2010 G1

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Let ABC be an acute triangle with D, E, F the feet of the altitudes lying on BC, CA, AB respectively. One of the intersection points of the line EF and the circumcircle is P. The lines BP and DF meet at point Q. Prove that $AP = AQ$.

We use directed angles $\mod 180^\circ$. From the cyclic quads we have from altitudes,

$$\measuredangle AFQ = \measuredangle AFD = \measuredangle ACD = \measuredangle ACB = \measuredangle APB = \measuredangle APQ,$$

so $AQPF$ is cyclic. Now, recall A is the $D-$excenter of $\triangle DEF$, so A lies on the exterior angle bisector of $\angle QFP$. Thus, A is the midpoint of $\widehat{PFQ}$ of $(AQPF)$. Hence, $AP = AQ$ as desired. $\blacksquare$