Body Modeling Techniques for String Instrument Synthesis

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Abstract

Techniques are described for obtaining efficient computational models of stringed instrument resonators such as guitar bodies. Warping the frequency axis to an approximate Bark scale using a first-order conformal map decreases the required body filter order by a factor of 5 to 10 for a given quality level. Structures which implement frequency-warped filters are described. Techniques are described for factoring a body resonator into its least-damped and most-damped modes so that the most-damped modes can be commuted with the string and stored in a shortened look-up table or approximated by a filtered noise burst for commuted synthesis.

1 Introduction

All acoustical string instruments have some kind of a body or soundboard which is the main source of sound radiation. Such an "amplification" is necessary since a vibrating string alone has a very limited capability to move air and radiate efficiently.

Another function of a body or a soundboard is to add coloration and reverberation to the radiated sound. Since acoustic amplification is more or less based on resonating structures, the spectral content of string output signals is thus changed. Due to a relatively slow decay of the body resonances, the temporal structure of the string signals is also changed to exhibit a reverberant quality.

The third acoustic role of a body or a soundboard is to create complex directivity patterns so that the intensity in the radiated sound field is a function of direction. Combined with room acoustics, this adds spaciousness to the perceived sound.

High-quality real-time sound synthesis of string instruments, based on physical modeling and DSP techniques, has been available for several years, [Smith 1983, Karjalainen and Laine 1991, Smith 1993a, Karjalainen *et al.* 1993]. The string itself is very efficiently modeled by digital waveguide filters and extensions of the Karplus-Strong model [Smith 1983, Karplus and Strong 1983, Jaffe and Smith 1983, Smith 1987, Smith 1992, Välimäki *et al.* 1996], and the excitation may be simply implemented as a wavetable or a set of wavetables. The body (or a soundboard), although in most cases a linear and time-invariant system, is found to be computationally very expensive if a full quality synthesis is desired.

Two kinds of efficient solutions are given in literature. The commuting of the body response and consolidation with the string excitation into a wavetable, [Smith 1993a, Karjalainen *et al.* 1993], is a very practical and straightforward method but lacks parametric control of body features. Body filters with a small or moderate number of resonating modes (e.g., [Smith 1983, Stonick and Massie 1992, Bradley and Stonick 1995]) provide another efficient method, but lacking the quality of full-featured instrument bodies. These two approaches can be blended to give parametric control over the most important resonances, while retaining the full richness of remaining resonances in wavetable form.

In this paper, we give a survey of body modeling techniques for model-based sound synthesis by covering methods from full-quality filter models to commuted synthesis, as well as hybrid methods, including new body filter designs. Both traditional and new filter structures are utilized, with discussion of estimation techniques for the calibration of model parameters from measured responses of acoustic instruments. The acoustic guitar is used as the primary example in these studies.

2 Body Impulse Responses

The acoustics of string instrument bodies and soundboards is a relatively widely studied topic [Fletcher and Rossing 1990].

A natural starting point for body modeling is to measure or compute the body impulse response. Figure 9a shows the first 100 milliseconds of the impulse response from the body of an acoustic guitar of classical design. The response was measured by tapping the bridge vertically with an impulse hammer (strings were damped) and by measuring the response with a microphone located one meter in front of the sound hole. Figure 2 depicts the magnitude behavior in the frequency domain for the full impulse response.

