2012 G2

Ezra Guerrero Alvarez

January 27, 2022

2012 G2

2012 G2

Let $ABCD$ be a cyclic quadrilateral whose diagonals AC and BD meet at E. The extensions of the sides AD and BC beyond A and B meet at F. Let G be the point such that $ECGD$ is a parallelogram, and let H be the image of E under reflection in AD. Prove that D, H, F, G are concyclic.

We use directed angles mod 180°. Let D', F', G', H' be images of D, F, G, H under a homothety with center E and ratio $\frac{1}{2}$. Since $ECGD$ is a parallelogram, G' is the midpoint of $\overline{\text{CD}}$. Thus, we want to show G' lies on the nine-point circle of $\triangle DEF$. Let M be the midpoint of $\overline{\text{DF}}$. Then, $(MD'F')$ is said nine-point circle. We have

$$\measuredangle MG'D' = \measuredangle(\overline{\text{BC}}, \overline{\text{EC}}) = \measuredangle BCA = \measuredangle BDA = \measuredangle D'DM = \measuredangle MF'D',$$

so G' lies on the circle as desired. ■