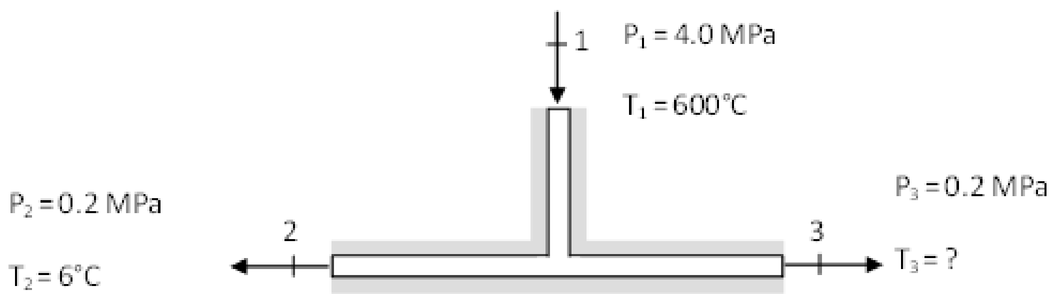
HW1.Pb2.Fall2014.soln

September 8, 2014

(2) An inventor has proposed the insulated device shown with air as the medium. Find T3 (°C) if the mass flow rate in ports 2 and 3 are equal. How would you determine if this process is possible?



The basis of this analysis is the first law for a steady state steady flow condition. In this scenario, the form of the energy equations is

$$\sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out} = \dot{q}_{out} - \dot{q}_{in}$$

We recognize the need for the mass conservation also:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

For this problem there is a single input port and two outlet ports:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

Energy conservation for the two port problem is:

$$\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 0$$

The left hand side is zero because of adiabatic operation. We now substitute

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

into energy and get:

$$(\dot{m}_2 + \dot{m}_3) h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 0$$

which can be simplified to

$$\dot{m}_2 (h_1 - h_2) + \dot{m}_3 (h_1 - h_3) = 0$$

For a non-reacting ideal gas the change in enthalpy can be expressed as:

$$\int dh = \int c_p(T) dT$$

If we assume that the temperature variation is not too large, we can use a constant/average specific heat capacity and we get:

$$\Delta h = c_p \Delta T$$

which we substitute into the energy equation:

$$\dot{m}_2 c_p (T_1 - T_2) + \dot{m}_3 c_p (T_1 - T_3) = 0$$

For the case in which $\dot{m}_2 = \dot{m}_3$, we get:

$$T_3 = 2T_1 - T_2$$

In [1]:
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T1=600; T2=6; T3=2*T1-T2;
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In [2]:
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T3
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Out[2]:
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1194
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We find that the node 3 temperature is 1194 ^oC

We can check whether the answer makes sense using the second law of thermodynamics. The second law for a steady state staedy flow process looks very much like the energy equation, but with the transported property being the entropy. The statement, in words, is that the entropy of the control volume (for an adiabatic process) must be greater than zero.

$$\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 \geq 0$$

which can then be written as:

$$\dot{m}_2 (s_1 - s_2) + \dot{m}_3 (s_1 - s_3) \geq 0$$

For constant specific heat capacities:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Check if this system violates the second law. Try some other combinations of flow rates.

In []: