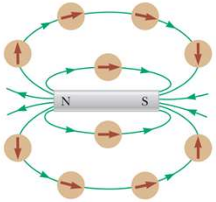
Electromagnetism



July, 2022, ASTU

Electric Field

- Electric field is a region of space around a charged object which exerts a force on other charged objects.

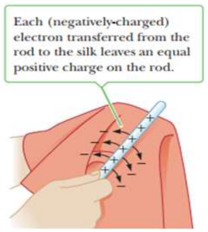
- There are two kinds of charge, **positive (proton)** and **negative (electron)**.

- Most objects are electrostatically neutral as they contain an equal number of positive and negative charges.

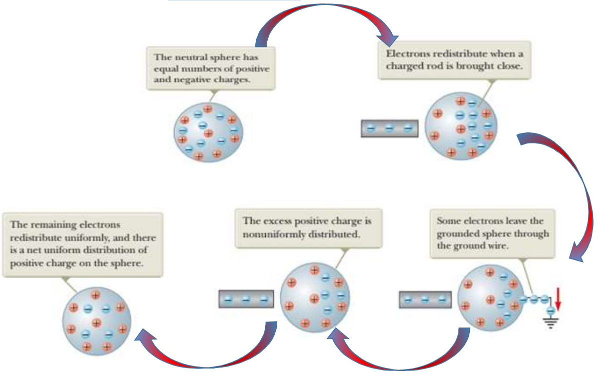
- Properties of electric charge
  - Electric charge is always conserved
  - Electric charge is quantized (Ne, N = integer and $e = 1.602 \times 10^{-19} C$)
  - Charges of the same signs ($+ve$ & $+ve$, $-ve$ & $-ve$) repel and charges with opposite signs ($-ve$ & $+ve$) attract one another.

Electric Field

- Formation of charge
  - Friction

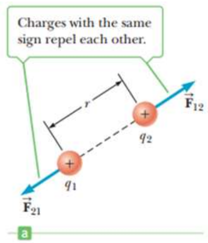


- Induction



Electric Field

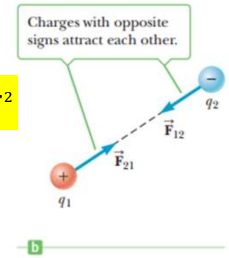
- The strength of the electrostatic force (repulsive or attractive force) between two charges q_1 and q_2 is given by Coulomb's law.



$$F = k_e \frac{|q_1||q_2|}{r^2}$$

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.9875 \times 10^9\ N.m^2/C^2$$

$$\varepsilon_0 = 8.8542 \times 10^{-12}\ C^2/N.m^2$$



- The direction of the force is along the joining line.

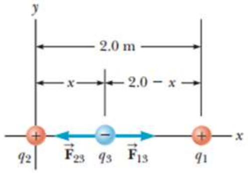
Electric Field

- An **electric force** has the following properties:

1. It is directed along a line joining the two particles and is inversely proportional to the square of the separation distance r, between them.
2. It is proportional to the product of the magnitudes of the charges, $|q_1|$ and $|q_2|$, of the two particles.
3. It is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

- Example 1:

The positive charge $q_1 = 15\ \mu C$, is at $x = 2\ m$, and the positive charge $q_2 = 6.0\ \mu C$ is at the origin. Where must a *negative* charge q_3 be placed on the x-axis so that the resultant electric force on it is zero?



Electric Field

$$F_{13x} = +k_e \frac{(15 \times 10^{-6}\ \text{C})|q_3|}{(2.0\ \text{m} - x)^2} \qquad F_{23x} = -k_e \frac{(6.0 \times 10^{-6}\ \text{C})|q_3|}{x^2}$$

$$\sum F_{net} = 0 \qquad k_e \frac{(15 \times 10^{-6}\ \text{C})|q_3|}{(2.0\ \text{m} - x)^2} - k_e \frac{(6.0 \times 10^{-6}\ \text{C})|q_3|}{x^2} = 0$$

$$6(2 - x)^2 = 15x^2$$

$$6(4 - 4x + x^2) = 15x^2 \rightarrow 2(4 - 4x + x^2) = 5x^2$$

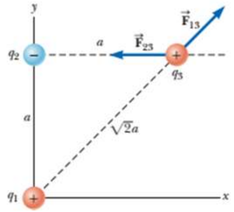
$$3x^2 + 8x - 8 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - (4)(3)(-8)}}{2 \cdot 3} = \frac{-4 \pm 2\sqrt{10}}{3} \qquad x = 0.77\ \text{m}$$

Electric Field

- Example 2:

Consider three point charges located at the corners of a right triangle, where $q_1 = q_3 = 5\ \mu C$, $q_2 = -2\ \mu C$, and $a = 0.1\ m$. Find the resultant force exerted on q_3.



