KMB, 03/10/18

M1F Foundations of Analysis, Problem Sheet 1

1. (a) F ($x = 2$ is also a root)

   (b) T (it doesn't matter that $x = 2$ is a root here)

   (c) F ($x = 2$ is a problem again)

   (d) T (the two roots are $x = 1$ and $x = 2$ – but can you *prove* that there are no others?)

   (e) T ($x = 3$ isn't a root but this doesn't matter)

   (f) F ($x = 3$ isn't a root and this time it matters).

   The key thing to understand here is that $P \Rightarrow Q$ means, and *only* means, that if P is true, then Q is true. So, for example, part (e) is true, even though in practice it's a bit weird and unhelpful; the point is that logically it's a true statement.

2. It is true that R implies P. Here's why. Let's assume R is true (with the goal of trying to deduce that P is true). Can Q be false? No! For if Q is false then we know from the question that R will also be false, but we're assuming R is true. So Q must be true as well. And then again from the question, P must be true. So if R is true then P is true too.

3. P is true and Q is false, so $(P \Rightarrow Q)$ is false. Similarly, R is false and S is true, so $(R \Rightarrow S)$ is true. So the question asks whether (false) $\Leftarrow$ (true), and this is false. So the answer to the question is "false".

4. It's not hard to check (do it!) that if $PQR = TTF$ then all the conditions are satisfied, and if $PQR = FFT$ then they all are too. So we cannot deduce anything about the truth of any of the individual statements.

5. TFTTTFFT.

   The tricky thing here is to understand that $\{1, 2, 1\} = \{1, 2\}$.

6. TFTTTFTT.

   Note: $\{1, 2\} \in A$ and $A \in B$, but $\{1, 2\} \notin B$.

7. (a) is false, because $B \subseteq \mathbf{Z}$ by definition, so $\frac{1}{2} \notin B$, so $\frac{1}{2} \notin A \cap B$.

   (b) is true, because $\frac{1}{2} \in \mathbf{R}$ and $\left(\frac{1}{2}\right)^2 = \frac{1}{4} < 3$, so $\frac{1}{2} \in A$ and hence $\frac{1}{2} \in A \cup B$.

   (c) is false, because $x = \frac{3}{2} \in \mathbf{R}$ satisfies $x^2 = 9/4 < 3$ and $x^3 = 27/8 > 3$, so $x \in A$ but $x \notin C$, which means $A \not\subseteq C$.

   (d) is true. In fact if $x \in \mathbf{Z}$ and $|x| \geq 2$ then $x^2 \geq 4$, meaning $x \notin B$. On the other hand ± 1 and $0 \in B$, meaning $B = \{-1, 0, 1\}$. All of these elements are easily checked to be in C.

   (e) is not true, because $-100 \in C$ (as its cube is less than zero) but $(-100)^2 = 10^4 > 3$ so $-100 \notin A$ and $-100 \notin B$.

   (f) is not true. To check that these sets are not equal, all we need to do is to find a real number which is in one but not the other. Again I claim $x = -100$ will work. For we've just seen it's in C, so it's definitely in $(A \cap B) \cup C$. However we've also just seen it's not in $A \cup B$, so it's also not in $(A \cup B) \cap C$.