Show me the Money! The Dynamics of Startups Competition for Venture Capital Financing, Survival and Growth

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Abstract

This paper proposes a new model to describe startups and venture capitalists interaction. We use evolutionary game theory in conjunction with Lotka-Volterra's predator-prey model to describe the dynamics of startups competition for venture capital financing, survival and growth in a hostile environment. The simulations show results in line with the typica "boom and bust" dynamics of the U.S. venture capital industry.

1. Introduction

The importance of startups is well documented in the literature. Highly innovative new ventures such as Google, Intel, Facebook, Amazon and Starbucks were responsible for society's revolution and are a driving force in the economic development (Bygrave and Timmons, 1992; Gompers, 1994; Gompers and Lerner, 2001a, 2001b, 2002; Metrick, 2007; Koellinger and Thurik, 2012). With Silicon Valley and Boston 128 as its beacons, the U.S. has long established a startup culture which brings a very competitive dynamics into the economy. Hence, startups are responsible for 3% of new jobs created in the U.S. each year (Haltiwanger et al, 2013; GEM USA, 2015).

The capability to access and to combine key resources (financial, human, technological, natural, etc) is critical for a company to survive and to grow in a competitive environment, especially in the beginning of its lifecycle (Porter, 1979; Prahalad and Hamel 1989; Gompers, 1994; Teece and Pisano 1994; Teece et al 1997). Nevertheless, many entrepreneurs are not

able to handle the capital constraint in the first years of operation and perish before cross the 'valley of death'. According to the GEM USA (2015), 30% of startups in the U.S. are closed due to unprofitability and 5% due to financing problems, not to mention the endogeneity effect between these two variables.

Venture capital is a valuable resource to startups which usually have limited access to bank financing due to their negative cash flows and lack of tangible assets. Thus, in the U.S it is one of the most important sources of financing to startups. Accordingly to GEM USA (2015), 39% of the startups with less than 3 months old and 19% of those with 3 months to 4 years old access this source of capital.

Different from bank financing, venture capital shares upside and downside risks with the entrepreneur by taking equity stake in the startup (Gompers, 1994; Gompers and Lerner 2001a; Gompers and Lerner 2001b). Moreover, venture capitalists provide startups with other resources that increase survivorship rates, such as budgeting and performance tools, compensation arrangements, high level networking, mentoring and advice to entrepreneurs (Gorman and Sahlman, 1989; Gompers and Lerner, 2001b; Gompers and Lerner 2002).

Notwithstanding its clear relevance, the literature describing the dynamics of startups financing by venture capitalists is still scarce. Most studies use steady-state equilibrium models and empirical research (Bengtsson and Hsu, 2010; Pasquini et al, 2016). Only a few propose new theoretical frameworks (De Bettignies, 2008). Papers using dynamic models such as two-sided matching games (Fox et al, 2012) or predator-prey competition (Brander and De Bettignies, 2009) are practically non-existent.

Encouraged by a long-time existent gap between Economics and Entrepreneurship, we propose a new theoretical model using evolutionary game theory that links Lotka-Volterra's predator-prey model (Lotka, 1925; Volterra, 1926) and Haldane's fitness in the natural selection (Haldane, 1957). More precisely, our work intends to shed a light on a critical question: what is the dynamics of startups competition for venture capital financing, survival and growth in a hostile environment? Furthermore, we aim to investigate whether in a scenario of limited capital the most prepared entrepreneurs can better succeed in a competitive environment.

Although we use Lotka-Volterra's model as our base framework, our work differs significantly from Brander and De Bettingnies (2009). We introduce a natural selection dynamics as we appropriate from Haldane (1957) idea of fitness, a set of characteristics which make the individual more likely to succeed in a competitive environment. It is perfectly linked to the idea that some of the entrepreneur's characteristics (resilience, experience, etc) make it more likely to raise funds, or some of the startups characteristics (innovation, industry, etc). For example, high potential entrepreneurs are only 4% and create 3 times more jobs

than average potential that corresponds to 6% of total entrepreneurs. Highly potential entrepreneurs create 15 times more jobs than low potential ones that correspond for 90% of the total amount of entrepreneurs (GEM Global 2015). The entrepreneur optimism level is important to the success rate of a startup (De Meza and Southey, 1996; Camerer and Lovallo, 1999). We consider the startups competing for a scarce and valuable resource: venture capitalist's money. The best startups (and entrepreneurs) have a higher chance to access the funding and, consequently, only those with the best fitness will survive.

