M1F Foundations of Analysis, Problem Sheet 1

1. (*)Which of the following statements involving an integer x are true and which are false? Just write T or F, and perhaps also one remark about why you think this is the answer.

   (a) $x^2 - 3x + 2 = 0 \Rightarrow x = 1$.

   (b) $x^2 - 3x + 2 = 0 \Leftarrow x = 1$.

   (c) $x^2 - 3x + 2 = 0 \Longleftrightarrow x = 1$.

   (d) $x^2 - 3x + 2 = 0 \Longleftrightarrow x = 1$ or $x = 2$.

   (e) $x^2 - 3x + 2 = 0 \Rightarrow x = 1$ or $x = 2$ or $x = 3$.

   (f) $x^2 - 3x + 2 = 0 \Leftarrow x = 1$ or $x = 2$ or $x = 3$.

2. Suppose P, Q and R are mathematical statements (so they are either true or false). Let's say we know that if Q is true then P is true, and that if Q is false then R is false. Does R imply P? Write down either a proof, or a counterexample.

3. (*)Say P is true, Q is false, R is false and S is true. Is $(P \Rightarrow Q) \Leftarrow (R \Rightarrow S)$ true or false?

4. Say P, Q and R are true/false mathematical statements, and we know the following:

   (a) $P \Rightarrow (Q \vee R)$,

   (b) $\neg Q \Rightarrow (R \vee \neg P)$

   (c) $(Q \wedge R) \Rightarrow \neg P$.

   Can we deduce *anything* about P, Q or R individually? For example, is R definitely false? Is P definitely true?

5. Let A be the set $\{1, 2, 3, 4, 5\}$. Which of the following statements are true and which are false? (just write T or F).

   (a) $1 \in A$.

   (b) $\{1\} \in A$.

   (c) $\{1\} \subseteq A$.

   (d) $\{1, 2\} \subseteq A$.

   (e) $\{1, 2, 1\} \subseteq A$.

   (f) $\{1, 1\} \in A$.

   (g) $A \in A$.

   (h) $A \supseteq A$.

6. Now let A be the slightly weirder set $\{1, 2, \{1, 2\}\}$ and let B be the even weirder set $\{1, 2, A\}$. Which of the following statements are true and which are false? (again just write T or F).

   (a) $1 \in A$.

   (b) $\{1\} \in A$.

   (c) $\{1, 2\} \in A$.

   (d) $\{1, 2\} \subseteq A$.

   (e) $1 \in B$.

   (f) $\{1\} \in B$.

   (g) $(\{1, 2\} \in B) \Rightarrow (1 \in A)$.

   (h) $(\{1, 2\} \subseteq B) \vee (1 \notin A)$.

7. Set $A = \{x \in \mathbf{R} : x^2 < 3\}$, $B = \{x \in \mathbf{Z} : x^2 < 3\}$ and $C = \{x \in \mathbf{R} : x^3 < 3\}$. For each statement below, either prove it or disprove it! Be careful with your logic and your exposition.

   (a) $\frac{1}{2} \in A \cap B$.

   (b) $\frac{1}{2} \in A \cup B$.

   (c) $A \subseteq C$.

   (d) $B \subseteq C$.

   (e) $C \subseteq A \cup B$.

   (f) $(A \cap B) \cup C = (A \cup B) \cap C$