S0804.

(i) We have $a \leq a$ for all a so $\sim$ is reflexive. We have $1 \leq 2$ but $2 \not\leq 1$, so $\sim$ is not symmetric. If $a \leq b$ and $b \leq c$ then $a \leq c$, so $\sim$ is transitive.

(ii) $a - a = 0 = 0^2$, so $\sim$ is reflexive. We have $2 \sim 1$ as $2 - 1 = 1^2$, but $1 \not\sim 2$ as -1 is not a square, so the relation is not symmetric. Finally we have $3 \sim 2$ and $2 \sim 1$ but $3 \not\sim 1$ as 2 is not a square, so $\sim$ is not transitive either.

(iii) $2 \neq 2^2$ so $2 \not\sim 2$, and $\sim$ is not reflexive. We have $4 \sim 2$ but $2 \not\sim 4$ as $2 \neq 4^2$, so the relationship is not symmetric. We have $4 \sim 2$ and $16 \sim 4$ but $16 \not\sim 2$ so the relation is not transitive.

(iv) We have $1 \not\sim 1$ so $\sim$ is not reflexive. If $a \sim b$ then $a + b = 0$, so $b + a = 0$, so $b \sim a$, hence $\sim$ is symmetric. Finally we have $1 \sim -1$ and $-1 \sim 1$ but $1 \not\sim 1$ so $\sim$ is not transitive.

(v) We have $a - a = 0$ is an integer, so $\sim$ is reflexive. If $a - b$ is an integer then so is $b - a$, so $\sim$ is symmetric. Finally if $a - b$ and $b - c$ are integers, then their sum is $a - c$ which is also an integer. So $a \sim b$ and $b \sim c$ implies $a \sim c$, and in particular $\sim$ is also transitive. So in fact this relation is an equivalence relation.

(vi) $2 \not\sim 2$ so $\sim$ is not reflexive. We know $1 \sim 3$ but $3 \not\sim 1$ so $\sim$ is not symmetric. It is however impossible to find $a, b, c \in S$ with $a \sim b$ and $b \sim c$ (because b would have to be 1 and 3) so the statement "$a \sim b$ and $b \sim c$ implies $a \sim c$" is true, as if P is false then "P implies Q" is always true whatever the truth value of Q. So this relation is transitive.

(vii) This relation is reflexive, symmetric and transitive, because it is impossible to find any counterexamples to these statements as S is empty (for example for $\sim$ not to be reflexive we would have to find $a \in S$ with $a \not\sim a$, but we can't find any $a \in S$ at all, so $\sim$ is reflexive etc etc).