$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2) \frac{(2.00 \times 10^{-6}\ \text{C})(5.00 \times 10^{-6}\ \text{C})}{(0.100\ \text{m})^2} = 8.99\ \text{N}$$

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}\,a)^2}$$

$$= (8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2) \frac{(5.00 \times 10^{-6}\ \text{C})(5.00 \times 10^{-6}\ \text{C})}{2(0.100\ \text{m})^2} = 11.2\ \text{N}$$

$$F_{13x} = (11.2\ \text{N}) \cos 45.0° = 7.94\ \text{N}$$

$$F_{13y} = (11.2\ \text{N}) \sin 45.0° = 7.94\ \text{N}$$

$$F_{3x} = F_{13x} + F_{23x} = 7.94\ \text{N} + (-8.99\ \text{N}) = -1.04\ \text{N}$$

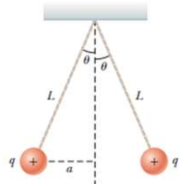
$$F_{3y} = F_{13y} + F_{23y} = 7.94\ \text{N} + 0 = 7.94\ \text{N}$$

$$\vec{\mathbf{F}}_3 = (-1.04\hat{\mathbf{i}} + 7.94\hat{\mathbf{j}})\ \text{N}$$

Electric Field

- Example 3:

Two identical small charged spheres, each having a mass of $3\ mg$, hang in equilibrium. The length L of each string is 0.15 m, and the angle $\theta = 5^0$. Find the magnitude of the charge on each sphere.



$$\sum F_x = T \sin\theta - F_e = 0 \rightarrow T \sin\theta = F_e$$

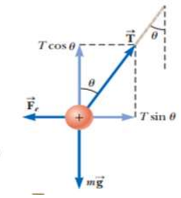
$$\sum F_y = T \cos\theta - mg = 0 \rightarrow T \cos\theta = mg$$

$$\tan\theta = \frac{F_e}{mg} \rightarrow F_e = mg \tan\theta$$

$$F_e = 3 \times 10^{-2} \times 10 \times 0.09 = 2.7 \times 10^{-2} N = \frac{k_e q^2}{(2L sin5)^2}$$

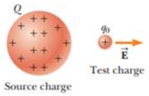
$$q^2 = \frac{4 \times 0.0225 \times 2.7 \times 10^{-2}}{0.09 \times 8.99 \times 10^9} = 3 \times 10^{-12}\ C^2$$

$$q = 1.73 \times 10^{-6} C = 1.73\ \mu C$$

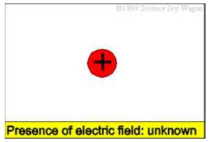


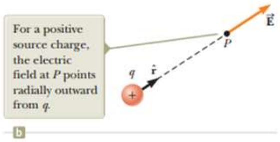
Electric Field

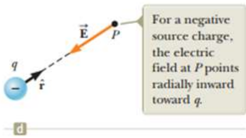
➢ Electric field ($\vec{E}$) is the region around a charged object where another charged object will experience a **force**.



$$\vec{E} = \frac{\vec{F_e}}{q_0} = \frac{k_e Q q_0}{r^2 . q_0} = \frac{k_e Q}{r^2}$$



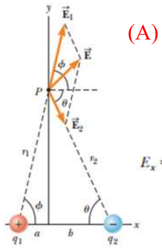




Electric Field

❖ Example 4:

Charges q_1 and q_2 are located on the x axis, at distances a and b, respectively, from the origin. **(A)** Find the components of the net electric field at the point P, which is at position $(0, y)$. **(B)** Evaluate the electric field at point P in the special case that $|q_1| = |q_2|$ and $a = b$. **(C)** Find the electric field due to the electric dipole when point P is a distance $y \gg a$ from the origin.



(A)

$$E_1 = k_e \frac{|q_1|}{r_1^2} = k_e \frac{|q_1|}{a^2 + y^2} \qquad E_2 = k_e \frac{|q_2|}{r_2^2} = k_e \frac{|q_2|}{b^2 + y^2}$$

$$\vec{\mathbf{E}}_1 = k_e \frac{|q_1|}{a^2 + y^2} \cos\phi \,\hat{\mathbf{i}} + k_e \frac{|q_1|}{a^2 + y^2} \sin\phi \,\hat{\mathbf{j}} \qquad \vec{\mathbf{E}}_2 = k_e \frac{|q_2|}{b^2 + y^2} \cos\theta \,\hat{\mathbf{i}} - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta \,\hat{\mathbf{j}}$$

$$E_x = E_{1x} + E_{2x} = k_e \frac{|q_1|}{a^2 + y^2} \cos\phi + k_e \frac{|q_2|}{b^2 + y^2} \cos\theta$$

$$E_y = E_{1y} + E_{2y} = k_e \frac{|q_1|}{a^2 + y^2} \sin\phi - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta$$

Electric Field

❖ Example 4: continued

(B)

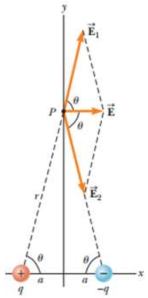
$$E_x = k_e \frac{q}{a^2 + y^2} \cos\theta + k_e \frac{q}{a^2 + y^2} \cos\theta = 2k_e \frac{q}{a^2 + y^2} \cos\theta$$

$$E_y = k_e \frac{q}{a^2 + y^2} \sin\theta - k_e \frac{q}{a^2 + y^2} \sin\theta = 0$$

$$\cos\theta = \frac{a}{r} = \frac{a}{(a^2 + y^2)^{1/2}} \qquad E_x = 2k_e \frac{q}{a^2 + y^2}\left[\frac{a}{(a^2 + y^2)^{1/2}}\right] = k_e \frac{2aq}{(a^2 + y^2)^{3/2}}$$

