  - An LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input.

LR Parsing Algorithm



A Configuration of LR Parsing Algorithm

- A configuration of a LR parsing is:

$$(\underbrace{S_o\ X_1\ S_1\ \ldots\ X_m\ S_m}_{\text{Stack}},\ \ \underbrace{a_i\ a_{i+1}\ \ldots\ a_n\ \$}_{\text{Rest of Input}})$$

- S_m and a_i decides the parser action by consulting the parsing action table. (*Initial Stack* contains just S_o)

- A configuration of a LR parsing represents the right sentential form:

$$X_1\ \ldots\ X_m\ a_i\ a_{i+1}\ \ldots\ a_n\ \$$$

Actions of A LR-Parser

1. **shift s** -- shifts the next input symbol and the state **s** onto the stack

   $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ ) \Rightarrow (S_o\ X_1\ S_1 \ldots X_m\ S_m\ a_i\ s,\ a_{i+1} \ldots a_n\ \$\ )$

2. **reduce A→β** (or **`rn`** where n is a production number)
   - pop 2|**β**| (=r) items from the stack;
   - then push **A** and **s** where **s=goto[s_{m-r},A]**

   $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ ) \Rightarrow (S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ A\ s,\ a_i \ldots a_n\ \$\ )$

   - Output is the reducing production reduce A→β

3. **Accept** – Parsing successfully completed

4. **Error** -- Parser detected an error (an empty entry in the action table)

Reduce Action

- pop 2|**β**| (=r) items from the stack; let us assume that **β** = $Y_1 Y_2 \ldots Y_r$
- then push **A** and **s** where **s=goto[s$_{m-r}$,A]**

$$(S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ Y_1\ S_{m-r} \ldots Y_r\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ )$$

$$\rightarrow (S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ A\ s,\ a_i \ldots a_n\ \$\ )$$

- In fact, $Y_1 Y_2 \ldots Y_r$ is a handle.

$$X_1 \ldots X_{m-r}\ A\ a_i \ldots a_n\ \$ \Rightarrow X_1 \ldots X_m\ Y_1 \ldots Y_r\ a_i\ a_{i+1} \ldots a_n\ \$$$

(SLR) Parsing Tables for Expression Grammar

1) $E \rightarrow E+T$
2) $E \rightarrow T$
3) $T \rightarrow T*F$
4) $T \rightarrow F$
5) $F \rightarrow (E)$
6) $F \rightarrow id$

	Action Table							Goto Table		
state	**id**	**+**	*****	**(**	**)**	**$**		**E**	**T**	**F**
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

Actions of A (S)LR-Parser -- Example

stack	**input**	**action**	**output**
0	id*id+id$	shift 5	
0id5	*id+id$	reduce by F→id	F→id
0F3	*id+id$	reduce by T→F	T→F
0T2	*id+id$	shift 7	
0T2*7	id+id$	shift 5	
0T2*7id5	+id$	reduce by F→id	F→id
0T2*7F10	+id$	reduce by T→T*F	T→T*F
0T2	+id$	reduce by E→T	E→T
0E1	+id$	shift 6	
0E1+6	id$	shift 5	
0E1+6id5	$	reduce by F→id	F→id
0E1+6F3	$	reduce by T→F	T→F
0E1+6T9	$	reduce by E→E+T	E→E+T
0E1	$	accept	

Constructing SLR Parsing Tables – LR(0) Item

- An **LR(0) item** of a grammar G is a production of G a dot at the some position of the right side.
- Ex: $A \rightarrow aBb$ Possible LR(0) Items: (four different possibility)

$$A \rightarrow \bullet aBb$$
$$A \rightarrow a \bullet Bb$$
$$A \rightarrow aB \bullet b$$
$$A \rightarrow aBb \bullet$$

- Sets of LR(0) items will be the states of action and goto table of the SLR parser.
- A collection of sets of LR(0) items (**the canonical LR(0) collection**) is the basis for constructing SLR parsers.
- *Augmented Grammar*:

  G' is G with a new production rule $S' \rightarrow S$ where S' is the new starting symbol.

The Closure Operation

- If ***I*** is a set of LR(0) items for a grammar G, then ***closure(I)*** is the set of LR(0) items constructed from I by the two rules:

1. Initially, every LR(0) item in I is added to closure(I).
2. If $A \rightarrow \alpha \bullet B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow \bullet \gamma$ will be in the closure(I).
   We will apply this rule until no more new LR(0) items can be added to closure(I).