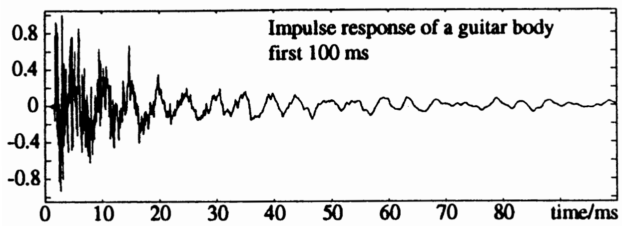

Figure 1: *An example of a body impulse response for an acoustic guitar.*

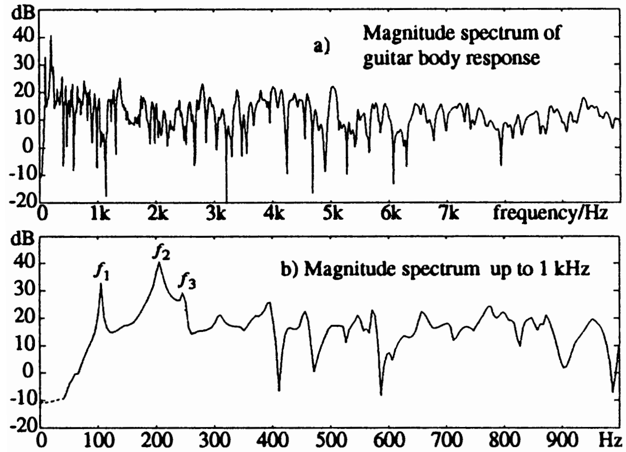

Figure 2: *Magnitude spectrum of the impulse response shown in Figure 1: a) full spectrum, b) low frequencies up to 1 kHz.*

Figure 1 suggests that the impulse response of the body is a combination of exponentially decaying sinusoids, i.e., signal components corresponding to the resonance frequencies of Figure 2. This must be the case if the transfer function of the body is linear, time-invariant, and finite order. In the frequency domain, the signal will be spectrally colored depending on how the harmonics of a string signal are located in relation to peaks or valleys of the body frequency response. It is obvious that both the magnitude spectrum shape and the temporal structure of the impulse response are important from the point of view of auditory perception.

A more comprehensive look at the body response characteristics may be achieved by a time-frequency representation which is more like the audio spectrum analysis of hearing. Figure 3 illustrates a short-time Fourier spectrum plot for the first 70 milliseconds of the response. A general increase in the decay rate at higher modal frequencies can be easily observed. We may also notice that resonance modes do not follow the simple exponential decay that would be linear curves on the dB scale. This ripple can be explained by multiple resonances that interact since they are closer together than the spectral resolution of the analysis. (At very low frequencies the alignment of window in relation to signal cycles may also be a source of ripple.[1])

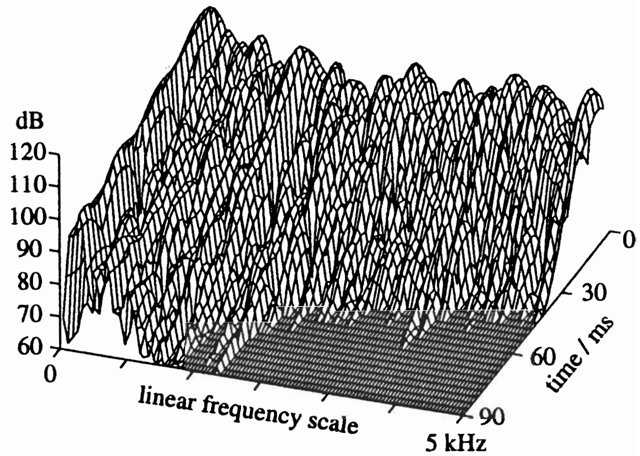

Figure 3: *Time-frequency plot of the guitar body response using short-time Fourier analysis. A Hamming window of 12 ms was used with a 3 ms hop size.*

3 Traditional Digital Filters as Body Models

The signal transfer properties from strings to radiated sound can be considered to be linear and time invariant (LTI) in most string instruments. In this case, an efficient way of implementing the body or soundboard for sound synthesis purposes is by means of digital filtering. Here we first consider the use of traditional filter structures—FIR and IIR filters—as direct implementations of body impulse responses. Then we introduce warped filter techniques and their application to body modeling.

