ELEMENTARY COUNTING PRINCIPLES

1.1 Basic Counting Principles

1.1.1 Addition principle (AP)

If a first task E can be per formed in n(E)= r ways, while a second task F can be per formed in n(F)= t ways, and the two tasks (E and F) can not be accomplished simultaneously, then either task E or F can be performed in:

$n(EVF) = n(E)+n\ (F)= r + t$ *ways.*

This principle is sometimes referred to as the sum Rule.

Examples

1. *Suppose there are 4 male and 3 female instructors teaching multivariable calculus in our college. In how many ways can a student choose a calculus instructor in order to take the course?*

Solution: - The student is faced with two tasks. The task of either

(i) choosing a female calculus instructor or

(ii) Choosing a male calculus instructor.

If E= the task of choosing a female instructor.
and F = the task of choosing a male instructor,
then:

n(E) = 3 --- Number of ways of doing task E, i.e., choosing a female instructor.
n(F) = 4 --- Number or ways of doing task F, i.e., choosing a male instructor.

Since the tasks E and F cannot be per formed simultaneously, then n(EVF) – the number of ways of accomplishing either task E or F, by addition principle (AP), is :

$$n(EVF) = n(E) + n(F) = 3+4 = 7.$$

Therefore, the student can choose one instructor teaching multivariable calculus in 7 ways ///

Generalization of AP

Suppose a first task E_1 can be performed in $n(E_1) = r_1$ ways, a second task E_2 can be performed in $n(E_2) = r_2$ ways, a third task E_3 can be performed in $n(E_3) = r_3$ ways, etc, and an n^{th} task E_n can be performed in $n(E_n) = r_n$ ways. If no two of the tasks can be performed at the same time, then the number of ways in which any one of tasks E_1 or E_2 or E_3 or ---, or E_n can be performed is:

$$n(E_1 \vee E_2 \vee \text{---} \vee E_n) = n(E_1) + n(E_2) + \text{---} + n(E_n)$$
$$= r_1 + r_2 + \text{---} + r_n$$
$$= \sum_{i=1}^{n} r_i$$

1.1.2 Multiplication principle (MP)

If an operation consists of two separate steps E and F, and if the first step E can be performed in n (E) = r ways and corresponding to each of these r ways , there are n (F) = t ways of performing the second step F, then the entire operation can be performed in:

$$n(E) \times n(F) = rt \text{ different ways}$$

Examples

1. *A room in a building has four doors that may be designated as Door A, B, C and D. If a person is interested in entering the room and leaving it by a different door, then in how many possible ways can be fulfill his interest?*

Solution: - To handle problems of this nature, it helps to have a TREE DIAGRAM of the following form.

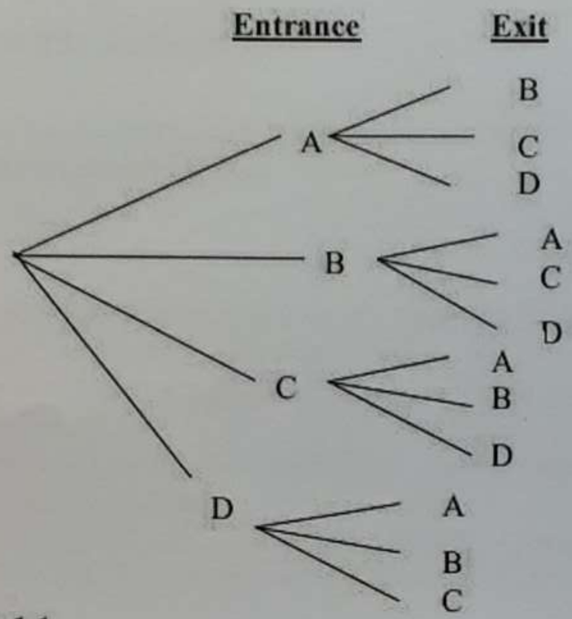

Figure 1.1.

Note that the tree diagram takes order to account. Thus, AB and BA count as two different possibilities or arrangements. The first one refers to the possibility that one enters the room through door A and leaves through door B, while the second describes the process of entering through door B and leaving through door A. Order is the essence of such arrangements, and any change in order yields a completely different arrangement.

The diagram shows that there are four branches corresponding to the entrances in to the room through the four doors A, B, C and D; and that subsequent to (or following) one's entry into the room through one of the doors, there are only three doors left and one can use any of the three doors for exit. For instance, if one enters through door A, then one is free to leave the room through one of the remaining doors B, C, or D. Thus, we find that there are in all 12 different paths along the branches of the tree corresponding to the 12 different ways in which one can enter the room by one door and leave by another.

