Top-Down Parsing

- The parse tree is created top to bottom.
- Top-down parser
  - Recursive-Descent Parsing
    - Backtracking is needed (If a choice of a production rule does not work, we backtrack to try other alternatives.)
    - It is a general parsing technique, but not widely used.
    - Not efficient
  - Predictive Parsing
    - no backtracking
    - efficient
    - needs a special form of grammars (LL(1) grammars).
    - Recursive Predictive Parsing is a special form of Recursive Descent parsing without backtracking.
    - Non-Recursive (Table Driven) Predictive Parser is also known as LL(1) parser.

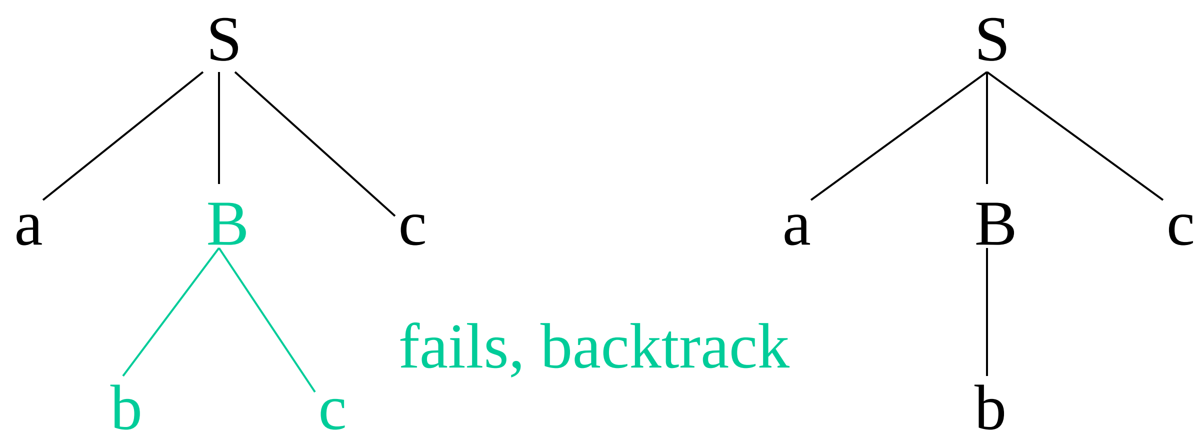
Recursive-Descent Parsing (uses Backtracking)

- Backtracking is needed.
- It tries to find the left-most derivation.

$S \rightarrow aBc$

$B \rightarrow bc \mid b$

input: abc



Predictive Parser

a grammar → eliminate left recursion → left factor → a grammar suitable for predictive parsing (a LL(1) grammar) no %100 guarantee.

- When re-writing a non-terminal in a derivation step, a predictive parser can uniquely choose a production rule by just looking the current symbol in the input string.

$A \rightarrow \alpha_1 \mid \ldots \mid \alpha_n$ input: ... a ....

current token

Predictive Parser (example)

stmt → `if` ...... |
    `while` ...... |
    `begin` ...... |
    `for` .....

- When we are trying to write the non-terminal *stmt*, if the current token is `if` we have to choose first production rule.
- When we are trying to write the non-terminal *stmt*, we can uniquely choose the production rule by just looking the current token.
- We eliminate the left recursion in the grammar, and left factor it. But it may not be suitable for predictive parsing (not LL(1) grammar).

Recursive Predictive Parsing

- Each non-terminal corresponds to a procedure.

Ex: $A \rightarrow aBb$ (This is only the production rule for A)

proc A {

- match the current token with a, and move to the next token;
- call ‘B’;
- match the current token with b, and move to the next token;

}

Recursive Predictive Parsing (cont.)

$A \rightarrow aBb \mid bAB$

```
proc A {
    case of the current token {
        'a':  - match the current token with a, and move to the next token;
              - call 'B';
              - match the current token with b, and move to the next token;
        'b':  - match the current token with b, and move to the next token;
              - call 'A';
              - call 'B';
    }
}
```

Recursive Predictive Parsing (cont.)

- When to apply ε-productions.

