

# MOSEK Optimizer API for Java

Release 8.1.0.83

MOSEK ApS

## **CONTENTS**

1	1.1 Why the Optimizer API for Java?	2
2	Contact Information	3
3	License Agreement	5
4	Installation 4.1 Building Examples and Testing the Installation	<b>7</b>
5	5.1 Modelling	11 11 11
6	6.1 Linear Optimization	13 13 19 26 31 35 38 43
7	7.1 Accessing the solution 7.2 Errors and exceptions 7.3 Input/Output 7.4 Setting solver parameters 7.5 Retrieving information items 7.6 Progress and data callback	49 53 53 55 57 60
8		<b>65</b> 65
9	9.1 Solving Linear Systems Involving the Basis Matrix	
10	10.1 Memory management and garbage collection	

		The license system         8'           Deployment         85
11		Studies Portfolio Optimization
19	Prob	lem Formulation and Solutions 109
14		Linear Optimization
		Conic Quadratic Optimization
		Semidefinite Optimization
		Quadratic and Quadratically Constrained Optimization
		General Convex Optimization
13	The	Optimizers for Continuous Problems 119
		Presolve
		Using Multiple Threads in an Optimizer
		Linear Optimization
		Conic Optimization
		Nonlinear Convex Optimization
<b>14</b>		Optimizer for Mixed-integer Problems 138
		The Mixed-integer Optimizer Overview
	14.2	Relaxations and bounds
	14.3	Termination Criterion
	14.4	Speeding Up the Solution Process
	14.5	Understanding Solution Quality
	14.6	The Optimizer Log
15		tional features 139
		Problem Analyzer
		Analyzing Infeasible Problems
	15.3	Sensitivity Analysis
16	API	Reference 159
		API Conventions
		Functions grouped by topic
		Class Env
		Class Task
		Exceptions
		Parameters grouped by topic
		Parameters (alphabetical list sorted by type)
		Response codes
		Enumerations
		Class types
		Nonlinear extensions
<b>17</b>		orted File Formats 378
	17.1	The LP File Format
	17.2	The MPS File Format
	17.3	The OPF Format
	17.4	The CBF Format
	17.5	The XML (OSiL) Format
	17.6	The Task Format
	17.7	The JSON Format
	17.8	The Solution File Format
18	$\mathbf{List}$	of examples 429
		face changes         43           Compatibility
	$_{1}y.1$	$\circ$ ониралиниу

_	Functions	_
	Parameters	
19.4	Constants	434
19.5	Response Codes	438
$\mathbf{Bibliog}$	aphy	441
Symbol	Index	443
$\mathbf{Index}$		459

## INTRODUCTION

The **MOSEK** Optimization Suite 8.1.0.83 is a powerful software package capable of solving large-scale optimization problems of the following kind:

- linear,
- conic quadratic (also known as second-order cone),
- convex quadratic,
- semidefinite,
- and general convex.

Integer constrained variables are supported for all problem classes except for semidefinite and general convex problems. In order to obtain an overview of features in the **MOSEK** Optimization Suite consult the product introduction guide.

The most widespread class of optimization problems is *linear optimization problems*, where all relations are linear. The tremendous success of both applications and theory of linear optimization can be ascribed to the following factors:

- The required data are simple, i.e. just matrices and vectors.
- Convexity is guaranteed since the problem is convex by construction.
- Linear functions are trivially differentiable.
- There exist very efficient algorithms and software for solving linear problems.
- Duality properties for linear optimization are nice and simple.

Even if the linear optimization model is only an approximation to the true problem at hand, the advantages of linear optimization may outweigh the disadvantages. In some cases, however, the problem formulation is inherently nonlinear and a linear approximation is either intractable or inadequate. *Conic optimization* has proved to be a very expressive and powerful way to introduce nonlinearities, while preserving all the nice properties of linear optimization listed above.

The fundamental expression in linear optimization is a linear expression of the form

$$Ax - b \in \mathcal{K}$$

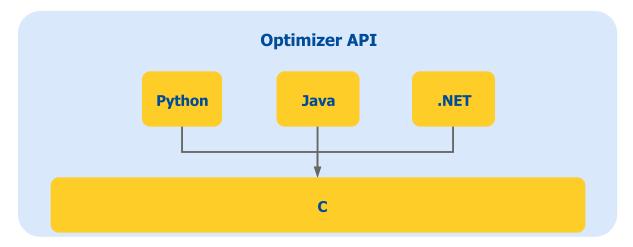
where  $\mathcal{K} = \{y : y \ge 0\}$ , i.e.,

$$Ax - b = y, y \in \mathcal{K}.$$

In conic optimization a wider class of convex sets  $\mathcal{K}$  is allowed, for example in 3 dimensions  $\mathcal{K}$  may correspond to an ice cream cone. The conic optimizer in **MOSEK** supports three structurally different types of cones  $\mathcal{K}$ , which allows a surprisingly large number of nonlinear relations to be modelled (as described in the **MOSEK** modeling cookbook), while preserving the nice algorithmic and theoretical properties of linear optimization.

## 1.1 Why the Optimizer API for Java?

The Optimizer API for Java provides an object-oriented interface to the **MOSEK** optimizers. This object oriented design is common to Java, Python and .NET and is based on a thin class-based interface to the native C optimizer API. The overhead introduced by this mapping is minimal.



The Optimizer API for Java can be used with any application running on the Oracle Java platform (and possibly other Java implementations). It consists of a single class library mosek.jar and a set of library files that must be available at runtime.

The Optimizer API for Java provides access to:

- Linear Optimization (LO)
- Conic Quadratic (Second-Order Cone) Optimization (CQO, SOCO)
- Convex Quadratic and Quadratically Constrained Optimization (QCQO)
- Semidefinite Optimization (SDO)

as well as to additional functions for

- problem analysis,
- sensitivity analysis,
- infeasibility diagnostics,
- BLAS/LAPACK linear algebra routines.

## **TWO**

## **CONTACT INFORMATION**

Phone	$+45\ 7174\ 9373$	
Website	mosek.com	
Email		
	sales@mosek.com	Sales, pricing, and licensing
	support@mosek.com	Technical support, questions and bug reports
	info@mosek.com	Everything else.
Mailing Address		
	MOSEK ApS	
	Fruebjergvej 3	
	Symbion Science Park, Box 16	
	2100 Copenhagen O	
	Denmark	

You can get in touch with  $\mathbf{MOSEK}$  using popular social media as well:

Blogger	http://blog.mosek.com/	
Google Group	https://groups.google.com/forum/#!forum/mosek	
Twitter	https://twitter.com/mosektw	
$\mathbf{Google} +$	$\rm https://plus.google.com/+Mosek/posts$	
Linkedin	https://www.linkedin.com/company/mosek-aps	

In particular **Twitter** is used for news, updates and release announcements.

## LICENSE AGREEMENT

Before using the MOSEK software, please read the license agreement available in the distribution at MOSEK website https://mosek.com/products/license-agreement.

MOSEK uses some third-party open-source libraries. Their license details follows.

#### zlib

**MOSEK** includes the zlib library obtained from the zlib website. The license agreement for zlib is shown in Listing 3.1.

#### Listing 3.1: zlib license.

zlib.h - interface of the 'zlib' general purpose compression library version 1.2.7, May 2nd, 2012

Copyright (C) 1995-2012 Jean-loup Gailly and Mark Adler

This software is provided 'as-is', without any express or implied warranty. In no event will the authors be held liable for any damages arising from the use of this software.

Permission is granted to anyone to use this software for any purpose, including commercial applications, and to alter it and redistribute it freely, subject to the following restrictions:

- The origin of this software must not be misrepresented; you must not claim that you wrote the original software. If you use this software in a product, an acknowledgment in the product documentation would be appreciated but is not required.
- 2. Altered source versions must be plainly marked as such, and must not be misrepresented as being the original software.
- 3. This notice may not be removed or altered from any source distribution.

Jean-loup Gailly Mark Adler

jloup@gzip.org madler@alumni.caltech.edu

## fplib

**MOSEK** includes the floating point formatting library developed by David M. Gay obtained from the netlib website. The license agreement for *fplib* is shown in Listing 3.2.

## Listing 3.2: fplib license.

**CHAPTER** 

**FOUR** 

## **INSTALLATION**

In this section we discuss how to install and setup the MOSEK Optimizer API for Java.

Important: Before running this MOSEK interface please make sure that you:

- Installed **MOSEK** correctly. Some operating systems require extra steps. See the Installation guide for instructions and common troubleshooting tips.
- Set up a license. See the Licensing guide for instructions.

## Compatibility

The Optimizer API for Java is compatible with Java version 1.7 or later.

## **Locating Files**

The files in Optimizer API for Java are organized as reported in Table 4.1.

Table 4.1: Relevant files for the Optimizer API for Java.

Relative Path	Description	Label
<pre><mskhome>/mosek/8/tools/platform/<platform>/bin</platform></mskhome></pre>	Libraries and jar file	<jardir></jardir>
<mskhome>/mosek/8/tools/examples/java</mskhome>	Examples	<exdir></exdir>
<mskhome>/mosek/8/tools/examples/data</mskhome>	Additional data	<miscdir></miscdir>

#### where

- $\bullet$  <MSKHOME> is the folder in which the  $\mathbf{MOSEK}$  package has been installed,
- <PLATFORM> is the actual platform among those supported by MOSEK, i.e. win32x86, win64x86, linux64x86 or osx64x86.

## Setting up paths

To compile and run a Java program using MOSEK the correct path to <JARDIR>/mosek.jar must be provided in the Java classpath. This is usually set with the command line option

```
javac -d . -classpath <JARDIR>/mosek.jar lo1.java
java -classpath .:<JARDIR>/mosek.jar com.mosek.example.lo1
```

Alternatively, this can be set with the environment variable CLASSPATH. For more information about specifying class libraries and compiling applications, see the full Java documentation at http://java.sun. com/.

## 4.1 Building Examples and Testing the Installation

This section describes how to verify that **MOSEK** has been installed correctly, and how to build and execute the Java examples distributed with **MOSEK**.

## 4.1.1 Windows

#### Building and executing a program

To compile the example lol. java distributed with MOSEK:

- Open a DOS prompt and go to the examples directory <EXDIR>.
- To compile a Java program and produce the class files in the current directory, type

```
javac -classpath <JARDIR>\mosek.jar -d . lo1.java
```

• To run the compiled program, type

```
java -classpath .:<JARDIR>\mosek.jar com.mosek.example.lo1
```

#### Compiling with Microsoft NMake

The distributed examples can also be compiled using Microsoft NMake. This requires that paths and environment is set up for Visual Studio tools (usually, the sub-menu containing Visual Studio also contains a Visual Studio Command Prompt which take care of all the necessary setup).

To build the examples, open a DOS box and change directory to the examples directory <EXDIR>. To compile all examples type

```
nmake /f Makefile
```

This will compile all the classes into a jar file. To run all the examples type

```
nmake /f Makefile test
```

## 4.1.2 Mac OS and Linux

## Building and executing a program

To compile the example lol.java distributed with MOSEK:

- Open a console and go to the examples directory <EXDIR>.
- To compile a Java program and produce the class files in the current directory, type

```
| javac -classpath <JARDIR>/mosek.jar -d . lo1.java
```

• To run the compiled program, type

```
java -classpath .:<JARDIR>/mosek.jar com.mosek.example.lo1
```

## Compiling examples using make

The example directory contains makefiles for use with GNU Make. To build the examples, open a prompt and change directory to the examples directory <EXDIR>. To compile all examples type

make -f Makefile

This will compile all the classes into a jar file. To run all the examples type

make test

**CHAPTER** 

**FIVE** 

## **DESIGN OVERVIEW**

## 5.1 Modelling

Optimizer API for Java is an interface for specifying optimization problems directly in matrix form. It means that an optimization problem such as:

minimize 
$$c^T x$$
  
subject to  $Ax \leq b$ ,  
 $x \in \mathcal{K}$ 

is specified by describing the matrix A, vectors b, c and a list of cones K directly.

The main characteristics of this interface are:

- Simplicity: once the problem data is assembled in matrix form, it is straightforward to input it into the optimizer.
- Exploiting sparsity: data is entered in sparse format, enabling huge, sparse problems to be defined and solved efficiently.
- Efficiency: the Optimizer API incurs almost no overhead between the user's representation of the problem and MOSEK's internal one.

Optimizer API for Java does not aid with modeling. It is the user's responsibility to express the problem in MOSEK's standard form, introducing, if necessary, auxiliary variables and constraints. See Sec. 12 for the precise formulations of problems MOSEK solves.

## 5.2 "Hello World!" in MOSEK

Here we present the most basic workflow pattern when using Optimizer API for Java.

## Creating an environment and task

Every interaction with **MOSEK** using Optimizer API for Java begins by creating a **MOSEK environment**. It coordinates the access to **MOSEK** from the current process.

In most cases the user does not interact directly with the environment, except for creating optimization **tasks**, which contain actual problem specifications and where optimization takes place. An environment can host multiple tasks.

#### **Defining tasks**

After a task is created, the input data can be specified. An optimization problem consists of several components; objective, objective sense, constraints, variable bounds etc. See Sec. 6 for basic tutorials on how to specify and solve various types of optimization problems.

## Retrieving the solutions

When the model is set up, the optimizer is invoked with the call to *Task.optimize*. When the optimization is over, the user can check the results and retrieve numerical values. See further details in Sec. 7.

We refer also to Sec. 7 for information about more advanced mechanisms of interacting with the solver

## Source code example

Below is the most basic code sample that defines and solves a trivial optimization problem

```
 \begin{array}{ll} \text{minimize} & x \\ \text{subject to} & 2.0 \le x \le 3.0. \end{array}
```

For simplicity the example does not contain any error or status checks.

Listing 5.1: "Hello World!" in MOSEK

```
package com.mosek.example;
import mosek.*;
public class helloworld {
 public static void main(String[] args) {
   double[] x = new double[1];
   Env env = null;
   Task task = null:
   try {
     env = new Env();
                                      // Create Environment
     task = new Task(env, 0, 1);
                                      // Create Task
     task.appendvars(1);
                                                  // 1 variable x
     task.putcj(0, 1.0);
                                                  // c_0 = 1.0
     task.putvarbound(0, boundkey.ra, 2.0, 3.0); // 2.0 <= x <= 3.0
     task.putobjsense(objsense.minimize);
                                                  // minimize
                                           // Optimize
     task.optimize();
     task.getxx(soltype.itr, x);
                                                    // Get solution
     System.out.println("Solution x = " + x[0]); // Print solution
                              // Dispose of env and task just to be sure
   finally {
     task.dispose();
     env.dispose();
   }
 }
```

## **OPTIMIZATION TUTORIALS**

In this section we demonstrate how to set up basic types of optimization problems. Each short tutorial contains a working example of formulating problems, defining variables and constraints and retrieving solutions.

## 6.1 Linear Optimization

The simplest optimization problem is a purely linear problem. A *linear optimization problem* is a problem of the following form:

Minimize or maximize the objective function

$$\sum_{j=0}^{n-1} c_j x_j + c^f$$

subject to the linear constraints

$$l_k^c \le \sum_{j=0}^{n-1} a_{kj} x_j \le u_k^c, \quad k = 0, \dots, m-1,$$

and the bounds

$$l_i^x \le x_j \le u_i^x, \quad j = 0, \dots, n - 1.$$

The problem description consists of the following elements:

- $\bullet$  m and n the number of constraints and variables, respectively,
- x the variable vector of length n,
- ullet c the coefficient vector of length n

$$c = \left[ \begin{array}{c} c_0 \\ \vdots \\ c_{n-1} \end{array} \right],$$

- $c^f$  fixed term in the objective,
- A an  $m \times n$  matrix of coefficients

$$A = \begin{bmatrix} a_{0,0} & \cdots & a_{0,(n-1)} \\ \vdots & \cdots & \vdots \\ a_{(m-1),0} & \cdots & a_{(m-1),(n-1)} \end{bmatrix},$$

- $l^c$  and  $u^c$  the lower and upper bounds on constraints,
- $l^x$  and  $u^x$  the lower and upper bounds on variables.

Please note that we are using 0 as the first index:  $x_0$  is the first element in variable vector x.

## 6.1.1 Example LO1

The following is an example of a small linear optimization problem:

under the bounds

$$\begin{array}{cccccc} 0 & \leq & x_0 & \leq & \infty, \\ 0 & \leq & x_1 & \leq & 10, \\ 0 & \leq & x_2 & \leq & \infty, \\ 0 & \leq & x_3 & \leq & \infty. \end{array}$$

#### Solving the problem

To solve the problem above we go through the following steps:

- 1. Create an environment.
- 2. Create an optimization task.
- 3. Load a problem into the task object.
- 4. Optimization.
- 5. Extracting the solution.

Below we explain each of these steps.

#### Create an environment.

Before setting up the optimization problem, a MOSEK environment must be created. All tasks in the program should share the same environment.

```
try (mosek.Env env = new Env();
```

## Create an optimization task.

Next, an empty task object is created:

```
mosek.Task task = new Task(env, 0, 0)) {
// Directs the log task stream to the user specified
// method task_msg_obj.stream
task.set_Stream(
   mosek.streamtype.log,
   new mosek.Stream()
{ public void stream(String msg) { System.out.print(msg); }});
```

We also connect a call-back function to the task log stream. Messages related to the task are passed to the call-back function. In this case the stream call-back function writes its messages to the standard output stream.

### Load a problem into the task object.

Before any problem data can be set, variables and constraints must be added to the problem via calls to the functions Task.appendcons and Task.appendvars.

```
// Append 'numcon' empty constraints.
// The constraints will initially have no bounds.
task.appendcons(numcon);

// Append 'numvar' variables.
// The variables will initially be fixed at zero (x=0).
task.appendvars(numvar);
```

New variables can now be referenced from other functions with indexes in  $0, \ldots, \mathtt{numvar} - 1$  and new constraints can be referenced with indexes in  $0, \ldots, \mathtt{numcon} - 1$ . More variables and/or constraints can be appended later as needed, these will be assigned indexes from  $\mathtt{numvar/numcon}$  and up.

Next step is to set the problem data. We loop over each variable index  $j=0,\ldots,\text{numvar}-1$  calling functions to set problem data. We first set the objective coefficient  $c_j=\mathtt{c}[\mathtt{j}]$  by calling the function Task.putcj.

```
task.putcj(j, c[j]);
```

#### Setting bounds on variables

The bounds on variables are stored in the arrays

```
mosek.boundkey
bkx[] = {mosek.boundkey.lo,
          mosek.boundkey.ra,
          mosek.boundkey.lo,
          mosek.boundkey.lo
         };
double blx[]
               = \{0.0,
                  0.0.
                  0.0,
                  0.0
                 }:
double bux[]
               = { +infinity,
                    10.0,
                    +infinity,
                    +infinity
                 };
```

and are set with calls to Task.putvarbound.

```
// Set the bounds on variable j.
// blx[j] <= x_j <= bux[j]
task.putvarbound(j, bkx[j], blx[j], bux[j]);</pre>
```

The Bound key stored in bkx specifies the type of the bound according to Table 6.1.

Bound key	Type of bound	Lower bound	Upper bound
boundkey.fx	$u_j = l_j$	Finite	Identical to the lower bound
boundkey.fr	Free	$-\infty$	$+\infty$
boundkey.lo	$l_j \leq \cdots$	Finite	$+\infty$
boundkey.ra	$l_j \leq \cdots \leq u_j$	Finite	Finite
boundkey.up	$\cdots \leq u_j$	$-\infty$	Finite

Table 6.1: Bound keys as defined in the enum boundkey.

For instance bkx[0] = boundkey. lo means that  $x_0 \ge l_0^x$ . Finally, the numerical values of the bounds on variables are given by

$$l_i^x = \mathtt{blx}[\mathtt{j}]$$

and

$$u_i^x = \text{bux}[j].$$

## Defining the linear constraint matrix.

Recall that in our example the A matrix is given by

$$A = \left[ \begin{array}{cccc} 3 & 1 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 3 \end{array} \right].$$

This matrix is stored in sparse format in the arrays:

```
int asub[][] = {
      {0, 1},
      {0, 1, 2},
      {0, 1},
      {1, 2}
    };
    double aval[][] = {
      {3.0, 2.0},
      {1.0, 1.0, 2.0},
      {2.0, 3.0},
      {1.0, 3.0}
};
```

The array aval[j] contains the non-zero values of column j and asub[j] contains the row index of these non-zeros.

Using the function Task.putacol we set column j of A

There are many alternative formats for entering the A matrix. See functions such as Task.putarow, Task.putarowlist, Task.putaijlist and similar.

Finally, the bounds on each constraint are set by looping over each constraint index  $i=0,\ldots,\mathtt{numcon}-1$ 

```
// Set the bounds on constraints.
// blc[i] <= constraint_i <= buc[i]
for (int i = 0; i < numcon; ++i)
   task.putconbound(i, bkc[i], blc[i], buc[i]);</pre>
```

## Optimization

After the problem is set-up the task can be optimized by calling the function Task.optimize.

```
task.optimize();
```

### Extracting the solution.

After optimizing the status of the solution is examined with a call to Task.getsolsta. If the solution status is reported as solsta.optimal or solsta.near\_optimal the solution is extracted in the lines below:

The Task. getxx function obtains the solution. MOSEK may compute several solutions depending on the optimizer employed. In this example the basic solution is requested by setting the first argument to soltype.bas.

#### **Catching exceptions**

We cache any exceptions thrown by MOSEK in the lines:

```
catch (mosek.Exception e) {
   System.out.println ("An error/warning was encountered");
   System.out.println (e.toString());
   throw e;
}
```

The types of exceptions that **MOSEK** can throw can be seen in Sec. 16.8.

#### Source code

The complete source code lol.java of this example appears below. See also lol.java for a version where the A matrix is entered row-wise.

Listing 6.1: Linear optimization example.

```
package com.mosek.example;
import mosek.*;
public class lo1 {
 static final int numcon = 3;
 static final int numvar = 4;
 public static void main (String[] args) {
   // Since the value of infinity is ignored, we define it solely
    // for symbolic purposes
   double infinity = 0;
                = {3.0, 1.0, 5.0, 1.0};
   double c[]
         asub[][] = {
    int.
      {0, 1},
     {0, 1, 2},
     {0, 1},
     {1, 2}
    double aval[][] = {
     {3.0, 2.0},
     {1.0, 1.0, 2.0},
     {2.0, 3.0},
     {1.0, 3.0}
   };
   mosek.boundkey[]
           = {mosek.boundkey.fx,
```

```
mosek.boundkey.lo,
          mosek.boundkey.up
         };
double blc[] = \{30.0,
                  15.0,
                  -infinity
                 }:
double buc[] = \{30.0,
                  +infinity,
                  25.0
                 };
mosek.boundkey
bkx[] = {mosek.boundkey.lo,
          mosek.boundkey.ra,
          mosek.boundkey.lo,
         mosek.boundkey.lo
         };
double blx[] = \{0.0,
                  0.0.
                  0.0.
                  0.0
                 };
double bux[] = { +infinity,
                   10.0,
                   +infinity,
                   +infinity
                 };
double[] xx = new double[numvar];
try (mosek.Env env = new Env();
     mosek.Task task = new Task(env, 0, 0)) {
  // Directs the log task stream to the user specified
  // method task_msg_obj.stream
  task.set_Stream(
   mosek.streamtype.log,
   new mosek.Stream()
  { public void stream(String msg) { System.out.print(msg); }});
  // Append 'numcon' empty constraints.
  // The constraints will initially have no bounds.
  task.appendcons(numcon);
  // Append 'numvar' variables.
  // The variables will initially be fixed at zero (x=0).
  task.appendvars(numvar);
  for (int j = 0; j < numvar; ++j) {
    // Set the linear term c_{-}j in the objective.
    task.putcj(j, c[j]);
    // Set the bounds on variable j.
    // blx[j]  <= x_j  <= bux[j]
    task.putvarbound(j, bkx[j], blx[j], bux[j]);
    // Input column j of A
                                         /* Variable (column) index.*/
    task.putacol(j,
                 asub[j],
                                        /* Row index of non-zeros in column j.*/
                                        /* Non-zero Values of column j. */
                 aval[j]);
  }
  // Set the bounds on constraints.
  // blc[i] <= constraint_i <= buc[i]</pre>
  for (int i = 0; i < numcon; ++i)</pre>
```

```
task.putconbound(i, bkc[i], blc[i], buc[i]);
    // Input the objective sense (minimize/maximize)
    task.putobjsense(mosek.objsense.maximize);
    // Solve the problem
    task.optimize();
    // Print a summary containing information
    // about the solution for debugging purposes
    task.solutionsummary(mosek.streamtype.msg);
    // Get status information about the solution
    mosek.solsta solsta[] = new mosek.solsta[1];
    task.getsolsta(mosek.soltype.bas, solsta);
    switch (solsta[0]) {
      case optimal:
      case near_optimal:
       task.getxx(mosek.soltype.bas, // Request the basic solution.
        System.out.println("Optimal primal solution\n");
        for (int j = 0; j < numvar; ++j)
          System.out.println ("x[" + j + "]:" + xx[j]);
        break;
      case dual_infeas_cer:
      case prim_infeas_cer:
      case near_dual_infeas_cer:
      case near_prim_infeas_cer:
        System.out.println("Primal or dual infeasibility certificate found.\n");
        break;
      case unknown:
        System.out.println("Unknown solution status.\n");
        System.out.println("Other solution status");
        break:
  catch (mosek.Exception e) {
    System.out.println ("An error/warning was encountered");
    System.out.println (e.toString());
}
```

## 6.2 Quadratic Optimization

**MOSEK** can solve quadratic and quadratically constrained problems, as long as they are convex. This class of problems can be formulated as follows:

minimize 
$$\frac{\frac{1}{2}x^{T}Q^{o}x + c^{T}x + c^{f}}{\text{subject to}}$$
 subject to 
$$l_{k}^{c} \leq \frac{\frac{1}{2}x^{T}Q^{k}x + \sum_{j=0}^{n-1}a_{k,j}x_{j}}{l_{j}^{x}} \leq u_{k}^{c}, \quad k = 0, \dots, m-1,$$
 
$$l_{j}^{x} \leq x_{j} \leq u_{j}^{x}, \quad j = 0, \dots, n-1.$$
 (6.2)

Without loss of generality it is assumed that  $Q^o$  and  $Q^k$  are all symmetric because

$$x^T Q x = \frac{1}{2} x^T (Q + Q^T) x.$$

This implies that a non-symmetric Q can be replaced by the symmetric matrix  $\frac{1}{2}(Q+Q^T)$ .

The problem is required to be convex. More precisely, the matrix  $Q^o$  must be positive semi-definite and the kth constraint must be of the form

$$l_k^c \le \frac{1}{2} x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \tag{6.3}$$

with a negative semi-definite  $Q^k$  or of the form

$$\frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \le u_k^c.$$

with a positive semi-definite  $Q^k$ . This implies that quadratic equalities are *not* allowed. Specifying a non-convex problem will result in an error when the optimizer is called.

A matrix is positive semidefinite if all the eigenvalues of Q are nonnegative. An alternative statement of the positive semidefinite requirement is

$$x^T Q x \ge 0, \quad \forall x.$$

If the convexity (i.e. semidefiniteness) conditions are not met **MOSEK** will not produce reliable results or work at all.

## 6.2.1 Example: Quadratic Objective

We look at a small problem with linear constraints and quadratic objective:

minimize 
$$x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2$$
 subject to 
$$1 \le x_1 + x_2 + x_3$$
 
$$0 < x.$$
 (6.4)

The matrix formulation (6.4) has:

$$Q^o = \left[ \begin{array}{ccc} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{array} \right], c = \left[ \begin{array}{c} 0 \\ -1 \\ 0 \end{array} \right], A = \left[ \begin{array}{ccc} 1 & 1 & 1 \end{array} \right],$$

with the bounds:

$$l^c = 1, u^c = \infty, l^x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
 and  $u^x = \begin{bmatrix} \infty \\ \infty \\ \infty \end{bmatrix}$ 

Please note the explicit  $\frac{1}{2}$  in the objective function of (6.2) which implies that diagonal elements must be doubled in Q, i.e.  $Q_{11} = 2$ , whereas the coefficient in (6.4) is 1 in front of  $x_1^2$ .

## Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

## Setting up quadratic objective

The quadratic objective is specified using the function Task.putqobj. Since  $Q^o$  is symmetric only the lower triangular part of  $Q^o$  is inputted. In fact entries from above the diagonal may not appear in the input.

The lower triangular part of the matrix  $Q^o$  is specified using an unordered sparse triplet format (for details, see Sec. 16.1.4):

```
int[] qsubi = {0, 1, 2, 2 };
int[] qsubj = {0, 1, 0, 2 };
double[] qval = {2.0, 0.2, -1.0, 2.0};
```

Please note that

- only non-zero elements are specified (any element not specified is 0 by definition),
- the order of the non-zero elements is insignificant, and
- only the lower triangular part should be specified.

Finally, this definition of  $Q^o$  is loaded into the task:

```
task.putqobj(qsubi, qsubj, qval);
```

#### Source code

Listing 6.2: Source code implementing problem (6.4).

```
package com.mosek.example;
import mosek.*;
public class qo1 {
 static final int numvar = 3;  /* Number of variables.
 static final int NUMANZ = 3;  /* Number of numzeros in A.
 static final int NUMQNZ = 4; /* Number of nonzeros in Q.
 public static void main (String[] args) {
   // Since the value infinity is never used, we define
   // 'infinity' symbolic purposes only
   double infinity = 0;
   double[] c = \{0.0, -1.0, 0.0\};
   mosek.boundkey[] bkc = { mosek.boundkey.lo };
   double[] blc = {1.0};
   double[] buc = {infinity};
   mosek.boundkey[] bkx = { mosek.boundkey.lo,
                             mosek.boundkey.lo,
                             mosek.boundkey.lo
                           };
   double[] blx = {0.0,
                   0.0,
                   0.0
                  };
   double[] bux = {infinity,
                   infinity,
                   infinity
                  };
   int[][] asub = { {0}, {0}, {0} };
double[][] aval = { {1.0}, {1.0}, {1.0} };
   double[] xx = new double[numvar];
   try (Env env = new Env();
        {\tt Task\ task\ =\ new\ Task(env,\ 0,\ 0))\ \{}
      // Directs the log task stream to the user specified
     // method task_msg_obj.stream
     task.set_Stream(
       mosek.streamtype.log,
```

```
new mosek.Stream()
{ public void stream(String msg) { System.out.print(msg); }});
/* Give MOSEK an estimate of the size of the input data.
This is done to increase the speed of inputting data.
However, it is optional. */
/* Append 'numcon' empty constraints.
The constraints will initially have no bounds. */
task.appendcons(numcon);
/* Append 'numvar' variables.
The variables will initially be fixed at zero (x=0). */
task.appendvars(numvar);
for (int j = 0; j < numvar; ++j) {
  /* Set the linear term c_j in the objective.*/
 task.putcj(j, c[j]);
  /* Set the bounds on variable j.
     blx[j] <= x_j <= bux[j] */
 task.putbound(mosek.accmode.var, j, bkx[j], blx[j], bux[j]);
  /* Input column j of A */
 task.putacol(j,
                                      /* Variable (column) index.*/
               asub[j],
                                      /* Row index of non-zeros in column j.*/
               aval[j]);
                                      /* Non-zero Values of column j. */
/* Set the bounds on constraints.
for i=1, ..., numcon: blc[i] \leftarrow constraint i \leftarrow buc[i] */
for (int i = 0; i < numcon; ++i)
 task.putbound(mosek.accmode.con, i, bkc[i], blc[i], buc[i]);
 The lower triangular part of the Q
matrix in the objective is specified.
int[]
         qsubi = {0, 1, 2, 2 };
int[]
         qsubj = \{0, 1,
                           0, 2 };
double[] qval = \{2.0, 0.2, -1.0, 2.0\};
/* Input the Q for the objective. */
task.putqobj(qsubi, qsubj, qval);
/* Solve the problem */
mosek.rescode r = task.optimize();
System.out.println (" Mosek warning:" + r.toString());
// Print a summary containing information
// about the solution for debugging purposes
task.solutionsummary(mosek.streamtype.msg);
mosek.solsta solsta[] = new mosek.solsta[1];
/* Get status information about the solution */
task.getsolsta(mosek.soltype.itr, solsta);
/* Get the solution */
task.getxx(mosek.soltype.itr, // Interior solution.
switch (solsta[0]) {
 case optimal:
  case near_optimal:
    System.out.println("Optimal primal solution\n");
    for (int j = 0; j < numvar; ++j)
     System.out.println ("x[" + j + "]:" + xx[j]);
```

```
break:
      case dual_infeas_cer:
      case prim_infeas_cer:
      case near_dual_infeas_cer:
      case near_prim_infeas_cer:
        System.out.println("Primal or dual infeasibility\n");
        break:
      case unknown:
        System.out.println("Unknown solution status.\n");
        break;
      default:
        System.out.println("Other solution status");
 }
  catch (mosek.Exception e) {
    System.out.println ("An error/warning was encountered");
    System.out.println (e.toString());
    throw e:
} /* Main */
```

## 6.2.2 Example: Quadratic constraints

In this section we show how to solve a problem with quadratic constraints. Please note that quadratic constraints are subject to the convexity requirement (6.3).

Consider the problem:

minimize 
$$x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2$$
 subject to 
$$1 \leq x_1 + x_2 + x_3 - x_1^2 - x_2^2 - 0.1x_3^2 + 0.2x_1x_3,$$
 
$$x > 0.$$

This is equivalent to

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}x^TQ^ox + c^Tx \\ \text{subject to} & \frac{1}{2}x^TQ^0x + Ax & \geq & b, \\ & & x \geq 0, \end{array} \tag{6.5}$$

where

$$Q^o = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{bmatrix}, c = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T, A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, b = 1.$$
 
$$Q^0 = \begin{bmatrix} -2 & 0 & 0.2 \\ 0 & -2 & 0 \\ 0.2 & 0 & -0.2 \end{bmatrix}.$$

The linear parts and quadratic objective are set up the way described in the previous tutorial.

## Setting up quadratic constraints

To add quadratic terms to the constraints we use the function Task.putqconk.

```
int[] qsubi = {0, 1, 2, 2 };
int[] qsubj = {0, 1, 2, 0 };
double[] qval = { -2.0, -2.0, -0.2, 0.2};
```

While Task.putqconk adds quadratic terms to a specific constraint, it is also possible to input all quadratic terms in one chunk using the Task.putqcon function.

#### Source code

Listing 6.3: Implementation of the quadratically constrained problem (6.5).

```
package com.mosek.example;
import mosek.*;
public class qcqo1 {
 static final int numcon = 1;  /* Number of constraints.
static final int numvar = 3;  /* Number of variables.
 static final int NUMANZ = 3; /* Number of numzeros in A.
 static final int NUMQNZ = 4; /* Number of nonzeros in Q.
 public static void main (String[] args) {
   // Since the value infinity is never used, we define
    // 'infinity' symbolic purposes only
   double infinity = 0;
   double[] c = \{0.0, -1.0, 0.0\};
    mosek.boundkey[]
                       bkc = {mosek.boundkey.lo};
    double[] blc = {1.0};
   double[] buc = {infinity};
   mosek.boundkey[] bkx
    = {mosek.boundkey.lo,
       mosek.boundkey.lo,
       mosek.boundkey.lo
      };
    double[] blx = {0.0,}
                    0.0.
                    0.0
                   };
    double[] bux = {infinity,
                    infinity,
                     infinity
                    };
    int[][]
             asub = \{ \{0\}, \{0\}, \{0\}, \{0\} \};
    double[][] aval = { {1.0}, {1.0}, {1.0} };
    double[] xx = new double[numvar];
    try (mosek.Env env = new mosek.Env();
         mosek.Task task = new mosek.Task(env, 0, 0)) {
      // Directs the log task stream to the user specified
      // method task_msg_obj.stream
      task.set_Stream(
        mosek.streamtype.log,
        new mosek.Stream()
      { public void stream(String msg) { System.out.print(msg); }});
```

```
/* Give MOSEK an estimate of the size of the input data.
This is done to increase the speed of inputting data.
However, it is optional. */
/* Append 'numcon' empty constraints.
The constraints will initially have no bounds. */
task.appendcons(numcon);
/* Append 'numvar' variables.
The variables will initially be fixed at zero (x=0). */
task.appendvars(numvar);
for (int j = 0; j < numvar; ++j) {
  /* Set the linear term c_j in the objective.*/
 task.putcj(j, c[j]);
 /* Set the bounds on variable j.
  blx[j] <= x_j <= bux[j] */
 task.putbound(mosek.accmode.var, j, bkx[j], blx[j], bux[j]);
 /* Input column j of A */
 task.putacol(j,
                                      /* Variable (column) index.*/
               asub[j],
                                      /* Row index of non-zeros in column j.*/
               aval[j]);
                                      /* Non-zero Values of column j. */
/* Set the bounds on constraints.
for i=1, ..., numcon : blc[i] \leftarrow constraint i \leftarrow buc[i] */
for (int i = 0; i < numcon; ++i)
 task.putbound(mosek.accmode.con, i, bkc[i], blc[i], buc[i]);
 * The lower triangular part of the Q
 * matrix in the objective is specified.
int[] qosubi = { 0, 1, 2, 2 };
        qosubj = { 0, 1, 0, 2 };
double[] qoval = { 2.0, 0.2, -1.0, 2.0 };
/* Input the Q for the objective. */
task.putqobj(qosubi, qosubj, qoval);
 * The lower triangular part of the Q^0
 * matrix in the first constraint is specified.
 * This corresponds to adding the term
 * x0^2 - x1^2 - 0.1 x2^2 + 0.2 x0 x2
int[]
         qsubi = {0, 1, 2, 2 };
int[] qsubj = {0, 1, 2, 0 };
double[] qval = { -2.0, -2.0, -0.2, 0.2};
/* put Q^0 in constraint with index 0. */
task.putqconk (0,
               qsubi,
               qsubj,
               qval);
task.putobjsense(mosek.objsense.minimize);
/* Solve the problem */
try {
```

```
mosek.rescode termcode = task.optimize();
    } catch (mosek.Warning e) {
      System.out.println (" Mosek warning:");
      System.out.println (e.toString ());
    // Print a summary containing information
    // about the solution for debugging purposes
   task.solutionsummary(mosek.streamtype.msg);
   mosek.solsta solsta[] = new mosek.solsta[1];
    /* Get status information about the solution */
   task.getsolsta(mosek.soltype.itr, solsta);
    task.getxx(mosek.soltype.itr, // Interior solution.
    switch (solsta[0]) {
     case optimal:
      case near_optimal:
       System.out.println("Optimal primal solution\n");
       for (int j = 0; j < numvar; ++j)
         System.out.println ("x[" + j + "]:" + xx[j]);
       break;
      case dual_infeas_cer:
      case prim_infeas_cer:
      case near_dual_infeas_cer:
      case near_prim_infeas_cer:
       System.out.println("Primal or dual infeasibility.\n");
       break:
      case unknown:
       System.out.println("Unknown solution status.\n");
       break;
        System.out.println("Other solution status");
        break;
   }
 catch (mosek.Exception e) {
   System.out.println ("An error/warning was encountered");
   System.out.println (e.msg);
   throw e:
} /* Main */
```

## 6.3 Conic Quadratic Optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{K}_t$$

where  $x^t$  is a subset of the problem variables and  $\mathcal{K}_t$  is a convex cone. Since the set  $\mathbb{R}^n$  of real numbers is also a convex cone, we can simply write a compound conic constraint  $x \in \mathcal{K}$  where  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_l$  is a product of smaller cones and x is the full problem variable.

MOSEK can solve conic quadratic optimization problems of the form

where the domain restriction,  $x \in \mathcal{K}$ , implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \dots, x^{p-1}), \text{ with } x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}.$$

For convenience, a user defining a conic quadratic problem only needs to specify subsets of variables  $x^t$  belonging to quadratic cones. These are:

• Quadratic cone:

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{0} \ge \sqrt{\sum_{j=1}^{n-1} x_{j}^{2}} \right\}.$$

• Rotated quadratic cone:

$$Q_r^n = \left\{ x \in \mathbb{R}^n : 2x_0 x_1 \ge \sum_{j=2}^{n-1} x_j^2, \quad x_0 \ge 0, \quad x_1 \ge 0 \right\}.$$

For example, the following constraint:

$$(x_4, x_0, x_2) \in \mathcal{Q}^3$$

describes a convex cone in  $\mathbb{R}^3$  given by the inequality:

$$x_4 \ge \sqrt{x_0^2 + x_2^2}.$$

Furthermore, each variable may belong to one cone at most. The constraint  $x_i - x_j = 0$  would however allow  $x_i$  and  $x_j$  to belong to different cones with same effect.

## 6.3.1 Example CQO1

Consider the following conic quadratic problem which involves some linear constraints, a quadratic cone and a rotated quadratic cone.

minimize 
$$x_4 + x_5 + x_6$$
  
subject to  $x_1 + x_2 + 2x_3 = 1$ ,  
 $x_1, x_2, x_3 \ge 0$ , (6.6)  
 $x_4 \ge \sqrt{x_1^2 + x_2^2}$ ,  
 $2x_5x_6 \ge x_3^2$ 

## Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

#### Setting up the conic constraints

A cone is defined using the function Task.appendcone:

The first argument selects the type of quadratic cone, in this case either conetype.quad for a quadratic cone or conetype.rquad for a rotated quadratic cone. The second parameter is currently ignored and passing 0.0 will work.

The last argument is a list of indexes of the variables appearing in the cone.

Variants of this method are available to append multiple cones at a time.