The Closure Operation -- Example

$E' \to E$	$\text{closure}(\{E' \to \bullet E\}) =$
$E \to E+T$	$\{\ E' \to \bullet E$ ← kernel items
$E \to T$	$E \to \bullet E+T$
$T \to T*F$	$E \to \bullet T$
$T \to F$	$T \to \bullet T*F$
$F \to (E)$	$T \to \bullet F$
$F \to id$	$F \to \bullet (E)$
	$F \to \bullet id\ \}$

Goto Operation

- If I is a set of LR(0) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:
  - If $A \rightarrow \alpha \bullet X\beta$ in I
    then every item in **closure({$A \rightarrow \alpha X \bullet \beta$})** will be in goto(I,X).

Example:

$I = \{ \quad E' \rightarrow \bullet E, \quad E \rightarrow \bullet E+T, \quad E \rightarrow \bullet T,$
$\quad T \rightarrow \bullet T*F, \quad T \rightarrow \bullet F,$
$\quad F \rightarrow \bullet (E), \quad F \rightarrow \bullet id \ \}$

$goto(I,E) = \{ E' \rightarrow E \bullet, E \rightarrow E \bullet +T \}$

$goto(I,T) = \{ E \rightarrow T \bullet, T \rightarrow T \bullet *F \}$

$goto(I,F) = \{ T \rightarrow F \bullet \}$

$goto(I,() = \{ F \rightarrow (\bullet E), E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F, T \rightarrow \bullet F,$
$F \rightarrow \bullet (E), F \rightarrow \bullet id \ \}$

$goto(I,id) = \{ F \rightarrow id \bullet \}$

Construction of The Canonical LR(0) Collection

- To create the SLR parsing tables for a grammar G, we will create the canonical LR(0) collection of the grammar G'.

- ***Algorithm***:

  C is { closure({S'→ $\bullet$ S}) }

  repeat the followings until no more set of LR(0) items can be added to ***C***.

      for each I in ***C*** and each grammar symbol X

          if goto(I,X) is not empty and not in ***C***

              add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

The Canonical LR(0) Collection -- Example

I_0: $E' \to .E$
$E \to .E+T$
$E \to .T$
$T \to .T*F$
$T \to .F$
$F \to .(E)$
$F \to .id$

I_1: $E' \to E.$
$E \to E.+T$

I_2: $E \to T.$
$T \to T.*F$

I_3: $T \to F.$

I_4: $F \to (.E)$
$E \to .E+T$
$E \to .T$
$T \to .T*F$
$T \to .F$
$F \to .(E)$
$F \to .id$