[1] Actually, there is no perfect window size for this analysis since low frequencies require better frequency resolution and high frequencies better temporal resolution. A constant-Q or "wavelet" short-time spectrogram would be closer to an audio transform than the standard short-time Fourier transform.

FIR and IIR Filters as Body Models

A discrete-time LTI system may be represented using the z transform by

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{i=0}^{N} \beta_i z^{-1}}{1 + \sum_{i=1}^{P} \alpha_i z^{-1}} \qquad (1)$$

The most straightforward way to realize a known body response is to use the samples of a measured or computed impulse response as taps in an *FIR filter*, for which the coefficients α_i in (1) are zero. If N is larger than the impulse-response duration, this implements the desired convolution of the string output and the body response yielding a full accuracy body model to the extent that the whole audible portion of the response is available free of noise and artifacts.

An obvious problem with FIR modeling is the filter length N and thus the computational expense of the method. In the current guitar example, in order to cover a period of a single decay time-constant for the lowest mode, an FIR filter of order $N = 5000$ taps is needed when a sampling rate of 22 kHz is used. For a 60 dB dynamic range and full audio bandwidth, an FIR order of about $N = 25000$ is required! In practice, using only the first 100 ms of the response is found to be quite satisfactory, which means a 2200 tap filter for a 22 kHz sampling rate. Even this is computationally much more expensive than a model for six guitar strings, and it may be more than a modern signal processing chip can do in real time. The conclusion is that FIR models are generally impractical unless very efficient FIR hardware is available.

The sharply resonating and exponentially decaying components of a body response imply that IIR filters are more appropriate for efficient synthesis models than FIR filters. In order to see how well straightforward all-pole modeling works, we may apply autoregressive (AR) modeling using the autocorrelation method of *linear prediction* (LP) [Markel and Gray 1976] to the impulse response shown in Figure 1. This yields an all-pole filter model where the coefficients β_i of (1), for $i = 2$ to N, are equal to zero, are the predictor coefficients, and P is the order of the filter. Experiments with all-pole modeling have shown that, in our example, an order of $P = 500$ to 1000 is needed to yield a well matched temporal response. Lower filter orders, although relatively good from a spectral point of view, make the lowest resonances decay too fast. This can be addressed using a spectral weighting function (preemphasis) at the expense of the high-frequency fit.

The next generalization with traditional digital filters is to model the impulse response with a pole-zero (or ARMA) model. We have tried this using Prony's method [Parks and Burrus 1987, pp. 206–209]. The results show that this does not relax the requirements for the order P, but adding 100 zeros or so to the model improves the fit of the transient attack of the impulse response. From the point of view of auditory perception, however, this has only a small effect.

4 Warped Filters

Many filter design and model estimation methods allow for an error weighting function versus frequency in order to control the varying importance of different frequencies. Here, however, we take a different approach: Instead of an explicit weighting function we use frequency scale *warping* that is in principle applicable to any design or estimation technique. The most popular warping method is to use the bilinear conformal mapping [Churchill 1960, Parks and Burrus 1987] since it is the most general conformal mapping that preserves order. It can be used to warp the impulse response, frequency response, or transfer function polynomials. The warped FFT was introduced by [Oppenheim *et al.* 1971] and warped linear prediction was developed by [Strube 1980]. Generalized methods using the FAM functions have been developed by [Laine *et al.* 1994]. Smith has applied the bilinear mapping in order to design filter models for the violin body [Smith 1983].

The bilinear warping is realized by substituting unit delays by first-order allpass sections, i.e.

$$z^{-1} \leftarrow D_1(z) = \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}} \qquad (2)$$

This means that the frequency-warped version of a filter can be implemented by such a simple replacement technique. (Modifications are needed to make warped IIR filters realizable.) The transfer function expressions after the substitution may also be expanded to yield an equivalent IIR filter of traditional form. It is easy to show that the inverse warping can be achieved with a similar substitution but using $-\lambda$ instead of λ.

