

Ants on a line*

There are N ants on a 1m line, where N is a positive and even integer. The $\frac{N}{2}$ left-most ants are moving right, and the $\frac{N}{2}$ right-most ants are moving left. When 2 ants collide, they will both reverse direction. How many collisions will there have been in total once all ants have fallen off the end of the line?

Source: Quant Trading Guide Callum McDougall (Nov 2020) 4.1 Q6

Solution 1

For $N = 2$, there is only 1 collision. Then $N = 4$, there are initially 1 collision then another 2 before the two ants will fall off. Notice that we return to the case of $N = 2$, so $3 + 1$ collisions. Then $N = 6$, similarly we have the 1 collision, then 2 more collisions then the last 2 collisions before two ants falling off so return to the case $N = 4$ until $N = 2$, so in total $5 + 3 + 1$ collisions. This goes to any N that satisfies the question's restrictions. Hence we can model the total amount of collisions such as,

$$\begin{aligned}
 \text{Total collisions} &= 1 + (2 + 1) + (2 * 2 + 1) + \cdots + (2 * (\frac{N}{2} - 1) + 1) \\
 &= \frac{N}{2} + \frac{N}{2} * (\frac{N}{2} - 1) \\
 &= (\frac{N}{2})^2
 \end{aligned}$$

Solution 2 (Given Sol)

For every collision, both ants reverse direction. Similarly, we can say that for every collision, ants pass each other. Therefore, every ant will collide with $\frac{N}{2}$ ants (the amount of opposite moving ants). However, the total amount of collisions is halved because we are double counting the amount collisions so,

$$\begin{aligned}
 \text{Total collisions} &= \frac{1}{2} N (\frac{N}{2}) \\
 &= (\frac{N}{2})^2
 \end{aligned}$$