(C)

$$E \approx k_e \frac{2aq}{y^3}$$



Electric Field

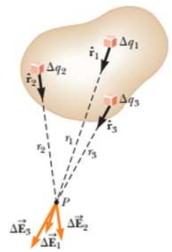
❖ **Electric Field of a Continuous Charge Distribution**

➢ The electric field at P due to one charge element carrying charge Δq is

$$\Delta \vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

➢ The total field at P in the limit $\Delta q_i \to 0$ is

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$



- If a charge Q is uniformly distributed throughout a volume V, the **volume charge density ρ** is defined by

$$\rho = \frac{Q}{V} = \frac{dq}{dV}$$

- If a charge Q is uniformly distributed on a surface of area A, the **surface charge density σ** is defined by

$$\sigma = \frac{Q}{A} = \frac{dq}{dA}$$

Electric Field

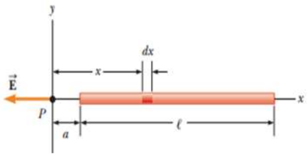
❖ Electric Field of a Continuous Charge Distribution

- If a charge Q is uniformly distributed along a line of length ,l, the **linear charge density** λ is defined by

$$\lambda = \frac{Q}{\ell} = \frac{dq}{d\ell}$$

❖ Example 5:

A rod of length ℓ, has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.



$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

$$E = \int_a^{\ell+a} k_e \lambda \frac{dx}{x^2} = k_e \lambda \int_a^{\ell+a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x}\right]_a^{\ell+a}$$

$$E = k_e \frac{Q}{\ell}\left(\frac{1}{a} - \frac{1}{\ell + a}\right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

Electric Field

❖ Example 6:

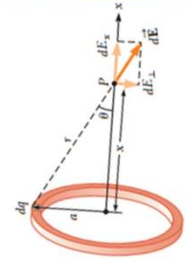
A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring.

$$dE_x = k_e \frac{dq}{r^2}\cos\theta = k_e \frac{dq}{a^2 + x^2}\cos\theta \qquad \cos\theta = \frac{x}{r} = \frac{x}{(a^2 + x^2)^{1/2}}$$

$$dE_x = k_e \frac{dq}{a^2 + x^2}\left[\frac{x}{(a^2 + x^2)^{1/2}}\right] = \frac{k_e x}{(a^2 + x^2)^{3/2}}\, dq$$

$$E_x = \int \frac{k_e x}{(a^2 + x^2)^{3/2}}\, dq = \frac{k_e x}{(a^2 + x^2)^{3/2}} \int dq$$

$$E = \boxed{\frac{k_e x}{(a^2 + x^2)^{3/2}}\, Q}$$



Electric Field

❖ Example 7:

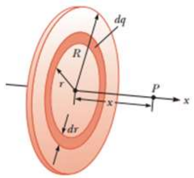
A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk.

$$dq = \sigma\, dA = \sigma(2\pi r\, dr) = 2\pi\sigma r\, dr$$

$$dE_x = \frac{k_e x}{(r^2 + x^2)^{3/2}}(2\pi\sigma r\, dr)$$

$$E_x = k_e x\pi\sigma \int_0^R \frac{2r\, dr}{(r^2 + x^2)^{3/2}} = k_e x\pi\sigma \int_0^R (r^2 + x^2)^{-3/2} d(r^2)$$

$$= k_e x\pi\sigma \left[\frac{(r^2 + x^2)^{-1/2}}{-1/2}\right]_0^R = \boxed{2\pi k_e\sigma\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]}$$



$R \gg x$

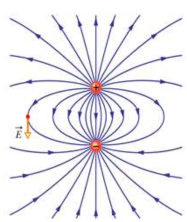
$$E = 2\pi k_e \sigma = \frac{\sigma}{2\epsilon_0}$$

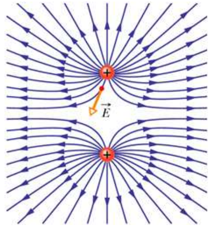
Electric Field

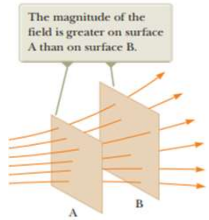
❖ Electric Field Lines

➢ Electric field lines are lines representing an electric field in a region of space.

1. The electric field vector $\vec{E}$ is tangent to the electric field lines at each point.
2. The number of lines per unit area through a surface perpendicular to the lines is proportional to the strength of the electric field in a given region.



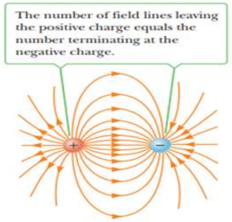


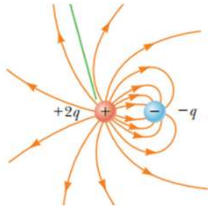


Electric Field

❖ The rules for drawing electric field lines for any charge distribution

1. The lines for a group of point charges must begin on positive charges and end on negative charges. In the case of an excess of charge, some lines will begin or end infinitely far away.
2. The number of lines drawn leaving a positive charge or ending on a negative charge is proportional to the magnitude of the charge.
3. No two field lines can cross each other.



