Modeling Electromagnetic Fields

EE4375 - FEM For EE Applications

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Overview

- Electric Fields
- Magnetic Fields
- Thermal Fields (thermal losses)
- Mechanical Fields (displacement, strain and stress) (deformation)
- Acoustic Fields (pressure) (noice, vibration)

Stationary Electric Fields (1/4)

Nomenclature for Stationary Electric Field

(requires physical units to be filled in)

- ϵ_0: permitivity of vacuum (scalar)
- $\epsilon(\mathbf{x})$: permitivity of material (scalar field)
- $\epsilon_r(\mathbf{x}) = \epsilon(\mathbf{x})/\epsilon_0$: relative permitivity (scalar field)
- $\rho(\mathbf{x})$: electrical charge density (scalar field)
- $\phi(\mathbf{x})$: electrical potential (scalar field)
- $\mathbf{E}(\mathbf{x})$: electric field (vector field)
- $\mathbf{D}(\mathbf{x})$: electric displacement field (vector field)

Stationary Electric Fields (2/4)

Maxwell Equation for Stationary Electric Field

1. $\nabla \times \mathbf{E} = \operatorname{curl} \mathbf{E} = \mathbf{0}$ (vector-valued equation)
2. $\nabla \cdot \mathbf{D} = \operatorname{div} \mathbf{D} = \rho$ (scalar equation)
3. $\mathbf{D} = \epsilon \, \mathbf{E}$ (vector-valued equation)

Stationary Electric Fields (3/4)

Maxwell Equation for Stationary Electric Field

1. From (1) follows that **E** is conservative, thus a potential ϕ exists such that $\mathbf{E} = -\nabla\phi = -\mathrm{grad}\,\phi$ (minus sign follows convention) (vector-valued equation)

2. From (3) then follows that $\mathbf{D} = \epsilon\,\mathbf{E} = -\epsilon\nabla\phi$ (vector-valued equation)

3. From (2) then follows that $\nabla \cdot \mathbf{D} = -\nabla \cdot (\epsilon\nabla\phi) = \rho$ (scalar equation)

Stationary Electric Fields (4/4)

Poisson Equation for the Electric Potential

- case that $\epsilon = \epsilon(\mathbf{x})$ is spatially dependent

$$-\nabla \cdot (\epsilon \nabla \phi) = -\text{div}\ (\epsilon\, \text{grad}\, \phi) = \rho$$

- case that ϵ is constant (independent of $\mathbf{x}$)

$$-\nabla \cdot (\nabla \phi) = -\nabla^2 \phi = -\triangle \phi = \rho$$

- Laplace equation in case that $\rho = 0$

Magnetic Fields

To be filled in

- quasi-stationary magnetic fields: time-harmonic and transient
- stranded conductor: current driven vs. voltage driven
- coupling with electrical fields
- Ohmic or copper losses (proportional to I^2)
- iron losses (Steinmetz formula with material data)

Stationary Magnetic Fields (1/)

Nomenclature for Stationary Magnetic Field

(requires physical units to be filled in)

- μ_0: permeability of vacuum (scalar)
- $\mu(\mathbf{x})$: permeability of material (scalar field)
- $\mu_r(\mathbf{x}) = \mu(\mathbf{x})/\mu_0$: relative permeability (scalar field)
- $\mathbf{J}(\mathbf{x})$: applied current density (vector field)
- $\mathbf{M}(\mathbf{x})$: magnetization (vector field)
- $V_m(\mathbf{x})$: magnetic scalar potential (scalar field)
- $\mathbf{A}(\mathbf{x})$: magnetic vector potential (vector field)
- $\mathbf{B}(\mathbf{x})$: magnetic flux (vector field)
- $\mathbf{H}(\mathbf{x})$: magnetic field (vector field)

Stationary Magnetic Fields (2/)

Maxwell Equation for Stationary Magnetic Field

1. $\nabla \times \mathbf{H} = \mathrm{curl}\,\mathbf{H} = \mathbf{J}$ (vector-valued equation)
2. $\nabla \cdot \mathbf{B} = \mathrm{div}\,\mathbf{B} = 0$ (scalar equation)
3. $\mathbf{B} = \mu\,\mathbf{H}$ (vector-valued equation)

Stationary Magnetic Fields (3/)

Maxwell Equation for Stationary Magnetic Field

1. From (2) follows that a vector potential **A** exists such that $\mathbf{B} = \nabla \times \mathbf{A}$ (vector-valued equation)
2. From (3) then follows that $\mathbf{H} = 1/\mu\, \mathbf{B} = 1/\mu\, \nabla \times \mathbf{A}$ (vector-valued equation)
3. From (1) then follows that $\nabla \times \mathbf{H} = \nabla \times (1/\mu \nabla \times \mathbf{A}) = \mathbf{J}$ (vector equation)

Stationary Magnetic Fields (4/)

Double Curl Equation for the Magnetic Vector Potential

1. $\nabla \times \mathbf{H} = \nabla \times (1/\mu \nabla \times \mathbf{A}) = \mathbf{J}$

Stationary Magnetic Fields (5/)

Two-Dimensional Perpendicular Current Formulation

- assume current perpendicular to 2D modeling plate $\mathbf{J} = (0, 0, J_z(x, y))$
- then $\mathbf{B} = (B_x(x, y), B_y(x, y), 0)$ and $\mathbf{H} = (H_x(x, y), H_y(x, y), 0)$
- then $\mathbf{A} = (0, 0, A_z(x, y))$
- $\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right) = J_z(x, y)$
- in post-processing $B_x(x, y) = -\frac{\partial A_z}{\partial y}$, $H_x = 1/\mu B_x$, $B_y(x, y) = \frac{\partial A_z}{\partial x}$, $H_y = 1/\mu B_y$

Time-Harmonic Magnetic Fields

Transient Magnetic Fields

Thermal Fields

- Poisson equation for the thermal field

References

- list references here