2001 C4

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A set of three nonnegative integers $\{x, y, z\}$ with $x < y < z$ is called historic if $\{z - y, y - x\} = \{1776, 2001\}$. Show that the set of all nonnegative integers can be written as the union of pairwise disjoint historic sets.

We use the following greedy algorithm: In each step we take the least nonnegative integer x that has not been selected and add the set $\{x, x+1776, x+1776+2001\}$ if it's available and $\{x, x+2001, x+1776, x+1776+2001\}$ otherwise. Also, when choosing a historic set, we color the least number red, the greatest blue and the middle one green. It suffices to show that if in a step we pick x, then $x + 1776 + 2001$ and one of $x + 1776, x + 2001$ have not been colored. Clearly they are not colored red, since they are greater than x. If $x + 1776 + 2001$ is green, then one of $x + 1776, x + 2001$ must be red. If it is blue, then x would have already been chosen. In conclusion, $x + 1776 + 2001$ is not colored. Now, if $x + 2001$ were green, $x + 225$ would be red, but this is not possible. Hence, if $x + 2001$ is colored, it is blue. Then, $x - 1776$ is red and x is verde, but x was not colored. Therefore, we can always choose $x + 2001$. Thus the algorithm never fails and $\mathbb{N}$ is partitioned. ■