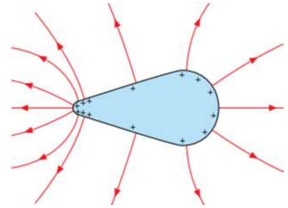


Electric Field

❖ Conductors in Electrostatic Equilibrium

➢ When no net motion of charge occurs within a conductor, the conductor is said to be in **electrostatic equilibrium.** An isolated conductor has the following properties:

1. The electric field is zero everywhere inside the conducting material.
2. Any excess charge on an isolated conductor resides entirely on its surface.
3. The electric field just outside a charged conductor is perpendicular to the conductor's surface.
4. On an irregularly shaped conductor, the charge accumulates at sharp points, where the radius of curvature of the surface is smallest.



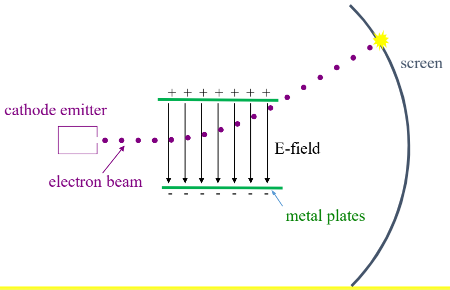
Electric Field

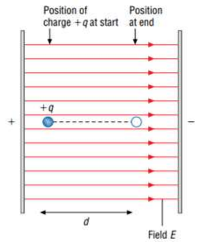
❖ **Motion of a Charged Particle in a Uniform Electric Field**

➢ When a particle of charge q and mass m is placed in an electric field $\vec{E}$, the electric force exerted on the charge is $q\vec{E}$.

$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \frac{q\vec{\mathbf{E}}}{m}$$

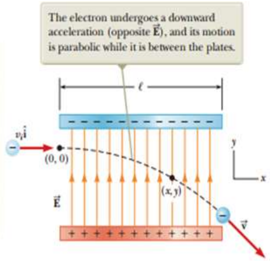




Electric Field

❖ Example 8:

? An electron enters the region of a uniform electric field with $v_i = 3 \times 10^6 m/s$ and $\vec{E} = 200N/C$. The horizontal length of the plates is $\ell = 0.100\ m$. **(A)** Find the acceleration of the electron while it is in the electric field. **(B)** Assuming the electron enters the field at time $t = 0$, find the time at which it leaves the field. **(C)** What is its vertical position when it leaves the field?



(A) $$a_y = -\frac{eE}{m_e} = -\frac{(1.60 \times 10^{-19}\ C)(200\ N/C)}{9.11 \times 10^{-31}\ kg} = \boxed{-3.51 \times 10^{13}\ m/s^2}$$

(B) $$x_f = x_i + v_x t \rightarrow t = \frac{x_f - x_i}{v_x}$$

$$t = \frac{\ell - 0}{v_x} = \frac{0.100\ m}{3.00 \times 10^6\ m/s} = \boxed{3.33 \times 10^{-8}\ s}$$

(C) $$y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2$$

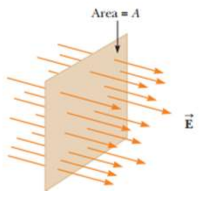
$$y_f = 0 + 0 + \tfrac{1}{2}(-3.51 \times 10^{13}\ m/s^2)(3.33 \times 10^{-8}\ s)^2$$

$$= -0.019\,5\ m = \boxed{-1.95\ cm}$$

Electric Field

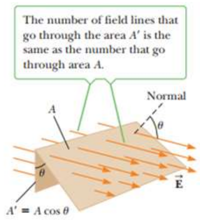
❖ Electric Flux and Gauss's Law

- Electric flux is a measure of the number of electric field lines passing through a surface.
- Therefore, the total number of lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field and surface area perpendicular to the field is called the **electric flux (ϕ_E).**



$$\phi_E = EA_\perp$$

$$\phi_E = EA\cos\theta$$

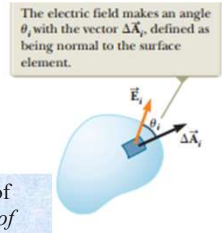


Electric Field

❖ Electric Flux and Gauss's Law

$$\Phi_{E,i} = E_i\,\Delta A_i\,\cos\theta_i = \vec{\mathbf{E}}_i\cdot\Delta\vec{\mathbf{A}}_i$$

$$\Phi_E \approx \sum \vec{\mathbf{E}}_i\cdot\Delta\vec{\mathbf{A}}_i \quad = \int_{\text{surface}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}$$



- The *net* flux through the surface is proportional to the net number of lines leaving the surface, where the net number means *the number of lines leaving the surface minus the number of lines entering the surface.*
- If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative.

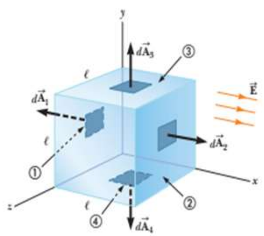
- We can write the net flux ϕ_E through a closed surface as

$$\Phi_E = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \oint E_n\,dA$$

Electric Field

❖ Example 9:

? Consider a uniform electric field $\vec{E}$ oriented in the x direction in empty space. A cube of edge length ℓ, is placed in the field. Find the net electric flux through the surface of the cube.



$$\Phi_{xy} = EA\cos(90°) = \boxed{0} \text{ (back and front surfaces)}$$

$$\Phi_{xz} = EA\cos(90°) = \boxed{0} \text{ (top and bottom surfaces)}$$

$$\int_1 \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_1 E(\cos 180°)\,dA = -E\int_1 dA = -EA = -E\ell^2$$

$$\int_2 \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_2 E(\cos 0°)\,dA = E\int_2 dA = +EA = E\ell^2$$

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

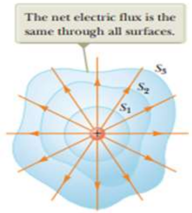
Electric Field

❖ Electric Flux and Gauss's Law

- Gauss's law is a law stating that the electric flux though any closed surface is proportional to he enclosed electric charge.

