Failure of Gypsum Wallboard Anchors at Elevated Temperatures

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PROBLEM STATEMENT

Insulation is placed in wall cavities to improve the thermal performance of homes and buildings. Most insulation material is combustible, and so it is imperative from a fire protection standpoint to provide a continuous barrier between the combustible insulation and any heat flux that might result from a compartment fire. In the vast majority of building walls, a layer of gypsum wallboard performs this fire protective role and has proven effective in fire testing as long as the gypsum layer remains intact.

Building users frequently install anchors into the gypsum wallboard to hang pictures and other objects. The anchors rely on the structural capacity of the gypsum board to resist the weight of the hanging object. When these anchors overload the gypsum and fail, they can create a hole in the fire protection layer and expose the underlying insulation.

Since gypsum wallboard is known to thermally degrade when exposed to elevated temperatures, it is important to determine if the gypsum wallboard can continue to support the applied anchor loads during a fire. In this experiment, we will heat a wallboard anchor system to various temperatures and load it to failure to examine whether the structural strength of the gypsum has a dependence on temperature.

BACKGROUND

Before discussing the specifics of our experiment, we will present some relevant background information. In the following sections gypsum, calcination, wall anchors, and heat transfer will be discussed. Each one of these topics plays a crucial role in the experiments that we executed.

Gypsum

Gypsum is the material that composes drywall used in homes and buildings. In order to understand how the gypsum will be affected by heating, it was necessary to understand the properties and behavior of unheated gypsum.

The gypsum association publishes data from laboratory tests following ASTM procedures to determine the various strength properties. For ½" regular gypsum wallboard, the listed compressive strength at 70°F is 350 psi (www.gypsum.org). Note that this is loaded perpendicularly to the paper facing, but we assume that the paper will contribute little in this direction of loading, so the value can be taken as the compressive strength of the gypsum itself, and used for comparative purposes in our study. The thermal properties of gypsum are presented in Table 2 in the heat transfer section.

Calcination

Calcination is the release of chemically bound water in a two stage endothermic process. Gypsum is approximately 21% water by mass (Cramer, 2003), and as it undergoes calcination during a fire, the released water provides a significant non-combustible sink for the fire heat release. The details of the chemical reaction are explained well by Charles Aire, whose work we cite below.

The first stage starts around 100°C. Gypsum is properly calcium sulphate dehydrate.

$CaSO_4 \cdot 2H_2O$ + heat = $CaSO_4 \cdot \frac{1}{2} H_2O$ + 3/2 H_2O

The first stage converts the gypsum to a hemihydrate, and the second reaction removes the remaining water to make an anhydrate form of calcium sulphate.

$CaSO_4 \cdot \frac{1}{2} H_2O$ + heat = $CaSO_4$ + ½ H_2O

Above 600°C, a decarbonation reaction occurs producing a calcium oxide and a significant mass loss. (Aire, 2013).

Since we hypothesize that the relevant strength loss from a structural perspective will occur in the calcination process, we did not pursue the higher temperature effects.

Aire studied gypsum samples exposed to a steadily increasing heating rate of 20°C per minute and found that the mass loss associated with calcination, 17%, started at 140°C and leveled off by about 200°C. Under these experimental conditions, the mass loss occurred between 6-9 minutes into the test. Aire correlated the mass loss to the calcination reaction, but did not directly compare it to a reduction in strength (Aire, 2013).

In a similar study, Cramer, et al. report that both the calcination reaction and the evaporation of the water produced occur between 100°C and 125°C, however they state that at the time of their study there was no conclusive information available about the rate of the calcination reaction. Due to the scatter in resulted calcination times and temperatures, Cramer et al. speculate that the reaction rate might be faster at higher temperatures (Cramer, 2003).

The same study found that gypsum samples exposed to 400°C underwent a complete calcination reaction after 20 to 30 minutes. At that point it was found that all of the moisture had been driven off and that the gypsum's bending strength fell to near zero. In their experiments, Cramer found that bending strength appeared to decrease linearly as the temperatures increased.

The Cramer study also investigated the strengthening due to the paper facing of the gypsum panel. They conclude that as long as the paper is intact, it appears to provide a significant role in the strength of the wall panel (Cramer, 2003).

Drywall Anchors

Another important consideration in our study is the physical mechanism that drywall anchors use to allow gypsum wallboard to resist a hanging load. There are many types of drywall anchors available on the commercial market, made typically of plastics or of metals. Since materials such as nylon have melting temperatures lower than that which the anchors would be exposed to in a compartment fire, we elected to use metallic anchors in order to ensure that the integrity of the drywall anchor would not have an effect on our results. This type of anchor is commonly found and rated for loads ranging from 25 to 50 pounds. These anchors consist of a nominal ½" diameter, hollow, cast-metal cylinder that has widely spaced male threads on its exterior and female threads on the interior. The device is pressed and screwed perpendicularly into the drywall; it cuts its own hole as it is screwed into the wall as shown in Figure 1. The male threads of the anchor engage with the gypsum and offer significant pull out resistance or a failure due to rotation. Once the anchor is secured flush to the drywall surface, a thinner diameter screw is screwed into the hollow shaft of the anchor. Typically a cable is attached to this smaller screw and an object is hung from the cable.