#### Source code

Listing 6.4: Source code solving problem (6.6).

```
package com.mosek.example;
import mosek.*;
public class cqo1 {
 static final int numcon = 1;
 static final int numvar = 6;
 public static void main (String[] args) throws java.lang.Exception {
   // Since the value infinity is never used, we define
   // 'infinity' symbolic purposes only
   double infinity = 0;
   mosek.boundkey[] bkc
                            = { mosek.boundkey.fx };
   double[] blc = { 1.0 };
   double[] buc = { 1.0 };
   mosek.boundkey[] bkx
    = {mosek.boundkey.lo,
      mosek.boundkey.lo,
      mosek.boundkey.lo,
      mosek.boundkey.fr,
      mosek.boundkey.fr,
      mosek.boundkey.fr
      };
    double[] blx = { 0.0,}
                     0.0,
                     0.0,
                     -infinity,
                     -infinity,
                     -infinity
                   };
   double[] bux = { +infinity,
                     +infinity,
                     +infinity,
                     +infinity,
                     +infinity,
                     +infinity
                   };
    double[]c = {0.0},
                     0.0,
```

```
0.0,
                 1.0,
                 1.0,
                 1.0
               };
double[][] aval = {
  \{1.0\},
  {1.0}.
  {2.0}
};
int[][]
           asub = {
 {0},
  {0},
  {0}
};
int[] csub = new int[3];
double[] xx = new double[numvar];
// create a new environment object
try (Env env = new Env();
     Task task = new Task(env, 0, 0)) {
  // Directs the log task stream to the user specified
  // method task_msg_obj.stream
  task.set_Stream(
    mosek.streamtype.log,
    new mosek.Stream()
  { public void stream(String msg) { System.out.print(msg); }});
  /* Give MOSEK an estimate of the size of the input data.
  This is done to increase the speed of inputting data.
  However, it is optional. */
  /* Append 'numcon' empty constraints.
     The constraints will initially have no bounds. */
  task.appendcons(numcon);
  /* Append 'numvar' variables.
     The variables will initially be fixed at zero (x=0). */
  task.appendvars(numvar);
  /* Optionally add a constant term to the objective. */
  task.putcfix(0.0);
  for (int j = 0; j < numvar; ++j) {
    /* Set the linear term c_j in the objective.*/
    task.putcj(j, c[j]);
    /* Set the bounds on variable j.
       blx[j] \ll x_j \ll bux[j] */
    task.putbound(mosek.accmode.var, j, bkx[j], blx[j], bux[j]);
  }
  for (int j = 0; j < aval.length; ++j)
    /* Input column j of A */
                                         /* Variable (column) index.*/
    task.putacol(j,
                 asub[j],
                                        /* Row index of non-zeros in column j.*/
                 aval[j]);
                                        /* Non-zero Values of column j. */
  /* Set the bounds on constraints.
  for i=1, ..., numcon : blc[i] \leftarrow constraint i \leftarrow buc[i] */
  for (int i = 0; i < numcon; ++i)
    task.putbound(mosek.accmode.con, i, bkc[i], blc[i], buc[i]);
  csub[0] = 3:
  csub[1] = 0;
```

```
csub[2] = 1;
    task.appendcone(mosek.conetype.quad,
                    0.0, /* For future use only, can be set to 0.0 */
                    csub);
    csub[0] = 4;
    csub[1] = 5;
    csub[2] = 2;
    task.appendcone(mosek.conetype.rquad, 0.0, csub);
    task.putobjsense(mosek.objsense.minimize);
    System.out.println ("optimize");
    /* Solve the problem */
    mosek.rescode r = task.optimize();
    System.out.println (" Mosek warning:" + r.toString());
    // Print a summary containing information
    // about the solution for debugging purposes
    task.solutionsummary(mosek.streamtype.msg);
    mosek.solsta solsta[] = new mosek.solsta[1];
    /* Get status information about the solution */
    task.getsolsta(mosek.soltype.itr, solsta);
    task.getxx(mosek.soltype.itr, // Interior solution.
    switch (solsta[0]) {
      case optimal:
      case near_optimal:
        System.out.println("Optimal primal solution\n");
        for (int j = 0; j < numvar; ++j)
          System.out.println ("x[" + j + "]:" + xx[j]);
        break;
      case dual_infeas_cer:
      case prim_infeas_cer:
      case near_dual_infeas_cer:
      case near_prim_infeas_cer:
        System.out.println("Primal or dual infeasibility.\n");
        break:
      case unknown:
        System.out.println("Unknown solution status.\n");
      default:
        System.out.println("Other solution status");
        break;
  } catch (mosek.Exception e) {
    System.out.println ("An error/warning was encountered");
    System.out.println (e.toString());
    throw e;
  }
}
```

# 6.4 Semidefinite Optimization

Semidefinite optimization is a generalization of conic quadratic optimization, allowing the use of matrix variables belonging to the convex cone of positive semidefinite matrices

$$S_+^r = \left\{ X \in S^r : z^T X z \ge 0, \quad \forall z \in \mathbb{R}^r \right\},$$

where  $S^r$  is the set of  $r \times r$  real-valued symmetric matrices.

MOSEK can solve semidefinite optimization problems of the form

$$\begin{array}{lll} \text{minimize} & \sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f \\ \text{subject to} & l_i^c & \leq & \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle & \leq & u_i^c, & i = 0, \dots, m-1, \\ l_j^x & \leq & x_j & \leq & u_j^x, & j = 0, \dots, n-1, \\ & & x \in \mathcal{K}, \overline{X}_j \in \mathcal{S}_+^{r_j}, & j = 0, \dots, p-1 \end{array}$$

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_+^{r_j}$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}^{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $A, B \in \mathbb{R}^{m \times n}$  we have

$$\langle A, B \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} A_{ij} B_{ij}.$$

## 6.4.1 Example SDO1

We consider the simple optimization problem with semidefinite and conic quadratic constraints:

The problem description contains a 3-dimensional symmetric semidefinite variable which can be written explicitly as:

$$\overline{X} = \begin{bmatrix} \overline{X}_{00} & \overline{X}_{10} & \overline{X}_{20} \\ \overline{X}_{10} & \overline{X}_{11} & \overline{X}_{21} \\ \overline{X}_{20} & \overline{X}_{21} & \overline{X}_{22} \end{bmatrix} \in \mathcal{S}_{+}^{3},$$

and a conic quadratic variable  $(x_0, x_1, x_2) \in \mathcal{Q}^3$ . The objective is to minimize

$$2(\overline{X}_{00} + \overline{X}_{10} + \overline{X}_{11} + \overline{X}_{21} + \overline{X}_{22}) + x_0,$$

subject to the two linear constraints

$$\begin{array}{rcl} \overline{X}_{00} + \overline{X}_{11} + \overline{X}_{22} + x_0 & = & 1, \\ \overline{X}_{00} + \overline{X}_{11} + \overline{X}_{22} + 2(\overline{X}_{10} + \overline{X}_{20} + \overline{X}_{21}) + x_1 + x_2 & = & 1/2. \end{array}$$

### Setting up the linear and quadratic part

The linear and quadratic parts (constraints, variables, objective, cones) are set up using the methods described in the relevant tutorials; Sec. 6.1 and Sec. 6.3. Here we only discuss the aspects directly involving semidefinite variables.

### Appending semidefinite variables

First, we need to declare the number of semidefinite variables in the problem, similarly to the number of linear variables and constraints. This is done with the function <code>Task.appendbarvars</code>.

```
task.appendbarvars(dimbarvar);
```

### Appending coefficient matrices

Coefficient matrices  $\overline{C}_j$  and  $\overline{A}_{ij}$  are constructed as weighted combinations of sparse symmetric matrices previously appended with the function Task.appendsparsesymmat.

The arguments specify the dimension of the symmetric matrix, followed by its description in the sparse triplet format. Only lower-triangular entries should be included. The function produces a unique index of the matrix just entered in the collection of all coefficient matrices defined by the user.

After one or more symmetric matrices have been created using Task.appendsparsesymmat, we can combine them to set up the objective matrix coefficient  $\overline{C}_j$  using Task.putbarcj, which forms a linear combination of one or more symmetric matrices. In this example we form the objective matrix directly, i.e. as a weighted combination of a single symmetric matrix.

```
task.putbarcj(0, idx, falpha);
```

Similarly, a constraint matrix coefficient  $\overline{A}_{ij}$  is set up by the function Task.putbaraij.

```
task.putbaraij(0, 0, idx, falpha);
```

### Retrieving the solution

After the problem is solved, we read the solution using Task.getbarxj:

```
task.getbarxj(mosek.soltype.itr, /* Request the interior solution. */
0,
barx);
```

The function returns the half-vectorization of  $\overline{X}_j$  (the lower triangular part stacked as a column vector), where the semidefinite variable index j is passed as an argument.

## Source code

Listing 6.5: Source code solving problem (6.7).

```
lenbarvar[] = \{3 * (3 + 1) / 2\}; /* Number of scalar SD variables */
mosek.boundkey bkc[] = { mosek.boundkey.fx,
                         mosek.boundkey.fx
                     = { 1.0, 0.5 };
double[]
             blc
double[]
                    = { 1.0, 0.5 };
             buc
int∏
             barc_i = {0, 1, 1, 2, 2},
             barc_j = \{0, 0, 1, 1, 2\};
             barc_v = {2.0, 1.0, 2.0, 1.0, 2.0};
double[]
int[][]
                     = {{0}, {1, 2}}; /* column subscripts of A */
double[][]
             aval
                     = \{\{1.0\}, \{1.0, 1.0\}\};
             bara_i = { {0, 1, 2}, {0, 1, 2, 1, 2, 2}},
int[][]
             bara_j = \{ \{0, 1, 2\}, \{0, 0, 0, 1, 1, \dots \} \}
                                                                    2 } };
double[][] bara_v = { {1.0, 1.0, 1.0}, {1.0, 1.0, 1.0, 1.0, 1.0, 1.0}};
             conesub = \{ 0, 1, 2 \};
int[]
try (Env env = new Env();
     Task task = new Task(env, 0, 0)) {
  // Directs the log task stream to the user specified
  //\ {\it method}\ task\_{\it msg\_obj.stream}
  task.set_Stream(
    mosek.streamtype.log,
   new mosek.Stream()
  { public void stream(String msg) { System.out.print(msg); }});
  /* Append 'NUMCON' empty constraints.
     The constraints will initially have no bounds. */
  task.appendcons(numcon);
  /* Append 'NUMVAR' variables.
     The variables will initially be fixed at zero (x=0). */
  task.appendvars(numvar);
  /* Append 'NUMBARVAR' semidefinite variables. */
  task.appendbarvars(dimbarvar);
  /* Optionally add a constant term to the objective. */
  task.putcfix(0.0);
  /* Set the linear term c_j in the objective.*/
  task.putcj(0, 1.0);
  for (int j = 0; j < numvar; ++j)
    task.putvarbound(j, mosek.boundkey.fr, -0.0, 0.0);
  /* Set the linear term barc_j in the objective.*/
    long[] idx = new long[1];
    double[] falpha = { 1.0 };
    idx[0] = task.appendsparsesymmat(dimbarvar[0],
                                     barc_i,
                                     barc_j,
                                     barc_v);
    task.putbarcj(0, idx, falpha);
  /* Set the bounds on constraints.
```

```
for i=1, ...,numcon : blc[i] <= constraint i <= buc[i] */</pre>
for (int i = 0; i < numcon; ++i)
                                 /* Index of constraint.*/
  task.putconbound(i,
                                /* Bound key.*/
                   bkc[i],
                   blc[i],
                                /* Numerical value of lower bound.*/
                   buc[i]);
                                /* Numerical value of upper bound.*/
/* Input A row by row */
for (int i = 0; i < numcon; ++i)
  task.putarow(i,
               asub[i],
               aval[i]);
/* Append the conic quadratic cone */
{\tt task.appendcone} ({\tt mosek.conetype.quad},
                0.0,
                conesub);
/* Add the first row of barA */
  long[] idx = new long[1];
  double[] falpha = {1.0};
  task.appendsparsesymmat(dimbarvar[0],
                           bara_i[0],
                           bara_j[0],
                           bara_v[0],
                           idx);
 task.putbaraij(0, 0, idx, falpha);
{
  long[] idx = new long[1];
  double[] falpha = {1.0};
  /* Add the second row of barA */
  {\tt task.appendsparsesymmat(dimbarvar[0],}
                          bara_i[1],
                           bara_j[1],
                           bara_v[1],
                           idx):
 task.putbaraij(1, 0, idx, falpha);
/* Run optimizer */
task.optimize();
/* Print a summary containing information
   about the solution for debugging purposes*/
task.solutionsummary (mosek.streamtype.msg);
mosek.solsta[] solsta = new mosek.solsta[1];
task.getsolsta (mosek.soltype.itr, solsta);
switch (solsta[0]) {
 case optimal:
  case near_optimal:
    double[] xx = new double[numvar];
    double[] barx = new double[lenbarvar[0]];
    task.getxx(mosek.soltype.itr, xx);
    task.getbarxj(mosek.soltype.itr,
                                         /* Request the interior solution.
```

```
0.
                       barx);
        System.out.println("Optimal primal solution");
        for (int i = 0; i < numvar; ++i)</pre>
          System.out.println("x[" + i + "]
                                             : " + xx[i]);
        for (int i = 0; i < lenbarvar[0]; ++i)</pre>
          System.out.println("barx[" + i + "]: " + barx[i]);
        break:
      case dual_infeas_cer:
      case prim_infeas_cer:
      case near_dual_infeas_cer:
      case near_prim_infeas_cer:
        System.out.println("Primal or dual infeasibility certificate found.");
      case unknown:
        System.out.println("The status of the solution could not be determined.");
        break:
      default:
        System.out.println("Other solution status.");
}
```

# 6.5 Integer Optimization

An optimization problem where one or more of the variables are constrained to integer values is called a (mixed) integer optimization problem. **MOSEK** supports integer variables in combination with linear and conic quadratic problems. See the previous tutorials for an introduction to how to model these types of problems.

### 6.5.1 Example MILO1

We use the example

maximize 
$$x_0 + 0.64x_1$$
  
subject to  $50x_0 + 31x_1 \le 250$ ,  
 $3x_0 - 2x_1 \ge -4$ ,  
 $x_0, x_1 \ge 0$  and integer (6.8)

to demonstrate how to set up and solve a problem with integer variables. It has the structure of a linear optimization problem (see Sec. 6.1) except for integrality constraints on the variables. Therefore, only the specification of the integer constraints requires something new compared to the linear optimization problem discussed previously.

First, the integrality constraints are imposed using the function Task.putvartype:

```
for (int j = 0; j < numvar; ++j)
  task.putvartype(j, mosek.variabletype.type_int);</pre>
```

Next, the example demonstrates how to set various useful parameters of the mixed-integer optimizer. See Sec. 14 for details.

```
/* Set max solution time */
task.putdouparam(mosek.dparam.mio_max_time, 60.0);
```

The complete source for the example is listed Listing 6.6. Please note that when Task. getsolutionslice is called, the integer solution is requested by using soltype.itg. No dual solution is defined for integer optimization problems.

Listing 6.6: Source code implementing problem (6.8).

```
package com.mosek.example;
import mosek.*;
public class milo1 {
 static final int numcon = 2;
 static final int numvar = 2;
 public static void main (String[] args) {
   // Since the value infinity is never used, we define
   // 'infinity' symbolic purposes only
   double infinity = 0;
   mosek.boundkey[] bkc
     = { mosek.boundkey.up, mosek.boundkey.lo };
   double[] buc = { 250.0,
                                     infinity };
   mosek.boundkey[] bkx
    = { mosek.boundkey.lo, mosek.boundkey.lo };
   double[] blx = { 0.0,
                           0.0 };
   double[] bux = { infinity,
                                    infinity };
   double[] c = {1.0, 0.64 };
   int[][] asub = { {0, 1},
                                 {0, 1} };
   double[][] aval = { {50.0, 3.0}, {31.0, -2.0} };
   int[] ptrb = { 0, 2 };
   int[] ptre = { 2, 4 };
   double[] xx = new double[numvar];
   try (Env env = new Env();
        Task task = new Task(env, 0, 0)) {
     // Directs the log task stream to the user specified
     // method task_msg_obj.stream
     task.set_Stream(
       mosek.streamtype.log,
       new mosek.Stream()
     { public void stream(String msg) { System.out.print(msg); }});
     task.set_ItgSolutionCallback(
     new mosek.ItgSolutionCallback() {
       public void callback(double[] xx) {
         System.out.print("New integer solution: ");
         for (double v : xx) System.out.print("" + v + " ");
         System.out.println("");
       }
     });
     /* Append 'numcon' empty constraints.
     The constraints will initially have no bounds. */
     task.appendcons(numcon);
     /* Append 'numvar' variables.
     The variables will initially be fixed at zero (x=0). */
     task.appendvars(numvar);
     for (int j = 0; j < numvar; ++j) {
```

```
/* Set the linear term c_j in the objective.*/
  task.putcj(j, c[j]);
  /* Set the bounds on variable j.
     blx[j] <= x_j <= bux[j] */
  task.putvarbound(j, bkx[j], blx[j], bux[j]);
  /* Input column j of A */
                                      /* Variable (column) index.*/
  task.putacol(j,
               asub[j],
                                      /* Row index of non-zeros in column j.*/
               aval[j]);
                                      /* Non-zero Values of column j. */
/* Set the bounds on constraints.
for i=1, ..., numcon : blc[i] <= constraint i <= buc[i] */
for (int i = 0; i < numcon; ++i)
 task.putconbound(i, bkc[i], blc[i], buc[i]);
/* Specify integer variables. */
for (int j = 0; j < numvar; ++j)
 task.putvartype(j, mosek.variabletype.type_int);
/* Set max solution time */
task.putdouparam(mosek.dparam.mio_max_time, 60.0);
/* A maximization problem */
task.putobjsense(mosek.objsense.maximize);
/* Solve the problem */
try {
 task.optimize();
} catch (mosek.Warning e) {
 System.out.println (" Mosek warning:");
  System.out.println (e.toString ());
// Print a summary containing information
// about the solution for debugging purposes
task.solutionsummary(mosek.streamtype.msg);
task.getxx(mosek.soltype.itg, // Integer solution.
           xx):
mosek.solsta solsta[] = new mosek.solsta[1];
/* Get status information about the solution */
task.getsolsta(mosek.soltype.itg, solsta);
switch (solsta[0]) {
  case integer_optimal:
  case near_integer_optimal:
    System.out.println("Optimal solution\n");
    for (int j = 0; j < numvar; ++j)
     System.out.println ("x[" + j + "]:" + xx[j]);
    break:
  case prim_feas:
    System.out.println("Feasible solution\n");
    for (int j = 0; j < numvar; ++j)
      System.out.println ("x[" + j + "]:" + xx[j]);
    break;
  case unknown:
    mosek.prosta prosta[] = new mosek.prosta[1];
    task.getprosta(mosek.soltype.itg, prosta);
    switch (prosta[0]) {
      case prim_infeas_or_unbounded:
        System.out.println("Problem status Infeasible or unbounded");
        break:
      case prim_infeas:
```

```
System.out.println("Problem status Infeasible.");
            break;
          case unknown:
            System.out.println("Problem status unknown.");
            break;
          default:
            System.out.println("Other problem status.");
        }
        break;
      default:
        System.out.println("Other solution status");
  }
  catch (mosek.Exception e) {
    System.out.println ("An error or warning was encountered");
    System.out.println (e.getMessage ());
    throw e:
  }
}
```

## 6.5.2 Specifying an initial solution

Solution time of can often be reduced by providing an initial solution for the solver. It is not necessary to specify the whole solution. By setting the <code>iparam.mio\_construct\_sol</code> parameter to <code>onoffkey.on</code> and inputting values for the integer variables only, <code>MOSEK</code> will be forced to compute the remaining continuous variable values. If the specified integer solution is infeasible or incomplete, <code>MOSEK</code> will simply ignore it.

We concentrate on a simple example below.

maximize 
$$7x_0 + 10x_1 + x_2 + 5x_3$$
  
subject to  $x_0 + x_1 + x_2 + x_3 \le 2.5$   
 $x_0, x_1, x_2 \in \mathbb{Z}$   
 $x_0, x_1, x_2, x_3 \ge 0$  (6.9)

Solution values can be set using Task.putxxslice and related methods.

Listing 6.7: Implementation of problem (6.9) specifying an initial solution.

The complete code is not very different from the first example and is available for download as mioinitsol.java. For more details about this process see Sec. 14.

# 6.6 Problem Modification and Reoptimization

Often one might want to solve not just a single optimization problem, but a sequence of problems, each differing only slightly from the previous one. This section demonstrates how to modify and re-optimize an existing problem. The example we study is a simple production planning model.

Problem modifications regarding variables, cones, objective function and constraints can be grouped in categories:

- add/remove,
- coefficient modifications,
- bounds modifications.

Especially removing variables and constraints can be costly. Special care must be taken with respect to constraints and variable indexes that may be invalidated.

Depending on the type of modification, **MOSEK** may be able to optimize the modified problem more efficiently exploiting the information and internal state from the previous execution. After optimization, the solution is always stored internally, and is available before next optimization. The former optimal solution may be still feasible, but no longer optimal; or it may remain optimal if the modification of the objective function was small. This special case is discussed in Sec. 15.3.

In general, **MOSEK** exploits dual information and availability of an optimal basis from the previous execution. The simplex optimizer is well suited for exploiting an existing primal or dual feasible solution. Restarting capabilities for interior-point methods are still not as reliable and effective as those for the simplex algorithm. More information can be found in Chapter 10 of the book |Chv83|.

Parameter settings (see Sec. 7.4) can also be changed between optimizations.

# 6.6.1 Example: Production Planning

A company manufactures three types of products. Suppose the stages of manufacturing can be split into three parts: Assembly, Polishing and Packing. In the table below we show the time required for each stage as well as the profit associated with each product.

Product no.	Assembly (minutes)	Polishing (minutes)	Packing (minutes)	Profit (\$)
0	2	3	2	1.50
1	4	2	3	2.50
2	3	3	2	3.00

With the current resources available, the company has 100,000 minutes of assembly time, 50,000 minutes of polishing time and 60,000 minutes of packing time available per year. We want to know how many items of each product the company should produce each year in order to maximize profit?

Denoting the number of items of each type by  $x_0, x_1$  and  $x_2$ , this problem can be formulated as a linear optimization problem:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2$$
  
subject to  $2x_0 + 4x_1 + 3x_2 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 \le 60000$ , (6.10)

and

$$x_0, x_1, x_2 \geq 0.$$

Code in Listing 6.8 loads and solves this problem.

Listing 6.8: Setting up and solving problem (6.10)

```
// Since the value infinity is never used, we define
// 'infinity' symbolic purposes only
double infinity = 0;
int numcon = 3;
int numvar = 3;
```

```
c[] = \{1.5, 2.5, 3.0\};
mosek.boundkey
                 bkc[] = { mosek.boundkey.up,
                           mosek.boundkey.up,
                           mosek.boundkey.up
                 blc[] = { -infinity,
double
                           -infinity,
                           -infinity
                         };
                 buc[] = { 100000,
double
                           50000,
                           60000
                         };
mosek.boundkey
                bkx[] = { mosek.boundkey.lo,
                          mosek.boundkey.lo,
                           mosek.boundkey.lo
                         };
                 blx[] = { 0.0, 0.0, 0.0 };
double
double
                 bux[] = { +infinity,
                           +infinity,
                           +infinity
int asub[][] = {
  {0, 1, 2},
  {0, 1, 2},
  {0, 1, 2}
};
double aval[][] = {
 { 2.0, 3.0, 2.0 },
 { 4.0, 2.0, 3.0 },
 { 3.0, 3.0, 2.0 }
double[] xx = new double[numvar];
try (Env env = new Env();
     Task task = new Task(env, 0, 0)) {
  /* Append the constraints. */
  task.appendcons(numcon);
  /* Append the variables. */
  task.appendvars(numvar);
  /* Put C. */
  for (int j = 0; j < numvar; ++j)
   task.putcj(j, c[j]);
  /* Put constraint bounds. */
  for (int i = 0; i < numcon; ++i)
    task.putbound(mosek.accmode.con, i, bkc[i], blc[i], buc[i]);
  /* Put variable bounds. */
  for (int j = 0; j < numvar; ++j)
    task.putbound(mosek.accmode.var, j, bkx[j], blx[j], bux[j]);
  /* Put A. */
  if ( numcon > 0 ) {
    for (int j = 0; j < numvar; ++j)
     task.putacol(j,
                   asub[j],
                   aval[j]);
```

# 6.6.2 Changing the Linear Constraint Matrix

Suppose we want to change the time required for assembly of product 0 to 3 minutes. This corresponds to setting  $a_{0,0} = 3$ , which is done by calling the function Task.putaij as shown below.

```
task.putaij(0, 0, 3.0);
```

The problem now has the form:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2$$
  
subject to  $3x_0 + 4x_1 + 3x_2 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 \le 60000$ , (6.11)

and

$$x_0, x_1, x_2 \ge 0.$$

After this operation we can reoptimize the problem.

### 6.6.3 Appending Variables

We now want to add a new product with the following data:

Product no.	Assembly (minutes)	Polishing (minutes)	Packing (minutes)	Profit (\$)
3	4	0	1	1.00

This corresponds to creating a new variable  $x_3$ , appending a new column to the A matrix and setting a new term in the objective. We do this in Listing 6.9

Listing 6.9: How to add a new variable (column)

After this operation the new problem is:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2 + 1.0x_3$$
  
subject to  $3x_0 + 4x_1 + 3x_2 + 4x_3 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 + 1x_3 \le 60000$ , (6.12)

and

$$x_0, x_1, x_2, x_3 \ge 0.$$

# 6.6.4 Appending Constraints

Now suppose we want to add a new stage to the production process called *Quality control* for which 30000 minutes are available. The time requirement for this stage is shown below:

Product no.	Quality control (minutes)
0	1
1	2
2	1
3	1

This corresponds to adding the constraint

$$x_0 + 2x_1 + x_2 + x_3 \le 30000$$

to the problem. This is done as follows.

Listing 6.10: Adding a new constraint.

Again, we can continue with re-optimizing the modified problem.

# 6.7 Solution Analysis

The main purpose of **MOSEK** is to solve optimization problems and therefore the most fundamental question to be asked is whether the solution reported by **MOSEK** is a solution to the desired optimization problem.

There can be several reasons why it might be not case. The most prominent reasons are:

- A wrong problem. The problem inputted to **MOSEK** is simply not the right problem, i.e. some of the data may have been corrupted or the model has been incorrectly built.
- Numerical issues. The problem is badly scaled or otherwise badly posed.
- Other reasons. E.g. not enough memory or an explicit user request to stop.

The first step in verifying that **MOSEK** reports the expected solution is to inspect the solution summary generated by **MOSEK** (see Sec. 6.7.1). The solution summary provides information about

- the problem and solution statuses,
- objective value and infeasibility measures for the primal solution, and
- objective value and infeasibility measures for the dual solution, where applicable.

By inspecting the solution summary it can be verified that **MOSEK** produces a feasible solution, and, in the continuous case, the optimality can be checked using the dual solution. Furthermore, the problem itself ca be inspected using the problem analyzer discussed in Sec. 15.1.

If the summary reports conflicting information (e.g. a solution status that does not match the actual solution), or the cause for terminating the solver before a solution was found cannot be traced back to the reasons stated above, it may be caused by a bug in the solver; in this case, please contact **MOSEK** support (see Sec. 2).

If it has been verified that **MOSEK** solves the problem correctly but the solution is still not as expected, next step is to verify that the primal solution satisfies all the constraints. Hence, using the original problem it must be determined whether the solution satisfies all the required constraints in the model. For instance assume that the problem has the constraints

$$x_1 + 2x_2 + x_3 \le 1,$$
  
 $x_1, x_2, x_3 \ge 0$ 

and MOSEK reports the optimal solution

$$x_1 = x_2 = x_3 = 1.$$

Then clearly the solution violates the constraints. The most likely explanation is that the model does not match the problem entered into **MOSEK**, for instance

$$x_1 - 2x_2 + x_3 \le 1$$

may have been inputted instead of

$$x_1 + 2x_2 + x_3 \le 1.$$

A good way to debug such an issue is to dump the problem to *OPF file* and check whether the violated constraint has been specified correctly.

Verifying that a feasible solution is optimal can be harder. However, for continuous problems, i.e. problems without any integer constraints, optimality can verified using a dual solution. Normally, **MOSEK** will report a dual solution; if that is feasible and has the same objective value as the primal solution, then the primal solution must be optimal.

An alternative method is to find another primal solution that has better objective value than the one reported to MOSEK. If that is possible then either the problem is badly posed or there is bug in MOSEK.

## 6.7.1 The Solution Summary

Due to **MOSEK** employs finite precision floating point numbers then reported solution is an approximate optimal solution. Therefore after solving an optimization problem it is relevant to investigate how good an approximation the solution is. For a convex optimization problem that is an easy task because the optimality conditions are:

- The primal solution must satisfy all the primal constraints.
- The dual solution much satisfy all the dual constraints.
- The primal and dual objective values must be identical.

Therefore, the **MOSEK** solution summary displays that information that makes it possible to verify the optimality conditions. Indeed the solution summary reports how much primal and dual solutions violate the primal and constraints respectively. In addition the objective values assoctated with each solution repoted.

In case of a linear optimization problem the solution summary may look like

```
Basic solution summary
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal. obj: -4.6475314286e+002 nrm: 5e+002 Viol. con: 1e-014 var: 1e-014
Dual. obj: -4.6475314543e+002 nrm: 1e+001 Viol. con: 4e-009 var: 4e-016
```

The interpretaion of the solution summary is as follows:

- Information for the basic solution is reported.
- The problem status is primal and dual feasible which means the problem has an optimal solution.
- The solution status is optimal.
- Next information about the primal solution is reported. The information consists of the objective value, the infinity norm of the primal solution and violation measures. The violation for the constraints (con:) is the maximal violation in any of the constraints. Whereas the violations for the variables (var:) is the maximal bound violation for any of the variables. In this case the primal violations for the constraints and variables are small meaning the solution is an almost feasible solution. Observe due to the rounding errors it can be expected that the violations are proportional to the size (nrm:) of the solution.
- Similarly for the dual solution the violations are small and hence the dual solution is almost feasible.
- Finally, it can be seen that the primal and dual objective values are almost identical.

To summarize in this case a primal and a dual solution only violate the primal and dual constraints slightly. Moreover, the primal and dual objective values are almost identical and hence it can be concluded that the reported solution is a good approximation to the optimal solution.

The reason the size (=norms) of the solution are shown is that it shows some about conditioning of the problem because if the primal and/or dual solution has very large norm then the violations and objective values are sensitive to small pertubations in the problem data. Therefore, the problem is unstable and care should be taken before using the solution.

Observe the function Task. solutionsummary will print out the solution summary. In addition

- the problem status can be obtained using Task. getprosta.
- the solution status can be obtained using Task. getsolsta.
- the primal constraint and variable violations can be obtained with Task. getpviolcon and Task. getpviolvar.
- the dual constraint and variable violations can be obtained with Task. getdviolcon and Task. getdviolvar respectively.
- the primal and dual objective values can be obtained with Task.getprimalobj and Task.getdualobj.

Now what happens if the problem does not have an optimal solution e.g. is primal infeasible. In such a case the solution summary may look like

```
Interior-point solution summary
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 6.7319732555e+000 nrm: 8e+000 Viol. con: 3e-010 var: 2e-009
```

i.e. MOSEK reports that the solution is a certificate of primal infeasibility but a certificate of primal infeasibility what does that mean? It means that the dual solution is a Farkas type certificate. Recall Farkas' Lemma says

$$\begin{array}{rcl} Ax & = & b, \\ x & > & 0 \end{array}$$

if and only if a y exists such that

$$\begin{array}{lcl}
A^T y & \leq & 0, \\
b^T y & > & 0.
\end{array}$$
(6.13)

Observe the infeasibility certificate has the same form as a regular dual solution and therefore the certificate is stored as a dual solution. In order to check quality of the primal infeasibility certificate it should be checked whether satisfies (6.13). Hence, the dual objective value is  $b^Ty$  should be strictly positive and the maximal violation in  $A^Ty \leq 0$  should be a small. In this case we conclude the certificate is of high quality because the dual objective is postive and large compared to the violations. Note the Farkas certificate is a ray so any postive multiple of that ray is also certificate. This implies the absolute of the value objective value and the violation is not relevant.

In the case a problem is dual infeasible then the solution summary may look like

```
Basic solution summary
Problem status : DUAL_INFEASIBLE
Solution status : DUAL_INFEASIBLE_CER
Primal. obj: -2.00000000000e-002 nrm: 1e+000 Viol. con: 0e+000 var: 0e+000
```

Observe when a solution is a certificate of dual infeasibility then the primal solution contains the certificate. Moreoever, given the problem is a minimization problem the objective value should be negative and large compared to the worst violation if the certificate is strong.

Listing 6.11 shows how to use these function to determine the quality of the solution.

Listing 6.11: An example of solution quality analysis.

```
package com.mosek.example;
import mosek.*;
public class solutionquality {
  public static void main (String[] args) {
    if (args.length == 0) {
       System.out.println ("Missing argument, syntax is:");
       System.out.println (" solutionquality inputfile");
    } else {
```

```
try (Env env = new Env();
          Task task = new Task(env, 0, 0)) {
       task.set_Stream (mosek.streamtype.log,
       new mosek.Stream() {
         public void stream(String msg) { System.out.print(msg); }
       });
       // We assume that a problem file was given as the first command
       // line argument (received in `args')
       task.readdata (args[0]);
       // Solve the problem
       task.optimize ();
       // System.Out.Println (a summary of the solution
       task.solutionsummary (mosek.streamtype.log);
       mosek.solsta solsta[] = new mosek.solsta[1];
       task.getsolsta(mosek.soltype.bas, solsta);
       double pobj[] = new double[1];
       double pviolcon[] = new double[1];
       double pviolvar[] = new double[1];
       double pviolbarvar[] = new double[1];
       double pviolcones[] = new double[1];
       double pviolitg[] = new double[1];
       double dobj[] = new double[1];
       double dviolcon[] = new double[1];
       double dviolvar[] = new double[1];
       double dviolbarvar[] = new double[1];
       double dviolcones[] = new double[1];
       task.getsolutioninfo(mosek.soltype.bas,
                            pobj, pviolcon, pviolvar, pviolbarvar, pviolcones, pviolitg,
                            dobj, dviolcon, dviolvar, dviolbarvar, dviolcones);
       switch (solsta[0]) {
         case optimal:
         case near_optimal:
           double abs_obj_gap
                                  = Math.abs(dobj[0] - pobj[0]);
           double rel_obj_gap
                                 = abs_obj_gap / (1.0 + Math.min(Math.abs(pobj[0]), Math.
\hookrightarrowabs(dobj[0]));
           double max_primal_viol = Math.max(pviolcon[0], pviolvar[0]);
           max_primal_viol = Math.max(max_primal_viol , pviolbarvar[0]);
           max_primal_viol = Math.max(max_primal_viol , pviolcones[0]);
                                 = Math.max(dviolcon[0], dviolvar[0]);
           double max_dual_viol
           max_dual_viol = Math.max(max_dual_viol , dviolbarvar[0]);
           max_dual_viol = Math.max(max_dual_viol , dviolcones[0]);
           // Assume the application needs the solution to be within
               1e-6 of optimality in an absolute sense. Another approach
           // would be looking at the relative objective gap
           System.out.println ("Customized solution information.\n");
           System.out.println (" Absolute objective gap: " + abs_obj_gap);
           System.out.println (" Relative objective gap: " + rel_obj_gap);
           System.out.println (" Max primal violation : " + max_primal_viol);
           System.out.println (" Max dual violation : " + max_dual_viol);
           boolean accepted = true;
           if ( rel_obj_gap > 1e-6 ) {
```

```
System.out.println ("Warning: The relative objective gap is LARGE.");
              accepted = false;
            // We will accept a primal infeasibility of 1e-8 and
            // dual infeasibility of 1e-6. These number should chosen problem
            // dependent.
            if ( max\_primal\_viol > 1e-8 ) {
              System.out.println ("Warning: Primal violation is too LARGE");
              accepted = false;
            if (max_dual_viol > 1e-6 ) {
              System.out.println ("Warning: Dual violation is too LARGE.");
              accepted = false;
            if ( accepted ) {
              int numvar = task.getnumvar();
              System.out.println ("Optimal primal solution");
              double xj[] = new double[1];
              for (int j = 0; j < numvar; j++) {
                {\tt task.getxxslice}({\tt mosek.soltype.bas}, \ {\tt j}, \ {\tt j} \ + \ 1, \ {\tt xj});\\
                System.out.println ("x[" + j + "]: " + xj[0]);
            } else {
               // print etailed information about the solution
              task.analyzesolution(mosek.streamtype.log, mosek.soltype.bas);
            break:
          case dual_infeas_cer:
          case prim_infeas_cer:
          case near_dual_infeas_cer:
          case near_prim_infeas_cer:
            {\bf System.out.println~("Primal~or~dual~infeasibility~certificate~found.")};\\
            break;
          case unknown:
            System.out.println ("The status of the solution is unknown.");
            break:
          default:
            System.out.println ("Other solution status");
      } catch (mosek.Exception e) {
        System.out.println ("An error/warning was encountered");
        System.out.println (e.toString());
        throw e;
      }
    }
 }
}
```

### 6.7.2 The Solution Summary for Mixed-Integer Problems

The solution summary for a mixed-integer problem may look like

Listing 6.12: Example of solution summary for a mixed-integer problem.

```
Integer solution solution summary
Problem status : PRIMAL_FEASIBLE
Solution status : INTEGER_OPTIMAL
```

```
Primal. obj: 3.4016000000e+005 nrm: 1e+000 Viol. con: 0e+000 var: 0e+000 itg: 3e-014
```

The main diffrence compared to the continous case covered previously is that no information about the dual solution is provided. Simply because there is no dual solution available for a mixed integer problem. In this case it can be seen that the solution is highly feasible because the violations are small. Moreoever, the solution is denoted integer optimal. Observe *itg:* 3e-014 implies that all the integer constrained variables are at most 3e – 014 from being an exact integer.

For a more in-depth treatment see the following sections:

- Case studies for more advanced and complicated optimization examples.
- Problem Formulation and Solutions for formal mathematical formulations of problems MOSEK can solve, dual problems and infeasibility certificates.

## SOLVER INTERACTION TUTORIALS

In this section we cover the interaction with the solver.

# 7.1 Accessing the solution

This section contains important information about the status of the solver and the status of the solution, which must be checked in order to properly interpret the results of the optimization.

### 7.1.1 Solver termination

The optimizer provides two status codes relevant for error handling:

- Response code of type rescode. It indicates if any unexpected error (such as an out of memory error, licensing error etc.) has occurred. The expected value for a successful optimization is rescode.ok.
- **Termination code**: It provides information about why the optimizer terminated, for instance if a predefined time limit has been reached. These are not errors, but ordinary events that can be expected (depending on parameter settings and the type of optimizer used).

If the optimization was successful then the method *Task.optimize* returns normally and its output is the termination code. If an error occurs then the method throws an exception, which contains the response code. See Sec. 7.2 for how to access it.

If a runtime error causes the program to crash during optimization, the first debugging step is to enable logging and check the log output. See Sec. 7.3.

If the optimization completes successfully, the next step is to check the solution status, as explained below.

### 7.1.2 Available solutions

MOSEK uses three kinds of optimizers and provides three types of solutions:

- basic solution (BAS, from the simplex optimizer),
- interior-point solution (ITR, from the interior-point optimizer),
- integer solution (ITG, from the mixed-integer optimizer).

Under standard parameters settings the following solutions will be available for various problem types:

	Simplex opti-	Interior-point opti-	Mixed-integer opti-
	mizer	mizer	mizer
Linear problem	soltype.bas	soltype.itr	
Nonlinear continuous prob-		soltype.itr	
lem			
Problem with integer vari-			soltype.itg

Table 7.1: Types of solutions available from MOSEK

For linear problems the user can force a specific optimizer choice making only one of the two solutions available. For example, if the user disables basis identification, then only the interior point solution will be available for a linear problem. Numerical issues may cause one of the solutions to be unknown even if another one is feasible.

Not all components of a solution are always available. For example, there is no dual solution for integer problems.

The user will always need to specify which solution should be accessed.

### 7.1.3 Problem and solution status

Assuming that the optimization terminated without errors, the next important step is to check the problem and solution status. There is one for every type of solution, as explained above.

### Problem status

ables

Problem status (*prosta*, retrieved with *Task.getprosta*) determines whether the problem is certified as feasible. Its values can roughly be divided into the following broad categories:

- **feasible** the problem is feasible. For continuous problems and when the solver is run with default parameters, the feasibility status should ideally be prosta.prim\_and\_dual\_feas.
- **primal/dual infeasible** the problem is infeasible or unbounded or a combination of those. The exact problem status will indicate the type of infeasibility.
- unknown the solver was unable to reach a conclusion, most likely due to numerical issues.

#### Solution status

Solution status (solsta, retrieved with Task.getsolsta) provides the information about what the solution values actually contain. The most important broad categories of values are:

- optimal (solsta.optimal) the solution values are feasible and optimal.
- near optimal (solsta.near\_optimal) the solution values are feasible and they were certified to be at least nearly optimal up to some accuracy.
- **certificate** the solution is in fact a certificate of infeasibility (primal or dual, depending on the solution).
- unknown/undefined the solver could not solve the problem or this type of solution is not available for a given problem.

The solution status determines the action to be taken. For example, in some cases a suboptimal solution may still be valuable and deserve attention. It is the user's responsibility to check the status and quality of the solution.

### Typical status reports

Here are the most typical optimization outcomes described in terms of the problem and solution statuses. Note that these do not cover all possible situations that can occur.

Table 7.2: Continuous problems (solution status for soltype.itr or soltype.bas)

Outcome	Problem status	Solution status
Optimal	prosta.	solsta.optimal
	$prim\_and\_dual\_feas$	
Primal infeasible	prosta.prim_infeas	solsta.
		prim_infeas_cer
Dual infeasible	prosta.dual_infeas	solsta.
		$dual\_infeas\_cer$
Uncertain (stall, numerical issues, etc.)	prosta.unknown	solsta.unknown

Table 7.3: Integer problems (solution status for soltype.itg, others undefined)

Outcome	Problem status	Solution status
Integer optimal	prosta.prim_feas	$solsta.integer\_optimal$
Infeasible	prosta.prim_infeas	solsta.unknown
Integer feasible point	prosta.prim_feas	solsta.prim_feas
No conclusion	prosta.unknown	solsta.unknown

## 7.1.4 Retrieving solution values

After the meaning and quality of the solution (or certificate) have been established, we can query for the actual numerical values. They can be accessed with methods such as:

- Task. getprimalobj, Task. getdualobj the primal and dual objective value.
- Task. getxx solution values for the variables.
- Task. getsolution a full solution with primal and dual values

and many more specialized methods, see the API reference.

## 7.1.5 Source code example

Below is a source code example with a simple framework for assessing and retrieving the solution to a conic quadratic optimization problem.

Listing 7.1: Sample framework for checking optimization result.

```
try (Env env = new Env();
       Task task = new Task(env, 0, 0)) {
    // (Optionally) attach the log handler to receive log information
    /*
    task.set_Stream(
      streamtype.log,
     new mosek.Stream()
    { public void stream(String msg) { System.out.print(msg); }});
    // (Optionally) uncomment this line to experience solution status Unknown
    // task.putintparam(iparam.intpnt_max_iterations, 1);
    // On this example we read an optimization problem from a file
    task.readdata(filename);
    // Perform optimization.
    rescode trm = task.optimize();
    // Handle solution status. We expect Optimal
    solsta solsta = task.getsolsta(soltype.itr);
    switch ( solsta ) {
      case optimal:
      case near_optimal:
        // Fetch and print the solution
        System.out.println("An optimal interior point solution is located.");
        int numvar = task.getnumvar();
        double[] xx = new double[numvar];
        task.getxx(soltype.itr, xx);
        for(int i = 0; i < numvar; i++)</pre>
          System.out.println("x[" + i + "] = " + xx[i]);
        break;
      case dual_infeas_cer:
      case near_dual_infeas_cer:
        System.out.println("Dual infeasibility certificate found.");
        break;
      case prim_infeas_cer:
      case near_prim_infeas_cer:
        System.out.println("Primal infeasibility certificate found.");
        break;
      case unknown:
        \ensuremath{//} The solutions status is unknown. The termination code
        // indicates why the optimizer terminated prematurely.
        System.out.println("The solution status is unknown.");
        Env.getcodedesc(trm, symname, desc);
        System.out.printf(" Termination code: %s %s\n", symname, desc);
        break;
        System.out.println("Unexpected solution status " + solsta + "\n");
        break;
    }
 }
 catch (mosek.Error e) {
    System.out.println("Unexpected error (" + e.code + ") " + e.msg);
  }
}
```

# 7.2 Errors and exceptions

### **Exceptions**

Almost every function in Optimizer API for Java can throw an exception informing that the requested operation was not performed correctly, and indicating the type of error that occurred. This is the case in situations such as for instance:

- referencing a nonexisting variable (for example with too large index),
- defining an invalid value for a parameter,
- accessing an undefined solution,
- repeating a variable name, etc.

It is therefore a good idea to catch exceptions of type *Error*. The one case where it is *extremely important* to do so is when *Task.optimize* is invoked. We will say more about this in Sec. 7.1.

The exception contains a  $response\ code$  (element of the enum rescode) and short diagnostic messages. They can be accessed as in the following example.

```
try {
   task.putdouparam(mosek.dparam.intpnt_co_tol_rel_gap, -1.0e-7);
}
catch (mosek.Exception e) {
   mosek.rescode res = e.code;
   System.out.println("Response code " + res + "\nMessage " + e.msg);
}
```

It will produce as output:

Another way to obtain a human-readable string corresponding to a response code is the method *Env.* getcodedesc. A full list of exceptions, as well as response codes, can be found in the *API reference*.

### Optimizer errors and warnings

The optimizer may also produce warning messages. They indicate non-critical but important events, that will not prevent solver execution, but may be an indication that something in the optimization problem might be improved. Warning messages are normally printed to a log stream (see Sec. 7.3). A typical warning is, for example:

```
MOSEK warning 53: A numerically large upper bound value 6.6e+09 is specified for constraint → 'C69200' (46020).
```

Warnings can also be suppressed by setting the <code>iparam.max\_num\_warnings</code> parameter to zero, if they are well-understood.

# 7.3 Input/Output

The logging and I/O features are provided mainly by the  $\mathbf{MOSEK}$  task and to some extent by the  $\mathbf{MOSEK}$  environment objects.

## 7.3.1 Stream logging

By default the solver runs silently and does not produce any output to the console or otherwise. However, the log output can be redirected to a user-defined output stream or stream callback function. The log output is analogous to the one produced by the command-line version of **MOSEK**.

The log messages are partitioned in three streams:

- messages, streamtype.msg
- warnings, streamtype.wrn
- errors, streamtype.err

These streams are aggregated in the *streamtype.log* stream. A stream handler can be defined for each stream separately.

The *Stream* class is used to receive text strings emitted to **MOSEK**'s output streams. Extending *Stream* is the way to customize the solver output. When a *Stream* object is attached to a *Task* stream, any text that is printed to that stream will be passed to the *Stream.stream* method. For example:

The stream can be detached by calling

After optimization is completed an additional short summary of the solution and optimization process can be printed to any stream using the method *Task.solutionsummary*.

## 7.3.2 Log verbosity

The logging verbosity can be controlled by setting the relevant parameters, as for instance

- iparam.log,
- iparam.log\_intpnt,
- iparam.log\_mio,
- iparam.log\_cut\_second\_opt,
- iparam.log\_sim, and
- iparam.log\_sim\_minor.

Each parameter controls the output level of a specific functionality or algorithm. The main switch is iparam.log which affect the whole output. The actual log level for a specific functionality is determined as the minimum between iparam.log and the relevant parameter. For instance, the log level for the output produce by the interior-point algorithm is tuned by the  $iparam.log\_intpnt$ ; the actual log level is defined by the minimum between iparam.log and  $iparam.log\_intpnt$ .

Tuning the solver verbosity may require adjusting several parameters. It must be noticed that verbose logging is supposed to be of interest during debugging and tuning. When output is no more of interest, the user can easily disable it globally with <code>iparam.log</code>. Larger values of <code>iparam.log</code> do not necessarily result in increased output.

By default **MOSEK** will reduce the amount of log information after the first optimization on a given problem. To get full log output on subsequent re-optimizations set  $iparam.log\_cut\_second\_opt$  to zero.

# 7.3.3 Saving a problem to a file

An optimization problem can be dumped to a file using the method <code>Task.writedata</code>. The file format will be determined from the filename's extension (unless the parameter <code>iparam.write\_data\_format</code> specifies something else). Supported formats are listed in Sec. 17 together with a table of problem types supported by each.

For instance the problem can be written to an OPF file with

```
task.writedata("data.opf");
task.optimize();
```

All formats can be compressed with gzip by appending the .gz extension, for example

```
task.writedata("data.task.gz");
```

Some remarks:

- Unnamed variables are given generic names. It is therefore recommended to use meaningful variable names if the problem file is meant to be human-readable.
- The task format is MOSEK's native file format which contains all the problem data as well as solver settings.

## 7.3.4 Reading a problem from a file

A problem saved in any of the supported file formats can be read directly into a task using <code>Task.readdata</code>. The task must be created in advance. Afterwards the problem can be optimized, modified, etc. If the file contained solutions, then are also imported, but the status of any solution will be set to <code>solsta.unknown</code> (solutions can also be read separately using <code>Task.readsolution</code>). If the file contains parameters, they will be set accordingly.

```
task = new mosek.Task(env, 0, 0);
try {
   task.readdata("file.task.gz");
   task.optimize();
} catch (mosek.Exception e) {
   System.out.println("Problem reading the file");
}
```

# 7.4 Setting solver parameters

**MOSEK** comes with a large number of parameters that allows the user to tune the behavior of the optimizer. The typical settings which can be changed with solver parameters include:

- choice of the optimizer for linear problems,
- choice of primal/dual solver,
- turning presolve on/off,
- turning heuristics in the mixed-integer optimizer on/off,
- level of multi-threading,
- feasibility tolerances,

- solver termination criteria,
- behaviour of the license manager,

and more. All parameters have default settings which will be suitable for most typical users.

The API reference contains:

- Full list of parameters
- List of parameters grouped by topic

### **Setting parameters**

Each parameter is identified by a unique name. There are three types of parameters depending on the values they take:

- Integer parameters. They take either either simple integer values or values from an enumeration provided for readability and compatibility of the code. Set with *Task.putintparam*.
- Double (floating point) parameters. Set with Task.putdouparam.
- String parameters. Set with Task.putstrparam.

There are also parameter setting functions which operate fully on symbolic strings containing commandline style names of parameters and their values. See the example below. The optimizer will try to convert the given argument to the exact expected type, and will error if that fails.

If an incorrect value is provided then the parameter is left unchanged.

For example, the following piece of code sets up parameters which choose and tune the interior point optimizer before solving a problem.

Listing 7.2: Parameter setting example.

```
// Set log level (integer parameter)
task.putintparam(mosek.iparam.log, 1);
// Select interior-point optimizer... (integer parameter)
task.putintparam(mosek.iparam.optimizer, mosek.optimizertype.intpnt.value);
// ... without basis identification (integer parameter)
task.putintparam(mosek.iparam.intpnt_basis, mosek.basindtype.never.value);
// Set relative gap tolerance (double parameter)
task.putdouparam(mosek.dparam.intpnt_co_tol_rel_gap, 1.0e-7);
// The same using explicit string names
               ("MSK_DPAR_INTPNT_CO_TOL_REL_GAP", "1.0e-7");
task.putparam
task.putnadouparam("MSK_DPAR_INTPNT_CO_TOL_REL_GAP", 1.0e-7);
// Incorrect value
try {
  task.putdouparam(mosek.dparam.intpnt_co_tol_rel_gap, -1.0);
catch (mosek.Error e) {
  System.out.println("Wrong parameter value");
```

### Reading parameter values

The functions Task.getintparam, Task.getdouparam, Task.getstrparam can be used to inspect the current value of a parameter, for example:

```
double param = task.getdouparam(mosek.dparam.intpnt_co_tol_rel_gap);
System.out.println("Current value for parameter intpnt_co_tol_rel_gap = " + param);
```

# 7.5 Retrieving information items

After the optimization the user has access to the solution as well as to a report containing a large amount of additional *information items*. For example, one can obtain information about:

- timing: total optimization time, time spent in various optimizer subroutines, number of iterations, etc.
- solution quality: feasibility measures, solution norms, constraint and bound violations, etc.
- problem structure: counts of variables of different types, constraints, nonzeros, etc.
- integer optimizer: integrality gap, objective bound, number of cuts, etc.

and more. Information items are numerical values of integer, long integer or double type. The full list can be found in the API reference:

- Double
- Integer
- Long

Certain information items make sense, and are made available, also *during* the optimization process. They can be accessed from a callback function, see Sec. 7.6 for details.

#### Remark

For efficiency reasons, not all information items are automatically computed after optimization. To force all information items to be updated use the parameter  $iparam.auto\_update\_sol\_info$ .

### Retrieving the values

Values of information items are fetched using one of the methods

- Task. getdouinf for a double information item,
- Task. getintinf for an integer information item,
- Task.getlintinf for a long integer information item.

Each information item is identified by a unique name. The example below reads two pieces of data from the solver: total optimization time and the number of interior-point iterations.

Listing 7.3: Information items example.

```
double tm = task.getdouinf(mosek.dinfitem.optimizer_time);
int iter = task.getintinf(mosek.iinfitem.intpnt_iter);

System.out.println("Time: " + tm);
System.out.println("Iterations: " + iter);
```

# 7.6 Progress and data callback

Callbacks are a very useful mechanism that allow the caller to track the progress of the **MOSEK** optimizer. A callback function provided by the user is regularly called during the optimization and can be used to

- obtain a customized log of the solver execution,
- collect information for debugging purposes or

• ask the solver to terminate.

Optimizer API for Java has the following callback mechanisms:

- progress callback, which provides only the basic status of the solver.
- data callback, which provides the solver status and a complete set of information items that describe the progress of the optimizer in detail.
- integer solution callback, for reporting progress on a mixed-integer problem.

#### Warning

The callbacks functions *must not* invoke any functions of the solver, environment or task. Otherwise the state of the solver and its outcome are undefined. The only exception is the possibility to retrieve an integer solution, see below.

### Retrieving mixed-integer solutions

If the mixed-integer optimizer is used, the callback will take place, in particular, every time an improved integer solution is found. In that case it is possible to retrieve the current values of the best integer solution from within the callback function. It can be useful for implementing complex termination criteria for integer optimization. Note that there is a specialized callback class for retrieving only the integer solution anyway.

### 7.6.1 Data callback

In the data callback **MOSEK** passes a callback code and values of all information items to a user-defined function. The callback function is called, in particular, at the beginning of each iteration of the interior-point optimizer. For the simplex optimizers  $iparam.log\_sim\_freq$  controls how frequently the call-back is called. Note that the callback is done quite frequently, which can lead to degraded performance. If the information items are not required, the simpler progress callback may be a better choice.

The callback is set by calling the method  $Task.set\_InfoCallback$ . The callback function must be implemented by extending the abstract class DataCallback and implementing the method DataCallback. callback.

Non-zero return value of the callback function indicates that the optimizer should be terminated.

### 7.6.2 Progress callback

In the progress callback  $\mathbf{MOSEK}$  provides a single code indicating the current stage of the optimization process.

The callback is set by calling the method  $Task.set\_Progress$ . The callback function must be implemented by extending the abstract class Progress and implementing the method Progress.progress.

Non-zero return value of the callback function indicates that the optimizer should be terminated.

### 7.6.3 Integer solution callback

In this type of callback the user-defined callback function receives an updated solution every time the mixed-integer optimizer improves the objective value. It can be useful for implementing complex termination criteria for integer optimization.

### **Syntax**

The callback is set by calling the method  $Task.set\_ItgSolutionCallback$ . The callback function must be implemented by extending the abstract class ItgSolutionCallback and implementing the method ItgSolutionCallback.callback.

### 7.6.4 Working example: Data callback

The following example defines a data callback function that prints out some of the information items. It interrupts the solver after a certain time limit.

Listing 7.4: An example of a data callback function.

```
private static DataCallback makeUserCallback(final double maxtime) {
 return new DataCallback() {
   public int callback(callbackcode caller,
                        double[]
                                      douinf.
                                      intinf,
                        int[]
                                      lintinf) {
                        long[]
      double opttime = 0.0;
      int itrn:
      double pobj, dobj, stime;
      Formatter f = new Formatter(System.out);
      switch (caller) {
        case begin_intpnt:
          f.format("Starting interior-point optimizer\n");
          break:
        case intpnt:
                 = intinf[iinfitem.intpnt_iter.value
                                                             ];
          itrn
                  = douinf[dinfitem.intpnt_primal_obj.value];
          pobj
                  = douinf[dinfitem.intpnt_dual_obj.value ];
                 = douinf[dinfitem.intpnt_time.value
                                                             1:
          opttime = douinf[dinfitem.optimizer_time.value
          f.format("Iterations: %-3d\n", itrn);
          f.format(" Time: %6.2f(%.2f) ", opttime, stime);
          f.format(" Primal obj.: %-18.6e Dual obj.: %-18.6e\n", pobj, dobj);
          break;
        case end_intpnt:
          \textbf{f}. \texttt{format}(\texttt{"Interior-point optimizer finished.} \\ \texttt{\colored-height});
          break:
        case begin_primal_simplex:
          f.format("Primal simplex optimizer started.\n");
          break;
        case update_primal_simplex:
                 = intinf[iinfitem.sim_primal_iter.value ];
          pobj
                  = douinf[dinfitem.sim_obj.value
                                                            ];
          stime
                 = douinf[dinfitem.sim_time.value
                                                             ];
          opttime = douinf[dinfitem.optimizer_time.value ];
          f.format("Iterations: %-3d\n", itrn);
          f.format(" Elapsed time: %6.2f(%.2f\n", opttime, stime);
          f.format(" Obj.: %-18.6e", pobj );
          break;
        case end_primal_simplex:
          f.format("Primal simplex optimizer finished.\n");
          break;
        case begin_dual_simplex:
          f.format("Dual simplex optimizer started.\n");
          break:
```

```
case update_dual_simplex:
                  = intinf[iinfitem.sim_dual_iter.value
                                                              ];
                   = douinf[dinfitem.sim_obj.value
                                                              ];
                  = douinf[dinfitem.sim_time.value
                                                              ];
          opttime = douinf[dinfitem.optimizer_time.value
                                                              ];
          f.format("Iterations: %-3d\n", itrn);
          \label{eq:f.format} \mbox{f.format(" Elapsed time: $\%6.2f(\%.2f)\n", opttime, stime);}
          f.format(" Obj.: %-18.6e\n", pobj);
          break:
        case end_dual_simplex:
          f.format("Dual simplex optimizer finished.\n");
          break:
        case begin_bi:
          f.format("Basis identification started.\n");
          break;
        case end_bi:
          f.format("Basis identification finished.\n");
          break:
        default:
      System.out.flush();
      if (opttime >= maxtime)
        // mosek is spending too much time. Terminate it.
        return 1:
      return 0;
    }
 };
}
```

Assuming that we have defined a task task and a time limit maxtime, the callback function is attached as follows:

Listing 7.5: Attaching the data callback function to the model.

```
task.set_InfoCallback(makeUserCallback(maxtime));
```

# 7.7 MOSEK OptServer

**MOSEK** provides an easy way to offload optimization problem to a remote server in both *synchronous* or *asynchronous* mode. This section describes related functionalities from the client side, i.e. sending optimization tasks to the remote server and retrieving solutions.

Setting up and configuring the remote server is described in a separate manual for the OptServer.

## 7.7.1 Synchronous Remote Optimization

In synchronous mode the client sends an optimization problem to the server and blocks, waiting for the optimization to end. Once the result has been received, the program can continue. This is the simplest mode and requires very few modifications to existing code: instead of <code>Task.optimize</code> the user must invoke <code>Task.optimizermt</code> with the host and port where the server is running and listening as additional arguments. The rest of the code remains untouched.

Note that it is impossible to recover the job in case of a broken connection.

### Source code example

Listing 7.6: Using the OptServer in synchronous mode.

```
package com.mosek.example;
import mosek.*;
public class opt_server_sync {
 public static void main (String[] args) {
    if (args.length == 0) {
     System.out.println ("Missing argument, syntax is:");
     System.out.println (" opt_server_sync inputfile host port numpolls");
    } else {
     String inputfile = args[0];
     String host
                     = args[1];
     String port
                      = args[2];
     rescode trm[] = new rescode[1];
      try (Env env = new Env();
          Task task = new Task(env, 0, 0)) {
        task.set_Stream (mosek.streamtype.log,
        new mosek.Stream() {
         public void stream(String msg) { System.out.print(msg); }
        }):
        task.readdata (inputfile);
        task.optimizermt (host, port, trm);
        task.solutionsummary (mosek.streamtype.log);
     7
   }
 }
```

## 7.7.2 Asynchronous Remote Optimization

In asynchronous mode the client sends a job to the remote server and the execution of the client code continues. In particular, it is the client's responsibility to periodically check the optimization status and, when ready, fetch the results. The client can also interrupt optimization. The most relevant methods are:

- $\bullet$   $Task.\,asyncoptimize$  : Offload the optimization task to a solver server.
- Task. asyncpoll: Request information about the status of the remote job.
- $\bullet$   $Task.\,asyncgetresult$  : Request the results from a completed remote job.
- Task. asyncstop: Terminate a remote job.

## Source code example

In the example below the program enters in a polling loop that regularly checks whether the result of the optimization is available.

Listing 7.7: Using the OptServer in asynchronous mode.

```
package com.mosek.example;
import mosek.*;
public class opt_server_async {
```

```
public static void main (String[] args) {
 if (args.length == 0) {
   System.out.println ("Missing argument, syntax is:");
   System.out.println (" opt_server_async inputfile host port numpolls");
 } else {
   String inputfile = args[0];
                  = args[1];
   String host
   String port
                    = args[2];
   int numpolls
                 = Integer.parseInt(args[3]);
   try (Env env = new Env()) {
     String token;
     try(Task task = new Task(env, 0, 0)) {
       task.readdata (inputfile);
       token = task.asyncoptimize (host, port);
     System.out.printf("Task token = %s\n", token);
     try(Task task = new Task(env, 0, 0)) {
       System.out.println("Reading input file...");
       task.readdata (inputfile);
       System.out.println("Setting log stream...");
       task.set_Stream (mosek.streamtype.log,
       new mosek.Stream() {
         public void stream(String msg) { System.out.print(msg); }
       });
       long start = System.currentTimeMillis();
       System.out.println("Starting polling loop...");
       int i = 0;
       while ( true ) {
         Thread.sleep(100);
          System.out.printf("poll %d...\n", i);
         rescode trm[] = new rescode[1];
         rescode resp[] = new rescode[1];
          boolean respavailable = task.asyncpoll( host,
                                                  port,
                                                  token.
                                                  resp,
                                                  trm);
          System.out.println("polling done");
          if (respavailable) {
           System.out.println("solution available!");
           task.asyncgetresult(host,
                                port,
                                token.
                                resp,
```

```
trm);

task.solutionsummary (mosek.streamtype.log);
break;
}

i++;

if (i == numpolls) {
    System.out.println("max num polls reached, stopping host.");
    task.asyncstop (host, port, token);
    break;
}

} catch (java.lang.Exception e) {
    System.out.println("Something unexpected happend...");
}

}

}

}
}
```

### NONLINEAR TUTORIALS

This chapter provides information about how to solve general convex nonlinear optimization problems using **MOSEK**. By general nonlinear problems we mean those that cannot be formulated in conic or convex quadratically constrained form.

In general we recommend not to use the general nonlinear optimizer unless absolutely necessary. The reasons are:

- The algorithm employed for nonlinear optimization problems is not as efficient as the one employed
  for conic problems. Conic problems have special structure that can be exploited to make the
  optimizer faster and more robust.
- MOSEK has no way of checking whether the formulated problem is convex and if this assumption is not satisfied the optimizer will not work.
- The nonlinear optimizer requires 1st and 2nd order derivative information which is often hard to provide correctly.

Instead, we advise:

- Consider reformulating the problem to a conic quadratic optimization problem if at all possible. In particular many problems involving polynomial terms can easily be reformulated to conic quadratic form.
- Consider reformulating the problem to a separable optimization problem because that simplifies the issue with verifying convexity and computing 1st and 2nd order derivatives significantly. In most cases problems in separable form also solve faster because of the simpler structure of the functions.
- Finally, if the problem cannot be reformulated in separable form use a modelling language like AMPL or GAMS, which will perform all the preprocessing, computing function values and derivatives. This eliminates an important source of errors. Therefore, it is strongly recommended to use a modelling language at the prototype stage.

The Optimizer API for Java provides the following nonlinear interfaces:

# 8.1 Separable Convex (SCopt) Interface

The Optimizer API for Java provides a way to add simple non-linear functions composed from a limited set of non-linear terms. Non-linear terms can be mixed with quadratic terms in objective and constraints. We consider problems which can be formulated as:

minimize 
$$z_0(x) + c^T x$$
subject to 
$$l_i^c \leq z_i(x) + a_i^T x \leq u_i^c \quad i = 1 \dots m$$

$$l^x \leq x \leq u^x,$$

where  $x \in \mathbb{R}^n$  and each  $z_i : \mathbb{R}^n \to \mathbb{R}$  is separable, that is can be written as a sum

$$z_i(x) = \sum_{j=1}^n z_{i,j}(x_j).$$

The interface implements a limited set of functions which can appear as  $z_{i,j}$ . They are:

Tr			
Separable function	Operator name	Name	
$fx \ln(x)$	ent	Entropy function	
$fe^{gx+h}$	exp	Exponential function	
$f \ln(gx+h)$	log	Logarithm	
$f(x+h)^g$	pow	Power function	

Table 8.1: Functions supported by the SCopt interface.

where  $f, g, h \in \mathbb{R}$  are constants. This formulation does not guarantee convexity. For **MOSEK** to be able to solve the problem, the following requirements must be met:

- If the objective is minimized, the sum of non-linear terms must be convex, otherwise it must be concave.
- Any constraint bounded below must be concave, and any constraint bounded above must be convex.
- Each separable term must be twice differentiable within the bounds of the variable it is applied to.

Some simple rules can be followed to ensure that the problem satisfies **MOSEK**'s convexity and differentiability requirements. First of all, for any variable  $x_i$  used in a separable term, the variable bounds must define a range within which the function is twice differentiable. These bounds are defined in Table 8.2.

		1
Separable function	Operator name	Safe $x$ bounds
$fx \ln(x)$	ent	0 < x.
$fe^{gx+h}$	exp	$-\infty < x < \infty$ .
$f \ln(gx+h)$	log	If $g > 0$ : $-h/g < x$ .
		If $g < 0$ : $x < -h/g$ .
$f(x+h)^g$	pow	If $g > 0$ and integer: $-\infty < x < \infty$ .
		If $g < 0$ and integer: either $-h < x$ or $x < -h$ .
		Otherwise: $-h < x$ .

Table 8.2: Safe bounds for functions in the SCopt interface.

To ensure convexity, we require that each  $z_i(x)$  is either a sum of convex terms or a sum of concave terms. Table 8.3 lists convexity conditions for the relevant ranges for f > 0 — changing the sign of f switches concavity/convexity.

Table 8.3: Convexity conditions for functions in the SCopt interface.

Separable function	Operator name	Convexity conditions
$fx \ln(x)$	ent	Convex within safe bounds.
$fe^{gx+h}$	exp	Convex for all $x$ .
$f \ln(gx+h)$	log	Concave within safe bounds.
$f(x+h)^g$	pow	If $g$ is even integer: convex
		within safe bounds.
		If $g$ is odd integer:
		• concave if $(-\infty, -h)$ ,
		• convex if $(-h, \infty)$
		If $0 < g < 1$ : concave within
		safe bounds.
		Otherwise: convex within safe
		bounds.

A problem involving linear combinations of variables (such as  $ln(x_1+x_2)$ ), can be converted to a separable problem using slack variables and additional equality constraints.

## 8.1.1 Example

Consider the following separable convex problem:

minimize 
$$\exp(x_2) - \ln(x_1)$$
  
subject to  $x_2 \ln(x_2) \le 0$   
 $x_1^{1/2} - x_2 \ge 0$   
 $\frac{1}{2} \le x_1, x_2 \le 1$ . (8.1)

Note that all nonlinear functions are well defined for x values satisfying the variable bounds strictly. This assures that function evaluation errors will not occur during the optimization process because  $\mathbf{MOSEK}$ .

The linear part of the problem is specified as usually. The nonlinear part is set using the function <code>Task.putSCeval</code>. See the <code>API reference</code> for a description of the format. After that a standard invocation of <code>Task.optimize</code> solves the problem. The <code>API reference</code> describes additional functions for reading and writing SCopt terms from/to a file.

Listing 8.1: Implementation of problem (8.1).

```
package com.mosek.example;
import mosek.*;
public class scopt1 {
 public static void main(String[] args) {
   try (Env env = new Env();
        Task task = new Task(env, 0, 0)) {
     task.set_Stream(
       mosek.streamtype.log,
        new mosek.Stream()
      { public void stream(String msg) { System.out.print(msg); }});
      int numvar = 2;
      int numcon = 2;
     double inf = 0.;
     mosek.boundkey[]
     bkc = new mosek.boundkey[] {
       mosek.boundkey.up,
       mosek.boundkey.lo
     double[] blc = new double[] { -inf, .0 };
     double[] buc = new double[] {    .0, inf};
     mosek.boundkey[] bkx = new mosek.boundkey[] {
       mosek.boundkey.ra, mosek.boundkey.ra
     };
     double[] blx = new double[] {0.5, 0.5};
     double[] bux = new double[] {1.0, 1.0};
      task.appendvars(numvar);
      task.appendcons(numcon);
      task.putvarboundslice(0, numvar, bkx, blx, bux);
      task.putconboundslice(0, numcon, bkc, blc, buc);
      task.putaij(1, 1, -1.0);
     mosek.scopr[] opro = new mosek.scopr[] {mosek.scopr.log, mosek.scopr.exp};
                         = new int[] { 0, 1};
      int[]
              oprjo
      double[] oprfo
                         = new double[] { -1.0, 1.0 };
      double[] oprgo
                         = new double[] { 1.0, 1.0 };
```

```
double[] oprho
                       = new double[] { 0.0, 0.0 };
    mosek.scopr[] oprc = new mosek.scopr[] { mosek.scopr.ent, mosek.scopr.pow };
                = new int[]
= new int[]
                                    { 0, 1};
    int[] opric
                                    { 1, 0};
    int[] oprjc
    double[] oprfc
                     = new double[] { 1.0, 1.0 };
                   = new double[] { .0, 0.5 };
    double[] oprgc
                     = new double[] {    .0, 0.0 };
    double[] oprhc
    task.putSCeval(opro, oprjo, oprfo, oprgo, oprho,
                  oprc, opric, oprjc, oprfc, oprgc, oprhc);
    task.putintparam(mosek.iparam.write_ignore_incompatible_items, 1);
    task.writeSC("scopt1.sco", "scopt1.opf");
    task.optimize();
    double[] res = new double[numvar];
    task.getsolutionslice(
     mosek.soltype.itr,
     mosek.solitem.xx,
     0, numvar,
     res);
    System.out.print("Solution is: [ " + res[0]);
    for (int i = 1; i < numvar; ++i) System.out.print(", " + res[i]);</pre>
   System.out.println(" ]");
  } catch (mosek.Exception e) {
    System.out.println ("An error/warning was encountered");
    System.out.println (e.toString());
    throw e;
 }
}
```

# ADVANCED NUMERICAL TUTORIALS

MOSEK provides access to numerical linear algebra tools essential for more advanced applications. They are described in this section.

# 9.1 Solving Linear Systems Involving the Basis Matrix

A linear optimization problem always has an optimal solution which is also a basic solution. In an optimal basic solution there are exactly m basic variables where m is the number of rows in the constraint matrix A. Define

$$B \in \mathbb{R}^{m \times m}$$

as a matrix consisting of the columns of A corresponding to the basic variables. The basis matrix B is always non-singular, i.e.

$$det(B) \neq 0$$

or, equivalently,  $B^{-1}$  exists. This implies that the linear systems

$$B\bar{x} = w \tag{9.1}$$

and

$$B^T \bar{x} = w \tag{9.2}$$

each have a unique solution for all w.

**MOSEK** provides functions for solving the linear systems (9.1) and (9.2) for an arbitrary w.

In the next sections we will show how to use  $\mathbf{MOSEK}$  to

- identify the solution basis,
- solve arbitrary linear systems.

### 9.1.1 Basis identification

To use the solutions to (9.1) and (9.2) it is important to know how the basis matrix B is constructed. Internally **MOSEK** employs the linear optimization problem

where

$$x^c \in \mathbb{R}^m$$
 and  $x \in \mathbb{R}^n$ .

The basis matrix is constructed of m columns taken from

$$\begin{bmatrix} A & -I \end{bmatrix}$$
.

If variable  $x_j$  is a basis variable, then the j-th column of A, denoted  $a_{:,j}$ , will appear in B. Similarly, if  $x_i^c$  is a basis variable, then the i-th column of -I will appear in the basis. The ordering of the basis variables and therefore the ordering of the columns of B is arbitrary. The ordering of the basis variables may be retrieved by calling the function

```
task.initbasissolve(basis);
```

This function initializes data structures for later use and returns the indexes of the basic variables in the array basis. The interpretation of the basis is as follows. If

then the *i*-th basis variable is  $x_i^c$ . Moreover, the *i*-th column in B will be the *i*-th column of -I. On the other hand if

$$\mathtt{basis}[i] \geq \mathtt{numcon},$$

then the i-th basis variable is the variable

$$x_{\mathtt{basis}[i]-\mathtt{numcon}}$$

and the i-th column of B is the column

$$A_{:,(basis[i]-numcon)}$$
.

For instance if basis[0] = 4 and numcon = 5, then since basis[0] < numcon, the first basis variable is  $x_4^c$ . Therefore, the first column of B is the fourth column of -I. Similarly, if basis[1] = 7, then the second variable in the basis is  $x_{basis[1]-numcon} = x_2$ . Hence, the second column of B is identical to  $a_{:,2}$ .

#### An example

Consider the linear optimization problem:

minimize 
$$x_0 + x_1$$
  
subject to  $x_0 + 2x_1 \le 2$ ,  
 $x_0 + x_1 \le 6$ ,  
 $x_0, x_1 \ge 0$ . (9.4)

Suppose a call to Task. initbasissolve returns an array basis so that

```
basis[0] = 1,
basis[1] = 2.
```

Then the basis variables are  $x_1^c$  and  $x_0$  and the corresponding basis matrix B is

$$\left[\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array}\right].$$

Please note the ordering of the columns in B.

Listing 9.1: A program showing how to identify the basis.

```
package com.mosek.example;
import mosek.*;
public class solvebasis {
```

```
public static void main(String[] args) {
          // Since the value infinity is never used, we define
           // 'infinity' symbolic purposes only
         double
          infinity = 0;
         double[] c = {1.0, 1.0};
                                                 ptrb = {0, 2};
         int∏
                                                       ptre = {2 , 4};
         int[]
                                                         asub = \{0, 1, \dots, 1, \dots,
                                                                                                 0, 1
                                                                                              };
          double[] aval = {1.0, 1.0,
                                                                                                  2.0, 1.0
                                                                                              };
         mosek.boundkey[] bkc = {
                   mosek.boundkey.up,
                   mosek.boundkey.up
          double[] blc = { -infinity,
                                                                                                            -infinity
                                                                                                   };
          double[] buc = {2.0,
                                                                                                    6.0
                                                                                                   };
         mosek.boundkey[] bkx = {
                   mosek.boundkey.lo,
                   mosek.boundkey.lo
          };
          double[] blx = \{0.0,
                                                                                                     0.0
                                                                                              };
          double[] bux = { +infinity,
                                                                                                              +infinity
                                                                                                   };
           int
                                              numvar = 2;
                                             numcon = 2;
          double[] w1 = {2.0, 6.0};
          double[] w2 = {1.0, 0.0};
         try (Env env = new Env();
                                    Task task = new Task(env, 0, 0)) {
                     task.inputdata(numcon, numvar,
                                                                                                    С,
                                                                                                    0.0,
                                                                                                   ptrb,
                                                                                                   ptre,
                                                                                                    asub,
                                                                                                    aval,
                                                                                                    bkc,
                                                                                                    blc,
                                                                                                    buc,
                                                                                                     bkx.
                                                                                                     blx,
```

```
bux):
task.putobjsense(mosek.objsense.maximize);
System.out.println("optimize");
 task.optimize();
} catch (mosek.Warning e) {
 System.out.println("Mosek warning:");
  System.out.println(e.toString());
int[] basis = new int[numcon];
task.initbasissolve(basis);
//List basis variables corresponding to columns of B
int[] varsub = {0, 1};
for (int i = 0; i < numcon; i++) {</pre>
  System.out.println("Basis i:" + i + " Basis:" + basis[i]);
 if (basis[varsub[i]] < numcon) {</pre>
   System.out.println("Basis variable no " + i + " is xc" +
                       basis[i]);
  } else {
    int index = basis[i] - numcon;
    System.out.println("Basis variable no " + i + " is x" +
                       index);
}
// solve Bx = w1
// varsub contains index of non-zeros in b.
// On return b contains the solution x and
// varsub the index of the non-zeros in x.
int[] nz = new int[1];
nz[0] = 2;
task.solvewithbasis(0, nz, varsub, w1);
System.out.println("nz =" + nz[0]);
System.out.println("\nSolution to Bx = w1:\n");
for (int i = 0; i < nz[0]; i++) {
  if (basis[varsub[i]] < numcon) {</pre>
    System.out.println("xc" + basis[varsub[i]] + "=" + w1[varsub[i]]);
    int index = basis[varsub[i]] - numcon;
    System.out.println("x" + index + " = " + w1[varsub[i]]);
// Solve B^Tx = w2
nz[0] = 2;
varsub[0] = 0;
varsub[1] = 1;
task.solvewithbasis(1, nz, varsub, w2);
System.out.println("\nSolution to B^Tx = w2:\n");
for (int i = 0; i < nz[0]; i++) {
  \verb|if (basis[varsub[i]] < numcon) | \{ \\
    System.out.println("xc" + basis[varsub[i]] + " = " + w2[varsub[i]]);
  } else {
    int index = basis[varsub[i]] - numcon;
```

```
System.out.println("x" + index + " = " + w2[varsub[i]]);
}
}
catch (mosek.Exception e)
   /* Catch both Error and Warning */
{
   System.out.println("An error was encountered");
   System.out.println(e.getMessage());
   throw e;
}
}
```

In the example above the linear system is solved using the optimal basis for (9.4) and the original right-hand side of the problem. Thus the solution to the linear system is the optimal solution to the problem. When running the example program the following output is produced.

```
basis[0] = 1
Basis variable no 0 is xc1.
basis[1] = 2
Basis variable no 1 is x0.

Solution to Bx = b:

x0 = 2.000000e+00
xc1 = -4.000000e+00

Solution to B^Tx = c:

x1 = -1.000000e+00
x0 = 1.000000e+00
```

Please note that the ordering of the basis variables is

$$\left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right]$$

and thus the basis is given by:

$$B = \left[ \begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array} \right]$$

It can be verified that

$$\left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right] = \left[\begin{array}{c} -4 \\ 2 \end{array}\right]$$

is a solution to

$$\left[\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array}\right] \left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right] = \left[\begin{array}{c} 2 \\ 6 \end{array}\right].$$

# 9.1.2 Solving arbitrary linear systems

MOSEK can be used to solve an arbitrary (rectangular) linear system

$$Ax = b$$

using the Task.solvewithbasis function without optimizing the problem as in the previous example. This is done by setting up an A matrix in the task, setting all variables to basic and calling the Task.solvewithbasis function with the b vector as input. The solution is returned by the function.

### An example

Below we demonstrate how to solve the linear system

$$\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$
 (9.5)

with two inputs b = (1, -2) and b = (7, 0).

```
package com.mosek.example;
import mosek.*;
public class solvelinear {
 static public void put_a(
   mosek. Task task,
   double[][] aval,
   int[][] asub,
   int[]
             ptrb,
   int[]
              ptre,
   int
              numvar,
             basis ) {
   // Since the value infinity is never used, we define
   // 'infinity' symbolic purposes only
   double
   infinity = 0;
   mosek.stakey[] skx = new mosek.stakey [numvar];
   mosek.stakey[] skc = new mosek.stakey [numvar];
   for (int i = 0; i < numvar ; ++i) {</pre>
     skx[i] = mosek.stakey.bas;
     skc[i] = mosek.stakey.fix;
   task.appendvars(numvar);
   task.appendcons(numvar);
   for (int i = 0; i < numvar; ++i)
     task.putacol(i,
                   asub[i],
                   aval[i]);
   for (int i = 0; i < numvar; ++i)
     task.putconbound(
       i,
       mosek.boundkey.fx,
        0.0,
        0.0);
   for (int i = 0; i < numvar; ++i)
     task.putvarbound(
        i,
       mosek.boundkey.fr,
        -infinity,
        infinity);
    //task.makesolutionstatusunknown(mosek.soltype.bas);
   /* Define a basic solution by specifying
       status keys for variables & constraints. */
   for (int i = 0; i < numvar; ++i)
```

```
task.putsolutioni (
      mosek.accmode.var,
     mosek.soltype.bas,
     skx[i],
     0.0,
     0.0,
      0.0,
      0.0);
 for (int i = 0; i < numvar; ++i)
    task.putsolutioni (
     mosek.accmode.con,
     mosek.soltype.bas,
      skc[i],
     0.0,
      0.0,
      0.0.
      0.0);
  task.initbasissolve(basis);
public static void main (String[] argv) {
 int numcon = 2;
  int numvar = 2;
 double[][] aval = {
   \{-1.0\},
   { 1.0, 1.0 }
 };
  int[][] asub = {
   { 1 },
   { 0, 1 }
  };
  int []
            ptrb = new int[] {0, 1};
  int []
            ptre = new int[] {1, 3};
  int[]
             bsub = new int[numvar];
             b = new double[numvar];
  double[]
             basis = new int[numvar];
 int[]
 try (Env env = new Env();
       Task task = new Task(env, 0, 0)) {
    // Directs the log task stream to the user specified
    // method task_msg_obj.streamCB
    task.set_Stream(
     mosek.streamtype.log,
     new mosek.Stream()
    { public void stream(String msg) { System.out.print(msg); }});
    /* Put A matrix and factor A.
       Call this function only once for a given task. */
    put_a(
     task,
      aval,
      asub,
      ptrb,
      ptre,
      numvar
```

```
basis
    /* now solve rhs */
    b[0] = 1;
    b[1] = -2;
    bsub[0] = 0;
    bsub[1] = 1;
    int[] nz_ = { 2 };
    task.solvewithbasis(0, nz_, bsub, b);
    int nz = nz_{0};
    System.out.println("\nSolution to Bx = b:\n");
    /* Print solution and show correspondents
       to original variables in the problem */
    for (int i = 0; i < nz; ++i) {
      if (basis[bsub[i]] < numcon)</pre>
        System.out.println ("This should never happen");
      else
        System.out.println("x" + (basis[bsub[i]] - numcon) + " = " + b[bsub[i]]);
    b[0] = 7;
    bsub[0] = 0;
    nz_[0] = 1;
    task.solvewithbasis(0, nz_, bsub, b);
    nz = nz_[0];
    System.out.println ("\nSolution to Bx = b:\n");
    /* Print solution and show correspondents
       to original variables in the problem */
    for (int i = 0; i < nz; ++i) {
      if (basis[bsub[i]] < numcon)</pre>
        System.out.println("This should never happen");
      else
        System.out.println("x" + (basis[bsub[i]] - numcon) + " = " + b[bsub[i]] );
    }
  }
}
```

The most important step in the above example is the definition of the basic solution, where we define the status key for each variable. The actual values of the variables are not important and can be selected arbitrarily, so we set them to zero. All variables corresponding to columns in the linear system we want to solve are set to basic and the slack variables for the constraints, which are all non-basic, are set to their bound.

The program produces the output:

```
Solution to Bx = b:

x1 = 1
x0 = 3

Solution to Bx = b:

x1 = 7
x0 = 7
```

# 9.2 Calling BLAS/LAPACK Routines from MOSEK

Sometimes users need to perform linear algebra operations that involve dense matrices and vectors. Also **MOSEK** extensively uses high-performance linear algebra routines from the BLAS and LAPACK packages and some of these routines are included in the package shipped to the users.

The  $\mathbf{MOSEK}$  versions of BLAS/LAPACK routines:

- use MOSEK data types and return value conventions,
- preserve the BLAS/LAPACK naming convention.

Therefore the user can leverage on efficient linear algebra routines, with a simplified interface, with no need for additional packages.

#### List of available routines

BLAS Name	MOSEK function	Math Expression		
AXPY	Env. axpy	$y = \alpha x + y$		
DOT	Env. dot	$  x^T y  $		
GEMV	Env.gemv	$y = \alpha Ax + \beta y$		
GEMM	Env.gemm	$C = \alpha AB + \beta C$		
SYRK	Env.syrk	$C = \alpha A A^T + \beta C$		

Table 9.1: BLAS routines available.

Table 9.2: LAPACK routines available.

LAPACK Name	MOSEK function	Description
POTRF	${\it Env.potrf}$	Cholesky factorization of a semidefinite symmetric matrix
SYEVD	Env.syevd	Eigenvalues and eigenvectors of a symmetric matrix
SYEIG	Env.syeig	Eigenvalues of a symmetric matrix

## Source code examples

In Listing 9.2 we provide a simple working example. It has no practical meaning except showing how to organize the input and call the methods.

Listing 9.2: Calling BLAS and LAPACK routines from Optimizer API for Java.

```
package com.mosek.example;

public class blas_lapack {
   static final int n = 3, m = 2, k = 3;

   public static void main (String[] args) {

        double alpha = 2.0, beta = 0.5;
        double[] x = {1., 1., 1.};
        double[] y = {1., 2., 3.};
        double[] z = {1.0, 1.0};

        /*A has m=2 rows and k=3 cols*/
        double[] A = {1., 1., 2., 2., 3., 3.};
        /*B has k=3 rows and n=3 cols*/
        double[] B = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
        double[] C = { 1., 2., 3., 4., 5., 6.};

        double[] D = {1.0, 1.0, 1.0, 1.0};
```

```
double[] Q = {1.0, 0.0, 0.0, 2.0};
  double[] v = new double[2];
 double[] xy = {0.};
  try (mosek.Env env = new mosek.Env()) {
    /* routines*/
    env.dot(n, x, y, xy);
    env.axpy(n, alpha, x, y);
    env.gemv(mosek.transpose.no, m, n, alpha, A, x, beta, z);
    env.gemm(mosek.transpose.no, mosek.transpose.no, m, n, k, alpha, A, B, beta, C);
    env.syrk(mosek.uplo.lo, mosek.transpose.no, m, k, alpha, A, beta, D);
    /* LAPACK routines*/
    env.potrf(mosek.uplo.lo, m, Q);
    env.syeig(mosek.uplo.lo, m, Q, v);
    env.syevd(mosek.uplo.lo, m, Q, v);
  } catch (mosek.Exception e) {
    System.out.println ("An error/warning was encountered");
    System.out.println (e.toString());
}
```

# 9.3 Computing a Sparse Cholesky Factorization

Given a positive semidefinite symmetric (PSD) matrix

$$A \in \mathbb{R}^{n \times n}$$

it is well known there exists a matrix L such that

$$A = LL^T$$
.

If the matrix L is lower triangular then it is called a *Cholesky factorization*. Given A is positive definite (nonsingular) then L is also nonsingular. A Cholesky factorization is useful for many reasons:

- A system of linear equations Ax = b can be solved by first solving the lower triangular system Ly = b followed by the upper triangular system  $L^Tx = y$ .
- A quadratic term  $x^T A x$  in a constraint or objective can be replaced with  $y^T y$  for  $y = L^T x$ , potentially leading to a more robust formulation (see [And13]).

Therefore, **MOSEK** provides a function that can compute a Cholesky factorization of a PSD matrix. In addition a function for solving linear systems with a nonsingular lower or upper triangular matrix is available.

In practice A may be very large with n is in the range of millions. However, then A is typically sparse which means that most of the elements in A are zero, and sparsity can be exploited to reduce the cost of computing the Cholesky factorization. The computational savings depend on the positions of zeros in

A. For example, below a matrix A is given together with a Cholesky factor up to 5 digits of accuracy:

$$A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 2.0000 & 0 & 0 & 0 \\ 0.5000 & 0.8660 & 0 & 0 \\ 0.5000 & -0.2887 & 0.8165 & 0 \\ 0.5000 & -0.2887 & -0.4082 & 0.7071 \end{bmatrix}.$$
(9.6)

However, if we symmetrically permute the rows and columns of A using a permutation matrix P

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad A' = PAP^T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 4 \end{bmatrix},$$

then the Cholesky factorization of  $A' = L'L'^T$  is

$$L' = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array} \right]$$

which is sparser than L.

Computing a permutation matrix that leads to the sparsest Cholesky factorization or the minimal amount of work is NP-hard. Good permutations can be chosen by using heuristics, such as the minimum degree heuristic and variants. The function <code>Env.computesparsecholesky</code> provided by <code>MOSEK</code> for computing a Cholesky factorization has a build in permutation aka. reordering heuristic. The following code illustrates the use of <code>Env.computesparsecholesky</code> and <code>Env.sparsetriangularsolvedense</code>.

Listing 9.3: How to use the sparse Cholesky factorization routine available in MOSEK.

```
env.computesparsecholesky(0,
                                             //Disable multithreading
                                             //Apply reordering heuristic
                                   1,
                                   1.0e-14, //Singularity tolerance
                                   anzc, aptrc, asubc, avalc,
                                   perm, diag,
                                   lnzc, lptrc, lensubnval, lsubc, lvalc);
       printsparse(n, perm[0], diag[0], lnzc[0], lptrc[0], lensubnval[0], lsubc[0], lvalc[0]);
        /* Permuted b is stored as x. */
       double[] x = new double[n];
       for (int i = 0; i < n; i++) x[i] = b[perm[0][i]];
       /*Compute inv(L)*x.*/
       env.sparsetriangularsolvedense(mosek.transpose.no, lnzc[0], lptrc[0], lsubc[0], u
\hookrightarrowlvalc[0], x);
        /*Compute inv(L^T)*x.*/
       env.sparsetriangularsolvedense(mosek.transpose.yes, lnzc[0], lptrc[0], lsubc[0], u
\hookrightarrowlvalc[0], x);
       System.out.print("\nSolution A x = b, x = [ ");
       for (int i = 0; i < n; i++)
         for (int j = 0; j < n; j++) if (perm[0][j] == i) System.out.print(x[j] + " ");</pre>
       System.out.println("]\n");
```

We can set up the data to recreate the matrix A from (9.6):

and we obtain the following output:

The output indicates that with the permutation matrix

$$P = \left[ \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right]$$

there is a Cholesky factorization  $PAP^T = LL^T$ , where

$$L = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1.4142 & 0 \\ 0 & 0 & 0.7071 & 0.7071 \end{array} \right]$$

The remaining part of the code solvers the linear system Ax = b for  $b = [13, 3, 4, 5]^T$ . The solution is reported to be  $x = [1, 2, 3, 4]^T$ , which is correct.

The second example shows what happens when we compute a sparse Cholesky factorization of a singular matrix. In this example A is a rank 1 matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^{T}$$
 (9.7)

Now we get the output

```
P = [ 0 2 1 ]

diag(D) = [ 0.00e+00 1.00e-14 1.00e-14 ]

L=

1.00e+00 0.00e+00 0.00e+00

1.00e+00 1.00e-07 0.00e+00

1.00e+00 0.00e+00 1.00e-07
```

which indicates the decomposition

$$PAP^T = LL^T - D$$

where

$$P = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{array} \right], \quad L = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 1 & 10^{-7} & 0 \\ 1 & 0 & 10^{-7} \end{array} \right], \quad D = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 10^{-14} & 0 \\ 0 & 0 & 10^{-14} \end{array} \right].$$

Since A is only positive semdefinite, but not of full rank, some of diagonal elements of A are boosted to make it truely positive definite. The amount of boosting is passed as an argument to Env. computesparsecholesky, in this case  $10^{-14}$ . Note that

$$PAP^T = LL^T - D$$

where D is a small matrix so the computed Cholesky factorization is exact of slightly perturbed A. In general this is the best we can hope for in finite precision and when A is singular or close to being singular.

We will end this section by a word of caution. Computing a Cholesky factorization of a matrix that is not of full rank and that is not sufficiently well conditioned may lead to incorrect results i.e. a matrix that is indefinite may declared positive semidefinite and vice versa.

# 9.4 Converting a quadratically constrained problem to conic form

MOSEK employs the following form of quadratic problems:

A conic quadratic constraint has the form

$$x \in \mathcal{Q}^n$$

in its most basic form where

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{1} \ge \sqrt{\sum_{j=2}^{n} x_{j}^{2}} \right\}.$$

A quadratic problem such as (9.8), if convex, can be reformulated in conic form. This is in fact the reformulation **MOSEK** performs internally. It has many advantages:

- ullet elegant duality theory for conic problems,
- reporting accurate dual information for quadratic inequalities is hard and/or computational expensive,
- it certifies that the original quadratic problem is indeed convex,
- modelling directly in conic form usually leads to a better model [And13] i.e. a faster solution time and better numerical properties.

In addition, there are more types of conic constraints that can be combined with a quadratic cone, for example semidefinite cones.

**MOSEK** offers a function that performs the conversion from quadratic to conic quadratic form explicitly. Note that the reformulation is not unique. The approach followed by **MOSEK** is to introduce additional variables, linear constraints and quadratic cones to obtain a larger but equivalent problem in which the original variables are preserved.

In particular:

- all variables and constraints are kept in the problem,
- each quadratic constraint and quadratic terms in the objective generate one rotated quadratic cone,
- each quadratic constraint will contain no coefficients and upper/lower bounds will be set to  $\infty, -\infty$  respectively.

This allows the user to recover the original variable and constraint values, as well as their dual values, with no conversion or additional effort.

**Note:** Task.toconic modifies the input task in-place: this means that if the reformulation is not possible, i.e. the problem is not conic representable, the state of the task is in general undefined. The user should consider cloning the original task.

# 9.4.1 Quadratic Constraint Reformulation

Let us assume we want to convert the following quadratic constraint

$$l \le \frac{1}{2}x^T Q x + \sum_{j=0}^{n-1} a_j x_j \le u$$

to conic form. We first check whether  $l = -\infty$  or  $u = \infty$ , otherwise either the constraint can be dropped, or the constraint is not convex. Thus let us consider the case

$$\frac{1}{2}x^T Q x + \sum_{j=0}^{n-1} a_j^T x_j \le u. \tag{9.9}$$

Introducing an additional variable w such that

$$w = u - \sum_{j=0}^{n-1} a_j^T x_j \tag{9.10}$$

we obtain the equivalent form

$$\begin{array}{rcl} \frac{1}{2}x^TQx & \leq & w, \\ u - \sum_{j=0}^{n-1} a_j x_j & = & w. \end{array}$$

If Q is positive semidefinite, then there exists a matrix F such that

$$Q = FF^T (9.11)$$

and therefore we can write

$$||Fx||^2 \le 2w,$$
  
 $u - \sum_{j=0}^{n-1} a_j^T x_j = w.$ 

Introducing an additional variable z = 1, and setting y = Fx we obtain the conic formulation

$$(w, z, y) \in \mathcal{Q}_r,$$

$$z = 1$$

$$y = Fx$$

$$w = u - a^T x.$$

$$(9.12)$$

Summarizing, for each quadratic constraint involving t variables, MOSEK introduces

- 1. a rotated quadratic cone of dimension t+2,
- 2. two additional variables for the cone roots,
- 3. t additional variables to map the remaining part of the cone,
- 4. t linear constraints.

A quadratic term in the objective is reformulated in a similar fashion. We refer to [And13] for a more thorough discussion.

### **Example**

Next we consider a simple problem with quadratic objective function:

```
minimize  \begin{array}{l} \frac{1}{2}(13x_0^2+17x_1^2+12x_2^2+24x_0x_1+12x_1x_2-4x_0x_2)-22x_0-14.5x_1+12x_2+1\\ \text{subject to} & -1\leq x_0,x_1,x_2\leq 1 \end{array}
```

We can specify it in the human-readable OPF format.

```
[comment]
An example of small QO problem from Boyd and Vandenberghe, "Convex Optimization", page 189 ex_u -- 4.3
The solution is (1,0.5,-1)
[/comment]
[variables]
x0 x1 x2
[/variables]
[objective min]
0.5 (13 x0^2 + 17 x1^2 + 12 x2^2 + 24 x0 * x1 + 12 x1 * x2 - 4 x0 * x2 ) - 22 x0 - 14.5 x1 +_u -- 12 x2 + 1
[/objective]
[bounds]
[b] -1 <= * <= 1 [/b]
[/bounds]
```

The objective function is convex, the minimum is attained for  $x^* = (1, 0.5, -1)$ . The conversion will introduce first a variable  $x_3$  in the objective function such that  $x_3 \ge 1/2x^TQx$  and then convert the latter directly in conic form. The converted problem follows:

```
minimize -22x_0 - 14.5x_1 + 12x_2 + x_3 + 1 subject to 3.61x_0 + 3.33x_1 - 0.55x_2 - x_6 = 0 +2.29x_1 + 3.42x_2 - x_7 = 0 0.81x_1 - x_8 = 0 -x_3 + x_4 = 0 x_5 = 1 (x_4, x_5, x_6, x_7, x_8) \in \mathcal{Q}_{\nabla} -1 \le x_0, x_1, x_2 \le 1
```

The model generated by Task. toconic is

```
[comment]
  Written by MOSEK version 8.1.0.19
   Date 21-08-17
   Time 10:53:36
[/comment]
[hints]
  [hint NUMVAR] 9 [/hint]
  [hint NUMCON] 4 [/hint]
  [hint NUMANZ] 11 [/hint]
  [hint NUMQNZ] 0 [/hint]
  [hint NUMCONE] 1 [/hint]
[/hints]
[variables disallow_new_variables]
 x0000_x0 x0001_x1 x0002_x2 x0003 x0004
 x0005 x0006 x0007 x0008
[/variables]
```

```
[objective minimize]
   - 2.2e+01 x0000_x0 - 1.45e+01 x0001_x1 + 1.2e+01 x0002_x2 + x0003
[/objective]
[constraints]
   [ {\tt con c0000} ] \quad 3.605551275463989e + 00 \ {\tt x0000\_x0} \ - \ 5.547001962252291e - 01 \ {\tt x0000\_x2} \ + \ 3. 
\Rightarrow 328201177351375e+00 x0001_x1 - x0006 = 0e+00 [/con]
 [con c0001] 3.419401657060442e+00 \times 0002_x2 + 2.294598480395823e+00 \times 0001_x1 - \times 0007 = 0e+00_0
\hookrightarrow [/con]
  [con c0002] 8.111071056538127e-01 \times 0001_x1 - \times 0008 = 0e+00 [/con]
  [con c0003] - x0003 + x0004 = 0e+00 [/con]
[/constraints]
[bounds]
  [b] -1e+00
                   <= x0000_x0,x0001_x1,x0002_x2 <= 1e+00 [/b]</pre>
                       x0003,x0004 free [/b]
  [b]
  [b]
                       x0005 = 1e+00 [/b]
  ГъТ
                       x0006,x0007,x0008 free [/b]
  [cone rquad k0000] x0004, x0005, x0006, x0007, x0008 [/cone]
[/bounds]
```

We can clearly see that constraints c0000, c0001 and c0002 represent the original linear constraints as in (9.11), while c0003 corresponds to (9.10). The cone roots are x0005 and x0004.

# **TECHNICAL GUIDELINES**

This section contains some technical guidelines for the Optimizer API for Java users.

For modelling guidelines check one of the following sections:

- Sec. 13 for how to address numerical issues in modelling and how to tune the continuous optimizers.
- Sec. 14 for how to tune the mixed-integer optimizer.

# 10.1 Memory management and garbage collection

Users who experience memory leaks, especially:

- memory usage not decreasing after the solver terminates,
- memory usage increasing when solving a sequence of problems,

should make sure that the <code>Task</code> objects are properly garbage collected. Since each <code>Task</code> object links to a <code>MOSEK</code> task resource in a linked library, it is sometimes the case that the garbage collector is unable to reclaim it automatically. This means that substantial amounts of memory may be leaked. For this reason it is very important to make sure that the <code>Task</code> object is disposed of, either automatically or manually, when it is not used any more.

It is recommended to use a construction such as

```
try {
  env = new mosek.Env();
  task = new mosek.Task(env, 0,0);
  // ...
  // ... optimization ...
  // ...
}
finally {
  if (task != null) task.dispose();
  if (env != null) env.dispose();
}
```

This construction assures that the <code>Task.dispose</code> method is called when the object goes out of scope, even if an exception occurred. If this approach cannot be used, e.g. if the <code>Task</code> object is returned by a factory function, one should explicitly call the <code>Task.dispose</code> method when the object is no longer used. The same applies to the environment object.

# 10.2 Multithreading

### Thread safety

Sharing a task between threads is safe, as long as it is not accessed from more than one thread at a time. Multiple tasks can be created and used in parallel without any problems.

#### **Parallelization**

The interior-point and mixed-integer optimizers in **MOSEK** are parallelized. By default **MOSEK** will automatically select the number of threads. However, the maximum number of threads allowed can be changed by setting the parameter *iparam.num\_threads* and related parameters. This should never exceed the number of cores. See Sec. 13 and Sec. 14 for more details for the two optimizer types.

The speed-up obtained when using multiple threads is highly problem and hardware dependent. We recommend experimenting with various thread numbers to determine the optimal settings. For small problems using multiple threads may be counter-productive because of the associated overhead.

By default the optimizer is run-to-run deterministic, which means that it will return the same answer each time it is run on the same machine with the same input, the same parameter settings (including number of threads) and no time limits.

# 10.3 Efficiency

Although MOSEK is implemented to handle memory efficiently, the user may have valuable knowledge about a problem, which could be used to improve the performance of MOSEK This section discusses some tricks and general advice that hopefully make MOSEK process your problem faster.

### Reduce the number of function calls and avoid input loops

For example, instead of setting the entries in the linear constraint matrix one by one (Task.putaij) define them all at once (Task.putaijlist) or in convenient large chunks (Task.putacollist etc.)

#### Use one environment only

If possible share the environment between several tasks. For most applications you need to create only a single environment.

### Read part of the solution

When fetching the solution, data has to be copied from the optimizer to the user's data structures. Instead of fetching the whole solution, consider fetching only the interesting part (see for example Task. getxxslice and similar).

## **Avoiding memory fragmentation**

MOSEK stores the optimization problem in internal data structures in the memory. Initially MOSEK will allocate structures of a certain size, and as more items are added to the problem the structures are reallocated. For large problems the same structures may be reallocated many times causing memory fragmentation. One way to avoid this is to give MOSEK an estimated size of your problem using the functions:

- Task. putmaxnumvar. Estimate for the number of variables.
- Task.putmaxnumcon. Estimate for the number of constraints.
- Task.putmaxnumcone. Estimate for the number of cones.
- Task.putmaxnumbarvar. Estimate for the number of semidefinite matrix variables.
- Task.putmaxnumanz. Estimate for the number of non-zeros in A.
- Task.putmaxnumqnz. Estimate for the number of non-zeros in the quadratic terms.

None of these functions changes the problem, they only serve as hints. If the problem ends up growing larger, the estimates are automatically increased.

### Do not mix put- and get- functions

MOSEK will queue put- requests internally until a get- function is called. If put- and get- calls are interleaved, the queue will have to be flushed more frequently, decreasing efficiency.

In general get- commands should not be called often (or at all) during problem setup.

## Use the LIFO principle

When removing constraints and variables, try to use a LIFO (Last In First Out) approach. MOSEK can more efficiently remove constraints and variables with a high index than a small index.

An alternative to removing a constraint or a variable is to fix it at 0, and set all relevant coefficients to 0. Generally this will not have any impact on the optimization speed.

### Add more constraints and variables than you need (now)

The cost of adding one constraint or one variable is about the same as adding many of them. Therefore, it may be worthwhile to add many variables instead of one. Initially fix the unused variable at zero, and then later unfix them as needed. Similarly, you can add multiple free constraints and then use them as needed.

#### Do not remove basic variables

When performing re-optimizations, instead of removing a basic variable it may be more efficient to fix the variable at zero and then remove it when the problem is re-optimized and it has left the basis. This makes it easier for **MOSEK** to restart the simplex optimizer.

# 10.4 The license system

MOSEK is a commercial product that always needs a valid license to work. MOSEK uses a third party license manager to implement license checking. The number of license tokens provided determines the number of optimizations that can be run simultaneously.

By default a license token remains checked out from the first optimization until the end of the **MOSEK** session, i.e.

- $\bullet$  a license token is checked out when  $Task.\,optimize$  is first called, and
- it is returned when the **MOSEK** environment is deleted.

Calling Task. optimize from different threads using the same MOSEK environment only consumes one license token.

Starting the optimization when no license tokens are available will result in an error.

Default behaviour of the license system can be changed in several ways:

- Setting the parameter *iparam.cache\_license* to *onoffkey.off* will force **MOSEK** to return the license token immediately after the optimization completed.
- Setting the license wait flag with the parameter <code>iparam.license\_wait</code> will force **MOSEK** to wait until a license token becomes available instead of returning with an error. The wait time between checks can be set with <code>Env.putlicensewait</code>.
- Additional license checkouts and checkins can be performed with the functions *Env. checkinlicense* and *Env. checkoutlicense*.
- Usually the license system is stopped automatically when the MOSEK library is unloaded. However, when the user explicitly unloads the library (using e.g. FreeLibrary), the license system must be stopped before the library is unloaded. This can be done by calling the function <code>Env.licensecleanup</code> as the last function call to MOSEK.

# 10.5 Deployment

When redistributing a Java application using the **MOSEK** Optimizer API for Java 8.1.0.83, the following libraries must be included:

64-bit Linux	64-bit Windows	32-bit Windows	64-bit Mac OS
libmosek64.so.8.1	mosek64_8_1.dll	mosek8_1.dll	libmosek64.8.1.dylib
libiomp5.so	libomp5md.dll	libomp5md.dll	
libcilkrts.so.5	cilkrts20.dll	cilkrts20.dll	libcilkrts.5.dylib
libmosekjava8_1.so	mosekjava8_1.dll	mosekjava8_1.dll	libmosekjava8_1.jnilib
libmosekxx8_1.so	mosekxx8_1.dll	mosekxx8_1.dll	libmosekxx8_1.dylib
libmosekscopt8_1.so	mosekscopt8_1.dll	mosekscopt8_1.dll	libmosekscopt8_1.dylib

By default the Java interface will look for the binaries in the same directory as the .jar file, so they should be placed in the same directory when redistributing.

# **CASE STUDIES**

In this section we present some case studies in which the Optimizer API for Java is used to solve real-life applications. These examples involve some more advanced modelling skills and possibly some input data. The user is strongly recommended to first read the basic tutorials of Sec. 6 before going through these advanced case studies.

Case Studies	J .	1	Keywords
Portofolio Optimization	CQO	NO	Markowitz, Slippage, Market Impact

# 11.1 Portfolio Optimization

In this section the Markowitz portfolio optimization problem and variants are implemented using the MOSEK optimizer API.

# 11.1.1 A Basic Portfolio Optimization Model

The classical Markowitz portfolio optimization problem considers investing in n stocks or assets held over a period of time. Let  $x_j$  denote the amount invested in asset j, and assume a stochastic model where the return of the assets is a random variable r with known mean

$$\mu = \mathbf{E}r$$

and covariance

$$\Sigma = \mathbf{E}(r - \mu)(r - \mu)^T.$$

The return of the investment is also a random variable  $y = r^T x$  with mean (or expected return)

$$\mathbf{E}y = \mu^T x$$

and variance (or risk)

$$(y - \mathbf{E}y)^2 = x^T \Sigma x.$$

The problem facing the investor is to rebalance the portfolio to achieve a good compromise between risk and expected return, e.g., maximize the expected return subject to a budget constraint and an upper bound (denoted  $\gamma$ ) on the tolerable risk. This leads to the optimization problem

maximize 
$$\mu^T x$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $x^T \Sigma x \leq \gamma^2$ ,  
 $x \geq 0$ . (11.1)

The variables x denote the investment i.e.  $x_j$  is the amount invested in asset j and  $x_j^0$  is the initial holding of asset j. Finally, w is the initial amount of cash available.

A popular choice is  $x^0 = 0$  and w = 1 because then  $x_j$  may be interpreted as the relative amount of the total portfolio that is invested in asset j.

Since e is the vector of all ones then

$$e^T x = \sum_{j=1}^n x_j$$

is the total investment. Clearly, the total amount invested must be equal to the initial wealth, which is

$$w + e^T x^0$$
.

This leads to the first constraint

$$e^T x = w + e^T x^0.$$

The second constraint

$$x^T \Sigma x < \gamma^2$$

ensures that the variance, or the risk, is bounded by  $\gamma^2$ . Therefore,  $\gamma$  specifies an upper bound of the standard deviation the investor is willing to undertake. Finally, the constraint

$$x_i \geq 0$$

excludes the possibility of short-selling. This constraint can of course be excluded if short-selling is allowed.

The covariance matrix  $\Sigma$  is positive semidefinite by definition and therefore there exist a matrix G such that

$$\Sigma = GG^T. \tag{11.2}$$

In general the choice of G is **not** unique and one possible choice of G is the Cholesky factorization of  $\Sigma$ . However, in many cases another choice is better for efficiency reasons as discussed in Sec. 11.1.3.

For a given G we have that

$$x^{T} \Sigma x = x^{T} G G^{T} x$$
$$= \|G^{T} x\|^{2}.$$

Hence, we may write the risk constraint as

$$\gamma \geq \left\|G^Tx\right\|$$

or equivalently

$$[\gamma; G^T x] \in \mathcal{Q}^{n+1}$$
.

where  $Q^{n+1}$  is the n+1 dimensional quadratic cone. Therefore, problem (11.1) can be written as

$$\begin{array}{lll} \text{maximize} & \mu^T x \\ \text{subject to} & e^T x & = & w + e^T x^0, \\ & [\gamma; G^T x] & \in & \mathcal{Q}^{n+1}, \\ & x & > & 0, \end{array} \tag{11.3}$$

which is a conic quadratic optimization problem that can easily be solved using MOSEK.

## **Example data**

Subsequently we will use the following sample input taken from [CT07]. We set

$$\mu = \begin{bmatrix} 0.1073 \\ 0.0737 \\ 0.0627 \end{bmatrix}$$

and

$$\Sigma = 0.1 \left[ \begin{array}{ccc} 0.2778 & 0.0387 & 0.0021 \\ 0.0387 & 0.1112 & -0.0020 \\ 0.0021 & -0.0020 & 0.0115 \end{array} \right]$$

This implies

$$G^T = \sqrt{0.1} \begin{bmatrix} 0.5271 & 0.0734 & 0.0040 \\ 0 & 0.3253 & -0.0070 \\ 0 & 0 & 0.1069 \end{bmatrix}$$

using 5 significant digits. Moreover, let

$$x^0 = \left[ \begin{array}{c} 0.0 \\ 0.0 \\ 0.0 \end{array} \right]$$

and

$$w = 1.0.$$

### Why a Conic Formulation?

Problem (11.1) is a convex quadratically constrained optimization problem that can be solved directly using MOSEK. Why then reformulate it as a conic quadratic optimization problem (11.3)? The main reason for choosing a conic model is that it is more robust and usually solves faster and more reliably. For instance it is not always easy to numerically validate that the matrix  $\Sigma$  in (11.1) is positive semidefinite due to the presence of rounding errors. It is also very easy to make a mistake so  $\Sigma$  becomes indefinite. These problems are completely eliminated in the conic formulation.

Moreover, observe the constraint

$$||G^Tx|| \le \gamma$$

more numerically robust than

$$x^T \Sigma x < \gamma^2$$

for very small and very large values of  $\gamma$ . Indeed, if say  $\gamma \approx 10^4$  then  $\gamma^2 \approx 10^8$ , which introduces a scaling issue in the model. Hence, using conic formulation we work with the standard deviation instead of variance, which usually gives rise to a better scaled model.

### Implementing the Portfolio Model

## Creating a matrix formulation

The Optimizer API for Java requires that an optimization problem is entered in the following standard form:

maximize 
$$c^{T}\hat{x}$$
subject to 
$$l^{c} \leq A\hat{x} \leq u^{c},$$

$$l^{x} \leq \hat{x} \leq u^{x},$$

$$\hat{x} \in \mathcal{K}.$$

$$(11.4)$$

We refer to  $\hat{x}$  as the API variable. It means we need to reformulate (11.3). The first step is to introduce auxiliary variables so that the conic constraint involves only unique variables:

maximize 
$$\mu^T x$$
 subject to  $e^T x = w + e^T x^0$ ,  $G^T x - t = 0$ ,  $[s;t] \in \mathcal{Q}^{n+1}$ ,  $x \geq 0$ ,  $s = \gamma$ . (11.5)

Here s is an additional scalar variable and t is a vector variable of dimension n. The next step is to concatenate all the variables into one long variable vector:

$$\hat{x} = [x; s; t] = \begin{bmatrix} x \\ s \\ t \end{bmatrix}$$
 (11.6)

The details of the concatenation are specified below.

Table 11.1: Storage layout of the  $\hat{x}$  variable.

Variable	Length	Offset
x	n	0
s	1	n
t	n	n+1

The offset determines where the variable starts. (Note that all variables are indexed from 0). For instance

$$\hat{x}_{n+1+i} = t_i.$$

because the offset of the t variable is n+1.

Given the ordering of the variables specified by (11.6) it is useful to visualize the linear constraints (11.4) in an explicit block matrix form:

$$\begin{bmatrix}
 & 1 & 0 & 0 \\
\hline
 & G^T & 0 & -1 \\
\hline
 & & & -1
\end{bmatrix} \cdot \begin{bmatrix} x \\
\hline
 & s \\
\hline
 & t \end{bmatrix} = \begin{bmatrix} w + e^T x_0 \\
\hline
 & 0 \end{bmatrix}.$$
(11.7)

In other words, we should define the specific components of the problem description as follows:

$$c = \begin{bmatrix} \mu^{T} & 0 & 0_{n} \end{bmatrix}^{T}, 
A = \begin{bmatrix} e^{T} & 0 & 0_{n} \\ G^{T} & 0_{n} & -I_{n} \end{bmatrix}, 
l^{c} = \begin{bmatrix} w + e^{T}x^{0} & 0_{n} \end{bmatrix}^{T}, 
u^{c} = \begin{bmatrix} w + e^{T}x^{0} & 0_{n} \end{bmatrix}^{T}, 
l^{x} = \begin{bmatrix} 0_{n} & \gamma & -\infty_{n} \end{bmatrix}^{T}, 
u^{x} = \begin{bmatrix} \infty_{n} & \gamma & \infty_{n} \end{bmatrix}^{T}.$$
(11.8)

#### Source code example

From the block matrix form (11.7) and the explicit specification (11.8), using the offset information in Table 11.1 it is easy to calculate the index and value of each entry of the linear constraint matrix. The code below sets up the general optimization problem (11.3) and solves it for the example data. Of course it is only necessary to set non-zero entries of the linear constraint matrix.

Listing 11.1: Code implementing model (11.3).

```
package com.mosek.example;
public class case_portfolio_1 {
 static final int n = 3;
 public static void main (String[] args) {
   // Since the value infinity is never used, we define
    // 'infinity' for symbolic purposes only
   double infinity = 0;
   double gamma = 0.05;
   double[] mu = {0.1073, 0.0737, 0.0627};
   double[][] GT = {
     \{0.1667, 0.0232, 0.0013\},\
     {0.0000, 0.1033, -0.0022},
     {0.0000, 0.0000, 0.0338}
   };
   double[] x0 = \{0.0, 0.0, 0.0\};
   double w = 1.0;
   double totalBudget;
   int numvar = 2 * n + 1;
   int numcon = n + 1;
   //Offset of variables into the API variable.
   int offsetx = 0;
    int offsets = n;
   int offsett = n + 1;
   try ( mosek.Env env = new mosek.Env ();
         mosek.Task task = new mosek.Task (env, 0, 0) ) {
      // Directs the log task stream to the user specified
      //\ {\it method}\ task\_{\it msg\_obj.stream}
      task.set_Stream(
       mosek.streamtype.log,
       new mosek.Stream()
      { public void stream(String msg) { System.out.print(msg); }});
      // Constraints.
      task.appendcons(numcon);
      // Constraint bounds. Compute total budget.
      totalBudget = w;
      for (int i = 0; i < n; ++i)
      {
       totalBudget += x0[i];
        /* Constraint bounds c^l = c^u = 0 */
       {\tt task.putconbound(i + 1, mosek.boundkey.fx, 0.0, 0.0);}
       task.putconname(i + 1, "GT[" + (i + 1) + "]");
      /* The total budget constraint c^l = c^u = totalBudget in first row of A. */
      task.putconbound(0, mosek.boundkey.fx, totalBudget, totalBudget);
      task.putconname(0, "budget");
      // Variables.
      task.appendvars(numvar);
      /* x variables. */
      for (int j = 0; j < n; ++j)
```

```
/* Return of asset j in the objective */
      task.putcj(offsetx + j, mu[j]);
      /* Coefficients in the first row of A */
      task.putaij(0, offsetx + j, 1.0);
      /* No short-selling - x^l = 0, x^u = inf */
      task.putvarbound(offsetx + j, mosek.boundkey.lo, 0.0, infinity);
      task.putvarname(offsetx + j, "x[" + (j + 1) + "]");
    /* s variable is a constant equal to gamma. */
    task.putvarbound(offsets, mosek.boundkey.fx, gamma, gamma);
    task.putvarname(offsets, "s");
    /* t variables (t = GT*x). */
    for (int j = 0; j < n; ++j)
      /* Copying the GT matrix in the appropriate block of A */
      for (int k = 0; k < n; ++k)
        if ( GT[k][j] != 0.0 )
          task.putaij(1 + k, offsetx + j, GT[k][j]);
      /* Diagonal -1 entries in a block of A */
      task.putaij(1 + j, offsett + j, -1.0);
      /* Free - no bounds */
      task.putvarbound(offsett + j, mosek.boundkey.fr, -infinity, infinity);
      {\tt task.putvarname}({\tt offsett} \; + \; j, \; {\tt "t["} \; + \; (j \; + \; 1) \; + \; {\tt "]"});
    /* Define the cone spanned by (s, t), i.e. of dimension n + 1 */
    int[] csub = new int[n + 1];
    csub[0] = offsets;
    for(int j = 0; j < n; j++) csub[j + 1] = offsett + j;
    task.appendcone( mosek.conetype.quad,
                     0.0, /* For future use only, can be set to 0.0 */
                      csub );
    task.putconename(0, "stddev");
    /* A maximization problem */
    task.putobjsense(mosek.objsense.maximize);
    task.optimize();
    /* Display solution summary for quick inspection of results */
    task.solutionsummary(mosek.streamtype.log);
    task.writedata("dump.opf");
    /* Read the results */
    double expret = 0.0, stddev = 0.0;
    double[] xx = new double[n + 1];
    task.getxxslice(mosek.soltype.itr, 0, offsets + 1, xx);
    for (int j = 0; j < n; ++j)
      expret += mu[j] * xx[j + offsetx];
    System.out.printf("\nExpected return %e for gamma %e\n", expret, xx[offsets]);
  }
}
```

The above code produces the result:

Listing 11.2: Output from the solver.

```
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 7.4766507287e-02
                                                 Viol. con: 2e-08
                                                                      var: 0e+00
                                   nrm: 1e+00
                                                                                    cones: 2e-
⇔08
 Dual.
          obj: 7.4766554102e-02
                                   nrm: 3e-01
                                                 Viol. con: 0e+00
                                                                      var: 3e-08
                                                                                    cones:
-0e+00
Expected return 7.476651e-02 for gamma 5.000000e-02
```

#### Source code comments

The source code is a direct translation of the model (11.5) using the explicit block matrix specification (11.8) but a few comments are nevertheless in place.

In the lines

```
//Offset of variables into the API variable.
int offsetx = 0;
int offsets = n;
int offsett = n + 1;
```

offsets into the MOSEK API variable are stored as in Table 11.1. The code

```
/* x variables. */
for (int j = 0; j < n; ++j)
{
    /* Return of asset j in the objective */
    task.putcj(offsetx + j, mu[j]);
    /* Coefficients in the first row of A */
    task.putaij(0, offsetx + j, 1.0);
    /* No short-selling - x^l = 0, x^u = inf */
    task.putvarbound(offsetx + j, mosek.boundkey.lo, 0.0, infinity);
    task.putvarname(offsetx + j, "x[" + (j + 1) + "]");
}</pre>
```

sets up the data for x variables. For instance

```
/* Return of asset j in the objective */
task.putcj(offsetx + j, mu[j]);
```

inputs the objective coefficients for the x variables. Moreover, the code

```
task.putvarname(offsetx + j, "x[" + (j + 1) + "]");
```

assigns meaningful names to the API variables. This is not needed but it makes debugging easier.

Note that the solution values are only accessed for the interesting variables; for instance the auxiliary variable t is omitted from this process.

#### **Debugging Tips**

Implementing an optimization model in Optimizer API for Java can be error-prone. In order to check the code for accidental errors it is very useful to dump the problem to a file in a human readable form for visual inspection. The line

```
task.writedata("dump.opf");
```

does that and it produces a file with the content:

Listing 11.3: Problem (11.5) stored in OPF format.

```
[comment]
   Written by MOSEK version 8.1.0.24
   Date 11-09-17
   Time 14:34:24
[/comment]
[hints]
  [hint NUMVAR] 7 [/hint]
  [hint NUMCON] 4 [/hint]
  [hint NUMANZ] 12 [/hint]
  [hint NUMQNZ] 0 [/hint]
  [hint NUMCONE] 1 [/hint]
[/hints]
[variables disallow_new_variables]
  'x[1]' 'x[2]' 'x[3]' s 't[1]'
  't[2]' 't[3]'
[/variables]
[objective maximize]
   1.073e-01 \ 'x[1]' + 7.37e-02 \ 'x[2]' + 6.270000000000001e-02 \ 'x[3]'
[/objective]
[constraints]
  [con 'budget'] 'x[1]' + 'x[2]' + 'x[3]' = 1e+00 [/con]
   [con 'GT[1]'] \quad 1.667e-01 'x[1]' + 2.32e-02 'x[2]' + 1.3e-03 'x[3]' - 't[1]' = 0e+00 [/con] 
   [con 'GT[2]'] \quad 1.033e-01 'x[2]' - 2.2e-03 'x[3]' - 't[2]' = 0e+00 [/con] 
  [con 'GT[3]'] 3.38e-02 'x[3]' - 't[3]' = 0e+00 [/con]
[/constraints]
[bounds]
                  <= 'x[1]','x[2]','x[3]' [/b]
  [b] 0e+00
  [b]
                    s = 5e-02 [/b]
                    't[1]', 't[2]', 't[3]' free [/b]
  [b]
  [cone quad 'stddev'] s, 't[1]', 't[2]', 't[3]' [/cone]
[/bounds]
```

Since the API variables have been given meaningful names it is easy to verify by hand that the model is correct.

### 11.1.2 The efficient Frontier

The portfolio computed by the Markowitz model is efficient in the sense that there is no other portfolio giving a strictly higher return for the same amount of risk. An efficient portfolio is also sometimes called a Pareto optimal portfolio. Clearly, an investor should only invest in efficient portfolios and therefore it may be relevant to present the investor with all efficient portfolios so the investor can choose the portfolio that has the desired tradeoff between return and risk. This leads to the concept of efficient frontier.

Given a nonnegative  $\alpha$  the optimization problem

maximize 
$$\mu^T x - \alpha s$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $[s; G^T x] \in \mathcal{Q}^{n+1}$ ,  
 $x \geq 0$ . (11.9)

computes an efficient portfolio which maximizes expected return while minimizing risk, where the tradeoff between the two is controlled by  $\alpha$ . Ideally the problem (11.9) should be solved for all values  $\alpha \geq 0$  but in practice that is impossible.

For the example data from Sec. 11.1.1, the optimal values of return and risk for a range of  $\alpha$ s are listed below:

Listing 11.4: Results obtained solving problem (11.9) for different values of  $\alpha$ .

```
alpha
                           std dev
             exp ret
0.000e+000
             1.073e-001
                           7.261e-001
2.500e-001
             1.033e-001
                           1.499e-001
5.000e-001
             6.976e-002
                           3.735e-002
7.500e-001
             6.766e-002
                           3.383e-002
1.000e+000
             6.679e-002 3.281e-002
1.500e+000
             6.599e-002
                          3.214e-002
             6.560e-002
2.000e+000
                          3.192e-002
             6.537e-002
2.500e+000
                          3.181e-002
3.000e+000
             6.522e-002
                           3.176e-002
3.500e+000
             6.512e-002
                           3.173e-002
                           3.170e-002
4.000e+000
             6.503e-002
4.500e+000
             6.497e-002
                           3.169e-002
```

### Source code example

The example code in Listing 11.5 demonstrates how to compute the efficient portfolios for several values of  $\alpha$ . The code is mostly similar to the one in Sec. 11.1.1, except the problem is re-optimized in a loop for varying  $\alpha$ .

Listing 11.5: Code implementing model (11.9).

```
package com.mosek.example;
import mosek.*;
public class case_portfolio_2 {
 static final int n = 3;
 public static void main (String[] args) {
    // Since the value infinity is never used, we define
    // 'infinity' symbolic purposes only
   double infinity = 0;
   double gamma = 0.05;
   double[] mu = {0.1073, 0.0737, 0.0627};
   double[][] GT = {
     \{0.1667, 0.0232, 0.0013\},\
     \{0.0000, 0.1033, -0.0022\},\
     {0.0000, 0.0000, 0.0338}
   };
   double[] x0 = \{0.0, 0.0, 0.0\};
   double w = 1.0;
   double[] alphas = {0.0, 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5};
   int numalphas = 12;
   int numvar = 2 * n + 1;
   int numcon = n + 1;
    //Offset of variables into the API variable.
    int offsetx = 0;
    int offsets = n;
    int offsett = n + 1;
    try ( Env env = new mosek.Env ();
          Task task = new mosek. Task (env, 0, 0) ) {
```

```
// Directs the log task stream to the user specified
// method task_msg_obj.stream
task.set_Stream(
 mosek.streamtype.log,
 new mosek.Stream()
{ public void stream(String msg) { System.out.print(msg); }});
//Constraints.
task.appendcons(numcon);
for ( int i = 1; i \le n; ++i) {
  w += x0[i - 1];
 task.putconbound(i, mosek.boundkey.fx, 0., 0.);
 task.putconname(i, "GT[" + i + "]");
task.putconbound(0, mosek.boundkey.fx, w, w);
task.putconname(0, "budget");
//Variables.
task.appendvars(numvar);
int[] xindx = {offsetx + 0, offsetx + 1, offsetx + 2};
task.putclist(xindx, mu);
for ( int i = 0; i < n; ++i) {
 for ( int j = i; j < n; ++j)
    task.putaij(i + 1, offsetx + j, GT[i][j]);
 task.putaij(i + 1, offsett + i, -1.0);
 task.putvarbound(offsetx + i, mosek.boundkey.lo, 0., 0.);
 task.putvarname(offsetx + i, "x[" + (i + 1) + "]");
 task.putvarname(offsett + i, "t[" + (i + 1) + "]");
  task.putvarbound(offsett + i, mosek.boundkey.fr, 0., 0.);
task.putvarbound(offsets, mosek.boundkey.fr, gamma, gamma);
task.putvarname(offsets, "s");
double[] e = {1.0, 1.0, 1.0};
task.putarow(0, xindx, e);
int[] csub = {offsets, offsett + 0, offsett + 1, offsett + 2};
task.appendcone( mosek.conetype.quad,
                 0.0, /* For future use only, can be set to 0.0 */
                 csub);
task.putconename(0, "stddev");
/* A maximization problem */
task.putobjsense(mosek.objsense.maximize);
//task.writedata("dump.opf");
  //Turn all log output off.
  task.putintparam(mosek.iparam.log, 0);
  System.out.printf("%-12s %-12s %-12s", "alpha", "exp ret", "std dev");
  for (int k = 0; k < numalphas; ++k) {
   task.putcj(offsets, -alphas[k]);
```

```
task.optimize();
          task.solutionsummary(mosek.streamtype.log);
          double expret = 0.0, stddev = 0.0;
          double[] xx = new double[numvar];
          task.getxx(mosek.soltype.itr, xx);
          for (int j = 0; j < n; ++j)
            expret += mu[j] * xx[j + offsetx];
          System.out.printf("%-12.3e %-12.3e %-12.3e\n", alphas[k], expret, xx[offsets]);
        System.out.println("");
      } catch (mosek.Warning mw) {
        System.out.println (" Mosek warning:");
        System.out.println (mw.toString ());
    } catch ( mosek.Exception e) {
      System.out.println ("An error/warning was encountered");
      System.out.println (e.toString ());
      throw e:
 }
}
```

# 11.1.3 Improving the Computational Efficiency

In practice it is often important to solve the portfolio problem very quickly. Therefore, in this section we discuss how to improve computational efficiency at the modelling stage.