The usefulness of frequency warping in our case comes from the fact that, given a target transfer function $H(z)$, we may find a lower order *warped filter* $H_w(D_1(z))$ that is a good approximation of $H(z)$. For an appropriate value of λ, the bilinear warping can fit the psychoacoustic Bark scale, based on the critical band concept, relatively accurately [Strube 1980, Zwicker 1990]. For this purpose, an approximate formula for the optimum value of λ as a function of sampling rate is given in [Smith and Abel 1995]. For a sampling rate of 44.1 kHz this yields $\lambda = 0.7233$ and for 22 kHz $\lambda = 0.6288$. When using the warping techniques, the optimality of λ in a specific application depends both on auditory aspects and the characteristics of the system to be modeled.

Warped FIR (WFIR) Filters

The principle of a warped FIR filter (WFIR) is shown in Figure 4a, which may be written as

$$B_w(z) = B\left(D_1^{-1}(z)\right) = \sum_{i=0}^{M} \beta_i [D_1(z)]^i \quad (3)$$

A more detailed filter structure for implementation is depicted in 4b. As the latter form shows, a warped FIR is actually recursive, i.e., an IIR filter with M poles at $z = \lambda$, where M is the order of the filter.

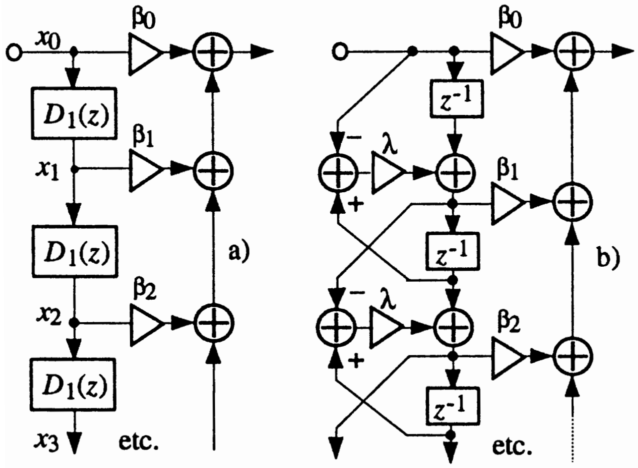

Figure 4: *Warped FIR modeling: (a) general principle, (b) detailed filter structure for implementation.*

A straightforward method to get the tap coefficients β_i for a WFIR filter is to warp the original impulse response and to truncate it by "windowing" the portion that has amplitude above a threshold of interest. (Notice that the bilinear mapping of a signal by (2) is linear but not shift invariant [Strube 1980]). There exist various formulations for computing a warped version of a signal [Strube 1980, Smith 1983, Laine *et al.* 1994]. An accurate and numerically stable method is to apply the FIR filter structure of Figure 4a or 4b with tap coefficients being the samples of the signal to be warped. When an impulse is fed to this filter, the response will be the warped signal. Figure 5 shows the warped ($\lambda = 0.63$) guitar body response as a time-frequency plot for comparison with the original one in Figure 3. As can easily be seen, the warping tends to balance the decay rates and resonance bandwidths for all frequency ranges.

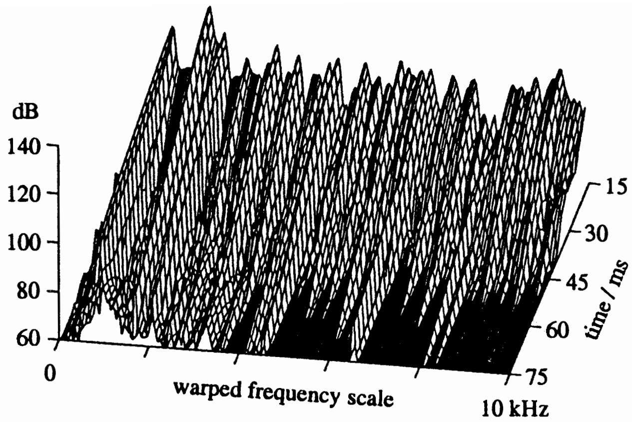

Figure 5: *Time-frequency plot of warped guitar body response. A Hamming window of 24 ms* was used with a 3 ms* hop size, where * denotes warped time.*