  $A \rightarrow aA \mid bB \mid \varepsilon$

- If all other productions fail, we should apply an ε-production. For example, if the current token is not a or b, we may apply the ε-production.
- Most correct choice: We should apply an ε-production for a non-terminal A when the current token is in the follow set of A (which terminals can follow A in the sentential forms).

Recursive Predictive Parsing (Example)

$A \rightarrow aBe \mid cBd \mid C$

$B \rightarrow bB \mid \varepsilon$

$C \rightarrow f$

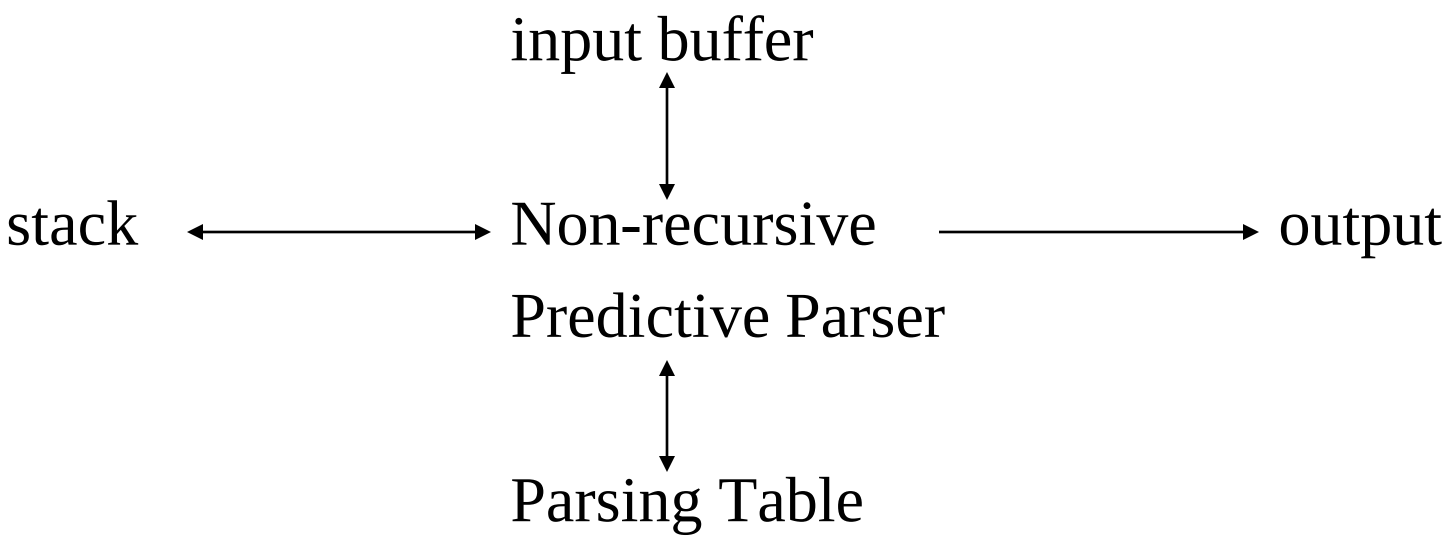
```
proc C {   match the current token with f,
           and move to the next token; }

proc A {
    case of the current token {
        a:  - match the current token with a,
              and move to the next token;
            - call B;
            - match the current token with e,
              and move to the next token;
        c:  - match the current token with c,
              and move to the next token;
            - call B;
            - match the current token with d,
              and move to the next token;
        f:  - call C
    }
}

proc B {
    case of the current token {
        b:  - match the current token with b,
              and move to the next token;
            - call B
        e,d:  do nothing
    }
}
```

first set of C

follow set of B

Non-Recursive Predictive Parsing -- LL(1) Parser

- Non-Recursive predictive parsing is a table-driven parser.
- It is a top-down parser.
- It is also known as LL(1) Parser.