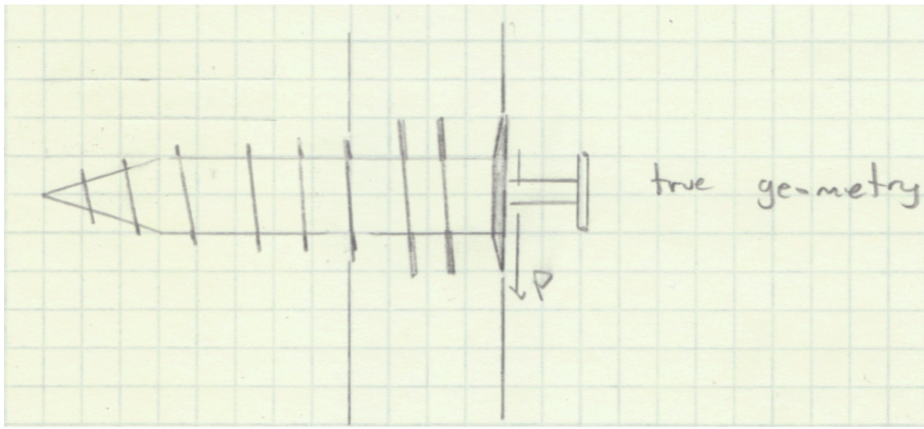


Figure 1. Schematic of the Drywall Anchor

When loads are applied to the drywall anchor, the line of action of the force is nearly in plane with the surface of the drywall, and the neutral axis is relatively close to the centerline of the wall panel. We performed an equilibrium force analysis assuming a linear response in the drywall, with the anchor rotating as a rigid body. In our analysis, we applied a 50-pound load, the maximum rated load, at 0.1" from the gypsum face. The free body diagram and stress distribution used for the analysis are shown in Figure 2 and Figure 3 respectively.

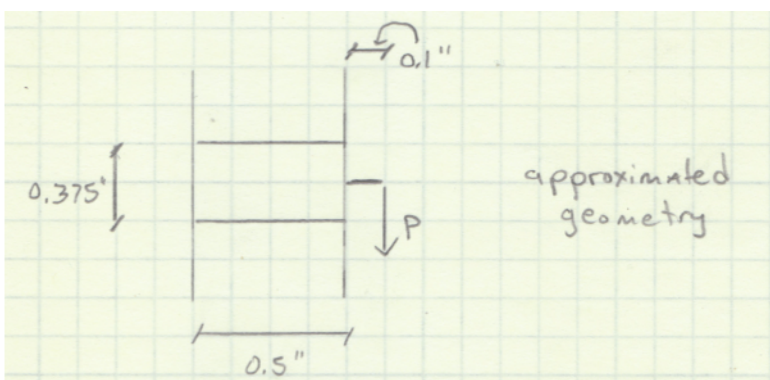


Figure 2. Free Body Diagram

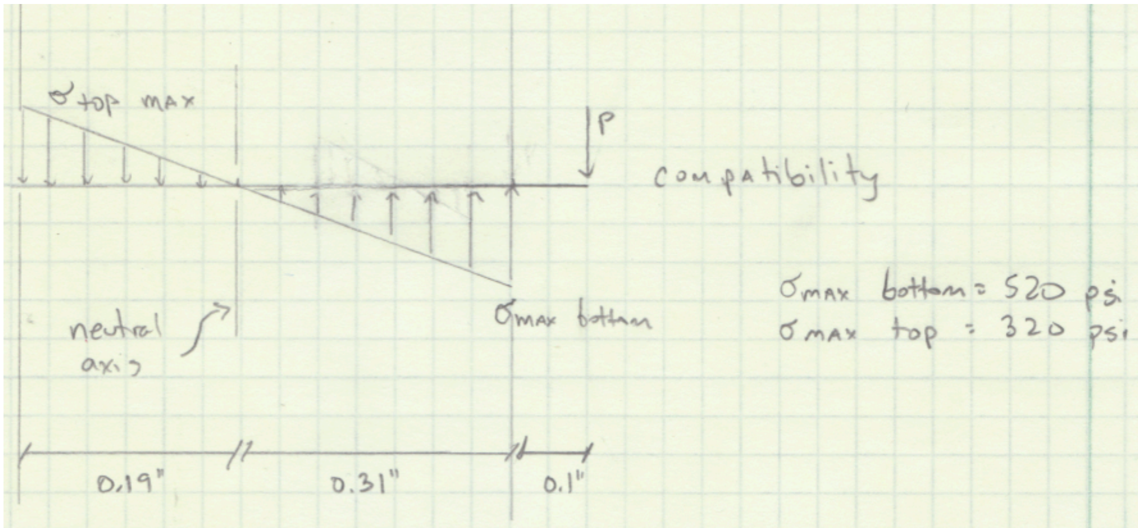


Figure 3. Stress Distribution Inside the Gypsum

$$\Sigma \mathrm{F}(\mathrm{vert}) = 0 \tag{1}$$

P + Resultant (top) = Resultant (bottom)

Resultant (top) = 3/8" * $\int_{lh}^{n.a.} \sigma\, dx$

Resultant (bottom) = 3/8" * $\int_{n.a.}^{rh} \sigma\, dx$

n.a = neutral axis
lh = left-hand side of drywall
rh = right-hand side of drywall
t_{dw} = drywall thickness

$$\Sigma M(n.a.) = 0 \tag{2}$$

P*(0.1"+t_{dw} –n.a.) = Resultant (top)*(n.a.) + Resultant(bottom) *(t_{dw} – n.a.)

