Ezra Rowitsch

MAE 529: Finite Element Structural Analysis

12/21/2020

Introduction:

In this report we will be examining the natural frequencies and associate mode shapes of several plates using the student version of the commercial finite element code Abaqus. This examination will look at plates of three different thickness ratios and the use of both shell and solid elements in the model.

Assumptions:

Several properties of these plates will be calculated using my person number, which is 37641406. If *mncd ijkl* represent the person number then we have k = 0 and l = 6. From this we know that the geometry and boundary conditions should consist of a rectangular plate with two opposite edges simply supported and the other two free. We can also calculate the aspect ration γ.

$$\gamma = \frac{k+4}{4} = \frac{0+4}{4} = 1$$

Now we know γ=1 which means our rectangular plate will in fact be a square. We arbitrarily choose that the sides of our square plate will be 4 units. Since we were given that 2a and 2b would represent the length in the x and y directions respectively we can see that both a = 2 and b = 2. We were given three thickness ratios to examine, which define the thickness h of the plate as follows.

$$\xi = \frac{h}{b}$$

We were given ξ_1 = 1/8, ξ_2 = 1/16, ξ_3 = 1/32, so from the preceding relation we can get h_1 = 1/4, h_2 = 1/8, h_3 = 1/16.

With all the dimensions of the plate found we need to assign some physical constraints. We were given Poisson's ration ν = .25, and we will put density ρ=.281 and elastic modulus E = 28000.

Plate 1, ξ = 1/8:

First we will look at examining the plate using solid elements. First we modelled the 4x4x1/4 plate in Abaqus and set the boundary conditions on the left and right side as XSYMM. This would represent the simply supported opposite edges. The model was then seeded such that it will always have at least 2 elements deep, as it was found that models only a single element deep give unsatisfactory results. We used 8-node linear brocks with incompatible modes for the element type. It was found that this element type resulted in much better convergence and allowed for reaching higher modes than using reduced integration elements.

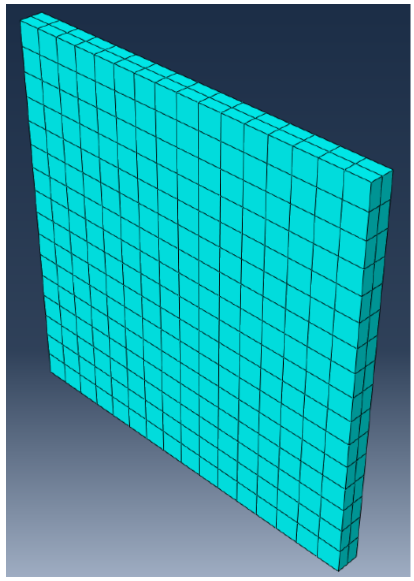

Figure 1: *Plate ξ=1/8 broken up into a 512 element mesh*

We took 3 different meshes, each with a different number of elements.

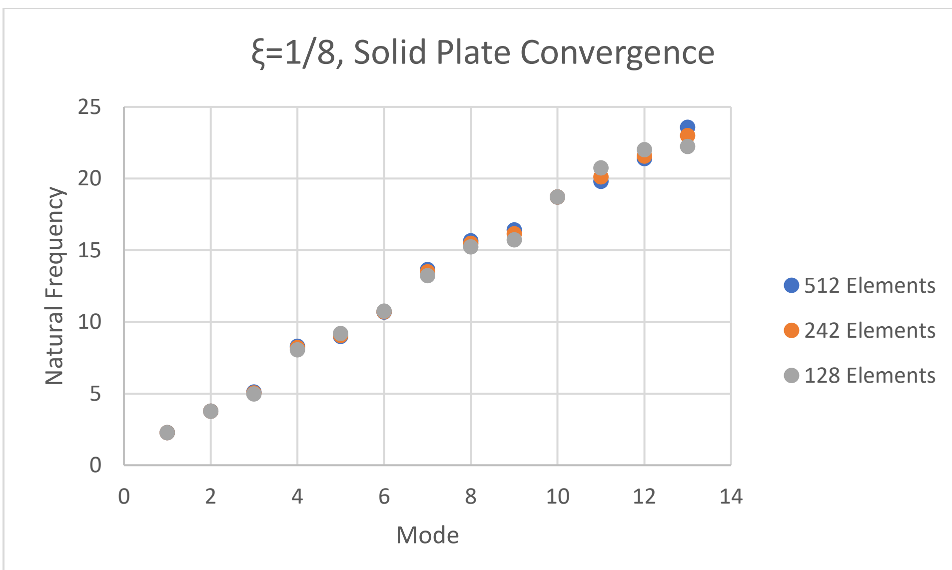

Figure 2: *Plate ξ=1/8, with solid elements natural frequency convergence*

As can be seen in Figure 2 this model converges very nicely.

