HW1.Pb1.Fall2014.soln

September 8, 2014

(1) A closed rigid container has 6 kg of air at an initial pressure of 100 kPa and volume of 25 m^3. The air undergoes a process to a final pressure of 200 kPa. The pressure increase was a result of heat transfer. How much heat (energy) was transferred (kJ)?

We use the first law for this problem:

$$\Delta u = q_{in} + w_{in}$$

where u is the mass specific internal energy with units of $\frac{kJ}{kg}$. The mass specific heat transfer in and work in also have the same units.

We model the air as an ideal gas and note that for an ideal gas that the internal energy can be modeled as a function of temperature only.

$$\Delta u = c_v \Delta T$$

The mass specific heat capacity at constant volume is c_v. This can be looked up at a representative temperature. We insert this into the first law and also note that there is no work being done on the air.

$$c_v \Delta T = q_{in}$$

We then use the ideal gas equation of state:

$$P = \rho R T$$

where we solve for the temperature as

$$T = \frac{P}{\rho R}$$

we get as a final form:

$$q_{in} = c_v \frac{\Delta P}{\rho R} = c_v \frac{P_f - P_o}{\rho R}$$

which we solve for q_{in}.

We perform a sanity check on units:

$$[J/kgK] \frac{[kJ/m^3]}{[kg/m^3][J/kgK]}$$

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In [28]: mass=6.; Po=100.; vol=25.; Pf=200.;
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The specific heat capacity at constant volume c_v is found using the universal gas constant $\bar{R}$ and the specific heat capacity at constant presssure c_p tabulated in Table 2.1. Since air is primarily N_2, we can use the N_2 data. Thermodynamics tells us that $R = c_p - c_v$ or that $c_v = c_p - R$. We find the initial temperature of the air from Ideal gas: $T = \frac{P}{\rho R}$

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In [29]: rho=mass/vol; Rg=8.314/28.; T=Po/(rho*Rg)
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In [30]: c_vmol=35.1-8.314;
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In [31]: c_v=c_vmol/28.
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In [32]:
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q_in=cv*(Pf-Po)/(rho*Rg);
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In [33]:
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q_in
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Out[33]:
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1080.5067757196696
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We find that the heat addition into the compartment is 1080 kJ/kg.