Part IV

Two player games, Minimax and Alpha-beta pruning

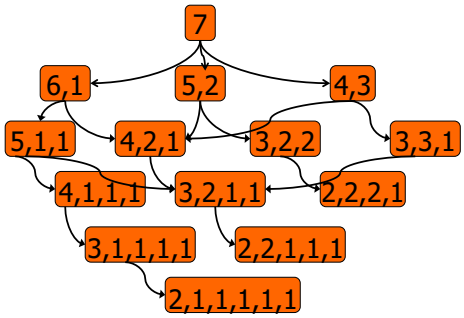
The Game of Jianshizi (Nim)

- There are two opponents
- A number of stones are placed on a table
- At each move the player must divide a pile of stones into two non-empty piles of different sizes
- Nim is a so-called misère (povery) game in that the first person who cannot move looses

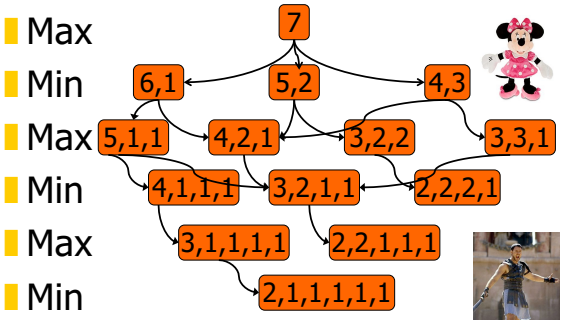
Example game play for 8 stones

- 1 ● ● ● ● ● ● ● ●
- 2 ● ● ● ● ● ● ● ●
- 1 ● ● ● ● ● ● ● ●
- 2 ● ● ● ● ● ● ● ●
- 1 ● ● ● ● ● ● ● ●
- Player 2 then wins

Every game played with 7 stones



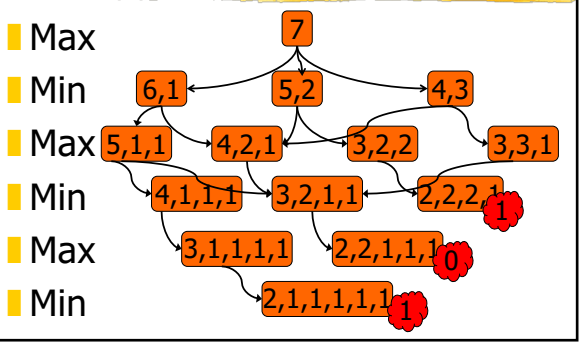
Max = Maximus (player) vs Min = Minny (opponent)



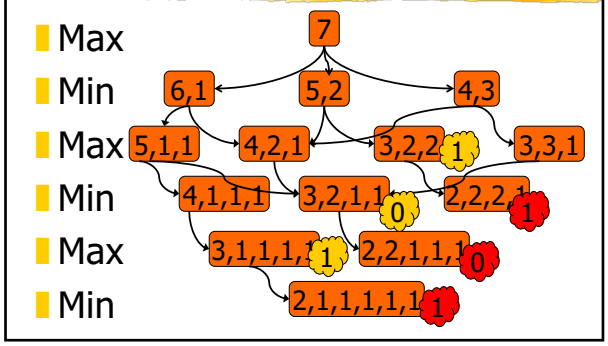
Minimax [von Neumann, 1928] for reasoning about who wins

- Max wants to maximise Max's score
- Min wants to minimise Max' s score
- Decorate the leaf nodes with a value that indicates who wins:
  - ie. 1 if Max wins
  - ie. 0 if Max loses
- Decorate the nodes of the tree by propagating values upwards

Decorating terminal nodes



Decorate nodes with just one successor



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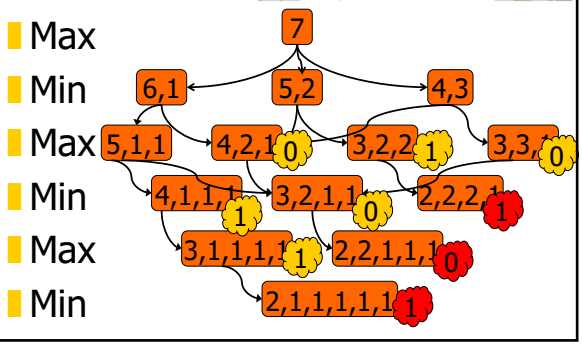
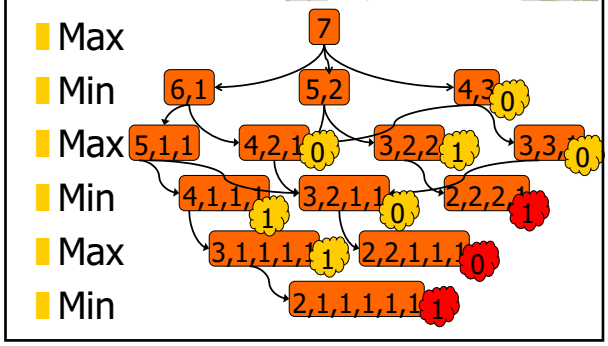
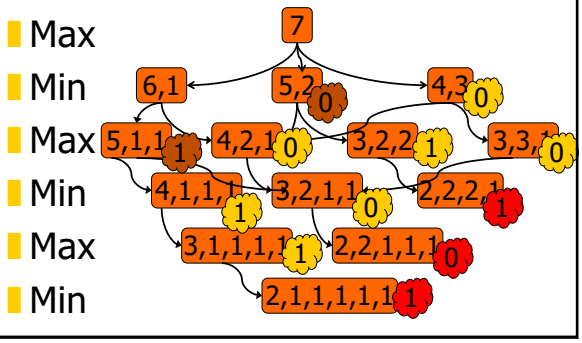


