Partial Solution of 3 x N Chomp boards in the form of [2, H, 2]

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Summer Research Project

- Summer Research 2020
- With Dr. B
- Combinatorial Games, specifically Chomp

1. What is the Combinatorial Game Chomp?
2. So What Did I Do?
3. Divide and Conquer
4. Postamble

What is the Combinatorial Game Chomp?

What Makes a Combinatorial Game?

- Deterministic, no randomness
- Perfect Information, nothing is hidden
- At one time only one of the players can be winning

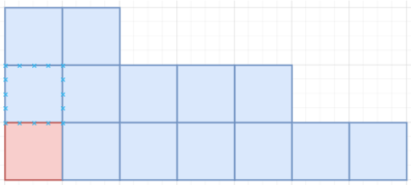
- Two types:
  - Finite, the game will end, you can't loop
  - Infinite, the game may not end, you can create loops

But what is Chomp?

- Played on a board like a chocolate bar where the lower left piece is poisoned
- You don't want to eat the poison, unless you've built up a resistance to iocane powder
- Alternating turns where you choose a piece and break off all pieces above and to the right of it

What Are The Different Board Types?

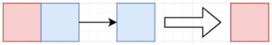
- Prescriptive: 2 x N, N x N, etc.
- Pictographic: just a picture



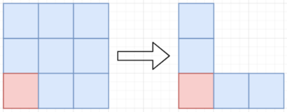
- Long form: truncated, lists columns by height
  - $\{3, 3, 2, 2, 2, 1, 1\}$
- Short Form: truncated, lists width of groups of column heights
  - $[2, 3, 2]$

What Has Already Been Done?

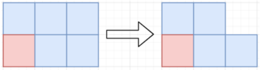
- 1 x N: Go first, leave only poisoned piece



- N x N: Go first, make an L, Tweedledee-Tweedledum

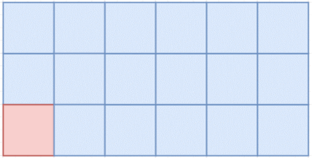


- 2 x N: Go first, take top right, Tweedledee-Tweedledum

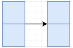


What Are The Board Positions?

- Winning and losing positions (P vs N)
  - Every board has a recursively defined Nim Number, but they blew up too fast
- Looking at 3 x N boards



  - Note the arrow can be any number of blocks including zero
  - I call this an “H-block”



So What Did I Do?

Data Mining with the Short Form

- Wrote a Python script to iterate through all boards from a maximum size to get its position state and those of its children recursively with memoization
  - Work toward the base case, can exit early
  - Code on my GitHub for the curious
- Noticed a pattern in the Long Form, but it was hard to see, so I made the Short Form
  - $\{3, 3, 2, \ldots 2, 1, 1\}$ was always a winning position
  - $[2, H, 2]$
- Saw a pattern, tried to prove it

Building a Seeded Kindergarten

Two ways to generate the children:

- Generating the first generation of children visually
- Generating the first generation of children algorithmically

Children of $[2, H, 2]$:

- In other words, if I gave my opponent a $[2, H, 2]$, what could they give me back?
- All 1st generation children are losing positions

The Children of $[2, H, 2]$

As generated by the almighty algorithm

- Top:
  - $[1, H + 1, 2]$
  - $[0, H + 2, 2]$
- Middle:
  - $[2, H - K, 2 + K]$
  - $[2, H - H, 2 + H]$
  - $[1, 0, 2 + H + 1]$
  - $[0, 0, 2 + H + 2]$
- Bottom:
  - $[2, H, 1]$ s.t. $H \geq 1$
  - $[2, H, 1]$ s.t. $H = 0$
  - $[2, H, 0]$
  - $[2, H - K, 0]$ s.t. $(H{-}K) \geq 2$
  - $[2, H - K, 0]$ s.t. $(H{-}K) = 1$
  - $[2, H - K, 0]$ s.t. $(H{-}K) = 0$
  - $[1, 0, 0]$
  - $[0]$