Mode	Natural Frequency
1	2.26
2	3.77
3	5.10
4	8.29
5	8.98
6	10.68
7	13.65
8	15.64
9	16.42
10	18.71
11	19.79
12	21.38
13	23.56

Chart 1: *Listed Natural Frequencies Found Using Solid Elements*

The list of natural frequencies extracted from running the model can be found in chart 1.

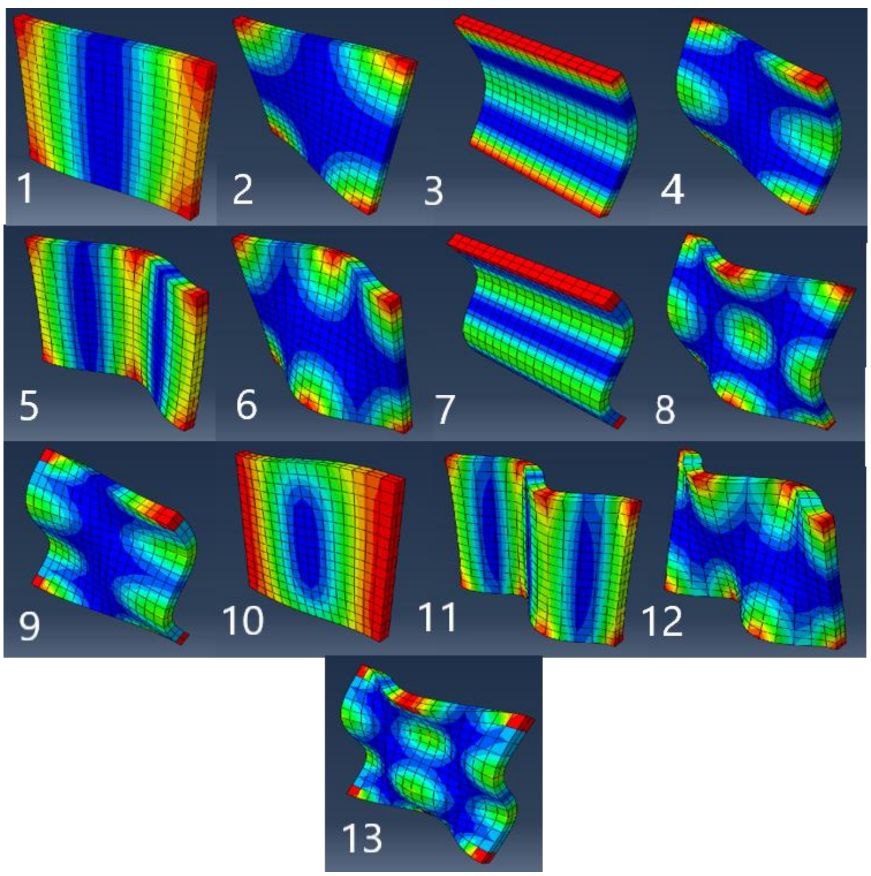

Figure 3: *Plate ξ=1/8 solid elements mode shapes*

We then want to compare using shell elements. For this we run a similar analysis using continuum shell elements instead.

