

**M1F Foundations of Analysis, Problem Sheet 1**

1. (\*) (PB0101) Which of the following statements involving an integer  $x$  are true and which are false? Just write T or F, and perhaps also one remark about why you think this is the answer.

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| (a) $x^2 - 3x + 2 = 0 \Rightarrow x = 1$ .     | (e) $x^2 - 3x + 2 = 0 \Rightarrow x = 1$ or $x = 2$ or $x = 3$ . |
| (b) $x^2 - 3x + 2 = 0 \Leftarrow x = 1$ .      |  |
| (c) $x^2 - 3x + 2 = 0 \iff x = 1$ .            | (f) $x^2 - 3x + 2 = 0 \Leftarrow x = 1$ or $x = 2$ or $x = 3$ .  |
| (d) $x^2 - 3x + 2 = 0 \iff x = 1$ or $x = 2$ . |  |

2. (PB0102) Suppose  $P$ ,  $Q$  and  $R$  are mathematical statements (so they are either true or false). Let's say we know that if  $Q$  is true then  $P$  is true, and that if  $Q$  is false then  $R$  is false. Does  $R$  imply  $P$ ? Write down either a proof, or a counterexample.

3. (\*) (PB0103) Say  $P$  is true,  $Q$  is false,  $R$  is false and  $S$  is true. Is  $(P \Rightarrow Q) \Leftarrow (R \Rightarrow S)$  true or false?

4. (PB0104) Say  $P$ ,  $Q$  and  $R$  are true/false mathematical statements, and we know the following:

- (a)  $P \Rightarrow (Q \vee R)$ ,  
 (b)  $\neg Q \Rightarrow (R \vee \neg P)$   
 (c)  $(Q \wedge R) \Rightarrow \neg P$ .

5. (PB0105) Let  $A$  be the set  $\{1, 2, 3, 4, 5\}$ . Which of the following statements are true and which are false? (just write T or F).

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|------------------------------|---------------------------------|
| (a) $1 \in A$ .              | (e) $\{1, 2, 1\} \subseteq A$ . |
| (b) $\{1\} \in A$ .          | (f) $\{1, 1\} \in A$ .          |
| (c) $\{1\} \subseteq A$ .    | (g) $A \in A$ .                 |
| (d) $\{1, 2\} \subseteq A$ . | (h) $A \supseteq A$ .           |

6. (PB0106) Now let  $A$  be the slightly weirder set  $\{1, 2, \{1, 2\}\}$  and let  $B$  be the even weirder set  $\{1, 2, A\}$ . Which of the following statements are true and which are false? (again just write T or F).

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|------------------------------|--|
| (a) $1 \in A$ .              | (e) $1 \in B$ .                                  |
| (b) $\{1\} \in A$ .          | (f) $\{1\} \in B$ .                              |
| (c) $\{1, 2\} \in A$ .       | (g) $(\{1, 2\} \in B) \Rightarrow (1 \in A)$ .   |
| (d) $\{1, 2\} \subseteq A$ . | (h) $(\{1, 2\} \subseteq B) \vee (1 \notin A)$ . |

7. (PB0107) Set  $A = \{x \in \mathbf{R} : x^2 < 3\}$ ,  $B = \{x \in \mathbf{Z} : x^2 < 3\}$  and  $C = \{x \in \mathbf{R} : x^3 < 3\}$ . For each statement below, either prove it or disprove it! Be careful with your logic and your exposition.

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|----------------------------------|---|
| (a) $\frac{1}{2} \in A \cap B$ . | (d) $B \subseteq C$ .                         |
| (b) $\frac{1}{2} \in A \cup B$ . | (e) $C \subseteq A \cup B$ .                  |
| (c) $A \subseteq C$ .            | (f) $(A \cap B) \cup C = (A \cup B) \cap C$ . |