The Children of $[2, H, 2]$

Simplified using two rules

- Top:
  - $[1, H, 2]$
  - $[0, H, 2]$
- Middle:
  - $[2, H, 2 + K]$
  - $\cancel{[2, 0, 2 + H]}$
  - $[1, 0, H]$
  - $[H]$
- Bottom:
  - $[2, H, 1]$
  - $[2, H, 1]$
  - $[2, H, 0]$
  - $\cancel{[2, H, 0]}$
  - $[2, 1, 0]$
  - $[2, 0, 0]$
  - $[1, 0, 0]$
  - $\cancel{[0]}$

The Children of $[2, H, 2]$

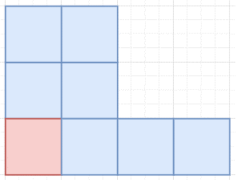
Reordered

- Group A:
  - $[H]$
  - $[1, 0, 0]$
- Group B:
  - $[1, H, 2]$
  - $[0, H, 2]$
  - $[2, 0, 0]$
- Group C:
  - $[1, 0, H]$
  - $[2, 0, 1]$
  - $[2, 1, 0]$
- Group D:
  - $[2, H, 2 + K]$
  - $[2, H, 1]$
  - $[2, H, 0]$

A Funky Dude

By which I mean the base case but who has time to say all that

- Aka $[2, H, 2]$ when $H = 0$
  - Just Follow the normal table and treat H as zero
  - It works fine, but I just wanted to address the base case real fast



Divide and Conquer

The Children of $[2, H, 2]$

Reordered

- Group A:
  - $[H]$
  - $[1, 0, 0]$
- Group B:
  - $[1, H, 2]$
  - $[0, H, 2]$
  - $[2, 0, 0]$
- Group C:
  - $[1, 0, H]$
  - $[2, 0, 1]$
  - $[2, 1, 0]$
- Group D:
  - $[2, H, 2 + K]$
  - $[2, H, 1]$
  - $[2, H, 0]$

Group A

Reduces to 1 x N

$[H]$

$[1, 0, 0]$

$[H]$

Group A: Reduces to 1 x N

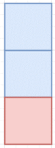
$[0, 0, H + 2 + 2]$;
Opponent left only the bottom row

$[1]$;
Leave only the last piece

$[1, 0, 0]$

Group A: Reduces to 1 x N

$[1, H - H, 2 - 2]$;
Opponent broke off all 2's, all 1's, and one 3



$[1]$;
Leave only the last piece

The Children of $[2, H, 2]$

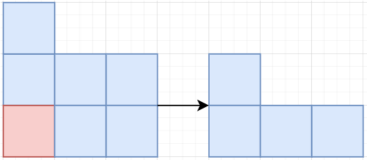
Reordered

- Reduces to 1 x N:
  - $[H]$
  - $[1, 0, 0]$
- Group B:
  - $[1, H, 2]$
  - $[0, H, 2]$
  - $[2, 0, 0]$
- Group C:
  - $[1, 0, H]$
  - $[2, 0, 1]$
  - $[2, 1, 0]$
- Group D:
  - $[2, H, 2 + K]$
  - $[2, H, 1]$
  - $[2, H, 0]$

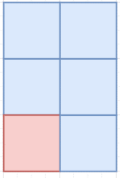
Group B

Reduces to 2 x N

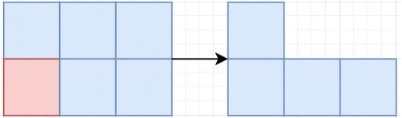
$[1, H, 2]$



$[2, 0, 0]$



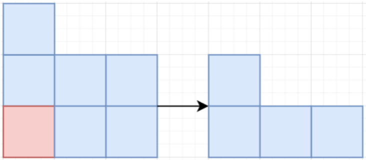
$[0, H, 2]$



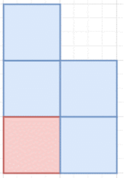
$[1, H, 2]$

Group B: Reduces to 2 x N

$[2 - 1, H + 1, 2]$;
Opponent removed one top block



$[1, 1, 0]$;
Break off all but one 2

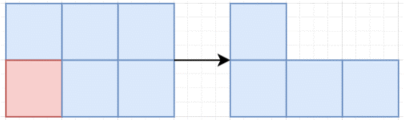


$[0, H, 2]$

Group B: Reduces to 2 x N

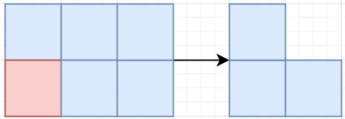
$[2 - 2, H + 2, 2 - 2]$;

