PHYSICAL MODELS OF INSTRUMENTS
A MODULAR APPROACH, APPLICATION TO STRINGS

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Abstract

This paper describes our work on the simulation of physical instruments, including a resonator, a string, and exciters like bows , hammers, plectra or fingers, as well as our effort toward a modular conception aimed at a better use and extension of the simulation programs.

1. Introduction

The essential characteristics of synthesis by physical models are that it offers a pre-organized material, and that physical rules which are involved in normal vibrating motion produce this structuring.

The pre-organization of the material is today widely considered as necessary because the raw floating sample does not in every case stimulate the composer efficently, and may produce unexpected musical results. Therefore, the creative energy of our composers is often exhausted before even being productive,and this is where pre-organization is useful. It offers the composer an environnment of a priori organized objects. Physical-model synthesis is, with respect to this first level of organisation, an original solution in which elementary structures are made with mechanical and acoustic rules.

Physical rules produce objects of the physical space, and, the interaction of these objects makes sounds which are strongly bound to their natural correlates; the physical description gives, for a sound, a reference to the producing mode. Our work takes place in the time domain much more than in frequency domain, in other words, the physical-model synthesis is more concerned with the cause than with the effect. The problem is that the first level of structure has to be wide and general enough to be used in a musical context, and precise enough to provide the objects with the necessary features; and one might regret, in this respect, the stiffness of physical rules.

We expect that this environment will stimulate the composers in the realm of structure, because the bonds between elementary objects produced by the same cause are a useful tool in constructing musical forms; and in the realm of expression as well because the perception mechanisms associate a given effect with a virtual cause whose characteristics are determinant in the expressive features of the effect. In many cases the subjective power, the weight of an object is closely correlated to the physical energy involved in its production.

Our sources for strings derive from the work of Smith [Smith83] on the violin, Weinreich [Weinreich74] and Boutillon [Boutillon99] on the piano. We must refer as well to the important work of Cadoz [Cadoz79] on physical model-synthesis.

2. Resonator

The direct approach for describing the string is to represent it as a series of springs and masses [Lazarus72] [Cadoz79]. We know that in good approximation such a physical system behaves like the physical mass continuum of a string. The integration of the equation of dynamics for each mass and each time step gives the position of the masses. It is possible, then, to introduce losses of energy in the string, as well as bending stiffness etc.. The method is physically very satisfying but its drawback is that it implies the calculation, for each time step, of the position of each mass, whereas only the masses concerned by the excitation, and the masses close to the bridge are really significant in a musical context, because they are highly responsible for the produced sound.

Another method consists in the representation of the resonator by a double delay line, each line supporting a wave propagating in the opposite direction, and reflected at the end of the line into the other line. A single delay line was the center element of the Karplus-Strong algorithm [Karplus99] as well as of the violin model of Smith, but both approaches involved more signal processing techniques than proper physical description, and were consequently limited in terms of excitations. We adapted some of Smith's solutions to our resonator, being always eager to preserve the physical significance of the model.

The movements involved are unidimentional, and the physical method chosen for calculating the shape of the free oscillating string at each time step is a method of integration by arbitrary functions. The shape of the string at any time is given by the sum of two waves $\varphi 1$ and $\varphi 2$ moving in opposite directions.

$$\varphi_1 = \left[\frac{1}{2}\varphi - \frac{1}{2c}\int \psi(x)\partial x \right](x - ct)$$

$$\varphi_2 = \left[\frac{1}{2}\varphi + \frac{1}{2c}\int \psi(x)\partial x \right](x + ct)$$

where $\varphi(x)$ and $\psi(x)$ are the distributions of displacement and of speed respectively at the initial time, and c the velocity of transverse waves on the string. Thus, each line supports one of the functions φ_1 and φ_2, and a simple translation of these functions gives the shape of the string. The drawback of the method is that we cannot introduce losses or stiffness along the string, and we have to simulate them at the reflection points. The period of transverse waves on the string is dependent on the longitudinal tension F_x, the lengh l and the lineic mass ε of the string. In our case, it depends on the sampling rate R and the total number of cells N. For an ideal string, the variation of tension or lengh should not affect the timbre. In other words, for an ideal string, the fundamental frequency is controlled indifferently by tension or lengh. We have the same situation in the model with possible variations of two parameters r and N. For trivial reasons, we choose to adjust the fundamental pitch of each string with the parameter N. Note that dynamic variations of pitch during performance will be produced by an excitation representing the left hand finger in contact with the string and the fingerbroad.

