BASIC ELECTRICITY AND ELECTRONICS

BY: TAMIRAT DEMELASH

tamirudemelash151@gmail.com

UNITY UNIVERSITY

COLLEGE OF COMPUTING

DEPARTMENT OF COMPUTER SCINCE (CS)

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Chapter three

3. Direct current circuits with resistances

3.1 Series circuits

Nodes, Branches, and Loops

- **Network** as an interconnection of elements or devices, whereas a circuit is a network providing one or more closed paths.
- **Network topology**; such elements include branches, nodes, and loops.
  - **A branch** represents a single element such as a voltage source or a resistor. A branch represents any two-terminal element.
  - The circuit in Fig. below has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.
  - A **node** is the point of connection between two or more branches. It is usually indicated by a dot in a circuit.
  - If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node.
  - The circuit in Fig. below has three nodes, and *c*. Notice that the three points that form node *b* are connected by perfectly conducting wires and therefore constitute a single point. The same is true of the four points forming node *c*.
  - The two circuits in Figs. 1 and 2 are identical. However, for the sake of clarity, nodes *b* and *c* are spread out with perfect conductors as in Fig. 1.

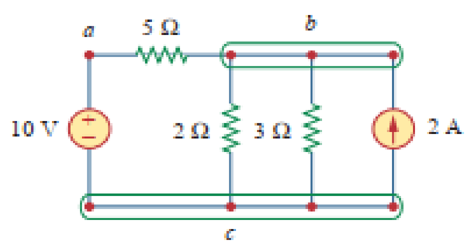


Fig.1

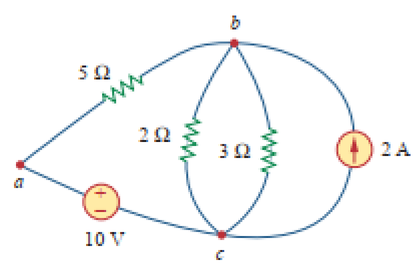


fig.2

- A **loop** is any closed path in a circuit. A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.

- A loop is said to be ***independent*** if it contains at least one branch which is not a part of any other independent loop.
- Independent loops or paths result in independent sets of equations.
- A network with *b* branches, *n* nodes, and *l* independent loops will satisfy the fundamental theorem of network topology: $b = l + n - 1$.

- Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.
- Elements are in series when they are chain-connected or connected sequentially, **end to end.** For example, two elements are in series if they share one common node and no other element is connected to that common node.
- Two or more elements are **in parallel** if they are connected to the same two nodes and consequently have the same voltage across them.
- Elements in parallel are connected to the same pair of terminals. Elements may be connected in a way that they are neither in series nor in parallel.
- In fig. above, the 5- and 2- resistors are neither in series nor in parallel with each other.

Example: Determine the number of branches and nodes in the circuit shown in Fig. below; Identify which elements are in series and which are in parallel.

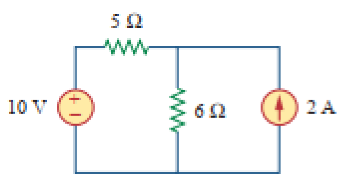


Figure 2.12
For Example 2.4.

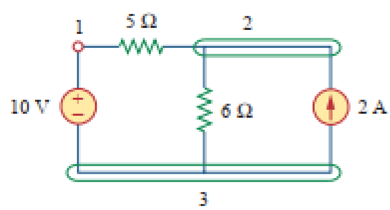


Figure 2.13
The three nodes in the circuit of Fig. 2.12.

Solution

- Since there are four elements in the circuit, the circuit has four branches: 10 V, , and 2 A.
- The circuit has three nodes as identified in Fig. 2.13. The 5- resistor is in series with the 10-V voltage source because the same current would flow in both.
- The 6- resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.

3.2. Current and resistance in series circuits

Kirchhoff's Laws

- Ohm's law by itself is not sufficient to analyze circuits.
- However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits.
- Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.
- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- Mathematically, KCL implies that:

$$\sum_{n=1}^{N} i_n = 0$$

Where N is the number of branches connected to the node and is the nth current entering (or leaving) the node.

- By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.
- To prove KCL, assume a set of currents $ik(t)$, k= 1, 2, …. Flow into a node. The algebraic sum of currents at the node is:

$$i_T(t)= i1(t) + i2(t) + i3(t) \ldots\ldots\ldots$$

Integrating both sides of Eq. above gives $q_T(t)= q1(t) \ +q2(t) \ +q3(t) \ldots\ldots$

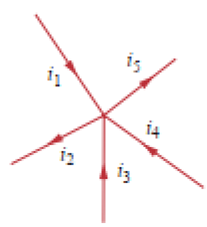


Figure 2.16
Currents at a node illustrating KCL.

