MA581 HW3

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U78022335/Discussion A2

3.7

a)
- The first four digits can be any of the digits 0-9, which means each of these positions has 10 possible values.
- The fifth digit can be any of the digits 1-8, meaning it has 8 possible values.

Then, we can get that

$$\text{Number of Ways} = 10^4 * 8 = 80000$$

b)
- The first three digits can be any of the digits 0-9, which means each of these positions has 10 possible values.
- The fourth digit can be any of the digits 1-9, meaning it has 9 possible values.

Then, we can get that

$$\text{Number of Ways} = 10^3 * 9 = 9000$$

c) The total can be calculated as the following:

$$\text{Total Number of Ways} = \text{Five-digits+Four-digits} = 80000*9000 = 72*10^7$$

3.17

There will be $\frac{n(n-1)}{2}$ times handshakes.
Each of the n people will shake hands with $n-1$ others (since no one shakes their own hand), but this counts each handshake twice (once for each person in the pair), so divide by 2 to get the correct count.

3.26

To determine the number of possibilities for investing in 4 out of 30 mutual funds, we use the permutation formula:

$$P(n, r) = \frac{n!}{(n-r)!}$$

where n is the total number of items to choose from, and r is the number of items to choose. For our case, $n = 30$ and $r = 4$, the formula becomes:

$$P(30, 4) = \frac{30!}{(30-4)!} = \frac{30!}{26!}$$

Evaluating this expression yields:

$$P(30, 4) = 657,720$$

Therefore, there are 657,720 possible ways to invest in 4 out of the 30 mutual funds, considering the order of investment.

3.32

To calculate the total number of possibilities in the Powerball lottery, we consider two selections: choosing 5 numbers out of 53 without regard to order, and choosing 1 Powerball number out of 42. The formula for combinations is used for the first selection:

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

where n is the total number of items to choose from, r is the number of items to choose, $n!$ denotes the factorial of n, $r!$ is the factorial of r, and $(n-r)!$ is the factorial of the difference between n and r. Thus, for choosing 5 numbers out of 53, we have:

$$C(53, 5) = \frac{53!}{5!(53-5)!}$$

For the Powerball number, since it is a single choice out of 42, there are 42 possible outcomes. The total number of possibilities for the Powerball lottery is given by the product of these two selections:

$$\text{Total Possibilities} = C(53, 5) \times 42$$

After evaluating the combination and multiplying by the number of Powerball options, we find:

$$\text{Total Possibilities} = 120,526,770$$

Therefore, there are 120,526,770 possible ways to select numbers for the Powerball game.

3.63

First, we consider the arrangement of the groups themselves. We have three groups (mathematicians, chemists, physicists), so we can arrange these groups in 3! ways.

Next, we consider the arrangements within each group. The four mathematicians can be arranged among themselves in 4! ways, the three chemists in 3! ways, and the five physicists in 5! ways.

The total number of ways to arrange all 12 people without any restrictions is 12!, since there are 12 individuals in total. The probability that all members of each discipline sit together is calculated as the ratio of favorable arrangements to the total number of arrangements. This is given by:

$$\text{Probability} = \frac{3! \times 4! \times 3! \times 5!}{12!} = 0.000216$$

Therefore, the probability that all members of each discipline (mathematicians, chemists, and physicists) sit together is approximately 0.0216%.