2012 G1

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Given triangle ABC the point J is the center of the excircle opposite the vertex A. This excircle is tangent to the side BC at M, and to the lines AB and AC at K and L, respectively. The lines LM and BJ meet at F, and the lines KM and CJ meet at G. Let S be the point of intersection of the lines AF and BC, and let T be the point of intersection of the lines AG and BC. Prove that M is the midpoint of ST.

Note that $AKJL$ is cyclic with diameter $\overline{\text{AJ}}$. Therefore, $\angle KJL = 180° - \angle KAL$. Now, since $\overline{\text{BJ}}$ is the perpendicular bisector of $\overline{\text{KM}}$, F is equidistant to K and M. Hence,

$$\angle KFL = \angle KFM = \angle 180° - 2\angle FMK = 2\angle KML - 180° = 180° - \angle KJL,$$

so F lies on this circle. Analogously, G lies on the circle as well, so $AFKJLG$ is cyclic. Now, let $X = \overline{\text{JM}} \cap (AKL)$ and $Y = \overline{\text{AM}} \cap (AKL)$. Since $\overline{\text{AJ}}$ is a diameter, $\overline{\text{AX}} \perp \overline{\text{XJ}} \perp \overline{\text{BC}}$. Let ∞ be the point at infinity of line $\overline{\text{BC}}$. Because A and J are the midpoints of the minor and major arcs $\widehat{KL}$ we find

$$-1 = (AJ; LK) \overset{M}{=} (YX; FG) \overset{A}{=} (M\infty, ST),$$

implying M is the midpoint of $\overline{\text{ST}}$, as desired. ■