Warped IIR (WIIR) Filters

When linear prediction is applied to a warped impulse response it yields a warped all-pole filter. Other methods for warped LP analysis (WLP) are studied in [Laine *et al.* 1994], including an efficient way to compute warped autocorrelation coefficients $r_w(i)$ directly from the original signal. This is based on the warped delay-line structure of Figure 4a, whereby

$$r_w(i) = \sum_n x_0(n) x_i(n) \quad (4)$$

is summed over a time interval or window of interest. After that, the warped predictor coefficients are achieved from warped autocorrelation coefficients as usual [Markel and Gray 1976] to yield a filter model

$$H_w(D_1(z)) = \frac{G_w}{1 + \sum_{i=1}^{R} \alpha_i [D_1(z)]^i} \quad (5)$$

A somewhat surprising observation is that the filter structure of (5) cannot be implemented directly since there will be delay-free loops in the structure for $\lambda \neq 0$. (Of course the bilinear mapping, inherent in the filter structure, may be expanded at design time to yield a normal IIR filter. This, however, can lead to numerical difficulties if the filter order exceeds about 20 to 30 in 64-bit double precision floating-point.) Figure 6 depicts two realizable forms of WIIR filters. Strube [1980] suggested an approximation in which low-pass sections are used instead of allpass sections (see Figure 6a). In practice, it works only for low orders and warping values λ due to excessive high-frequency attenuation otherwise. The version in Figure 6b is more general for warped pole-zero modeling but it has also a more complex structure. The coefficients σ_i can be computed from coefficients α_i of (5) using a recursion or matrix operation as shown in [Karjalainen 1996].

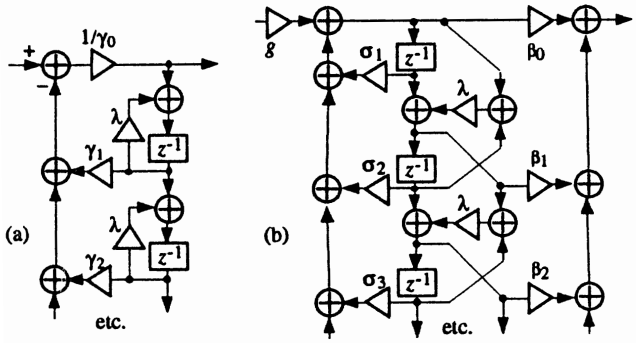

Figure 6: *Filter structures for implementation of warped IIR filters: (a) lowpass structure that does not work with high orders, and (b) modified allpass structure (warped pole-zero filter).*

Body Modeling with Warped Filters

For a given quality level, the warped filter strategy using a Bark-scale warping yields a reduction in filter order by a factor 5 to 10. This means that a WFIR of order M less than 500 gives results similar to those of an FIR filter of order $N = 2000$. For warped all-pole filters, an order of about $R = 100$ is equivalent to normal IIR order of about $P = 500$ to 1000. A body filter resulting from using Prony's method for a warped filter, with $M = 50$ to 100 and $R = 100$ to 200, will represent both transient and decay properties relatively well, although a Bark warping ($\lambda = 0.63$, sampling rate 22 kHz) has a tendency to shorten the impulse response of the highest frequencies a bit too much.