2. The Model

As already mentioned, the venture capital financing is a highly valuable resource to the startups struggling to survive in a competitive environment. Lotka-Volterra's predator-prey model can provide useful insights regarding the dynamics of startups competition, financing, survival and growth. The assumption of an endogenous, self-sustaining dynamic has kept the model at bay from the mainstream and, consequently, out of the radar of the majority of the researches from those fields. Its application in Economics, Business and Finance has been timid over the years with only few authors venturing into the discussions (Goodwin, 1965; Brander and Taylor, 1998; Pezzey and Anderies, 2003; Hoekstra and Van Den Bergh, 2005, Gracia, 2005; Kremer and Olken, 2009; Brander and De Bettingnies, 2009).

The startups prey the capital available by the venture capitalists to new investments, called dry powder. Equations (1) and (2) refer to the predator-prey dynamics where the startups are searching for venture capital financing.

$$\frac{\mathrm{d}X}{\mathrm{d}t} = rX(1 - \frac{X}{K}) - X.\sum_{i=1}^{n} \beta_i.Y_i \tag{1}$$

$$\frac{\mathrm{d}Y_i}{\mathrm{d}t} = \gamma f_i.X.Y_i. - Y_i\phi \tag{2}$$

X: Amount of preys (Dry Powder) - in dollars

Y: Amount of predators (Startups) - in units

K: Capital available in the market (Committed Capital) - in dollars

r: Investment birthrate

β: Predation efficiency measure

γ: Average Investment per Startup - in 1/dollars

In addition to hunt venture capitalists' money, the startups also need to survive against their peers. The main assumption here is that the startups are not the same. Thus, the survival likelihood of a startup will be given by its ability to compete in a hostile environment (Hvide and Panos, 2014; Buera et al, 2015; McKelvey, 2016). In our model we set two fitness that when combined result in its global fitness for each startup: g_i represents the fitness related to the startup itself (e.g. business plan, innovation, industry) and h_i represents the fitness related to the entrepreneur (e.g. track record, experience, resilience, optimism and motivation).

$$\phi = \frac{\sum_{i=1}^{n} f_i Y_i}{\sum_{i=1}^{n} Y_i} \tag{3}$$

f_i: Fitness of Startup i

ϕ: Average fitness

Our model considers three different populations of startups: HQ (high quality), AV (average quality), LQ (low quality). The result of the interaction among these startups is given by a payoff matrix.

$$A.P = F \tag{4}$$

A: Matrix of payoff

P: Matrix of amount of predators

F: Matrix of fitness

3. Discussions

When our model is analyzed in the steady-state equilibrium, the variables X (dry powder amount) and Y (number of startups) present the following characteristic:

$$X* = \frac{\phi}{\gamma.f_i} \tag{5}$$

$$Y_i* = \frac{K.f_i.r - \phi.r}{k.\beta_i.f_i} \tag{6}$$

The parameters are β, δ, ϕ, X, Y_i, K, r, f_i, and the formal conditions are as follows.
(C1) X, $Y_i \geq 0$ and all $\in \mathbb{R}$
(C2) f_i, β, δ, $\phi \geq 0$ and all $\in \mathbb{R}$

We are interested in the dynamics of the model. With r=5, K=20, β=2.5, f=10, ϕ=10 and γ=1 we simulate the dynamics of the system. It presents a cyclical dynamics towards convergence.

