11/1/2010

THIS LECTURE

- COMPLETE IGNITION DISCUSSION
- BEGIN FLAME SPREAD
  (CONTINUOUS IGNITION)

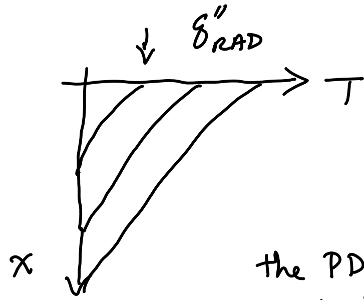
REVIEW SOLID IGNITION

- SOLID IGNITION IS TREATED AS A HEAT TRANSFER PROBLEM.

- WE USE TWO LIMITING CASE APPROXS TO ANALYZE IGNITION.

  - THERMALLY THIN IGNITION & THERMALLY THICK IGNITION.

LAST TIME



$x=0$ surface of material

$$q''_{IN} = -k \left.\frac{\partial T}{\partial x}\right|_{x=0}$$

the PDE that governs the temp evolution in the solid is

$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}}$$

The solution is

$$T(x,t) = T_0 + \frac{q''}{k}\left[\left(\frac{4\alpha t}{\pi}\right)^{1/2} \exp\left(\frac{-x^2}{4\alpha t}\right) - x \,\mathrm{erfc}\left(\frac{x}{\sqrt{4\alpha t}}\right)\right]$$

Soln is arrived at by Laplace transform....

$$\mathrm{erfc}(\eta) = 1 - \mathrm{erf}(\eta)$$

complementary error funct — error function

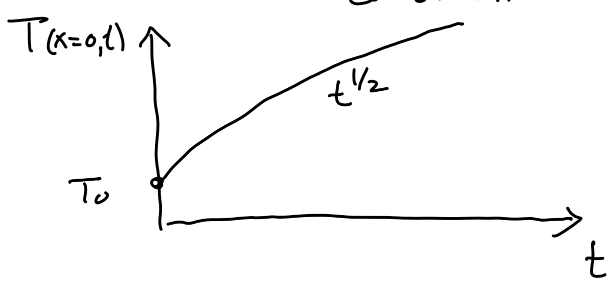
The parameters of interest from a fire perspective are the ignition time & ignition temperature.

$$T(x=0,t) = T_o + \frac{q''}{K}(\alpha t)^{1/2}\left(\frac{4}{\pi}\right)^{1/2}$$

where $$\frac{\alpha^{1/2}}{K} = \left(\frac{K}{\rho c K^2}\right)^{1/2} = \left(\frac{1}{K\rho c}\right)^{1/2}$$

$$\boxed{T_{ign} = T_o + q''\left(\frac{t_{ign}}{K\rho c}\right)^{1/2}\left(\frac{4}{\pi}\right)^{1/2}} \Leftarrow \text{JQ}$$

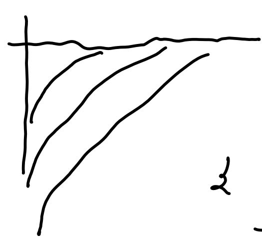
constant heat flux ignition.



We didn't include convective losses.

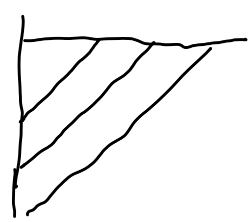
We can arrive at the same basic solution as the above if we use one important relationship.

δ



$\delta \approx \sqrt{\alpha t}$

& curved temp profile is linear

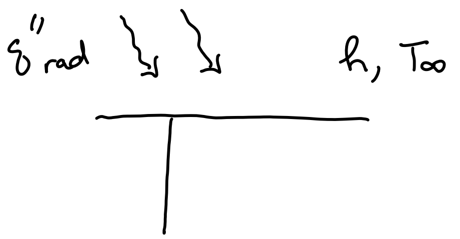


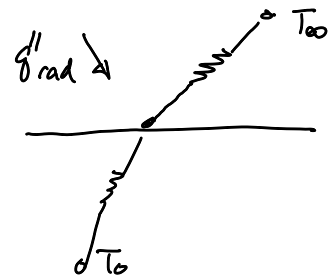
$$q''_{rad} = \underbrace{\frac{K(T_{surf}-T_o)}{\delta(t)}}_{-K\frac{\partial T}{\partial x}}$$