- Consider the node in Fig. 2.16. Applying KCL gives:

$$i1 + (-i2) + i3 + i4 + (-i5) = 0$$

Since currents, and are entering the node, while currents and are leaving it. By rearranging the terms, we get: $i1 + i3 + i4 = i2 + i5$

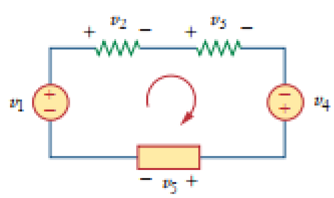
- *The sum of the currents entering a node is equal to the sum of the currents leaving the node.*

- Kirchhoff's second law is based on the principle of conservation of energy.
- **Kirchhoff's voltage law (KVL)** states that the algebraic sum of all voltages around a closed path (or loop) is zero. Mathematically, KVL states that:

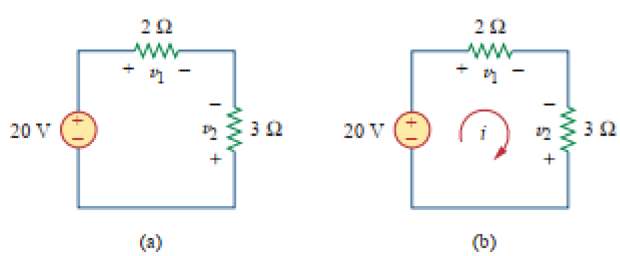
$$\sum_{m=1}^{M} v_m = 0$$

Where M is the number of voltages in the loop (or the number of branches in the loop) and is the m^{th} voltage.

- KVL can be applied in two ways: by taking either a clockwise or a counterclockwise trip around the loop. Either
  way, the algebraic sum of voltages around the loop is zero.
- To illustrate KVL, consider the circuit in Fig. below. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.
- We can start with any branch and go around the loop either clockwise or counterclockwise.
- Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be –V1, +V2,+V3, -V4 and +V5 in that order.
- For example, as we reach branch 3, the positive terminal is met first; hence, we have +V3. For branch 4, we reach the negative terminal first; hence –V4, Thus, KVL yields: **-V1 +*V2* + *V3* - *V4* +*V5* =0** Rearranging terms gives: ***V2* + *V3* +*V5* =*V4* + V1** which may be interpreted as: ***Sum of voltage drops =Sum of voltage rises.***



- For the circuit in Fig. (a) Given below find voltages and *v*1 and v2.



Solution: To find and we apply Ohm's law and Kirchhoff's voltage law. Assume that current *i* flows through the loop as shown in Fig. (b). From Ohm's law, V1=2*i; V2=-3*i;

Applying KVL around the loop gives:

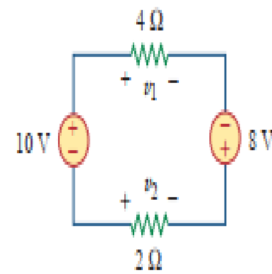
-20+ *V*1 -*V*2 =0 substituting and rearranging -20+ 2*i --3*i =0 implies 5*i=20

i=4A Substituting *i* in Eq. of V finally gives: *V*1 =8 V, *V*2 = -12

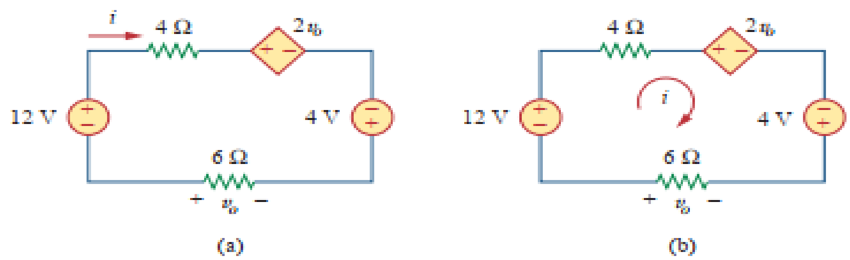
Find v_1 and v_2 in the circuit of Fig. 2.22.

Practice Problem 2.5

Answer: 12 V, −6 V.



Determine v_o and i in the circuit shown in Fig. 2.23(a).



Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad \textbf{(2.6.1)}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad \textbf{(2.6.2)}$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

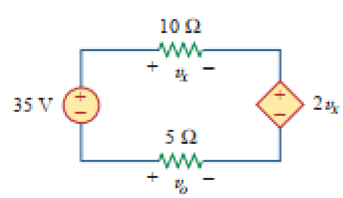
$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Find v_x and v_o in the circuit of Fig. 2.24.

Practice Problem 2.6

Answer: 10 V, −5 V.



Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

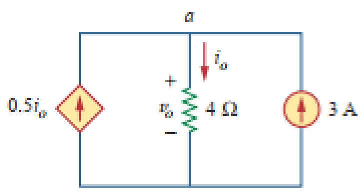


Figure 2.25
For Example 2.7.

Solution:
Applying KCL to node a, we obtain

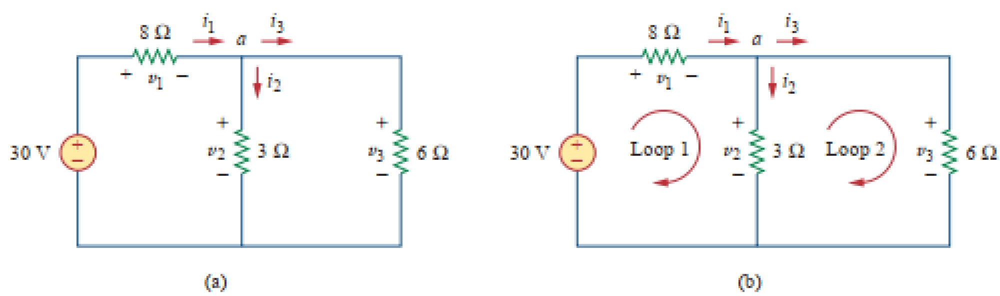
$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6\text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24\text{ V}$$

Example 2.8

Find currents and voltages in the circuit shown in Fig. 2.27(a).



Solution:
We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \qquad \textbf{(2.8.1)}$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \tag{2.8.2}$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \quad \Rightarrow \quad v_3 = v_2 \tag{2.8.4}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \quad \Rightarrow \quad i_3 = \frac{i_2}{2} \tag{2.8.5}$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3\text{ A}, \quad i_3 = 1\text{ A}, \quad v_1 = 24\text{ V}, \quad v_2 = 6\text{ V}, \quad v_3 = 6\text{ V}$$

Series Resistors and Voltage Division

- The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.
- Resistors in series behave as a single resistor whose resistance is equal to the sum of the resistances of the individual resistors. For *N* resistors in series then:

$$\boxed{R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n}$$

The two resistors are in series, since the same current *i* flows in both of them. Applying Ohm's law to each of the resistors, we obtain: $v1 = iR1$, $v2 = iR2$

If we apply KVL to the loop (moving in the clockwise direction), we have $-v + v1 + v2 = 0$

Combining the above equations, we get:

$v = v1 + v2 = i\,(R1 + R2)$ or $i = \dfrac{v}{R_1 + R_2}$

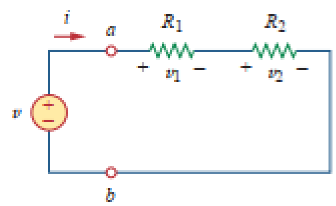
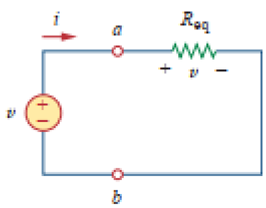


Fig (a)

We can be written as ***v =i*Req;*** implying that the two resistors can be replaced by an equivalent resistor; that is,

$$R_{eq} = R_1 + R_2$$



To determine the voltage across each resistor in the above fig (a), we obtain:

$$\boxed{v_1 = \frac{R_1}{R_1 + R_2}v, \qquad v_2 = \frac{R_2}{R_1 + R_2}v}$$

Notice that the source voltage *v* is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop.

- This is called the *principle of voltage division*, and the circuit in Fig. (a) is called a *voltage divider*. In general, if a voltage divider has *N* resistors in series with the source voltage *v*, the *n*th resistor ( ) will have a voltage drop of:

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N}v$$

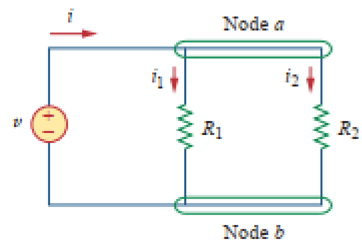
Total Power in a Series Circuit

The power needed to produce current in each series resistor is used up in the form of heat. Therefore, the *total power* used is the sum of the individual values of power dissipated in each part of the circuit. As a formula,

