SOUND SYNTHESIS BY PHYSICAL MODELS,
APPLICATION TO STRINGS.

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Abstract: **This paper deals with our latest results concerning sound synthesis by physical models, in the case of a three dimensional string set in motion by exciters. Our work is concerned with both the physical and the computer science aspects of the problem, and we describe several improvements and extensions of the programs presented at the International Computer Music Conference 85 [1].**

1. Introduction

The basic principle of sound synthesis by physical models is that sound samples are computed with a time domain model of a mechanical system, that is the time quantisized description of the interaction between a resonator and different exciters; whereas most other sound synthesis systems are concerned with frequency domain computations. The main features of the physical model method in the scientific field are that it produces results of good quality when used with aperiodic movements and non linear coupling between exciters and resonators, behaviour which is difficult to describe in terms of frequency; and that the synthesiser is controlled, even at low level, with parameters of the physical space such as masses or forces which are very different from the abstract mathematical tools of the frequency domain methods. On the other hand, the control of the resonance frequencies in the time domain might be less accurate than in the frequency domain, because of computation costs and the relative lack of experience in physical model synthesis.
The corresponding features of this method in the musical field are the good quality of transients and sustained sounds, the ability to synthesize a succession of excitations on the same resonator and to produce a musical phrase perceptible as a coherent object and not as a patchwork of sounds with no acoustic relation between one another; and the caracteristics of the mechanical parameters which might be more familiar to composers than the mathematical ones. Other effects of physical rules are the high level structuration of the elementary sounds and their clear relation with the natural correlates.
The digital model can be used for both physical and musical aims, for the investigation of badly understood complex vibrations which are difficult to discribe fully in an analytical way, such as the movement of a bowed string , or for the computation of strongly featured sounds for musical purposes and the integration of the model in a pool of synthesizers used in a musical context.

For these aims, a compromise must be found at all steps among the level of physical approximation, the corresponding caracteristics of the sounds and the computation costs. Our experience is that pure digital processing methods with no direct physical correlates often has undesirable effects. Musical use requires the ability to describe a very large number of situations which might be sometimes insignificant for physical matters and this is why we have chosen a modular approach, each module corresponding to a specific mechanical part of the system, which allows the illustration of various situations. We concentrated our work on the description of the interaction of a string and different exciters as hammers, plectrums, bows and fingers. Each one of these elements is represented by a module. This paper relates the extensions of the one dimensional model we presented in [1] as well as our efforts toward the improvments and the generalisation of the modules. The sources of this study were mostly the works of Cremer [2] J.Smith [3] and G.Weinreich [4] on the topic.

2. Method

We have chosen for the representation of the string, a method of integration by arbitrary functions. The displacement of a point of the string is given by the sum at this point of two waves propagating in opposite directions. These waves, which contain the information of initial displacement and velocity of the two transverse vibration modes, are moved along a delay line, without affecting their shape. We made the assumption that reflections at the end of the string are perfect and only affect the sign of the displacement. Thus, the string is a set of two discrete delay lines in which the information is translated from one cell to the next at the sample rate R and shifted at the ends of the lines from one line to the other with a change of sign. The lines are symmetrical and a pair of symmetrical cells on each line corresponds to a given abcissa on the string. One can access the displacement information of a given point of the string either directly, if that point corresponds to a pair of cells by summing the information contained by the cells, or indirectly if that point does not correspond to any pair of cells by interpolating the information given by the two nearest pairs of cells. These interpolations are provided by generators of delays smaller than a sample; a set of which is active at one end of the lines to give to the string a non-integral lengh. As a result the model behaves like a perfect harmonic resonator with two transverse modes of vibration, and the effects of spatial discretisation of the matter continuum are limited to the side effects of the interpolations between the pairs of cells. The period of the string is given by $T=2N/R$ if N is the floating-point number of cells in one line, and this corresponds to the quantity$2L/(e/F_x)^{1/2}$ of the ideal string theory., where e is the lineic mass and F_x the tension of the string. e and Fx are inputs to the program because they are involved in the interaction with exciters but do not affect direcly the period of the resonator. We detailed this method in [1] in the case of our one-dimensional model. Its advantages are that it easily provides information at any point of the string, which is not the case, as far as we know, with a spring/mass model; and it is very cheap in computation costs. Its drawback is that the modelized resonator is perfect, and its timbre is therefore rather uninteresting. This is why we introduced a second tranverse mode of vibration and coupled the resonator with a bridge model. Smith [3], Schumacher and Woodhouse [5] have used delay lines, and Cadoz [6] has used the mass/spring model.