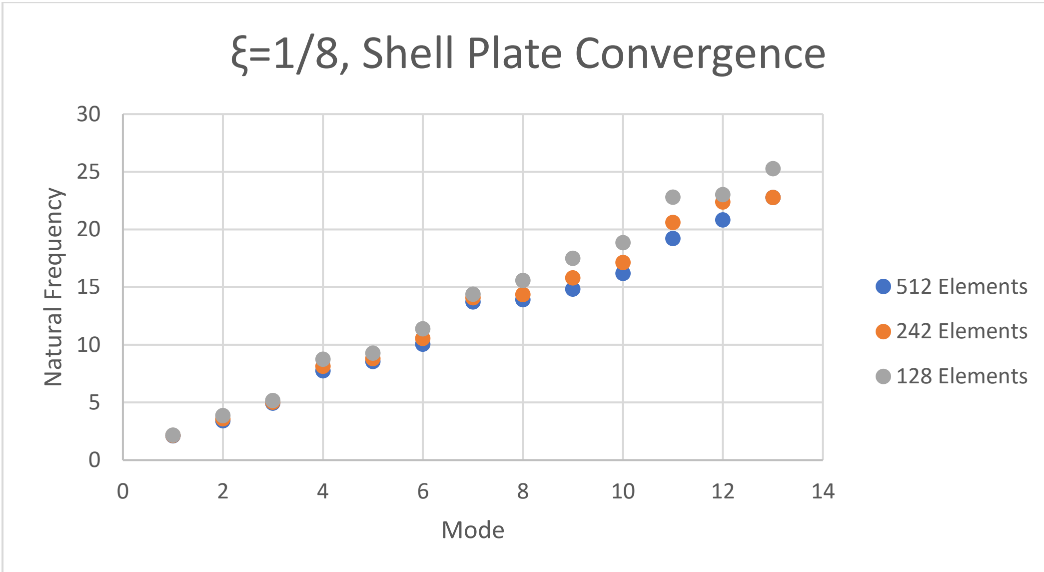

Figure 4: *Plate ξ=1/8, with shell elements natural frequency convergence*

As can be seen in Figure 4 we also got pretty good convergence with the shell elements, although it looks like it is not quite as good as the solid elements.

Mode	Natural Frequency
1	2.12
2	3.41
3	4.94
4	7.75
5	8.57
6	10.07
7	13.72
8	13.91
9	14.83
10	16.21
11	19.24
12	20.83
13	22.77

Chart 2: *Listed Natural Frequencies Found Using Shell Elements*

As can be seen in Chart 2 the natural frequencies found using shell elements end up being very close to what we found with solid elements, with the exception of modes 8, 9, 10, and 13. It also seems that these are resolving to generally be slightly lower than on the solid elements.

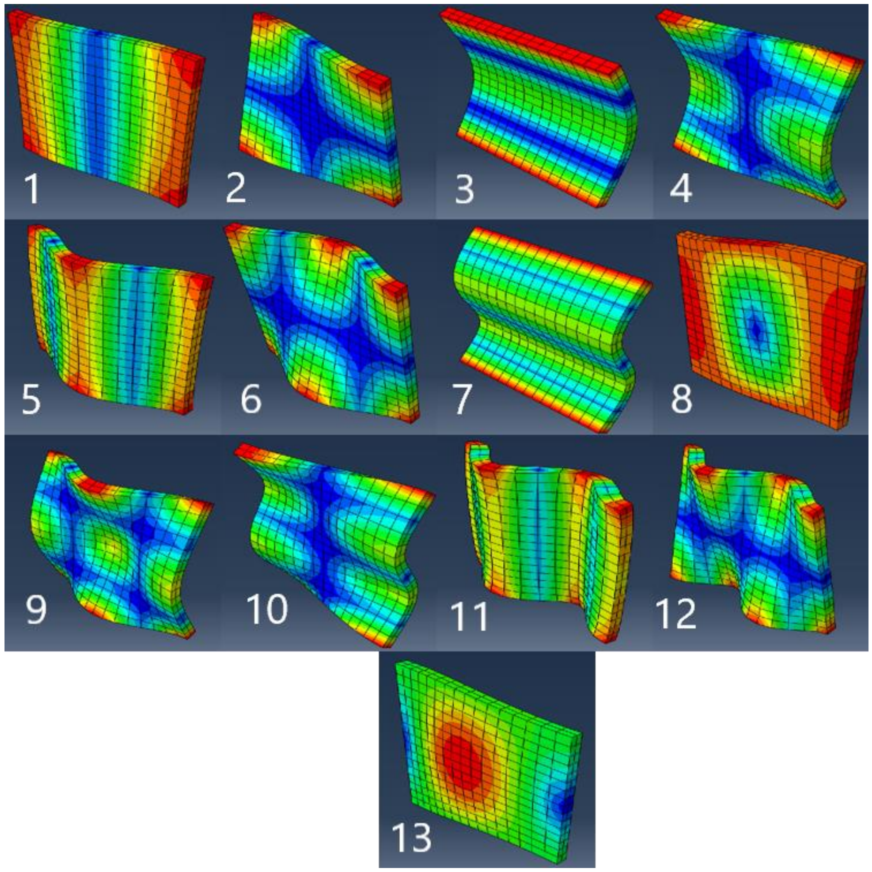

Figure 5: *Plate ξ=1/8 shell element mode shapes*

As can be seen in Figure 5 we get similar mode shapes from the shell elements that we were getting from the solid elements although it looks like modes 8, 9, 10 and 13 have different shapes. Looking at the natural frequency charts it looks as though these also are significantly different so it seems that the solid elements and shell elements are unable to resolve the same mode shapes. It also looks like the blue areas of low displacement are much thinner and more concentrated than in the solid elements.

It is also worth noting that for both solid and shell elements for all three thicknesses we received three mode shapes that were attached to frequencies that were very low and close to zero. These were not shown in either the charts or the figures as they appeared in all plates and element types and are likely related to the rigid body motion that is possible with the boundary conditions being used.

