2018 G2

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Let ABC be a triangle with $AB = AC$, and let M be the midpoint of BC. Let P be a point such that $PB < PC$ and PA is parallel to BC. Let X and Y be points on the lines PB and PC, respectively, so that B lies on the segment PX, C lies on the segment PY, and $\angle PXM = \angle PYM$. Prove that the quadrilateral $APXY$ is cyclic.

Let X' and Y' be points on $\overline{\text{PX}}$ and $\overline{\text{PY}}$ respectively such that $BX = XX'$ and $CY = YY'$. Then, from the midpoints,

$$\angle BX'C = \angle PXM = \angle PYM = \angle BY'C,$$

so $BX'Y'C$ is cyclic. Let O be its center. Then, $\overline{\text{OX}} \perp \overline{\text{XX}'}$ and $\overline{\text{OY}} \perp \overline{\text{YY}'}$ giving that (PXY) is the circle with diameter $\overline{\text{OP}}$. It suffices to show that A is on this circle. However, note that $A - M - O$, so

$$\angle PAO = \angle PAM = 90^\circ,$$

as desired. ■