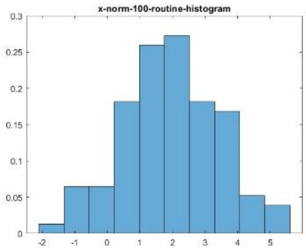
# Project Report (ECE 514) Part 1

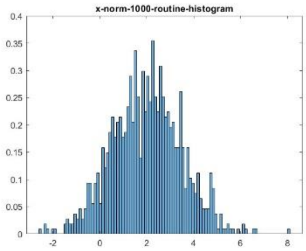
Somshubhra Roy
sroy22@ncsu.edu
200483305

1.

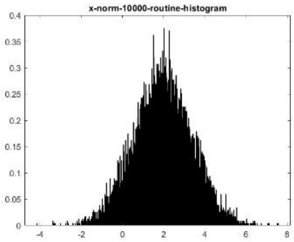
- R.V.s were generated according by both MATLAB routines as well as rejection method using a Uniform (0,1) distribution as the generating function for sample size T=100,1000,10000 observations using the following distributions: Normal (2,2), Uniform (2,4), Exponential (2).The code is attached below.
- Histograms and parameters for each case of MATLAB routine generation and rejection method simulation of R.V.s:
  - MATLAB routine generated:



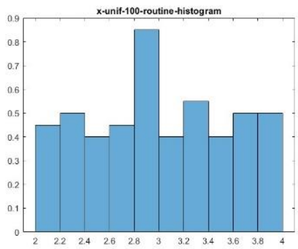

```
Normal
distribution
mu = 1.93893
sigma = 1.46218
```



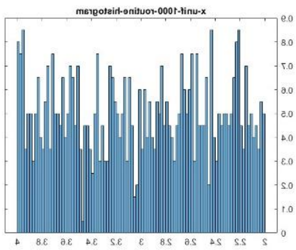

```
Normal
distribution
mu = 2.00671
sigma = 1.48065
```



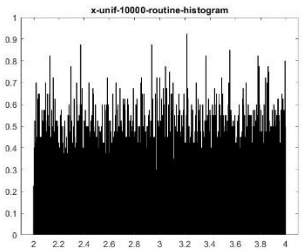

```
Normal
distribution
mu = 1.97787
sigma = 1.41098
```



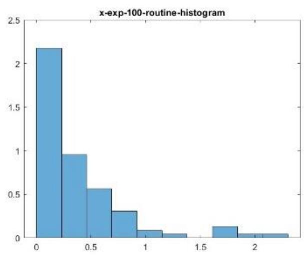

```
Uniform
distribution
a = 1.995877
b = 3.999427
```



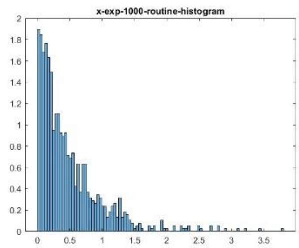
```
Uniform
distribution
a = 1.998317
b = 4.000669
```



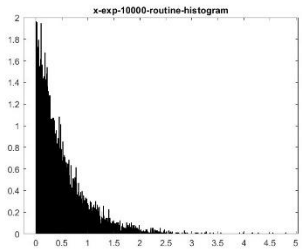

```
Uniform
distribution
a = 1.999896
b = 4.000050
```




```
Exponential
distribution
mu = 0.390299
```

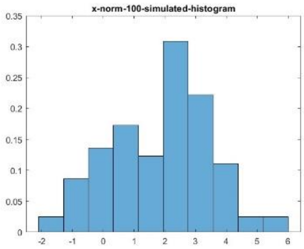



```
Exponential
distribution
mu = 0.485077
```

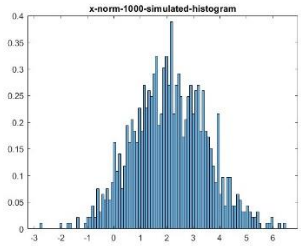



```
Exponential
distribution
mu = 0.501019
```

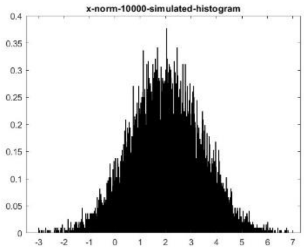
- Rejection Method Simulated:



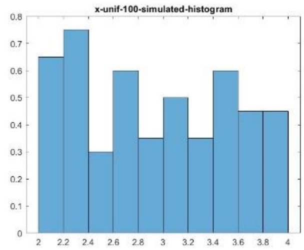

Normal
distribution
mu = 1.87097
sigma = 1.60734



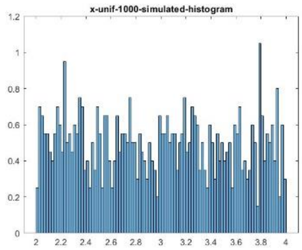

Normal
distribution
mu = 2.10932
sigma = 1.38467



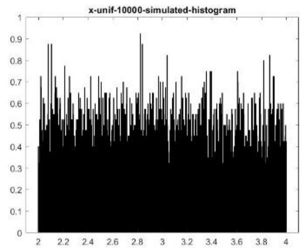

Normal
distribution
mu = 2.00971
sigma = 1.40842



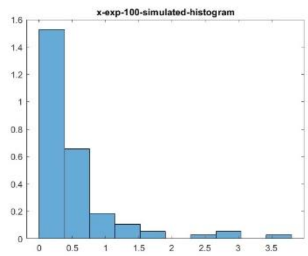

Uniform
distribution
a = 1.991417
b = 3.993401



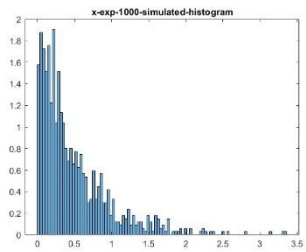

Uniform
distribution
a = 2.007274
b = 4.001426



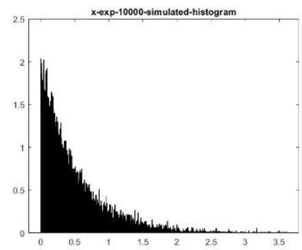

Uniform
distribution
a = 1.999823
b = 3.999635




Exponential
distribution
mu = 0.517511




Exponential
distribution
mu = 0.483193




Exponential
distribution
mu = 0.504432

- For Normal Distribution:
  Given,
  Mu=2, var=2
  Sigma = sqrt(2)=1.414

  For Routine generated RVs,
  At T=100,
  Normal distribution
  mu = 1.93893
  sigma = 1.46218
  At T=1000,
  Normal distribution
  mu = 2.00671
  sigma = 1.48065
  At T=10000,
  Normal distribution
  mu = 1.97787
  sigma = 1.41098

  For Rejection method simulation of R.V.s,
  At T=100,
  Normal distribution
  mu = 1.87097
  sigma = 1.60734
  At T=1000,
  Normal distribution
  mu = 2.10932
  sigma = 1.38467
  At T=10000,
  Normal distribution
  mu = 2.00971
  sigma = 1.40842

  For Uniform Distribution:
  Given,
  a=2
  b=4

  For Routine generated RVs,
  At T=100,
  Uniform distribution
  a = 1.995877
  b = 3.999427
  AT T=1000,
  Uniform distribution
  A = 1.998317
  b = 4.000669
  At T=10000,
  Uniform distribution
  a = 1.999896
  b = 4.000050

  For Rejection method simulation of R.V.s,
  At T=100,
  Uniform distribution
  a = 1.991417
  b = 3.993401
  At T=1000,
  Uniform distribution
  a = 2.007274
  b = 4.001426
  At T=10000,
  Uniform distribution
  a = 1.999823
  b = 3.999635

For Exponential Distribution:
Given,
Lambda=2
Mu=1/2=0.5

For Routine generated RVs
At T=100,
Exponential distribution
mu = 0.390299
At T=1000,
Exponential distribution
mu = 0.485077
At T=10000,
Exponential distribution
mu = 0.501019

For Rejection method simulation of R.V.s,
At T=100,
Exponential distribution
mu = 0.517511
At T=1000,
Exponential distribution
mu = 0.483193
At T=10000,
Exponential distribution
mu = 0.504432

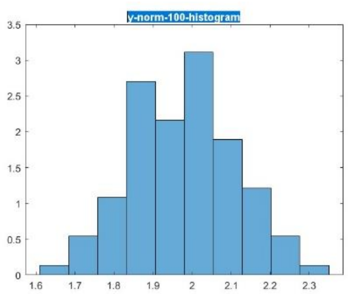
Thus, it can be observed that the empirically estimated parameters differ from the theoretical parameters.