The computational cost is of course to some extent dependent on the number of constraints and variables in the optimization problem. However, in practice a more important factor is the sparsity: the number of nonzeros used to represent the problem. Indeed it is often better to focus on the number of nonzeros in G see (11.2) and try to reduce that number by for instance changing the choice of G.

In other words if the computational efficiency should be improved then it is always good idea to start with focusing at the covariance matrix. As an example assume that

$$\Sigma = D + VV^T$$

where D is a positive definite diagonal matrix. Moreover, V is a matrix with n rows and p columns. Such a model for the covariance matrix is called a factor model and usually p is much smaller than n. In practice p tends to be a small number independent of n, say less than 100.

One possible choice for G is the Cholesky factorization of  $\Sigma$  which requires storage proportional to n(n+1)/2. However, another choice is

$$G^T = \left[ \begin{array}{c} D^{1/2} \\ V^T \end{array} \right]$$

because then

$$GG^T = D + VV^T$$
.

This choice requires storage proportional to n + pn which is much less than for the Cholesky choice of G. Indeed assuming p is a constant storage requirements are reduced by a factor of n.

The example above exploits the so-called factor structure and demonstrates that an alternative choice of G may lead to a significant reduction in the amount of storage used to represent the problem. This will in most cases also lead to a significant reduction in the solution time.

The lesson to be learned is that it is important to investigate how the covariance matrix is formed. Given this knowledge it might be possible to make a special choice for G that helps reducing the storage requirements and enhance the computational efficiency. More details about this process can be found in [And13].

# 11.1.4 Slippage Cost

The basic Markowitz model assumes that there are no costs associated with trading the assets and that the returns of the assets are independent of the amount traded. Neither of those assumptions is usually valid in practice. Therefore, a more realistic model is

maximize 
$$\mu^{T} x$$
subject to  $e^{T}x + \sum_{j=1}^{n} C_{j}(x_{j} - x_{j}^{0}) = w + e^{T}x^{0},$ 

$$x^{T}\sum_{j=1}^{n} x_{j} \leq \gamma^{2},$$

$$x \geq 0,$$

$$(11.10)$$

where the function

$$C_j(x_j-x_j^0)$$

specifies the transaction costs when the holding of asset j is changed from its initial value.

# 11.1.5 Market Impact Costs

If the initial wealth is fairly small and no short selling is allowed, then the holdings will be small and the traded amount of each asset must also be small. Therefore, it is reasonable to assume that the prices of the assets are independent of the amount traded. However, if a large volume of an asset is sold or purchased, the price, and hence return, can be expected to change. This effect is called market impact costs. It is common to assume that the market impact cost for asset j can be modelled by

$$C_j = m_j \sqrt{|x_j - x_j^0|}$$

where  $m_j$  is a constant that is estimated in some way by the trader. See [GK00] [p. 452] for details. Hence, we have

$$C_j(x_j - x_j^0) = m_j |x_j - x_j^0| \sqrt{|x_j - x_j^0|} = m_j |x_j - x_j^0|^{3/2}.$$

From [MOSEKApS12] it is known that

$$\{(c,z): c \ge z^{3/2}, z \ge 0\} = \{(c,z): (v,c,z), (z,1/8,v) \in \mathcal{Q}_r^3\}$$

where  $\mathcal{Q}_r^3$  is the 3-dimensional rotated quadratic cone. Hence, it follows

$$\begin{aligned} z_j &= |x_j - x_j^0|, \\ (v_j, c_j, z_j), (z_j, 1/8, v_j) &\in \mathcal{Q}_r^3, \\ \sum_{j=1}^n C_j (x_j - x_j^0) &= \sum_{j=1}^n c_j. \end{aligned}$$

Unfortunately this set of constraints is nonconvex due to the constraint

$$z_j = |x_j - x_j^0| \tag{11.11}$$

but in many cases the constraint may be replaced by the relaxed constraint

$$z_j \ge |x_j - x_j^0|,\tag{11.12}$$

which is equivalent to

$$\begin{aligned}
 z_j &\geq x_j - x_j^0, \\
 z_j &\geq -(x_j - x_j^0).
 \end{aligned}
 \tag{11.13}$$

For instance if the universe of assets contains a risk free asset then

$$z_j > |x_j - x_j^0| \tag{11.14}$$

cannot hold for an optimal solution.

If the optimal solution has the property (11.14) then the market impact cost within the model is larger than the true market impact cost and hence money are essentially considered garbage and removed by generating transaction costs. This may happen if a portfolio with very small risk is requested because the only way to obtain a small risk is to get rid of some of the assets by generating transaction costs. We generally assume that this is not the case and hence the models (11.11) and (11.12) are equivalent.

The above observations lead to

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x + m^{T}c = w + e^{T}x^{0},$   
 $[\gamma; G^{T}x] \in \mathcal{Q}^{n+1},$   
 $z_{j} \geq x_{j} - x_{j}^{0}, \quad j = 1, \dots, n,$   
 $z_{j} \geq x_{j}^{0} - x_{j}, \quad j = 1, \dots, n,$   
 $[v_{j}; c_{j}; z_{j}], [z_{j}; 1/8; v_{j}] \in \mathcal{Q}_{r}^{3}, \quad j = 1, \dots, n,$   
 $x > 0.$  (11.15)

The revised budget constraint

$$e^T x + m^T c = w + e^T x^0$$

specifies that the initial wealth covers the investment and the transaction costs. Moreover, v and z are auxiliary variables that model the market impact cost so that  $z_j \ge |x_j - x_j^0|$  and  $c_j \ge z_j^{3/2}$ .

It should be mentioned that transaction costs of the form

$$c_j \ge z_j^{p/q}$$

where p and q are both integers and  $p \ge q$  can be modelled using quadratic cones. See [MOSEKApS12] for details.

# Creating a matrix formulation

One more reformulation of (11.15) is needed to bring it to the standard form (11.4).

where  $f, g \in \mathbb{R}^{n \times 3}$ . The additional variables f and g are introduced to ensure that each variable appears at most once in any cone.

The formulation (11.16) is not the most compact possible, but it is easy to implement. **MOSEK** presolve will automatically simplify it.

The first step in developing the implementation is to chose an ordering of the variables. We will choose the following ordering:

$$\hat{x} = [x; s; t; c; v; z; f; g]$$

Table 11.2 shows the mapping between the  $\hat{x}$  vector and the model variables.

Table 11.2: Storage layout for the  $\hat{x}$ 

Variable	Length	Offset
x	n	0
s	1	n
t	n	n+1
c	n	2n + 1
v	n	3n+1
z	n	4n + 1
$f(:)^T$	3n	5n + 1
$g(:)^T$	3n	8n + 1

The next step is to consider how the linear constraint matrix A and the remaining data vectors are laid out. Reusing the idea in Sec. 11.1.1 we can write the data in block matrix form and read off all the required coordinates. This extension of the code setting up the constraint  $G^T x - t = 0$  from Sec. 11.1.1 is shown below.

### Source code example

The example code in Listing 11.6 demonstrates how to implement the model (11.16).

Listing 11.6: Code implementing model (11.16).

```
package com.mosek.example;
public class case_portfolio_3 {
 static final int n = 3;
 public static void main (String[] args) {
    // Since the value infinity is never used, we define
    // 'infinity' symbolic purposes only
   double infinity = 0;
   double gamma = 0.05;
   double[] mu = {0.1073, 0.0737, 0.0627};
   double[][] GT = {
     \{0.1667, 0.0232, 0.0013\},\
     \{0.0000, 0.1033, -0.0022\},\
     {0.0000, 0.0000, 0.0338}
   };
   double[] x0 = \{0.0, 0.0, 0.0\};
   double w = 1.0;
   double[] m = \{0.01, 0.01, 0.01\};
   int offsetx = 0;
    int offsets = offsetx + n;
   int offsett = offsets + 1;
   int offsetc = offsett + n;
   int offsetv = offsetc + n;
    int offsetz = offsetv + n;
    int offsetf = offsetz + n;
    int offsetg = offsetf + 3 * n;
   int numvar = offsetg + 3 * n;
   int offset_con_budget = 0;
   int offset_con_gx_t = offset_con_budget + 1;
   int offset_con_abs1 = offset_con_gx_t + n;
    int offset_con_abs2 = offset_con_abs1 + n;
```

```
int offset_con_f = offset_con_abs2 + n;
int offset_con_g = offset_con_f + 3 * n;
int numcon = 1 + 3 * n + 2 * 3 * n;
try ( mosek.Env env = new mosek.Env ();
      mosek.Task task = new mosek.Task (env, 0, 0) ) {
  // Directs the log task stream to the user specified
  // method task_msq_obj.stream
  task.set_Stream(
   mosek.streamtype.log,
   new mosek.Stream()
  { public void stream(String msg) { System.out.print(msg); }});
  //Set up constraint bounds, names and variable coefficients
  task.appendcons(numcon);
  for ( int i = 0; i < n; ++i) {
    w += x0[i];
    task.putconbound(offset_con_gx_t + i, mosek.boundkey.fx, 0., 0.);
    task.putconname(offset_con_gx_t + i, "GT[" + (i + 1) + "]");
    task.putconbound(offset_con_abs1 + i, mosek.boundkey.lo, -x0[i], infinity);
    task.putconname(offset_con_abs1 + i, "zabs1[" + (i + 1) + "]");
    task.putconbound(offset_con_abs2 + i, mosek.boundkey.lo, x0[i], infinity);
    task.putconname(offset_con_abs2 + i, "zabs2[" + (i + 1) + "]");
    for (int j = 0; j < 3; ++j) {
      task.putconbound(offset_con_f + 3 * i + j, mosek.boundkey.fx, 0., 0.);
      task.putconname(offset\_con\_f \ + \ 3 \ * \ i \ + \ j, \ "f[" \ + \ (i \ + \ 1) \ + \ "," \ + \ (j \ + \ 1) \ + \ "]");
      task.putconbound(offset_con_g + 3 * i + j, mosek.boundkey.fx, 0., 0.);
      task.putconname(offset\_con\_g + 3 * i + j, "g[" + (i + 1) + "," + (j + 1) + "]");
    task.putconbound(offset_con_g + 3 * i + 1, mosek.boundkey.fx, -1. / 8, -1. / 8.);
  // e x = w + e x0
  task.putconbound(offset_con_budget, mosek.boundkey.fx, w, w);
  task.putconname(offset_con_budget, "budget");
  //Variables.
  task.appendvars(numvar);
  //the objective function coefficients
  int[] xindx = {offsetx + 0, offsetx + 1, offsetx + 2};
  task.putclist(xindx, mu);
  double[] one_m_one = {1.0, -1.0};
  double[] one_one = {1.0, 1.0};
  //set up variable bounds and names
  for ( int i = 0; i < n; ++i) {
    task.putvarbound(offsetx + i, mosek.boundkey.lo, 0., infinity);
    task.putvarbound(offsett + i, mosek.boundkey.fr, infinity);
    task.putvarbound(offsetc + i, mosek.boundkey.fr, infinity);
    task.putvarbound(offsetz + i, mosek.boundkey.fr, infinity, infinity);
    task.putvarbound(offsetv + i, mosek.boundkey.fr, infinity, infinity);
    for (int j = 0; j < 3; ++j) {
     task.putvarbound(offsetf + j + i * 3, mosek.boundkey.fr, infinity);
      task. \underline{putvarbound} (offsetg + j + i * 3, \ \underline{mosek.boundkey.fr}, \ \underline{infinity});
```

```
task.putvarname(offsetx + i, "x[" + (i + 1) + "]");
  task.putvarname(offsett + i, "t[" + (i + 1) + "]");
  task.putvarname(offsetc + i, "c[" + (i + 1) + "]");
  task.putvarname(offsetz + i, "z[" + (i + 1) + "]");
  task.putvarname(offsetv + i, "v[" + (i + 1) + "]");
  for (int j = 0; j < 3; ++j) {
   task.putvarname(offsetf + j + i * 3, "f[" + (i + 1) + "," + (j + 1) + "]");
   task.putvarname(offsetg + j + i * 3, "g[" + (i + 1) + "," + (j + 1) + "]");
  for ( int j = i; j < n; ++j)
    task.putaij(offset_con_gx_t + i, j, GT[i][j]);
  task.putaij(offset_con_gx_t + i, offsett + i, -1.0);
  task.putaij(offset_con_budget, offsetx + i, 1.0);
  task.putaij(offset_con_budget, offsetc + i, m[i]);
  // z_{-}j - x_{-}j >= -x0_{-}j
  int[] indx1 = {offsetz + i, offsetx + i};
  task.putarow(offset_con_abs1 + i, indx1, one_m_one);
  // z_j + x_j > = +x0_j
  int[] indx2 = {offsetz + i, offsetx + i};
  task.putarow(offset_con_abs2 + i, indx2, one_one);
  int[] indxf1 = { offsetv + i, offsetf + i * 3};
  task.putarow(offset_con_f + 3 * i, indxf1, one_m_one);
  int[] indxf2 = {offsetc + i, offsetf + i * 3 + 1};
  task.putarow(offset_con_f + 1 + 3 * i, indxf2, one_m_one);
  int[] indxf3 = {offsetz + i, offsetf + i * 3 + 2};
  task.putarow(offset_con_f + 2 + 3 * i, indxf3, one_m_one);
  int[] indxg1 = {offsetz + i, offsetg + i * 3};
 task.putarow(offset_con_g + 3 * i, indxg1, one_m_one);
  task.putaij(offset_con_g + 3 * i + 1, offsetg + i * 3 + 1, -1.);
 int[] indxg3 = {offsetv + i, offsetg + i * 3 + 2};
 task.putarow(offset_con_g + 3 * i + 2, indxg3, one_m_one);
task.putvarbound(offsets, mosek.boundkey.fx, gamma, gamma);
task.putvarname(offsets, "s");
//Cones.
int conecount = 0;
int[] csub = {offsets, offsett + 0, offsett + 1, offsett + 2};
task.appendcone(mosek.conetype.quad, 0.0, csub);
task.putconename(conecount, "stddev");
++conecount;
for (int j = 0; j < n; ++j, ++conecount) {
  \verb"int[]" cone \verb"indx" = \{ \verb"offsetf" + j * 3 , \verb"offsetf" + j * 3 + 1, "" offsetf" + j * 3 + 2 \};
 task.appendcone(mosek.conetype.rquad, 0.0, coneindx);
 task.putconename(conecount, "f[" + (j + 1) + "]");
for (int j = 0; j < n; ++j, ++conecount) {
 \verb|int[]| coneindx = \{ \verb|offsetg + j * 3 , \verb|offsetg + j * 3 + 1, \verb|offsetg + j * 3 + 2 \}; \\
 task.appendcone(mosek.conetype.rquad, 0.0, coneindx);
 task.putconename(conecount, "g[" + (j + 1) + "]");
/* A maximization problem */
```

```
task.putobjsense(mosek.objsense.maximize);
    /* Solve the problem */
    try {
      //Turn all log output off.
      //task.putintparam(mosek.iparam.log,0);
      //task.writedata("dump.opf");
      task.optimize();
      task.solutionsummary(mosek.streamtype.log);
      double expret = 0.0, stddev = 0.0;
      double[] xx = new double[numvar];
      task.getxx(mosek.soltype.itr, xx);
      for (int j = 0; j < n; ++j)
        expret += mu[j] * xx[j + offsetx];
      System.out.printf("Expected return %e for gamma %e\n\n", expret, xx[offsets]);
    } catch (mosek.Warning mw) {
      System.out.println (" Mosek warning:");
      System.out.println (mw.toString ());
 } catch ( mosek.Exception e) {
    System.out.println ("An error/warning was encountered");
    System.out.println (e.toString ());
    throw e;
 }
}
```

The example code above produces the result

```
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 7.4390660847e-02
                                                Viol. con: 6e-09
                                                                   var: 0e+00
                                                                                   cones: 4e-
                                   nrm: 1e+00
⇔09
 Dual.
          obj: 7.4390675795e-02
                                   nrm: 3e-01
                                                Viol. con: 1e-19
                                                                     var: 8e-09
                                                                                   cones:
-0e+00
Expected return 7.439066e-02 for gamma 5.000000e-02
```

If the problem is dumped to an OPF file, it has the following content.

Listing 11.7: OPF file for problem (11.16).

```
[comment]
  Written by MOSEK version 8.1.0.24
  Date 12-09-17
  Time 12:34:27
[/comment]

[hints]
  [hint NUMVAR] 34 [/hint]
  [hint NUMCON] 28 [/hint]
  [hint NUMMONZ] 60 [/hint]
  [hint NUMQNZ] 0 [/hint]
  [hint NUMQNZ] 7 [/hint]
```

```
[/hints]
[variables disallow_new_variables]
 'x[1]' 'x[2]' 'x[3]' s 't[1]'
 't[2]' 't[3]' 'c[1]' 'c[2]' 'c[3]'
 'v[1]' 'v[2]' 'v[3]' 'z[1]' 'z[2]
 'z[3]' 'f[1,1]' 'f[1,2]' 'f[1,3]' 'f[2,1]'
 'f[2,2]' 'f[2,3]' 'f[3,1]' 'f[3,2]' 'f[3,3]'
 'g[1,1]' 'g[1,2]' 'g[1,3]' 'g[2,1]' 'g[2,2]'
 'g[2,3]' 'g[3,1]' 'g[3,2]' 'g[3,3]'
[/variables]
[objective maximize]
  1.073e-01 'x[1]' + 7.37e-02 'x[2]' + 6.27000000000001e-02 'x[3]'
[constraints]
 [con 'budget'] 'x[1]' + 'x[2]' + 'x[3]' + 1e-02 'c[1]' + 1e-02 'c[2]'
    + 1e-02 'c[3]' = 1e+00 [/con]
  [con 'GT[1]'] \quad 1.667e-01 'x[1]' + 2.32e-02 'x[2]' + 1.3e-03 'x[3]' - 't[1]' = 0e+00 [/con] 
 [con 'GT[2]'] 1.033e-01 'x[2]' - 2.2e-03 'x[3]' - 't[2]' = 0e+00 [/con] [con 'GT[3]'] 3.38e-02 'x[3]' - 't[3]' = 0e+00 [/con]
 [con 'zabs1[1]'] Oe+OO <= - 'x[1]' + 'z[1]' [/con]
 [con 'zabs1[2]'] 0e+00 <= - 'x[2]' + 'z[2]' [/con]
 [con 'zabs1[3]'] 0e+00 <= - 'x[3]' + 'z[3]' [/con]
 [con 'zabs2[1]'] 0e+00 \le 'x[1]' + 'z[1]' [/con]
 [con 'zabs2[2]'] 0e+00 \le 'x[2]' + 'z[2]' [/con]
 [con 'zabs2[3]'] 0e+00 \le 'x[3]' + 'z[3]' [/con]
 [con 'f[1,1]'] 'v[1]' - 'f[1,1]' = 0e+00 [/con]
 [con 'f[1,2]'] 'c[1]' - 'f[1,2]' = 0e+00 [/con]
 [con 'f[1,3]'] 'z[1]' - 'f[1,3]' = 0e+00 [/con]
 [con 'f[2,1]'] 'v[2]' - 'f[2,1]' = 0e+00 [/con]
 [con 'f[2,2]'] 'c[2]' - 'f[2,2]' = 0e+00 [/con]
 [con 'f[2,3]'] 'z[2]' - 'f[2,3]' = 0e+00 [/con]
 [con 'f[3,1]'] 'v[3]' - 'f[3,1]' = 0e+00 [/con]
 [con 'f[3,2]'] 'c[3]' - 'f[3,2]' = 0e+00 [/con]
 [con 'f[3,3]'] 'z[3]' - 'f[3,3]' = 0e+00 [/con]
 [con 'g[1,1]'] 'z[1]' - 'g[1,1]' = 0e+00 [/con]
 [con 'g[1,2]'] - 'g[1,2]' = -1.25e-01 [/con]
 [con 'g[1,3]'] 'v[1]' - 'g[1,3]' = 0e+00 [/con]
 [con 'g[2,1]'] 'z[2]' - 'g[2,1]' = 0e+00 [/con]
 [con 'g[2,2]'] - 'g[2,2]' = -1.25e-01 [/con]
 [con 'g[2,3]'] 'v[2]' - 'g[2,3]' = 0e+00 [/con]
 [con 'g[3,1]']   'z[3]' - 'g[3,1]' = 0e+00 [/con]
 [/constraints]
[bounds]
                 <= 'x[1]','x[2]','x[3]' [/b]
 [b] 0e+00
                   s = 5e-02 [/b]
 ГъТ
 [b]
                    't[1]','t[2]','t[3]','c[1]','c[2]','c[3]' free [/b]
                    'v[1]','v[2]','v[3]','z[1]','z[2]','z[3]' free [/b]
 ГъЪ
                    'f[1,1]','f[1,2]','f[1,3]','f[2,1]','f[2,2]','f[2,3]' free [/b]
 ГъЪ
                    'f[3,1]','f[3,2]','f[3,3]','g[1,1]','g[1,2]','g[1,3]' free [/b]
 ГъТ
                    'g[2,1]','g[2,2]','g[2,3]','g[3,1]','g[3,2]','g[3,3]' free [/b]
 [cone quad 'stddev'] s, 't[1]', 't[2]', 't[3]' [/cone]
 [cone rquad 'f[1]'] 'f[1,1]', 'f[1,2]', 'f[1,3]' [/cone]
 [cone rquad 'f[2]'] 'f[2,1]', 'f[2,2]', 'f[2,3]' [/cone]
 [cone rquad 'f[3]'] 'f[3,1]', 'f[3,2]', 'f[3,3]' [/cone]
 [cone rquad 'g[1]'] 'g[1,1]', 'g[1,2]', 'g[1,3]' [/cone]
 [cone rquad 'g[2]'] 'g[2,1]', 'g[2,2]', 'g[2,3]' [/cone]
  [cone rquad 'g[3]'] 'g[3,1]', 'g[3,2]', 'g[3,3]' [/cone]
```

## [/bounds]

The file verifies that the correct problem has been set up.

## PROBLEM FORMULATION AND SOLUTIONS

In this chapter we will discuss the following issues:

- The formal, mathematical formulations of the problem types that MOSEK can solve and their duals.
- The solution information produced by MOSEK.
- The infeasibility certificate produced by **MOSEK** if the problem is infeasible.

# 12.1 Linear Optimization

A linear optimization problem can be written as

where

- $\bullet$  m is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

A primal solution (x) is (primal) feasible if it satisfies all constraints in (12.1). If (12.1) has at least one primal feasible solution, then (12.1) is said to be (primal) feasible.

In case (12.1) does not have a feasible solution, the problem is said to be (primal) infeasible

### 12.1.1 Duality for Linear Optimization

Corresponding to the primal problem (12.1), there is a dual problem

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ & A^T y + s_l^x - s_u^x &= c, \\ \text{subject to} & -y + s_l^c - s_u^c &= 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x &\geq 0. \end{array} \tag{12.2}$$

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. E.g.

$$l_i^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0 \text{ and } l_i^x \cdot (s_l^x)_j = 0.$$

This is equivalent to removing variable  $(s_l^x)_j$  from the dual problem. A solution

$$(y, s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x})$$

to the dual problem is feasible if it satisfies all the constraints in (12.2). If (12.2) has at least one feasible solution, then (12.2) is (dual) feasible, otherwise the problem is (dual) infeasible.

#### A Primal-dual Feasible Solution

A solution

$$(x, y, s_l^c, s_u^c, s_l^x, s_u^x)$$

is denoted a *primal-dual feasible solution*, if (x) is a solution to the primal problem (12.1) and  $(y, s_l^c, s_u^c, s_l^x, s_u^x)$  is a solution to the corresponding dual problem (12.2).

#### The Duality Gap

Let

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

be a primal-dual feasible solution, and let

$$(x^c)^* := Ax^*.$$

For a primal-dual feasible solution we define the duality gap as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] \ge 0$$

$$(12.3)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (12.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

### **An Optimal Solution**

It is well-known that a linear optimization problem has an optimal solution if and only if there exist feasible primal and dual solutions so that the duality gap is zero, or, equivalently, that the *complementarity conditions* 

$$\begin{array}{rclcrcl} (s_{u}^{c})_{i}^{*}((x_{i}^{c})^{*}-l_{i}^{c}) & = & 0, & i=0,\ldots,m-1, \\ (s_{u}^{c})_{i}^{*}(u_{i}^{c}-(x_{i}^{c})^{*}) & = & 0, & i=0,\ldots,m-1, \\ (s_{l}^{x})_{j}^{*}(x_{j}^{*}-l_{j}^{x}) & = & 0, & j=0,\ldots,n-1, \\ (s_{u}^{x})_{j}^{*}(u_{j}^{x}-x_{j}^{*}) & = & 0, & j=0,\ldots,n-1, \end{array}$$

are satisfied.

If (12.1) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

## 12.1.2 Infeasibility for Linear Optimization

#### **Primal Infeasible Problems**

If the problem (12.1) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

to (12.4) so that

$$(l^c)^T(s_l^c)^* - (u^c)^T(s_u^c)^* + (l^x)^T(s_l^x)^* - (u^x)^T(s_u^x)^* > 0.$$

Such a solution implies that (12.4) is unbounded, and that its dual is infeasible. As the constraints to the dual of (12.4) are identical to the constraints of problem (12.1), we thus have that problem (12.1) is also infeasible.

#### **Dual Infeasible Problems**

If the problem (12.2) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

minimize 
$$c^T x$$
  
subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ ,  $\hat{l}^x \leq x \leq \hat{u}^x$ , (12.5)

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_j^x = \left\{ \begin{array}{ll} 0 & \text{if } l_j^x > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_j^x := \left\{ \begin{array}{ll} 0 & \text{if } u_j^x < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that

$$c^T x < 0$$
.

Such a solution implies that (12.5) is unbounded, and that its dual is infeasible. As the constraints to the dual of (12.5) are identical to the constraints of problem (12.2), we thus have that problem (12.2) is also infeasible.

#### Primal and Dual Infeasible Case

In case that both the primal problem (12.1) and the dual problem (12.2) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

#### Minimalization vs. Maximalization

When the objective sense of problem (12.1) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (12.2). The dual problem thus takes the form

This means that the duality gap, defined in (12.3) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$A^{T}y + s_{l}^{x} - s_{u}^{x} = 0,$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \leq 0,$$
(12.6)

such that the objective value is strictly negative

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* < 0.$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (12.5) such that  $c^T x > 0$ .

# 12.2 Conic Quadratic Optimization

Conic quadratic optimization is an extension of linear optimization (see Sec. 12.1) allowing conic domains to be specified for subsets of the problem variables. A conic quadratic optimization problem can be written as

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  
 $l^x \le x \le u^x$ ,  
 $x \in \mathcal{K}$ , (12.7)

where set  $\mathcal{K}$  is a Cartesian product of convex cones, namely  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_p$ . Having the domain restriction,  $x \in \mathcal{K}$ , is thus equivalent to

$$x^t \in \mathcal{K}_t \subset \mathbb{R}^{n_t}$$
,

where  $x = (x^1, ..., x^p)$  is a partition of the problem variables. Please note that the *n*-dimensional Euclidean space  $\mathbb{R}^n$  is a cone itself, so simple linear variables are still allowed.

MOSEK supports only a limited number of cones, specifically:

- The  $\mathbb{R}^n$  set.
- The quadratic cone:

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{1} \ge \sqrt{\sum_{j=2}^{n} x_{j}^{2}} \right\}.$$

• The rotated quadratic cone:

$$Q_r^n = \left\{ x \in \mathbb{R}^n : 2x_1 x_2 \ge \sum_{j=3}^n x_j^2, \quad x_1 \ge 0, \quad x_2 \ge 0 \right\}.$$

Although these cones may seem to provide only limited expressive power they can be used to model a wide range of problems as demonstrated in |MOSEKApS12|.

## 12.2.1 Duality for Conic Quadratic Optimization

The dual problem corresponding to the conic quadratic optimization problem (12.7) is given by

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f$$
 subject to 
$$A^T y + s_l^x - s_u^x + s_n^x = c - y + s_l^c - s_u^c = 0,$$
 
$$s_l^c, s_u^c, s_l^x, s_u^x \ge 0,$$
 
$$s_n^c \in \mathcal{K}^*,$$
 (12.8)

where the dual cone  $\mathcal{K}^*$  is a Cartesian product of the cones

$$\mathcal{K}^* = \mathcal{K}_1^* \times \cdots \times \mathcal{K}_n^*$$

where each  $\mathcal{K}_t^*$  is the dual cone of  $\mathcal{K}_t$ . For the cone types **MOSEK** can handle, the relation between the primal and dual cone is given as follows:

• The  $\mathbb{R}^n$  set:

$$\mathcal{K}_t = \mathbb{R}^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \{ s \in \mathbb{R}^{n_t} : \quad s = 0 \}.$$

• The quadratic cone:

$$\mathcal{K}_t = \mathcal{Q}^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \mathcal{Q}^{n_t} = \left\{ s \in \mathbb{R}^{n_t} : s_1 \ge \sqrt{\sum_{j=2}^{n_t} s_j^2} \right\}.$$

• The rotated quadratic cone:

$$\mathcal{K}_t = \mathcal{Q}_r^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \mathcal{Q}_r^{n_t} = \left\{ s \in \mathbb{R}^{n_t} : 2s_1 s_2 \ge \sum_{j=3}^{n_t} s_j^2, \quad s_1 \ge 0, \quad s_2 \ge 0 \right\}.$$

Please note that the dual problem of the dual problem is identical to the original primal problem.

# 12.2.2 Infeasibility for Conic Quadratic Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. This works exactly as for linear problems (see Sec. 12.1.2).

#### **Primal Infeasible Problems**

If the problem (12.7) is infeasible, **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the problem

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & \\ & &$$

such that the objective value is strictly positive.

#### **Dual infeasible problems**

If the problem (12.8) is infeasible, **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{ll} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{ll} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that the objective value is strictly negative.

# 12.3 Semidefinite Optimization

Semidefinite optimization is an extension of conic quadratic optimization (see Sec. 12.2) allowing positive semidefinite matrix variables to be used in addition to the usual scalar variables. A semidefinite optimization problem can be written as

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f$$
subject to  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq u_i^c, \quad i = 0, \dots, m-1$ 

$$l_j^x \leq x_j \leq x_j \qquad \leq u_j^x, \quad j = 0, \dots, n-1$$

$$x \in \mathcal{K}, \overline{X}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, p-1$$

$$(12.9)$$

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_+^{r_j}$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}^{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $U, V \in \mathbb{R}^{m \times n}$  we have

$$\langle U, V \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} U_{ij} V_{ij}.$$

With semidefinite optimization we can model a wide range of problems as demonstrated in [MOSEKApS12].

## 12.3.1 Duality for Semidefinite Optimization

The dual problem corresponding to the semidefinite optimization problem (12.9) is given by

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f$$
 subject to 
$$\frac{c - A^T y + s_u^x - s_l^x = s_n^x,}{\overline{C}_j - \sum_{i=0}^m y_i \overline{A}_{ij} = \overline{S}_j,} \qquad j = 0, \dots, p-1$$
  $s_l^c - s_u^c = y,$   $s_l^c, s_u^c, s_l^x, s_u^x \ge 0,$   $s_n^c \in \mathcal{K}^*, \quad \overline{S}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, p-1$ 

where  $A \in \mathbb{R}^{m \times n}$ ,  $A_{ij} = a_{ij}$ , which is similar to the dual problem for conic quadratic optimization (see Sec. 12.2.1), except for the addition of dual constraints

$$\left(\overline{C}_j - \sum_{i=0}^m y_i \overline{A}_{ij}\right) \in \mathcal{S}_+^{r_j}.$$

Note that the dual of the dual problem is identical to the original primal problem.

# 12.3.2 Infeasibility for Semidefinite Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of the infeasibility. This works exactly as for linear problems (see Sec. 12.1.2).

#### **Primal Infeasible Problems**

If the problem (12.9) is infeasible, **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is a certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the problem

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & \\ & A^T y + s_l^x - s_u^x + s_n^x = 0, \\ & \sum_{i=0}^{m-1} y_i \overline{A}_{ij} + \overline{S}_j = 0, \\ & -y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\ & s_n^c \in \mathcal{K}^*, \quad \overline{S}_j \in \mathcal{S}_+^{r_j}, & j = 0, \dots, p-1 \\ \end{array}$$

such that the objective value is strictly positive.

#### **Dual Infeasible Problems**

If the problem (12.10) is infeasible, **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

$$\begin{array}{lll} \text{minimize} & \sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle \\ \text{subject to} & \hat{l}_i^c & \leq & \sum_{j=1}^n a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle & \leq & \hat{u}_i^c, \quad i = 0, \dots, m-1 \\ & \hat{l}^x & \leq & x & \leq & \hat{u}^x, \\ & x \in \mathcal{K}, \quad \overline{X}_j \in \mathcal{S}_+^{r_j}, & j = 0, \dots, p-1 \end{array}$$

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c >; -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c <; \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

and

$$\hat{l}_j^x = \left\{ \begin{array}{ll} 0 & \text{if } l_j^x >; -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \quad \text{and} \quad \hat{u}_j^x := \left\{ \begin{array}{ll} 0 & \text{if } u_j^x <; \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

such that the objective value is strictly negative.

# 12.4 Quadratic and Quadratically Constrained Optimization

A convex quadratic and quadratically constrained optimization problem has the form

where  $Q^o$  and all  $Q^k$  are symmetric matrices. Moreover, for convexity,  $Q^o$  must be a positive semidefinite matrix and  $Q^k$  must satisfy

$$\begin{array}{rcl} -\infty < l_k^c & \Rightarrow & Q^k \text{ is negative semidefinite,} \\ u_k^c < \infty & \Rightarrow & Q^k \text{ is positive semidefinite,} \\ -\infty < l_k^c \leq u_k^c < \infty & \Rightarrow & Q^k = 0. \end{array}$$

The convexity requirement is very important and MOSEK checks whether it is fulfilled.

### 12.4.1 A Recommendation

Any convex quadratic optimization problem can be reformulated as a conic quadratic optimization problem, see [MOSEKApS12] and in particular [And13]. In fact MOSEK does such conversion internally as a part of the solution process for the following reasons:

- the conic optimizer is numerically more robust than the one for quadratic problems.
- the conic optimizer is usually faster because quadratic cones are simpler than quadratic functions, even though the conic reformulation usually has more constraints and variables than the original quadratic formulation.
- it is easy to dualize the conic formulation if deemed worthwhile potentially leading to (huge) computational savings.

However, instead of relying on the automatic reformulation we recommend to formulate the problem as a conic problem from scratch because:

- it saves the computational overhead of the reformulation including the convexity check. A conic problem is convex by construction and hence no convexity check is needed for conic problems.
- usually the modeller can do a better reformulation than the automatic method because the modeller can exploit the knowledge of the problem at hand.

To summarize we recommend to formulate quadratic problems and in particular quadratically constrained problems directly in conic form.

## 12.4.2 Duality for Quadratic and Quadratically Constrained Optimization

The dual problem corresponding to the quadratic and quadratically constrained optimization problem (12.11) is given by

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + \frac{1}{2} x^T \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x = c, \\ & - y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0. \end{array}$$

The dual problem is related to the dual problem for linear optimization (see Sec. 12.1.1), but depends on the variable x which in general can not be eliminated. In the solutions reported by **MOSEK**, the value of x is the same for the primal problem (12.11) and the dual problem (12.12).

## 12.4.3 Infeasibility for Quadratic and Quadratically Constrained Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. This works exactly as for linear problems (see Sec. 12.1.2).

#### **Primal Infeasible Problems**

If the problem (12.11) with all  $Q^k = 0$  is infeasible, **MOSEK** will report a certificate of primal infeasibility. As the constraints are the same as for a linear problem, the certificate of infeasibility is the same as for linear optimization (see Sec. 12.1.2).

#### **Dual Infeasible Problems**

If the problem (12.12) with all  $Q^k = 0$  is dual infeasible, **MOSEK** will report a certificate of dual infeasibility. The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{ll} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{ll} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that the objective value is strictly negative.

# 12.5 General Convex Optimization

The general nonlinear optimizer (which may be available for all or some types of nonlinear problems depending on the interface), solves smooth (twice differentiable) convex nonlinear optimization problems of the form

$$\begin{array}{lll} \text{minimize} & & f(x) + c^T x + c^f \\ \text{subject to} & l^c & \leq & g(x) + Ax & \leq & u^c, \\ & l^x & \leq & x & \leq & u^x, \end{array}$$

where

- *m* is the number of constraints.
- n is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part objective function.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.

- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $f: \mathbb{R}^n \to \mathbb{R}$  is a nonlinear function.
- $g: \mathbb{R}^n \to \mathbb{R}^m$  is a nonlinear vector function.

This means that the i-th constraint has the form

$$l_i^c \le g_i(x) + \sum_{j=1}^n a_{ij} x_j \le u_i^c.$$

The linear term Ax is not included in g(x) since it can be handled much more efficiently as a separate entity when optimizing.

The nonlinear functions f and g must be smooth in all  $x \in [l^x; u^x]$ . Moreover, f(x) must be a convex function and  $g_i(x)$  must satisfy

$$\begin{array}{rcl} -\infty < l_i^c & \Rightarrow & g_i(x) \text{ is concave,} \\ u_i^c < \infty & \Rightarrow & g_i(x) \text{ is convex,} \\ -\infty < l_i^c \leq u_i^c < \infty & \Rightarrow & g_i(x) = 0. \end{array}$$

## 12.5.1 Duality for General convex Optimization

Similarly to the linear case, **MOSEK** reports dual information in the general nonlinear case. Indeed in this case the Lagrange function is defined by

$$\begin{array}{lcl} L(x,s_{l}^{c},s_{u}^{c},s_{u}^{x},s_{u}^{x}) & := & f(x)+c^{T}x+c^{f} \\ & -(s_{l}^{c})^{T}(g(x)+Ax-l^{c})-(s_{u}^{c})^{T}(u^{c}-g(x)-Ax) \\ & -(s_{l}^{x})^{T}(x-l^{x})-(s_{u}^{x})^{T}(u^{x}-x), \end{array}$$

and the dual problem is given by

$$\begin{array}{lll} \text{maximize} & L(x, s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x}) \\ \text{subject to} & \nabla_{x} L(x, s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x})^{T} & = & 0, \\ & s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0, \end{array}$$

which is equivalent to

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ & + f(x) - g(x)^T y - (\nabla f(x)^T - \nabla g(x)^T y)^T x \\ \text{subject to} & A^T y + s_l^x - s_u^x - (\nabla f(x)^T - \nabla g(x)^T y) & = \ c, \\ & - y + s_l^c - s_u^c & = \ 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0. \end{array}$$

In this context we use the following definition for scalar functions

$$\nabla f(x) = \left[\frac{\partial f(x)}{\partial x_1}, \dots, \frac{\partial f(x)}{\partial x_n}\right]$$

and accordingly for vector functions

$$\nabla g(x) = \begin{bmatrix} \nabla g_1(x) \\ \vdots \\ \nabla g_m(x) \end{bmatrix}.$$

## THE OPTIMIZERS FOR CONTINUOUS PROBLEMS

The most essential part of **MOSEK** are the optimizers. This chapter describes the optimizers for the class of *continuous problems* without integer variables, that is:

- linear problems,
- conic problems (quadratic and semidefinite),
- general convex problems.

**MOSEK** offers an interior-point optimizer for each class of problems and also a simplex optimizer for linear problems. The structure of a successful optimization process is roughly:

#### • Presolve

- 1. Elimination: Reduce the size of the problem.
- 2. Dualizer: Choose whether to solve the primal or the dual form of the problem.
- 3. Scaling: Scale the problem for better numerical stability.

#### • Optimization

- 1. Optimize: Solve the problem using selected method.
- 2. Terminate: Stop the optimization when specific termination criteria have been met.
- 3. Report: Return the solution or an infeasibility certificate.

The preprocessing stage is transparent to the user, but useful to know about for tuning purposes. The purpose of the preprocessing steps is to make the actual optimization more efficient and robust. We discuss the details of the above steps in the following sections.

## 13.1 Presolve

Before an optimizer actually performs the optimization the problem is preprocessed using the so-called presolve. The purpose of the presolve is to

- 1. remove redundant constraints,
- 2. eliminate fixed variables,
- 3. remove linear dependencies,
- 4. substitute out (implied) free variables, and
- 5. reduce the size of the optimization problem in general.

After the presolved problem has been optimized the solution is automatically postsolved so that the returned solution is valid for the original problem. Hence, the presolve is completely transparent. For further details about the presolve phase, please see [AA95] and [AGMX96].

It is possible to fine-tune the behavior of the presolve or to turn it off entirely. If presolve consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This is done

by setting the parameter *iparam.presolve\_use* to *presolvemode.off*. The two most time-consuming steps of the presolve are

- the eliminator, and
- the linear dependency check.

Therefore, in some cases it is worthwhile to disable one or both of these.

#### Numerical issues in the presolve

During the presolve the problem is reformulated so that it hopefully solves faster. However, in rare cases the presolved problem may be harder to solve then the original problem. The presolve may also be infeasible although the original problem is not. If it is suspected that presolved problem is much harder to solve than the original, we suggest to first turn the eliminator off by setting the parameter <code>iparam.presolve\_eliminator\_max\_num\_tries</code> to 0. If that does not help, then trying to turn entire presolve off may help.

Since all computations are done in finite precision, the presolve employs some tolerances when concluding a variable is fixed or a constraint is redundant. If it happens that **MOSEK** incorrectly concludes a problem is primal or dual infeasible, then it is worthwhile to try to reduce the parameters *dparam. presolve\_tol\_x* and *dparam.presolve\_tol\_s*. However, if reducing the parameters actually helps then this should be taken as an indication that the problem is badly formulated.

#### Eliminator

The purpose of the eliminator is to eliminate free and implied free variables from the problem using substitution. For instance, given the constraints

$$\begin{array}{rcl} y & = & \sum_j x_j, \\ y, x & \geq & 0, \end{array}$$

y is an implied free variable that can be substituted out of the problem, if deemed worthwhile. If the eliminator consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This can be done by setting the parameter <code>iparam.presolve\_eliminator\_max\_num\_tries</code> to 0. In rare cases the eliminator may cause that the problem becomes much hard to solve.