∴ There are 12 ways of entering by one door and leaving by another.

On the other hand, the data in the problem implies that there are

- 4 Possible entrances into the room.

Corresponding to each of these entrances, there are

- 3 possible exits from the room.

Thus, using the following rule called multiplication principle, we find that there are:

$4 \times 3 = 12$ ways of entering the room by a door and leaving by another.

Generalization of MP

If an operation consists of n separate steps, of which a first step E_1 can be performed in $n(E_1) = r_1$ ways; following this, a second step E_2 can be performed in $n(E_2)= r_2$ ways, and, following the 2^{nd} step E_2, a third step E_3 can be performed in $n(E_3)= r_3$ ways, etc., and following all the previous steps, an nth step E_n can be performed in $n(E_n)= r_n$ ways, then the entire operation can be performed and completed in:

$n(E_1). n(E_2). n(E_3)\text{---} n(E_n) = r_1 r_2 r_3 \ldots r_n$ different ways.

1.2 The pigeon hole principle

Definition 1: if n pigeonholes are occupied by n+1 pigeons then at least one pigeonhole is occupied by more than one pigeon.

Examples:

1. If a computer department consists of 13 doctors then two of the doctors (pigeons) were born in the same month (pigeonholes).
2. Suppose in a dormitory of 8 beds there are 9 students. Then at least two students share the same bed.
3. If there are 366 people then at least two people must have the same birth day as there 365 days in a year.

Definition 2: if n pigeonholes are occupied by k*n+1 or more pigeons, where k is a positive integer, then at least one pigeonhole is occupied by k+1 or more pigeons.

Examples:

1. *Find the minimum number of students in a class to be sure that three of them are born in the same month.*

   Solution:

   n = 12, number of months in a year (pigeonholes)
   $k+1 = 3 \rightarrow k = 2$.
   Hence $k*n+1 = 2*12+1 = 25$
   Therefore the minimum number of students in a class to be sure that three of them are in the same class is 25.

2. *Suppose a laundry bag consists of many red, white, and blue socks. Find the minimum number of socks that one need to grip in order to get two pairs (four socks) of the same color.*

   Solution:

   Pigeonhole: number of colors. That is, n = 3
   Pigeons: socks.
   Then, $k+1 = 4$.
   Hence $k = 3$
   $k*n+1 = 10$.
   Therefore one needs to grip at least 4 socks of the same color

1.3 Permutations

In this section, we will show that the multiplication rule (multiplication principle) provides a general method for finding the number of permutations of n different things taken r at a time. Many types of problems of permutations can be shortened by means of convenient symbols and formulas we now introduce.

1.3.1. Factorial Notations

Definition 1:- The product of the first n consecutive positive integers is called n-factorial denoted by n! and defined as:

$n! = 1x2x3x4x \text{---} x(n-1)x\, n = nx(n-1)x \text{---} x3x2x1$

Note that:

(i) $6! = 6x5!$ (ii) $7! = 7x6!$ (iii) $200! = 200x199!$

In general;

$n! = n(n-1)!$

- If n=0, then we define 0! =1.

In particular, observe that

$1! = 1$

$2! = 2x1=2$

$3! = 3x2x1 = 6$

$4! = 4x3x2x1 = 24$

$5! = 5x4x3x2x1 = 120$

$6! = 6x5x4x3x2x1 = 720$

$7! = 7x6x5x4x3x2x1 = 5040$, etc.

The factorial notation is very useful for representing large numbers of the type frequently encountered in the study of permutations and related topics.

1.3.2. Permutation principles

Definition 2:- Any arrangement of r objects taken from a collection of n objects is called a permutation of n objects taken r at a time or an r-permutation of n objects.

<u>Notation</u>: The number of permutations (or possible arrangements in any order) of n objects taken r at a time is denoted by ${}^np_r$ or p (n, r) frequently; where $0 \leq r \leq n$. Other notations are $p_{n,r}$ and $(n)_r$.

