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1. Linear Algebra

1.1. Introduction

Linear systems of equations form a core topic in linear algebra with widespread applications in mathematics, science, engineering, and technology. Solving linear systems efficiently is crucial, and among various techniques, *Gaussian elimination* is one of the most effective and widely used.

1.2. Linear Systems

A linear system of equations consists of multiple linear equations involving the same set of variables:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_1nx_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_2nx_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases} \tag{1.1}$$

Where:

- a_{ii} are the coefficients,
- x_i are the unknowns,
- b_i are the constants.

1.3. Matrix Representation

A linear system can be written compactly in matrix form as:

$$A \cdot X = B \tag{1.2}$$

Where:

- $A: m \times n$ coefficient matrix
- $X: n \times 1$ unknown vector
- $B: m \times 1$ result vector

This representation is ideal for applying numerical methods.

1.3.1. Determinant of a Matrix

The determinant of a square matrix A, denoted $\det(A)$ or |A|, helps determine invertibility. For a 2×2 matrix:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \operatorname{ad} - \operatorname{bc} \tag{1.3}$$

1.3.2. Transpose of a Matrix

The transpose of a matrix A, denoted A^T , swaps its rows and columns. Example:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \tag{1.4}$$

$$A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \tag{1.5}$$

1.3.3. Inverse of a Matrix

A square matrix A has an inverse A^{-1} if:

$$A \cdot A^{-1} = I \tag{1.6}$$

- , where $\det(A) \neq 0$
 - Methods to compute A^{-1} :

Gaussian Elimination 1.4.

A step-by-step method to reduce a matrix to upper triangular form:

1.4.1. Steps:

- 1. Form the augmented matrix: $[A \mid B]$
- 2. Forward elimination:
 - · Identify pivot elements along the main diagonal
 - Use row operations to zero out entries below each pivot
- 3. Back substitution:
 - · Solve for variables starting from the last row upward

1.4.2. Example

Given the system:

$$\begin{cases} 2x + 3y - z &= 5\\ 4x + y + 2z &= 6\\ -2x + 5y + 3z = 7 \end{cases}$$

1.4.2.1. **Step 1: Augmented Matrix**

$$\begin{pmatrix} 2 & 3 & -1 & | & 5 \\ 4 & 1 & 2 & | & 6 \\ -2 & 5 & 3 & | & 7 \end{pmatrix}$$

1.4.2.2. **Step 2: Row Reduction**

- · Normalize the first row
- Eliminate entries below the pivot
- · Repeat for next rows

1.4.2.3. **Step 3: Back Substitution**

Find values of z, then y, and finally x.

1.5. **Gauss-Jordan Elimination**

An extension of Gaussian Elimination that produces Reduced Row Echelon Form (RREF):

1.5.1. Steps:

- Form the augmented matrix $[A \mid B]$
- Use row operations to form leading 1s (pivots)
- Zero out both above and below the pivot in each column

1.5.2. Resulting Form:

$$\begin{pmatrix} 1 & 0 & 0 & | & x_1 \\ 0 & 1 & 0 & | & x_2 \\ 0 & 0 & 1 & | & x_3 \end{pmatrix}$$

1.6. Row Echelon Form (REF)

A matrix is in row echelon form if:

- All nonzero rows are above zero rows
- The leading entry (pivot) in each nonzero row is 1
- Each pivot is to the right of the one in the row above
- All entries below a pivot are 0

1.6.1. Example

$$\begin{pmatrix} 1 & 2 & -1 & | & 4 \\ 0 & 1 & 3 & | & -2 \\ 0 & 0 & 1 & | & 5 \end{pmatrix}$$

1.6.2. Advantages

- Simplifies solving linear systems
- Easy to implement algorithmically
- Helps identify inconsistent or dependent systems

1.6.3. Visual Representation

$$\begin{pmatrix} 1 & & \\ 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Interpretation:

- Zero row at the bottom
- Leading 1s at $A_{\{11\}}$ and $A_{\{22\}}$
- All entries below pivots are 0

1.6.4. Solving with Row Echelon Form (REF)

We are given the system:

$$\begin{cases} x + y + z &= 6 \\ 2x + 3y + 7z = 20 \\ x + 3y + 4z &= 13 \end{cases}$$

Step 1: Write the Augmented Matrix

$$\begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 2 & 3 & 7 & | & 20 \\ 1 & 3 & 4 & | & 13 \end{pmatrix}$$

Step 2: Row Reduction to Row Echelon Form

- Keep Row 1 as is.
- Eliminate below the first pivot (Row 1, Col 1):

$$\begin{array}{l} R_2 \coloneqq R_2 - 2 \cdot R_1 \\ R_3 \coloneqq R_3 - R_1 \\ \text{New matrix:} \\ \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 5 & | & 8 \\ 0 & 2 & 3 & | & 7 \end{pmatrix} \end{array}$$

• Eliminate below the second pivot (Row 2, Col 2):

$$R_3 \coloneqq R_3 - 2 \cdot R_2$$
 Resulting Row Echelon Form:

$$\begin{pmatrix}
1 & 1 & 1 & | & 6 \\
0 & 1 & 5 & | & 8 \\
0 & 0 & -7 & | & -9
\end{pmatrix}$$

Step 3: Back Substitution

From the last row:

$$-7z = -9 \to z = \frac{9}{7} \tag{1.7}$$

Second row:

$$y + 5z = 8 \to y = 8 - 5 \cdot \left(\frac{9}{7}\right) = \frac{11}{7} \tag{1.8}$$

First row:

$$x+y+z=6 \to x=6-\frac{11}{7}-\frac{9}{7}=\frac{22}{7} \eqno(1.9)$$

1.6.5. Final Answer:

$$x = \frac{22}{7}, y = \frac{11}{7}, z = \frac{9}{7} \tag{1.10}$$

- 1.7 Vector
- 1.8 Vector Space
- 1.9 Linear Transformation
- 1.10 Introduction to SVMs
- **1.11** Basis
- 1.12 Eigenvalues and Eigenvectors

2. Calculus

- **2.1** Limit
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3. Probability and Statistics

- 3.1 Probability
- 3.2 Probability Distributions
- 3.3 Sample
- 3.4 Estimating
- 3.5 Hypothesis Testing