Assuming that the stress distribution will be linear prior to yield, and solving in equations (1) and (2), we determined a maximum compressive stress on the front face of 195 psi, and a maximum compressive stress of 120 psi on the back face, with the neutral axis located at 0.19", or slightly left of the midpoint. The full analysis is shown in Appendix I.

In this analysis, the maximum stress was less than the listed 350psi compressive strength, giving a factor of safety of about 1.8 before first yield. However, if the point of loading is extended to ½" from the front face, the maximum compressive stress becomes 375 psi. In this equally likely loading condition the gypsum anchor could yield, or possibly fail. Because of the variability of loading conditions, even in a "typical" application, we decided to concentrate our analysis on a comparison of the percentage reduction in strength of observed failures rather than trying to report precise values for stresses at failure. A diagram showing the observed failure mechanism for the wall anchor is illustrated in Figure 4.

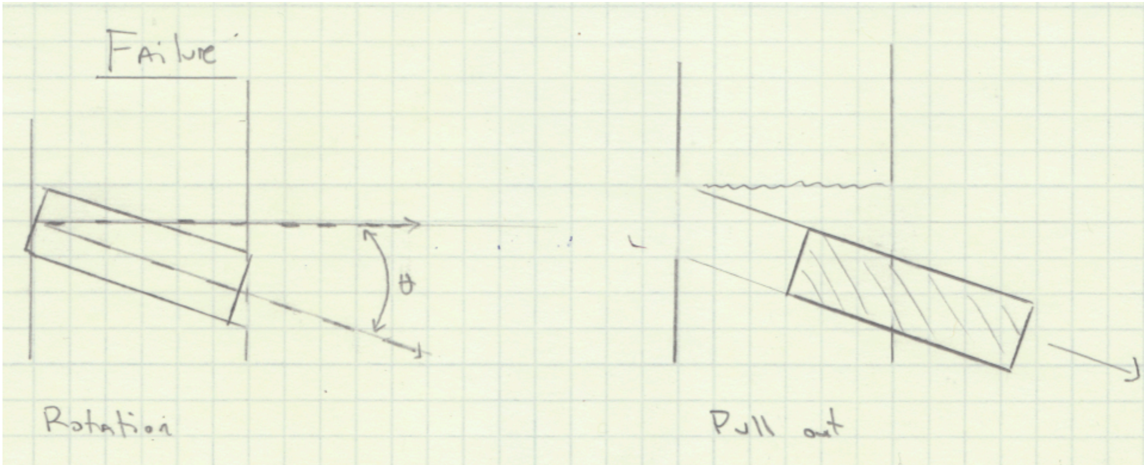


Figure 4. Failure Mode of Wall Anchors

Heat Transfer

As previously mentioned, gypsum serves as the primary fire barrier between a flame and the interior insulation in a home or building. However, if the gypsum fails the fire risk will be increased. We hypothesize that the gypsum will be susceptible to failure at smaller loads when heated above the calcination temperature. Therefore, it is necessary to determine how quickly gypsum will heat when exposed to those temperatures that it would be exposed to during a compartment fire. In order to make this determination, heat transfer models were utilized.

When considering how to model the heat transfer to the gypsum, a primary consideration was whether a thermally thin, lumped capacitance method, or thermally thick, semi-infinite, model should be used. Considering that standard gypsum is about 1/2" thick, it was determined that a thermally thin model would not suffice. Further, the Biot number was calculated in Equation 1 and determined to be greater than 0.1. This gives even further reason that a lumped capacitance model would not be appropriate for the gypsum samples of interest. On the other hand, it is not reasonable that the gypsum would fall under the semi-infinite case, due to the fact that the entire sample could eventually be heated to an external temperature. Therefore, a one-term approximation to the series solutions for transient one-dimensional conduction was used. This is demonstrated in the following section.

In order to determine the time required for the gypsum to heat to the calcination temperature, we selected compartment conditions consistent with a fully developed compartment fire near flashover. Under this assumption the hot gas layer in the room would be near 800°C and the combined heat transfer coefficient would be on the order of 30 W/m^2K. These values are demonstrated in Table 1, and the thermal properties of gypsum are shown in Table 2.

