

Birthday Problem

You and your colleagues know that your boss A's birthday is one of the following 10 dates:

Mar 4, Mar 5, Mar 8

Jun 4, Jun 7

Sep 1, Sep 5

Dec 1, Dec 2, Dec 8

A told you only the month of his birthday, and told your colleague C only the day. After that, you first said: "I don't know A's birthday; C doesn't know it either." After hearing what you said, C replied: "I didn't know A's birthday, but now I know it." You smiled and said: "Now I know it, too" After looking at the 10 dates and hearing your comments, your administrative assistant wrote down A's birthday without asking any questions. So what did the assistant write?

Source: A Practical Guide to Quantitative Finance Interviews, Xinfeng Zhou (2008) 2.2 Logic Reasoning: Birthday problem

Solution 1

You know the month of the birthday and C knows the day of the birthday.

You said "I don't know A's birthday; C doesn't know it either." which implies that you can't deduce the birthday date only knowing the month and given the month you know, C can't either. The first part is straight-forward as each month there are atleast 2 different dates. Next, notice that there are two unique days that are only in one month i.e. 2nd and 7th. If the birthday is on either of the two days then C can deduce the birthday date. However, the second part implies that this is impossible, so the month is not either June nor December, otherwise C would've knew the birthday date.

Now our list is as below

Mar 4, Mar 5, Mar 8

Sep 1, Sep 5

Then, C says that he knows the date given the information implied by us. So it must that the day on our new list is unique otherwise its still impossible for C to know the date. So this eliminates Mar 5 and Sep 5.

Now our list is as below

Mar 4, Mar 8

Sep 1

Then you figured the birthday date this could only mean that the answer is 1st September, as if the date is either the day in March its still impossible for you to figure out which one.