$$T = 2\frac{N}{R} = 2l\sqrt{\frac{\varepsilon}{F_x}}$$

We use a set of two allpass filters to generate delays shorter than a sample. We need two filters in order to preserve the symetry of the lines and to be able to interpolate between a situation with $2N$ cells to a situation with $2(N+1)$ cells, using the filters so that they do not affect the timbre. The transfer function $H_a(z)$ and the delay $\tau(C)$ are dependent on a parametre C. For values of C included in $[-1,0]$, the function $H_a(z)$ is heavily dependent on frequency, and, consequently the string becomes dispersive. The produced effect, a dilation of the spectrum for high frequencies, is qualitatively the same as the effect of bending stiffness. The resonator is then provided with the properties of a metallic or wood bar. This makes very easily homegenous scales of inharmonic sounds, with the whole range of possible excitations. We have therefore included in the model a module consisting of a pair of additional allpass filters.

$$H_a(z) = \frac{C + z^{-1}}{1 + Cz^{-1}} \qquad \tau(C) = \frac{1 - C}{1 + C}$$

The absorption for high frequencies is obtained with a single zero low-pass filter at the bridge. This is a first approximation of a physical bridge which involves few calculations. Through the parameter ρ, we can control the relative absorption of high and low partials, and simulate strings of various kinds. The global absorption of the string is controlled with a real reflection coefficient at the nut.

$$H_p(z) = (1 - \rho) + \rho z^{-1}$$

The significant points for the user are the last cells of the delay lines at the bridge. These two cells are in fact the output and input of the string. Output is sound, and input is usually information coming from other strings through the bridge. As a result, for the user, the string is an input/output module, provided with a dynamic control system (*fig* 2). Note that the integration by arbitrary functions does not imply any discretisation of the mass along the string, and that the string is, in our model, a continuum.

3. Excitations

The general mechanical system used to model fingers, plectrums, hammers or dampers is represented in *fig* 2. The mass m is in contact with *one* point of the string and the operator applies a force F_o to the mass M. The two masses are bound with a spring k and we have included losses B and C. In the case of a harpsichord, the masses M and m may represent the mass of the jack and of the feather which plucks the string; for the piano, they are the masses of the hammer and of the felt, and one can easily imagine in the case of the finger, the same kind of mechanical model, where k represents the stiffness of the pad.

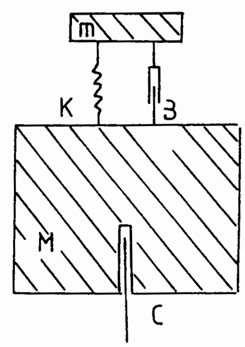

Figure 1

Mechanical model for the exciter:
Two masses m and M, a spring K and resistances B and C. The mass m is in contact with the string, and the operator applies a force F_o on mass M.

During the excitation, we consider that the signals leaving and arriving at the point of excitation are the incident transmitted and reflected waves from each side of this point. If we consider then $y_1(t)$, $y_2(t)$, $z_1(t)$ and $z_2(t)$, the waves on the resonator from each side of the mass m, and $y(t)$ the abcissa of the mass m, we get the expression of the force applied by the string to the mass m and the set of differential equations for the abcissa $Y(t)$ and $y(t)$ of the masses M and m where $z_1(t)$ and $z_2(t)$ are the instantaneous inputs at the point of excitation. Integration of this set of equations gives at each time step the values of $Y(t)$, $y(t)$, $y_1(t)$ and $y_2(t)$, as a function of the instantaneous inputs $z_1(t)$ and $z_2(t)$.

$$F_c = -A\frac{\partial y}{\partial t} + A\frac{\partial(z_1 + z_2)}{\partial t}$$

$$\begin{cases} \dfrac{\partial^2 Y}{\partial t^2} = \dfrac{k}{M}(y(t) - Y(t)) - \dfrac{B}{M}\dfrac{\partial}{\partial t}(Y(t) - y(t)) - \dfrac{C}{M}\dfrac{\partial Y}{\partial t} + \dfrac{F_0}{M} \\ \dfrac{\partial^2 y}{\partial t^2} = \dfrac{k}{m}(Y(t) - y(t)) + \dfrac{B}{m}\dfrac{\partial}{\partial t}(Y(t) - y(t)) - \dfrac{A}{m}\dfrac{\partial y}{\partial t} + \dfrac{A}{m}\dfrac{\partial}{\partial t}(z_1(t) + z_2(t)) \end{cases}$$

