(PB0107)

(a) is false, because $B \subseteq \mathbf{Z}$ by definition, so $\frac{1}{2} \notin B$, so $\frac{1}{2} \notin A \cap B$.

(b) is true, because $\frac{1}{2} \in \mathbf{R}$ and $\left(\frac{1}{2}\right)^2 = \frac{1}{4} < 3$, so $\frac{1}{2} \in A$ and hence $\frac{1}{2} \in A \cup B$.

(c) is false, because $x = \frac{3}{2} \in \mathbf{R}$ satisfies $x^2 = 9/4 < 3$ and $x^3 = 27/8 > 3$, so $x \in A$ but $x \notin C$, which means $A \nsubseteq C$.

(d) is true. In fact if $x \in \mathbf{Z}$ and $|x| \geq 2$ then $x^2 \geq 4$, meaning $x \notin B$. On the other hand ± 1 and $0 \in B$, meaning $B = \{-1, 0, 1\}$. All of these elements are easily checked to be in C.

(e) is not true, because $-100 \in C$ (as its cube is less than zero) but $(-100)^2 = 10^4 > 3$ so $-100 \notin A$ and $-100 \notin B$.

(f) is not true. To check that these sets are not equal, all we need to do is to find a real number which is in one but not the other. Again I claim $x = -100$ will work. For we've just seen it's in C, so it's definitely in $(A \cap B) \cup C$. However we've also just seen it's not in $A \cup B$, so it's also not in $(A \cup B) \cap C$.