3. Exitations

Exciters, in the one dimensional model, were acting at one point of the resonator, and it seemed necessary, in some cases, to be able to simulate extended excitations. An exciter module is now a set of excitation points, acting close to each other. It is then possible to build a discrete representation of an extended excitation if all the excitation points are associated with the same mechanical excitation system, or to build a hybrid exciter zone including different kinds of excitations.The exciter module is connected to the resonator delay lines at the points corresponding to the ends of the excitation zone. The pair of lines at these points supports the incident, reflected, and transmited waves from each side of the exciter.. The generators of short delays which are used to access any point of the string are controlled dynamically, as in [1], so that it is possible to move the excitation zone continuously along the string during playing.This is important for bow and finger playing. The minimal distance between two points of an exciter is greater than N/R, thus excitation points act independently during the time interval $1/R$. For closer points the discrete representation is no longer efficient, and it would be more convienient to use an analytical integral representation of the extended excitations. It is possible to introduce from each side of a given point losses corresponding to a lowpass filter with transfer function $H(Z)=\rho-\rho Z^{-1}$. These filters are very useful for timbre adjustment in some cases.

The general model for an excitation point is a two mass spring and loss device with either one or two degrees of freedom. The first case is illustrated in Fig.1 with the corresponding set of equations

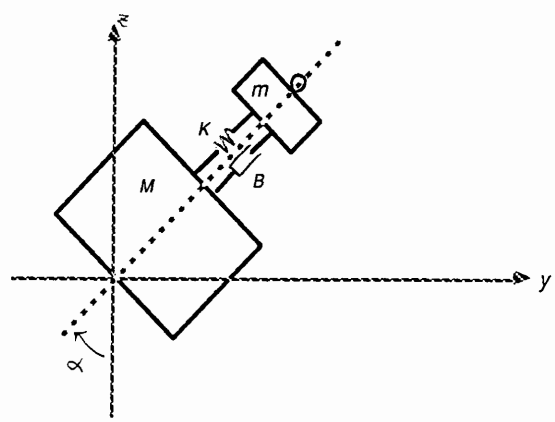


Fig 1
Mechanical model for the exciter.

$$
\begin{aligned}
MY''(t)^2 = \; & -K\,(Y(t)-y(t)) - B\,(Y'(t)-y'(t)) - C\,Y'(t) + \\
& \sin\alpha\,(F_{oy}\sin\alpha+F_{oz}\cos\alpha) \\
my''(t) = \; & K\,(Y(t)-y(t))) + B\,(Y'(t)-y'(t)) - \;A\,y'(t) + \\
& A\sin\alpha\,(\sin\alpha\; i'_y(t) + \cos\alpha\; i'_z(t)t) \\
M''Z(t) = \; & -K\,(Z(t)-z(t)) - B\,(Z'(t)-z'(t)) - C\,Z'(t) + \\
& \cos\alpha\,(F_{oy}\sin\alpha+F_{oz}\cos\alpha) \\
mz''(t) = \; & K\,(Z(t)-z(t))) + B\,(Z'(t)-z'(t)) - \;A\,z'(t) + \\
& A\cos\alpha\,(\sin\alpha\; i'_y(t) + \cos\alpha i'_z(t))
\end{aligned}
\tag{1}
$$

where M, m, K, B, C and α represent the masses, stiffness, and losses indicated; $Y(t)$, $Z(t)$, $y(t)$ and $z(t)$ are respectively the coordinates of the masses M and m in the Oyz plane; F_{oy} ,F_{oz} are the coordinates of the force applied by the operator on the mass M, A is twice the string impedance and $i_y(t)$, $i_z(t)$ are the incoming waves at each side of the excitation point. The terms of interaction between the string and the mass m clearly appears in the system with the factor A. The coordinates of the mass M and m are computed at each time step as functions of insant inputs on the string, $i_y(t)$ and $i_z(t)$, and instant input force F_{oy} F_{oz}. The outputs on the string $o_y(t)$ and $o_z(t)$ and computed from the relation between incident and reflected waves on the resonator from each side of the excitation point and the coordinates of the mass m.

$$
\begin{gathered}
y(t)=i_y(t)+o_y(t) \\
z(t)=i_z(t)+o_z(t)
\end{gathered}
\tag{2}
$$

It is clear in the system (1) that inputs to the string in the Oy direction will produce a movement of the masses in the Oy and Oz directions and consequently outputs both in the Oy and Oz directions. This is a first approach describing a simple coupling of the transverse modes of vibration. This system might be used to describe the movements of bridges, fingers, hammers or plectrums. Note that the bridge, on a real instrument, with its assymetrical two footed structure might be assumed, in a first approximation, as having one degree of freedom in the Oyz plan [4].

