MA581 HW5

Yunzhe Yu

U78022335/Discussion A2

4.45

Given:

- $P(A_2) = 0.401$ (Probability of being 25-44 years old)
- $P(L_1) = 0.126$ (Probability of living alone)
- $P(A_2 \cap L_1) = 0.038$ (Joint probability of being 25-44 years old and living alone)

We want to check if:

$$P(A_2 \cap L_1) = P(A_2) \times P(L_1)$$

Calculating the product:

$$P(A_2) \times P(L_1) = 0.401 \times 0.126 = 0.050526$$

Comparing the product with the joint probability:

$$P(A_2 \cap L_1) \neq P(A_2) \times P(L_1)$$

Since $0.038 \neq 0.050526$, events A_2 and L_1 are not independent.

4.47

Let A be an event of a sample space. We verify the following statements:

a) If $P(A) = 0$ or $P(A) = 1$, then for each event B of the sample space, A and B are independent events.

- If $P(A) = 0$, then A never occurs, thus $P(A \cap B) = 0$ for any event B, satisfying $P(A \cap B) = P(A) \times P(B)$.
- If $P(A) = 1$, then A always occurs, hence $P(A \cap B) = P(B)$ for any event B, which is equal to $P(A) \times P(B)$ since $P(A) = 1$.

b) If A and A^c are independent events, then $P(A) = 0$ or $P(A) = 1$.

- For A and A^c to be independent, $P(A \cap A^c) = P(A) \times P(A^c)$. However, $A \cap A^c$ is the null set, so $P(A \cap A^c) = 0$.
- Since $P(A^c) = 1 - P(A)$, we have $P(A) \times (1 - P(A)) = 0$. This implies $P(A) = 0$ or $P(A) = 1$.

4.55

a) If two events are mutually exclusive, this means they cannot occur simultaneously. Therefore, the joint probability of two mutually exclusive events is:

$$P(A \cap B) = 0$$

b) If two events, A and B, with positive probabilities are independent, the occurrence of one event does not affect the occurrence of the other. Their joint probability is given by the product of their individual probabilities, which is not 0 since both events have positive probabilities. This can be expressed as:

$$P(A \cap B) = P(A) \times P(B)$$

Since $P(A) > 0$ and $P(B) > 0$, it follows that $P(A \cap B) > 0$. This implies that two events cannot be both independent and mutually exclusive if they both have positive probabilities.

c) Consider the events of selecting a card from a standard deck:

- Event A: Drawing a red card.
- Event B: Drawing a king.

These events are neither mutually exclusive nor independent:

- They are not mutually exclusive because it is possible to draw a card that is both red and a king (e.g., the King of Diamonds or the King of Hearts).
- They are not independent because the probability of drawing a king changes if we know the card is red. Specifically, there are 2 red kings in a deck of 52 cards, so knowing a card is red makes it more likely to be a king compared to any random card from the deck.

4.68

Given:

- The probability of being a Democrat $P(D) = 0.40$.
- The probability of being a Republican $P(R) = 0.32$.
- The probability of being an Independent $P(Ind) = 0.28$.
- The conditional probability of favoring increased spending given the person is a Democrat $P(I|D) = 0.60$.
- The conditional probability of favoring increased spending given the person is a Republican $P(I|R) = 0.80$.
- The conditional probability of favoring increased spending given the person is an Independent $P(I|Ind) = 0.30$.

To find the total probability of favoring increased spending $P(I)$, we use the law of total probability:

$$P(I) = P(I|D) \cdot P(D) + P(I|R) \cdot P(R) + P(I|Ind) \cdot P(Ind)$$

$$P(I) = 0.60 \cdot 0.40 + 0.80 \cdot 0.32 + 0.30 \cdot 0.28 = 0.58$$

Using Bayes' Theorem, we calculate the probability that a person is a Democrat given they favor increased spending $P(D|I)$:

$$P(D|I) = \frac{P(I|D) \cdot P(D)}{P(I)}$$

$$P(D|I) = \frac{0.60 \cdot 0.40}{0.58} \approx 0.414$$

Therefore, the probability that a person is a Democrat given they favor increased spending is approximately 41.4%.

4.72

Let S_2 be the event that the other drawer contains a silver coin, and S_1 be the event that the first drawer opened contains a silver coin. We seek $P(S_2|S_1)$, the probability that the other drawer contains a silver coin given that we found a silver coin in the first drawer. The total possibilities of finding a silver coin are as follows:

$$\begin{aligned}
&\text{Chest A: 1 (1 silver, 1 gold)}\\
&\text{Chest B: 1 (1 silver, 1 gold)}\\
&\text{Chest C: 2 (2 silver)}\\
&\text{Chest D: 0 (2 gold)}
\end{aligned}$$

Total ways to find a silver coin: $1 + 1 + 2 = 4$.

The possibilities where finding a silver leads to another silver only happen with Chest C, which has 2 ways to find a silver coin initially.

Thus, the probability that the other drawer also contains a silver coin is calculated as:

$$P(S_2|S_1) = \frac{\text{Ways to find another silver after finding one}}{\text{Total ways to find a silver coin}} = \frac{2}{4} = 0.5$$

Therefore, the probability that the other drawer contains a silver coin, given that the first drawer opened contains a silver coin, is 0.5 or 50%.

5.7

a) The sample space S for this experiment can be defined as follows, where U_i represents Urn i, R represents a red ball, and W represents a white ball:

$$S = \{(U_1, R, R), (U_1, R, W), (U_1, W, R), (U_1, W, W), (U_2, R, R), (U_2, R, W), (U_2, W, R), (U_2, W, W)\}$$

b) We construct a table where X represents the number of red balls drawn:

Outcome	X
(U_1, R, R)	2
(U_1, R, W)	1
(U_1, W, R)	1
(U_1, W, W)	0
(U_2, R, R)	2
(U_2, R, W)	1
(U_2, W, R)	1
(U_2, W, W)	0