JQ.2.4.soln

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(2.4) Show that the pressure rise $P - P_o$ in a closed rigid vessel of volume V for net heat addition $\dot{Q}$ is

$$\frac{P - P_o}{P_o} = \frac{\dot{Q}t}{\rho_o V c_v T_o}$$

Start with the fixed mass form of the first law (see problems 1 and 3 in HW1) in an unsteady formulation. Recognize that there is no work being done.

$$\frac{dU}{dt} = \dot{Q}_{in} + \dot{W}_{in}$$

$$\frac{dU}{dt} = \dot{Q}_{in}$$

where U is the total internal energy with units of kJ. We model the air as an ideal gas and note that for an ideal gas that the internal energy can be modeled as a function of temperature only. Model the change in internal energy as the specific heat capacity multiplied by the change in tempeature.

$$dU = mc_v dT$$

The mass specific heat capacity at constant volume is c_v. This can be looked up at a representative temperature. We insert this into the first law.

$$mc_v \frac{dT}{dt} = \dot{Q}_{in}$$

We then use the ideal gas equation of state:

$$P = \rho RT$$

where we solve for the temperature as

$$T = \frac{P}{\rho R}$$

$$\frac{dT}{dt} = \frac{V}{mR}\frac{dP}{dt} = \frac{\dot{Q}_{in}}{mc_v}$$

Because the heat addition rate is constant, perform simple integration of the 1st law. We use ideal gas equation of state to simplify the expression.

$$P(t) - P_o = \frac{\dot{Q}_{in} tR}{Vc_v} = \frac{\dot{Q}_{in} tP_o}{mc_v T_o}$$

$$\frac{P - P_o}{P_o} = \frac{\dot{Q}t}{\rho_o V c_v T_o}$$