#### Linear dependency checker

The purpose of the linear dependency check is to remove linear dependencies among the linear equalities. For instance, the three linear equalities

$$\begin{array}{rcl} x_1 + x_2 + x_3 & = & 1, \\ x_1 + 0.5x_2 & = & 0.5, \\ 0.5x_2 + x_3 & = & 0.5. \end{array}$$

contain exactly one linear dependency. This implies that one of the constraints can be dropped without changing the set of feasible solutions. Removing linear dependencies is in general a good idea since it reduces the size of the problem. Moreover, the linear dependencies are likely to introduce numerical problems in the optimization phase. It is best practice to build models without linear dependencies, but that is not always easy for the user to control. If the linear dependencies are removed at the modelling stage, the linear dependency check can safely be disabled by setting the parameter <code>iparam.presolve\_lindep\_use</code> to <code>onoffkey.off</code>.

## Dualizer

All linear, conic, and convex optimization problems have an equivalent dual problem associated with them. **MOSEK** has built-in heuristics to determine if it is more efficient to solve the primal or dual

problem. The form (primal or dual) is displayed in the **MOSEK** log and available as an information item from the solver. Should the internal heuristics not choose the most efficient form of the problem it may be worthwhile to set the dualizer manually by setting the parameters:

- iparam. intpnt\_solve\_form: In case of the interior-point optimizer.
- *iparam.sim\_solve\_form*: In case of the simplex optimizer.

Note that currently only linear and conic quadratic problems may be automatically dualized.

#### **Scaling**

Problems containing data with large and/or small coefficients, say 1.0e+9 or 1.0e-7, are often hard to solve. Significant digits may be truncated in calculations with finite precision, which can result in the optimizer relying on inaccurate data. Since computers work in finite precision, extreme coefficients should be avoided. In general, data around the same *order of magnitude* is preferred, and we will refer to a problem, satisfying this loose property, as being *well-scaled*. If the problem is not well scaled, **MOSEK** will try to scale (multiply) constraints and variables by suitable constants. **MOSEK** solves the scaled problem to improve the numerical properties.

The scaling process is transparent, i.e. the solution to the original problem is reported. It is important to be aware that the optimizer terminates when the termination criterion is met on the scaled problem, therefore significant primal or dual infeasibilities may occur after unscaling for badly scaled problems. The best solution of this issue is to reformulate the problem, making it better scaled.

By default MOSEK heuristically chooses a suitable scaling. The scaling for interior-point and simplex optimizers can be controlled with the parameters <code>iparam.intpnt\_scaling</code> and <code>iparam.sim\_scaling</code> respectively.

# 13.2 Using Multiple Threads in an Optimizer

### Multithreading in interior-point optimizers

The interior-point optimizers in MOSEK have been parallelized. This means that if you solve linear, quadratic, conic, or general convex optimization problem using the interior-point optimizer, you can take advantage of multiple CPU's. By default MOSEK will automatically select the number of threads to be employed when solving the problem. However, the maximum number of threads employed can be changed by setting the parameter <code>iparam.num\_threads</code>. This should never exceed the number of cores on the computer.

The speed-up obtained when using multiple threads is highly problem and hardware dependent, and consequently, it is advisable to compare single threaded and multi threaded performance for the given problem type to determine the optimal settings. For small problems, using multiple threads is not be worthwhile and may even be counter productive because of the additional coordination overhead. Therefore, it may be advantageous to disable multithreading using the parameter <code>iparam.intpnt\_multi\_thread</code>.

The interior-point optimizer parallelizes big tasks such linear algebra computations.

### **Thread Safety**

The **MOSEK** API is thread-safe provided that a task is only modified or accessed from one thread at any given time. Also accessing two or more separate tasks from threads at the same time is safe. Sharing an environment between threads is safe.

#### **Determinism**

The optimizers are run-to-run deterministic which means if a problem is solved twice on the same computer using the same parameter setting and exactly the same input then exactly the same results is obtained. One restriction is that no time limits must be imposed because the time taken to perform an operation on a computer is dependent on many factors such as the current workload.

# 13.3 Linear Optimization

## 13.3.1 Optimizer Selection

Two different types of optimizers are available for linear problems: The default is an interior-point method, and the alternative is the simplex method (primal or dual). The optimizer can be selected using the parameter *iparam.optimizer*.

#### The Interior-point or the Simplex Optimizer?

Given a linear optimization problem, which optimizer is the best: the simplex or the interior-point optimizer? It is impossible to provide a general answer to this question. However, the interior-point optimizer behaves more predictably: it tends to use between 20 and 100 iterations, almost independently of problem size, but cannot perform warm-start. On the other hand the simplex method can take advantage of an initial solution, but is less predictable from cold-start. The interior-point optimizer is used by default.

#### The Primal or the Dual Simplex Variant?

**MOSEK** provides both a primal and a dual simplex optimizer. Predicting which simplex optimizer is faster is impossible, however, in recent years the dual optimizer has seen several algorithmic and computational improvements, which, in our experience, make it faster on average than the primal version. Still, it depends much on the problem structure and size. Setting the *iparam.optimizer* parameter to *optimizertype.free\_simplex* instructs **MOSEK** to choose one of the simplex variants automatically.

To summarize, if you want to know which optimizer is faster for a given problem type, it is best to try all the options.

### 13.3.2 The Interior-point Optimizer

The purpose of this section is to provide information about the algorithm employed in the **MOSEK** interior-point optimizer for linear problems and about its termination criteria.

### The homogeneous primal-dual problem

In order to keep the discussion simple it is assumed that  $\mathbf{MOSEK}$  solves linear optimization problems of standard form

$$\begin{array}{lll} \text{minimize} & c^T x \\ \text{subject to} & Ax & = & b, \\ & x \geq 0. & \end{array} \tag{13.1}$$

This is in fact what happens inside MOSEK; for efficiency reasons MOSEK converts the problem to standard form before solving, then converts it back to the input form when reporting the solution.

Since it is not known beforehand whether problem (13.1) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason why **MOSEK** solves the so-called homogeneous model

$$\begin{array}{rcl}
Ax - b\tau & = & 0, \\
A^{T}y + s - c\tau & = & 0, \\
-c^{T}x + b^{T}y - \kappa & = & 0, \\
x, s, \tau, \kappa & \geq & 0,
\end{array}$$
(13.2)

where y and s correspond to the dual variables in (13.1), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (13.2) always has solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (13.2) satisfies

$$x_i^* s_i^* = 0$$
 and  $\tau^* \kappa^* = 0$ .

Moreover, there is always a solution that has the property  $\tau^* + \kappa^* > 0$ .

First, assume that  $\tau^* > 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left\{ \frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*} \right\}$$

is a primal-dual optimal solution (see Sec. 12.1 for the mathematical background on duality and optimality).

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This implies that at least one of

$$c^T x^* < 0 \tag{13.3}$$

or

$$b^T y^* > 0 (13.4)$$

is satisfied. If (13.3) is satisfied then  $x^*$  is a certificate of dual infeasibility, whereas if (13.4) is satisfied then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

#### **Interior-point Termination Criterion**

For efficiency reasons it is not practical to solve the homogeneous model exactly. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In the k-th iteration of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to homogeneous model is generated, where

$$x^k, s^k, \tau^k, \kappa^k > 0.$$

#### Optimal case

Whenever the trial solution satisfies the criterion

$$\left\| A \frac{x^{k}}{\tau^{k}} - b \right\|_{\infty} \leq \epsilon_{p} (1 + \|b\|_{\infty}),$$

$$\left\| A^{T} \frac{y^{k}}{\tau^{k}} + \frac{s^{k}}{\tau^{k}} - c \right\|_{\infty} \leq \epsilon_{d} (1 + \|c\|_{\infty}), \text{ and}$$

$$\min \left( \frac{(x^{k})^{T} s^{k}}{(\tau^{k})^{2}}, \left| \frac{c^{T} x^{k}}{\tau^{k}} - \frac{b^{T} y^{k}}{\tau^{k}} \right| \right) \leq \epsilon_{g} \max \left( 1, \frac{\min(\left| c^{T} x^{k} \right|, \left| b^{T} y^{k} \right|)}{\tau^{k}} \right), \tag{13.5}$$

the interior-point optimizer is terminated and

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is reported as the primal-dual optimal solution. The interpretation of (13.5) is that the optimizer is terminated if

- $\frac{x^k}{\tau^k}$  is approximately primal feasible,
- $\left\{\frac{y^k}{\tau^k}, \frac{s^k}{\tau^k}\right\}$  is approximately dual feasible, and
- the duality gap is almost zero.

### **Dual infeasibility certificate**

On the other hand, if the trial solution satisfies

$$-\epsilon_{i}c^{T}x^{k} > \frac{\|c\|_{\infty}}{\max\left(1, \|b\|_{\infty}\right)} \|Ax^{k}\|_{\infty}$$

then the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows: First assume that  $\|Ax^k\|_{\infty} = 0$ ; then  $x^k$  is an exact certificate of dual infeasibility. Next assume that this is not the case, i.e.

$$||Ax^k||_{\infty} > 0,$$

and define

$$\bar{x} := \epsilon_i \frac{\max(1, \|b\|_{\infty})}{\|Ax^k\|_{\infty} \|c\|_{\infty}} x^k.$$

It is easy to verify that

$$\|A\bar{x}\|_{\infty} = \epsilon_i \frac{\max\left(1, \|b\|_{\infty}\right)}{\|c\|_{\infty}} \text{ and } -c^T\bar{x} > 1,$$

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation. A smaller value means a better approximation.

#### Primal infeasibility certificate

Finally, if

$$\epsilon_i b^T y^k > \frac{\|b\|_{\infty}}{\max\left(1, \|c\|_{\infty}\right)} \left\|A^T y^k + s^k\right\|_{\infty}$$

then  $y^k$  is reported as a certificate of primal infeasibility.

#### Adjusting optimality criteria and near optimality

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$  using parameters; see table for details.

Table 13.1: Parameters employed in termination criterion

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (13.5) reveals that the quality of the solution depends on  $\|b\|_{\infty}$  and  $\|c\|_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

In some cases the interior-point method terminates having found a solution not too far from meeting the optimality condition (13.5). A solution is defined as  $near\ optimal$  if scaling the termination tolerances  $\varepsilon_p,\ \varepsilon_d,\ \varepsilon_g$  and  $\varepsilon_g$  by the same factor  $\varepsilon_n\in[1.0,+\infty]$  makes the condition (13.5) satisfied. A near optimal solution is therefore of lower quality but still potentially valuable. If for instance the solver stalls, i.e. it can make no more significant progress towards the optimal solution, a near optimal solution could be available and be good enough for the user. Near infeasibility certificates are defined similarly. The value of  $\varepsilon_n$  can be adjusted with the parameter  $dparam.intpnt\_co\_tol\_near\_rel$ .

The basis identification discussed in Sec. 13.3.2 requires an optimal solution to work well; hence basis identification should be turned off if the termination criterion is relaxed.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

#### **Basis Identification**

An interior-point optimizer does not return an optimal basic solution unless the problem has a unique primal and dual optimal solution. Therefore, the interior-point optimizer has an optimal post-processing step that computes an optimal basic solution starting from the optimal interior-point solution. More information about the basis identification procedure may be found in [AY96]. In the following we provide an overall idea of the procedure.

There are some cases in which a basic solution could be more valuable:

- a basic solution is often more accurate than an interior-point solution,
- a basic solution can be used to warm-start the simplex algorithm in case of reoptimization,
- a basic solution is in general more sparse, i.e. more variables are fixed to zero. This is particularly appealing when solving continuous relaxations of mixed integer problems, as well as in all applications in which sparser solutions are preferred.

To illustrate how the basis identification routine works, we use the following trivial example:

$$\begin{array}{lll} \text{minimize} & x+y \\ \text{subject to} & x+y & = & 1, \\ & x,y \geq 0. & \end{array}$$

It is easy to see that all feasible solutions are also optimal. In particular, there are two basic solutions, namely

$$\begin{array}{rcl} (x_1^*,y_1^*) & = & (1,0), \\ (x_2^*,y_2^*) & = & (0,1). \end{array}$$

The interior point algorithm will actually converge to the center of the optimal set, i.e. to  $(x^*, y^*) = (1/2, 1/2)$  (to see this in **MOSEK** deactivate *Presolve*).

In practice, when the algorithm gets close to the optimal solution, it is possible to construct in polynomial time an initial basis for the simplex algorithm from the current interior point solution. This basis is used to warm-start the simplex algorithm that will provide the optimal basic solution. In most cases the constructed basis is optimal, or very few iterations are required by the simplex algorithm to make it optimal and hence the final *clean-up* phase be short. However, for some cases of ill-conditioned problems the additional simplex clean up phase may take of lot a time.

By default **MOSEK** performs a basis identification. However, if a basic solution is not needed, the basis identification procedure can be turned off. The parameters

- iparam.intpnt\_basis,
- iparam.bi\_iqnore\_max\_iter, and
- iparam.bi\_ignore\_num\_error

control when basis identification is performed.

The type of simplex algorithm to be used (primal/dual) can be tuned with the parameter <code>iparam.bi\_clean\_optimizer</code>, and the maximum number of iterations can be set with <code>iparam.bi\_max\_iterations</code>.

Finally, it should be mentioned that there is no guarantee on which basic solution will be returned.

### The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer - threads
                                  : 1
Optimizer - solved problem
                                  : the dual
Optimizer - Constraints
                                  : 2
Optimizer - Cones
                                  : 0
Optimizer - Scalar variables
                                 : 6
                                                     conic
                                                                            : 0
Optimizer - Semi-definite variables: 0
                                                     scalarized
                                                                            : 0
Factor
          - setup time
                                 : 0.00
                                                     dense det. time
                                                                            : 0.00
Factor
          - ML order time
                                  : 0.00
                                                     GP order time
                                                                            : 0.00
          - nonzeros before factor : 3
                                                      after factor
                                                                            : 3
Factor
Factor
          - dense dim.
                              : 0
                                                      flops
                                                                            : 7.00e+001
ITE PFEAS
           DFEAS GFEAS PRSTATUS
                                        POBJ
                                                         DOBJ
                                                                           MU
  1.0e+000 8.6e+000 6.1e+000 1.00e+000 0.000000000e+000 -2.208000000e+003 1.0e+000 0.00
   1.1e+000\ 2.5e+000\ 1.6e-001\ 0.00e+000\ -7.901380925e+003\ -7.394611417e+003\ 2.5e+000\ 0.00e+000
   1.4e-001 3.4e-001 2.1e-002 8.36e-001 -8.113031650e+003 -8.055866001e+003 3.3e-001 0.00
   2.4e-002 5.8e-002 3.6e-003 1.27e+000 -7.777530698e+003 -7.766471080e+003 5.7e-002 0.01
  1.3e-004 3.2e-004 2.0e-005 1.08e+000 -7.668323435e+003 -7.668207177e+003 3.2e-004 0.01
   1.3e-008 3.2e-008 2.0e-009 1.00e+000 -7.668000027e+003 -7.668000015e+003 3.2e-008 0.01
  1.3e-012 3.2e-012 2.0e-013 1.00e+000 -7.667999994e+003 -7.667999994e+003 3.2e-012 0.01
```

The first line displays the number of threads used by the optimizer and the second line tells that the optimizer chose to solve the dual problem rather than the primal problem. The next line displays the

problem dimensions as seen by the optimizer, and the Factor... lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 13.3.2 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $\|A^Ty^k + s^k c\tau^k\|_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to -1 if that is not the case.
- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- $\bullet$  MU:  $\frac{(x^k)^Ts^k+\tau^k\kappa^k}{n+1}$  . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started.

## 13.3.3 The Simplex Optimizer

An alternative to the interior-point optimizer is the simplex optimizer. The simplex optimizer uses a different method that allows exploiting an initial guess for the optimal solution to reduce the solution time. Depending on the problem it may be faster or slower to use an initial guess; see Sec. 13.3.1 for a discussion. **MOSEK** provides both a primal and a dual variant of the simplex optimizer.

#### **Simplex Termination Criterion**

The simplex optimizer terminates when it finds an optimal basic solution or an infeasibility certificate. A basic solution is optimal when it is primal and dual feasible; see Sec. 12.1 for a definition of the primal and dual problem. Due to the fact that computations are performed in finite precision **MOSEK** allows violations of primal and dual feasibility within certain tolerances. The user can control the allowed primal and dual tolerances with the parameters <code>dparam.basis\_tol\_x</code> and <code>dparam.basis\_tol\_s</code>.

Setting the parameter *iparam.optimizer* to *optimizertype.free\_simplex* instructs **MOSEK** to select automatically between the primal and the dual simplex optimizers. Hence, **MOSEK** tries to choose the best optimizer for the given problem and the available solution. The same parameter can also be used to force one of the variants.

#### Starting From an Existing Solution

When using the simplex optimizer it may be possible to reuse an existing solution and thereby reduce the solution time significantly. When a simplex optimizer starts from an existing solution it is said to perform a *warm-start*. If the user is solving a sequence of optimization problems by solving the problem, making modifications, and solving again, **MOSEK** will warm-start automatically.

By default **MOSEK** uses presolve when performing a warm-start. If the optimizer only needs very few iterations to find the optimal solution it may be better to turn off the presolve.

#### Numerical Difficulties in the Simplex Optimizers

Though MOSEK is designed to minimize numerical instability, completely avoiding it is impossible when working in finite precision. MOSEK treats a "numerically unexpected behavior" event inside the optimizer as a *set-back*. The user can define how many set-backs the optimizer accepts; if that number is exceeded, the optimization will be aborted. Set-backs are a way to escape long sequences where the optimizer tries to recover from an unstable situation.

Examples of set-backs are: repeated singularities when factorizing the basis matrix, repeated loss of feasibility, degeneracy problems (no progress in objective) and other events indicating numerical difficulties. If the simplex optimizer encounters a lot of set-backs the problem is usually badly scaled; in such a situation try to reformulate it into a better scaled problem. Then, if a lot of set-backs still occur, trying one or more of the following suggestions may be worthwhile:

- Raise tolerances for allowed primal or dual feasibility: increase the value of
  - dparam.basis\_tol\_x, and
  - $dparam.basis_tol_s.$
- Raise or lower pivot tolerance: Change the dparam.simplex\_abs\_tol\_piv parameter.
- Switch optimizer: Try another optimizer.
- Switch off crash: Set both iparam.sim\_primal\_crash and iparam.sim\_dual\_crash to 0.
- Experiment with other pricing strategies: Try different values for the parameters
  - iparam.sim\_primal\_selection and
  - iparam.sim\_dual\_selection.
- If you are using warm-starts, in rare cases switching off this feature may improve stability. This is controlled by the <code>iparam.sim\_hotstart</code> parameter.
- Increase maximum number of set-backs allowed controlled by <code>iparam.sim\_max\_num\_setbacks</code>.
- If the problem repeatedly becomes infeasible try switching off the special degeneracy handling. See the parameter <code>iparam.sim\_degen</code> for details.

#### The Simplex Log

Below is a typical log output from the simplex optimizer:

Optimi Optimi	-		: the pr	rimal			
Optimi	izer - Scalar v	ariables	: 1424	conic		: 0	
Optimi	izer - hotstart	;	: no				
ITER	DEGITER(%)	PFEAS	DFEAS	POBJ	DOBJ		TIME
$\hookrightarrow$	TOTTIME						
0	0.00	1.43e+05	NA	6.5584140832e+03	NA		0.00 <sub>L</sub>
$\hookrightarrow$	0.02						
1000	1.10	0.00e+00	NA	1.4588289726e+04	NA		0.13 <mark>⊔</mark>
$\hookrightarrow$	0.14						
2000	0.75	0.00e+00	NA	7.3705564855e+03	NA		0.21 <sub>L</sub>
$\hookrightarrow$	0.22						
3000	0.67	0.00e+00	NA	6.0509727712e+03	NA		0.29 <sub>L</sub>
$\hookrightarrow$	0.31						
4000	0.52	0.00e+00	NA	5.5771203906e+03	NA		0.38 <sub>L</sub>
$\hookrightarrow$	0.39						
4533	0.49	0.00e+00	NA	5.5018458883e+03	NA		0.42 <mark>u</mark>
$\hookrightarrow$	0.44						

The first lines summarize the problem the optimizer is solving. This is followed by the iteration log, with the following meaning:

- ITER: Number of iterations.
- DEGITER(%): Ratio of degenerate iterations.
- PFEAS: Primal feasibility measure reported by the simplex optimizer. The numbers should be 0 if the problem is primal feasible (when the primal variant is used).
- DFEAS: Dual feasibility measure reported by the simplex optimizer. The number should be 0 if the problem is dual feasible (when the dual variant is used).
- POBJ: An estimate for the primal objective value (when the primal variant is used).
- DOBJ: An estimate for the dual objective value (when the dual variant is used).
- TIME: Time spent since this instance of the simplex optimizer was invoked (in seconds).
- TOTTIME: Time spent since optimization started (in seconds).

# 13.4 Conic Optimization

For conic optimization problems only an interior-point type optimizer is available.

### 13.4.1 The Interior-point optimizer

#### The homogeneous primal-dual problem

The interior-point optimizer is an implementation of the so-called homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [ART03]. In order to keep our discussion simple we will assume that **MOSEK** solves a conic optimization problem of the form:

minimize 
$$c^T x$$
  
subject to  $Ax = b$ ,  $x \in \mathcal{K}$  (13.6)

where K is a convex cone. The corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & b^T y \\ \text{subject to} & A^T y + s & = & c, \\ & x \in \mathcal{K}^* \end{array} \tag{13.7}$$

where  $\mathcal{K}^*$  is the dual cone of  $\mathcal{K}$ . See Sec. 12.2 for definitions.

Since it is not known beforehand whether problem (13.6) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason that **MOSEK** solves the so-called homogeneous model

$$Ax - b\tau = 0,$$

$$A^{T}y + s - c\tau = 0,$$

$$-c^{T}x + b^{T}y - \kappa = 0,$$

$$x \in \mathcal{K},$$

$$s \in \mathcal{K}^{*},$$

$$\tau, \kappa \geq 0,$$

$$(13.8)$$

where y and s correspond to the dual variables in (13.6), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (13.8) always has a solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*,y^*,s^*,\tau^*,\kappa^*)$$

to the homogeneous model (13.8) satisfies

$$(x^*)^T s^* + \tau^* \kappa^* = 0$$

i.e. complementarity. Observe that  $x^* \in \mathcal{K}$  and  $s^* \in \mathcal{K}^*$  implies

$$(x^*)^T s^* \ge 0$$

and therefore

$$\tau^* \kappa^* = 0$$

since  $\tau^*, \kappa^* \geq 0$ . Hence, at least one of  $\tau^*$  and  $\kappa^*$  is zero.

First, assume that  $\tau^* > 0$  and hence  $\kappa^* = 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*/\tau^* & \in & \mathcal{K}, \\ s^*/\tau^* & \in & \mathcal{K}^*. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left(\frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*}\right)$$

is a primal-dual optimal solution.

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^* & \in & \mathcal{K}, \\ s^* & \in & \mathcal{K}^* \end{array}$$

This implies that at least one of

$$c^T x^* < 0 \tag{13.9}$$

or

$$b^T y^* > 0 (13.10)$$

holds. If (13.9) is satisfied, then  $x^*$  is a certificate of dual infeasibility, whereas if (13.10) holds then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

## **Interior-point Termination Criterion**

Since computations are performed in finite precision, and for efficiency reasons, it is not possible to solve the homogeneous model exactly in general. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In every iteration k of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to the homogeneous model is generated, where

$$x^k \in \mathcal{K}, s^k \in \mathcal{K}^*, \tau^k, \kappa^k > 0.$$

Therefore, it is possible to compute the values:

$$\begin{array}{lll} \rho_p^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \leq \rho \varepsilon_p (1 + \|b\|_{\infty}) \right\}, \\ \rho_d^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T \frac{y^k}{\tau^k} + \frac{s^k}{\tau^k} - c \right\|_{\infty} \leq \rho \varepsilon_d (1 + \|c\|_{\infty}) \right\}, \\ \rho_g^k &=& \arg\min_{\rho} \left\{ \rho \mid \left( \frac{(x^k)^T s^k}{(\tau^k)^2}, \left| \frac{c^T x^k}{\tau^k} - \frac{b^T y^k}{\tau^k} \right| \right) \leq \rho \varepsilon_g \max \left( 1, \frac{\min\left(\left| c^T x^k \right|, \left| b^T y^k \right| \right)}{\tau^k} \right) \right\}, \\ \rho_{pi}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T y^k + s^k \right\|_{\infty} \leq \rho \varepsilon_i b^T y^k, \, b^T y^k > 0 \right\} \text{ and } \\ \rho_{di}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A x^k \right\|_{\infty} \leq -\rho \varepsilon_i c^T x^k, \, c^T x^k < 0 \right\}. \end{array}$$

Note  $\varepsilon_p, \varepsilon_d, \varepsilon_q$  and  $\varepsilon_i$  are nonnegative user specified tolerances.

#### **Optimal Case**

Observe  $\rho_p^k$  measures how far  $x^k/\tau^k$  is from being a good approximate primal feasible solution. Indeed if  $\rho_p^k \leq 1$ , then

$$\left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \le \varepsilon_p (1 + \|b\|_{\infty}). \tag{13.11}$$

This shows the violations in the primal equality constraints for the solution  $x^k/\tau^k$  is small compared to the size of b given  $\varepsilon_p$  is small.

Similarly, if  $\rho_d^k \leq 1$ , then  $(y^k, s^k)/\tau^k$  is an approximate dual feasible solution. If in addition  $\rho_g \leq 1$ , then the solution  $(x^k, y^k, s^k)/\tau^k$  is approximate optimal because the associated primal and dual objective values are almost identical.

In other words if  $\max(\rho_p^k, \rho_d^k, \rho_q^k) \leq 1$ , then

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is an approximate optimal solution.

#### **Dual Infeasibility Certificate**

Next assume that  $\rho_{di}^k \leq 1$  and hence

$$||Ax^k||_{\infty} \le -\varepsilon_i c^T x^k$$
 and  $-c^T x^k > 0$ 

holds. Now in this case the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{x} := \frac{x^k}{-c^T x^k}$$

and it is easy to verify that

$$||A\bar{x}||_{\infty} \leq \varepsilon_i$$
 and  $c^T\bar{x} = -1$ 

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

### **Primal Infeasiblity Certificate**

Next assume that  $\rho_{pi}^k \leq 1$  and hence

$$||A^T y^k + s^k||_{\infty} \le \varepsilon_i b^T y^k \text{ and } b^T y^k > 0$$

holds. Now in this case the problem is declared primal infeasible and  $(y^k, s^k)$  is reported as a certificate of primal infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{y} := \frac{y^k}{b^T y^k}$$
 and  $\bar{s} := \frac{s^k}{b^T y^k}$ 

and it is easy to verify that

$$||A^T \bar{y} + \bar{s}||_{\infty} \le \varepsilon_i \text{ and } b^T \bar{y} = 1$$

which shows  $(y^k, s^k)$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

#### Adjusting optimality criteria and near optimality

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_q$  and  $\varepsilon_i$  using parameters; see table for details.

Table 13.2: Parameters employed in termination criterion

ToleranceParameter	name		
$arepsilon_p$	$dparam.intpnt\_co\_tol\_pfeas$		
$arepsilon_d$	$dparam.intpnt\_co\_tol\_dfeas$		
$arepsilon_g$	$dparam.intpnt\_co\_tol\_rel\_gap$		
$\varepsilon_i$	$dparam.intpnt\_co\_tol\_infeas$		

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (13.11) reveals that the quality of the solution depends on  $||b||_{\infty}$  and  $||c||_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_q$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

In some cases the interior-point method terminates having found a solution not too far from meeting the optimality condition (13.11). A solution is defined as near optimal if scaling the termination tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_g$  by the same factor  $\varepsilon_n \in [1.0, +\infty]$  makes the condition (13.11) satisfied. A near optimal solution is therefore of lower quality but still potentially valuable. If for instance the solver stalls, i.e. it can make no more significant progress towards the optimal solution, a near optimal solution could be available and be good enough for the user. Near infeasibility certificates are defined similarly. The value of  $\varepsilon_n$  can be adjusted with the parameter dparam.intpnt\_co\_tol\_near\_rel.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

## The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer - threads : 20
Optimizer - solved problem : the primal
Optimizer - Constraints : 1
Optimizer - Cones : 2
```

```
Optimizer - Scalar variables
                                   : 6
                                                       conic
          - Semi-definite variables: 0
Optimizer
                                                       scalarized
                                                                              : 0
                                                       dense det. time
          - setup time
                                   : 0.00
                                                                              : 0.00
Factor
Factor
          - ML order time
                                   : 0.00
                                                       GP order time
                                                                              : 0.00
Factor
          - nonzeros before factor : 1
                                                       after factor
                                                                              : 1
                                                                              : 1.70e+01
Factor
          - dense dim.
                                   : 0
                                                       flops
ITE PFEAS
            DFEAS
                   GFEAS
                              PRSTATUS
                                         POBJ
                                                           DOBJ
                                                                             MU
                                                                                      TIME
   1.0e+00 2.9e-01 3.4e+00 0.00e+00
                                         2.414213562e+00
                                                           0.000000000e+00
                                                                             1.0e+00
                                                                                      0.01
1
   2.7e-01 7.9e-02 2.2e+00 8.83e-01
                                         6.969257574e-01
                                                           -9.685901771e-03 2.7e-01
   6.5e-02 1.9e-02 1.2e+00 1.16e+00
                                         7.606090061e-01
                                                           6.046141322e-01
                                                                             6.5e-02
   1.7e-03 5.0e-04 2.2e-01 1.12e+00
                                         7.084385672e-01
                                                           7.045122560e-01
                                                                             1.7e-03
   1.4e-08 4.2e-09
                     4.9e-08
                             1.00e+00
                                         7.071067941e-01
                                                           7.071067599e-01
                                                                             1.4e-08
```

The first line displays the number of threads used by the optimizer and the second line tells that the optimizer chose to solve the dual problem rather than the primal problem. The next line displays the problem dimensions as seen by the optimizer, and the Factor... lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 13.4.1 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $\|A^Ty^k + s^k c\tau^k\|_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to −1 if that is not the case.
- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- MU:  $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$ . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started (in seconds).

# 13.5 Nonlinear Convex Optimization

## 13.5.1 The Interior-point Optimizer

For general convex optimization problems an interior-point type optimizer is available. The interior-point optimizer is an implementation of the homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [AY98], [AY99].

### The Convexity Requirement

Continuous nonlinear problems are required to be convex. For quadratic problems **MOSEK** tests this requirement before optimizing. Specifying a non-convex problem results in an error message.

The following parameters are available to control the convexity check:

- iparam.check\_convexity: Turn convexity check on/off.
- dparam.check\_convexity\_rel\_tol: Tolerance for convexity check.
- iparam. log\_check\_convexity: Turn on more log information for debugging.

#### The Differentiability Requirement

The nonlinear optimizer in **MOSEK** requires both first order and second order derivatives. This of course implies care should be taken when solving problems involving non-differentiable functions.

For instance, the function

$$f(x) = x^2$$

is differentiable everywhere whereas the function

$$f(x) = \sqrt{x}$$

is only differentiable for x>0. In order to make sure that  $\mathbf{MOSEK}$  evaluates the functions at points where they are differentiable, the function domains must be defined by setting appropriate variable bounds.

In general, if a variable is not ranged  $\mathbf{MOSEK}$  will only evaluate that variable at points strictly within the bounds. Hence, imposing the bound

$$x \ge 0$$

in the case of  $\sqrt{x}$  is sufficient to guarantee that the function will only be evaluated in points where it is differentiable.

However, if a function is defined on a closed range, specifying the variable bounds is not sufficient. Consider the function

$$f(x) = \frac{1}{x} + \frac{1}{1 - x}. (13.12)$$

In this case the bounds

will not guarantee that  $\mathbf{MOSEK}$  only evaluates the function for x strictly between 0 and 1. To force  $\mathbf{MOSEK}$  to strictly satisfy both bounds on ranged variables set the parameter  $iparam.intpnt\_starting\_point$  to  $startpointtype.satisfy\_bounds$ .

For efficiency reasons it may be better to reformulate the problem than to force **MOSEK** to observe ranged bounds strictly. For instance, (13.12) can be reformulated as follows

$$\begin{array}{rcl} f(x) & = & \frac{1}{x} + \frac{1}{y} \\ 0 & = & 1 - x - y \\ 0 & \leq & x \\ 0 & \leq & y. \end{array}$$

### Interior-point Termination Criteria

The parameters controlling when the general convex interior-point optimizer terminates are shown in Table 13.3.

Table 13.3: Parameters employed in termination criteria.

Parameter name	Purpose
$dparam.intpnt\_nl\_tol\_pfeas$	Controls primal feasibility
$dparam.intpnt\_nl\_tol\_dfeas$	Controls dual feasibility
$dparam.intpnt\_nl\_tol\_rel\_gap$	Controls relative gap
$dparam.intpnt\_tol\_infeas$	Controls when the problem is declared infeasible
$dparam.intpnt_nl_tol_mu_red$	Controls when the complementarity is reduced enough

## THE OPTIMIZER FOR MIXED-INTEGER PROBLEMS

A problem is a mixed-integer optimization problem when one or more of the variables are constrained to be integer valued. Readers unfamiliar with integer optimization are recommended to consult some relevant literature, e.g. the book [Wol98] by Wolsey.

# 14.1 The Mixed-integer Optimizer Overview

MOSEK can solve mixed-integer

- linear,
- quadratic and quadratically constrained, and
- conic quadratic

problems, at least as long as they do not contain both quadratic objective or constraints and conic constraints at the same time. The mixed-integer optimizer is specialized for solving linear and conic optimization problems. Pure quadratic and quadratically constrained problems are automatically converted to conic form.

By default the mixed-integer optimizer is run-to-run deterministic. This means that if a problem is solved twice on the same computer with identical parameter settings and no time limit then the obtained solutions will be identical. If a time limit is set then this may not be case since the time taken to solve a problem is not deterministic. The mixed-integer optimizer is parallelized i.e. it can exploit multiple cores during the optimization.

The solution process can be split into these phases:

- 1. Presolve: See Sec. 13.1.
- 2. Cut generation: Valid inequalities (cuts) are added to improve the lower bound.
- 3. **Heuristic:** Using heuristics the optimizer tries to guess a good feasible solution. Heuristics can be controlled by the parameter <code>iparam.mio\_heuristic\_level</code>.
- 4. Search: The optimal solution is located by branching on integer variables.

## 14.2 Relaxations and bounds

It is important to understand that, in a worst-case scenario, the time required to solve integer optimization problems grows exponentially with the size of the problem (solving mixed-integer problems is NP-hard). For instance, a problem with n binary variables, may require time proportional to  $2^n$ . The value of  $2^n$  is huge even for moderate values of n.

In practice this implies that the focus should be on computing a near-optimal solution quickly rather than on locating an optimal solution. Even if the problem is only solved approximately, it is important to know how far the approximate solution is from an optimal one. In order to say something about the quality of an approximate solution the concept of *relaxation* is important.

Consider for example a mixed-integer optimization problem

$$z^* = \underset{\text{subject to}}{\text{minimize}} c^T x$$

$$subject to \quad Ax = b,$$

$$x \ge 0$$

$$x_j \in \mathbb{Z}, \qquad \forall j \in \mathcal{J}.$$

$$(14.1)$$

It has the continuous relaxation

obtained simply by ignoring the integrality restrictions. The relaxation is a continuous problem, and therefore much faster to solve to optimality with a linear (or, in the general case, conic) optimizer. We call the optimal value  $\underline{z}$  the *objective bound*. The objective bound  $\underline{z}$  normally increases during the solution search process when the continuous relaxation is gradually refined.

Moreover, if  $\hat{x}$  is any feasible solution to (14.1) and

$$\bar{z} := c^T \hat{x}$$

then

$$z < z^* < \bar{z}.$$

These two inequalities allow us to estimate the quality of the integer solution: it is no further away from the optimum than  $\bar{z} - \underline{z}$  in terms of the objective value. Whenever a mixed-integer problem is solved **MOSEK** reports this lower bound so that the quality of the reported solution can be evaluated.

## 14.3 Termination Criterion

In general, it is time consuming to find an exact feasible and optimal solution to an integer optimization problem, though in many practical cases it may be possible to find a sufficiently good solution. The issue of terminating the mixed-integer optimizer is rather delicate and the user has numerous possibilities of influencing it with various parameters. The mixed-integer optimizer employs a relaxed feasibility and optimality criterion to determine when a satisfactory solution is located.

A candidate solution that is feasible for the continuous relaxation is said to be an *integer feasible solution* if the criterion

$$\min(x_i - |x_i|, \lceil x_i \rceil - x_i) \le \delta_1 \quad \forall j \in \mathcal{J}$$

is satisfied, meaning that  $x_i$  is at most  $\delta_1$  from the nearest integer.

Whenever the integer optimizer locates an integer feasible solution it will check if the criterion

$$\bar{z} - \underline{z} \le \max(\delta_2, \delta_3 \max(10^{-10}, |\bar{z}|))$$

is satisfied. If this is the case, the integer optimizer terminates and reports the integer feasible solution as an optimal solution. If an optimal solution cannot be located after the time specified by the parameter <code>dparam.mio\_disable\_term\_time</code> (in seconds), it may be advantageous to relax the termination criteria, and they become replaced with

$$\bar{z} - \underline{z} \le \max(\delta_4, \delta_5 \max(10^{-10}, |\bar{z}|)).$$

Any solution satisfying those will now be reported as **near optimal** and the solver will be terminated (note that since this criterion depends on timing, the optimizer will not be run to run deterministic).

All the  $\delta$  tolerances discussed above can be adjusted using suitable parameters — see Table 14.1.

Table 14.1: Tolerances for the mixed-integer optimizer.

Tolerance	Parameter name
$\delta_1$	$dparam.mio\_tol\_abs\_relax\_int$
$\delta_2$	$dparam.mio\_tol\_abs\_gap$
$\delta_3$	$dparam.mio\_tol\_rel\_gap$
$\delta_4$	$dparam.mio\_near\_tol\_abs\_gap$
$\delta_5$	$dparam.mio\_near\_tol\_rel\_gap$

In Table 14.2 some other common parameters affecting the integer optimizer termination criterion are shown. Please note that if the effect of a parameter is delayed, the associated termination criterion is applied only after some time, specified by the <code>dparam.mio\_disable\_term\_time</code> parameter.

Table 14.2: Other parameters affecting the integer optimizer termination criterion.

Parameter name	De-	Explanation
	layed	
iparam.mio_max_num_branches	Yes	Maximum number of branches allowed.
iparam.mio_max_num_relaxs	Yes	Maximum number of relaxations allowed.
iparam.	Yes	Maximum number of feasible integer solutions allowed.
mio_max_num_solutions		

# 14.4 Speeding Up the Solution Process

As mentioned previously, in many cases it is not possible to find an optimal solution to an integer optimization problem in a reasonable amount of time. Some suggestions to reduce the solution time are:

- Relax the termination criterion: In case the run time is not acceptable, the first thing to do is to relax the termination criterion see Sec. 14.3 for details.
- Specify a good initial solution: In many cases a good feasible solution is either known or easily computed using problem-specific knowledge. If a good feasible solution is known, it is usually worthwhile to use this as a starting point for the integer optimizer.
- Improve the formulation: A mixed-integer optimization problem may be impossible to solve in one form and quite easy in another form. However, it is beyond the scope of this manual to discuss good formulations for mixed-integer problems. For discussions on this topic see for example [Wol98].

# 14.5 Understanding Solution Quality

To determine the quality of the solution one should check the following:

- The problem status and solution status returned by MOSEK, as well as constraint violations in case of suboptimal solutions.
- ullet The  $optimality\ gap$  defined as

$$\epsilon = |(\text{objective value of feasible solution}) - (\text{objective bound})| = |\bar{z} - \underline{z}|.$$

which measures how much the located solution can deviate from the optimal solution to the problem. The optimality gap can be retrieved through the information item dinfitem.  $mio\_obj\_abs\_gap$ . Often it is more meaningful to look at the relative optimality gap normalized against the magnitude of the solution.

$$\epsilon_{\rm rel} = \frac{|\bar{z} - \underline{z}|}{\max(10^{-10}, |\bar{z}|)}.$$

The relative optimality gap is available in  $dinfitem.mio\_obj\_rel\_gap$ .

# 14.6 The Optimizer Log

Below is a typical log output from the mixed-integer optimizer:

	-			, 35728 constraints, 3 ger, 4294 binary, 2279			
	e table si	-	rar ince	ger, 4294 Dinary, 227	9 Continuous		
-	HES RELAXS		DEPTH	BEST_INT_OBJ	BEST_RELAX_OBJ	REL_GAP(%)	TIME
0	1 0 0		NA	1.8218819866e+07	NA	1.6	
0	1	0	0	1.8331557950e+07	1.8218819866e+07	0.61	3.5
0	1	0	0	1.8300507546e+07	1.8218819866e+07	0.45	4.3
-	eneration	started.	· ·	2.000000.0100 0.	110210010000	0.7.10	
0	2	0	0	1.8300507546e+07	1.8218819866e+07	0.45	5.3
Cut ge	eneration	terminated	. Time =	1.43			
0	3	0	0	1.8286893047e+07	1.8231580587e+07	0.30	7.5
15	18	1	0	1.8286893047e+07	1.8231580587e+07	0.30	10.5
31	34	1	0	1.8286893047e+07	1.8231580587e+07	0.30	11.1
51	54	1	0	1.8286893047e+07	1.8231580587e+07	0.30	11.6
91	94	1	0	1.8286893047e+07	1.8231580587e+07	0.30	12.4
171	174	1	0	1.8286893047e+07	1.8231580587e+07	0.30	14.3
331	334	1	0	1.8286893047e+07	1.8231580587e+07	0.30	17.9
[	]						
Object	tive of be	st integer	solution	n : 1.825846762609e+0	7		
-	objective	_		: 1.823311032986e+0			
	•	ion object:	ive	: Not employed			
		ion # roun		: 0			
User objective cut value		Ü	: 0				
Number	r of cuts	generated		: 117			
Number of Gomory cuts				: 108			
Number of CMIR cuts				: 9			
Number of branches				: 4425			
Number of relaxations solved			ved	: 4410			
Number	r of inter	ior point	iteration	ns: 25			
		ex iterati		: 221131			

The first lines contain a summary of the problem as seen by the optimizer. This is followed by the iteration log. The columns have the following meaning:

- BRANCHES: Number of branches generated.
- RELAXS: Number of relaxations solved.
- ACT\_NDS: Number of active branch bound nodes.
- DEPTH: Depth of the recently solved node.
- $\bullet$  BEST\_INT\_OBJ: The best integer objective value,  $\bar{z}.$
- BEST\_RELAX\_OBJ: The best objective bound,  $\underline{z}$ .
- REL\_GAP(%): Relative optimality gap,  $100\% \cdot \epsilon_{\rm rel}$
- TIME: Time (in seconds) from the start of optimization.