Table 1. Typical Values During a Compartment Fire

Typical Overall Heat Transfer Coefficient	30 [W/(m^2K)]
Typical Room Temperature	1073 [K]

Table 2. Thermal Properties of Gypsum

Thermal Conductivity	0.5 [W/(mK]
kρc	10^5 [W^2s/(m^4K^2)]
Thermal Diffusivity	4*10^7 [m/s^2]

In the following calculations it is assumed that the gypsum is insulated on one side, and the time calculated is the time required for the backside of the gypsum to reach the calcination temperature.

$$Bi = \frac{hL_c}{K} \tag{3}$$

Where h is the heat transfer coefficient, L_c is the characteristic length, which in this case in the thickness of the gypsum, and k is the thermal conductivity of the gypsum

$$Bi = \frac{30\frac{W}{m^2K}*0.0127\ m}{0.5\frac{W}{mK}} = 0.762 \tag{4}$$

$$\theta^* = \frac{T_0 - T_\infty}{T_i - T_\infty} \tag{5}$$

Where T_0 is the final temperature of the gypsum, T_∞ is the ambient temperature, and T_i is the initial temperature of the gypsum. Using the value for a room temperature shown in Table 1, an initial temperature of 298K and the temperature of interest being 110°C or 383 K, θ^* was determined.

$$\theta^* = \frac{383 - 1073}{298 - 1073} = 0.8903 \tag{6}$$

Using a one-term approximation to the series solutions for transient one-dimensional conduction the following equation was obtained.

$$\theta^* = C_1 \exp\left(-(\xi_1^2) * Fo\right) \tag{7}$$

Where Fo is the Fourier number, and C_1 and ξ_1 are constants determined as a function of Biot number. For a Biot number of 0.762 values of C_1 = 1.0979 and ξ_1 = 0.7756 rad were found using interpolation [Fundamentals of Heat and Mass Transfer]. Equation 8 was used to find the Fo number from Equation 7 and the values of the constants.

$$Fo = \frac{\ln(\theta^*) - \ln(C_1)}{-(\xi_1^2)} \tag{8}$$

$$Fo = \frac{\ln(0.8903) - \ln(1.0979)}{-(0.7756^2)} = 0.3484 \tag{9}$$

It can be seen from Equation 9 that Fo > 0.2, which is a requirement of this approximation. A relationship between the characteristic length L_c, the thermal diffusivity, α, time, t, and Fourier number, Fo, was used to determine the amount of time that would be required for the gypsum to heat to a temperature where it could fail under a reduced load. In this case, the temperature of interested was estimated to be 383K. This is demonstrated in the following equations.

$$Fo = \frac{\alpha t}{L^2} \tag{10}$$

Solving the following equation for time, Equation XX was obtained.

$$t = \frac{FoL^2}{\alpha} \qquad 11)$$

$$t = \frac{0.3484*(0.0127m)^2}{4*10^{-7}\frac{m^2}{s}} = 140.49\ s \tag{12}$$

As can be seen from Equation 12, the gypsum could reach critical temperatures in roughly under three minutes when exposed to a compartment fire. Therefore, it is important to determine what type of loads the gypsum will fail under when exposed to a compartment fire.

HYPOTHESIS

With some understanding of the processes involved in this problem, we entered into this research with some expectations about the results. We hypothesized that the strength of the gypsum samples could be correlated to the degree of calcination that occurred across the gypsum sample. For calcination to occur, a minimum temperature would need to be maintained at each point through the cross section. Knowing that the thermal penetration depth would be time dependent under an external heat flux, we hypothesized that the degree of calcination would depend on time. Further, since calcination is an endothermic reaction we assumed that at higher temperatures the kinetics of the reaction could either occur more quickly or proceed further. Based on that assumption, we hypothesized that the strength of the samples should show temperature dependence. Lastly, we assumed that a significant portion of the tensile strength of the gypsum board is contributed by the paper facing. While paper doesn't calcinate, it is likely to burn under the heat flux from a compartment fire. We hypothesized that without the front-side paper facing, the gypsum samples would fail at much smaller loads.

EXPERIMENTAL SETUP

In order to conduct the following experiment it was necessary to do some preliminary calculations and to design and build a loading frame, which could be used to test the samples. In addition to this it was necessary to prepare the gypsum samples, and gather all needed materials.

Prior to conducting the experiment, we deemed it necessary to calculate the time required to heat the gypsum samples to the required temperatures using an oven. In this case, the one-term approximation to the series solution for transient one-dimensional conduction was used as in the calculations for conditions of a compartment fire. However, in this case it was determined that the sample would be heated from two sides and we were interested in the time for the centerline of the gypsum samples to reach the required temperatures. Since the samples were ½" thick the characteristic length was assumed to be ¼." The heat transfer coefficient shown in Table 1 and the thermal properties of gypsum shown in Table 2 were used in the following calculations. In order to show that the appropriate correlation was used the Biot number was calculated to ensure that it was greater than 0.1 and the Fourier number was calculated to ensure that it was greater than 0.2. This is shown in Equation 13 and Equation 15 respectively.

$$Bi = \frac{30\frac{W}{m^2K}*0.00635\,m}{0.5\,\frac{W}{mK}} = 0.381 \qquad (13)$$

The dimensionless temperature calculated in Equation 14 is based on the assumption that the oven will be raised to a temperature of 1K higher than that of the desired sample temperature. If it were assumed that the oven temperature were equal to that of the desired sample temperature an undefined value would be reached for the Fourier number. The following calculations are used to determine the time to heat a gypsum sample to 110°C. Table 3 illustrates the required times to heat the gypsum samples to temperatures of 110°C, 140°C, and 250°C. The same procedure was used in determining each of the required times.

$$\theta^* = \frac{383-384}{298-384} = 0.01163 \qquad (14)$$

Using the table provided [Fundamentals of Heat and Mass Transfer] and the calculated Biot number, values of C_1 = 1.0555 and ξ_1 = 0.5796 rad were determined and the Fourier number was calculated.

$$Fo = \frac{\ln(0.01163) - \ln(1.0555)}{-(0.5796^2)} = 13.42 \qquad (15)$$

The Fourier number was then used to calculate the time required to heat each of the samples.

$$t = \frac{13.42*(0.00635m)^2}{4*10^{-7}\frac{m^2}{s}} = 1352.8\,s \qquad (16)$$

Table 3. Required Times to Heat Gypsum Samples to Desired Temperatures

Desired Temperature [°C]	Required Heating Time [s]
110	1352.8
140	1442.7
250	1642.8

Due to the minimal thickness of the paper, we assumed that the time to heat the gypsum samples will be independent of the paper on the sample, therefore the time will be the same once the paper is removed.

