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Titolo

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Dedica

Acknowledgments

Abstract

Qui va l'abstract

Contents

Introduction

Studio delle geodetiche in metrica di Schwarzschild. Partendo da una delle più semplici geometrie dello spazio-tempo che offre la relatività generale, la metrica di Schwarzschild, vengono studiati alcuni dei fenomeni più comuni ad essa associati dal punto di vista teorico. Nella seconda parte vengono fatte delle simulazioni numeriche

gravitational physics

Chapter 1

Theory

1.1 Introduction

1.1.1 Why the Schwarzschild Geometry

Newtonian mechanics is built upon the concept of absolute time and space. Once the concept of *inertial frame* is well defined, physics can be done on a space described by Euclidean geometry. Free particles (particles on which no forces are acting) move in a straight line, which is the shortest distance between two points in a three-dimensional space, measured as:

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 \, . \tag{1.1}$$

On the other hand, time is *just* seen as a parameter, common to every inertial frame, that can be used to determine the particle velocity and acceleration.

With the appearance of Maxwell's Equations it became clear that what they predicted (the speed of light being constant in every inertial frame) was in contrast with the description of our space given by Newtonian Mechanics, where the speed of anything changes with respect to the inertial frame chosen. Between Maxwell's Equations end Newtonian mechanics Einstein chose to modify the latter and wrote his two postulates:

- The laws of physics are invariant (identical) in all inertial frames of reference;
- The speed of light in vacuum, $c = 299\,792\,458$m/s, is the same for all observers, regardless of the motion of light source or observer.

The postulates may or may not be intuitive, but simple observations based on them bring us to abandon the idea of absolute space and time and to introduce the concept of *spacetime*, together with a new way of measuring distances

$$\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \, . \tag{1.2}$$

The appearance of time in a formula that is supposed to give us the distance between 2 objects is surely destabilizing at first, but geometry teaches us that fixing the way we calculate Δs^2, more properly referred to as the *line element* $\mathrm{d}s^2$, it's enough to describe the geometry of the space that we are using. Since eq. 1.2 is different from eq. 1.1, in particular there is a minus sign in front of Δt^2, we moved away from the familiar three-dimensional Euclidean geometry and are now in four-dimensional spacetime, usually referred to as *flat spacetime* or *Minkowski space*.

This new geometry allowed for a reformulation of Maxwell's Equations and brought (and explained) fenomena like time dilation, length contraction and the relativity of simultaneity. The last one in particular, the concept that the simultaneity of two events depends on the frame of reference, poses a thread to the *force* of gravity. Up until this point gravity was defined as the istantaneous force F_{12} acting on a mass m_1 at time t due to a second mass m_2:

$$F_{12} = G\frac{m_1 m_2}{|r_1(t) - r_2(t)|^2} \tag{1.3}$$

The abjective *istantaneous* in a theory where nothing can travel faster then the speed of light should already raise some concern. But looking at $r_1(t)$ and $r_2(t)$ in eq. 1.3, that are supposed to indicate the positions of the masses in the same istant of time, makes it even more clear that the force F_{12} can't be the same in all frames of reference.

Solving this issue gave birth to the theory of general relativity, where a mass is not a source of gravitational force, but it's responsible for bending the four-dimensional spacetime. This implies that when we observe a particle deviating its trajectory from a straight line in the presence of a massive object, it's not because of a force acting on it. In fact we can consider the particle free and moving from point A to point B along the shortest path, it's just that in a curved surface the shortest path is not a straight line.

These idea surely doesn't make things more intuitive to imagine, but the implication on the formalism that is needed to discuss the theory are even worst. Bending the space we are working on because of the presence of a mass means that the line element $\mathrm{d}s^2$ must change. The implication of this and the theories that lie behind (Einstein field equations most notably) are behind the scope of this thesis.

Here we will focus on a specific study case: the geometry of empty space outside a spherically symmetric source of curvature, for example, a spherical star. This is one of the simplest curved spacetimes of general relativity because of the many symmetries that presents and, luckily, is also one of the most useful. The line element of what is called Schwarzschild geometry is

$$\mathrm{d}s^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(c\mathrm{d}t)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2) \tag{1.4}$$

expressed in spherical coordinates centered in the center of the mass.

1.1.2 Notation and formalisms

In the *flat spacetime*, chosen a particular inertial frame, we introduce a basis for four-vectors $\{\mathbf{e_t}, \mathbf{e_x}, \mathbf{e_y}, \mathbf{e_z}\}$, or equivalently $\{\mathbf{e_0}, \mathbf{r_r}, \mathbf{e_2}, \mathbf{e_3}\}$, of unit length. Any four-vector $\mathbf{a}$ can then be written as

$$\mathbf{a} = a^t\mathbf{e_r} + a^x\mathbf{e_x} + a^y\mathbf{e_y} + a^z\mathbf{e_z} = a^0\mathbf{e_0} + a^1\mathbf{e_1} + a^2\mathbf{e_2} + a^3\mathbf{e_3} \tag{1.5}$$

where $(a_t,\ a_x,\ a_y,\ a_z)$, or $(a_0,\ a_1,\ a_2,\ a_3)$, are the components of the four-vector. Both notations will be used.

Another useful convention is to use Roman letters (usually i or j) to refer to indices 1, 2, 3 and Greek letters (usually μ or ν) to refer to indices 0, 1, 2, 3. Using Einstein notation the expression in eq. 1.5, can be rewritten simply as $\mathbf{a} = a^\mu e_\mu$. Other useful ways to specify the components of $\mathbf{a}$ are

$$a^\mu = (a^t, a^x, a^y, a^z) \quad a^\mu = (a^t, a^i) \quad a^\mu = (a^t, \vec{a}) \tag{1.6}$$

where $\vec{a} = a^i e_i$ is the tree-dimensional vector (e_1, e_2, e_3)

1.2 Conserved Quantities

Sets

Chapter 2

Due

Compute

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Conclusions

Appendix A

Albero

A.1 Prova

Come funziona un'appendice

Appendix B

Barca

B.1 Prova

Appendice B

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