LL(1) Parser

input buffer

- our string to be parsed. We will assume that its end is marked with a special symbol $.

output

- a production rule representing a step of the derivation sequence (left-most derivation) of the string in the input buffer.

stack

- contains the grammar symbols
- at the bottom of the stack, there is a special end marker symbol $.
- initially the stack contains only the symbol $ and the starting symbol S. $S ← initial stack
- when the stack is emptied (ie. only $ left in the stack), the parsing is completed.

parsing table

- a two-dimensional array M[A,a]
- each row is a non-terminal symbol
- each column is a terminal symbol or the special symbol $
- each entry holds a production rule.

LL(1) Parser – Parser Actions

- The symbol at the top of the stack (say X) and the current symbol in the input string (say a) determine the parser action.
- There are four possible parser actions.

1. If X and a are \$ ➔ parser halts (successful completion)
2. If X and a are the same terminal symbol (different from \$)

   ➔ parser pops X from the stack, and moves the next symbol in the input buffer.

3. If X is a non-terminal

   ➔ parser looks at the parsing table entry M[X,a]. If M[X,a] holds a production rule $X \rightarrow Y_1 Y_2 \ldots Y_k$, it pops X from the stack and pushes $Y_k, Y_{k-1}, \ldots, Y_1$ into the stack. The parser also outputs the production rule $X \rightarrow Y_1 Y_2 \ldots Y_k$ to represent a step of the derivation.

4. none of the above ➔ error
   - all empty entries in the parsing table are errors.
   - If X is a terminal symbol different from a, this is also an error case.

LL(1) Parser – Example1

$S \rightarrow aBa$

$B \rightarrow bB \mid \varepsilon$

	a	**b**	**$**
S	$S \rightarrow aBa$		
B	$B \rightarrow \varepsilon$	$B \rightarrow bB$	

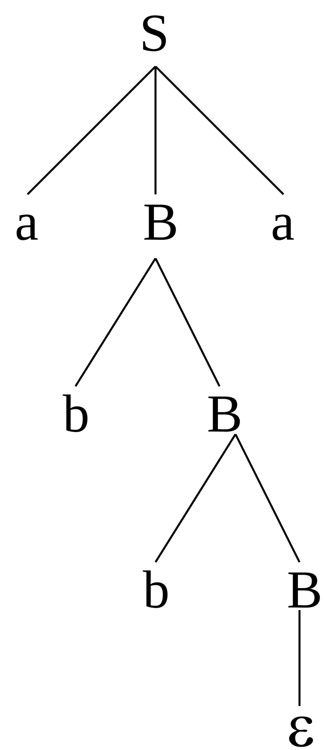
LL(1) Parsing Table

stack	**input**	**output**
$S	abba$	$S \rightarrow aBa$
$aBa	abba$	
$aB	bba$	$B \rightarrow bB$
$aBb	bba$	
$aB	ba$	$B \rightarrow bB$
$aBb	ba$	
$aB	a$	$B \rightarrow \varepsilon$
$a	a$	
$	$	accept, successful completion

LL(1) Parser – Example1 (cont.)

Outputs: $S \rightarrow aBa$ $\quad B \rightarrow bB$ $\quad B \rightarrow bB$ $\quad B \rightarrow \varepsilon$

Derivation(left-most): $S \Rightarrow aBa \Rightarrow abBa \Rightarrow abbBa \Rightarrow abba$



parse tree

LL(1) Parser – Example2

$E \rightarrow TE'$
$E' \rightarrow +TE' \;|\; \varepsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT' \;|\; \varepsilon$
$F \rightarrow (E) \;|\; id$

	id	**+**	*****	**(**	**)**	**$**
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \varepsilon$	$E' \rightarrow \varepsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \varepsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \varepsilon$	$T' \rightarrow \varepsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

LL(1) Parser – Example2

stack	input	output
$E	id+id$	$E \rightarrow TE'$
$E'T	id+id$	$T \rightarrow FT'$
$E' T'F	id+id$	$F \rightarrow id$
$ E' T'id	id+id$	
$ E' T'	+id$	$T' \rightarrow \varepsilon$
$ E'	+id$	$E' \rightarrow +TE'$
$ E' T+	+id$	
$ E' T	id$	$T \rightarrow FT'$
$ E' T' F	id$	$F \rightarrow id$
$ E' T'id	id$	
$ E' T'	$	$T' \rightarrow \varepsilon$
$ E'	$	$E' \rightarrow \varepsilon$
$	$	accept

Constructing LL(1) Parsing Tables

- Two functions are used in the construction of LL(1) parsing tables:
  - FIRST FOLLOW

- **FIRST(α)** is a set of the terminal symbols which occur as first symbols in strings derived from α where α is any string of grammar symbols.
- if α derives to ε, then ε is also in FIRST(α) .

