# 2004 G1

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October 16, 2021

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Let $ABC$ be an acute-angled triangle with $AB \neq AC$. The circle with diameter $BC$ intersects the sides $AB$ and $AC$ at $M$ and $N$ respectively. Denote by $O$ the midpoint of the side $BC$. The bisectors of the angles $\angle BAC$ and $\angle MON$ intersect at $R$. Prove that the circumcircles of the triangles $BMR$ and $CNR$ have a common point lying on the side $BC$.

Note that $R$ is the intersection of the interior angle bisector of $\angle MAN$ and the perpendicular bisector of $\overline{\text{MN}}$. Thus, $R$ lies on $(AMN)$. Then, consider the intersection, $L$, of $(BMR)$ with $\overline{\text{BC}}$. By Miquel point, $(CNL)$ passes through $R$ implying the result. ■