Further, we found it interesting to have a simple on-mass model for the excitation. It is actually identical with the two-mass model provided with a spring of infinite stiffness. For a given sampling rate, the quotient of mechanical stiffness by mass is limited in the same way as frequency. The set of differential equations reduces then to a single equation describing the movement of the single mass m in contact with the string.

$$\frac{\partial^2 y}{\partial t^2} + \frac{A+B}{m}\frac{\partial y}{\partial t} = \frac{A}{m}\frac{\partial}{\partial t}(z_1 + z_2) + \frac{F_0}{m}$$

For the bow, we have adopted the well known functional description using the static friction characteristic [Lazarus72], and describing the motion of the bowed string as a free oscillating motion. The expressions of the force F_c applied from the string to the bow and of the string's continuity at the point of contact lead to the set of equations giving the outputs $y_1(t)$ and $y_2(t)$ as a function of the instantaneous inputs $z_1(t)$ and $z_2(t)$.

$$\begin{cases} y_1(t) = \displaystyle\int_0^t \frac{cF_c}{2F_z}\partial t + z_2(t) \\ y_2(t) = \displaystyle\int_0^t \frac{cF_c}{2F_z}\partial t + z_1(t) \end{cases}$$

In all the cases, we may represent the exciter as a double input/output module (*fig.* 2) which computes the instantaneous outputs as a function of instantaneous inputs at the point of excitation, and which is not concerned with anything else happening on the string. This representation has two advantages: The first point is that the exciter takes into account all the past information present in the string as it begins to act; in other words, we may play two pizzicati one after the other in a natural way, which was not the case in previous models. The second point is that we may have many excitations on the same string at the same time, each one of them computing as if it was alone, i.e. rather independently from the others, and this is a great help in modular conception. Besides, two pairs of allpass filters allow the exciter to act at any place on the string, and to move dynamically during the excitation.

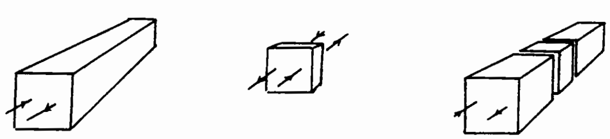

Figure 2

Resonator (input/output) and exciter (double input/output) modules.

4. **Results**

For the plucked string, *fig.* 3 shows the movement of the plectrum during the excitation. The dotted straight line is the position of the string at rest, dotted and solid curves are the trajectories of the masses M and m. From the contact to the moment of release, the mass m pulls the string away from its equilibrum position, and the sound produced in this time interval is a good approximation of the natural attack sound. Time of release is determined with a condition $F_c < F_{max}$ on the force applied by the string to the plectrum. After release, the mass m oscillates around M. We have chosen a rapid plucking in order to make evident that the shape of the string at release is, in the case of a dynamic attack, quite different from the well known triangle shaped string of ideal plucking. *Fig.* 4*a* shows the shape of the string after release and 4*b* the same string after a twelfth of the fundamental period.

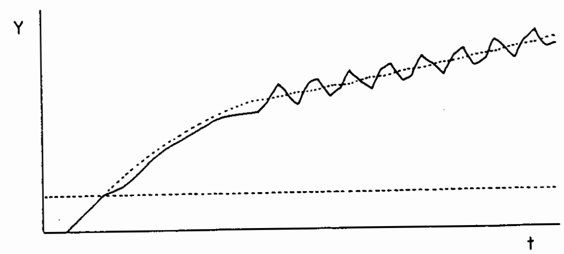

Figure 3

Trajectory of the plectrum:
The dotted straight line is the $y = 0$ line, dotted and solid curves are the trajectories of the masses M and m. From the contact to the moment of release, the mass m pulls the string away from its equilibrum position. After release, m and M oscillate around each other.

