Project Report (ECE 514) Part 1

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1.

ECE 514 PROJECT

PART 2

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X, Y are 2 jointly continious R.V.s

$$f_{XY}(x,y) = \begin{cases} x + \frac{3}{2}y^2 & , 0 < x, y \leq 1 \\ 0 & , \text{otherwise} \end{cases}$$

$$U = \begin{bmatrix} X \\ Y \end{bmatrix}$$

1) a) $R_U = E(U \cdot U^T)$

$$= \begin{bmatrix} E(X^2) & E(XY) \\ E(XY) & E(Y^2) \end{bmatrix}$$

i) $$E(X^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2\left(x + \frac{3}{2}y^2\right) dx\, dy$$

$$= \int_0^1\int_0^1 x^3 + \frac{3}{2}y^2 x^2\, dx\, dy$$

$$= \int_0^1 \left[\frac{x^4}{4} + \frac{\cancel{3}}{2}y^2\frac{x^3}{\cancel{3}}\right]_0^1 dy.$$

$$= \int_0^1 \frac{1}{4} + \frac{y^2}{2}\, dy$$

$$= \left[\frac{y}{4} + \frac{y^3}{6}\right]_0^1 = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

ii) $E(Y^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y^2 (x + \frac{3}{2} y^2)\, dx\, dy.$

$= \int_0^1\int_0^1 (y^2 x + \frac{3}{2} y^4)\, dy\, dx.$

$= \int_0^1 \left[\frac{y^3}{3} x + \frac{3}{2}\frac{y^5}{5} \right]_0^1 dx.$

$= \int_0^1 \left[\frac{x}{3} + \frac{3}{10} \right] dx.$

$= \left[\frac{x^2}{6} + \frac{3}{10} x. \right]_0^1$

$= \frac{1}{6} + \frac{3}{10}$

$= \frac{5+9}{30} = \frac{14}{30} = \frac{7}{15}$

iii) $E(XY) = \int_0^1\int_0^1 xy\ (x + \frac{3}{2} y^2) dx\, dy$

$= \int_0^1\int_0^1 x^2 y + \frac{3}{2} x y^3\ dx\, dy$

$= \int_0^1\int_0^1 x^2 y\, dx\, dy + \frac{3}{2}\int_0^1\int_0^1 x y^3\, dx\, dy.$

$= \frac{1}{3}\int_0^1 y\, dy + \frac{3}{4}\int_0^1 y^3 dy = \frac{17}{48}$

$\therefore \quad R_U = \begin{bmatrix} 5/12 & 17/48 \\ 17/48 & 7/15 \end{bmatrix} = \begin{bmatrix} 0.41667 & 0.3514667 \\ 0.3514667 & 0.4667 \end{bmatrix}$

[Ans:]

b) $C_U = R_U - E(U)\,E(U^T)$

$E(U) = \begin{bmatrix} E(X) \\ E(Y) \end{bmatrix}$ $\qquad E(U)\cdot E(U^T) = \begin{bmatrix} E(X) \\ E(Y) \end{bmatrix} \begin{bmatrix} E(X) & E(\end{bmatrix}$

$= \begin{bmatrix} (E(X))^2 & E(X)\,E(\\ E(X)E(Y) & (E(Y))^2 \end{bmatrix}$

i) $E(X) = \int_0^1\int_0^1 x\left(x + \frac{3}{2}y^2\right) dx\,dy$

$= \int_0^1\int_0^1 x^2 + \frac{3}{2}y^2 x \, dx\, dy$

$= \int_0^1 \left[\frac{x^3}{3} + \frac{3}{2}y^2\frac{x^2}{2}\right]_0^1 dy$

$= \int_0^1 \left[\frac{1}{3} + \frac{3y^2}{4}\right] dy$

$= \left[\frac{1}{3}y + \frac{3}{4}\frac{y^3}{3}\right]_0^1 = \cancel{\frac{1}{3} + \frac{3}{10}}\ \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

ii) $E(Y) = \int_0^1\int_0^1 y\left(x + \frac{3}{2}y^2\right) dy\, dx.$

$= \int_0^1\int_0^1 xy + \frac{3}{2}y^3\, dy\, dx.$

$= \int_0^1 \left[x\frac{y^2}{2} + \frac{3}{2}\frac{y^4}{4} \right]_0^1 dx.$

$= \int_0^1 \left[\frac{x}{2} + \frac{3}{8}\right] dx$

$= \left[\frac{x^2}{4} + \frac{3}{8}x.\right]_0^1$

$= \frac{1}{4} + \frac{3}{8}$

$= \frac{5}{8}$

$\therefore \quad E(U) = \begin{bmatrix} \frac{7}{12} \\ \frac{5}{8} \end{bmatrix}$

$\therefore \quad C_U = \begin{bmatrix} 5/12 & 17/48 \\ 17/48 & 7/15 \end{bmatrix} - \begin{bmatrix} \left(\frac{7}{12}\right)^2 & \frac{7}{12}\cdot\frac{5}{8} \\ \frac{7}{12}\cdot\frac{5}{8} & \left(\frac{5}{8}\right)^2 \end{bmatrix}$

$= \begin{bmatrix} \frac{11}{144} & -\frac{1}{96} \\ -\frac{1}{96} & \frac{73}{960} \end{bmatrix} = \begin{bmatrix} 0.0763889 & -0.01041667 \\ -0.01041667 & 0.07604167 \end{bmatrix}$

[Ans.]

2. The MATLAB code for generating X_s is attached below.
3. The MATLAB code for generating $Cov(X_s)$ is attached below.

```
Cov_X_s =

    0.0758   -0.0111
   -0.0111    0.0767
```

4. Cov_U =

```
    0.0764   -0.0104
   -0.0104    0.0760
```

Cov_X_s =

```
    0.0758   -0.0111
   -0.0111    0.0767
```

It is observed that Cov (X_s) is not exactly same as Cov (U) but the estimate can be improved by increasing the sample size of the generated vector X_s