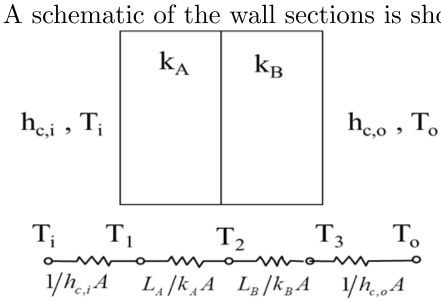
HW1.Pb4.Fall2014.soln

September 8, 2014

(4) A composite wall for a furnace is made of two materials, an insulating material with thermal conductivity k_A and an exterior skin with thermal conductivity k_B. Within the furnace there is an internal heat transfer coefficient $h_{c,i}$ and internal temperature T_i. Outside of this wall there is a heat transfer coefficient $h_{c,o}$ and external temperature T_o.

A schematic of the wall sections is shown below;



For this problem the left fluid temperature is set to be $T_i = 1000^oC$ while the right fluid temperature is set to be $T_o = 300^oC$. You are asked to find an equivalent thermal resistance for the wall section and to also find the heat flux through the wall.

In problem (3), we identified Newton's law of cooling as one way of modeling convective heat transfer. Remember that convective heat transfer is heat transfer between a fluid and a solid. In electrical circuit systems we are familiar with the concept of resistance. A resistance can be thought of as the proportionality between a potential difference and a transfer rate. In electrical systems the electrical resistance is the proportionality between the charge transfer rate (i.e., the current) and the voltage potential.

$$\Delta V = R_E i$$

For thermal system obeying Newton's law of cooling, we can devise a convective thermal resistance as the proportionality between the thermal potential difference (i.e., temperature difference) and the heat transfer rate.

$$\Delta T = R_T q$$

Viewed from the perspective of Newton's law of cooling which is $q = hA\Delta T$, we can restate the dependence as:

$$\Delta T = \frac{q}{hA}$$

and identify the convective thermal resistance to be

$$R = \frac{1}{hA}$$

. For conduction heat transfer in a simple steady-state, one-dimensional system, the conduction law (Fourier's law) is specified as:

$$q = kA\frac{\Delta T}{L}$$

where k is the thermal conductivity, A is the cross-sectional area, and L is the thickness of the slab/wall. For this simple form of Fourier's law, the conductive thermal resistance can be shown to be:

$$R = \frac{L}{KA}$$

. In electrical circuits in which the resistances are organized in a series manner, the current (heat flux) is the same through each element.

$$q = \frac{\Delta T_1}{R_1} = \frac{\Delta T_2}{R_2} = \frac{\Delta T_3}{R_3} = ...$$

We can now imagine that when we add each of the ΔT elements, we get the total temperature difference across the circuit and we also get the product of the heat transfer rate and the sum of the reistances.

$$q \sum R_i = T_1 - T_N$$

In terms of the total heat transfer rate, we get:

$$q = \frac{(T_1 - T_N)}{\sum R_i}$$

Recognize that although the total heat transfer rate is defined using the area of the wall, the heat flux typically denoted as $q" = \frac{q}{A}$ is the heat transfer per unit area.

$k_A = 1W/(mK), k_B = 20W/(mK), L_A = 2cm, L_B = 0.2cm, T_i = 1000^oC, T_o = 300^oC, h_{ci} = 30W/(m^2K), h_co = 10W/(m^2K)$

a) Find the equivalent resistance.

b) Find the heat flux through the wall.

In [6]:
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k_A= 1; k_B=20; L_A= 2; L_B= 0.2; T_i=1000; T_o=300; h_ci =30; h_co=10;
```

In [7]:
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Requiv=1/h_ci + L_A/k_A + L_B/k_B + 1/h_co;
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In [8]:
```
Requiv
```

Out[8]:
```
2.01
```

In [9]:
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qflux=(T_i-T_o)/Requiv;
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In [10]:
```
qflux
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Out[10]:
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348.25870646766174
```

We find that the equivalent resistance is $R''_{EQV} = 2.01 Km^2/W$. The heat flux is $q'' = 348.3 W/m^2$.

In []: