2019 G1

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Let ABC be a triangle. A circle passes through A, intersects $\overline{AB}$ and $\overline{AC}$ again at D and E respectively, and intersects $\overline{BC}$ at F and G, with $BF < BG$. Let T be a point such that $\overline{FT}$ is tangent to the circumcircle of $\triangle BDF$ and $\overline{GT}$ is tangent to the circumcircle of $\triangle CEG$. Prove that $\overline{AT} \parallel \overline{BC}$.

We use directed angles $\mod 180^\circ$. Using the tangency and cyclic pentagon $ADFGE$, we have

$$\measuredangle GFT = \measuredangle BFT = \measuredangle BDF = \measuredangle ADF = \measuredangle AGF$$
$$\measuredangle TGF = \measuredangle TGC = \measuredangle GEC = \measuredangle GEA = \measuredangle GFA$$

Adding these, we obtain $-\measuredangle FTG = -\measuredangle FAG$, so T lies on the circumcircle of $\triangle AFG$. However, we also have $\measuredangle GFT = \measuredangle AGF$, so $ATGF$ must be an isosceles trapezoid with $\overline{AT} \parallel \overline{GF}$, which is what we wanted to prove. $\blacksquare$