Decorate nodes with just one successor



Can a player win *if* they make good choices? (Minimax)



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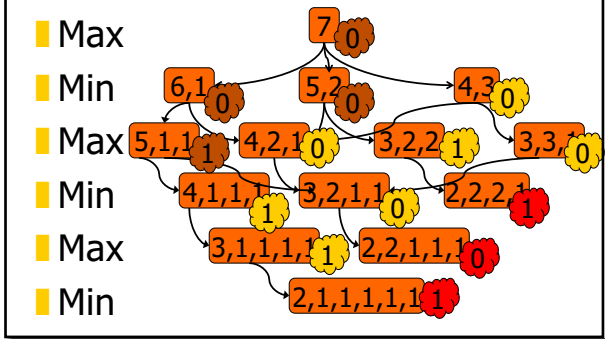


Reviewing the complete search tree

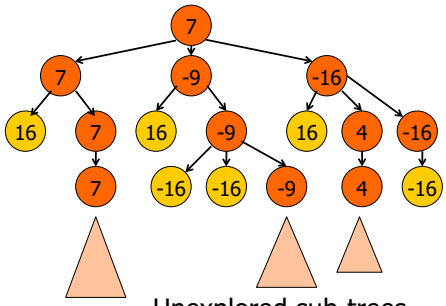
- What are Max's prospects for winning?
- What move should Min do if she has piles of 5, 2 stones?
- Suppose Min makes the wrong move into 5, 1, 1 stones. What should Max do next?
- The decoration represents the best that the player (in that state) can achieve

Other games

- Theoretically perfect: 0-1 games where one player has as much information as their opponent (Go, Othello)
- Theoretically imperfect: poker
- Practically imperfect: cannot see to game end
- At a Max ply in Chess:
  - ie. if Max checkmates Min, then value of +32
  - ie. if Min checkmates Max, value of -32
  - ie. else heuristic strength of -32 ≤ value ≤ 32

Heuristic Minimax for Chess with 4-ply look ahead

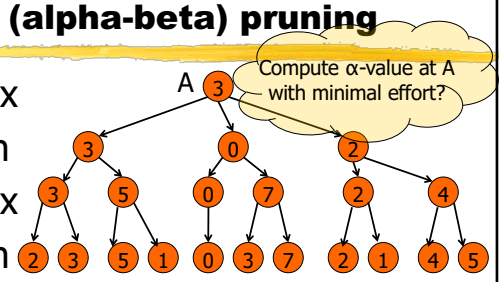
- Max
- Min
- Max
- Min



Unexplored sub-trees

α–β (alpha-beta) pruning

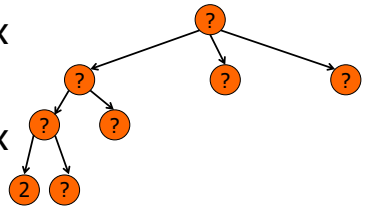
- Max
- Min
- Max
- Min



- α-values in Max ply never decrease when another child is known
- β-values in Min ply never increase

α–β pruning (step 1)

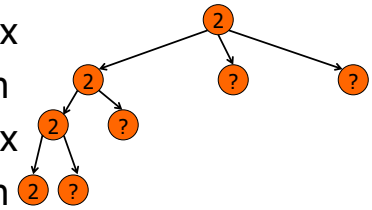
- Max
- Min
- Max
- Min



Descend to depth of 4-ply and calculate heuristic of 2, say, for leaf

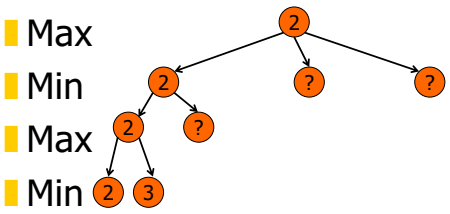
α–β pruning (step 2)

- Max
- Min
- Max
- Min



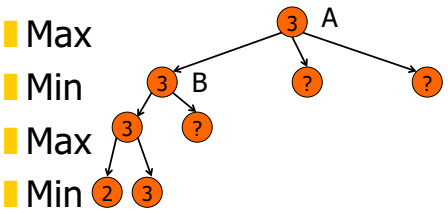
Propagate heuristic value up to root

α–β pruning (step 3)