The case with two degrees of freedom is illustrated in fig.2. The corresponding set of differential equations can be obtained from (1) with α taking the values 0 and $\pi/2$.

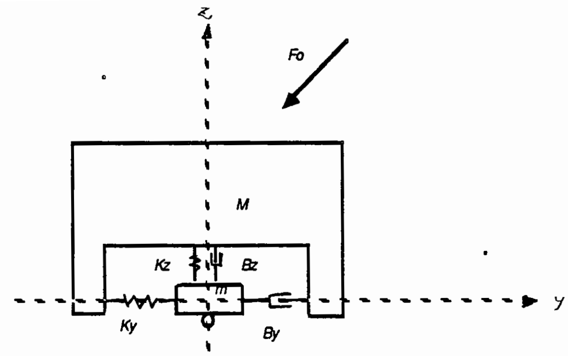


Fig.2
Mechanical model of the exciter.

$$
\begin{aligned}
MY''(t) &= -K_y(Y(t)-y(t)) - B_y(Y'(t)-y'(t)) - C_yY'(t)+F_{oy} \\
my''(t) &= K_y(Y(t)-y(t)) + B_y(Y'(t)-y'(t)) - Ay'(t)+Ai'_y(t) \\
MZ''(t) &= -K_z(Z(t)-z(t)) - B_z(Z'(t)-z'(t)) - C_zZ'(t)+F_{oz} \\
mz''(t) &= K_z(Z(t)-z(t)) + B_z(Z'(t)-z'(t)) - Az'(t) + Ai'_z(t)
\end{aligned}
\tag{3}
$$

In that case, the coordinates in the directions Oy and Oz are independent and the exciter does not introduce any coupling between the modes. This system might be used like system (1), to describe fingers, hammers or plectrums. It is useful as well to describe the bow movement in the sticking phase of friction. The bow is pushed or pulled in the Oy direction, m is then the mass of the bow hairs, M the mass of the bow wood, K_y and B_y the longitudinal stiffness and losses of the bow hairs and K_z B_z the stiffness and losses in the Oz direction. Relations (2) are unchanged in this case, thus the M, m coordinates and outputs are computed in the same way as before. In the sliding friction phase, it is necessary to introduce a sliding friction force F_{cy} applied from the bow hair to the string in the Oy direction. This force is dependent on the "bow pressure" F_z applied from the bow hair to the string in the Oz direction. In this respect, the movements of the mass m in both directions are coupled during sliding friction. The first, third and fourth equations of (1) are unchanged and the expression for the second equation, taking account for the Fcy force is

$$
\text{(4)} \quad my''(t) = K_y(Y(t)-y(t)) + B_y(Y'(t)-y'(t)) - (i'_y(t)-y'(t)+V)/(V/F_z\Delta\mu-1/A)
$$

where the last term represents the sliding friction force between the bow hair and the string. $\Delta\mu$ is an undimensional friction coefficient such that $F_z\Delta\mu$ is the maximum sticking force in the Oy direction. The sticking/sliding friction phenomena are nowdays badly understood, and physicists do not agree on the analytical expression of F_{cy}. We have introduced a programmable function V which has the dimension of a velocity and is dependent on the bowhair speed $\partial y/\partial t$. In the analytical description of the specific case of the Helmholtz motion one can the assume that V is a linear function of $\partial y/\partial t$, and that the ratio of V to $\partial y/\partial t$ depends on the abcissa of the bowing point on the string [2]. The system (2) is inappropriate in the case of sliding friction, and we use for the computation of the instantaneous outputs the following equation on each line

$$(5) \quad F_{cy}=A(o'_y(t)-i'_y(t))$$

Note that the only differences between the case of the bow and the cases of fingers, hammers or plectrums is the sliding force introduced in (4) and the equation (5) used for the output computation. We would also like to introduce friction in the case of plucking because it seems that the sliding phase of plucking is a very important feature with regard to the differentiation of hammered and plucked attacks. We present now results concerning elementary excitations.