The reduction in filter order due to warping translates to a similar reduction in computational complexity only if unit delays and allpass sections are equally complex computationally. In reality, many digital signal processors have hardware support to run ordinary FIR and IIR filters very efficienly. The complexity of the first-order allpass section used as a warped delay is several times higher than that of a unit delay. Due to this complexity of realization, reduction in computational cost with warped all-pole and IIR structures remains smaller than indicated by the order savings. In a typical case, for the TMS320C30 floating-point signal processor, a WIIR body filter model is only about two times faster than an equivalent normal IIR filter. On the other hand, if the warped filters are converted to conventional structures (using ultra-high precision computations, e.g., in Mathematica), the extra complexity disappears; since the warping preserves order, it does not have to increase filter complexity except when the number of poles is different from the number of zeros in which case the warping (or unwarping) will introduce new poles or zeros so that their number is the same.

A nice benefit of implementing the body filter in warped form is that λ is available as a qualitative *body size* parameter. The size parameter can be modulated to obtain new kinds of effects, or it can be used to "morph" among different members of an instrument family.

5 Body-Model Factoring

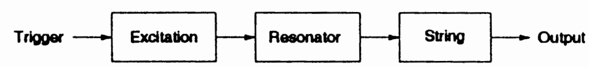

Figure 7: *Schematic model of a stringed instrument in which the string and resonator are commuted relative to their natural ordering.*

Commuted synthesis is a technique in which the body resonator is commuted with the string model, as shown in Fig. 7, in order to avoid having to implement a large body filter at all [Smith 1993a, Karjalainen *et al.* 1993]. In commuted synthesis, the excitation (e.g., plucking force versus time) can be convolved with the resonator impulse response to provide a single aggregate excitation signal. This signal is short enough to store in a look-up table, and a note is played by simply summing it into the string.

A valuable way of shortening the excitation table in commuted synthesis is to *factor* the body resonator into its *most-damped* and *least-damped* modes. The most-damped modes are then commuted and combined with the excitation in impulse-response form. The least-damped modes can be left in parametric form as recursive digital filter sections. Advantages of this factoring include the following:

- The excitation table is shortened.
- The excitation table signal-to-quantization-noise ratio is improved.
- The most important resonances remain *parametric*, facilitating real-time control.
- Multiple body outputs become available.
- Resonators may be already available in a separate effects unit, making them "free."
- A memory vs. computation trade-off is available for cost optimization.

Mode Extraction Techniques

The goal of resonator factoring is to identify and remove the least-damped resonant modes of the impulse response. In principle, this means ascertaining the precise resonance frequencies and bandwidths associated with each of the narrowest "peaks" in the resonator frequency response, and dividing them out via inverse filtering, so they can be implemented separately as resonators in cascade. If in addition the amplitude and phase of a resonance peak are accurately measurable in the complex frequency response, the mode can be removed by complex spectral subtraction (equivalent to subtracting the impulse-response of the resonant mode from the total impulse response); in this case, the parametric modes are implemented in a parallel bank as in [Bradley and Stonick 1995]. However, in the parallel case, the residual impulse response is not readily commuted with the string.

Various methods are available for estimating the mode parameters for inverse filtering:

- Amplitude response peak measurement
- Weighted digital filter design
- Linear prediction
- Sinusoidal modeling
- Late impulse-response analysis

In the body factoring example presented below, the frequency and bandwidth of the main Helmholtz air mode were measured manually using an interactive spectrum analysis tool. It is also easy to automate peak-finding in FFT magnitude data, as is routinely done in sinusoidal modeling, discussed further below.

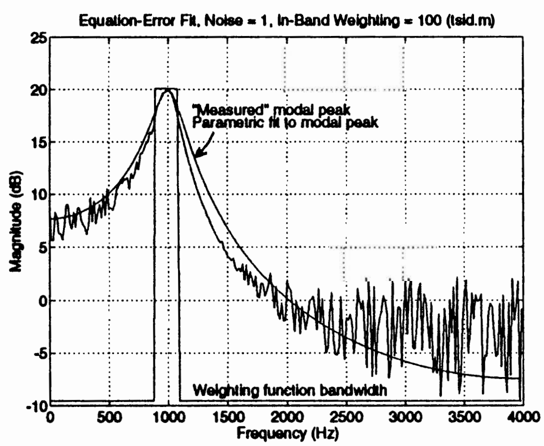

Figure 8: *Illustration of one way to determine the parameters of a least-damped resonant mode.*