I_5: $F \to id.$

I_6: $E \to E+.T$
$T \to .T*F$
$T \to .F$
$F \to .(E)$
$F \to .id$

I_7: $T \to T*.F$
$F \to .(E)$
$F \to .id$

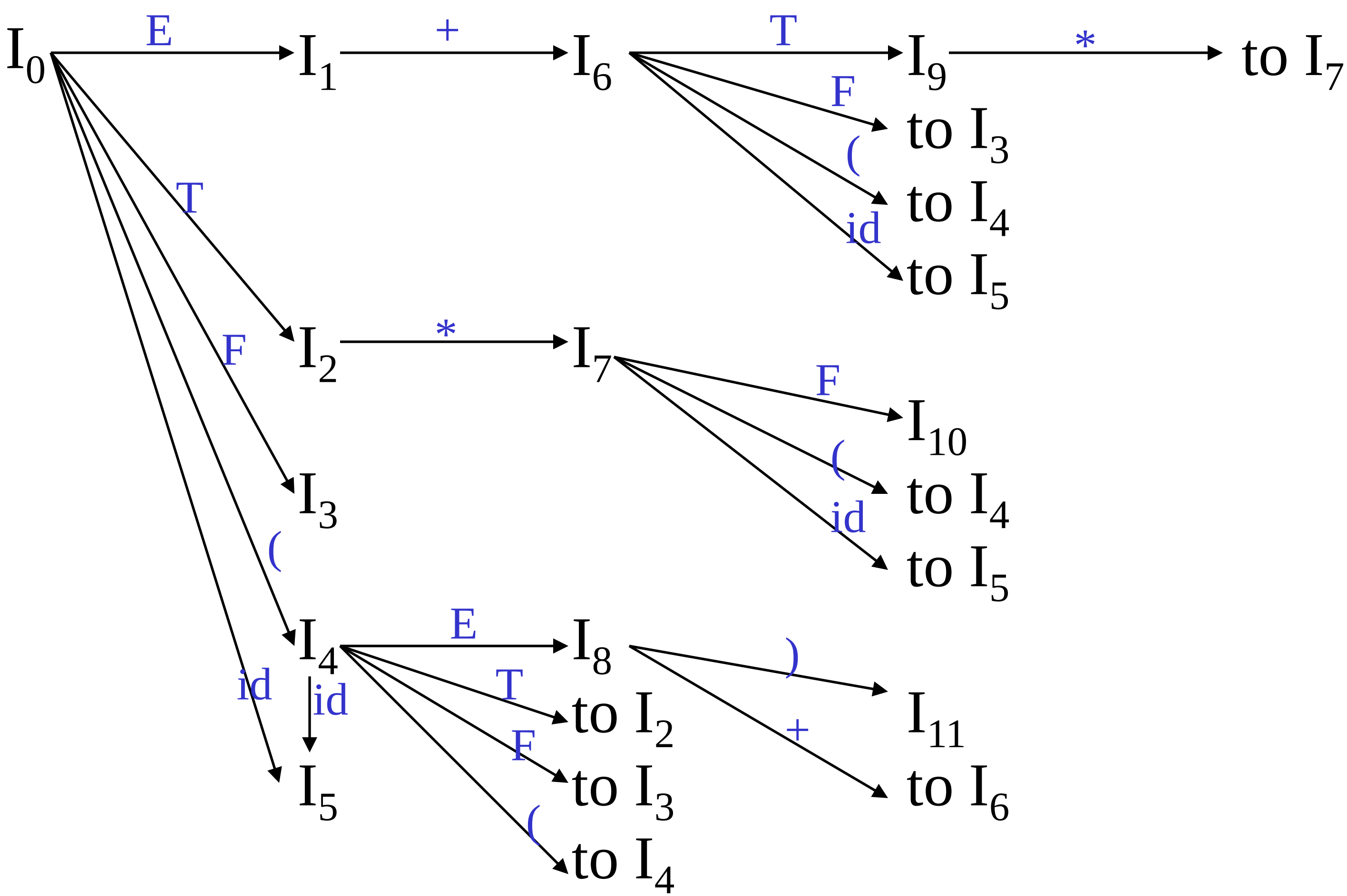
I_8: $F \to (E.)$
$E \to E.+T$

I_9: $E \to E+T.$
$T \to T.*F$

I_{10}: $T \to T*F.$

I_{11}: $F \to (E).$

Transition Diagram (DFA) of Goto Function



Constructing SLR Parsing Table

(of an augumented grammar G')

1. Construct the canonical collection of sets of LR(0) items for G'.

   $C \leftarrow \{I_0,...,I_n\}$

2. Create the parsing action table as follows
   - If a is a terminal, $A \rightarrow \alpha.a\beta$ in I_i and $goto(I_i,a)=I_j$ then action[i,a] is ***shift j.***
   - If $A \rightarrow \alpha.$ is in I_i, then action[i,a] is ***reduce*** $\boldsymbol{A \rightarrow \alpha}$ for all a in FOLLOW(A) where $A \neq S'$.
   - If $S' \rightarrow S.$ is in I_i, then action[i,$] is ***accept***.
   - If any conflicting actions generated by these rules, the grammar is not SLR(1).

3. Create the parsing goto table
   - for all non-terminals A, if $goto(I_i,A)=I_j$ then goto[i,A]=j

4. All entries not defined by (2) and (3) are errors.

5. Initial state of the parser contains $S' \rightarrow .S$

Parsing Tables of Expression Grammar

Action Table / Goto Table

state	id	+	*	(	)	$		E	T	F
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

SLR(1) Grammar

- An LR parser using SLR(1) parsing tables for a grammar G is called as the SLR(1) parser for G.
- If a grammar G has an SLR(1) parsing table, it is called SLR(1) grammar (or SLR grammar in short).
- Every SLR grammar is unambiguous, but every unambiguous grammar is not a SLR grammar.

shift/reduce and reduce/reduce conflicts

- If a state does not know whether it will make a shift operation or reduction for a terminal, we say that there is a **shift/reduce conflict**.