4. Results

In every case, exept for bridges and loads, the first part of the excitation consists of the motion of the exciter toward the resonator. The systems (1) and (3) are then integrated without the terms of interaction with the string. For an excitation point, the condition of contact is determined by a relation between the coordinates of the mass m and the displacement of the corresponding point on the string. It is impossible to neglect durations smaller than $1/R$ for mechanical reasons. Another aspect is that, especially in the case of a discrete representation of an extended excitation, more than one contact between an elementary mass and the string may occur in the same time interval $1/R$. Therefore, all the routines have a parametrable sample rate, to permit the interpolation of the whole system from a given non-integral normalized time τ/R to another time $(\tau+\partial t)/R$, $\partial t<R$, in an iterative way. As one can see in (1) or (3), the exciters are sensitive to the derivatives of the displacement information propagated along the lines, and, in this respect, the second order side effects of the interpolations must be minimal. In the case where only one contact occurs between the two integral normalized times t_1/R and $(t_1+R)/R$ for example, the programs interpolate the whole system from t_1/R to the time of contact τ/R, change the mode of the exciter and in some cases the mode of the resonator as well, and interpolates again from t/R to $(t_1+R)/R$. Changing of modes occurs rather rarely, and faster routines, with R as sample rate are running most of the time. After the contact, equations (1) or (3) are integrated taking into account for the interaction terms between the masses and the string. The system runs until a condition of release is reached. This phase of contact, in the case of fingers, plectrums or hammers is the transient phase.

For the plectrum, the condition of release is $F_c<F_{max}$ where F_c, the force applied by the plectrum on the string, reaches a maximum force F_{max}. We illustrate this situation in fig.3.a with a plot of the velocity along the string for five samples after release. The set of higher steps is the projection of the velocity on the Oxz plan and the set of lower steps is the projection on the Oxy plan. The maximum projected velocity is $-3.77m/s$. The plectrum, initially in contact with the string, has been pressed down with an increasing force F_{oy}, F_{oz} until release. M, m, Ky, Kz and F_{max} are respectively $0.01\ kg$, $0.005\ kg$, $10^2 N/m$, $10^4 N/m$ and $4.0N$.

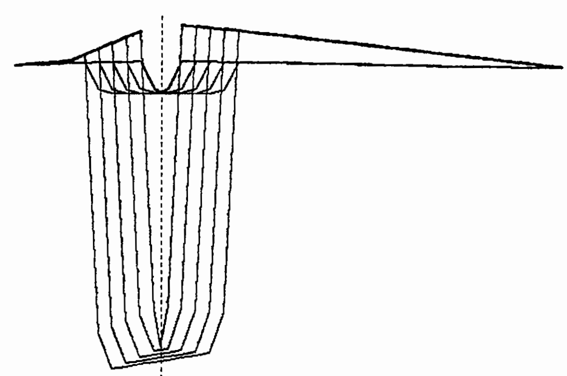


Fig 3
-a-
Velocity along the plucked string after release.
(Projections on Oy and Oz)

The brigde is always in contact mode. Fig.3.b shows the reflection of the velocity step on the bridge. One can see on Fig 3.c two sets of curves corresponding to the projections of the trajectories of the masses M and m of the bridge on the O_y and O_z axis, plotted against time. The whole sequence is about *15ms* long. The maximum deviation of the smaller mass is *0.2mm*, and the values of M, m, K and α are respectively *0.05kg*, *0.003kg*, *90000N/m* and *0.5Rd*. This plot is given as an example, and one can control the deviations of m and M with the loss parameters.

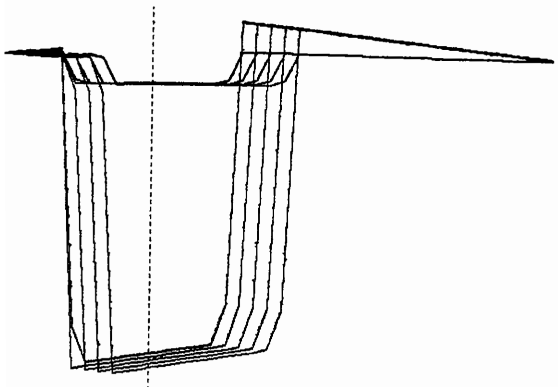
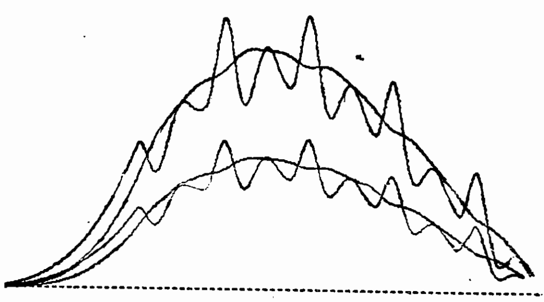


Fig 3

-b-

Velocity along the string, reflection on the bridge.
(Projections on Oy and Oz)



-c-

Velocities of the bridge masses against time.
(Projections on Oy and Oz)

In the case of a hammer, the condition of release, *Fc=0*, indicates that the string has pushed the hammer during a single attack, and we illustrate in fig.4 the situation of release in the case of a single contact, with the plot of velocity along the string after release. As before, the two sets of curves represent the projections of velocity on the O_{xy} and O_{xz} plan.