- The difference decreases by increasing the total number of samples, increasing the range to match the theoretical range of the distribution, and increasing the upper boundary of the envelope formed by the generating distribution such that it is greater than the maximum value of the pdf of the simulated distribution. These are the factors that contribute to the difference of the estimated and theoretical parameters.

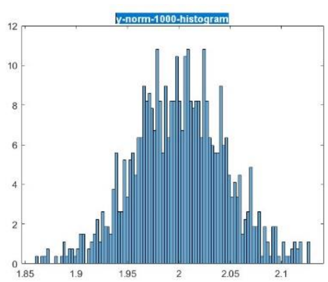
2.

- $Y_i = \frac{1}{T}\sum_i X_i$ , $i = 1,2,....,$T
  For T = 100, 1000, 1000
  The corresponding Histogram plots for Y are:

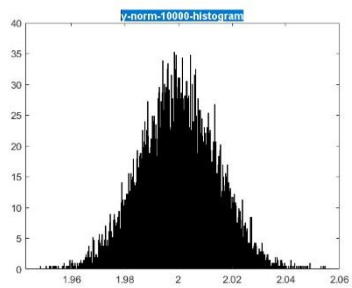
  I. When X ~ Normal (2,2):




T=100

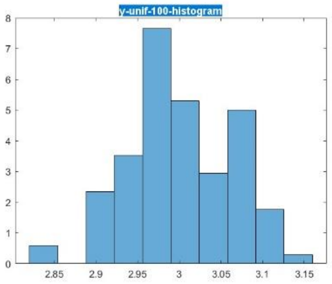



T=1000

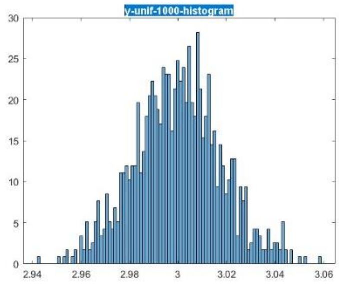



T=10000

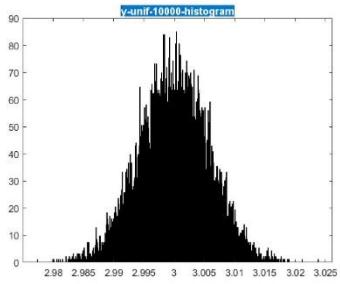
II. When X ~ Uniform (2,4):




T=100

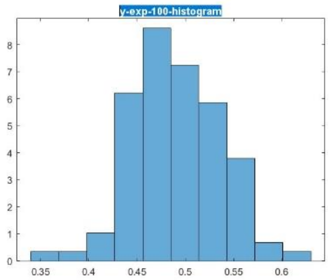



T=1000

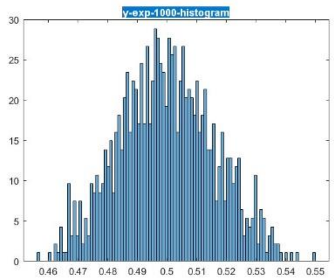



T=10000

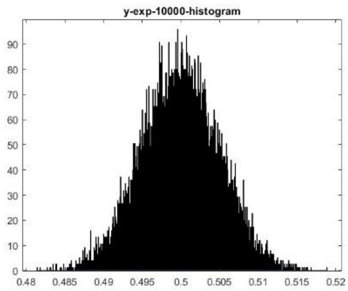
III. When X ~ Exponential (2):




T=100




T=1000




T=10000

- By consulting standard PDFs the histograms of Y for different number of observations appear to math the PDF curve of a gaussian distribution according to the Central Limit Theorem.