Plate 2, ξ = 1/16:

Now we will examine the plate with the 1/16 thickness ratio using similar elements and boundary conditions as for the 1/8 thickness ratio plate. So first we will look at the results we received using the 8-node linear brocks with incompatible modes solid elements.

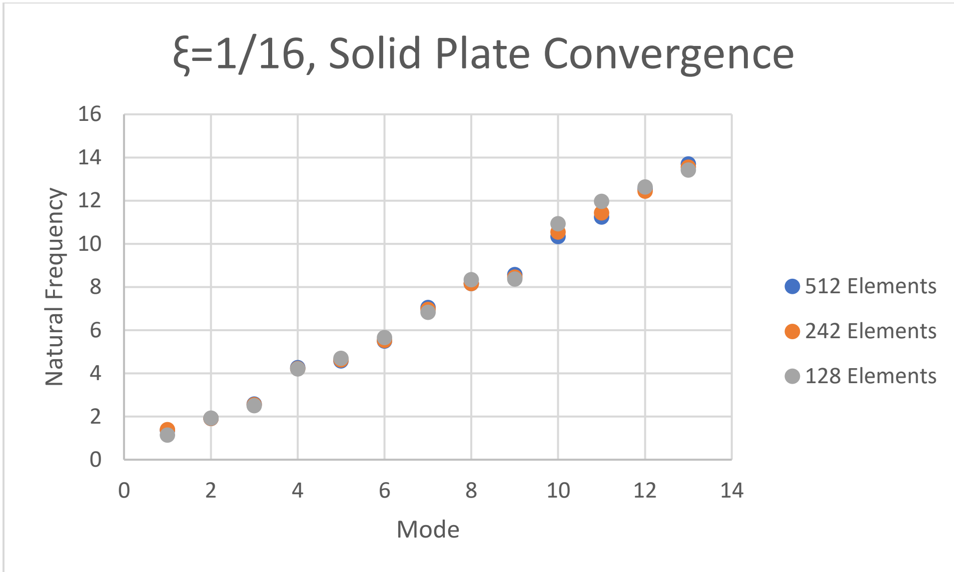

Figure 6: *Plate ξ=1/16, with solid elements natural frequency convergence*

As can be seen in Figure 6 this set up also converges very nicely.

Mode	Natural Frequency
1	1.36
2	1.91
3	2.58
4	4.26
5	4.58
6	5.49
7	7.05
8	8.17
9	8.57
10	10.34
11	11.24
12	12.55
13	13.70

Chart 3: *Listed Natural Frequencies Found Using Solid Elements*

Chart 3 shows the listed natural frequencies from this analysis. It seems that the natural frequencies are roughly half of what we received from the $\xi = 1/8$ plate analysis.

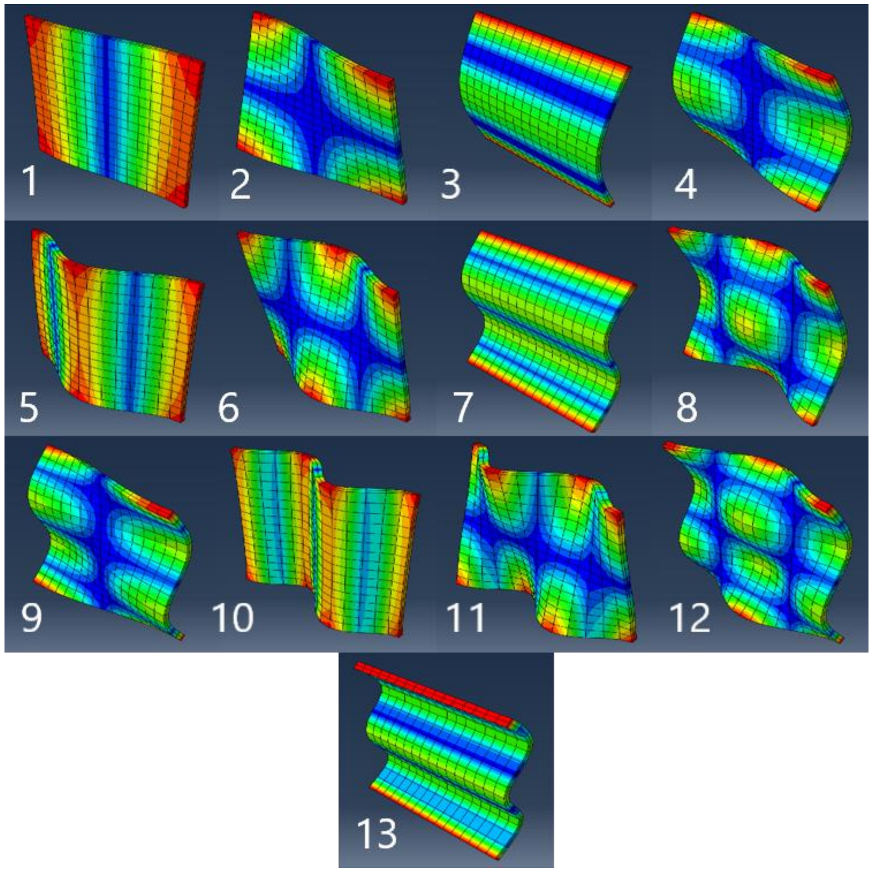

Figure 7: *Plate ξ=1/16 solid element mode shapes*

The mode shapes from this analysis can be found in Figure 7. They seem to follow a similar pattern as what was found in the ξ = 1/8 plate.

