2005 G3

Ezra Guerrero Alvarez

March 3, 2022

2005 G3

2005 G3

Let $ABCD$ be a parallelogram. A variable line g through the vertex A intersects the rays BC and DC at the points X and Y, respectively. Let K and L be the A-excenters of the triangles ABX and ADY. Show that the angle $\measuredangle KCL$ is independent of the line g.

Note that

$$\angle BKA = \frac{1}{2}\angle BXA = \angle DAL.$$

Analogously, $\angle LDA = \angle ABK$. Therefore, $\triangle ALD \sim \triangle KAB$. This implies

$$\frac{KA}{AL} = \frac{KB}{AD} = \frac{KB}{BC}.$$

Now, since $\angle KAL = \angle KBC = \frac{1}{2}\angle DAB$, it follows $\triangle KAL \sim \triangle KBC$. Thus, K is the center of the spiral similarity mapping $\overline{\text{AC}}$ to $\overline{\text{BL}}$. Thus, it is also the center of the spiral similarity mapping $\overline{\text{AB}}$ to $\overline{\text{CL}}$, so $\triangle KAB \sim \triangle KLC$. But then,

$$\angle KCL = \angle KBA = 180^\circ - \frac{1}{2}\angle DAB,$$

which is independent of g. ■