Note that the shape of the string is very different in that case from the case of the plucked string, and this will produce a difference of spectra between the two sounds. This difference is however not significant enough to characterize the attacks. More significant is the transient sound produced in the contact phase. Release occurs at a *0.37* fraction of sample. The maximum projected velocity is *1.1m/s*, and the values of M, m, K and α are *0.005kg*, *0.001kg*, *50000N/m* and *1rd*.

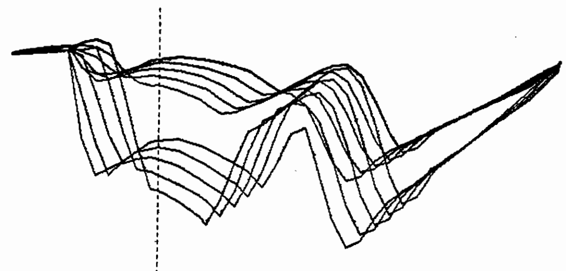


Fig 4

Velocity of a hammered string after release.
(Projections on Oy and Oz)

The contact of the finger with the fingerboard is obtained through a suddent increase of the mass of the finger when a condition $d>d_{max}$, where d is the displacement and the distance from the rest position of the string to the fingerboard has been reached. In this way we can avoid the introduction of an additive module simulating the behavior of the fingerboard. During the contact, the forces exerted on the fingerboard are equalized to avoid a shift of its position away from d_{max}. We show in the next figure the velocity bump resulting from the contact of a single left hand finger with the fingerboard, on a string initialy at rest.

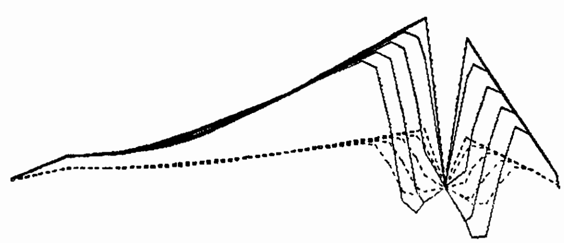


Fig 5

Contact of a finger with the fingerboard. Distribution of velocities.

The bow is pressed down in the Oz direction with the force F_{oz}, and pushed or pulled in the Oy direction with the force Foy. Both forces F_{oy} and F_{oz} are applied to the mass M and are controlled by the operator. Fig.6 shows the velocity of the bowed string projected in the Oy direction plotted against time. The bow has three bowhairs, and the two solid curves are corresponding to the points of contact between the string and the outer bowhairs. The dotted curve corresponds to the point of contact of the string with the inner bowhair. F_{oy} and F_{oz} are respectively $3N$ and $-4.7N$, the resulting speed of the bowhairs in the Oy direction is around $1.5 m/s$. This corresponds to the top value of the curves.

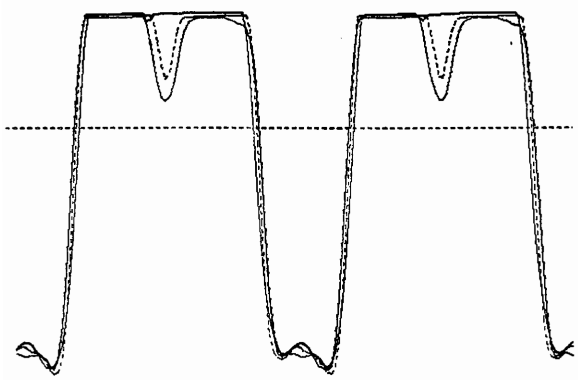


Fig 6
Bowed string. Velocities of the points of contact between the string and the bowhairs.

We have shown the simplest movements of the elementary exciters. It is possible to combine the excitation modules and to increase the sophistication of the input forces envelopes to obtain interesting behaviour of the model.

5. Conclusion

We have presented a first approach to the two dimensional model of a string excited by various objects. This is a basis for a digital investigation of the bowed string motion in the transverse plan. It would be interesting to study and interpret the behaviour of the model concerning the longitudinal stiffness of the bow hairs as well as the phenomena of coupling. Immediate improvements of the model would consist of the introduction of a more sophisticated description of coupling phenomena, and in the implementation of a general two mass stick/slide exciter model taking advantage of the analogies between all excitations situations. In addition, we would like to work on input parameter envelopes and involve the model in a musical production.

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