Now we need to look at this plate using shell elements.

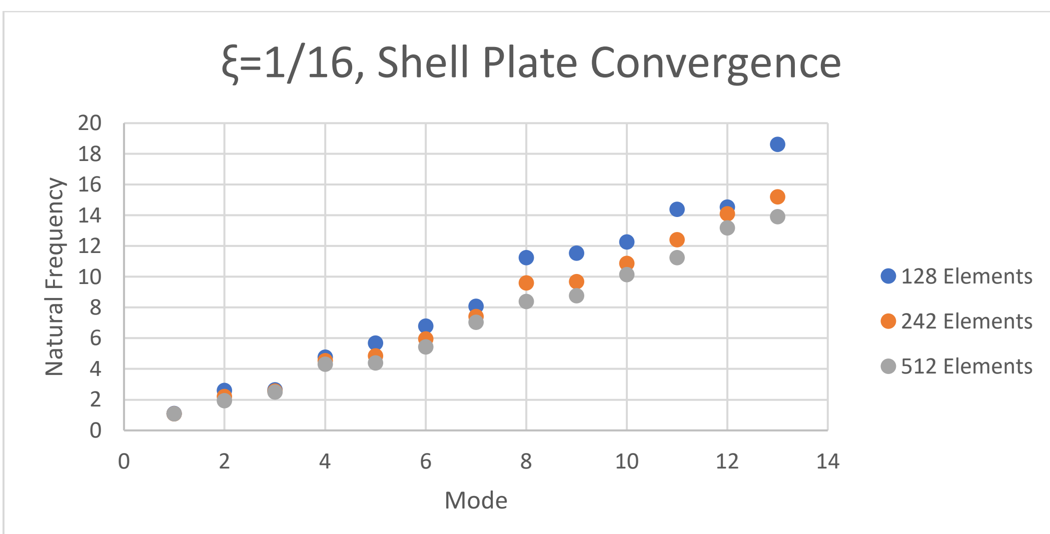

Figure 8: *Plate ξ=1/16, with shell elements natural frequency convergence*

As can be seen from Figure 8 we also are approaching convergence with using shell elements. We are using the student version of Abaqus so we were limited to how many nodes we could go to but likely one step smaller would have converged very well.

Mode	Natural Frequency
1	1.07
2	1.92
3	2.50
4	4.31
5	4.38
6	5.43
7	7.03
8	8.39
9	8.76
10	10.13
11	11.24
12	13.18
13	13.91

Chart 4: *Listed Natural Frequencies Found Using Shell Elements*

As can be seen from comparing Chart 4 and Chart 3 both elements seem to lead to similar natural frequencies, although mode 12 seems to be particularly off and modes 4 and 5 came to a much closer values than they did in the solid element analysis.