PT= P1+ P2 +P3 +…………+Pn

Parallel circuit

Voltage and resistance in parallel circuits: Parallel Resistors and Current Division
If two branches share two common nodes, they are called parallel connected. Where two resistors are connected
in parallel and therefore have the same voltage across them. Consider the figure given below:



From Ohm's law

$$v = i_1R_1 = i_2R_2$$

or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

Applying KCL at node *a* gives the total current *i* as **i= i1+i2;** Substituting the above Equation in to it, we get:

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

Where is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1R_2}$$

$$\boxed{R_{eq} = \frac{R_1R_2}{R_1 + R_2}}$$

Thus, the equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.
To the general case of a circuit with *N* resistors in parallel. The equivalent resistance is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Note that is always smaller than the resistance of the smallest resistor in the parallel combination.

If R1=R2=…….= R_N , then $R_{eq} = \frac{R}{N}$

Conductances in parallel behave as a single conductance whose value is equal to the sum of the individual conductance.

$$\boxed{G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N}$$

$G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, … , $G_N = 1/R_N$.

The equivalent conductance of resistors connected in parallel is the sum of their individual conductance.

Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

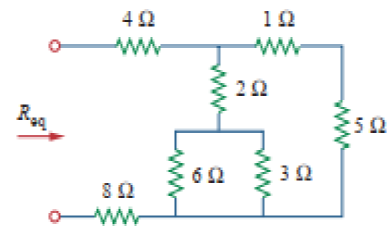


Figure 2.34
For Example 2.9.

Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

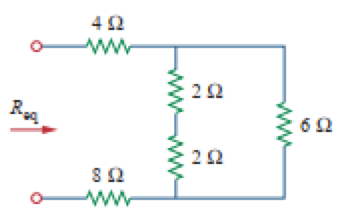
$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

(The symbol ‖ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

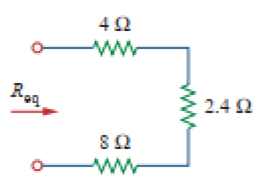
$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$



(a)



(b)

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Practice Problem 2.9

By combining the resistors in Fig. 2.36, find R_{eq}.

Answer: 6 Ω.

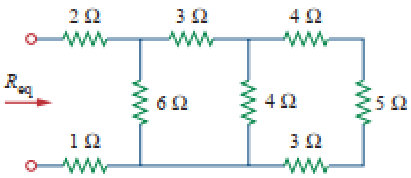


Figure 2.36
For Practice Prob. 2.9.

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

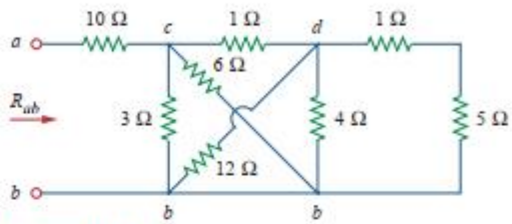


Figure 2.37
For Example 2.10.

Solution:
The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes *c* and *b*. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega \qquad \textbf{(2.10.1)}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\ \Omega \| 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega \qquad \textbf{(2.10.2)}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega \qquad \textbf{(2.10.3)}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of $1\ \Omega + 2\ \Omega = 3\ \Omega$. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \| 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

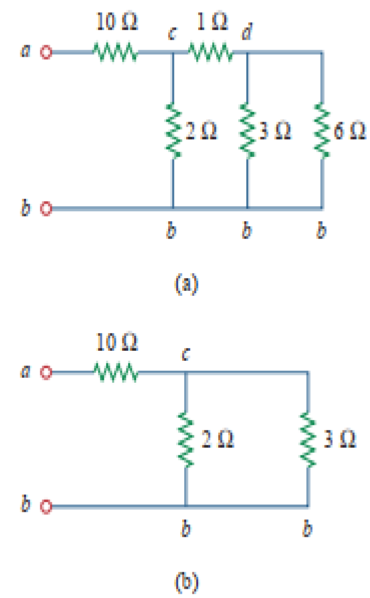
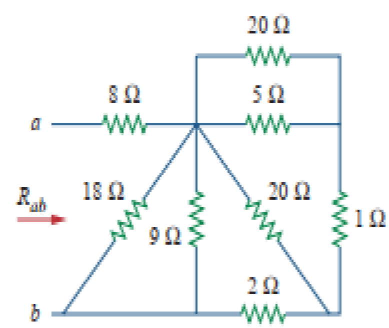


Figure 2.38
Equivalent circuits for Example 2.10.