- If a state does not know whether it will make a reduction operation using the production rule `i` or `j` for a terminal, we say that there is a **reduce/reduce conflict**.

- If the SLR parsing table of a grammar G has a conflict, we say that that grammar is not SLR grammar.

Conflict Example

$S \rightarrow L=R$
$S \rightarrow R$
$L \rightarrow *R$
$L \rightarrow id$
$R \rightarrow L$

I_0: $S' \rightarrow .S$
$S \rightarrow .L=R$
$S \rightarrow .R$
$L \rightarrow .*R$
$L \rightarrow .id$
$R \rightarrow .L$

I_1: $S' \rightarrow S.$

I_2: $S \rightarrow L.=R$
$R \rightarrow L.$

I_3: $S \rightarrow R.$

I_4: $L \rightarrow *.R$
$R \rightarrow .L$
$L \rightarrow .*R$
$L \rightarrow .id$

I_5: $L \rightarrow id.$

I_6: $S \rightarrow L=.R$
$R \rightarrow .L$
$L \rightarrow .*R$
$L \rightarrow .id$

I_7: $L \rightarrow *R.$

I_8: $R \rightarrow L.$

I_9: $S \rightarrow L=R.$

Problem

FOLLOW(R)={=,$}

= → shift 6

→ reduce by $R \rightarrow L$

shift/reduce conflict

Conflict Example2

$S \rightarrow AaAb$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$

$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S$

$S \rightarrow .AaAb$

$S \rightarrow .BbBa$

$A \rightarrow .$

$B \rightarrow .$

Problem

FOLLOW(A)={a,b}

FOLLOW(B)={a,b}

a → reduce by $A \rightarrow \varepsilon$
→ reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

b → reduce by $A \rightarrow \varepsilon$
→ reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

Constructing Canonical LR(1) Parsing Tables

- In SLR method, the state i makes a reduction by $A \rightarrow \alpha$ when the current token is a:
  - if the $A \rightarrow \alpha.$ in the I_i and a is FOLLOW(A)
- In some situations, βA cannot be followed by the terminal a in a right-sentential form when $\beta\alpha$ and the state i are on the top stack. This means that making reduction in this case is not correct.

$S \rightarrow AaAb$	$S \Rightarrow AaAb \Rightarrow Aab \Rightarrow ab$	$S \Rightarrow BbBa \Rightarrow Bba \Rightarrow ba$
$S \rightarrow BbBa$		
$A \rightarrow \varepsilon$	$Aab \Rightarrow \varepsilon\ ab$	$Bba \Rightarrow \varepsilon\ ba$
$B \rightarrow \varepsilon$	$AaAb \Rightarrow Aa\ \varepsilon\ b$	$BbBa \Rightarrow Bb\ \varepsilon\ a$

LR(1) Item

- To avoid some of invalid reductions, the states need to carry more information.
- Extra information is put into a state by including a terminal symbol as a second component in an item.
- A LR(1) item is:

$A \rightarrow \alpha \bullet \beta, a$ where **a** is the look-head of the LR(1) item

(**a** is a terminal or end-marker.)

LR(1) Item (cont.)

- When β (in the LR(1) item $A \rightarrow \alpha\bullet\beta,a$) is not empty, the look-head does not have any affect.
- When β is empty ($A \rightarrow \alpha\bullet,a$), we do the reduction by $A\rightarrow\alpha$ only if the next input symbol is **a** (not for any terminal in FOLLOW(A)).

- A state will contain $A \rightarrow \alpha\bullet,a_1$ where $\{a_1,...,a_n\} \subseteq$ FOLLOW(A)

$$
\begin{array}{l}
A \rightarrow \alpha\bullet,a_1 \\
\quad ... \\
A \rightarrow \alpha\bullet,a_n
\end{array}
$$

Canonical Collection of Sets of LR(1) Items

- The construction of the canonical collection of the sets of LR(1) items are similar to the construction of the canonical collection of the sets of LR(0) items, except that *closure* and *goto* operations work a little bit different.

closure(I) is: (where I is a set of LR(1) items)

- every LR(1) item in I is in closure(I)
- if $A \rightarrow \alpha \bullet B\beta, a$ in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow .\gamma, b$ will be in the closure(I) for each terminal b in FIRST(βa) .

goto operation

- If I is a set of LR(1) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:
  - If $A \rightarrow \alpha.X\beta,a$ in I
    then every item in **closure({$A \rightarrow \alpha X.\beta,a$})** will be in goto(I,X).

