2005 G2

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March 4, 2022

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Six points are chosen on the sides of an equilateral triangle ABC: A_1, A_2 on BC, B_1, B_2 on CA and C_1, C_2 on AB, such that they are the vertices of a convex hexagon $A_1A_2B_1B_2C_1C_2$ with equal side lengths.

Prove that the lines A_1B_2, B_1C_2 and C_1A_2 are concurrent.

Let P be the point inside the hexagon such that $\triangle A_1PA_2$ is equilateral. Then, since $\overline{\mathrm{PA_2}} \parallel \overline{\mathrm{B_1B_2}}$ and $PA_2 = B_1B_2$, it follows $PB_2B_1A_2$ is a rhombus. analogously, $PC_1C_2A_1$ is a rhombus. Thus, P is equidistant to A_1, A_2, B_2, C_1, so $A_1A_2B_2C_1$ is cyclic. Since $A_1A_2 = B_2C_1$, it is an isosceles trapezoid. Note that B_1 and C_2 lie on the perpendicular bisector of $\overline{\mathrm{A_2B_2}}$ and $\overline{\mathrm{A_1C_1}}$. Since the diagonals $\overline{\mathrm{A_2C_1}}, \overline{\mathrm{A_1B_2}}$ of the isosceles trapezoid meet at a point on this perpendicular bisector, it follows the three diagonals of the hexagon concur. ■