Many methods for digital filter design support spectral weighting functions that can be used to focus in on the least-damped modes in the frequency response. One is the *weighted equation-error* method which is available in the matlab `invfreqz()` function. Figure 8 illustrates use of it in a simple synthetic example with only one frequency-response peak in the presence of noise. Unless the weighting function is very tight around the peak, its bandwidth tends to be overestimated.

Another well known method for converting the least-damped modes into parametric form is Linear Prediction (LP) followed by polynomial factorization to obtain resonator poles. LP is particularly good at fitting spectral peaks due to the nature of the error criterion it minimizes [Smith 1983, pp. 43–50]. The poles closest to the unit circle in the z plane can be chosen as the least-damped part of the resonator. It is well known that when using LP to model spectral peaks for extraction, orders substantially higher than twice the number of spectral peaks should be used.

Another way to find the least-damped mode parameters is by means of a *sinusoidal model* of the body impulse response such as is often used to determine the string loop filter in waveguide string models [Smith 1993b, Välimäki *et al.* 1996]. (See, e.g., [Serra and Smith 1990] for further details on sinusoidal modeling and supporting C software). The sinusoidal amplitude envelopes yield a particularly robust measurement of bandwidth. Theoretically, the modal decay should be exponential. Therefore, on a dB scale, the amplitude envelope should decay *linearly*. Linear regression can be used to fit a straight line to the measured log-amplitude envelope of the impulse response of each long-ringing mode. Note that even when amplitude modulation is present due to mode couplings, the ripples tend to average out in the regression and have little effect on the slope measurement. This robustness can be enhanced by starting and ending the linear regression on local maxima in the amplitude envelope.

All methods useable with inverse filtering can be modified based on the observation that late in the impulse response, the damped modes have died away, and the least-damped modes dominate. Therefore, by discarding initial impulse-response data, the problem in some sense becomes "easier" at the price of working closer to the noise floor.

High-Frequency Modes ≈ Noise

Figure 9b suggests that the many damped modes remaining in the shortened body impulse response may not be clearly audible as resonances since their decay time is so short. This is confirmed by listening to shortened and spectrally flattened body responses which sound somewhere between a click and a noise burst. These observations suggest that the shortened, flattened, body response can be replaced by a psychoacoustically equivalent *noise burst*, as suggested for modeling piano soundboards [Van Duyne and Smith 1995]. Thus, the signal of Fig. 9b can be synthesized qualitatively by a white noise generator multiplied by an amplitude envelope. In this technique, it is helpful to use LP on the residual signal to flatten it. As a refinement, the noise burst can be time-varying filtered so that high frequencies decay faster [Van Duyne and Smith 1995]. Thus, the stringed instrument model may consist of *noise generator → excitation amplitude-shaping → time-varying lowpass → string model → parametric resonators → multiple outputs*. In addition, properties of the physical excitation may be incorporated, such as comb filtering to obtain a virtual pick or hammer position. Multiple outputs provide for individual handling of the most important resonant modes and facilitate simulation of directional radiation characteristics [Välimäki *et al.* 1996].