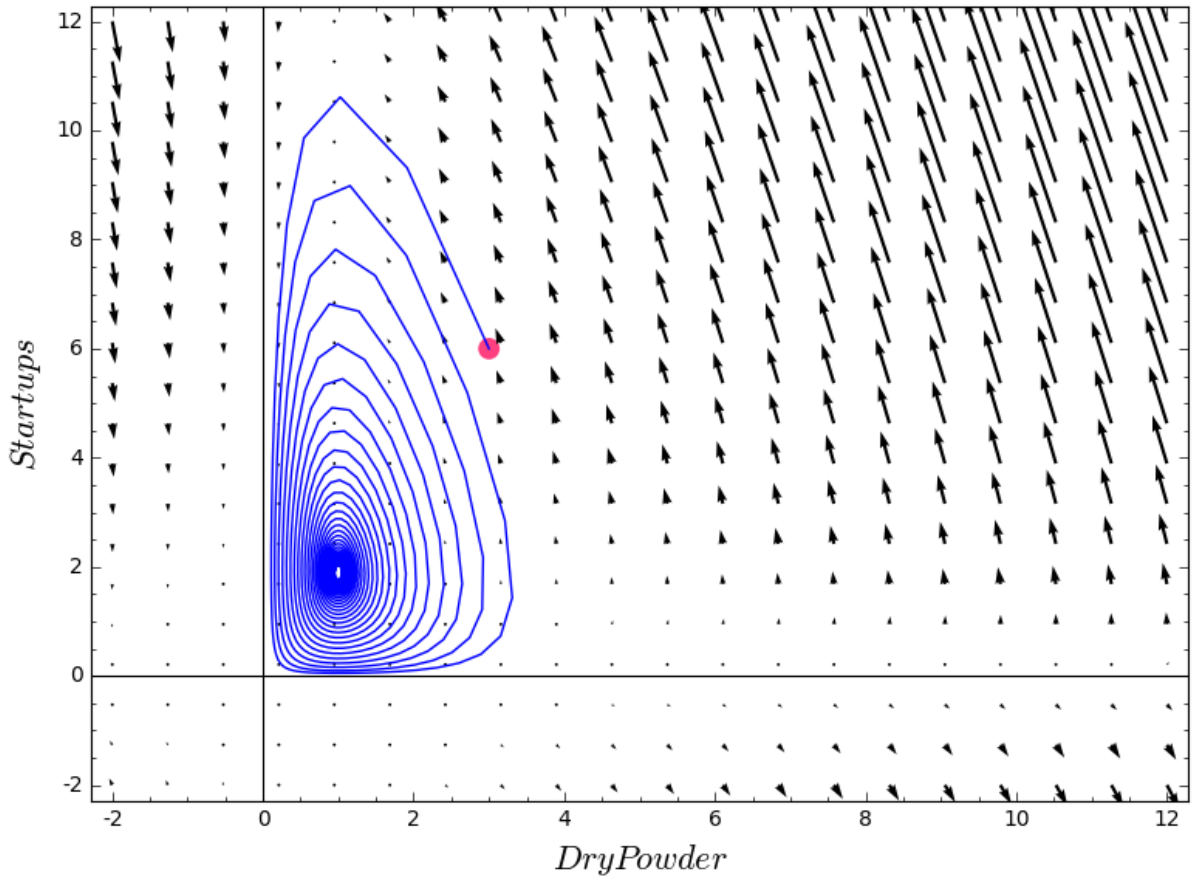

Fig. 1. Cyclical Dynamics

When the competition is given by fixed fitness, only the startup with the highest fitness survive. It is in line with the natural selection rule (see figure 2).