We also deemed it necessary to design and build a loading frame that could be used in the experimental setup. A CAD drawing of the loading frame can be seen in Figure 5.

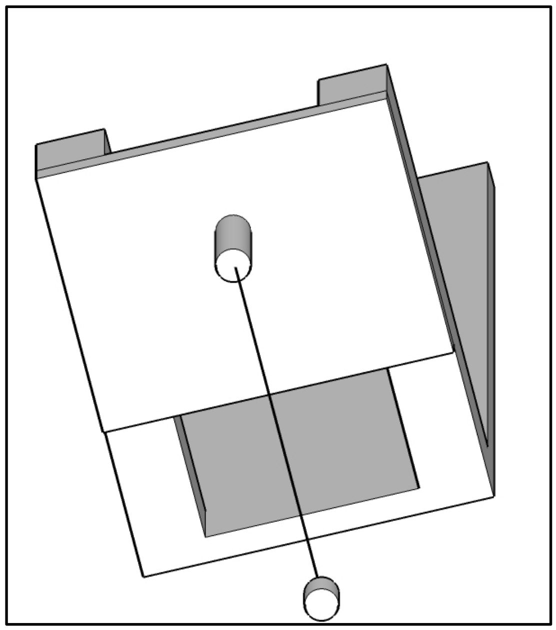


Figure 5. Schematic of Loading Frame

At this point, we moved forward to generating a list of supplies that would be needed for our experimental setup. These supplies are as follows:

- A rigid loading frame of welded steel angles clamped to a table top
- A linear spring scale ("hanging fish scale") rated to 22 kg
  (We calibrated the fish scale and determined its accuracy as ± 2 kg)
- 4"X4" samples of ½" thick drywall, regular
- Metal through-wall gypsum anchors, screw-in type, female threaded to accept #10 screws - rated by the manufacturer to hold 50 lbs (22.67 kg)
- 1/8" steel cable
- Hanging weights
- An Oven to heat the samples
- A thermocouple to verify the oven temperature

To prepare for the testing we cut 4 samples for each of the conditions we intended to test, 20 samples total. Care was taken in cutting the samples to ensure that minimal stress was imposed on the samples that might have caused cracking or strength loss. A drywall anchor was inserted into the middle of each sample. To insert the anchor, the anchor is pushed through the paper facing and then screwed in with a screwdriver until the anchor is flush with the paper facing. The anchors are 1.25" long, so the anchors protruded 0.75" past the backside paper facing. On four of the samples, the front-side paper facing was torched for several minutes until the paper had fully charred as shown in Figure 6 and Figure 7.

Figure 6. Charring Process

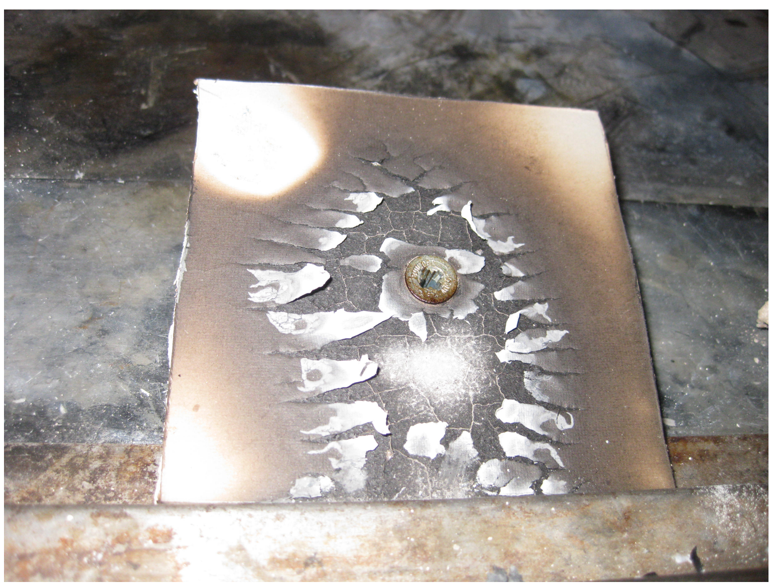


Figure 7. Gypsum Sample with Charred Paper

Next, the samples were placed in ovens and exposed to a constant temperature for approximately one hour. Four samples were heated to each temperature (110°C, 140°C, 250°C, and 250°C with paper burned). The oven temperatures were verified using a hand-held thermocouple. Temperatures were recorded before placing the samples in the ovens, and when the samples were removed as shown in Table 4.

