2019 A1

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Solve over $\mathbb{Z}$ the functional equation $f(2a) + 2f(b) = f(f(a+b))$.

We claim the only solutions are $f(x) = 2x + c$ for any integer c, which clearly work. Setting $(a, b) = (0, n)$ and $(1, n-1)$ we obtain

$$f(0) + 2f(n) = f(f(n)) = f(2) + 2f(n-1),$$

so $f(n) - f(n-1) = \frac{f(2)-f(0)}{2}$. Since the right hand side is a constant and $f\colon \mathbb{Z} \to \mathbb{Z}$, it follows that $f(x) = mx + c$. Then, substituting in the original equation we find $m = 2$ and c can be anything. $\blacksquare$