...Machine Learning...

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titulo

titulo

Fork e Join

- (\triangle) :: Cartesian k \Rightarrow (a 'k' c) \rightarrow (a 'k' d) \rightarrow (a 'k' (c \times d))
- (\bigtriangledown) :: Cartesian $k \Rightarrow (c \ 'k' \ a) \rightarrow (d \ 'k' \ a) \rightarrow ((c \times d) \ 'k' \ a)$

instancia de \rightarrow^+

```
newtype a \rightarrow<sup>+</sup> b = AddFun (a \rightarrow b)
instance Category (\rightarrow^+) where
       type Obj (\rightarrow^+) = Additive
       id = AddFun id
       AddFun g \circ AddFun f = AddFun (g \circ f)
instance Monoidal (\rightarrow^+) where
       AddFun f \times AddFun g = AddFun (f \times g)
instance Cartesian (\rightarrow<sup>+</sup>) where
       exl = AddFun exl
       exr = AddFun exr
       dup = AddFun dup
```

instancia de \rightarrow^+

```
instance Cocartesian (\rightarrow^+) where
          inl = AddFun inlF
          inr = AddFun inrF
          jam = AddFun jamF
in F:: Additive b \Rightarrow a \rightarrow a \times b
inrF :: Additive a \Rightarrow b \rightarrow a \times b
jamF :: Additive a \Rightarrow a \times a \rightarrow a
inlF = \lambda a \rightarrow (a, 0)
inrF = \lambdab \rightarrow (0, b)
jamF = \lambda(a, b) \rightarrow a + b
```

definição de NumCat

```
class NumCat k a where
      negateC :: a 'k' a
      addC :: (a \times a) 'k' a
      mulC :: (a \times a) 'k' a
       . . .
instance Num a \Rightarrow NumCat (\rightarrow) a where
      negateC = negate
      addC = uncurry (+)
      mulC = uncurry(\cdot)
```

$$D (negate u) = negate (D u)$$

$$D (u + v) = D u + D v$$

$$D(u \cdot v) = u \cdot Dv + v \cdot Du$$

- Impreciso na natureza de u e v.
- Algo mais preciso seria defenir a diferenciação das operações em si.

class Scalable k a **where**

scale :: $a \rightarrow (a 'k' a)$

instance Num a \Rightarrow Scalable (\rightarrow ⁺) a where scale a = AddFun (λ da \rightarrow a \cdot da)

instance NumCat D where

negateC = linearD negateC

addC = linearD addC

 $\mathsf{mulC} = \mathsf{D} \; (\lambda(\mathsf{a}, \mathsf{b}) \to (\mathsf{a} \cdot \mathsf{b}, \, \mathsf{scale} \, \mathsf{b} \, \bigtriangledown \, \mathsf{scale} \, \mathsf{a}))$

Generalizing Automatic Differentiation

```
newtype D_k a b = D (a \rightarrow b \times (a 'k' b))
```

linearD ::
$$(a \rightarrow b) \rightarrow (a 'k' b) \rightarrow D_k a b$$

linearD f f'= D (
$$\lambda a \rightarrow (f a, f')$$
)

instance Category
$$k \Rightarrow$$
 Category D_k where type Obj $D_k =$ Additive \land Obj $k \dots$

instance Monoidal
$$k \Rightarrow$$
 Monoidal D_k where ...

instance Cartesian
$$k \Rightarrow$$
 Cartesian D_k where ...

instance Cocartesian
$$k \Rightarrow$$
 Cocartesian D_k where

instance Scalable k s \Rightarrow NumCat D_k s where negateC = linearD negateC negateC addC = linearD addC addC mulC = D (λ (a, b) \rightarrow (a · b, scale b ∇ scale a))

Exemplos

Generalizar