$$\frac{q''\sqrt{\alpha t}}{K} = T_{surf} - T_o \quad \text{or} \quad T_{surf} = T_o + q''\left(\frac{t}{K\rho c}\right)^{1/2}$$

looks like the exact solution but off by $\left(\frac{4}{\pi}\right)^{1/2}$

For convective boundary problem...





energy balance is

$$q''_{rad} = \underbrace{h\left(T_{surf} - T_\infty\right)}_{\text{surf conv. loss}} + \underbrace{\frac{K\left(T_{surf} - T_o\right)}{\delta(t)}}_{\text{in-depth conduction.}}$$

solve for $T_{surf}(t)$

$$\frac{q''\delta(t)}{K} = \frac{h\delta}{K}\left(T_{surf} - T_\infty\right) + T_{surf} - T_o$$

$$T_{surf}\left(1 + \frac{h\delta}{K}\right) = \frac{q''\delta(t)}{K} + T_o + \frac{h\delta}{K}T_\infty$$

$$T_{surf}(t) = \frac{T_o + \frac{h\delta}{K}T_\infty}{1 + \frac{h\delta}{K}} + \frac{q''\delta(t)}{K\left(1 + \frac{h\delta}{K}\right)}$$

we identify $\frac{h\delta}{K}$ as a local Biot #.

$T_o = T_\infty$

$$T_{surf}(t) = T_o + \left(\frac{t}{K\rho c}\right)^{\frac{1}{2}} \frac{q''}{1 + \frac{hL}{K} \cdot \frac{\delta}{L}}$$

where $\frac{\delta}{L} = \frac{\sqrt{\alpha t}}{L} = F_o^{1/2}$

$$T_{ign}(t) = T_o + \left(\frac{t_{ign}}{K\rho c}\right)^{\frac{1}{2}} \frac{q''}{1 + \frac{hL}{K}\left(\frac{\alpha t_{ign}}{L^2}\right)^{1/2}}$$

Textbook

t_{ign} is considered in the small time limit & the large time limit.

The comparison is based on $-K \frac{\partial T}{\partial x}\Big|_{x=0} + h(T_{surf} - T_\infty)$

$$= q''_{rad}$$

small or short time is when

$$q''_{rad} \cong -K \frac{\partial T}{\partial x}\Big|_{x=0}$$ neglected convect losses

$$t_{ign} \cong \left(\frac{\Delta T}{q''}\right)^2 \cdot (K\rho c) \quad \text{ad-hoc}$$

$$\doteq \frac{\pi}{4} K\rho c \left(\frac{\Delta T}{q''}\right)^2 \quad \text{exact}$$

long time solution. include convective effects...

can argue that $h(T_{surf} - T_\infty) = q''_{rad} \quad t \to \infty$

The critical heat flux is the heat flux that maintains a convective-radiative surface balance.

$$q''_{critical} = h(T_{ign} - T_\infty)$$

$$\boxed{t_{ign} = \frac{K\rho c}{\pi} \frac{(q''_{rad}/h)^2}{\left[q''_{rad} - h(T_{ign} - T_\infty)\right]^2}}$$

text book soln for long time

approx surface balance

$$q''_{rad} \simeq h(T_{ign} - T_\infty) + \underbrace{\frac{K}{(\alpha t)^{1/2}}}(T_{ign} - T_\infty)$$

$$q''_{rad} - h(T_{ign} - T_\infty) = \frac{k}{(\alpha t)^{1/2}}(T_{ign} - T_\infty)$$

$$\frac{1}{\left[q''_r - h(T_{ign} - T_\infty)\right]^2} = \frac{\alpha t}{k^2}\frac{1}{(T_{ign} - T_\infty)^2}$$