<u>Preposition 1</u> (Permutations without repeating objects)

The number of permutations of n different objects taken r at a time, when none of the objects is repeated in an arrangement is:

$${}^np_r = n(n-1)(n-2)\text{---}(n-r+1) = \frac{n!}{(n-r)!}$$

Examples

1. *Evaluate:* *(a)* ${}^{8}p_5$ *(b)* ${}^{6}p_4$

Solution: - (a) $8p_5 = \dfrac{8!}{(8-5)!} = \dfrac{8!}{3!} = 8x7x6x5x4 = 6720 ///$

(b) ${}^{6}p_4 = \dfrac{6!}{(6-4)!} = \dfrac{6!}{2!} = 6x5x4x3 = 360 ///$

2. *Solve for n in each of the following*

(a) ${}^{n}p_2 = 56$ *(b)* ${}^{n}p_3 = 20\,n$ ***(Exercice)***

Solution : $(a)\ {}^{n}p_2 = 56 \Leftrightarrow \dfrac{n!}{(n-2)!} = 56 ...$ *by* definition

$$\Leftrightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 56$$

$$\Leftrightarrow n^2 - n = 56$$

$$\Leftrightarrow n^2 - n - 56 = 0$$

$$\Leftrightarrow (n+7)(n-8) = 0$$

$$\Leftrightarrow n = -7 \vee n = 8$$

n = -7 is rejected since $n \in N \cup \{0\}$. Thus, the value of n= 8 ///

3. *How many "words" of three letters can be formed from the letters a,b,c,d, and e, using each letter only once?*

Solution:- Since the letters a,b,c,d,e in different orders constitute different "words", the result is the number of permutations of five objects taken three at a time. Thus, by principle of permutation without repetition, there are:

$${}^{5}p_3 = \frac{5}{(5-3)!} = \frac{5!}{2!} = 5x4x3 = 60 \text{ words} ///$$

4. *What is the number of ways in which six students be seated in a classroom with 25 desks?*

Solution: - There are six students that are going to occupy six desks at a time. Thus, there are six seats to fill and 25 desks to choose from. The result is the number of permeations of 25 different objects taken six at a time, i.e., n= 25 r=6 and

$${}^{25}p_6 = \frac{25!}{(25-6)!} = \frac{25!}{19!} = 25x24x23x22x21x120$$

$$= 127{,}512{,}000 \text{ ways} ///$$

5. *In how many ways can six pupils stand in a line or linearly arranged to pay their college fees at the finance office counter?*

Solution: - This corresponds to the case r = n in proposition 1. Thus the number of permutations of a set of six different objects, taken altogether is:

$$^{6}P_{6} = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6x5x4x3x2x1}{1} = 720 \text{ ways///}$$

$$\Leftrightarrow {}^{6}P_{6} = 6!$$

Remark:- In general, the number of permutations of n objects taken all at time (or taken altogether) is:

$$\boxed{^{n}P_{n} = n!}$$

Proposition 2 (Permutations with objects repeated)

The number of permutations of n different objects taken r at a time, when each object can be repeated any number of times in an arrangements is:

$n \times n \times \ldots \times n = n^r$, since n is used r-times as a factor.

Examples

1. *A multiple-choice test has 100 questions with four possible answers for each question. How many different sets of 100 answers are possible?*

Solution: - Here, n= 4 and r= 100. Thus, the required permutation is 4^{100}.

2. *In how many ways can five prizes be given away to four boys*
   (a) When each boy is eligible for all the prizes?
   (b) When any boy may win all but one of the prizes?

Solution: - (a) There are 4 ways of giving away the first prize (to either boy1, boy2, boy3, or boy 4). There are again four ways of disposing the second prize since it can be given to any one of the four students; and so on. Finally, the 5^{th} prize can be disposed in 4 ways. Thus, the required number of permutations is 4^5= 1024 ///

(b) Since there are only four possibilities in which a student may have all the prizes, the number of permutations in this case is four less than that for case (a), that is:

$4^5 - 4 = 1024 - 4 = 1020$ ///

Preposition 3 (Permutations with alike objects)

The number of permutation of n objects taken all together and where p of the n objects are alike and of one kind; q others are alike and of another kind, and so on, up to t others alike and of still another kind such that p+q+---+t = n, is given by:

$$p(n;p,q,...,t)=\frac{n!}{p!q!..t!}$$

Examples

1. *In how many ways can 10 cars be placed in a stock car race if three of them are Chevrolet, four are Fords, two are Plymouths, and one is a Buick?*

Solution:- Here, n = 10, p = 3, q=4, r = 2, and t =1. The number of distinct arrangements or permutations is given by:

$$P(10;3,4,2,1)=\frac{10!}{3!4!2!1!}=1260///$$

2. *How many signals can be given using 10 flags of which two are red, five are blue, and three are yellow?*

Solution: - Clearly, n=10, p =2, q= 5 and r=3. Hence, the number of signals is:

$$P(10;2,5,3)=\frac{10!}{2!5!3!}=2520///$$

Proposition 4 (Circular permutations)

The number of permutations of n objects around a circle, taken altogether, is given by:
C= (n-1)!