Following that a summary of the optimization process is printed.

## ADDITIONAL FEATURES

In this section we describe additional features and tools which enable more detailed analysis of optimization problems with  $\mathbf{MOSEK}$ .

## 15.1 Problem Analyzer

The problem analyzer prints a detailed survey of the

- linear constraints and objective
- quadratic constraints
- conic constraints
- variables

of the model.

In the initial stages of model formulation the problem analyzer may be used as a quick way of verifying that the model has been built or imported correctly. In later stages it can help revealing special structures within the model that may be used to tune the optimizer's performance or to identify the causes of numerical difficulties.

The problem analyzer is run using *Task. analyzeproblem*. It produces output similar to the one below (this is the problem survey of the aflow30a problem from the MIPLIB 2003 collection).

```
Analyzing the problem
Constraints
                         Bounds
                                                   Variables
upper bd:
                421
                          ranged : all
                                                    cont:
                                                                421
fixed
                 58
                                                    bin :
                                                                 421
Objective, min cx
   range: min |c|: 0.00000 min |c|>0: 11.0000
                                                   max |c|: 500.000
distrib:
                |c|
                           vars
                  0
                            421
           [11, 100)
                            150
          [100, 500]
                            271
Constraint matrix A has
      479 rows (constraints)
      842 columns (variables)
     2091 (0.518449%) nonzero entries (coefficients)
Row nonzeros, A_i
  range: min A_i: 2 (0.23753%)
                                  max A_i: 34 (4.038%)
```

distrib:		A_i	rows	rows%	acc%		 <u>.</u>
		2	421	87.89	87.89		
	[8,	15]	20	4.18	92.07		
	[16,	31]	30	6.26	98.33		
	[32,	34]	8	1.67	100.00		
G - 7		A 1 -2					
Column nor		-	(0.447507%)	41. 0	(0.000005%)		
_			(0.417537%)	-		)	
distrib:		1 J	cols	cols%			
		2		51.66			
		3	407	48.34	100.00		
A nonzeros	s. A(ii`	)					
	_		: 1.00000	max  A(ii) .	100.000		
distrib:		-	coeffs	/II(±J//.	100.000		
dibuilb.			1670				
	[10, :						
	L10, .	100]	121				
Constraint	bound	s, lb	<= Ax <= ub				
distrib:		b	lb	s	ubs		
		0			421		
	[1,	10]	5	8	58		
Variable h	oounde	1h <=	= v <= ub				
distrib:	ounus,	b	- x <= ub 1b	e	ubs		
distiid:					นบธ		
	Га	0	84	2	101		
		10)			421		
	[10, 3	100]			421		
	[10,	100]			421		

The survey is divided into six different sections, each described below. To keep the presentation short with focus on key elements. The analyzer generally attempts to display information on issues relevant for the current model only: e.g., if the model does not have any conic constraints (this is the case in the example above) or any integer variables, those parts of the analysis will not appear.

#### **General Characteristics**

The first part of the survey consists of a brief summary of the model's linear and quadratic constraints (indexed by i) and variables (indexed by j). The summary is divided into three subsections:

## Constraints

- upper bd The number of upper bounded constraints,  $\sum_{j=0}^{n-1} a_{ij}x_j \leq u_i^c$
- $\bullet$  lower bd The number of lower bounded constraints,  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j$
- ranged The number of ranged constraints,  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j \leq u_i^c$
- fixed The number of fixed constraints,  $l_i^c = \sum_{j=0}^{n-1} a_{ij} x_j = u_i^c$
- free The number of free constraints

#### Bounds

• upper bd The number of upper bounded variables,  $x_j \leq u_j^x$ 

- ullet lower bd The number of lower bounded variables,  $l_k^x \leq x_j$
- ranged The number of ranged variables,  $l_k^x \leq x_j \leq u_j^x$
- fixed The number of fixed variables,  $l_k^x = x_j = u_i^x$
- free The number of free variables

#### Variables

- cont The number of continuous variables,  $x_i \in \mathbb{R}$
- bin The number of binary variables,  $x_j \in \{0, 1\}$
- int The number of general integer variables,  $x_i \in \mathbb{Z}$

Only constraints, bounds and domains actually in the model will be reported on; if all entities in a section turn out to be of the same kind, the number will be replaced by all for brevity.

## **Objective**

The second part of the survey focuses on (the linear part of) the objective, summarizing the optimization sense and the coefficients' absolute value range and distribution. The number of 0 (zero) coefficients is singled out (if any such variables are in the problem).

The range is displayed using three terms:

- min |c| The minimum absolute value among all coeffecients
- min |c|>0 The minimum absolute value among the nonzero coefficients
- max |c| The maximum absolute value among the coefficients

If some of these extrema turn out to be equal, the display is shortened accordingly:

- $\bullet$  If min |c| is greater than zero, the min |c|>0 term is obsolete and will not be displayed
- If only one or two different coefficients occur this will be displayed using all and an explicit listing of the coefficients

The absolute value distribution is displayed as a table summarizing the numbers by orders of magnitude (with a ratio of 10). Again, the number of variables with a coefficient of 0 (if any) is singled out. Each line of the table is headed by an interval (half-open intervals including their lower bounds), and is followed by the number of variables with their objective coefficient in this interval. Intervals with no elements are skipped.

#### **Linear Constraints**

The third part of the survey displays information on the nonzero coefficients of the linear constraint matrix

Following a brief summary of the matrix dimensions and the number of nonzero coefficients in total, three sections provide further details on how the nonzero coefficients are distributed by row-wise count (A\_i), by column-wise count (A|j), and by absolute value (|A(ij)|). Each section is headed by a brief display of the distribution's range (min and max), and for the row/column-wise counts the corresponding densities are displayed too (in parentheses).

The distribution tables single out three particularly interesting counts: zero, one, and two nonzeros per row/column; the remaining row/column nonzeros are displayed by orders of magnitude (ratio 2). For each interval the relative and accumulated relative counts are also displayed.

Note that constraints may have both linear and quadratic terms, but the empty rows and columns reported in this part of the survey relate to the linear terms only. If empty rows and/or columns are found in the linear constraint matrix, the problem is analyzed further in order to determine if the

corresponding constraints have any quadratic terms or the corresponding variables are used in conic or quadratic constraints.

The distribution of the absolute values, |A(ij)|, is displayed just as for the objective coefficients described above.

#### Constraint and Variable Bounds

The fourth part of the survey displays distributions for the absolute values of the finite lower and upper bounds for both constraints and variables. The number of bounds at 0 is singled out and, otherwise, displayed by orders of magnitude (with a ratio of 10).

#### **Quadratic Constraints**

The fifth part of the survey displays distributions for the nonzero elements in the gradient of the quadratic constraints, i.e. the nonzero row counts for the column vectors Qx. The table is similar to the tables for the linear constraints' nonzero row and column counts described in the survey's third part.

Quadratic constraints may also have a linear part, but that will be included in the linear constraints survey; this means that if a problem has one or more pure quadratic constraints, part three of the survey will report the number of linear constraint rows with 0 (zero) nonzeros. Likewise, variables that appear in quadratic terms only will be reported as empty columns (0 nonzeros) in the linear constraint report.

#### **Conic Constraints**

The last part of the survey summarizes the model's conic constraints. For each of the two types of cones, quadratic and rotated quadratic, the total number of cones are reported, and the distribution of the cones' dimensions are displayed using intervals. Cones dimensions of 2, 3, and 4 are singled out.

# 15.2 Analyzing Infeasible Problems

When developing and implementing a new optimization model, the first attempts will often be either infeasible, due to specification of inconsistent constraints, or unbounded, if important constraints have been left out.

In this section we will

- go over an example demonstrating how to locate infeasible constraints using the **MOSEK** infeasibility report tool,
- discuss in more general terms which properties may cause infeasibilities, and
- present the more formal theory of infeasible and unbounded problems.

## 15.2.1 Example: Primal Infeasibility

A problem is said to be *primal infeasible* if no solution exists that satisfies all the constraints of the problem.

As an example of a primal infeasible problem consider the problem of minimizing the cost of transportation between a number of production plants and stores: Each plant produces a fixed number of goods, and each store has a fixed demand that must be met. Supply, demand and cost of transportation per unit are given in Fig. 15.1.

The problem represented in Fig. 15.1 is infeasible, since the total demand

$$2300 = 1100 + 200 + 500 + 500$$

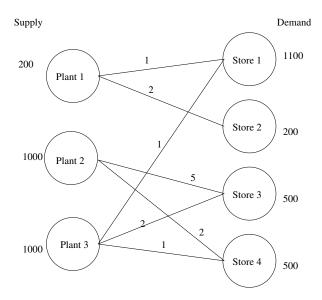


Fig. 15.1: Supply, demand and cost of transportation.

exceeds the total supply

$$2200 = 200 + 1000 + 1000$$

If we denote the number of transported goods from plant i to store j by  $x_{ij}$ , the problem can be formulated as the LP:

minimize 
$$x_{11}$$
 +  $2x_{12}$  +  $5x_{23}$  +  $2x_{24}$  +  $x_{31}$  +  $2x_{33}$  +  $x_{34}$  subject to  $x_{11}$  +  $x_{12}$   $\leq 200$ ,  $\leq 1000$ ,  $\leq 1000$ ,  $x_{23}$  +  $x_{24}$   $\leq 1000$ ,  $x_{31}$  +  $x_{33}$  +  $x_{34}$   $\leq 1000$ ,  $x_{11}$   $= 1100$ ,  $x_{12}$   $= 200$ ,  $x_{23}$  +  $x_{24}$  +  $x_{31}$   $= 500$ ,  $x_{24}$  +  $x_{31}$   $= 500$ ,  $x_{34}$  =  $x_{$ 

Solving problem (15.1) using **MOSEK** will result in a solution, a solution status and a problem status. Among the log output from the execution of **MOSEK** on the above problem are the lines:

```
Basic solution
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
```

The first line indicates that the problem status is primal infeasible. The second line says that a *certificate* of the infeasibility was found. The certificate is returned in place of the solution to the problem.

## 15.2.2 Locating the cause of Primal Infeasibility

Usually a primal infeasible problem status is caused by a mistake in formulating the problem and therefore the question arises: What is the cause of the infeasible status? When trying to answer this question, it is often advantageous to follow these steps:

- Remove the objective function. This does not change the infeasibility status but simplifies the problem, eliminating any possibility of issues related to the objective function.
- Consider whether your problem has some necessary conditions for feasibility and examine if these are satisfied, e.g. total supply should be greater than or equal to total demand.

• Verify that coefficients and bounds are reasonably sized in your problem.

If the problem is still primal infeasible, some of the constraints must be relaxed or removed completely. The **MOSEK** infeasibility report (Sec. 15.2.4) may assist you in finding the constraints causing the infeasibility.

Possible ways of relaxing your problem nclude:

- Increasing (decreasing) upper (lower) bounds on variables and constraints.
- Removing suspected constraints from the problem.

Returning to the transportation example, we discover that removing the fifth constraint

$$x_{12} = 200$$

makes the problem feasible.

## 15.2.3 Locating the Cause of Dual Infeasibility

A problem may also be *dual infeasible*. In this case the primal problem is often unbounded, meaning that feasible solutions exists such that the objective tends towards infinity. An example of a dual infeasible and primal unbounded problem is:

minimize 
$$x_1$$
 subject to  $x_1 \le 5$ .

To resolve a dual infeasibility the primal problem must be made more restricted by

- Adding upper or lower bounds on variables or constraints.
- Removing variables.
- Changing the objective.

## A cautionary note

The problem

minimize 
$$0$$
 subject to  $0 \le x_1$ ,  $x_j \le x_{j+1}$ ,  $j = 1, \ldots, n-1$ ,  $x_n \le -1$ 

is clearly infeasible. Moreover, if any one of the constraints is dropped, then the problem becomes feasible.

This illustrates the worst case scenario where all, or at least a significant portion of the constraints are involved in causing infeasibility. Hence, it may not always be easy or possible to pinpoint a few constraints responsible for infeasibility.

## 15.2.4 The Infeasibility Report

MOSEK includes functionality for diagnosing the cause of a primal or a dual infeasibility. It can be turned on by setting the <code>iparam.infeas\_report\_auto</code> to <code>onoffkey.on</code>. This causes MOSEK to print a report on variables and constraints involved in the infeasibility.

The *iparam.infeas\_report\_level* parameter controls the amount of information presented in the infeasibility report. The default value is 1.

#### **Example: Primal Infeasibility**

We will keep working with the problem (15.1) written in LP format:

Listing 15.1: The code for problem (15.1).

```
minimize
obj: + 1 \times 11 + 2 \times 12
     + 5 x23 + 2 x24
     + 1 x31 + 2 x33 + 1 x34
 s0: + x11 + x12
                    <= 200
 s1: + x23 + x24
                    <= 1000
 s2: + x31 + x33 + x34 \le 1000
 d1: + x11 + x31
                 = 1100
                     = 200
 d2: + x12
 d3: + x23 + x33
                     = 500
 d4: + x24 + x34
                     = 500
bounds
end
```

## **Example: Dual Infeasibility**

The following problem is dual to (15.1) and therefore it is dual infeasible.

Listing 15.2: The dual of problem (15.1).

```
maximize + 200 y1 + 1000 y2 + 1000 y3 + 1100 y4 + 200 y5 + 500 y6 + 500 y7
subject to
  x11: y1+y4 < 1
  x12: y1+y5 < 2
  x23: y2+y6 < 5
  x24: y2+y7 < 2
  x31: y3+y4 < 1
  x33: y3+y6 < 2
  x34: y3+y7 < 1
  -inf <= y1 < 0
   -\inf <= y2 < 0
  -inf <= y3 < 0
  y4 free
  y5 free
  y6 free
  y7 free
```

This can be verified by proving that

$$(y_1,\ldots,y_7)=(-1,0,-1,1,1,0,0)$$

is a certificate of dual infeasibility (see Sec. 12.1.2) as we can see from this report:

```
MOSEK DUAL INFEASIBILITY REPORT.

Problem status: The problem is dual infeasible

The following constraints are involved in the infeasibility.
```

Index	Name	Activity	Objective	Lower bound	Upper bound
5	x33	-1.00000e+00		NONE	2.000000e+00
6	x34	-1.000000e+00		NONE	1.000000e+00
The fol	lowing variab	les are involved in t	he infeasibility	·.	
Index	Name	Activity	Objective	Lower bound	Upper bound
0	y1	-1.00000e+00	2.000000e+02	NONE	0.000000e+00
2	у3	-1.00000e+00	1.000000e+03	NONE	0.000000e+00
3	y4	1.000000e+00	1.100000e+03	NONE	NONE
4	y5	1.000000e+00	2.000000e+02	NONE	NONE

Let  $y^*$  denote the reported primal solution. **MOSEK** states

- that the problem is *dual infeasible*,
- that the reported solution is a certificate of dual infeasibility, and
- ullet that the infeasibility measure for  $y^*$  is approximately zero.

Since the original objective was maximization, we have that  $c^Ty^* > 0$ . See Sec. 12.1.2 for how to interpret the parameter values in the infeasibility report for a linear program. We see that the variables y1, y3, y4, y5 and the constraints x33 and x34 contribute to infeasibility with non-zero values in the Activity column.

One possible strategy to fix the infeasibility is to modify the problem so that the certificate of infeasibility becomes invalid. In this case we could do one the following things:

- Add a lower bound on y3. This will directly invalidate the certificate of dual infeasibility.
- Increase the object coefficient of y3. Changing the coefficients sufficiently will invalidate the inequality  $c^T y^* > 0$  and thus the certificate.
- Add lower bounds on x11 or x31. This will directly invalidate the certificate of infeasibility.

Please note that modifying the problem to invalidate the reported certificate does *not* imply that the problem becomes dual feasible — the reason for infeasibility may simply *move*, resulting a problem that is still infeasible, but for a different reason.

More often, the reported certificate can be used to give a hint about errors or inconsistencies in the model that produced the problem.

## 15.2.5 Theory Concerning Infeasible Problems

This section discusses the theory of infeasibility certificates and how MOSEK uses a certificate to produce an infeasibility report. In general, MOSEK solves the problem

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  $l^x \le x \le u^x$  (15.2)

where the corresponding dual problem is

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c}$$

$$+ (l^{x})^{T} s_{l}^{r} - (u^{x})^{T} s_{u}^{x} + c^{f}$$
subject to 
$$A^{T} y + s_{l}^{x} - s_{u}^{x} = c,$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \leq 0.$$

$$(15.3)$$

We use the convension that for any bound that is not finite, the corresponding dual variable is fixed at zero (and thus will have no influence on the dual problem). For example

$$l_j^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0$$

## 15.2.6 The Certificate of Primal Infeasibility

A certificate of primal infeasibility is any solution to the homogenized dual problem

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & A^T y + s_l^x - s_u^x & = & 0, \\ & -y + s_l^c - s_u^c & = & 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \leq 0. \end{array}$$

with a positive objective value. That is,  $(s_l^{c*}, s_u^{c*}, s_u^{r*}, s_u^{r*})$  is a certificate of primal infeasibility if

$$(l^c)^T s_l^{c*} - (u^c)^T s_u^{c*} + (l^x)^T s_l^{x*} - (u^x)^T s_u^{x*} > 0$$

and

$$\begin{array}{lll} A^Ty + s_l^{x*} - s_u^{x*} & = & 0, \\ -y + s_l^{c*} - s_u^{c*} & = & 0, \\ s_l^{c*}, s_u^{x*}, s_l^{x*}, s_u^{x*} \leq 0. \end{array}$$

The well-known Farkas Lemma tells us that (15.2) is infeasible if and only if a certificate of primal infeasibility exists.

Let  $(s_l^{c*}, s_u^{c*}, s_l^{x*}, s_u^{x*})$  be a certificate of primal infeasibility then

$$(s_l^{c*})_i > 0((s_u^{c*})_i > 0)$$

implies that the lower (upper) bound on the i th constraint is important for the infeasibility. Furthermore,

$$(s_l^{x*})_i > 0((s_u^{x*})_i > 0)$$

implies that the lower (upper) bound on the j th variable is important for the infeasibility.

## 15.2.7 The certificate of dual infeasibility

A certificate of dual infeasibility is any solution to the problem

with negative objective value, where we use the definitions

$$\bar{l}_i^c := \left\{ \begin{array}{ll} 0, & l_i^c > -\infty, \\ -\infty, & \text{otherwise,} \end{array} \right\}, \ \bar{u}_i^c := \left\{ \begin{array}{ll} 0, & u_i^c < \infty, \\ \infty, & \text{otherwise,} \end{array} \right\}$$

and

$$\bar{l}^x_i := \left\{ \begin{array}{ll} 0, & l^x_i > -\infty, \\ -\infty, & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \bar{u}^x_i := \left\{ \begin{array}{ll} 0, & u^x_i < \infty, \\ \infty, & \text{otherwise.} \end{array} \right\}$$

Stated differently, a certificate of dual infeasibility is any  $x^*$  such that

$$c^{T}x^{*} < 0,$$

$$\bar{l}^{c} \leq Ax^{*} \leq \bar{u}^{c},$$

$$\bar{l}^{x} < x^{*} < \bar{u}^{x}$$

$$(15.4)$$

The well-known Farkas Lemma tells us that (15.3) is infeasible if and only if a certificate of dual infeasibility exists.

Note that if  $x^*$  is a certificate of dual infeasibility then for any j such that

$$x_i^* \le 0,$$

variable j is involved in the dual infeasibility.

## 15.3 Sensitivity Analysis

Given an optimization problem it is often useful to obtain information about how the optimal objective value changes when the problem parameters are perturbed. E.g, assume that a bound represents the capacity of a machine. Now, it may be possible to expand the capacity for a certain cost and hence it is worthwhile knowing what the value of additional capacity is. This is precisely the type of questions the sensitivity analysis deals with.

Analyzing how the optimal objective value changes when the problem data is changed is called *sensitivity* analysis.

#### References

The book [Chv83] discusses the classical sensitivity analysis in Chapter 10 whereas the book [RTV97] presents a modern introduction to sensitivity analysis. Finally, it is recommended to read the short paper [Wal00] to avoid some of the pitfalls associated with sensitivity analysis.

Warning: Currently, sensitivity analysis is only available for continuous linear optimization problems. Moreover, MOSEK can only deal with perturbations of bounds and objective function coefficients.

## 15.3.1 Sensitivity Analysis for Linear Problems

## The Optimal Objective Value Function

Assume that we are given the problem

$$z(l^{c}, u^{c}, l^{x}, u^{x}, c) = \underset{\text{subject to}}{\text{minimize}} c^{T}x$$

$$subject to \quad l^{c} \leq Ax \leq u^{c},$$

$$l^{x} \leq x \leq u^{x},$$

$$(15.5)$$

and we want to know how the optimal objective value changes as  $l_i^c$  is perturbed. To answer this question we define the perturbed problem for  $l_i^c$  as follows

$$\begin{array}{lll} f_{l_i^c}(\beta) & = & \text{minimize} & & c^T x \\ & & \text{subject to} & l^c + \beta e_i & \leq & Ax & \leq u^c, \\ & & l^x & \leq & x \leq & u^x, \end{array}$$

where  $e_i$  is the *i*-th column of the identity matrix. The function

$$f_{l_i^c}(\beta) \tag{15.6}$$

shows the optimal objective value as a function of  $\beta$ . Please note that a change in  $\beta$  corresponds to a perturbation in  $l_i^c$  and hence (15.6) shows the optimal objective value as a function of varying  $l_i^c$  with the other bounds fixed.

It is possible to prove that the function (15.6) is a piecewise linear and convex function, i.e. its graph may look like in Fig. 15.2 and Fig. 15.3.

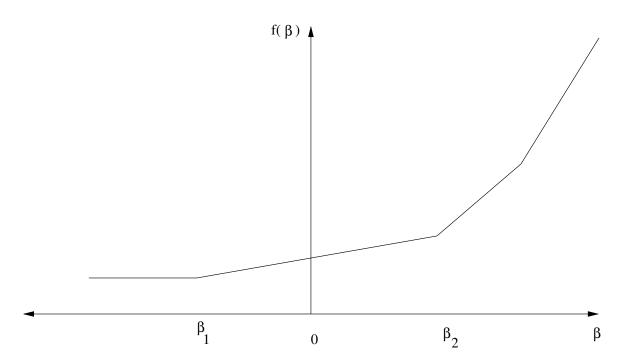


Fig. 15.2:  $\beta=0$  is in the interior of linearity interval.

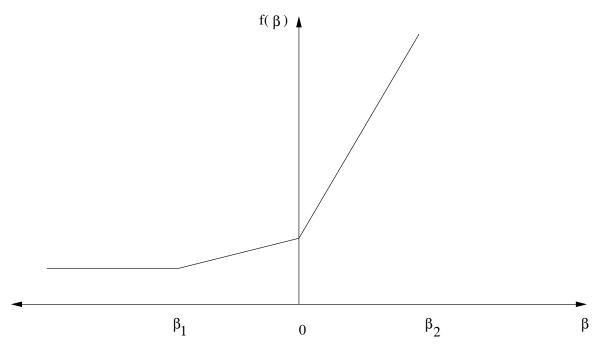


Fig. 15.3:  $\beta=0$  is a breakpoint.

Clearly, if the function  $f_{l_i^c}(\beta)$  does not change much when  $\beta$  is changed, then we can conclude that the optimal objective value is insensitive to changes in  $l_i^c$ . Therefore, we are interested in the rate of change in  $f_{l_i^c}(\beta)$  for small changes in  $\beta$  — specifically the gradient

$$f'_{l_i^c}(0),$$

which is called the *shadow price* related to  $l_i^c$ . The shadow price specifies how the objective value changes for small changes of  $\beta$  around zero. Moreover, we are interested in the *linearity interval* 

$$\beta \in [\beta_1, \beta_2]$$

for which

$$f'_{l^c}(\beta) = f'_{l^c}(0).$$

Since  $f_{l_i^c}$  is not a smooth function  $f'_{l_i^c}$  may not be defined at 0, as illustrated in Fig. 15.3. In this case we can define a left and a right shadow price and a left and a right linearity interval.

The function  $f_{l_i^c}$  considered only changes in  $l_i^c$ . We can define similar functions for the remaining parameters of the z defined in (15.5) as well:

$$f_{l_i^c}(\beta) = z(l^c + \beta e_i, u^c, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{u_i^c}(\beta) = z(l^c, u^c + \beta e_i, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{l_j^x}(\beta) = z(l^c, u^c, l^x + \beta e_j, u^x, c), \quad j = 1, \dots, n,$$

$$f_{u_j^x}(\beta) = z(l^c, u^c, l^x, u^x + \beta e_j, c), \quad j = 1, \dots, n,$$

$$f_{c_j}(\beta) = z(l^c, u^c, l^x, u^x, c + \beta e_j), \quad j = 1, \dots, n.$$

Given these definitions it should be clear how linearity intervals and shadow prices are defined for the parameters  $u_i^c$  etc.

## **Equality Constraints**

In **MOSEK** a constraint can be specified as either an equality constraint or a ranged constraint. If some constraint  $e_i^c$  is an equality constraint, we define the optimal value function for this constraint as

$$f_{e_i^c}(\beta) = z(l^c + \beta e_i, u^c + \beta e_i, l^x, u^x, c)$$

Thus for an equality constraint the upper and the lower bounds (which are equal) are perturbed simultaneously. Therefore, **MOSEK** will handle sensitivity analysis differently for a ranged constraint with  $l_i^c = u_i^c$  and for an equality constraint.

#### The Basis Type Sensitivity Analysis

The classical sensitivity analysis discussed in most textbooks about linear optimization, e.g. [Chv83], is based on an optimal basic solution or, equivalently, on an optimal basis. This method may produce misleading results [RTV97] but is **computationally cheap**. Therefore, and for historical reasons, this method is available in **MOSEK**.

We will now briefly discuss the basis type sensitivity analysis. Given an optimal basic solution which provides a partition of variables into basic and non-basic variables, the basis type sensitivity analysis computes the linearity interval  $[\beta_1, \beta_2]$  so that the basis remains optimal for the perturbed problem. A shadow price associated with the linearity interval is also computed. However, it is well-known that an optimal basic solution may not be unique and therefore the result depends on the optimal basic solution employed in the sensitivity analysis. This implies that the computed interval is only a subset of the largest interval for which the shadow price is constant. Furthermore, the optimal objective value function might have a breakpoint for  $\beta=0$ . In this case the basis type sensitivity method will only provide a subset of either the left or the right linearity interval.

In summary, the basis type sensitivity analysis is computationally cheap but does not provide complete information. Hence, the results of the basis type sensitivity analysis should be used with care.

#### The Optimal Partition Type Sensitivity Analysis

Another method for computing the complete linearity interval is called the *optimal partition type sensitivity analysis*. The main drawback of the optimal partition type sensitivity analysis is that it is computationally expensive compared to the basis type analysis. This type of sensitivity analysis is currently provided as an experimental feature in **MOSEK**.

Given the optimal primal and dual solutions to (15.5), i.e.  $x^*$  and  $((s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^x)^*, (s_u^x)^*)$  the optimal objective value is given by

$$z^* := c^T x^*$$
.

The left and right shadow prices  $\sigma_1$  and  $\sigma_2$  for  $l_i^c$  are given by this pair of optimization problems:

$$\begin{array}{lll} \sigma_1 & = & \text{minimize} & e_i^T s_l^c \\ & & \text{subject to} & A^T (s_l^c - s_u^c) + s_l^x - s_u^x & = c, \\ & & (l^c)^T (s_l^c) - (u^c)^T (s_u^c) + (l^x)^T (s_l^x) - (u^x)^T (s_u^x) & = z^*, \\ & & s_l^c, s_u^c, s_l^c, s_u^c \geq 0 \end{array}$$

and

$$\begin{array}{lll} \sigma_2 & = & \text{maximize} & e_l^T s_l^c \\ & \text{subject to} & A^T (s_l^c - s_u^c) + s_l^x - s_u^x & = & c, \\ & & (l^c)^T (s_l^c) - (u^c)^T (s_u^c) + (l^x)^T (s_l^x) - (u^x)^T (s_u^x) & = & z^*, \\ & & s_l^c, s_u^c, s_l^c, s_u^x \geq 0. \end{array}$$

These two optimization problems make it easy to interpret the shadow price. Indeed, if  $((s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*)$  is an arbitrary optimal solution then

$$(s_l^c)_i^* \in [\sigma_1, \sigma_2].$$

Next, the linearity interval  $[\beta_1, \beta_2]$  for  $l_i^c$  is computed by solving the two optimization problems

and

$$\beta_2 = \underset{\text{subject to}}{\text{maximize}} \qquad \beta \\ \text{subject to} \quad l^c + \beta e_i \leq \underset{c}{Ax} \leq u^c, \\ c^T x - \sigma_2 \beta = z^*, \\ l^x \leq x \leq u^x.$$

The linearity intervals and shadow prices for  $u_i^c$ ,  $l_i^x$ , and  $u_i^x$  are computed similarly to  $l_i^c$ .

The left and right shadow prices for  $c_j$  denoted  $\sigma_1$  and  $\sigma_2$  respectively are computed as follows:

$$\begin{array}{llll} \sigma_1 & = & \text{minimize} & & e_j^T x \\ & & \text{subject to} & l^c + \beta e_i & \leq & Ax & \leq & u^c, \\ & & & c^T x & = & z^*, \\ & & l^x & \leq & x & \leq & u^x, \end{array}$$

and

$$\begin{array}{llll} \sigma_2 & = & \text{maximize} & & e_j^T x \\ & \text{subject to} & l^c + \beta e_i & \leq & Ax & \leq & u^c, \\ & & & c^T x & = & z^*, \\ & l^x & \leq & x & \leq & u^x. \end{array}$$

Once again the above two optimization problems make it easy to interpret the shadow prices. Indeed, if  $x^*$  is an arbitrary primal optimal solution, then

$$x_j^* \in [\sigma_1, \sigma_2].$$

The linearity interval  $[\beta_1, \beta_2]$  for a  $c_j$  is computed as follows:

$$\begin{array}{lll} \beta_1 & = & \text{minimize} & \beta \\ & \text{subject to} & A^T(s_l^c - s_u^c) + s_l^x - s_u^x & = & c + \beta e_j, \\ & & (l^c)^T(s_l^c) - (u^c)^T(s_u^c) + (l^x)^T(s_l^x) - (u^x)^T(s_u^x) - \sigma_1 \beta & \leq & z^*, \\ & & s_l^c, s_u^c, s_l^c, s_u^x \geq 0 \end{array}$$

and

$$\begin{array}{lll} \beta_2 & = & \text{maximize} & \beta \\ & & \text{subject to} & A^T(s_l^c - s_u^c) + s_l^x - s_u^x & = & c + \beta e_j, \\ & & & (l^c)^T(s_l^c) - (u^c)^T(s_u^c) + (l^x)^T(s_l^x) - (u^x)^T(s_u^x) - \sigma_2\beta & \leq & z^*, \\ & & & s_l^c, s_u^c, s_l^c, s_u^x \geq 0. \end{array}$$

## **Example: Sensitivity Analysis**

As an example we will use the following transportation problem. Consider the problem of minimizing the transportation cost between a number of production plants and stores. Each plant supplies a number of goods and each store has a given demand that must be met. Supply, demand and cost of transportation per unit are shown in Fig. 15.4.

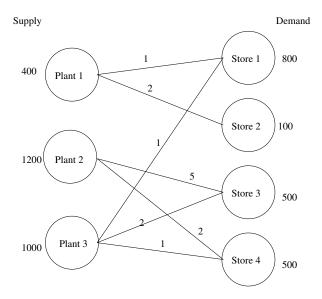


Fig. 15.4: Supply, demand and cost of transportation.

If we denote the number of transported goods from location i to location j by  $x_{ij}$ , problem can be formulated as the linear optimization problem of minimizing

$$1x_{11} + 2x_{12} + 5x_{23} + 2x_{24} + 1x_{31} + 2x_{33} + 1x_{34}$$

subject to

The sensitivity parameters are shown in Table 15.1 and Table 15.2 for the basis type analysis and in Table 15.3 and Table 15.4 for the optimal partition type analysis.

Table 15.1: Ranges and shadow prices related to bounds on constraints and variables: results for the basis type sensitivity analysis.

Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
1	-300.00	0.00	3.00	3.00
2	-700.00	$+\infty$	0.00	0.00
3	-500.00	0.00	3.00	3.00
4	-0.00	500.00	4.00	4.00
5	-0.00	300.00	5.00	5.00
6	-0.00	700.00	5.00	5.00
7	-500.00	700.00	2.00	2.00
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$x_{11}$	$-\infty$	300.00	0.00	0.00
$x_{12}$	$-\infty$	100.00	0.00	0.00
$x_{23}$	$-\infty$	0.00	0.00	0.00
$x_{24}$	$-\infty$	500.00	0.00	0.00
$x_{31}$	$-\infty$	500.00	0.00	0.00
$x_{33}$	$-\infty$	500.00	0.00	0.00
$x_{34}$	-0.000000	500.00	2.00	2.00

Table 15.2: Ranges and shadow prices related to bounds on constraints and variables: results for the optimal partition type sensitivity analysis.

Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
1	-300.00	500.00	3.00	1.00
2	-700.00	$+\infty$	-0.00	-0.00
3	-500.00	500.00	3.00	1.00
4	-500.00	500.00	2.00	4.00
5	-100.00	300.00	3.00	5.00
6	-500.00	700.00	3.00	5.00
7	-500.00	700.00	2.00	2.00
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$x_{11}$	$-\infty$	300.00	0.00	0.00
$x_{12}$	$-\infty$	100.00	0.00	0.00
$x_{23}$	$-\infty$	500.00	0.00	2.00
$x_{24}$	$-\infty$	500.00	0.00	0.00
$x_{31}$	$-\infty$	500.00	0.00	0.00
$x_{33}$	$-\infty$	500.00	0.00	0.00
$x_{34}$	$-\infty$	500.00	0.00	2.00

Table 15.3: Ranges and shadow prices related to the objective coefficients: results for the basis type sensitivity analysis.

Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Table 15.4: Ranges and shadow prices related to the objective coefficients: results for the optimal partition type sensitivity analysis.

Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Examining the results from the optimal partition type sensitivity analysis we see that for constraint number 1 we have  $\sigma_1 = 3$ ,  $\sigma_2 = 1$  and  $\beta_1 = -300$ ,  $\beta_2 = 500$ . Therefore, we have a left linearity interval of [-300, 0] and a right interval of [0, 500]. The corresponding left and right shadow prices are 3 and 1 respectively. This implies that if the upper bound on constraint 1 increases by

$$\beta \in [0, \beta_1] = [0, 500]$$

then the optimal objective value will decrease by the value

$$\sigma_2\beta = 1\beta$$
.

Correspondingly, if the upper bound on constraint 1 is decreased by

$$\beta \in [0, 300]$$

then the optimal objective value will increase by the value

$$\sigma_1\beta=3\beta.$$

## 15.3.2 Sensitivity Analysis with MOSEK

MOSEK provides the functions Task. primalsensitivity and Task. dualsensitivity for performing sensitivity analysis. The code in Listing 15.3 gives an example of its use.

Listing 15.3: Example of sensitivity analysis with the MOSEK Optimizer API for Java.

```
package com.mosek.example;
import mosek.*;
public class sensitivity {
 public static void main (String[] args) {
    // Since the value infinity is never used, we define
   // 'infinity' symbolic purposes only
   double
   infinity = 0;
   try (Env env = new Env();
         Task task = new Task(env, 0, 0)) {
      mosek.boundkey[] bkc = {
        mosek.boundkey.up, mosek.boundkey.up,
        mosek.boundkey.up, mosek.boundkey.fx,
       mosek.boundkey.fx, mosek.boundkey.fx,
       mosek.boundkey.fx
      };
      mosek.boundkey[] bkx = {
       mosek.boundkey.lo, mosek.boundkey.lo,
       mosek.boundkey.lo, mosek.boundkey.lo,
       mosek.boundkey.lo, mosek.boundkey.lo,
       mosek.boundkey.lo
      int[] ptrb = {0, 2, 4, 6, 8, 10, 12};
      int[] ptre = {2, 4, 6, 8, 10, 12, 14};
      int[] sub = {0, 3, 0, 4, 1, 5, 1, 6, 2, 3, 2, 5, 2, 6};
      double[] blc = { -infinity, -infinity,
                       -infinity, 800, 100, 500, 500
                     };
      double[] buc = {400, 1200, 1000, 800, 100, 500, 500};
      double[] c = {1.0, 2.0, 5.0, 2.0, 1.0, 2.0, 1.0};
      double[] blx = {0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0};
      double[] bux = {infinity, infinity,
                      infinity, infinity,
                      infinity, infinity,
                      infinity
                     };
      double[] val = {1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
                      1.0, 1.0, 1.0, 1.0, 1.0, 1.0
                     };
      int numcon = 7;  /* Number of constraints.
int numvar = 7;  /* Number of variables.
      int NUMANZ = 14; /* Number of non-zeros in A.
      // Directs the log task stream to the user specified
      // method task_msg_obj.print
      task.set_Stream(
       mosek.streamtype.log,
       new mosek.Stream()
      { public void stream(String msg) { System.out.print(msg); }});
      task.inputdata(numcon, numvar,
                     С.
                     0.0,
                     ptrb,
                     ptre,
                     sub,
                     val,
```

```
bkc,
               blc,
               buc,
               bkx,
               blx,
               bux);
/* A maximization problem */
task.putobjsense(mosek.objsense.minimize);
task.optimize();
/* Analyze upper bound on c1 and the equality constraint on c4 */
int subi[] = {0, 3};
mosek.mark marki[] = {mosek.mark.up, mosek.mark.up};
/* Analyze lower bound on the variables x12 and x31 */
int subj[] = {1, 4};
mosek.mark markj[] = {mosek.mark.lo, mosek.mark.lo};
double[] leftpricei = new double[2];
double[] rightpricei = new double[2];
double[] leftrangei = new double[2];
double[] rightrangei = new double[2];
double[] leftpricej = new double[2];
double[] rightpricej = new double[2];
double[] leftrangej = new double[2];
double[] rightrangej = new double[2];
task.primalsensitivity( subi,
                        marki,
                        subj,
                        markj,
                        leftpricei,
                        rightpricei,
                        leftrangei,
                        rightrangei,
                        leftpricej,
                        rightpricej,
                        leftrangej,
                        rightrangej);
System.out.println("Results from sensitivity analysis on bounds:\n");
System.out.println("For constraints:\n");
for (int i = 0; i < 2; ++i)
  System.out.print("leftprice = " + leftpricei[i] +
                   " rightprice = " + rightpricei[i] +
                   " leftrange = " + leftrangei[i] +
                   " rightrange = " + rightrangei[i] + "\n");
System.out.print("For variables:\n");
for (int i = 0; i < 2; ++i)
  System.out.print("leftprice = " + leftpricej[i] +
                   " rightprice = " + rightpricej[i] +
                   " leftrange = " + leftrangej[i] +
                   " rightrange = " + rightrangej[i] + "\n");
double[] leftprice = new double[2];
double[] rightprice = new double[2];
double[] leftrange = new double[2];
```

```
double[] rightrange = new double[2];
    int subc[] = {2, 5};
    task.dualsensitivity( subc,
                           leftprice,
                           rightprice,
                           leftrange,
                           rightrange
                        );
    System.out.println(
      "Results from sensitivity analysis on objective coefficients:"
    for (int i = 0; i < 2; ++i)
      System.out.print("leftprice = " + leftprice[i] +
                       " rightprice = " + rightprice[i] +
                       " leftrange = " + leftrange[i] +
                       " rightrange = " + rightrange[i] + "\n");
 } catch (mosek.Exception e)
    /* Catch both mosek. Error and mosek. Warning */
    System.out.println ("An error or warning was encountered");
    System.out.println (e.getMessage ());
    throw e;
 }
}
```

## **SIXTEEN**

## **API REFERENCE**

This section contains the complete reference of the **MOSEK** Optimizer API for Java. It is organized as follows:

- General API conventions.
- Methods:
  - Class Env (The MOSEK environment)
  - Class Task (An optimization task)
  - Browse by topic
- Optimizer parameters:
  - Double, Integer, String
  - Full list
  - Browse by topic
- Optimizer information items:
  - Double, Integer, Long
- Optimizer response codes
- Enumerations
- Exceptions
- User-defined class types
- Nonlinear API (SCopt)

## 16.1 API Conventions

## 16.1.1 Function arguments

## **Naming Convention**

In the definition of the **MOSEK** Optimizer API for Java a consistent naming convention has been used. This implies that whenever for example numcon is an argument in a function definition it indicates the number of constraints. In Table 16.1 the variable names used to specify the problem parameters are listed.

711 1 101	AFI for Java.					
API name	API type	Dimension	Related problem parameter			
numcon	int		m			
numvar	int		n			
numcone	int		t			
numqonz	int		$q_{ij}^o$			
qosubi	int[]	numqonz	$q_{ij}^o$			
qosubj	int[]	numqonz	$q_{ij}^o$			
qoval	double[]	numqonz	$q_{ij}^{\circ}$			
С	double[]	numvar	$c_j$ $c^f$			
cfix	double					
numqcnz	int		$q_{ij}^k$			
qcsubk	int[]	qcnz	$q_{ij}^{ec{k}}$			
qcsubi	int[]	qcnz	$q_{ij}^{ ilde{k}}$			
qcsubj	int[]	qcnz	$q_{ij}^{ ilde{k}}$			
qcval	double[]	qcnz	$q_{ij}^k$			
aptrb	int[]	numvar	$a_{ij}$			
aptre	int[]	numvar	$a_{ij}$			
asub	int[]	aptre[numvar-1]	$\mid a_{ij} \mid$			
aval	double[]	aptre[numvar-1]	$a_{ij}$			
bkc	int[]	numcon	$l_k^c$ and $u_k^c$			
blc	double[]	numcon	$l_k^c$			
buc	double[]	numcon	$u_k^c$			
bkx	int[]	numvar	$l_k^x$ and $u_k^x$			
blx	double[]	numvar	$l_k^x$			
bux	double[]	numvar	$u_k^x$			

Table 16.1: Naming conventions used in the **MOSEK** Optimizer API for Java.