- The mathematical form of **Gauss's law** is a generalization of what we have just described and states that the net flux through *any* closed surface is

$$\Phi_E = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\epsilon_0}$$



❖ The net flux through *any* closed surface surrounding a point charge q is given by q/ϵ_0 and is independent of the shape of that surface.

Electric Field

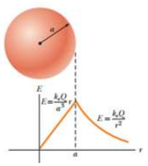
❖ Example 10:

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q. **(A)** Calculate the magnitude of the electric field at a point outside the sphere. **(B)** Find the magnitude of the electric field at a point inside the sphere.

(A) $\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = \frac{Q}{\epsilon_0}$

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \boxed{k_e \frac{Q}{r^2} \quad (\text{for } r > a)}$$

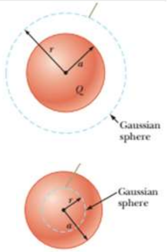


(B) $q_{in} = \rho V' = \rho(\frac{4}{3}\pi r^3)$

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{q_{in}}{4\pi\epsilon_0 r^2} = \frac{\rho(\frac{4}{3}\pi r^3)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} r$$

$$E = \frac{Q/\frac{4}{3}\pi a^3}{3(1/4\pi k_e)} r = \boxed{k_e \frac{Q}{a^3} r \quad (\text{for } r < a)}$$



Electric Field

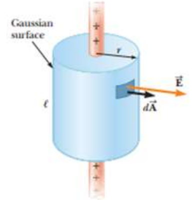
❖ Example 11:

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oint dA = EA = \frac{q_{in}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2k_e \frac{\lambda}{r}}$$



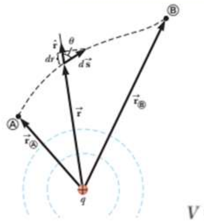
Electric Potential

➢ Electrical potential is the work done per unit positive charge to move a positive test charge from infinity to its current position within an electric field

$$\boxed{\Delta V = \frac{\Delta U}{q} = -\int_{Ⓐ}^{Ⓑ} \vec{\mathbf{E}} \cdot d\vec{s}}$$

$$\vec{\mathbf{E}} \cdot d\vec{s} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\vec{s}$$

$$\hat{\mathbf{r}} \cdot d\vec{s} = ds \cos\theta, \qquad ds \cos\theta = dr.$$

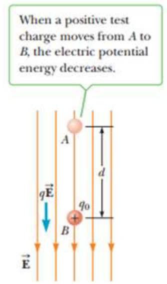


$$V_Ⓑ - V_Ⓐ = -k_e q \int_{r_Ⓐ}^{r_Ⓑ} \frac{dr}{r^2} = k_e \frac{q}{r} \Big|_{r_Ⓐ}^{r_Ⓑ}$$

$$V_Ⓑ - V_Ⓐ = k_e q \left[\frac{1}{r_Ⓑ} - \frac{1}{r_Ⓐ}\right]$$

$V = 0$ at $r_Ⓐ = \infty$.

$$\boxed{V = k_e \frac{q}{r}}$$



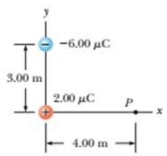
Electric Potential

➢ For a group of point charges, we can write the total electric potential at *given point* as

$$V = k_e \sum_i \frac{q_i}{r_i}$$

❖ Example 12:

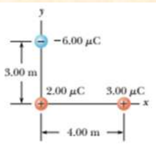
For the following figures: (A) Find the total electric potential due to these charges at the point P. **(B)** Find the change in potential energy of the system of two charges plus a third charge q_3 as the latter charge moves from infinity to point P



$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.988 \times 10^9\ N \cdot m^2/C^2)\left(\frac{2.00 \times 10^{-6}\ C}{4.00\ m} + \frac{-6.00 \times 10^{-6}\ C}{5.00\ m}\right)$$

$$= \boxed{-6.29 \times 10^3\ V}$$



$$\Delta U = U_f - U_i = q_3 V_P - 0 = (3.00 \times 10^{-6}\ C)(-6.29 \times 10^3\ V)$$

$$= \boxed{-1.89 \times 10^{-2}\ J}$$

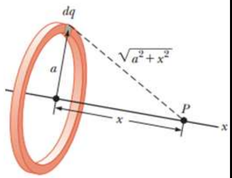
Electric Potential

❖ Obtaining the Value of the Electric Field from the Electric Potential

$$dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \qquad dV = k_e \frac{dq}{r} \qquad V = k_e \int \frac{dq}{r}$$

$$E_x = -\frac{dV}{dx} \qquad \vec{\mathbf{E}} = -\nabla V = -\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)V$$

❖ Example 13:



(A) Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q. **(B)** Find an expression for the magnitude of the electric field at point P.