Construction of The Canonical LR(1) Collection

- ***Algorithm***:

  C is { closure({S'→.S,$}) }

  repeat the followings until no more set of LR(1) items can be added to ***C***.

      for each I in ***C*** and each grammar symbol X

          if goto(I,X) is not empty and not in ***C***

              add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

A Short Notation for The Sets of LR(1) Items

- A set of LR(1) items containing the following items

$$A \rightarrow \alpha \bullet \beta, a_1$$

$$\ldots$$

$$A \rightarrow \alpha \bullet \beta, a_n$$

can be written as

$$A \rightarrow \alpha \bullet \beta, a_1/a_2/\ldots/a_n$$

Canonical LR(1) Collection -- Example

$S \rightarrow AaAb$
$S \rightarrow BbBa$
$A \rightarrow \varepsilon$
$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S\ ,\$$
$S \rightarrow .AaAb\ ,\$$
$S \rightarrow .BbBa\ ,\$$
$A \rightarrow .\ ,a$
$B \rightarrow .\ ,b$

$I_0 \xrightarrow{S} I_1$: $S' \rightarrow S.\ ,\$$

$I_0 \xrightarrow{A} I_2$: $S \rightarrow A.aAb\ ,\$$ $\xrightarrow{a}$ to I_4

$I_0 \xrightarrow{B} I_3$: $S \rightarrow B.bBa\ ,\$$ $\xrightarrow{b}$ to I_5

I_4: $S \rightarrow Aa.Ab\ ,\$$ $\xrightarrow{A}$ I_6: $S \rightarrow AaA.b\ ,\$$ $\xrightarrow{a}$ I_8: $S \rightarrow AaAb.\ ,\$$
$A \rightarrow .\ ,b$

I_5: $S \rightarrow Bb.Ba\ ,\$$ $\xrightarrow{B}$ I_7: $S \rightarrow BbB.a\ ,\$$ $\xrightarrow{b}$ I_9: $S \rightarrow BbBa.\ ,\$$
$B \rightarrow .\ ,a$

Canonical LR(1) Collection – Example2

$S' \rightarrow S$
1) $S \rightarrow L=R$
2) $S \rightarrow R$
3) $L \rightarrow *R$
4) $L \rightarrow id$
5) $R \rightarrow L$

I_0: $S' \rightarrow .S,\$$
$S \rightarrow .L=R,\$$
$S \rightarrow .R,\$$
$L \rightarrow .*R,\$/=$
$L \rightarrow .id,\$/=$
$R \rightarrow .L,\$$

$I_0 \xrightarrow{S} I_1$, $I_0 \xrightarrow{*} I_4$, $I_0 \xrightarrow{L} I_2$, $I_0 \xrightarrow{R} I_3$, $I_0 \xrightarrow{id} I_5$

I_1: $S' \rightarrow S.,\$$

I_2: $S \rightarrow L.=R,\$$ $\longrightarrow$ to I_6
$R \rightarrow L.,\$$

I_3: $S \rightarrow R.,\$$

I_4: $L \rightarrow *.R,\$/=$
$R \rightarrow .L,\$/=$
$L \rightarrow .*R,\$/=$
$L \rightarrow .id,\$/=$

$\xrightarrow{R}$ to I_7
$\xrightarrow{L}$ to I_8
$\xrightarrow{*}$ to I_4
$\xrightarrow{id}$ to I_5

I_5: $L \rightarrow id.,\$/=$

I_6: $S \rightarrow L=.R,\$$
$R \rightarrow .L,\$$
$L \rightarrow .*R,\$$
$L \rightarrow .id,\$$

$\xrightarrow{R}$ to I_9
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_7: $L \rightarrow *R.,\$/=$

I_8: $R \rightarrow L.,\$/=$

I_9: $S \rightarrow L=R.,\$$

I_{10}: $R \rightarrow L.,\$$

I_{11}: $L \rightarrow *.R,\$$
$R \rightarrow .L,\$$
$L \rightarrow .*R,\$$
$L \rightarrow .id,\$$

$\xrightarrow{R}$ to I_{13}
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_{12}: $L \rightarrow id.,\$$

