# MA581 HW4

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## 4.9

a) Given that there are 13 spades in a deck of 52 cards, the probability that the first card is a spade can be calculated as follows:

$$P(\text{first card is a spade}) = \frac{\text{Number of spades}}{\text{Total number of cards}}$$

Substituting the values:

$$P(\text{first card is a spade}) = \frac{13}{52}$$

This simplifies to:

$$P(\text{first card is a spade}) = 0.25$$

Therefore, the probability that the first card is a spade, given that the four cards dealt have different face values, is 0.25 or 25%.

b) Given that the four cards are dealt from different suits in a standard deck of 52 cards, we want to find the probability that the first card is a face card (Jack, Queen, or King).

Each suit (spades, hearts, diamonds, clubs) contains 3 face cards, making a total of 12 face cards in the deck. The probability that the first card is a face card can be calculated as follows:

$$P(\text{first card is a face card}) = \frac{\text{Number of face cards}}{\text{Total number of cards}}$$

Given the numbers:

$$P(\text{first card is a face card}) = \frac{12}{52}$$

This simplifies to:

$$P(\text{first card is a face card}) \approx 0.231$$

Therefore, the probability that the first card is a face card, given that the four cards come from different suits, is approximately 23.08%.

## 4.11

a) Given that the first child born is a boy, the possible outcomes are:

- Boy, Boy (BB)
- Boy, Girl (BG)

Thus, the probability that both children are boys is:

$$P(\text{both are boys} \mid \text{first is a boy}) = \frac{1}{2}$$

b) Given that at least one child is a boy, the possible outcomes are:

- Boy, Boy (BB)
- Boy, Girl (BG)
- Girl, Boy (GB)

Therefore, the probability that both children are boys, given that at least one is a boy, is:

$$P(\text{both are boys} \mid \text{at least one is a boy}) = \frac{1}{3}$$

## 4.12

A balanced die is rolled until the first 6 occurs. If that happens on the eighth toss, we want to find the probability that there are exactly two 4s among the first seven tosses. The binomial probability formula is used for this calculation:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $n$ is the number of trials (7), $k$ is the number of successful outcomes we're interested in (2 fours), $p$ is the probability of success on each trial (rolling a 4, which is $\frac{1}{6}$), and $\binom{n}{k}$ calculates the number of ways to choose $k$ successes out of $n$ trials.

Given these, the probability of getting exactly two 4s in the first seven tosses is calculated as:

$$P(\text{exactly two 4s in first seven tosses}) = \binom{7}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5$$

This results in a probability of approximately 0.234 or 23.44%.

## 4.19

To prove that $P(C|A \cap B) = \frac{P(B \cap C|A)}{P(B|A)}$, we start by using the definition of conditional probability:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Given this, we can express the probabilities as follows:

1. $P(C|A \cap B) = \frac{P(C \cap (A \cap B))}{P(A \cap B)}$
2. $P(B \cap C|A) = \frac{P((B \cap C) \cap A)}{P(A)}$
3. $P(B|A) = \frac{P(B \cap A)}{P(A)}$

By substituting these expressions, we aim to show:

$$\frac{P(B \cap C|A)}{P(B|A)} = \frac{\frac{P(A \cap B \cap C)}{P(A)}}{\frac{P(B \cap A)}{P(A)}}$$

Simplifying the right-hand side, we find:

$$\frac{P(A \cap B \cap C)}{P(A \cap B)}$$

This matches the expression for $P(C|A \cap B)$, thus proving:

$$P(C|A \cap B) = \frac{P(B \cap C|A)}{P(B|A)}$$

## 4.23

a) To determine the probability that a randomly selected American with Internet access is a regular Internet user who feels that the Web has reduced his or her social contact, we calculate:

$$P(\text{Regular and Reduced Social Contacts}) =$$

$$P(\text{Regular}) \times P(\text{Reduced Social Contacts — Regular}) = 0.36 \times 0.25 = 0.09$$

b) This means that 9% of Americans with Internet access are regular Internet users who believe that the Web has reduced their social contacts. In other words, out of every 100 Americans with Internet access, approximately 9 feel their social interactions have decreased due to their web usage.

## 4.31

Let $X$ be the outcome of the roll of the balanced die and $Y$ is the number of heads on $X$ toss of the balanced coin. The probability mass function of $X$ for a balanced die is

$$P(X = x) = \frac{1}{6} \text{ for } x = 1, 2, 3, 4, 5, 6$$

Probability of heads on a toss of balanced coin $= 0.5$. The conditional distribution of $Y$ given $X = x$ follows Binomial distribution with parameter $p = 0.5$ and $n = x$. Then

$$Y|X = x \ \text{ Bin } (n = x, p = 0.5)$$

The probability mass function of $Y|X$ is,

$$P(Y = y|X = x) = \binom{x}{y} 0.5^y (1 - 0.5)^{x-y} = \binom{x}{y} 0.5^x$$

for $y = 0, ..., x$

The number of heads is at most the number of tosses. That is, $y \leq x$. For $y = 2$, the possible values of $x$ are $2, 3, 4, 5, 6$. By law of total probability, probability of obtaining exactly two heads $= P(Y = 2)$

$$\sum_{x=2}^{6} P(Y = 2|X = x)P(X = x) = \sum_{n=2}^{6} \binom{x}{2} 0.5^x \frac{1}{6} = \frac{1}{6} \sum_{x=2}^{6} \binom{x}{2} 0.5^x = 1.546875/6 = 0.2578125$$

## 4.44

a) To assess whether events $C_1$ (injuries at work) and $S_2$ (female injuries) are independent, we examine if the product of their individual probabilities equals the probability of their intersection:

$$P(C_1 \cap S_2) = P(C_1)P(S_2)$$

Given data:

1. $P(C_1) = \frac{\text{Injuries at work}}{\text{Total injuries}} = \frac{9.3}{61.4}$
2. $P(S_2) = \frac{\text{Female injuries}}{\text{Total injuries}} = \frac{25.8}{61.4}$
3. $P(C_1 \cap S_2) = \frac{\text{Female injuries at work}}{\text{Total injuries}} = \frac{1.3}{61.4}$

After calculation, we find that $P(C_1 \cap S_2) \neq P(C_1)P(S_2)$, indicating that $C_1$ and $S_2$ are not independent. This means the occurrence of injuries at work is in some way related to the gender of the injured person, specifically females in this case.

b) To assess if the event of an injured person being male is independent of the event that an injury occurred at home, we verify if:

$$P(\text{Male} \cap \text{Injury at Home}) = P(\text{Male})P(\text{Injury at Home})$$

The findings reveal that these events are not independent, suggesting a relationship between the gender of the injured person and the circumstance of the injury.