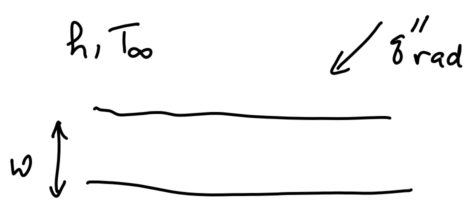
$$t = \frac{(T_{ign} - T_\infty)^2 (k\rho c)}{\left[q''_{rad} - h(T_{ign} - T_\infty)\right]^2}$$

in the long time approx

$$q''_{rad} \approx h(T_{ign} - T_\infty)$$

$$t_{ign} = \frac{\left[q''_{rad}/h\right]^2 (k\rho c)}{\left[q''_{rad} - h(T_{ign} - T_\infty)\right]^2}$$

Thin case is much easier .... it is a 1st order ODE.



the governing eqn is

IN − OUT = STORAGE − ~~GEN~~

approx

$$q''_{rad} A_s - h A_s (T - T_\infty) = mc\frac{dT}{dt}$$

short time neglect convection...

$$\int \frac{q''_{rad} A_s}{mc}\, dt = \int dT$$

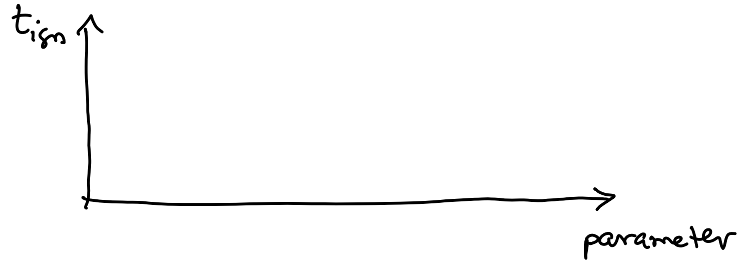
or

$$t_{ign} = \frac{(T_{ign} - T_0)}{q''_{rad}} \cdot \frac{\rho c w \cdot \cancel{A_s}}{\cancel{A_s}}$$

short time

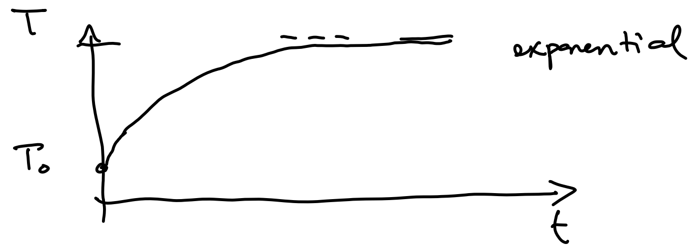
$$t_{ign} = \left(\frac{T_{ign} - T_0}{q''_{rad}}\right)\rho c w$$

w = thickness



in either thick or thin limit...

Thin sample
long time solution



$$\frac{dT}{dt} = \underbrace{\frac{q''A}{mc} - \frac{hA}{mc}(T - T_\infty)}_{\text{long time, this is zero}}$$

$$\underbrace{\phantom{\frac{q''A}{mc} - \frac{hA}{mc}(T - T_\infty)}}_{\text{call}} \quad \theta = \frac{q''A}{mc} - \frac{hA}{mc}(T - T_\infty)$$

$$\frac{d\theta}{dt} = 0 - \frac{hA}{mc}\frac{dT}{dt}$$

$$-\frac{mc}{hA}\frac{d\theta}{dt} = \theta$$

$$\theta = \theta_0 \exp\left[-\underbrace{\frac{hA}{mc}}_{\text{time const}} t\right]$$

$$\theta_0 = \frac{q''A}{mc} - \frac{hA}{mc}(T_0 - T_\infty) \qquad \tau = \frac{\rho c v w}{h}$$

$$\boxed{\ln\left(\frac{\theta}{\theta_0}\right) = -\frac{hA}{mc} t_{ign}}$$

solve for t_{ign}.