I_{13}: $L \rightarrow *R.,\$$

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

Construction of LR(1) Parsing Tables

1. Construct the canonical collection of sets of LR(1) items for G'.

   $C \leftarrow \{I_0,...,I_n\}$

2. Create the parsing action table as follows
   - If a is a terminal, $A \rightarrow \alpha \bullet a\beta, b$ in I_i and $goto(I_i,a)=I_j$ then action[i,a] is ***shift j.***
   - If $A \rightarrow \alpha \bullet, a$ is in I_i , then action[i,a] is ***reduce*** $\boldsymbol{A \rightarrow \alpha}$ where $A \neq S'$.
   - If $S' \rightarrow S \bullet, \$$ is in I_i , then action[i,$] is ***accept***.
   - If any conflicting actions generated by these rules, the grammar is not LR(1).

3. Create the parsing goto table
   - for all non-terminals A, if $goto(I_i,A)=I_j$ then goto[i,A]=j

4. All entries not defined by (2) and (3) are errors.

5. Initial state of the parser contains $S' \rightarrow .S, \$$

LR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3	r3			
8			r5	r5			
9				r1			
10				r5			
11	s12	s11				10	13
12				r4			
13				r3			

no shift/reduce or
no reduce/reduce conflict

$\Downarrow$

so, it is a LR(1) grammar

LALR Parsing Tables

- **LALR** stands for **LookAhead LR.**

- LALR parsers are often used in practice because LALR parsing tables are smaller than LR(1) parsing tables.
- The number of states in SLR and LALR parsing tables for a grammar G are equal.
- But LALR parsers recognize more grammars than SLR parsers.
- ***yacc*** creates a LALR parser for the given grammar.
- A state of LALR parser will be again a set of LR(1) items.

Creating LALR Parsing Tables

Canonical LR(1) Parser $\rightarrow$ LALR Parser

shrink # of states

- This shrink process may introduce a **reduce/reduce** conflict in the resulting LALR parser (so the grammar is NOT LALR)
- But, this shrink process does not produce a **shift/reduce** conflict.

The Core of A Set of LR(1) Items

- The core of a set of LR(1) items is the set of its first component.

Ex: $S \rightarrow L \bullet =R,\$$ ➔ $S \rightarrow L \bullet =R$ ← Core

$R \rightarrow L \bullet ,\$$ $R \rightarrow L \bullet$

- We will find the states (sets of LR(1) items) in a canonical LR(1) parser with same cores. Then we will merge them as a single state.

I_1:$L \rightarrow id \bullet ,=$ A new state: I_{12}: $L \rightarrow id \bullet ,=$

➔ $L \rightarrow id \bullet ,\$$

I_2:$L \rightarrow id \bullet ,\$$ have same core, merge them

- We will do this for all states of a canonical LR(1) parser to get the states of the LALR parser.
- In fact, the number of the states of the LALR parser for a grammar will be equal to the number of states of the SLR parser for that grammar.

Creation of LALR Parsing Tables

- Create the canonical LR(1) collection of the sets of LR(1) items for the given grammar.
- Find each core; find all sets having that same core; replace those sets having same cores with a single set which is their union.

  $C=\{I_0,...,I_n\}$ ➔ $C'=\{J_1,...,J_m\}$ where $m \leq n$
- Create the parsing tables (action and goto tables) same as the construction of the parsing tables of LR(1) parser.
  – Note that: If $J=I_1 \cup ... \cup I_k$ since $I_1,...,I_k$ have same cores
    ➔ cores of $goto(I_1,X),...,goto(I_2,X)$ must be same.
  – So, $goto(J,X)=K$ where K is the union of all sets of items having same cores as $goto(I_1,X)$.
- If no conflict is introduced, the grammar is LALR(1) grammar. (We may only introduce reduce/reduce conflicts; we cannot introduce a shift/reduce conflict)

Shift/Reduce Conflict

- We say that we cannot introduce a shift/reduce conflict during the shrink process for the creation of the states of a LALR parser.
- Assume that we can introduce a shift/reduce conflict. In this case, a state of LALR parser must have:

$$A \to \alpha \bullet ,a \qquad \text{and} \qquad B \to \beta \bullet a\gamma ,b$$

- This means that a state of the canonical LR(1) parser must have:

$$A \to \alpha \bullet ,a \qquad \text{and} \qquad B \to \beta \bullet a\gamma ,c$$

But, this state has also a shift/reduce conflict. i.e. The original canonical LR(1) parser has a conflict.

(Reason for this, the shift operation does not depend on lookaheads)

Reduce/Reduce Conflict

- But, we may introduce a reduce/reduce conflict during the shrink process for the creation of the states of a LALR parser.

$$I_1 : A \to \alpha\bullet,a \qquad\qquad I_2 : A \to \alpha\bullet,b$$
$$B \to \beta\bullet,b \qquad\qquad B \to \beta\bullet,c$$

$$\Downarrow$$

$$I_{12} : A \to \alpha\bullet,a/b$$ ➔ reduce/reduce conflict
$$B \to \beta\bullet,b/c$$

Canonical LALR(1) Collection – Example2

$S' \to S$
1) $S \to L=R$
2) $S \to R$
3) $L \to *R$
4) $L \to id$
5) $R \to L$

I_0: $S' \to \bullet S,\$$
$S \to \bullet L=R,\$$
$S \to \bullet R,\$$
$L \to \bullet *R,\$/=$
$L \to \bullet id,\$/=$
$R \to \bullet L,\$$

$I_0 \xrightarrow{S} I_1$: $S' \to S\bullet,\$$

$I_0 \xrightarrow{L} I_2$: $S \to L\bullet=R,\$$ $\to$ to I_6
$R \to L\bullet,\$$

$I_0 \xrightarrow{R} I_3$: $S \to R\bullet,\$$

$I_0 \xrightarrow{*} I_{411}$: $L \to *\bullet R,\$/=$
$R \to \bullet L,\$/=$
$L \to \bullet *R,\$/=$
$L \to \bullet id,\$/=$

$\xrightarrow{R}$ to I_{713}
$\xrightarrow{L}$ to I_{810}
$\xrightarrow{*}$ to I_{411}
$\xrightarrow{id}$ to I_{512}

$I_0 \xrightarrow{id} I_{512}$: $L \to id\bullet,\$/=$

I_6: $S \to L=\bullet R,\$$
$R \to \bullet L,\$$
$L \to \bullet *R,\$$
$L \to \bullet id,\$$

$\xrightarrow{R}$ to I_9
$\xrightarrow{L}$ to I_{810}
$\xrightarrow{*}$ to I_{411}
$\xrightarrow{id}$ to I_{512}

I_9: $S \to L=R\bullet,\$$

I_{713}: $L \to *R\bullet,\$/=$

I_{810}: $R \to L\bullet,\$/=$

Same Cores
I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

LALR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3	r3			
8			r5	r5			
9				r1			

