Visual and Numerical Proofs and Reasoning

Benjamin Sanders, MS November 19, 2019

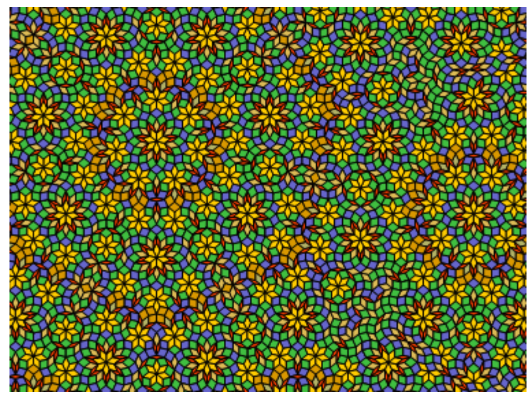
1 Visual Assignment Description

Prove that the inner, clear ball can or cannot be removed from the outer, colorful ball.

- Address each ring size.
- Address whether the ring pattern across the ball is uniform or not.
  - If it is uniform, this would imply that there is a single, small representative arrangement of rings whose pattern can be mapped across the surface.
  - If it is not uniform, this would imply that there is not a single, small representative arrangement of rings whose pattern can be mapped across the surface.

Here is the item in question. Further, here are two examples, one of a uniform pattern, and another of a not uniform pattern, over finite alphabets of symbols.





- In the proof, we have an alphabet composed of a small ring, and a large ring.
- In the soccer ball, we have an alphabet composed of a pentagon and a hexagon.
- In the quasicrystal, we have an alphabet composed of five shapes, denoted by colors red, orange, yellow, green, and violet.

Equivalently, is there a DFA that can accept the ring pattern across the outer, colorful ball? Therefore, is the ring pattern regular?

2 Numerical Assignment Description

Prove that an inner, solid ball of radius 1 unit can or cannot be removed from an outer, inflexible wire mesh of radius 5 and p of 41. Each wire is connected to another wire or wires in a length-minimizing fashion. All connections are designated strictly over the p points of a Fibonacci sphere. No points in p are void of a connection, and no connections are outside of p.

Hint: I have given you the pseudocode for implementing a Fibonacci sphere on the following page.

Procedural Fibonacci Sphere Generation

Ben Sanders · 17 Nov 2017

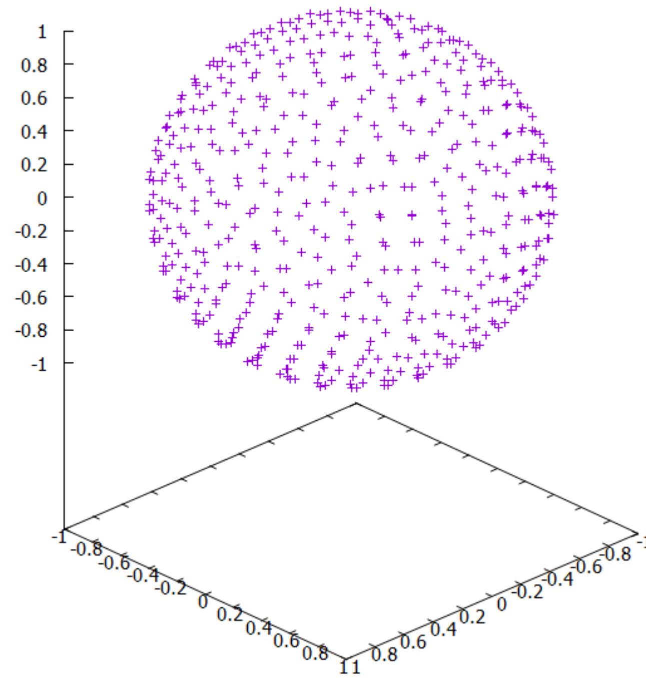
The first two function definitions [1] generate latitude and longitude angles in radians for p points about a sphere, where $p = 2n+1$. The set of points F represents points about the surface of a sphere such that $F_i = \{\text{LATITUDE}(i,n), \text{LONGITUDE}(i)\}$. F relies upon Fibonacci's golden ratio, ϕ and should therefore be considered a *Fibonacci Sphere* point configuration. The subsequent four function definitions [2] map F to coordinates in Cartesian space, centered about $(0,0,0)$ with radius r.

function LATITUDE(i:integer, n:integer)
    $\pi \leftarrow 3.141\,592\,653\,59\ldots$
    if $i \leq 0$ **then**
        return ARCSINE($\frac{2i}{2n+1}$)
    else
        return $\pi -$ARCSINE($\frac{2i}{2n+1}$)
    end if
end function

function LONGITUDE(i:integer)
    $\pi \leftarrow 3.141\,592\,653\,59\ldots$
    $\phi \leftarrow 1.618\,033\,988\,75\ldots$
    return $\frac{2\pi i}{\phi}$
end function

function GENERATE_X(n:integer, r:real)
    $a \leftarrow \{\emptyset\}$
    $i \leftarrow -n$
    while $i \leq n$ **do**
        $a \leftarrow a + \{-r * \text{COSINE}(\text{LATITUDE}(i,n)) * \text{COSINE}(\text{LONGITUDE}(i))\}$
        $i \leftarrow i+1$
    end while
    return a
end function

function GENERATE_Y(n:integer, r:real)
    $a \leftarrow \{\emptyset\}$
    $i \leftarrow -n$
    while $i \leq n$ **do**
        $a \leftarrow a + \{r * \text{SINE}(\text{LATITUDE}(i,n))\}$
        $i \leftarrow i+1$
    end while
    return a
end function



function GENERATE_Z(n:integer, r:real)
    $a \leftarrow \{\emptyset\}$
    $i \leftarrow -n$
    while $i \leq n$ **do**
        $a \leftarrow a + \{r * \text{COSINE}(\text{LATITUDE}(i,n)) * \text{SINE}(\text{LONGITUDE}(i))\}$
        $i \leftarrow i+1$
    end while
    return a
end function

function GENERATE_COORDINATES(n:integer, r:real)
    $a \leftarrow \{\{\emptyset\}\}$
    $x \leftarrow \text{GENERATE\_X}(n,r)$
    $y \leftarrow \text{GENERATE\_Y}(n,r)$
    $z \leftarrow \text{GENERATE\_Z}(n,r)$
    $i \leftarrow 0$
    while $i < \text{SIZEOF}(x)$ **do**
        $b \leftarrow \{\emptyset\}$
        $b \leftarrow \{x[i], y[i], z[i]\}$
        $a \leftarrow a + \{b\}$
        $i \leftarrow i+1$
    end while
    return a
end function

References

• [1] Measurement of areas on a sphere using Fibonacci and latitude-longitude lattices. Álvaro González. 23 December 2009.
• [2] Converting latitude and longitude to x y z coordinates in Partiview. Cosmus. Accessed 16 November 2017. http://astro.uchicago.edu/cosmus/tech/latlong.html.