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2 + x^2}} \qquad V = \frac{k_e}{\sqrt{a^2 + x^2}} \int dq = \frac{k_e Q}{\sqrt{a^2 + x^2}}$$

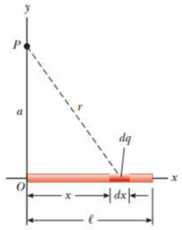
Electric Potential

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}(a^2 + x^2)^{-1/2}$$

$$= -k_e Q(-\tfrac{1}{2})(a^2 + x^2)^{-3/2}(2x) \qquad E_x = \frac{k_e x}{(a^2 + x^2)^{3/2}} Q$$

❖ Example 14:

A rod of length ℓ, located along the x axis has a total charge Q and a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance a from the origin.



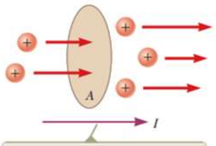
$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}} \qquad V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2 + x^2}} = k_e \frac{Q}{\ell} \ln\left(x + \sqrt{a^2 + x^2}\right)\Big|_0^{\ell}$$

$$V = \int_0^{\ell} k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}} \qquad V = k_e \frac{Q}{\ell}\left[\ln\left(\ell + \sqrt{a^2 + \ell^2}\right) - \ln a\right] = k_e \frac{Q}{\ell} \ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right)$$

Current and Resistance

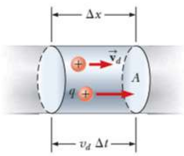
➢ Current (I (unit ampere (A))) quantitatively, suppose charges (Q) are moving perpendicular to a surface of area A.

➢ The **current** is defined as the rate at which charge flows through this surface.



The direction of the current is the direction in which positive charges flow when free to do so.

$$I_{avg} = \frac{\Delta Q}{\Delta t} \qquad I \equiv \frac{dQ}{dt}$$



$$\Delta Q = (nA\,\Delta x)q \qquad \Delta Q = (nAv_d\,\Delta t)q$$

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nqv_d A$$

✓ n is charge carrier density
✓ v_d is drift speed

Current and Resistance

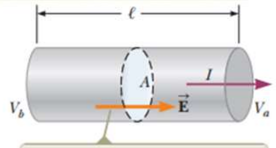
➢ Consider a conductor of cross-sectional area A carrying a current I. The **current density J** in the conductor is defined as the current per unit area.

$$J \equiv \frac{I}{A} = nqv_d \qquad J = \sigma E$$

✓ σ is conductivity

$$\Delta V = E\ell \qquad J = \sigma \frac{\Delta V}{\ell}$$

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = R\,I$$



A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\vec{\mathbf{E}}$, and this field produces a current I that is proportional to the potential difference.

❖ The quantity $R = \frac{\ell}{\sigma A}$ is called the **resistance (unit: Ohm (Ω))** of the conductor. We define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

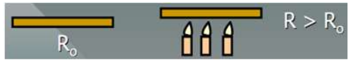
$$R \equiv \frac{\Delta V}{I}$$

✓ Ohm's law

Current and Resistance

❖ The inverse of conductivity is **resistivity** ρ:

$$\rho = \frac{1}{\sigma} \qquad R = \rho\frac{\ell}{A}$$

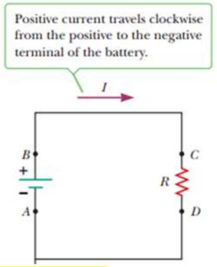


$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad R = R_0[1 + \alpha(T - T_0)]$$

❖ Example 15:

? If the radius of Nichrome wire is 0.32 mm. Calculate the resistance per unit length of this wire.

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{\rho}{\pi r^2} = \frac{1.0 \times 10^{-6}\,\Omega \cdot \text{m}}{\pi(0.32 \times 10^{-3}\,\text{m})^2} = 3.1\ \Omega/\text{m}$$



Current and Resistance

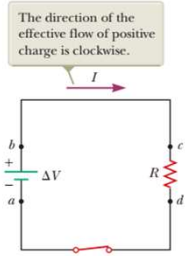
❖ **Electrical Power**

➢ As the charge moves from *a* to *b* through the battery, the electric potential energy of the system *increases* by an amount $Q\Delta V$, while the chemical potential energy in the battery *decreases* by the same amount.

▪ Let's now investigate the rate at which the electric potential energy of the system decreases as the charge Q passes through the resistor:

$$\frac{dU}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V$$

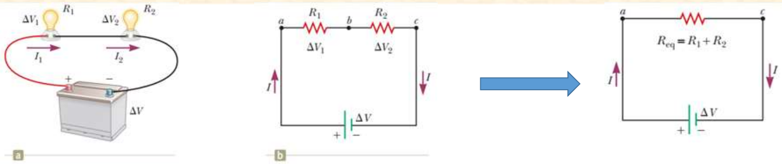
$$P = I\Delta V \qquad P = I^2R = \frac{(\Delta V)^2}{R}$$



Resistors in Series and Parallel

❖ **Resistors in Series**

➢ When two or more resistors are connected together as are the incandescent lightbulbs in Figure below (*a*), they are said to be in a **series combination.** Figure (b) is the circuit diagram for the lightbulbs, shown as resistors, and the battery.



