NUST SEECS
Multivariable Calculus

Assignment 3 | **Individual Assignment**
BSCS-AB | **Summer 2024**
Due Date: 16th August 2024 Friday (Before 4pm) | **Marks: 60**
[CLO - 3]Evaluate Fourier series, Fourier integral and Fourier-transforms of a given function.
Note: Hard copy submission is compulsory at 2pm to 4pm in CR-02

Suppose that $f(x)$ be any periodic function with period 2π, which can be represented in the form

$$\boldsymbol{f(x) = a_0 + a_1 cosx + b_1 sinx + a_2 cos2x + b_2 sin2x + \ldots + a_n cosnx + b_n sinnx + \ldots}$$

This representation is called Fourier series of $f(x)$. The coefficients a_0, a_n and b_n are known as Fourier coefficients and can be obtain by the formulas given in the following definition.

Question: Find the Fourier series of the following function. Also check whether the following function are even or odd.

(i) $f(x) = x, \ -2 < x < 2$

(ii) $f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 \le x < \pi \end{cases}$

(iii) $f(x) = x^2, 0 < x < \pi$

(iv) $f(x) = x|sinx|, \ -\pi < x < \pi$

DEFINITION 11.2.1 Fourier Series

The **Fourier series** of a function f defined on the interval $(-p, p)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{p}x + b_n \sin\frac{n\pi}{p}x\right), \tag{8}$$

where

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(x)\,dx \tag{9}$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi}{p}x\,dx \tag{10}$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi}{p}x\,dx. \tag{11}$$

Note: For more details on the topic you can consult the following:
1. Advanced Engineering Mathematics by Erwin Kreyszig, 3rd edition, Chapter 9.
2. Differential Equations with Boundary Value problems by D G Zill, 7th edition, Chapter 11.