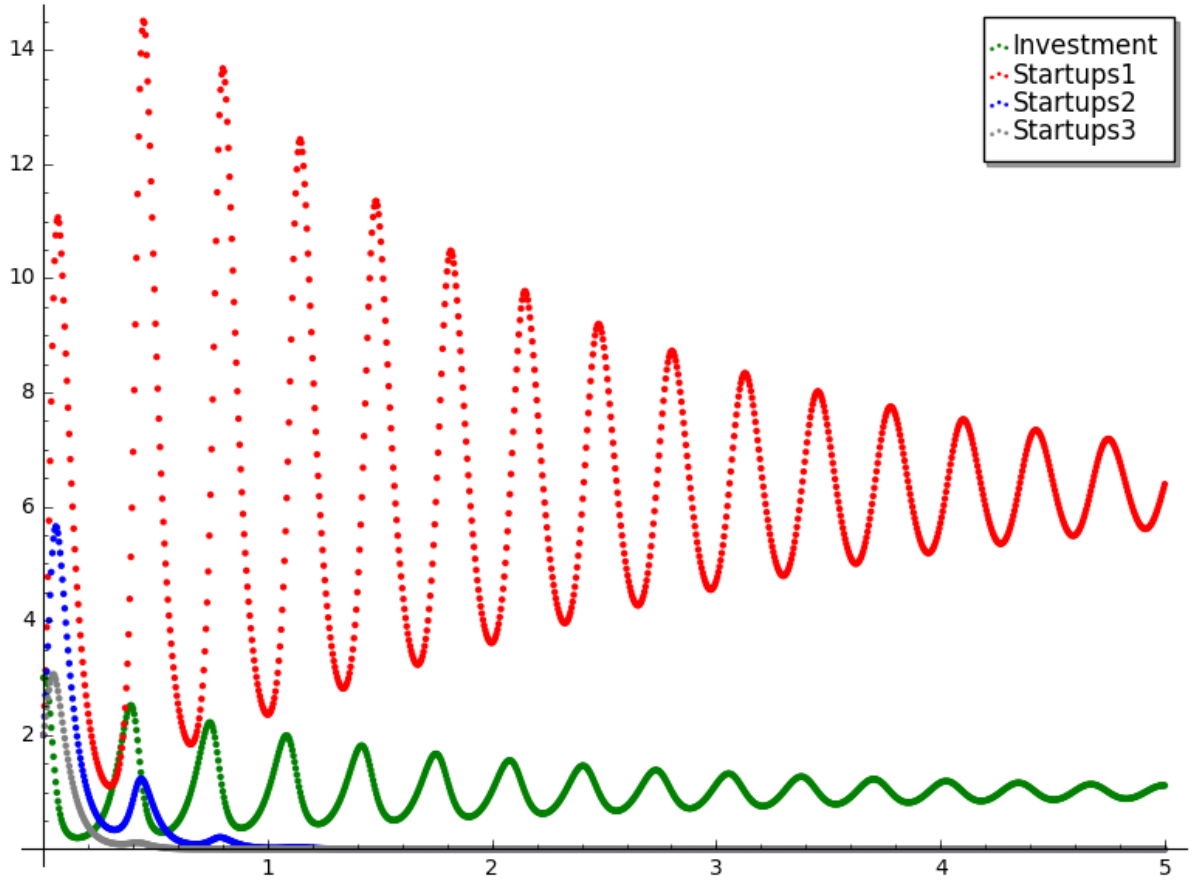

Fig. 2. Fixed fitness

When an evolutionary game is included in the model, the dynamics change. If the determinant of the payoff matrix is positive the system only has one global stable equilibrium. On the other hand, if the determinant is negative the system presents an unstable equilibrium, which means that the competition does not have a clear winner.

In figure 3, we simulate a positive determinant. As expected, AQ and LQ startups die. In figure 4, since the determinant is negative we observe that AQ and LQ startups coexhist with HQ startups in a constant rate.

