For an excellent grade you have to get minimum of 9 questions accepted of each set, for a 'good' - at least 7, and for 'passed' - 5 questions.

## Group product. Conjugacy class. Solvability.

**Problem 2.1:** Show that if  $G = N \rtimes H$  then  $G/N \simeq H$ .

**Problem 2.2:** Describe conjugacy classes of O(2)

**Problem 2.3:** Show that any subgroup of index 2 is normal.

**Problem 2.4:** Show that center Z(G) of G is an abelian invariant subgroup of G.

**Defenition 2.1.** Let p be a prime number. A p-group is a group whose order is some natural power of p.

**Problem 2.5:** Show that center of *p*-group is non-trivial..

**Problem 2.6:** Show that all Sylow p-subgroups are conjugate to each other

**Problem 2.7:** Show that each element is represented in only one conjugacy class with identity element forming its own class.

**Problem 2.8:** Show that  $p^2$ -group is abelian.

**Problem 2.9:** Let  $G = \langle a, b | ab = b^m a, ba = ab^n, m, n \in \mathbb{Z} \rangle$ . Show that G is solvable.

**Problem 2.10:** Show that for  $n \neq 6$  there's no outer automorphism of  $S_n$ . Construct the outer automorphism for n = 6.