2000 N1

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Determine all positive integers $n \geq 2$ that satisfy the following condition: for all a and b relatively prime to n we have

$$a \equiv b \pmod{n} \qquad \text{if and only if} \qquad ab \equiv 1 \pmod{n}.$$

We claim the answer is $n \in \{2, 3, 4, 6, 8, 12, 24\}$. Let $n = 2^k \cdot m$ where $k \geq 0$ and m is odd. Note that $m + 2 \perp n$. Therefore, if n works, we must have

$$(m+2)^2 \equiv 1 \pmod{n}.$$

Reducing mod m, this implies $m \mid 3$. Now, if $m = 1$ it follows $n \mid 8$. If $m = 3$, then $n \mid 24$. Thus, n is a divisor of 24. Manually checking each of its divisors, we see the only n satisfying the condition are those mentioned above. ■