Table 4. Oven Temperatures Measured with Thermocouple

Oven Temps [°C]		
Initial	Final	Target
281.7	239	250
260.2	250.2	250
121	141.6	140
114.5	110.9	110

Once the samples reached their desired temperatures, they were removed from the ovens and allowed to cool to room temperature. Once the samples returned to room temperature they were loaded to failure. The loading process was common to all of the samples, and is described below:

- Sample is attached to frame, using continuous restraint on both vertical sides as shown in Figure 8. Woodworking clamps were used to secure the sample to the frame.
- The attachment cable was hung from the screw. Care was taken to position the attachment cable flush with the drywall face.
- Pre-loading weights were hung from the cable. *
- The fish scale was positioned at the working point on the attachment cable.
- An increasing downward force was applied by hand on the fish scale until the sample failed.
- The failure load was recorded.**

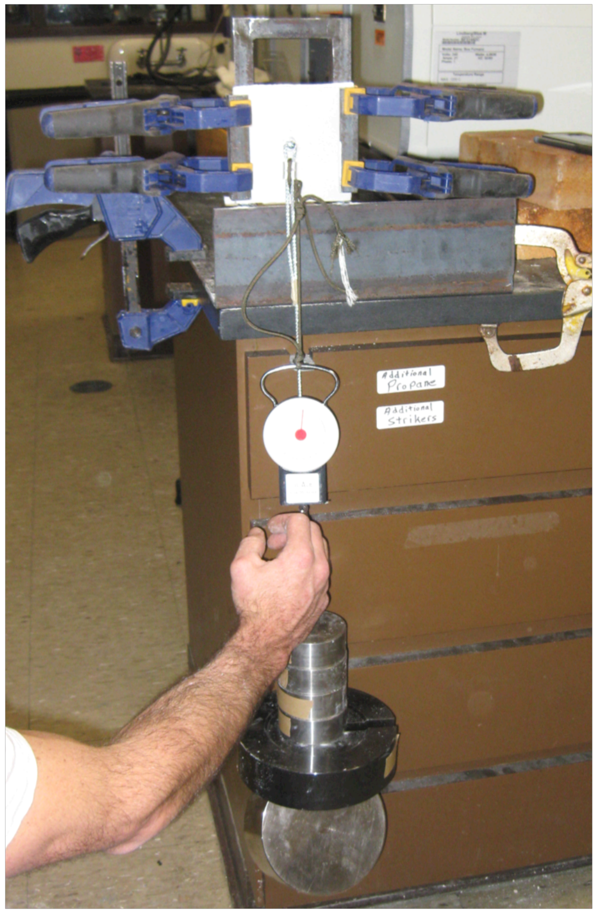


Figure 8. Test Setup

Finally, all specimens were loaded to failure*** using the procedure described above, and the failure loads were recorded.

* The pre-loading weights were necessary because the fish scale could only measure up to 22 kg. In our first attempt we loaded the scale past that point and broke it. After buying a second scale, we decided to pre-load the anchors with static weights up to near capacity, and then apply an incremental load by hand. The reported failure loads comprise the total load on the anchor at the point of failure.

** Each loading was captured on video, and the footage was reviewed to verify the load at failure.

***In our preliminary tests we determined that a two-part failure mechanism occurs when the drywall fails. First the anchor rotates approximately 30°, and then under a negligible increase in load, the anchor slips out of the wall. For the purposes of testing, we declared failure to be the point when the anchor had rotated and begun to slip. We did not "fully fail" the anchors, due to concerns of damaging the experimental equipment.

EXPERIMENTAL RESULTS

The load at failure was recorded for each of the gypsum samples. At each testing condition there is some variation at the failure load, however this is explained by the fact that some of the samples experienced a greater degree of failure than others. In some of the samples, the drywall anchor simply rotated and began to slip out of the drywall; this type of failure is demonstrated in Figure 9. In other cases, the drywall anchor completely slipped out of the sample leaving a hole in the gypsum, as shown in Figure 10, which would expose insulation in a real world scenario. We worked to be consistent in failing the drywall samples all to the same point. Due to safety concerns, our aim was to only fail the samples to the point shown in Figure 9, however occasionally the failure was brought to the point shown in Figure 10. The type of failure shown in Figure 10 would be that which would be more important to examine for real world scenarios.