Opponent broke off both 3's



$[H, 1]$;

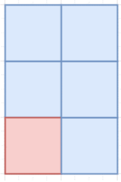
Break off the 1 one the end, staircase it



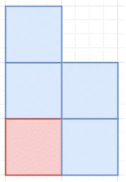
$[2, 0, 0]$

Group B: Reduces to 2 x N

$[2, H - H, 2 - 2]$;
Opponent broke off all 2's and all 1's



$[1]$;
Break off one 3, Sideways Staircase



The Children of $[2, H, 2]$

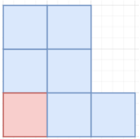
Reordered

- Reduces to 1 x N:
  - $[H]$
  - $[1, 0, 0]$
- Reduces to 2 x N:
  - $[1, H, 2]$
  - $[0, H, 2]$
  - $[2, 0, 0]$
- Group C:
  - $[1, 0, H]$
  - $[2, 0, 1]$
  - $[2, 1, 0]$
- Group D:
  - $[2, H, 2 + K]$
  - $[2, H, 1]$
  - $[2, H, 0]$

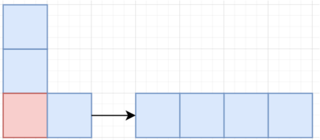
Group C

Reduces to N x N

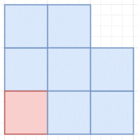
$[2, 0, 1]$



$[1, 0, H]$



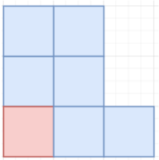
$[2, 1, 0]$



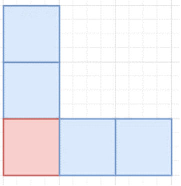
$[2, 0, 1]$

Group C: Reduces to N x N

$[2, 0, 2 - 1]$;
Opponent broke off an end piece from $H = 0$



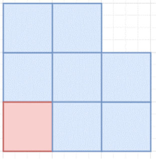
$[1, 0, 2]$;
Leave only the last piece



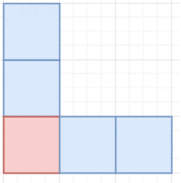
$[2, 1, 0]$

Group C: Reduces to N x N

$[2, 1, 2 - 2]$;
Opponent broke off both end pieces when $H = 1$



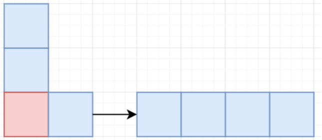
$[1, 0, 2]$;
Break out the center three pieces



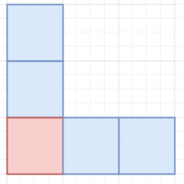
$[1, 0, H]$

Group C: Reduces to N x N

$[1, 0, H + 2 + 1]$;
Opponent all the 2's and one 3 to 1's



$[1, 0, 2]$;
Break off the long part of the foot



The Children of $[2, H, 2]$

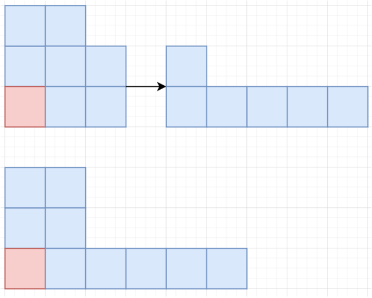
Reordered

- Reduces to 1 x N:
  - $[H]$
  - $[1, 0, 0]$
- Reduces to 2 x N:
  - $[1, H, 2]$
  - $[0, H, 2]$
  - $[2, 0, 0]$
- Reduces to N x N:
  - $[1, 0, H]$
  - $[2, 0, 1]$
  - $[2, 1, 0]$
- Group D:
  - $[2, H, 2 + K]$
  - $[2, H, 1]$
  - $[2, H, 0]$