- **FOLLOW(A)** is the set of the terminals which occur immediately after (follow) the *non-terminal A* in the strings derived from the starting symbol.
  - a terminal a is in FOLLOW(A) if $S \overset{*}{\Rightarrow} \alpha A a \beta$
  - \$ is in FOLLOW(A) if $S \overset{*}{\Rightarrow} \alpha A$

Compute FIRST for Any String X

- If X is a terminal symbol ➔ FIRST(X)={X}
- If X is a non-terminal symbol and $X \rightarrow \varepsilon$ is a production rule
  ➔ ε is in FIRST(X).
- If X is a non-terminal symbol and $X \rightarrow Y_1Y_2..Y_n$ is a production rule
  ➔ if a terminal **a** in FIRST(Y_i) and ε is in all FIRST(Y_j) for j=1,...,i-1 then **a** is in FIRST(X).
  ➔ if ε is in all FIRST(Y_j) for j=1,...,n then ε is in FIRST(X).
- If X is ε ➔ FIRST(X)={ε}
- If X is $Y_1Y_2..Y_n$
  ➔ if a terminal **a** in FIRST(Y_i) and ε is in all FIRST(Y_j) for j=1,...,i-1 then **a** is in FIRST(X).
  ➔ if ε is in all FIRST(Y_j) for j=1,...,n then ε is in FIRST(X).

FIRST Example

$E \rightarrow TE'$
$E' \rightarrow +TE' \mid \varepsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT' \mid \varepsilon$
$F \rightarrow (E) \mid id$

$\text{FIRST}(F) = \{(, id\}$
$\text{FIRST}(T') = \{*, \varepsilon\}$
$\text{FIRST}(T) = \{(, id\}$
$\text{FIRST}(E') = \{+, \varepsilon\}$
$\text{FIRST}(E) = \{(, id\}$

$\text{FIRST}(TE') = \{(, id\}$
$\text{FIRST}(+TE') = \{+\}$
$\text{FIRST}(\varepsilon) = \{\varepsilon\}$
$\text{FIRST}(FT') = \{(, id\}$
$\text{FIRST}(*FT') = \{*\}$
$\text{FIRST}(\varepsilon) = \{\varepsilon\}$
$\text{FIRST}((E)) = \{(\}$
$\text{FIRST}(id) = \{id\}$

Compute FOLLOW (for non-terminals)

- If S is the start symbol ➔ \$ is in FOLLOW(S)

- if $A \rightarrow \alpha B \beta$ is a production rule
  ➔ everything in FIRST(β) is FOLLOW(B) except ε

- If ($A \rightarrow \alpha B$ is a production rule) or
  ($A \rightarrow \alpha B \beta$ is a production rule and ε is in FIRST(β))
  ➔ everything in FOLLOW(A) is in FOLLOW(B).

We apply these rules until nothing more can be added to any follow set.

FOLLOW Example

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \varepsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \varepsilon$

$F \rightarrow (E) \mid id$

FOLLOW(E) = { \$,) }

FOLLOW(E') = { \$,) }

FOLLOW(T) = { +,), \$ }

FOLLOW(T') = { +,), \$ }

FOLLOW(F) = {+, *,), \$ }

Constructing LL(1) Parsing Table -- Algorithm

- for each production rule $A \rightarrow \alpha$ of a grammar G
  - for each terminal a in FIRST(α)
    ➔ add $A \rightarrow \alpha$ to M[A,a]
  - If ε in FIRST(α)
    ➔ for each terminal a in FOLLOW(A) add $A \rightarrow \alpha$ to M[A,a]
  - If ε in FIRST(α) and \$ in FOLLOW(A)
    ➔ add $A \rightarrow \alpha$ to M[A,\$]

- All other undefined entries of the parsing table are error entries.