▪ In a series connection, if an amount of charge Q exits resistor $R1$, charge Q must also enter the second resistor $R2$. Otherwise, charge would accumulate on the wire between the resistors. Therefore, the same amount of charge passes through both resistors in a given time interval and the currents are the same in both resistors: :

Resistors in Series and Parallel

❖ **Resistors in Series**

$$I = I_1 = I_2$$

➢ The potential difference applied across the series combination of resistors divides between the resistors

$$\Delta V = \Delta V_1 + \Delta V_2 = I_1R_1 + I_2R_2$$

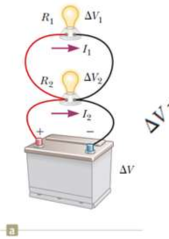
▪ The potential difference across the battery is also applied to the **equivalent resistance** R_{eq} in Figure c

$$\Delta V = IR_{eq} \Rightarrow IR_{eq} = I_1R_1 + I_2R_2 \rightarrow R_{eq} = R_1 + R_2$$

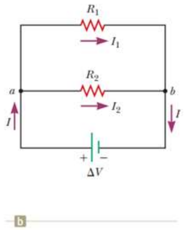
Resistors in Series and Parallel

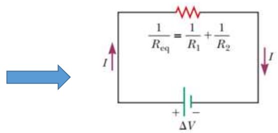
❖ **Resistors in Parallel**

➢ The potential difference applied across the series combination of resistors are the same.



$$\Delta V = \Delta V_1 = \Delta V_2$$





$$I = I_1 + I_2 = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$$

- Because electric charge is conserved, the current I that enters point a must equal the total current leaving that point

Resistors in Series and Parallel

➢ The current in the **equivalent resistance R_{eq}**.

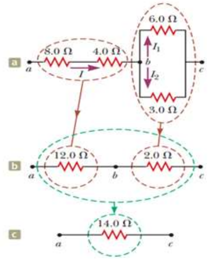
$$I = \frac{\Delta V}{R_{eq}}$$

$$\frac{\Delta V}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

the equivalent resistance of a parallel combination of resistors

Example

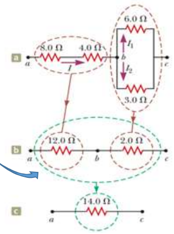
- Four resistors are connected as shown in Figure below. **(A)** Find the equivalent resistance between points a and c **(B)** What is the current in each resistor if a potential difference of 42 V is maintained between a and c?



Resistors in Series and Parallel

(A)

$$R_{eq} = 8.0\,\Omega + 4.0\,\Omega = 12.0\,\Omega$$



$$\frac{1}{R_{eq}} = \frac{1}{6.0\,\Omega} + \frac{1}{3.0\,\Omega} = \frac{3}{6.0\,\Omega}$$

$$R_{eq} = \frac{6.0\,\Omega}{3} = 2.0\,\Omega$$

$$R_{eq} = 12.0\,\Omega + 2.0\,\Omega = \boxed{14.0\,\Omega}$$

(B)

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\text{ V}}{14.0\,\Omega} = \boxed{3.0\text{ A}}$$

$$\Delta V_1 = \Delta V_2 \rightarrow (6.0\,\Omega)I_1 = (3.0\,\Omega)I_2 \rightarrow I_2 = 2I_1$$

$$I_1 + I_2 = 3.0\text{ A} \rightarrow I_1 + 2I_1 = 3.0\text{ A} \rightarrow I_1 = \boxed{1.0\text{ A}}$$

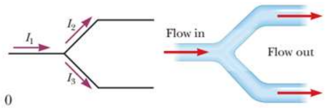
$$I_2 = 2I_1 = 2(1.0\text{ A}) = \boxed{2.0\text{ A}}$$

Kirchhoff's Rules

➢ The procedure for analyzing more complex circuits is made possible by using the following two principles, called **Kirchhoff's rules**.

1. Junction rule. At any junction, the sum of the currents must equal zero:

$$\sum_{\text{junction}} I = 0$$



$$I_1 - I_2 - I_3 = 0$$

- Kirchhoff's first rule is a statement of conservation of electric charge.

2. Loop rule. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\text{closed loop}} \Delta V = 0$$

- Kirchhoff's second rule follows from the law of conservation of energy for an isolated system.

Kirchhoff's Rules

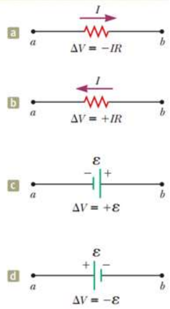
- Charges move from the high-potential end of a resistor toward the low potential end, so if a resistor is traversed in the direction of the current, the potential difference ΔV across the resistor is $-IR$ (Fig. a).

- If a resistor is traversed in the direction *opposite* the current, the potential difference ΔV across the resistor is $+IR$ (Fig. b).

- If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from negative to positive), the potential difference ΔV is $+\mathcal{E}$ (Fig. c).

$\Delta V = \mathcal{E} - Ir$ $\qquad \mathcal{E} = IR + Ir$

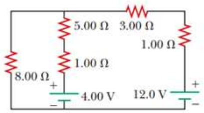
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from positive to negative), the potential difference ΔV is $-\mathcal{E}$ (Fig. d).