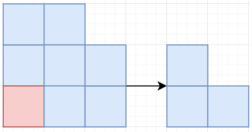
Group D

Maintain $[2, H, 2]$

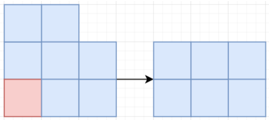
$[2, H, 2 + K]$



$[2, H, 1]$



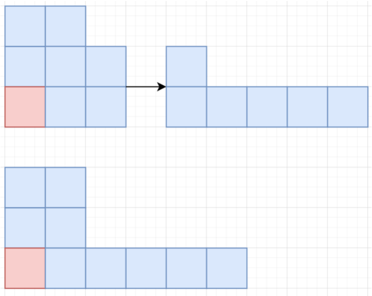
$[2, H, 0]$



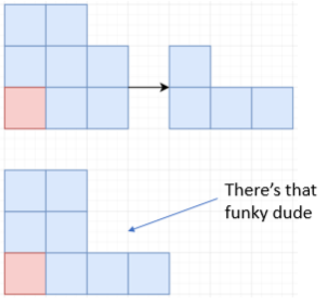
$[2, H, 2 + K]$

Group D: Maintain $[2, H, 2]$

$[2, H - K, 2 + K]$;
Opponent reduced some/all of the 2's



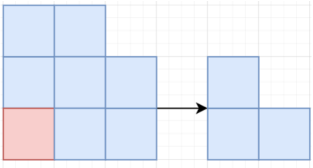
$[2, H, 2 - K]$;
Remove K 1's



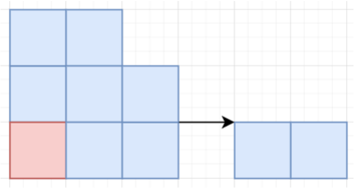
$[2, H, 1]$

Group D: Maintain $[2, H, 2]$

$[2, H, 2-1]$;
Opponent broke off one 1



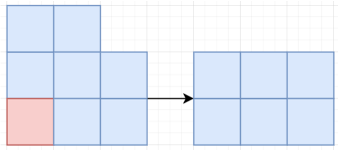
$[2, H-1, 2]$;
Reduce one 2 to a 1



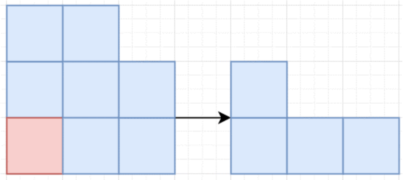
$[2, H, 0]$

Group D: Maintain $[2, H, 2]$

$[2, H, 2-2]$;
Opponent broke off both end pieces



$[2, H-2, 2]$;
Reduce two 2's to 1's



The Children of $[2, H, 2]$

Reordered

- Reduces to 1 x N:
  - $[H]$
  - $[1, 0, 0]$
- Reduces to 2 x N:
  - $[1, H, 2]$
  - $[0, H, 2]$
  - $[2, 0, 0]$
- Reduces to N x N:
  - $[1, 0, H]$
  - $[2, 0, 1]$
  - $[2, 1, 0]$
- Maintain $[2, H, 2]$:
  - $[2, H, 2 + K]$
  - $[2, H, 1]$
  - $[2, H, 0]$

Postamble

Summary and Current Problems

- As far as I can tell, no one else used the truncated forms before
- Partial solution and stable boards

- Not very elegant, cases may reduce
- Can't reach in one move from 3 x N, but it is stable

Slide Template Credit

I (Matt Torrence) who has been speaking to you through these slides designed this template myself. Feel free to delete this slide in your presentation.

I was inspired by the following stack overflow response and used it as a starting point for this template: `tex.stackexchange.com/a/146682/188835`

Typography used includes 4 typefaces:

- Charis SIL, for the default serif type
- `IBM Plex Mono` for monospace text
- Carlito for the default sans-serif type
- TeX Gyre Termes Math for all math mode text