2003 G2

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Three distinct points A, B, and C are fixed on a line in this order. Let Γ be a circle passing through A and C whose center does not lie on the line AC. Denote by P the intersection of the tangents to Γ at A and C. Suppose Γ meets the segment PB at Q. Prove that the intersection of the bisector of $\angle AQC$ and the line AC does not depend on the choice of Γ.

By properties of harmonic quads we find $\overline{\text{BQ}}$ is the $Q-$symmedian of $\triangle AQC$. Therefore,

$$\frac{AB}{BC} = \left(\frac{AQ}{QC}\right)^2.$$

Thus, if the bisector of $\angle AQC$ meets $\overline{\text{AC}}$ at T, then

$$\frac{AT}{TC} = \frac{AQ}{QC} = \sqrt{\frac{AB}{BC}},$$

which does not depend on the choice of Γ, as we wanted to show. $\blacksquare$