Kirchhoff's Rules

Example

1. The circuit shown in Figure below is connected for 2.00 min. (a) Determine the current in each branch of the circuit. (b) Find the energy delivered by each battery. (c) Find the energy delivered to each resistor. (d) Identify the type of energy storage transformation that occurs in the operation of the circuit. (e) Find the total amount of energy transformed into internal energy in the resistors.

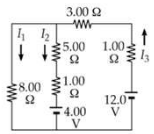


From Kirchhoff's current rule, $I_3 = I_1 + I_2$.

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$$12.0\text{ V} - (4.00\ \Omega)I_3 - (6.00\ \Omega)I_2 - 4.00\text{ V} = 0$$

$$8.00 = (4.00)I_3 + (6.00)I_2$$



Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00\ \Omega)I_2 - 4.00\text{ V} + (8.00\ \Omega)I_1 = 0$$

or $\quad (8.00\ \Omega)I_1 = 4.00 + (6.00\ \Omega)I_2$

Kirchhoff's Rules

Solving the above linear system (by substituting $I_1 + I_2$ for I_3), we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = \frac{4}{3}I_1 - \frac{2}{3} \end{cases}$$

and to the single equation

$$8 = 4I_1 + 10\left(\frac{4}{3}I_1 - \frac{2}{3}\right) = \frac{52}{3}I_1 - \frac{20}{3}$$

which gives

$$I_1 = \frac{3}{52}\left(8 + \frac{20}{3}\right) = 0.846\text{ A}$$

Then $\quad I_2 = I_2 = \frac{4}{3}(0.846) - \frac{2}{3} = 0.462$

and $\quad I_3 = I_1 + I_2 = 1.31\text{ A}$

give $\quad \boxed{I_1 = 846\text{ mA}, I_2 = 462\text{ mA}, I_3 = 1.31\text{ A}}$

(b) For 4.00-V battery:

$$\Delta U = P\Delta t = (\Delta V)I\Delta t = (4.00\text{ V})(-0.462\text{ A})(120\text{ s}) = -222\text{ J}$$

For 12.0-V battery:

$$\Delta U = (12.0\text{ V})(1.31\text{ A})(120\text{ s}) = 1.88\text{ kJ}$$

The results are: $\boxed{\text{-222 J by the 4.00-V battery and 1.88 kJ by the 12.0-V battery}}$.

Kirchhoff's Rules

(c) To the 8.00-Ω resistor:

$$\Delta U = I^2R\Delta t = (0.846\text{ A})^2(8.00\ \Omega)(120\text{ s}) = \boxed{687\text{ J}}$$

To the 5.00-Ω resistor:

$$\Delta U = (0.462\text{ A})^2(5.00\ \Omega)(120\text{ s}) = \boxed{128\text{ J}}$$

To the 1.00-Ω resistor in the center branch:

$$(0.462\text{ A})^2(1.00\ \Omega)(120\text{ s}) = \boxed{25.6\text{ J}}$$

To the 3.00-Ω resistor:

$$(1.31\text{ A})^2(3.00\ \Omega)(120\text{ s}) = \boxed{616\text{ J}}$$

To the 1.00-Ω resistor in the right-hand branch:

$$(1.31\text{ A})^2(1.00\ \Omega)(120\text{ s}) = \boxed{205\text{ J}}$$

Kirchhoff's Rules

(d) Chemical energy in the 12.0-V battery is transformed into internal energy in the resistors. The 4.00-V battery is being charged, so its chemical potential energy is increasing at the expense of some of the chemical potential energy in the 12.0-V battery.

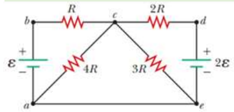
(e) Either sum the results in part (b): $-222\text{ J} + 1.88\text{ kJ} = 1.66\text{ kJ}$,

or in part (c): $687\text{ J} + 128\text{ J} + 25.6\text{ J} + 616\text{ J} + 205\text{ J} = 1.66\text{ kJ}$

The total amount of energy transformed is $\boxed{1.66\text{ kJ}}$.

Kirchhoff's Rules

2. Taking $R = 1.00\ kV$ and $\varepsilon = 250\ V$ in Figure below, determine the direction and magnitude of the current in the horizontal wire between *a* and *e*.



Label the currents in the branches as shown in FIG.(a). Reduce the circuit by combining the two parallel resistors as shown in. FIG. (b).

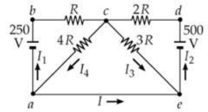


FIG.(a)

Apply Kirchhoff's loop rule to both loops in FIG. (b) to obtain

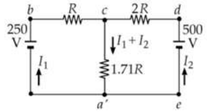


FIG.(b)

$$(2.71R)I_1 + (1.71R)I_2 = 250\text{ V}$$

$$(1.71R)I_1 + (3.71R)I_2 = 500\text{ V}$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

$I_1 = 10.0\text{ mA}$

$I_2 = 130.0\text{ mA}$

$$V_c - V_a = (I_1 + I_2)(1.71R) = 240\text{ V}$$

$$I_4 = \frac{V_c - V_a}{4R} = \frac{240\text{ V}}{4\,000\ \Omega} = 60.0\text{ mA}$$

$$I = I_4 - I_1 = 60.0\text{ mA} - 10.0\text{ mA} = +50.0\text{ mA}$$

or $I = \boxed{50.0\text{ mA from point a to point e}}$

End

To be continue