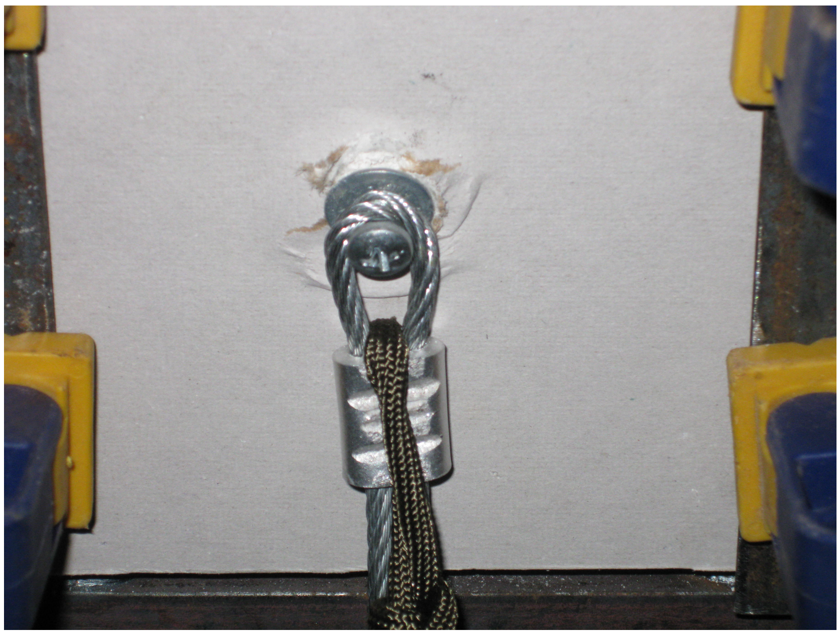
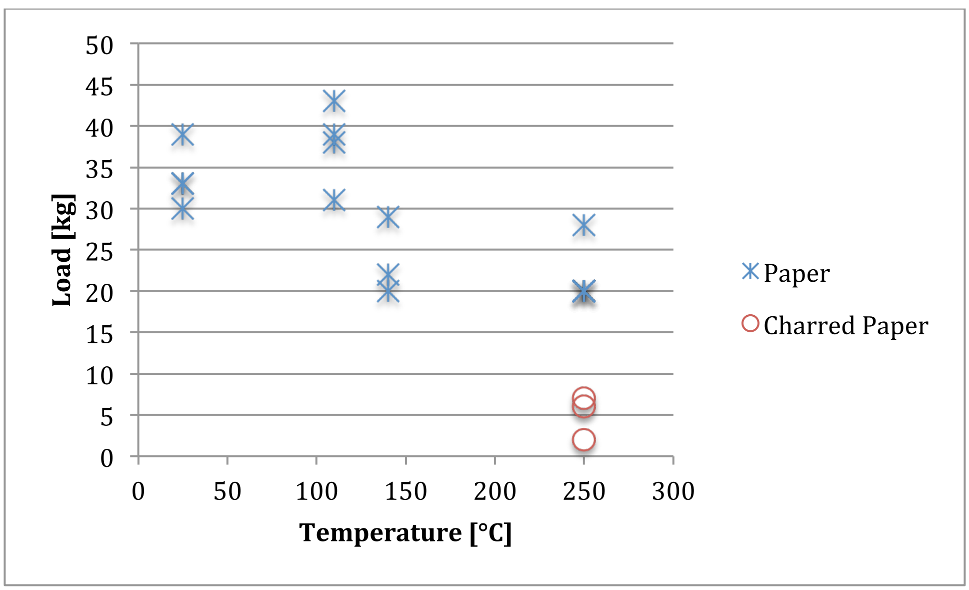


Figure 9. Rotational Failure of Drywall Anchor

Figure 10. Complete Failure of Drywall Anchor

The results of the experiment can be seen in Plot 1. Table 5 gives a more in depth look at the results presented in Plot 1.



Plot 1. Resulting Failure Loads for Gypsum Samples

Table 5. Mean Failure Loads Compared to Unheated Samples

Mean Failure Load			
Temperature [°C]	Paper	Mean Failure Load [kg]	Percent of Unheated Sample
25	Yes	33.75	100
110	Yes	37.75	112
140	Yes	24.75	73
250	Yes	20	59
250	No	5	15

As can be seen from Plot 1 and Table 5, there was no significant loss in strength between the room temperature samples and those heated to 110°C. This result was not as we had expected, and could still benefit from further investigation. One possible cause of this result is that the samples were driven to a more complete failure at these temperatures. Further it is possible, that the calcination simply did not come to completion in these samples, and slightly higher temperatures would be required in order to reduce the ability of the gypsum to carry a load. As expected the samples heated to 140°C and 250°C failed at lower loads. In addition to this, the samples in which the paper was removed and were heated to 250°C were able to resist only 15 percent of the load of an unheated gypsum sample. This further justifies our hypothesis that the paper on the gypsum is a significant factor the strength of the samples.

CONCLUSION AND FUTURE WORK

The results of this study show that failure of loaded gypsum board anchors could pose a realistic mechanism for compartment fire spread at shorter times than previously considered. Future studies could build upon the conclusions of this study to better characterize the fire risks of these anchor systems. Several directions for further investigation are mentioned below.

This study only investigated the temperature dependence of the strength loss in gypsum. From a fire risk perspective, the more important variable is the time dependence, since the fire safety community is interested to know how long the gypsum can be expected to keep the structure isolated from the fire's effects. A relevant study would be to load a gypsum anchor to its rated capacity and then expose it to a characteristic heat flux (or to elevated temperatures in an oven) and investigate the time to failure.

In our study, we determined that at 110°C, the gypsum samples did not appear to loose any strength. This result was surprising, since in the literature we reviewed, the calcination process was typically assumed to start around 100° – 110°C. Further study could be done to better characterize the calcination reaction, and the associated strength loss. By continuing the test regime started in this study, but increasing the exposure time from one hour, it could be determined if calcination does occur at this temperature, but takes a longer time to proceed through to the point where the strength is lost.

Finally, and perhaps of most interest to the fire safety community, various other types of gypsum drywall anchors should be tested. There are dozens of types of commercially available drywall anchors. The products available vary in loading capacity and in the way they distribute the forces. Some of the products on the market would produce significant holes in the drywall upon failure, and that behavior should be characterized and made part of the product documentation.