Examples

1. *In how many ways can eight gents (gentlemen) and eight ladies be seated for a round-table conference so that no two ladies sit together?*

Solution: - The number of ways in which the eight gents can be seated at a round table occupying alternate seats is given by:
C= (n-1)! = (8-1)!= 7! Ways
Then the ladies have a choice of eight remaining seats and this arrangement can be completed in 8! different ways. Now, using the fundamental principle of multiplication, we conclude that the required number is:

$$\begin{aligned}7!\times 8! &= 5040 \times 40320\\ &= 203{,}212{,}800 \text{ ways}.\end{aligned}$$

1.4. Combinations

Definition: - Any subset of r objects selected with complete disregard to their order from a collection of n different objects is called an r-combination of the n objects or a combination of n objects taken r at a time.

The number of r- combination of n objects is frequently denoted by either $\binom{n}{r}$ or $^{n}c_{r}$. The symbols, C(n,r), $C_{n,r}$ and C_r^n also appear in various texts.

Examples

1. *Find the number of combinations of the four objects a, b, c, & d taken three at a time.*

Solution: - Each combination consisting of the three objects determines 3!=6 permutations of the objects in the combination as shown in table 1.1. Thus the number of combinations multiplied by 3! equals the number of permutations, that is,

$$\binom{4}{3}.3! = p(4,3)$$

$$\Leftrightarrow \binom{4}{3} = \frac{p(4,3)}{3!}$$

$$But\ p(4,3) = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4x3x2 = 24 \text{ and } 3! = 1x2x3 = 6$$

There fore,

$$\binom{4}{3} = \frac{p(4,3)}{3!} = \frac{24}{6} = 4 \textit{ as noted in table } 1.1 \text{ beneath.}$$

Combination	Permutations
abc	abc, acb, bac, bca, cab, cba
abd	abd, adb, bad, bda, dab, dba
acd	acd, adc, cad, cda, dac, dca
bcd	bcd, bdc, cbd, cdb, dbc, dcb

Proposition 1 (number of r-combinations)

The number of combinations of n different objects taken r at a time (i.e., the number of r- combinations of n objects) is given by.

$$\binom{n}{r} = \frac{^{n}P_{r}}{r!} = \frac{n!}{r!(n-r)!}; where\ 0 \le r \le n.$$

Proof: - Note that $\binom{n}{r}$ represents the number of combinations of n objects taken r at a time. Each such combination of r objects can be arranged in r! different ways. There fore, $r!\binom{n}{r}$ yields the total number of permutations of n objects taken r a time, or

$$r!\binom{n}{r} = {}^{n}p_{r}$$

By proposition 1.1 and dividing throughout by r!, we obtain:

$$\binom{n}{r} = \frac{{}^{n}p_{r}}{r!} = \frac{n!}{r!(n-r)!}$$

2. *In how many ways can a committee of five be chosen from a group of 10 members of an association?*

Solution:- From proposition 1, the number of possibilities of selecting five-person committee from a group of 10 association members is given by the formula (Where n= 10 and r = 5):

$$\binom{10}{5} = \frac{10!}{5!(10-5)!} = \frac{10x9x8x7x6}{5x4x3x2x1} = 252///$$

3. *In how many ways can 11 players be chosen from a group of 13 players if*
   (a) *the players are selected at random?*
   (b) *a particular player must be included?*
   (c) *a certain player must be excluded?*

Solution: - (a) The number of combinations of 13 players, taken 11 at a time, is given by:

$$\binom{13}{11} = \frac{13!}{11!2!} = \frac{13x12}{2x1} = 78 \text{ ways}///$$

(b) If one particular player is always to be included, we need to select 10 more out of the remaining 12. This can be accomplished in:

$$\binom{12}{10} = \frac{12!}{10!2!} = \frac{12x11}{2x1} = 66 \text{ ways}///$$

(c) If one player should be excluded from the team, we need a selection of 11 players from the remaining group of 12. Thus, the required number of such combinations is:

$$\binom{12}{11} = \frac{12!}{11!1!} = \frac{12}{1} = 12 \text{ ways} ///$$

1.6. The Binomial Theorem

The quantities $\binom{n}{r}$ are called binomial coefficients because of the fundamental role these quantities play in the formulation of the binomial theorem. Expansions of positive integral powers of $(a+b)^n$, where $n=0,1,2, \ldots$, are of frequent occurrence in algebra and are beginning to appear in all phases of mathematics. Moreover, expansions of this nature are important because of their close relationship with the binomial distribution studied in statistics and related fields. We shall, therefore, under take a systematic development of the formula that produces such expansions.

