HW1.Pb3.Fall.2014.setup

September 2, 2014

(3) A sprinkler head fuse can be modeled as a cylinder of diameter 4 mm and length 12 mm. The density can be approximated as being 1000 kg/m^3. The specific heat capacity is approximately 1 kJ/kgK. The heat transfer coefficient of the smoke gases is 20 W/m^2K. If the smoke gases are 200 C and the fuse is initially at 20 C, how long will it take for the fuse to open if the activation temperature is 80C?

Sprinklers are considered to be one of the significant innovations in active fire protection systems. A fire sprinkler is a fused valve that releases water once the cylindrical fuse element fails when exposed to sufficiently high temperature conditions.

The basic model here is that the energy equation (first law) for an unsteady process can be written as:

$$\frac{dU}{dt} = \dot{Q}_{in} + \dot{W}_{in}$$

U is the internal energy with units of Joules (J), $\dot{Q}_{in}$ is the heat transfer rate in with units of watts (W), $\dot{W}_{in}$ is the rate of work done on the solid with units of watts (W).

As in problem (1) there is no work being done on the system, so $\dot{W}_{in} = 0$. The heat transfer into the cylinder is called convective heat transfer. Convection heat transfer is heat transfer between the fluid surrounding the solid and teh solid. While the detailed model of this heat transfer process require detailed analysis of several conservation equations (mass, momentum, and energy), there is a simple correlation called Newton's law of cooling that allows us to predict the heat transfer process.

Newton's law of cooling statest that the heat transfer rate $\dot{Q}_{in} = hA(T_\infty - T_s)$. In words, the heat transfer rate with units of Watts (W) is proporrtional to a temperature difference between the fluid temperature far from the solid (T_∞) and teh surface temperature of the solid (T_s). The area of contact between the fluid and solid is A. The details of the fluid mechanics and the type of fluid are contained in the parameter h called the heat transfer coefficient with units $\frac{W}{m^2K}$.

For this problem, we use the so-called "thermally lumped approximation" that considers a solid to have a single uniform temperature during a heat transfer process. You can think about this as being true when the solid internal heat transfer processes are very fast in smoothening out the internal temperatures relative to the rate at which the external fluid can heat up the surface of the solid. Later we will define a so-called Biot number that tells us when this is true. The Biot number depends on the thermal conductivity of the solid, the size of the solid, and the heta transfer coefficient between the fluid and solid. For now, we assume that this is a valid approximation.

Just like for problem (10 we can ssume that there is a relationship between the specific internal energy u and temperature. For a solid, the relationship is the simiar to what we specified for an ideal gas: $du = mcdT$. Note that we ddn't explicitly identify the specific heat capacity as being at constant volume (c_v) or at constant pressure (c_p).Note that m is the mass of the solid and is the product of the solid density and volume: $m = \rho V$. Using this approximation and also recognizing Newton's law of cooling, we get:

$$\rho c V \frac{dT_s}{dt} = hA(T_\infty - T_s)$$

We can integrate this equation to get:

$$\ln(\frac{T_s(t)}{T_s(0)}) = \frac{-hA}{\rho c V}t$$

For this problem, we need to calculate the time for sprinkler activation.
1. Rewrite the equation above in terms of time.
2. Identify the various parameters in the equation as specified in the problem statement. 3. Compute for the time.

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