Constructing LL(1) Parsing Table -- Example

$E \rightarrow TE'$	FIRST(TE')={(,id}	➔ $E \rightarrow TE'$ into M[E,(] and M[E,id]
$E' \rightarrow +TE'$	FIRST($+TE'$)={+}	➔ $E' \rightarrow +TE'$ into M[E',+]
$E' \rightarrow \varepsilon$	FIRST(ε)={ε}	➔ none
	but since ε in FIRST(ε)	
	and FOLLOW(E')={\$,)}	➔ $E' \rightarrow \varepsilon$ into M[E',\$] and M[E',)]
$T \rightarrow FT'$	FIRST(FT')={(,id}	➔ $T \rightarrow FT'$ into M[T,(] and M[T,id]
$T' \rightarrow *FT'$	FIRST($*FT'$)={*}	➔ $T' \rightarrow *FT'$ into M[T',*]
$T' \rightarrow \varepsilon$	FIRST(ε)={ε}	➔ none
	but since ε in FIRST(ε)	
	and FOLLOW(T')={\$,),+}	➔ $T' \rightarrow \varepsilon$ into M[T',\$], M[T',)] and M[T',+]
$F \rightarrow (E)$	FIRST((E))={(}	➔ $F \rightarrow (E)$ into M[F,(]
$F \rightarrow id$	FIRST(id)={id}	➔ $F \rightarrow id$ into M[F,id]

LL(1) Grammars

- A grammar whose parsing table has no multiply-defined entries is said to be LL(1) grammar.

one input symbol used as a look-head symbol do determine parser action

LL(1) left most derivation

input scanned from left to right

- The parsing table of a grammar may contain more than one production rule. In this case, we say that it is not a LL(1) grammar.

A Grammar which is not LL(1)

$S \rightarrow i\ C\ t\ S\ E \mid a$

$E \rightarrow e\ S \mid \varepsilon$

$C \rightarrow b$

FOLLOW(S) = { \$,e }

FOLLOW(E) = { \$,e }

FOLLOW(C) = { t }

FIRST(iCtSE) = {i}

FIRST(a) = {a}

FIRST(eS) = {e}

FIRST(ε) = {ε}

FIRST(b) = {b}

	a	**b**	**e**	**i**	**t**	**$**
S	$S \rightarrow a$			$S \rightarrow iCtSE$		
E			$E \rightarrow e\ S$ $E \rightarrow \varepsilon$			$E \rightarrow \varepsilon$
C		$C \rightarrow b$				

two production rules for M[E,e]

Problem ➔ ambiguity

A Grammar which is not LL(1) (cont.)

- What do we have to do it if the resulting parsing table contains multiply defined entries?
  - If we didn't eliminate left recursion, eliminate the left recursion in the grammar.
  - If the grammar is not left factored, we have to left factor the grammar.
  - If its (new grammar's) parsing table still contains multiply defined entries, that grammar is ambiguous or it is inherently not a LL(1) grammar.
- A left recursive grammar cannot be a LL(1) grammar.
  - $A \rightarrow A\alpha \mid \beta$
    - ➔ any terminal that appears in FIRST(β) also appears FIRST($A\alpha$) because $A\alpha \Rightarrow \beta\alpha$.
    - ➔ If β is ε, any terminal that appears in FIRST(α) also appears in FIRST($A\alpha$) and FOLLOW(A).
- A grammar is not left factored, it cannot be a LL(1) grammar
  - $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$
    - ➔ any terminal that appears in FIRST($\alpha\beta_1$) also appears in FIRST($\alpha\beta_2$).
- An ambiguous grammar cannot be a LL(1) grammar.

Properties of LL(1) Grammars

- A grammar G is LL(1) if and only if the following conditions hold for two distinctive production rules $A \rightarrow \alpha$ and $A \rightarrow \beta$

1. Both α and β cannot derive strings starting with same terminals.

2. At most one of α and β can derive to ε.

3. If β can derive to ε, then α cannot derive to any string starting with a terminal in FOLLOW(A).