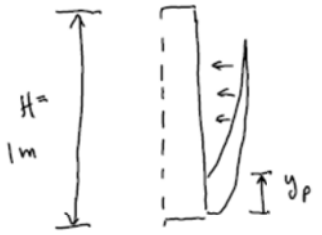
8.2 A 1m high slab of PMMA is ignited at bottom & spreads upward. Ignition is applied over a length of 5cm. If the slab is 1mm thick & ignited on both sides, how long will it take to be fully involved in pyrolysis.



$y_p(t^*) = H$ find t^*

Assume the flame height is governed by $y_f \cong 0.01 \dot{Q}'$ where y_f is in meters & $\dot{Q}'$ is KW/m.

The flame heat flux $\dot{q}''_f = 25 \frac{KW}{m^2}$ assume

$\dot{m}'' = \dot{q}/L$ $L = 1.6 \frac{KJ}{g}$. Let $T_\infty = 20^\circ C$.

Is the sample thick or thin?

Biot $Bi = \frac{hd}{K}$ $h \approx \frac{\dot{q}''_f}{T_{ig} - T_\infty} \cong 100 \frac{W}{m^2 K}$

$= 0.373 < 1$ Thin $Bi \equiv \frac{R_{conduction}}{R_{convection}}$

if $Bi \ll 1$ lump spatially

$h(T_2 - T_\infty) = K \frac{(T_1 - T_2)}{L}$

$$\frac{T_1 - T_2}{T_2 - T_0} = \frac{hL}{K} \ll 1$$

T_1 max temp in solid

T_2 min " "

$$\frac{T_2 - T_\infty}{T_1 - T_2} = \frac{K}{hL}$$

$\frac{T_2 - T_\infty}{T_1 - T_2} \gg 1$ when $\frac{hL}{K} \ll 1$

continuous ignition

$$\frac{dy_p}{dt} = \frac{y_f - y_p}{t_{ign}}$$

$$t_{ign} = \frac{\rho c d (T_{ign} - T_\infty)}{\dot{q}''}$$

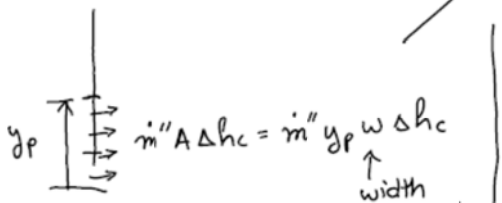
↑ chp 7 result

heat release rate ↓

$$\dot{Q} = \dot{m}'' A \Delta h_c = \frac{\dot{q}''}{L} A \Delta h_c \qquad \dot{Q}' = \frac{\dot{q}''}{L} y_p \Delta h_c$$

flame length →

$$y_f = 0.01 \dot{Q}'$$
$$= 0.01 \frac{\dot{q}''}{L} y_p \Delta h_c$$
$$= K y_p$$



$\dot{m}'' A \Delta h_c = \dot{m}'' y_p w \Delta h_c$

↑ width of sample

$$\frac{dy_p}{dt} = \frac{K y_p - y_p}{t_{ign}}$$
$$= \frac{(K-1) y_p}{t_{ign}}$$

$$y_p(t) = y_p(0) \exp\left(\frac{(K-1)t}{t_{ign}}\right) \quad \text{or}$$

$$t = \frac{t_{ign}}{K-1} \ln\left(\frac{y_p}{y_{p0}}\right) \qquad \text{case } y_p = 1\text{m}, \; t = 17 \text{ sec.}$$

use Tables 2.3, 7.5, 7.6 & 8.1 for PMMA properties

$\Delta h_c = 24{,}200$