Of course, the following identities, for example, could be established by direct multiplication:

$(a+b)^0 = 1$
$(a+b)^1 = a+b$
$(a+b)^2 = a^2 + 2ab + b^2$
$(a+b)^3 = a^3+3a^2 b+ 3ab^2+ b^3$.
$(a+b)^4 = a^4 + 4a^3 b + 6a^2 b^2 + 4ab^3 + b^4$
$(a+b)^5 = a^5 + 5a^4 b+ 10a^3 b^2 +10a^2b^3+ 5ab^4+ b^5$
$(a+b)^6 = a^6+6a^5 b+ 15a^4 b^2 + 20a^3 b^3 + 15a^2 b^4+ 6ab^5+ b^5$.

Note that as we continue expanding larger and larger powers of (a+b), several patterns emerge, leading to a part of the solution. The following patterns may be evident from the above process of multiplication:

(a) The coefficients of the first and last terms are both 1.
(b) There are n+1 terms in the expansion of $(a+b)^n$.
(c) The exponent of a starts with n and then decrease by 1 until the exponent of a has decreased to 0 in the last term, and exponent of b is 0 in the first term and then continues to increase by 1 with the exponent of b is n in the last term.
(d) The sum of the exponents of a and b in a given term is n.

Remark: - The binomial theorem gives the coefficient of terms in the expansion of powers of binomial expressions. Binomial expression is an expression which contains two terms.

Proposition (The Binomial Theorem)

If n, r are non-negative integers, where $0 \le r \le n$, then

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^4$$
$$= \sum_{r=0}^{n}\binom{n}{r} a^{n-r}b^r$$

Note that the (n+1) terms in the expansion of $(a+b)^n$, without their coefficients, are

$a^n ; a^{n-1}b,\ a^{n-2} b^2 ; \ldots a^{n-r} b^r, \ldots, a^2 b^{n-2} ; ab^{n-1} ; b^n$

In other words, each term in the expansion is of the form:

$a^{n-r}\ b^r$, where r= 0,1,2,…, n.

The coefficient of this general term is $\binom{n}{r}$, since this corresponds to the number of ways in which r b's and (n-r) a's can be selected, and thus the complete general term is;

$$\binom{n}{r}a^{n-r}b^{r}$$

A summation of this general term for r= 0, 1,2, ..., n yields the above assertion.

Corollary 1: From proposition above, with a=1, it follows that:

$$(1+b)^{n}=\binom{n}{0}+\binom{n}{1}b+\binom{n}{2}b^{2}+\binom{n}{3}b^{3}+\ldots+\binom{n}{r}b^{r}+\ldots+\binom{n}{n}b^{n}.$$

Corollary 2: With a=b=1 in proposition above, it follows that:

$$2^{n}=(1+1)^{n}=\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\ldots+\binom{n}{n}$$

Corollary 3: With a=1 and b=-1 in proposition above, we have:

$$0=[1+(-1)]^{n}=\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+\binom{n}{4}-\binom{n}{5}+\ldots+(-1)^{n}\binom{n}{n}$$

For even values of n, corollary 3 yields:

$$\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots+\binom{n}{n}=\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\ldots+\binom{n}{n-1}=2^{n-1}.$$

Examples

1. *Expand* $(x+2y)^{7}$.

Solution:-

$$\begin{aligned}(x+2y)^{7}&=\binom{7}{0}x^{7}+\binom{7}{1}x^{6}(2y)+\binom{7}{2}x^{5}(2y)^{2}+\binom{7}{3}x^{4}(2y)^{3}\\&\quad+\binom{7}{4}x^{3}(2y)^{4}+\binom{7}{5}x^{2}(2y)^{5}+\binom{7}{6}x(2y)^{6}+\binom{7}{7}(2y)^{7}\\&=x^{7}+14x^{6}y+84x^{5}y^{2}+280x^{4}y^{3}+560x^{3}y^{4}+672x^{2}y^{5}+448xy^{6}+128y^{7}.\ ///\end{aligned}$$

Remark:

The calculation of the coefficients is simplified by making use of the complementary combinations $\binom{n}{r}=\binom{n}{n-r}$.