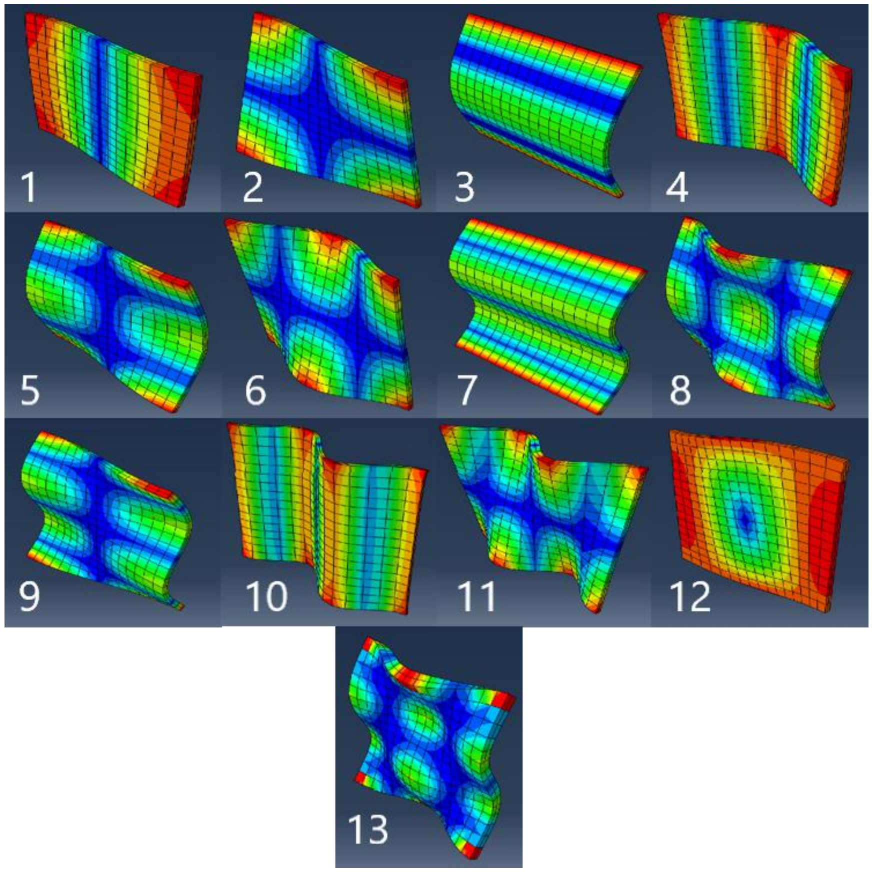

Figure 9: *Plate ξ=1/16 shell element mode shapes*

Figure 9 Shows the associated shell element mode shapes. If we compare to Figure 7 we can see some pretty noticeable differences. Mainly it seems as though modes 4 and 5 switched places, and mode 12 does not have a comparable shape from the solid element analysis. These misses are noticeable as well in the frequency charts 3 and 4. I believe that the solid elements are having trouble recreating the kind of shape shown in figure 9 mode 12. For this plate the solid elements did not show any of these kinds of modes and for the ξ = 1/8 plate the solid elements had 1 similar mode and the shell elements had 2.

Plate 3, ξ = 1/32:

Now we will examine the plate with the 1/32 thickness ratio using similar elements and boundary conditions as for the 1/8 thickness ratio plate. So first we will look at the results we received using the 8-node linear bricks with incompatible modes solid elements.

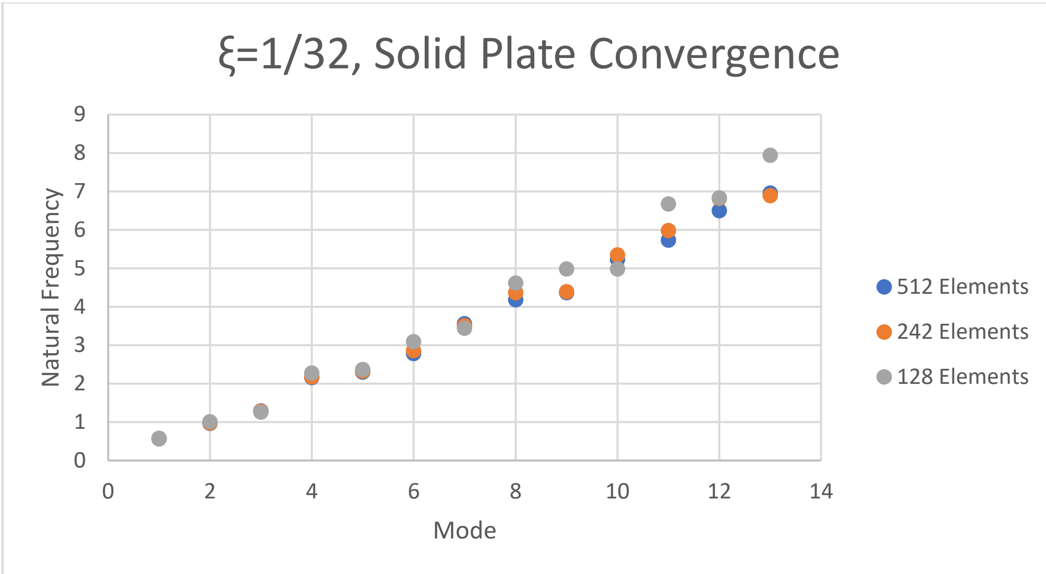

Figure 10: *Plate ξ=1/32, with solid elements natural frequency convergence*

As can be seen from Figure 10 these values also converge nicely.

Mode	Natural Frequency
1	0.57
2	0.96
3	1.29
4	2.16
5	2.30
6	2.78
7	3.55
8	4.18
9	4.36
10	5.23
11	5.74
12	6.50
13	6.96

Chart 5: *Listed Natural Frequencies Found Using Solid Elements*

The above Chart 5 shows the natural frequencies found using solid analysis. It can be seen immediately that again the frequencies are roughly half what we got from the ξ = 1/16 plate.