Practice Problem 2.10

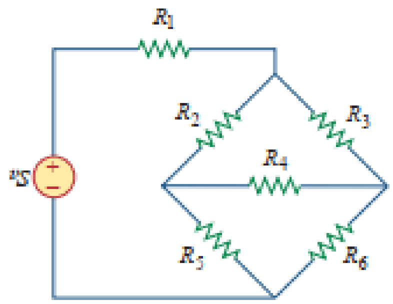
Find R_{ab} for the circuit in Fig. 2.39.

Answer: 11 Ω.



Wye-Delta Transformations

- Situations often arise in circuit analysis when the resistors are neither in parallel nor in series.
- For example, consider the bridge circuit in Fig. below. How do we combine resistors through when the resistors are neither in series nor in parallel?



- Many circuits of the type shown in Fig. above can be simplified by using three-terminal equivalent networks.
- These are the wye (Y) or tee (T) network shown in Fig. 2.47 and the delta ( ) or pi ( ) network shown in Fig. 2.48.

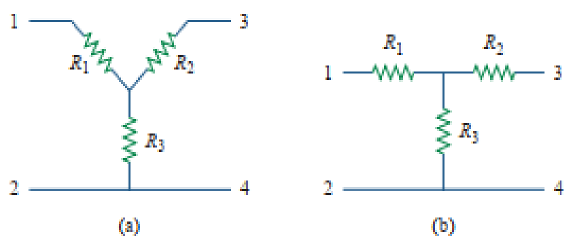


Figure 2.47
Two forms of the same network: (a) Y, (b) T.

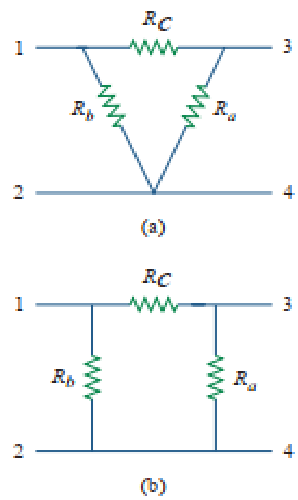


Figure 2.48
Two forms of the same network: (a) Δ, (b) Π.

- These networks occur by themselves or as part of a larger network.
- They are used in three-phase networks, electrical filters, and matching networks.
- Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

Delta to Wye Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$ $$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$ $$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

❖ Each resistor in the Y network is the product of the resistors in the two adjacent branches, divided by the sum of the three resistors.

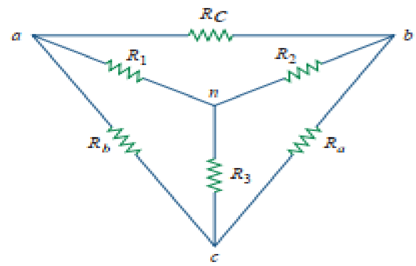


Fig. Superposition of Y and networks as an aid in transforming one to the other

Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Each resistor in the network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor. The Y and networks are said to be *balanced* when: ***R*1 =*R*2 =*R*3 = *R*Y, *Ra* =*Rb* =*Rc* =*R*¢**. Under these conditions, conversion formulas become:

$$R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y$$

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network. **Example**

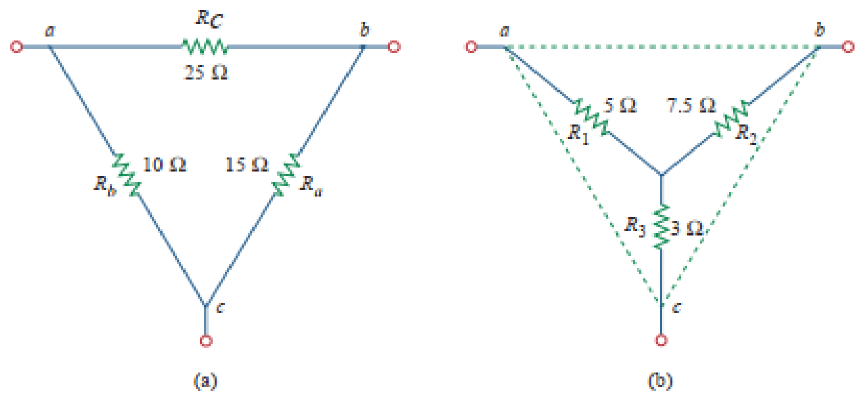


Figure 2.50
For Example 2.14: (a) original Δ network, (b) Y equivalent network.

Solution:

Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).