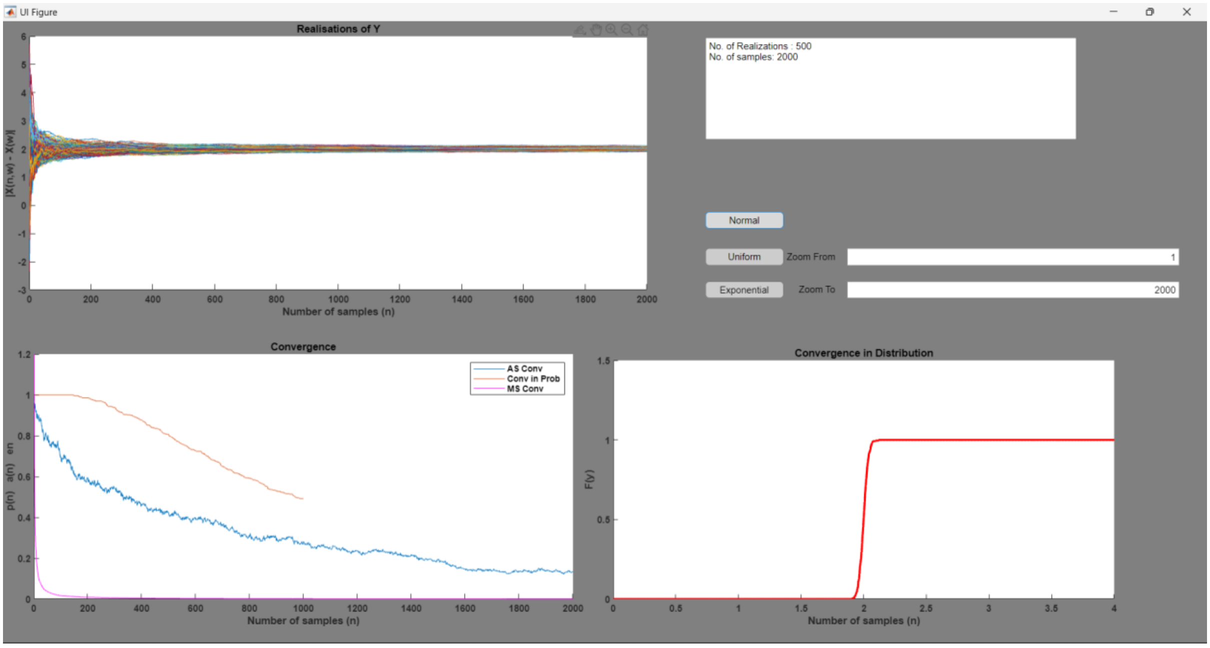
- However, it is observed that the generated histograms of Y from different variables have a reasonably stable mean $\mu_Y = \mu_X$ with changing variance $\sigma_Y = \sigma_X\sqrt{T}$, Thus, As $\lim_{T\to\infty} Y_i \sim Gaussian\ (\mu_Y, \sigma_Y^2)$

3.

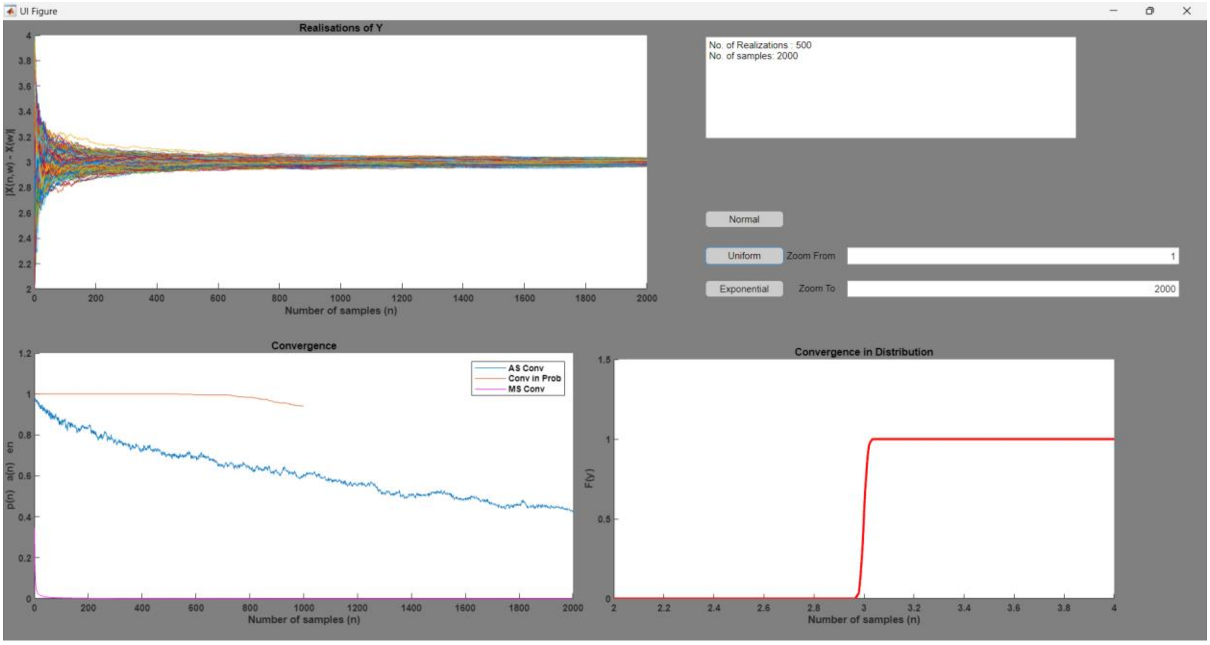
Based on the paper “Understanding Convergence Concepts: A Visual-Minded and Graphical Simulation Based Approach”, a demo using MATLAB is generated to answer the following questions:

1. $Y_T{}^{P} \rightarrow$ $\text{Mu}_y$
2. $Y_T{}^{A\cdot S} \rightarrow$ $\text{Mu}_y$
3. $Y_T{}^{M\cdot S} \rightarrow$ $\text{Mu}_y$
4. $Y_T{}^{L} \rightarrow X$

As the in the above sections, this demo is created for proving convergence of Y generate using three different distribution schemes namely, Normal, Uniform and Exponential. Following the paper closely the sample size T for the distribution of $Y$ is chosen to be a value of $T$ = 2000 and the number of realizations is chosen to be $M$ = 500. The convergence of the distribution of Y is consistent with the law of large numbers which states that for sufficiently large n of random variables; the normalized mean $\text{Mu}_y$ with a high probability takes values close to the mean of the distribution from which you sample. It is observed that in the case of normal distribution Y converge to the mean value of the distribution of X, i.e. $E[X]$ =2. It should be noted that in the case of uniform and exponential distributions $Y_T \overset{P}{\rightarrow} E[X]$ = 3 and $Y_T \overset{P}{\rightarrow} E[X]$ = 0.5 respectively. Therefore, to study the convergence in probability of a random variable $Y_T$ to $E[X]$ , we can define the random variable $Z_T = Y_T - E[X]$ and study the convergence in probability of $Z_T$ to the constant 0. This remark is also valid for almost sure convergence and convergence in $r^{th}$ mean. The screenshot of the GUI display below depicts the different convergence modes of the distribution Y generated using Normal samples. The In-Probability convergence (blue curve), the Almost Sure convergence (red curve) and the Mean Square convergence to the value zero is clearly observed in the centred graph. The right most graph depicts the convergence in distribution of Y to the normal distribution which has a mean value of zero. This is consistent with the expected result. The M = 500 realizations for Y from Normal variates is plotted in the first plot shown below




The screenshot of the GUI display below depicts the different convergence modes of the distribution Y generated using Uniform samples. It should be noted that as mentioned before, the In-Probability convergence (blue curve), the Almost Sure convergence (red curve) and the Mean Square convergence (green curve) depicted in the centred graph is for $Z_T = Y_T - E[X]$ where $E[X]$ = 3. The convergence of $Z_T$ to the constant 0 is observed. The right most graph depicts the convergence in distribution of Y to the normal distribution which has a mean value of $E[X]$ = 3. The M = 500 realizations for Y from Uniform variates is plotted in the first plot shown below. It is clearly observed that $Y_T$ converges to the value 3, which in turn implies the convergence of $Z_T = Y_T - 3$ to 0.




The screenshot of the GUI display below depicts the different convergence modes of the distribution Y generated using Exponential samples. It should be noted that as mentioned before, the In-Probability convergence (blue curve), the Almost Sure convergence (red curve) and the Mean Square convergence (green curve) depicted in the centred graph is for $Z_T$ = $Y_T$ − $E[X]$ where $E[X]$ = 0.5. The convergence of $Z_T$ to the constant 0 is observed. The right most graph depicts the convergence in distribution of Y to the normal distribution which has a mean value of $E[X]$ = 0.5. The M = 500 realizations for Y from Exponential variates is plotted in the first plot show below. It is clearly observed that $Y_T$ converges to the value 1, which in turn implies the convergence of $Z_T$ = $Y_T$ – 0.5 to 0.