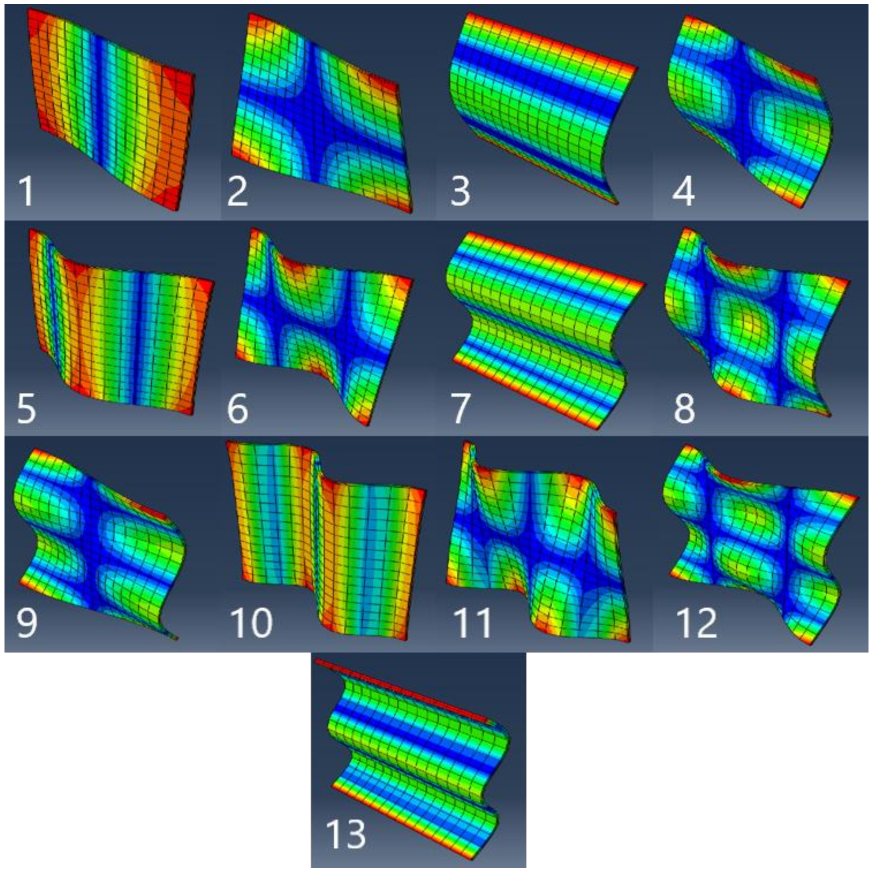

Figure 11: *Plate ξ=1/32 solid element mode shapes*

The above Figure 11 shows the solid element mode shapes.

Now we do a similar analysis using shell elements.

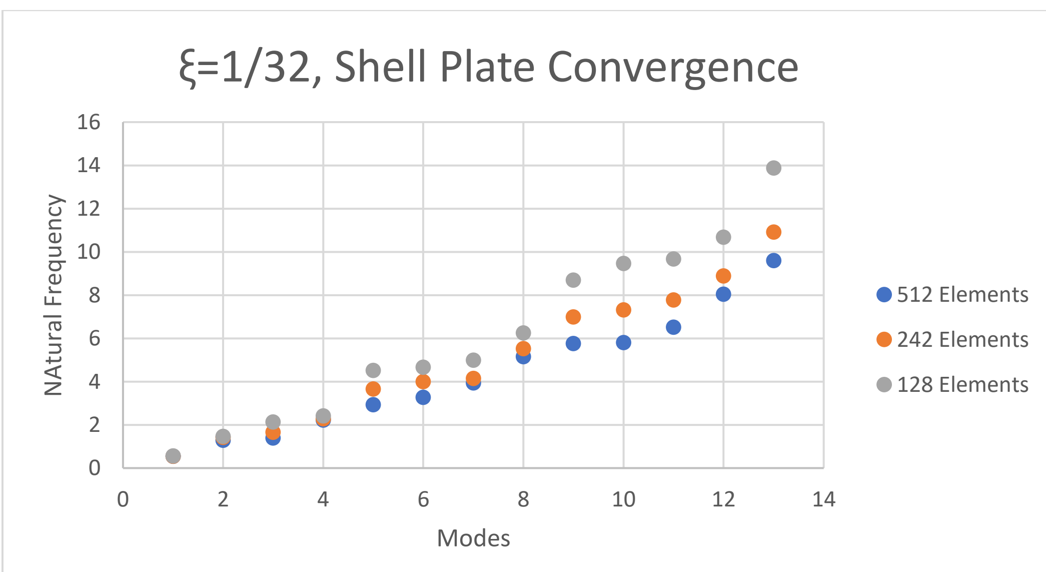

Figure 12: *Plate ξ=1/32, with shell elements natural frequency convergence*

As can be seen from the above, this method definitely needs a little more convergence and we could probably get that done if we were not using the Abaqus student version. With that said we can still get some relatively close values.