In the preceding example, we needed to calculate only up to $\binom{7}{3}$ and then recognize that $\binom{7}{0}=\binom{7}{7};\binom{7}{1}=\binom{7}{6};\binom{7}{2}=\binom{7}{5};\binom{7}{3}=\binom{7}{4}$

2. *Expand $(1+2x)^6$.*

Solution: - Letting n=6, a=1 and b=2x in corollary 1, we have:

$$(1+2x)^6=\binom{6}{0}+\binom{6}{1}(2x)+\binom{6}{2}(2x)^2+\binom{6}{3}(2x)^3+\binom{6}{4}(2x)^4+\binom{6}{5}(2x)^5+\binom{6}{6}(2x)^6.$$
$$=1+12x+60x^2+160x^3+240x^4+192x^5+64x^6 \;///$$

Using the proposition, find the numerical value of $(1.04)^{10}$.

Solution: - Let n =10, a=1 and b= 0.04 in corollary 1, then

$$(1+0.04)^{10}=\binom{10}{0}+\binom{10}{1}(0.04)+\binom{10}{2}(0.04)^3+\ldots+\binom{10}{10}(0.04)^{10}$$
$$=1+10\,(0.04)+45(0.04)+45(0.04)^2+120(0.04)^3+\ldots+(0.04)^{10}$$
$$=1+0.4+0.072+0.00768+\ldots$$
$$\approx 1.47968 \;///$$

Remark:

The $(r+1)^{st}$ term in the binomial expansion of $(a+b)^n$ is given by:

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r.$$

5. *Without an actual expansion, find the 10^{th} term of $(2x-y)^{11}$.*

Solution:

Using the above remark, we have:

$$T_{10}=\binom{11}{9}(2x)^{11-9}(-y)^9$$
$$=\binom{11}{9}(-1)^9\,2^2x^2y^9$$
$$=-\frac{11!}{9!\,2!}.4x^2y^9$$
$$=-220\,x^2y^9 \;///$$

6. *Find the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{10}$ without expanding it.*

Solution:

Since n=10, it follows that there are 11 terms in the binomial expansion of $\left(x-\frac{1}{x}\right)^{10}$.

Accordingly, the 6^{th} term represents the middle term. Thus,

$$T_6=\binom{10}{5}x^{10-5}\left(-\frac{1}{x}\right)^5=252x^5(-x^{-1})^5=-252 \;///$$

1.6. The inclusion-Exclusion Principle and Derangements

1.6.1. The Principle of Inclusion and Exclusion

In this section we develop and state a new counting technique called the Inclusion-exclusion Principle. Examples will be used to develop the technique and as well as to further demonstrate how the principle is applied.

Proposition 1: Let A,B and C be any finite sets. then:

1. $n(A\cup B) = n(A)+n(B)-n(A\cap B)$
2. $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-$ $n(B\cap C)+n(A\cap B\cap C)$

Proposition 2: Let A,B and C be any finite sets and U a universal set. If $n(U) = N$, then:

1. $n(\overline{A}\,\overline{B}) = N-[n(A)+n(B)]+n(AB) = N - n(A\cup B)$
2. $n(\overline{A}\,\overline{B}\,\overline{C}) = N-[n(A)+n(B)+n(C)]+[n(AB)+n(AC)+n(BC)]-n(ABC)$ $= N - n(A\cup B\cup C)$

Note: $\overline{A}\,\overline{B} = A^C\cap B^C$ *and* $AB = A\cap B$ where $(\overline{A})$ or A^c denotes the complement of set A.

Examples

1. *Suppose that in a group of 100 students; 50 taking mathematics, 40 taking computer science, 35 taking information science, 12 taking maths and computer science, 10 taking maths and information, 11 taking computer science and information science and 5 taking all.*
   a. *How many students are taking at least one subject*
   b. *How many of them taking none of the subjects*

   Solution:

   a. Let M = students taking mathematics
   C = students taking computer science
   I = students taking information

   Then by proposition 1

   $$n(M\cup C\cup I) = n(M)+n(C)+n(I)-n(M\cap C)-n(M\cap I)-n(C\cap I)+n(M\cap C\cap I)$$
   $$= 50+40+35-12-11-10+5$$
   $$= 97$$

   b. using proposition 2

   $$n(\overline{M}\,\overline{C}\,\overline{I}) = N - n(M\cup C\cup I)$$
   $$= 100-97$$
   $$= 3$$