no shift/reduce or
no reduce/reduce conflict

$\Downarrow$

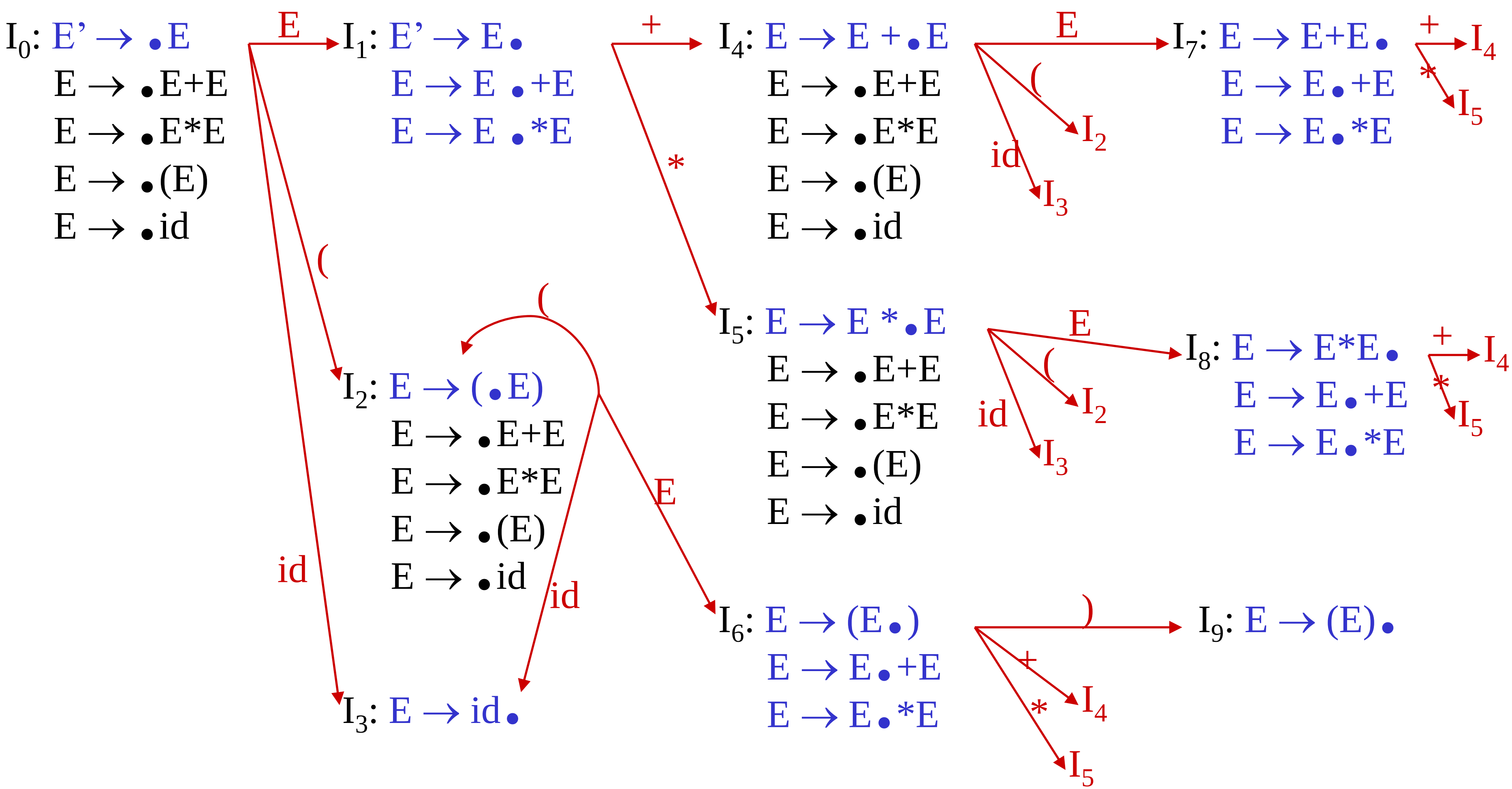
so, it is a LALR(1) grammar

Using Ambiguous Grammars

- All grammars used in the construction of LR-parsing tables must be un-ambiguous.
- Can we create LR-parsing tables for ambiguous grammars ?
  - Yes, but they will have conflicts.
  - We can resolve these conflicts in favor of one of them to disambiguate the grammar.
  - At the end, we will have again an unambiguous grammar.
- Why we want to use an ambiguous grammar?
  - Some of the ambiguous grammars are **much natural**, and a corresponding unambiguous grammar can be very complex.
  - Usage of an ambiguous grammar may **eliminate unnecessary reductions**.
- Ex.

$$E \rightarrow E+E \mid E^*E \mid (E) \mid id \quad \Rightarrow \quad \begin{array}{l} E \rightarrow E+T \mid T \\ T \rightarrow T^*F \mid F \\ F \rightarrow (E) \mid id \end{array}$$

Sets of LR(0) Items for Ambiguous Grammar



SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, +, *,) }

State I_7 has shift/reduce conflicts for symbols + and *.

$$I_0 \xrightarrow{E} I_1 \xrightarrow{+} I_4 \xrightarrow{E} I_7$$

when current token is +
- shift ➔ + is right-associative
- reduce ➔ + is left-associative

when current token is *
- shift ➔ * has higher precedence than +
- reduce ➔ + has higher precedence than *

SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, +, *,) }

State I_8 has shift/reduce conflicts for symbols + and *.

$$I_0 \xrightarrow{E} I_1 \xrightarrow{*} I_5 \xrightarrow{E} I_7$$

when current token is *

shift ➔ * is right-associative

reduce ➔ * is left-associative

when current token is +

shift ➔ + has higher precedence than *

reduce ➔ * has higher precedence than +

SLR-Parsing Tables for Ambiguous Grammar

	Action							Goto
	id	**+**	*****	**(**	**)**	**$**		**E**
0	s3			s2				1
1		s4	s5			acc		
2	s3			s2				6
3		r4	r4		r4	r4		
4	s3			s2				7
5	s3			s2				8
6		s4	s5		s9			
7		r1	s5		r1	r1		
8		r2	r2		r2	r2		
9		r3	r3		r3	r3		

End of ch3….

Thank You

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