Mode	Natural Frequency
1	0.54
2	1.29
3	1.39
4	2.22
5	2.92
6	3.27
7	3.94
8	5.16
9	5.76
10	5.81
11	6.51
12	8.04
13	9.60

Chart 6: *Listed Natural Frequencies Found Using Shell Elements*

As can be seen from Chart 6 the natural frequency mode 1 is close to the value we got for the solid elements. Mode 2 does not match up with the Mode 2 from the solid elements in chart 5, but it does seem to match up with Mode 3. Several other values seem to line up between the two charts but the higher modes definitely have significantly higher and different frequencies.

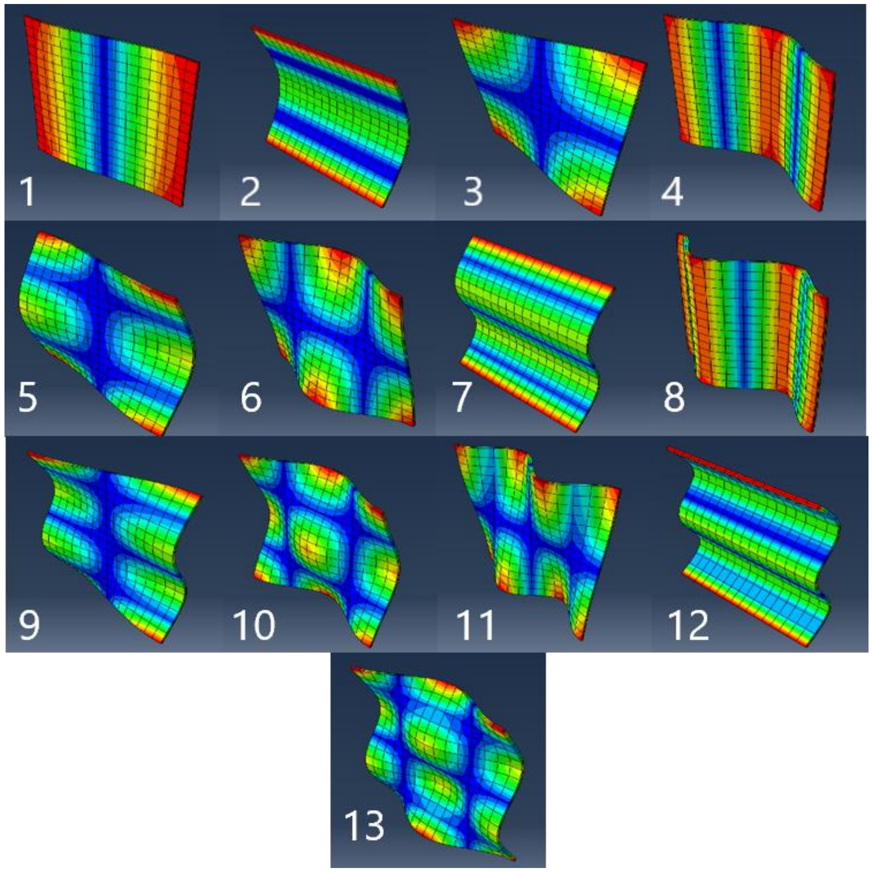

Figure 13: *Plate ξ=1/32 shell element mode shapes*

Figure 13 shows the associated mode shapes of the natural frequencies. It looks like the Mode 2 shape with the shell elements match with Mode 3 from the solid elements, so it would seem the shell elements model was having trouble generating the results for that mode shape. Another similar situation seems to be happening between Mode 10 from the solid elements and Mode 8 from the shell elements. This would explain why there were such disparate results between the natural frequencies and mode shapes for the solid and shell elements.

Conclusion:

From the results we found that for the thicker plates we had both shell and solid element models returning similar natural frequencies and mode shapes. It seems as though each time we halved the thickness ratio the associated natural frequencies were also halved. Interestingly we found that the solid elements with incompatible modes converged quicker and a little tighter than the shell elements did. It did seem that particularly for the thinnest plate we looked at that the different types of elements would simply not return some mode shapes leading to us getting matching results for some frequencies and shapes and missing others. If we were not using the Abaqus student version we would have liked to reduce the element size even smaller to see if we could resolve that issue or get the shell elements to converge at least a little nicer on the thinner plates.