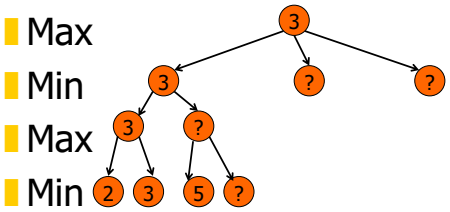
Descend and calculate next heuristic value, say, 3

α–β pruning (step 4)



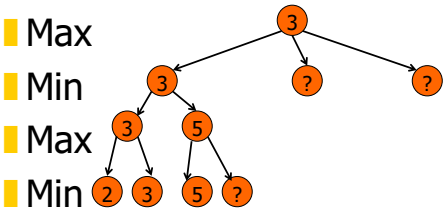
Propagate 3 up to root
Update A and B since they only have one evaluated successor

α–β pruning (step 5)



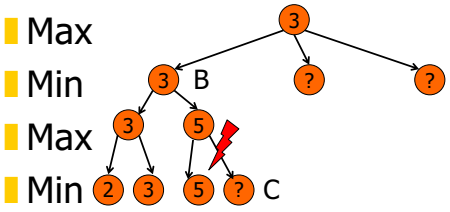
Descend to depth of 4-ply and assign heuristic of 5, say, to next leaf

α–β pruning (step 6)



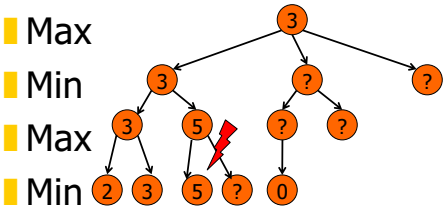
Propagate 5 back up to root

α–β pruning (step 7)



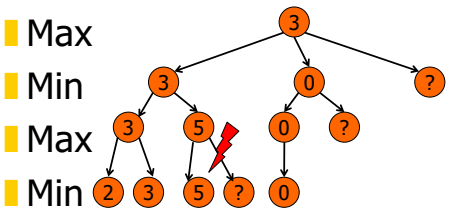
No need to compute heuristic for C since this cannot effect B

α–β pruning (step 8)



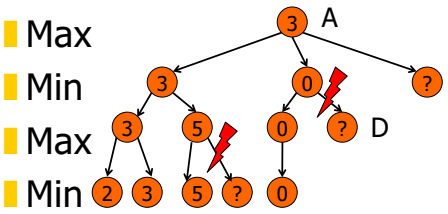
Descend to depth of 4-ply and assign heuristic of 0, say, to next leaf

α–β pruning (step 9)



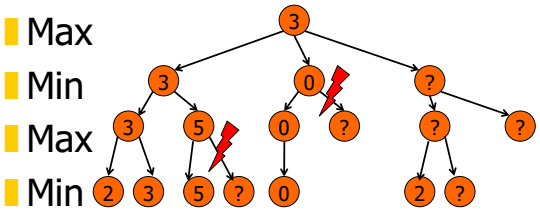
Propagate 0 up the tree

α–β pruning (step 10)



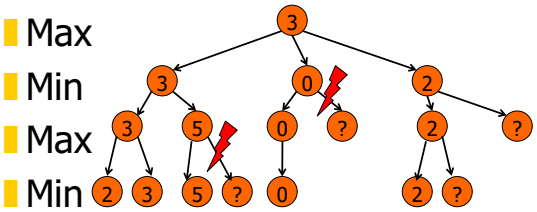
No need to compute heuristic for D
since this cannot effect A

α–β pruning (step 11)



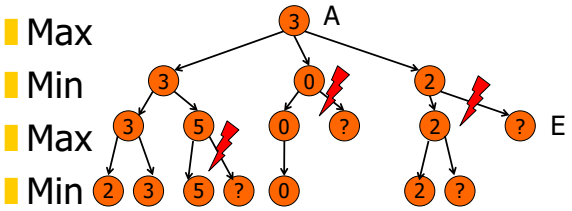
Descend to depth of 4-ply and assign
heuristic of 2, say, to next leaf

α–β pruning (step 12)



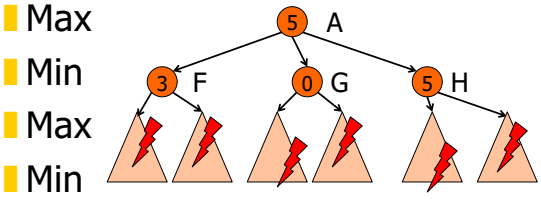
Propagate 2 up the tree

α–β pruning (step 13)



No need to compute heuristic for E
since this cannot effect A

How can Max make use of α–β pruning when at A?



Find β-values at F, G and H
Move to H as best chance of winning

Summary

- Focus is on two player games
- Use min or max to decorate alternate plys
- Use heuristics when trees are deep
- Do not need to decorate every sub-trees