Body Factoring Example

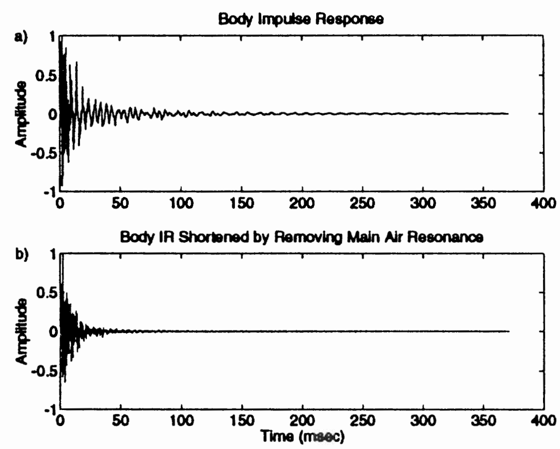

Figure 9: *Impulse response of a classical guitar body before and after removing the first peak (main air resonance) via the inverse filter defined by Eq. (6), with* $a_1 = -1.9963$ *and* $a_2 = 0.9972$.

Figure 9a shows the measured guitar-body impulse-response data plotted in Fig. 1 but extended to its full duration. Figure 9b shows the same impulse response after factoring out a single resonating mode near 100 Hz (the main Helmholtz air mode). As can be seen, the residual response is considerably shorter than the original.

Figure 10a shows the measured guitar-body amplitude response after warping to an approximate Bark frequency axis. Figure 10b shows the Bark-warped amplitude response after the main Helmholtz air mode is removed by inverse filtering. On the Bark frequency scale, it is much easier numerically to eliminate the main air mode.

The modal bandwidth used in the inverse filtering was chosen to be 10 Hz* which corresponds

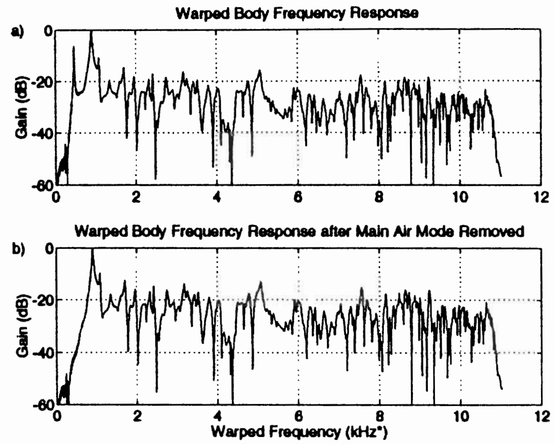

Figure 10: *Normalized Bark-warped amplitude response of a classical guitar body before and after removing the first peak (main air mode) via Eq. (6), with* $a_1 = -1.9801$ *and* $a_2 = 0.9972$.

to a Q of 46 for the main air mode. If the Bark-warping is done using a first-order conformal map [Smith and Abel 1995], its inverse preserves filter order [Smith 1983, pp. 61–67]. Applying the inverse warping to the parametric resonator drives its pole radius from 0.99858 in the Bark-warped z plane to 0.99997 in the unwarped z plane.

Note that if the inverse filter consists only of two zeros determined by the spectral peak parameters, other portions of the spectrum will be modified by the inverse filtering, especially at the next higher resonance, and in the linear trend of the overall spectral shape. To obtain a more *localized* mode extraction (useful when the procedure is to be repeated), we define the inverse filter as

$$H_r(z) \triangleq \frac{A(z)}{A(z/r)} \triangleq \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 + a_1 r z^{-1} + a_2 r^2 z^{-2}} \qquad (6)$$

where $A(z)$ is the inverse filter determined by the peak frequency and bandwidth, and $A(z/r)$ is the same polynomial with its roots contracted by the factor r. If r is close to but less than 1, the poles and zeros substantially cancel far away from the removed modal frequency so that the inverse filter has only a local effect on the frequency response. In the computed examples, r was arbitrarily set to 0.9, but it is not critical.

6 Conclusions

A number of techniques were discussed for improving the quality of virtual stringed instrument resonators and reducing implementation cost. These methods make it possible to simulate high quality stringed instruments using inexpensive software algorithms. As an example, a basic classical guitar model requires less than two percent of a 120

MHz Pentium processor for each actively vibrating string. Therefore, such models are practical today, even for low-cost multimedia applications. For the future, physical models suggest how to make good use of increased processing power as it comes along.

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