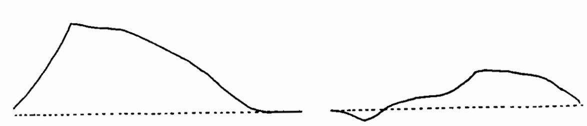

Figure 4

Shape of the plucked string:
Moment of release (left) and a twelfth of period later (right).

For the hammer,the trajectory of the contact point *fig* 7. (solid curve) shows the effect of the first and second reflected waves on the bridge, which try to push the hammer away. They do not achieve this, in fact,and we have to wait until the third reflected wave. At this moment, the hammer has lost its energy and is pushed away by the string. This appears clearly in the plot of the force applied by the string to the hammer during the interaction (*fig* 5). On the left side is displayed an experimental measurement performed by Boutillon [Boutillon84], and of the right results given by the model. In the modelized situation, you can see at the end of the interaction a second interaction. This means that after the first release, the hammer which has lost its energy is moving slowly, and the string which is faster pushes it away a second time. In any case, the second interaction is not very important with respect to the first, and it cannot be seen either in the experimental measurement or in the model trajectories. The corresponding shapes of the string are given in *fig*. 6.*a*.*b*.*c*.*d* for the first contact, first release, second contact and second release. Further mesurements by Boutillon helped us in the adjustment of the mechanical parameters.

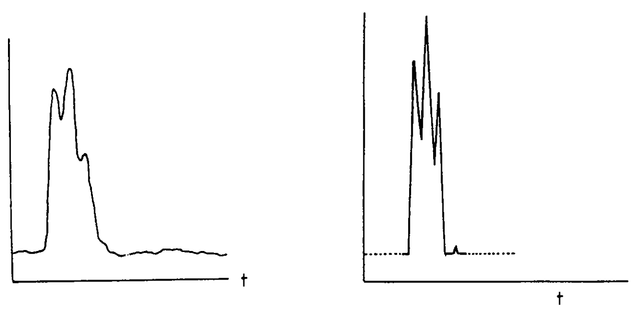

Figure 5

Acceleration of the hammer:
Second interaction produced by the third reflected waves after the first release.

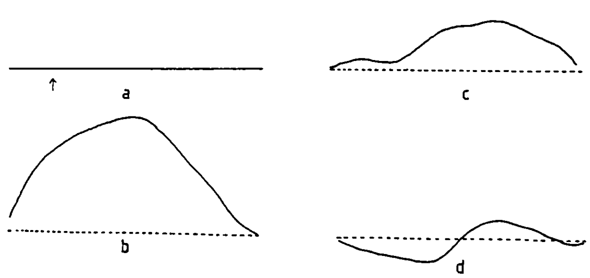

Figure 6

Shape of the hammered string:
fig *a*.*b*.*c*.*d* for the first contact, first release, second contact and second release.

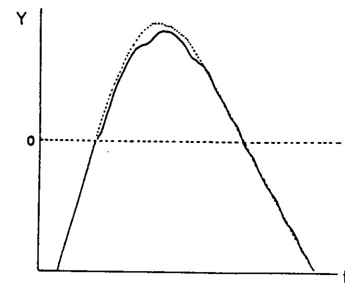

Figure 7

Trajectory of the Hammer:
Effect of the first and second reflected waves on the trajectory of the mass m.

We have included modules representing loads of different kinds (masses, springs, losses) for the string, in order to provide the model with a wider range of inharmonic sounds. The mechanical properties and the position of the loads can vary dynamically [Adrien85].
For the finger, the different mechanical parameters are adjusted by ear and with patience, and the contact with the fingerbroad is obtained in giving an important value to the mass m. At this moment, the finger, the string and the instrument are in contact, and the string is actually loaded with an important mass at this point. We considered that the damper of the piano, responsible for the notes cut off, is a very soft and absorbing hammer pushed against the string.
The bow model gives the well known results of the Helmoltz motion. We are not yet satisfied with its musical qualities, and intend to model a more realistic one in order to account for sticking and sliding friction [Cremer84]. This is possible within our double input/output representation of the excitation.

5. Conclusion

We have concentrated our work on excitation, and we wish to investigate further the properties of the resonator, in order to obtain, for example, a more realistic bridge or stiff string. In every situation, a compromise between physical realism and computation costs has to be found. The synthesisor is then to be scheduled in a process-oriented programming environment [Cointe84] at Ircam. We expect that high level control of these physical objects will produce interesting situations in a musical context.

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