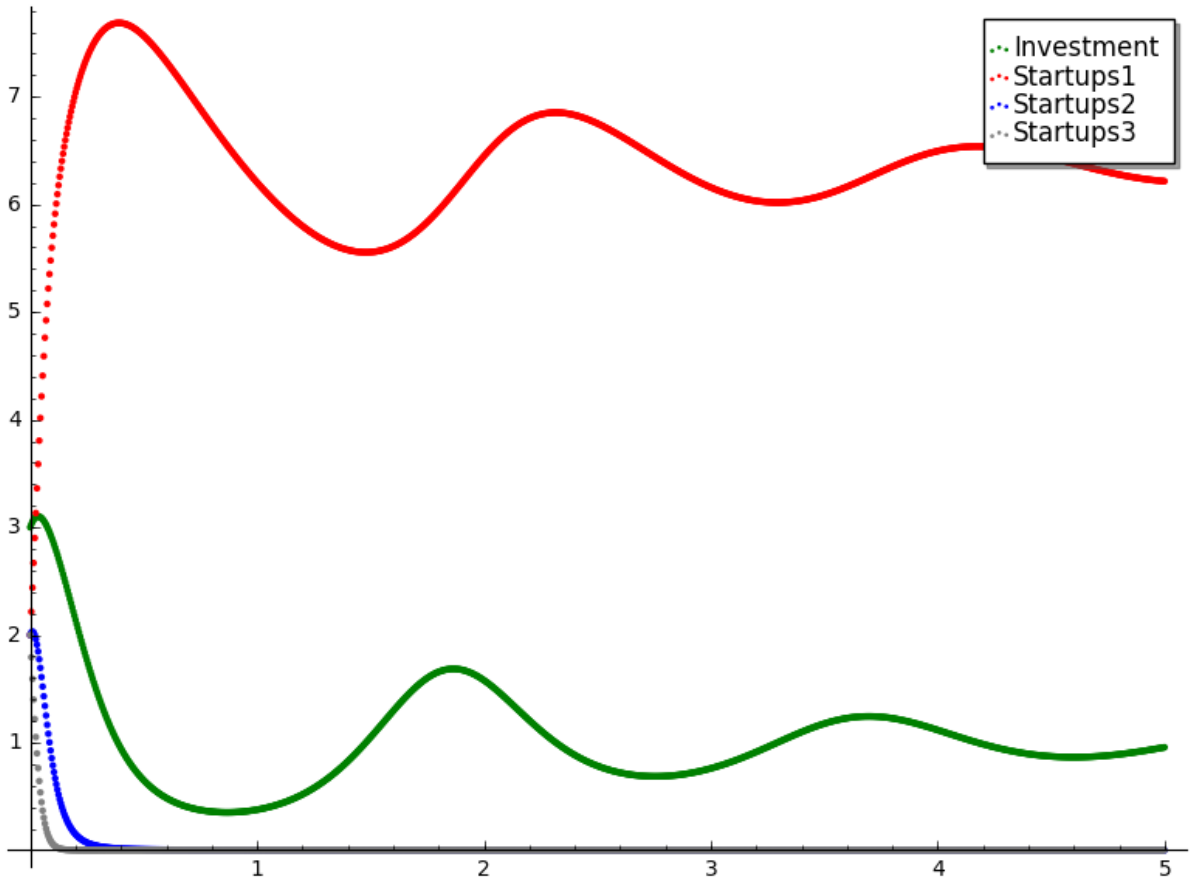

Fig. 3. Determinant positive

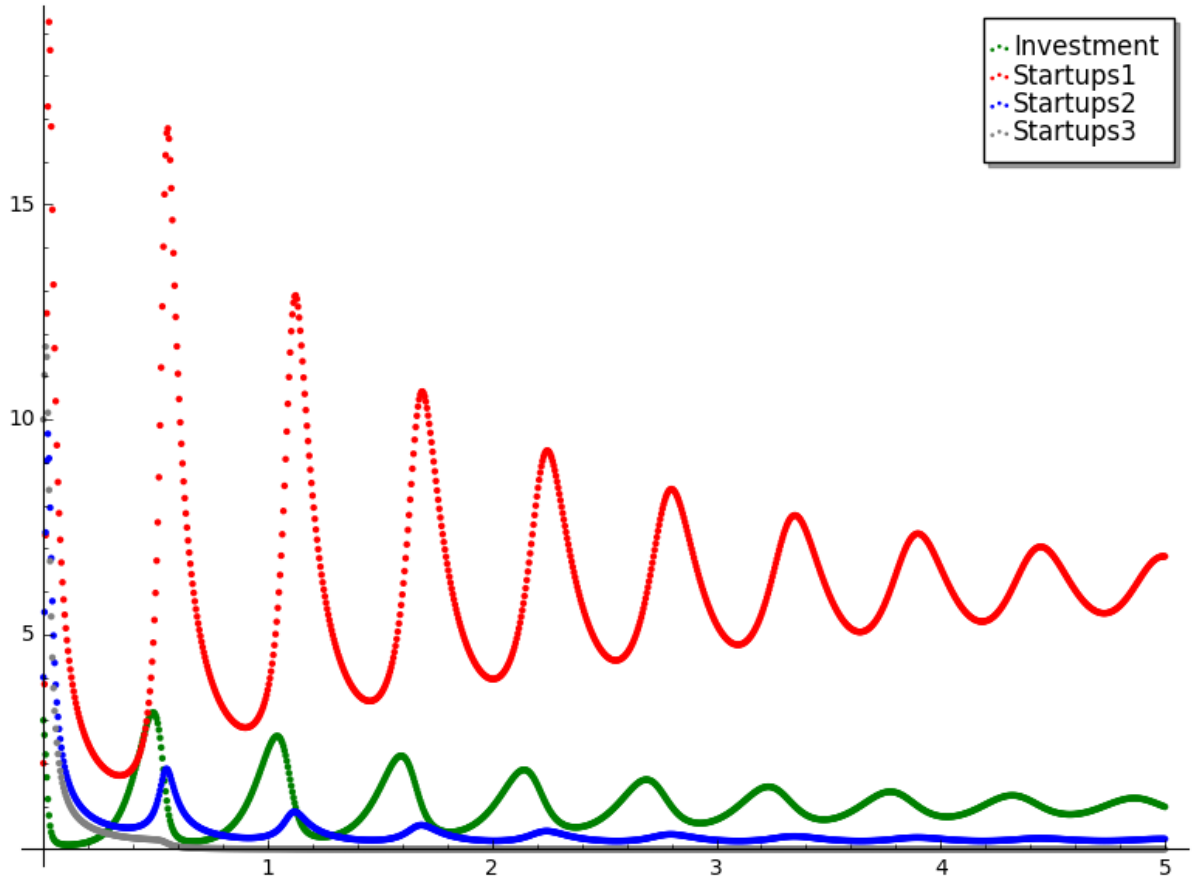

Fig. 4. Determinant negative

4. Conclusion

The main contribution of this work is to explore a new theoretical framework to describe the dynamics of startups financing by venture capitalists.

In line with Brander and De Bettignies (2009) we show that this interaction shows a predator-prey behavior. However, our model introduces a evolutionary game to model the startups competition among themselves. Those startups with better fitness have a higher chance to succeed in a competitive environment.

The results of our model are consistently with the venture capital "boom and bust" dynamics, where the dry powder and the number of startups oscillate throughout time.

An extension of the present work would introduce a stochastic dynamic to the evolutionary game. It could give a better insight regarding the survival of LQ and AQ startups competing against HQ startups. A second extension would include competition among the venture capitalists. A third extension would be include mutations in the startups (e.g. new technologies arise).

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