Modeling Electromagnetic Fields

EE4375 - FEM For EE Applications

Domenico Lahaye

DIAM - Delft Institute of Applied Mathematics

Last updated May 8, 2024

Overview

- Electric Fields
- Magnetic Fields
- Thermal Fields (thermal losses)
- Mechanical Fields (displacement, strain and stress) (deformation)
- Acoustic Fields (pressure) (noice, vibration)

Electric Fields (1/)

Nomenclature for Stationary Electric Field

(requires physical units to be filled in)

- ϵ_0: permitivity of vacuum (scalar)
- $\epsilon(\mathbf{x})$: permitivity of material (scalar field)
- $\epsilon_r(\mathbf{x}) = \epsilon(\mathbf{x})/\epsilon_0$: relative permeability (scalar field)
- $\rho(\mathbf{x})$: electrical charge density (scalar field)
- $\phi(\mathbf{x})$: electrical potential (scalar field)
- $\mathbf{E}(\mathbf{x})$: electric field (vector field)
- $\mathbf{D}(\mathbf{x})$: electric displacement field (vector field)

Electric Fields (2/)

Maxwell Equation for Stationary Electric Field

1. $\nabla \times \mathbf{E} = \operatorname{curl} \mathbf{E} = \mathbf{0}$ (vector-valued equation)
2. $\nabla \cdot \mathbf{D} = \operatorname{div} \mathbf{D} = \rho$ (scalar equation)
3. $\mathbf{D} = \epsilon \, \mathbf{E}$ (vector-valued equation)

Electric Fields (3/)

Maxwell Equation for Stationary Electric Field

1. From (1) follows that **E** is conservative, thus a potential ϕ exists such that $\mathbf{E} = -\nabla\phi = -\text{grad}\,\phi$ (minus sign follows convention) (vector-valued equation)

2. From (3) then follows that $\mathbf{D} = \epsilon\,\mathbf{E} = -\epsilon\nabla\phi$ (vector-valued equation)

3. From (2) then follows that $\nabla \cdot \mathbf{D} = -\nabla \cdot (\epsilon\nabla\phi) = \rho$ (scalar equation)

Electric Fields (4/)

Poisson Equation for the Electric Potential

- case that $\epsilon = \epsilon(\mathbf{x})$ is spatially dependent

$$-\nabla \cdot (\epsilon \nabla \phi) = -\text{div}\,(\epsilon\, \text{grad}\, \phi) = \rho$$

- case that ϵ is constant (independent of $\mathbf{x}$)

$$-\nabla \cdot (\nabla \phi) = -\nabla^2 \phi = -\triangle\, \phi = \rho$$

- Laplace equation in case that $\rho = 0$

Magnetic Fields

Overview

- stationary magnetic fields
- quasi-stationary magnetic fields: time-harmonic and transient
- stranded conductor: current driven vs. voltage driven
- coupling with electrical fields
- copper and iron losses

Magnetic Fields

Stationary Magnetic Fields

- double curl equation for the magnetic vector potential
- 2D perpendicular current configuration

Thermal Fields

- Poisson equation for the thermal field

References

- list references here