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Appendix I
Stress Calculations in Gypsum Under Applied Maximum Load

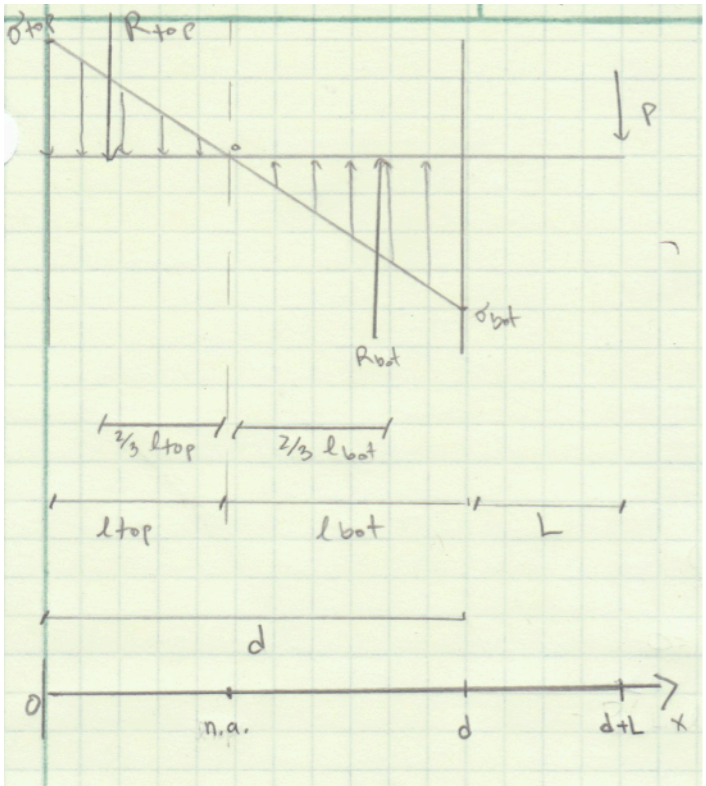

Figure 11: Free body Diagram

We solve the equilibrium equations treating σ as a distributed linear load, with units of lbs/inch, and then multiply by the projected area of bearing (3/8" diameter of the anchor), at the end of the analysis.

$\mathbf{\Sigma F(vert) = 0}$

P + R(top) = R(bot)

R(top) $= \int_0^{n.a.} \sigma \, dx$

$= (\frac{1}{2} \, l_{top} \, \sigma(top))$

R (bot) = $\int_{n.a.}^{d} \sigma \, dx$

$= (\frac{1}{2} \, l_{tbot} \, \sigma(bot))$

n.a = neutral axis

$P = \frac{1}{2} * (l_{bot} \, \sigma(bot) - \sigma(top) \, l_{top})$ (A)

$\boldsymbol{\Sigma M(n.a.) = 0}$

$P*(L+l_{bot}) = R(top)*(2l_{top}/3) + R(bot)*(2l_{bot}/3)$

$P = -1/3 * (l_{bot}^2 * \sigma(bot) + \sigma(top) * l_{top}^2) / (L + l_{bot})$ (B)

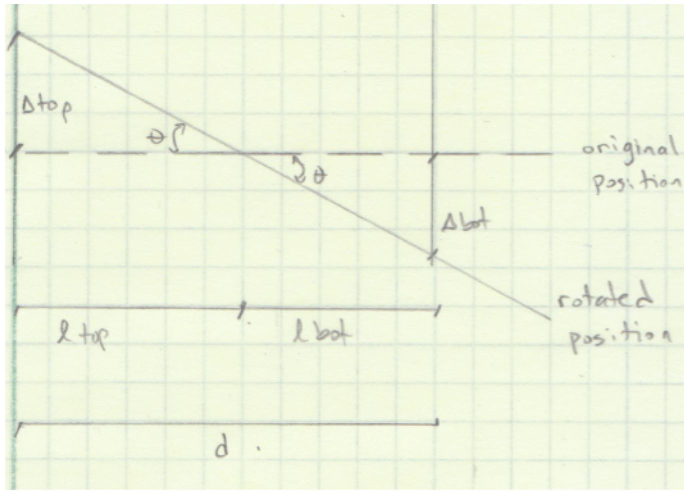

Figure 12: Compatibility Diagram of Rotations

Geometrically, we have:

$\Delta_{top}/l_{top} = \Delta_{bot}/l_{bot}$ (C)

Assuming a linear elastic response, we have:

$F = k * \Delta$, so
$\sigma = k * \Delta$ with σ in (lb/in) (D)

substituting (D) into (C), we get $\sigma_{bot} = \sigma_{top} * l_{bot}/l_{top}$ (E)

and we have $l_{top} = d - l_{bot}$ (F)

Using (E) and (F) in (A) and (B), and substituting P = 50 lbs, L =0.1", d = 0.5", we have a system of two equations in two unknowns. Solving, we find that:

l_{bot} = 0.19"
l_{top} = 0.31"
σ_{top} = 320 lb/inch
σ_{bot} = 520 lb/inch

Across the 3/8" width of the anchor we find that maximum stresses at this loading are 195 psi on the front face, and 120 psi on the back face.