The relation between the variable names and the problem parameters is as follows:

- $\bullet \ \ \text{The quadratic terms in the objective:} \ \ q^o_{\texttt{qosubi[t]},\texttt{qosubj[t]}} = \texttt{qoval[t]}, \quad t = 0, \dots, \texttt{numqonz} 1.$
- The linear terms in the objective :  $c_j = c[j], \quad j = 0, \dots, numvar 1$
- The fixed term in the objective :  $c^f = cfix$ .
- $\bullet \ \ \text{The quadratic terms in the constraints:} \ \ q_{\mathtt{qcsubi}[\mathtt{t}],\mathtt{qcsubj}[\mathtt{t}]}^{\mathtt{qcsubk}[\mathtt{t}]} = \mathtt{qcval}[\mathtt{t}], \quad t = 0, \dots, \mathtt{numqcnz} 1$
- The linear terms in the constraints:  $a_{\tt asub[t],j} = \tt aval[t], \quad t = \tt ptrb[j], \ldots, ptre[j] 1, \quad j = 0, \ldots, numvar 1$

## Passing arguments by reference

An argument described as  $\mathbf{T}$  by reference indicates that the function interprets its given argument as a reference to a variable of type  $\mathbf{T}$ . This usually means that the argument is used to output or update a value of type  $\mathbf{T}$ . For example, suppose we have a function documented as

```
void foo (..., int[] nzc, ...)
```

• nzc (int by reference) – The number of nonzero elements in the matrix. (output)

Then it could be called as follows.

```
int nzc = new int[1];
foo (..., nzc, ...)
System.out.println("The number of nonzero elements: ", nzc[0])
```

#### Information about input/output arguments

The following are purely informational tags which indicate how MOSEK treats a specific function argument.

- (input) An input argument. It is used to input data to MOSEK.
- (output) An output argument. It can be a user-preallocated data structure, a reference, a string buffer etc. where **MOSEK** will output some data.
- (input/output) An input/output argument. **MOSEK** will read the data and overwrite it with new/updated information.

## 16.1.2 Bounds

The bounds on the constraints and variables are specified using the variables bkc, blc, and buc. The components of the integer array bkc specify the bound type according to Table 16.2

v	v	
Symbolic constant	Lower bound	Upper bound
boundkey.fx	finite	identical to the lower bound
boundkey.fr	minus infinity	plus infinity
boundkey.lo	finite	plus infinity
boundkey.ra	finite	finite
boundkey.up	minus infinity	finite

Table 16.2: Symbolic key for variable and constraint bounds.

For instance bkc[2]=boundkey.lo means that  $-\infty < l_2^c$  and  $u_2^c = \infty$ . Even if a variable or constraint is bounded only from below, e.g.  $x \ge 0$ , both bounds are inputted or extracted; the irrelevant value is ignored.

Finally, the numerical values of the bounds are given by

$$l_k^c = \mathrm{blc}[\mathtt{k}], \quad k = 0, \dots, \mathrm{numcon} - 1$$

$$u_k^c = \operatorname{buc}[k], \quad k = 0, \dots, \operatorname{numcon} - 1.$$

The bounds on the variables are specified using the variables bkx, blx, and bux in the same way. The numerical values for the lower bounds on the variables are given by

$$l_i^x = \mathtt{blx}[\mathtt{j}], \quad j = 0, \dots, \mathtt{numvar} - 1.$$

$$u^x_j = \mathtt{bux[j]}, \quad j = 0, \dots, \mathtt{numvar} - 1.$$

### 16.1.3 Vector Formats

Three different vector formats are used in the **MOSEK** API:

#### Full (dense) vector

This is simply an array where the first element corresponds to the first item, the second element to the second item etc. For example to get the linear coefficients of the objective in task with numvar variables, one would write

```
double[] c = new double[numvar];
task.getc(c);
```

16.1. API Conventions 161

#### Vector slice

A vector slice is a range of values from first up to and **not including last** entry in the vector, i.e. for the set of indices i such that first <= i < last. For example, to get the bounds associated with constrains 2 through 9 (both inclusive) one would write

#### Sparse vector

A sparse vector is given as an array of indexes and an array of values. The indexes need not be ordered. For example, to input a set of bounds associated with constraints number 1, 6, 3, and 9, one might write

#### 16.1.4 Matrix Formats

The coefficient matrices in a problem are inputted and extracted in a sparse format. That means only the nonzero entries are listed.

#### **Unordered Triplets**

In unordered triplet format each entry is defined as a row index, a column index and a coefficient. For example, to input the A matrix coefficients for  $a_{1,2} = 1.1, a_{3,3} = 4.3$ , and  $a_{5,4} = 0.2$ , one would write as follows:

```
int[] subi = { 1, 3, 5 };
int[] subj = { 2, 3, 4 };
double[] cof = { 1.1, 4.3, 0.2 };
task.putaijlist(subi,subj,cof);
```

Please note that in some cases (like Task.putaijlist) only the specified indexes are modified — all other are unchanged. In other cases (such as Task.putqconk) the triplet format is used to modify all entries — entries that are not specified are set to 0.

## Column or Row Ordered Sparse Matrix

In a sparse matrix format only the non-zero entries of the matrix are stored. **MOSEK** uses a sparse packed matrix format ordered either by columns or rows. Here we describe the column-wise format. The row-wise format is based on the same principle.

## Column ordered sparse format

A sparse matrix in column ordered format is essentially a list of all non-zero entries read column by column from left to right and from top to bottom within each column. The exact representation uses four arrays:

- asub: Array of size equal to the number of nonzeros. List of row indexes.
- aval: Array of size equal to the number of nonzeros. List of non-zero entries of A ordered by columns.
- ptrb: Array of size numcol, where ptrb[j] is the position of the first value/index in aval/ asub for the j-th column.
- ptre: Array of size numcol, where ptre[j] is the position of the last value/index plus one in aval / asub for the j-th column.

With this representation the values of a matrix A with numcol columns are assigned using:

$$a_{\mathtt{asub}[k],j} = \mathtt{aval}[k] \quad \text{for} \quad j = 0, \dots, \mathtt{numcol} - 1, \ k = \mathtt{ptrb}[j], \dots, \mathtt{ptre}[j] - 1.$$

As an example consider the matrix

$$A = \begin{bmatrix} 1.1 & 1.3 & 1.4 \\ & 2.2 & & 2.5 \\ 3.1 & & 3.4 \\ & & 4.4 \end{bmatrix}$$
 (16.1)

which can be represented in the column ordered sparse matrix format as

$$\begin{array}{lll} \mathtt{ptrb} &=& [0,2,3,5,7], \\ \mathtt{ptre} &=& [2,3,5,7,8], \\ \mathtt{asub} &=& [0,2,1,0,3,0,2,1], \\ \mathtt{aval} &=& [1.1,3.1,2.2,1.3,4.4,1.4,3.4,2.5]. \end{array}$$

Fig. 16.1 illustrates how the matrix A in (16.1) is represented in column ordered sparse matrix format.

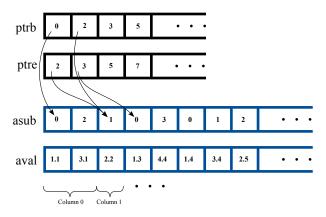


Fig. 16.1: The matrix A (16.1) represented in column ordered packed sparse matrix format.

## Column ordered sparse format with nonzeros

Note that nzc[j] := ptre[j]-ptrb[j] is exactly the number of nonzero elements in the j-th column of A. In some functions a sparse matrix will be represented using the equivalent dataset asub, aval, ptrb, nzc. The matrix A (16.1) would now be represented as:

```
\begin{array}{lll} \mathtt{ptrb} &=& [0,2,3,5,7], \\ \mathtt{nzc} &=& [2,1,2,2,1], \\ \mathtt{asub} &=& [0,2,1,0,3,0,2,1], \\ \mathtt{aval} &=& [1.1,3.1,2.2,1.3,4.4,1.4,3.4,2.5]. \end{array}
```

16.1. API Conventions 163

#### Row ordered sparse matrix

The matrix A (16.1) can also be represented in the row ordered sparse matrix format as:

```
\begin{array}{lll} \mathtt{ptrb} &=& [0,3,5,7],\\ \mathtt{ptre} &=& [3,5,7,8],\\ \mathtt{asub} &=& [0,2,3,1,4,0,3,2],\\ \mathtt{aval} &=& [1.1,1.3,1.4,2.2,2.5,3.1,3.4,4.4]. \end{array}
```

## 16.2 Functions grouped by topic

#### **Basis matrix**

• Infrequent: Task.basiscond, Task.initbasissolve, Task.solvewithbasis

#### Bound data

- Task.putconbound Changes the bound for one constraint.
- Task.putconboundlist Changes the bounds of a list of constraints.
- Task. putconboundslice Changes the bounds for a slice of the constraints.
- Task.putvarbound Changes the bound for one variable.
- Task. putvarboundlist Changes the bounds of a list of variables.
- Infrequent: Task.chgconbound, Task.chgvarbound, Task.getconbound, Task.getconbound, Task.getvarbound, Task.getvarboundslice
- Deprecated: Task.chgbound, Task.getbound, Task.getboundslice, Task.putbound, Task.putboundlist, Task.putboundslice

#### Conic constraint data

- Task. appended Appends a new conic constraint to the problem.
- Task.putcone Replaces a conic constraint.
- $\bullet$   $\it Task.\, remove cones$  Removes a number of conic constraints from the problem.
- Infrequent: Task.appendconeseq, Task.appendconesseq, Task.getcone, Task.getconeinfo, Task.qetnumcone, Task.qetnumconemem

#### Data file

- Task.readsolution Reads a solution from a file.
- Task.writedata Writes problem data to a file.
- Task.writesolution Write a solution to a file.
- $\begin{array}{ll} \bullet \ \, \textit{Infrequent:} & \textit{Task.readdata}, & \textit{Task.readdataformat}, & \textit{Task.readparamfile}, & \textit{Task.write} \\ \textit{write} \textit{jsonsol}, \textit{Task.write} \textit{paramfile} \\ \end{array}$

#### **Environment management**

- Env. licensecleanup Stops all threads and delete all handles used by the license system.
- Env. putlicensedebug Enables debug information for the license system.
- Env. putlicensepath Set the path to the license file.
- Env. putlicensewait Control whether mosek should wait for an available license if no license is available.
- Infrequent: Env.checkinall, Env.checkinlicense, Env.checkoutlicense, Env. putlicensecode

## Infeasibility diagnostics

- Task. getinfeasible subproblem Obtains an infeasible subproblem.
- Task.primalrepair Repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables.

#### Linear algebra

- Env. axpy Computes vector addition and multiplication by a scalar.
- Env. computesparsecholesky Computes a Cholesky factorization of sparse matrix.
- $\bullet$  Env. dot Computes the inner product of two vectors.
- Env. gemm Performs a dense matrix multiplication.
- Env. gemu Computes dense matrix times a dense vector product.
- Env. potrf Computes a Cholesky factorization of a dense matrix.
- Env. sparsetriangular solvedense Solves a sparse triangular system of linear equations.
- Env. syeig Computes all eigenvalues of a symmetric dense matrix.
- Env. syevd Computes all the eigenvalues and eigenvectors of a symmetric dense matrix, and thus its eigenvalue decomposition.
- Env. syrk Performs a rank-k update of a symmetric matrix.

#### Linear constraint data

- Task. appends on a number of constraints to the optimization task.
- Task. getnumcon Obtains the number of constraints.
- $\bullet$   $Task.\,putconboundslice$  Changes the bounds for a slice of the constraints.
- Task. removecons Removes a number of constraints.
- Infrequent: Task.getmaxnumcon

## Logging

- Task. linkfiletostream Directs all output from a task stream to a file.
- Infrequent: Env. linkfiletostream

#### Memory

• Infrequent: Task.checkmem, Task.getmemusage

#### **Naming**

- $\bullet$   $\it Task.putbarvarname$  Sets the name of a semidefinite variable.
- Task.putconename Sets the name of a cone.
- Task.putconname Sets the name of a constraint.
- Task. putobjname Assigns a new name to the objective.
- Task. puttaskname Assigns a new name to the task.
- Task.putvarname Sets the name of a variable.
- Infrequent: Task.getbarvarname, Task.getbarvarnameindex, Task.getbarvarnamelen, Task.getconename, Task.getconenameindex, Task.getconenamelen, Task.getconnameindex, Task.getconnamelen, Task.getobjname, Task.getobjnamelen, Task.gettaskname, Task.gettasknamelen, Task.getvarname, Task.getvarnameindex, Task.getvarnameindex, Task.getvarnamelen

#### Objective data

- Task. putcfix Replaces the fixed term in the objective.
- Task. putobjsense Sets the objective sense.
- Infrequent: Task.getobjsense

## Optimization

• Task.optimize - Optimizes the problem.

## **Optimizer statistics**

- Task. getdouinf Obtains a double information item.
- Task. getintinf Obtains an integer information item.
- $\bullet$   $\it Task.getlintinf$  Obtains a long integer information item.
- Infrequent: Task.getinfindex, Task.getinfmax, Task.getinfname

#### Parameter management

 $\bullet \ In frequent: \ Task. getnumparam, \ Task. getparammax, \ Task. getparammame, \ Task. is douparname, \ Task. is intparname, \ Task. is strparname, \ Task. set defaults$ 

## Parameters (get)

 $\bullet \ \textit{Infrequent: Task.getdouparam, Task.getintparam, Task.getstrparam, Task.getst$ 

## Parameters (put)

- Task.putdouparam Sets a double parameter.
- Task.putintparam Sets an integer parameter.
- Task.putstrparam Sets a string parameter.
- Infrequent: Task.putnadouparam, Task.putnaintparam, Task.putnastrparam, Task.putparam

#### Scalar variable data

- Task. appendvars Appends a number of variables to the optimization task.
- Task. getnumvar Obtains the number of variables.
- Task.putacol Replaces all elements in one column of the linear constraint matrix.
- Task. putaij Changes a single value in the linear coefficient matrix.
- Task. putarow Replaces all elements in one row of the linear constraint matrix.
- Task.putcj Modifies one linear coefficient in the objective.
- Task.putqcon Replaces all quadratic terms in constraints.
- $\bullet$   $\it Task.putqconk$  Replaces all quadratic terms in a single constraint.
- Task.putqobj Replaces all quadratic terms in the objective.
- Task. put qob ji j Replaces one coefficient in the quadratic term in the objective.
- Task.putvarboundslice Changes the bounds for a slice of the variables.
- Task. putvartype Sets the variable type of one variable.
- Task. removevars Removes a number of variables.
- Infrequent: Task.commitchanges, Task.getacol, Task.getacolnumnz, Task.  ${\it Task.getarow}\,,$ getacolslicetrip, Task.getaij, Task.getarownumnz, getarowslicetrip, Task.getc, Task.getcfix, Task.getcj, Task.getcslice, Task.getlenbarvarj, Task.getmaxnumanz, Task.getmaxnumqnz, Task.getmaxnumvar, Task. getnumanz, Task.getnumanz64, Task.getnumintvar, Task.getnumgconknz, qetnumqobjnz, Task.qetnumsymmat, Task.qetqconk, Task.qetqobj, Task.qetqobjij, Task.getsparsesymmat, Task.getsymmatinfo, Task.getvartype, Task.getvartypelist, Task.putacollist, Task.putacolslice, Task.putaijlist, Task.putarowlist, Task. putarowslice, Task.putclist, Task.putcslice, Task.putmaxnumanz, Task.putmaxnumqnz, Task.putmaxnumvar, Task.putvartypelist
- Deprecated: Task.getaslice

## Sensitivity analysis

- Task. dualsensitivity Performs sensitivity analysis on objective coefficients.
- Task.primalsensitivity Perform sensitivity analysis on bounds.
- Task.sensitivityreport Creates a sensitivity report.

## Solution (get)

- Task. getbarsj Obtains the dual solution for a semidefinite variable.
- Task. getbarxj Obtains the primal solution for a semidefinite variable.

- Task. qetskcslice Obtains the status keys for a slice of the constraints.
- Task. qetskxslice Obtains the status keys for a slice of the scalar variables.
- Task.getslcslice Obtains a slice of the slc vector for a solution.
- Task.getslxslice Obtains a slice of the slx vector for a solution.
- Task. getsnxslice Obtains a slice of the snx vector for a solution.
- Task. getsucslice Obtains a slice of the suc vector for a solution.
- Task.getsuxslice Obtains a slice of the sux vector for a solution.
- Task.getxcslice Obtains a slice of the xc vector for a solution.
- Task. getxxslice Obtains a slice of the xx vector for a solution.
- Task. getyslice Obtains a slice of the y vector for a solution.
- Infrequent: Task.getreducedcosts, Task.getskc, Task.getskx, Task.getslc, Task.getslx, Task.getsnx, Task.getsolution, Task.getsolutionslice, Task.getsuc, Task.getsux, Task.getxx, Task.getx
- Deprecated: Task.getsolutioni

### Solution (put)

- Task. putbars j Sets the dual solution for a semidefinite variable.
- Task. putbarxj Sets the primal solution for a semidefinite variable.
- Task.putskcslice Sets the status keys for a slice of the constraints.
- Task.putskxslice Sets the status keys for a slice of the variables.
- Task.putslcslice Sets a slice of the slc vector for a solution.
- $\bullet$  Task.putslxslice Sets a slice of the slx vector for a solution.
- Task.putsnxslice Sets a slice of the snx vector for a solution.
- Task.putsolution Inserts a solution.
- Task.putsucslice Sets a slice of the suc vector for a solution.
- $\bullet$   $\it Task.putsuxslice$  Sets a slice of the sux vector for a solution.
- Task.putxcslice Sets a slice of the xc vector for a solution.
- Task.putxxslice Obtains a slice of the xx vector for a solution.
- Task.putyslice Sets a slice of the y vector for a solution.
- Infrequent: Task.putskc, Task.putskx, Task.putslc, Task.putslx, Task.putsnx, Task.putsuc, Task.putsux, Task.putxx, Task.putxx, Task.puty
- Deprecated: Task.putsolutioni

## **Solution information**

- Task. getdualobj Computes the dual objective value associated with the solution.
- Task. getdualsolutionnorms Compute norms of the dual solution.
- Task. qetdviolbarvar Computes the violation of dual solution for a set of semidefinite variables.
- Task. getdviolcon Computes the violation of a dual solution associated with a set of constraints.
- Task. qetdviolcones Computes the violation of a solution for set of dual conic constraints.

- Task.getdviolvar Computes the violation of a dual solution associated with a set of scalar variables.
- Task.getprimalobj Computes the primal objective value for the desired solution.
- Task. getprimalsolutionnorms Compute norms of the primal solution.
- Task. getprosta Obtains the problem status.
- Task. getpuiolbarvar Computes the violation of a primal solution for a list of semidefinite variables.
- Task. getpuiolcon Computes the violation of a primal solution associated to a constraint.
- Task.getpviolcones Computes the violation of a solution for set of conic constraints.
- Task.getpviolvar Computes the violation of a primal solution for a list of scalar variables.
- Task. getsolsta Obtains the solution status.
- Task. getsolutioninfo Obtains information about of a solution.
- Task.solutiondef Checks whether a solution is defined.

#### Symmetric matrix variable data

- Task. appends arvars Appends semidefinite variables to the problem.
- Task. appendsparsesymmat Appends a general sparse symmetric matrix to the storage of symmetric matrices.
- Task. putbaraij Inputs an element of barA.
- Task.putbarcj Changes one element in barc.
- Infrequent: Task.getbarablocktriplet, Task.getbaraidx, Task.getbaraidxij, Task.getbaraidxinfo, Task.getbarasparsity, Task.getbarcblocktriplet, Task.getbarcidx, Task.getbarcidxinfo, Task.getbarcidxj, Task.getbarcsparsity, Task.getdimbarvarj, Task.getmaxnumbarvar, Task.getnumbarablocktriplets, Task.getnumbarcaz, Task.getnumbarcblocktriplets, Task.getnumbarvar, Task.putbarablocktriplet, Task.putbarcblocktriplet, Task.putmaxnumbarvar, Task.removebarvars

#### Task diagnostics

- Task.checkconvexity Checks if a quadratic optimization problem is convex.
- Task.getprobtype Obtains the problem type.
- Task. one solution summary Prints a short summary of a specified solution.
- Task.optimizersummary Prints a short summary with optimizer statistics from last optimization
- Task. solutionsummary Prints a short summary of the current solutions.
- Task.updatesolutioninfo Update the information items related to the solution.
- Infrequent: Task.analyzenames, Task.analyzeproblem, Task.analyzesolution, Env. echointro, Task.readsummary

#### Task management

• Infrequent: Task.deletesolution, Env.getcodedesc, Task.getmaxnumcone, Task.inputdata, Task.putmaxnumcon, Task.putmaxnumcone

#### Other

- Task.asyncgetresult Request a response from a remote job.
- Task. asyncoptimize Offload the optimization task to a solver server.
- Task. asyncpoll Requests information about the status of the remote job.
- Task. asyncstop Request that the job identified by the token is terminated.
- Env. dispose Free the underlying native allocation.
- Task. dispose Free the underlying native allocation.
- Env. getversion Obtains MOSEK version information.
- Task.optimizermt Offload the optimization task to a solver server.
- Task.putsolutionyi Inputs the dual variable of a solution.
- Task.readtask Load task data from a file.
- Task.resizetask Resizes an optimization task.
- Task.set\_InfoCallback Receive callbacks with solver status and information during optimization.
- Task.set\_ItgSolutionCallback Receive callbacks with solution updates from the mixed-integer optimizer.
- Task.set\_Progress Receive callbacks about current status of the solver during optimization.
- Env. set\_Stream Directs all output from an environment stream to a callback object.
- Task.set\_Stream Directs all output from a task stream to a callback object.
- Task. toconic In-place reformulation of a QCQP to a COP
- $\bullet$   $Task.unset\_Progress$  Deactivates all user callback functions.
- Task.writetask Write a complete binary dump of the task data.
- Infrequent: Task.getapiecenumnz, Task.strtoconetype, Task.strtosk
- Deprecated: Task.getaslicenumnz

## 16.3 Class Env

mosek.Env

The **MOSEK** global environment.

Env.Env

Env()

Env(String dbgfile)

Constructor of a new environment.

Parameters dbgfile (String) – File where the memory debugging log is written. (input)

Env.axpy

```
void axpy
  (int n,
   double alpha,
   double[] x,
   double[] y)
```

Adds  $\alpha x$  to y, i.e. performs the update

```
y := \alpha x + y.
```

Note that the result is stored overwriting y.

#### Parameters

- n (int) Length of the vectors. (input)
- alpha (double) The scalar that multiplies x. (input)
- x (double[]) The x vector. (input)
- y (double[]) The y vector. (input/output)

Groups Linear algebra

Env.checkinall

```
void checkinall ()
```

Check in all unused license features to the license token server.

Groups Environment management

Env.checkinlicense

```
void checkinlicense (mosek.feature feature)
```

Check in a license feature to the license server. By default all licenses consumed by functions using a single environment are kept checked out for the lifetime of the **MOSEK** environment. This function checks in a given license feature back to the license server immediately.

If the given license feature is not checked out at all, or it is in use by a call to <code>Task.optimize</code>, calling this function has no effect.

Please note that returning a license to the license server incurs a small overhead, so frequent calls to this function should be avoided.

Parameters feature (mosek.feature) - Feature to check in to the license system. (input)

Groups Environment management

Env.checkoutlicense

```
void checkoutlicense (mosek.feature feature)
```

Checks out a license feature from the license server. Normally the required license features will be automatically checked out the first time they are needed by the function <code>Task.optimize</code>. This function can be used to check out one or more features ahead of time.

The feature will remain checked out until the environment is deleted or the function Env. checkinlicense is called.

If a given feature is already checked out when this function is called, the call has no effect.

16.3. Class Env 171

Parameters feature (mosek.feature) - Feature to check out from the license system. (input)

Groups Environment management

Env.computesparsecholesky

```
void computesparsecholesky
  (int multithread,
   int ordermethod,
   double tolsingular,
   int[] anzc,
   long[] aptrc,
   int[] asubc,
   double[] avalc,
   int[][] perm,
   double[][] diag,
   int[][] lnzc,
   long[][] lptrc,
   long[][] lensubnval,
   int[][] lsubc,
   double[][] lvalc)
```

The function computes a Cholesky factorization of a sparse positive semidefinite matrix. Sparsity is exploited during the computations to reduce the amount of space and work required. Both the input and output matrices are represented using the sparse format.

To be precise, given a symmetric matrix  $A \in \mathbb{R}^{n \times n}$  the function computes a nonsingular lower triangular matrix L, a diagonal matrix D and a permutation matrix P such that

$$LL^T - D = PAP^T.$$

If ordermethod is zero then reordering heuristics are not employed and P is the identity.

If a pivot during the computation of the Cholesky factorization is less than

$$-\rho \cdot \max((PAP^T)_{jj}, 1.0)$$

then the matrix is declared negative semidefinite. On the hand if a pivot is smaller than

$$\rho \cdot \max((PAP^T)_{ij}, 1.0),$$

then  $D_{jj}$  is increased from zero to

$$\rho \cdot \max((PAP^T)_{jj}, 1.0).$$

Therefore, if A is sufficiently positive definite then D will be the zero matrix. Here  $\rho$  is set equal to value of tolsingular.

## Parameters

- multithread (int) If nonzero then the function may exploit multiple threads. (input)
- ordermethod (int) If nonzero, then a sparsity preserving ordering will be employed. (input)
- tolsingular (double) A positive parameter controlling when a pivot is declared zero. (input)
- anzc (int[]) anzc[j] is the number of nonzeros in the j-th column of A. (input)
- aptrc (long[]) aptrc[j] is a pointer to the first element in column j of A. (input)

- asubc (int[]) Row indexes for each column stored in increasing order. (input)
- avalc (double[]) The value corresponding to row indexed stored in asubc. (input)
- perm (int[] by reference) Permutation array used to specify the permutation matrix P computed by the function. (output)
- diag (double[] by reference) The diagonal elements of matrix D. (output)
- lnzc (int[] by reference) lnzc[j] is the number of non zero elements in column j of L. (output)
- lptrc (long[] by reference) lptrc[j] is a pointer to the first row index and value in column j of L. (output)
- lensubnval (long by reference) Number of elements in lsubc and lvalc. (output)
- lsubc (int[] by reference) Row indexes for each column stored in increasing order. (output)
- lvalc (double[] by reference) The values corresponding to row indexed stored in lsubc. (output)

Groups Linear algebra

Env.dispose

```
void dispose()
```

Free the underlying native allocation.

Env.dot

```
void dot
  (int n,
  double[] x,
  double[] y,
  double[] xty)
```

Computes the inner product of two vectors x, y of length  $n \geq 0$ , i.e

$$x \cdot y = \sum_{i=1}^{n} x_i y_i.$$

Note that if n = 0, then the result of the operation is 0.

### Parameters

- n (int) Length of the vectors. (input)
- x (double[]) The x vector. (input)
- y (double[]) The y vector. (input)
- xty (double by reference) The result of the inner product between x and y. (output)

Groups Linear algebra

Env.echointro

```
void echointro (int longver)
```

Prints an intro to message stream.

16.3. Class Env 173

Parameters longver (int) – If non-zero, then the intro is slightly longer. (input) Groups Task diagnostics

Env.gemm

```
void gemm
  (mosek.transpose transa,
  mosek.transpose transb,
  int m,
  int n,
  int k,
  double alpha,
  double[] a,
  double[] b,
  double beta,
  double[] c)
```

Performs a matrix multiplication plus addition of dense matrices. Given A, B and C of compatible dimensions, this function computes

$$C := \alpha op(A)op(B) + \beta C$$

where  $\alpha, \beta$  are two scalar values. The function op(X) denotes X if transX is transpose.no, or  $X^T$  if set to transpose.yes. The matrix C has m rows and n columns, and the other matrices must have compatible dimensions.

The result of this operation is stored in C.

#### **Parameters**

- transa (mosek.transpose) Indicates whether the matrix A must be transposed. (input)
- transb (mosek.transpose) Indicates whether the matrix B must be transposed. (input)
- m (int) Indicates the number of rows of matrix C. (input)
- n (int) Indicates the number of columns of matrix C. (input)
- k (int) Specifies the common dimension along which op(A) and op(B) are multiplied. For example, if neither A nor B are transposed, then this is the number of columns in A and also the number of rows in B. (input)
- alpha (double) A scalar value multiplying the result of the matrix multiplication. (input)
- a (double[]) The pointer to the array storing matrix A in a column-major format. (input)
- b (double[]) The pointer to the array storing matrix B in a column-major format. (input)
- ullet beta (double) A scalar value that multiplies C. (input)
- c (double[]) The pointer to the array storing matrix C in a column-major format. (input/output)

Groups Linear algebra

Env.gemv

```
void gemv
  (mosek.transpose transa,
  int m,
```

```
int n,
double alpha,
double[] a,
double[] x,
double beta,
double[] y)
```

Computes the multiplication of a scaled dense matrix times a dense vector, plus a scaled dense vector. Precisely, if trans is transpose.no then the update is

$$y := \alpha Ax + \beta y$$
,

and if trans is transpose. yes then

$$y := \alpha A^T x + \beta y,$$

where  $\alpha, \beta$  are scalar values and A is a matrix with m rows and n columns.

Note that the result is stored overwriting y.

### Parameters

- transa (mosek.transpose) Indicates whether the matrix A must be transposed. (input)
- m (int) Specifies the number of rows of the matrix A. (input)
- n (int) Specifies the number of columns of the matrix A. (input)
- alpha (double) A scalar value multiplying the matrix A. (input)
- a (double[]) A pointer to the array storing matrix A in a column-major format. (input)
- x (double[]) A pointer to the array storing the vector x. (input)
- beta (double) A scalar value multiplying the vector y. (input)
- y (double[]) A pointer to the array storing the vector y. (input/output)

Groups Linear algebra

Env.getcodedesc

```
static void getcodedesc
(mosek.rescode code,
StringBuffer symname,
StringBuffer str)
```

Obtains a short description of the meaning of the response code given by code.

### **Parameters**

- code (mosek.rescode) A valid MOSEK response code. (input)
- symname (StringBuffer) Symbolic name corresponding to code. (output)
- str (StringBuffer) Obtains a short description of a response code. (output)

Groups Task management

Env.getversion

```
static void getversion
  (int[] major,
  int[] minor,
```

16.3. Class Env 175

```
int[] build,
int[] revision)
```

Obtains MOSEK version information.

### **Parameters**

- major (int by reference) Major version number. (output)
- minor (int by reference) Minor version number. (output)
- build (int by reference) Build number. (output)
- revision (int by reference) Revision number. (output)

Env.licensecleanup

```
static void licensecleanup ()
```

Stops all threads and deletes all handles used by the license system. If this function is called, it must be called as the last **MOSEK** API call. No other **MOSEK** API calls are valid after this.

Groups Environment management

Env.linkfiletostream

```
void linkfiletostream
(mosek.streamtype whichstream,
String filename,
int append)
```

Sends all output from the stream defined by whichstream to the file given by filename.

### Parameters

- whichstream (mosek.streamtype) Index of the stream. (input)
- filename (String) A valid file name. (input)
- append (int) If this argument is 0 the file will be overwritten, otherwise it will be appended to. (input)

Groups Logging

Env.potrf

```
void potrf
  (mosek.uplo uplo,
   int n,
   double[] a)
```

Computes a Cholesky factorization of a real symmetric positive definite dense matrix.

### **Parameters**

- uplo (mosek.uplo) Indicates whether the upper or lower triangular part of the matrix is stored. (input)
- n (int) Dimension of the symmetric matrix. (input)
- a (double[]) A symmetric matrix stored in column-major order. Only the lower or the upper triangular part is used, accordingly with the uplo parameter. It will contain the result on exit. (input/output)

Groups Linear algebra

Env.putlicensecode

```
void putlicensecode (int[] code)
```

Input a runtime license code.

Parameters code (int[]) - A runtime license code. (input)

Groups Environment management

Env.putlicensedebug

```
void putlicensedebug (int licdebug)
```

Enables debug information for the license system. If licdebug is non-zero, then MOSEK will print debug info regarding the license checkout.

Parameters licdebug (int) – Whether license checkout debug info should be printed. (input)

Groups Environment management

Env.putlicensepath

```
void putlicensepath (String licensepath)
```

Set the path to the license file.

Parameters licensepath (String) – A path specifying where to search for the license. (input)

**Groups** Environment management

Env.putlicensewait

```
void putlicensewait (int licwait)
```

Control whether **MOSEK** should wait for an available license if no license is available. If licwait is non-zero, then **MOSEK** will wait for licwait-1 milliseconds between each check for an available license.

Parameters licwait (int) – Whether MOSEK should wait for a license if no license is available. (input)

Groups Environment management

Env.set\_Stream

```
void set_Stream
(mosek.streamtype whichstream,
mosek.Stream callback)
```

Directs all output from an environment stream to a callback object.

Can for example be called as:

Parameters

• whichstream (streamtype) - Index of the stream. (input)

16.3. Class Env 177

• callback (Stream) - The callback object. (input)

Env.sparsetriangularsolvedense

```
void sparsetriangularsolvedense
  (mosek.transpose transposed,
   int[] lnzc,
   long[] lptrc,
   int[] lsubc,
   double[] lvalc,
   double[] b)
```

The function solves a triangular system of the form

Lx = b

or

$$L^T x = b$$

where L is a sparse lower triangular nonsingular matrix. This implies in particular that diagonals in L are nonzero.

### Parameters

- transposed (mosek. transpose) Controls whether to use with L or  $L^T$ . (input)
- lnzc (int[]) lnzc[j] is the number of nonzeros in column j. (input)
- lptrc (long[]) lptrc[j] is a pointer to the first row index and value in column j. (input)
- lsubc (int[]) Row indexes for each column stored sequentially. Must be stored in increasing order for each column. (input)
- lvalc (double[]) The value corresponding to the row index stored in lsubc. (input)
- b (double[]) The right-hand side of linear equation system to be solved as a dense vector. (input/output)

Groups Linear algebra

Env.syeig

```
void syeig
  (mosek.uplo uplo,
  int n,
  double[] a,
  double[] w)
```

Computes all eigenvalues of a real symmetric matrix A. Given a matrix  $A \in \mathbb{R}^{n \times n}$  it returns a vector  $w \in \mathbb{R}^n$  containing the eigenvalues of A.

# Parameters

- uplo (mosek.uplo) Indicates whether the upper or lower triangular part is used. (input)
- n (int) Dimension of the symmetric input matrix. (input)
- a (double[]) A symmetric matrix A stored in column-major order. Only the part indicated by uplo is used. (input)

• w (double[]) - Array of length at least n containing the eigenvalues of A. (output)

# Groups Linear algebra

Env.syevd

```
void syevd
  (mosek.uplo uplo,
   int n,
   double[] a,
   double[] w)
```

Computes all the eigenvalues and eigenvectors a real symmetric matrix. Given the input matrix  $A \in \mathbb{R}^{n \times n}$ , this function returns a vector  $w \in \mathbb{R}^n$  containing the eigenvalues of A and it also computes the eigenvectors of A. Therefore, this function computes the eigenvalue decomposition of A as

$$A = UVU^T$$
.

where  $V = \mathbf{diag}(w)$  and U contains the eigenvectors of A.

Note that the matrix U overwrites the input data A.

#### **Parameters**

- uplo (mosek.uplo) Indicates whether the upper or lower triangular part is used. (input)
- n (int) Dimension of the symmetric input matrix. (input)
- a (double[]) A symmetric matrix A stored in column-major order. Only the part indicated by uplo is used. On exit it will be overwritten by the matrix U. (input/output)
- w (double[]) Array of length at least n containing the eigenvalues of A. (output)

Groups Linear algebra

Env.syrk

```
void syrk
  (mosek.uplo uplo,
  mosek.transpose trans,
  int n,
  int k,
  double alpha,
  double[] a,
  double beta,
  double[] c)
```

Performs a symmetric rank-k update for a symmetric matrix.

Given a symmetric matrix  $C \in \mathbb{R}^{n \times n}$ , two scalars  $\alpha, \beta$  and a matrix A of rank  $k \leq n$ , it computes either

$$C := \alpha A A^T + \beta C$$
,

when trans is set to transpose.no and  $A \in \mathbb{R}^{n \times k}$ , or

$$C := \alpha A^T A + \beta C,$$

when trans is set to transpose. yes and  $A \in \mathbb{R}^{k \times n}$ .

Only the part of C indicated by uplo is used and only that part is updated with the result.

16.3. Class Env 179

### Parameters

- uplo (mosek.uplo) Indicates whether the upper or lower triangular part of C is used. (input)
- trans (mosek. transpose) Indicates whether the matrix A must be transposed. (input)
- n (int) Specifies the order of C. (input)
- k (int) Indicates the number of rows or columns of A, depending on whether or not it is transposed, and its rank. (input)
- alpha (double) A scalar value multiplying the result of the matrix multiplication. (input)
- a (double[]) The pointer to the array storing matrix A in a column-major format. (input)
- beta (double) A scalar value that multiplies C. (input)
- c (double[]) The pointer to the array storing matrix C in a column-major format. (input/output)

Groups Linear algebra

# 16.4 Class Task

mosek.Task

Represents an optimization task.

Task.Task

```
Task(mosek.Env env)
```

```
Task(
  mosek.Env env,
  int numcon,
  int numvar)
```

```
Task(mosek.Task task)
```

Constructor of a new optimization task.

### **Parameters**

- env (Env) Parent environment. (input)
- numcon (int) An optional hint about the maximal number of constraints in the task. (input)
- numvar (int) An optional hint about the maximal number of variables in the task. (input)
- task (Task) A task that will be cloned. (input)

Task.analyzenames

```
void analyzenames
(mosek.streamtype whichstream,
mosek.nametype nametype)
```

The function analyzes the names and issues an error if a name is invalid.

### **Parameters**

- whichstream (mosek.streamtype) Index of the stream. (input)
- nametype (mosek.nametype) The type of names e.g. valid in MPS or LP files. (input)

Groups Task diagnostics

Task.analyzeproblem

```
void analyzeproblem (mosek.streamtype whichstream)
```

The function analyzes the data of a task and writes out a report.

```
Parameters whichstream (mosek.streamtype) - Index of the stream. (input)
Groups Task diagnostics
```

Task.analyzesolution

```
void analyzesolution
(mosek.streamtype whichstream,
mosek.soltype whichsol)
```

Print information related to the quality of the solution and other solution statistics.

By default this function prints information about the largest infeasibilities in the solution, the primal (and possibly dual) objective value and the solution status.

Following parameters can be used to configure the printed statistics:

- *iparam. ana\_sol\_basis* enables or disables printing of statistics specific to the basis solution (condition number, number of basic variables etc.). Default is on.
- *iparam.ana\_sol\_print\_violated* enables or disables listing names of all constraints (both primal and dual) which are violated by the solution. Default is off.
- dparam.ana\_sol\_infeas\_tol is the tolerance defining when a constraint is considered violated. If a constraint is violated more than this, it will be listed in the summary.

# Parameters

- whichstream (mosek.streamtype) Index of the stream. (input)
- whichsol (mosek.soltype) Selects a solution. (input)

Groups Task diagnostics

Task.appendbarvars

```
void appendbarvars (int[] dim)
```

Appends positive semidefinite matrix variables of dimensions given by dim to the problem.

Parameters dim (int[]) - Dimensions of symmetric matrix variables to be added. (input)

Groups Symmetric matrix variable data

Task.appendcone

```
void appendcone
  (mosek.conetype ct,
  double conepar,
  int[] submem)
```

Appends a new conic constraint to the problem. Hence, add a constraint

$$\hat{x} \in \mathcal{K}$$

to the problem where K is a convex cone.  $\hat{x}$  is a subset of the variables which will be specified by the argument submem.

Depending on the value of ct this function appends a normal (conetype.quad) or rotated quadratic cone (conetype.rquad).

Define

$$\hat{x} = x_{\mathtt{submem}[0]}, \dots, x_{\mathtt{submem}[\mathtt{nummem}-1]}.$$

Depending on the value of ct this function appends one of the constraints:

• Quadratic cone (conetype.quad):

$$\hat{x}_0 \geq \sqrt{\sum_{i=1}^{i < \text{nummem}} \hat{x}_i^2}$$

• Rotated quadratic cone (conetype.rquad):

$$2\hat{x}_0\hat{x}_1 \geq \sum_{i=2}^{i<\text{nummem}} \hat{x}_i^2, \quad \hat{x}_0, \hat{x}_1 \geq 0$$

Please note that the sets of variables appearing in different conic constraints must be disjoint.

For an explained code example see Section Conic Quadratic Optimization.

### **Parameters**

- ct (mosek.conetype) Specifies the type of the cone. (input)
- conepar (double) This argument is currently not used. It can be set to 0 (input)
- submem (int[]) Variable subscripts of the members in the cone. (input)

Groups Conic constraint data

Task.appendconeseq

```
void appendconeseq
  (mosek.conetype ct,
  double conepar,
  int nummem,
  int j)
```

Appends a new conic constraint to the problem, as in *Task.appendcone*. The function assumes the members of cone are sequential where the first member has index j and the last j+nummem-1.

### Parameters

- ct (mosek.conetype) Specifies the type of the cone. (input)
- conepar (double) This argument is currently not used. It can be set to 0 (input)

- nummem (int) Number of member variables in the cone. (input)
- j (int) Index of the first variable in the conic constraint. (input)

Groups Conic constraint data

Task.appendconesseq

```
void appendconesseq
  (mosek.conetype[] ct,
  double[] conepar,
  int[] nummem,
  int j)
```

Appends a number of conic constraints to the problem, as in Task.appendcone. The kth cone is assumed to be of dimension nummem[k]. Moreover, it is assumed that the first variable of the first cone has index j and starting from there the sequentially following variables belong to the first cone, then to the second cone and so on.

#### Parameters

- ct (mosek.conetype[]) Specifies the type of the cone. (input)
- conepar (double[]) This argument is currently not used. It can be set to 0 (input)
- nummem (int[]) Numbers of member variables in the cones. (input)
- j (int) Index of the first variable in the first cone to be appended. (input)

Groups Conic constraint data

Task.appendcons

```
void appendcons (int num)
```

Appends a number of constraints to the model. Appended constraints will be declared free. Please note that **MOSEK** will automatically expand the problem dimension to accommodate the additional constraints.

Parameters num (int) - Number of constraints which should be appended. (input)

Groups Linear constraint data

Task.appendsparsesymmat

```
void appendsparsesymmat
  (int dim,
  int[] subi,
  int[] subj,
  double[] valij,
  long[] idx)
```

```
long appendsparsesymmat
  (int dim,
  int[] subi,
  int[] subj,
  double[] valij)
```

**MOSEK** maintains a storage of symmetric data matrices that is used to build  $\overline{C}$  and  $\overline{A}$ . The storage can be thought of as a vector of symmetric matrices denoted E. Hence,  $E_i$  is a symmetric matrix of certain dimension.

This function appends a general sparse symmetric matrix on triplet form to the vector E of symmetric matrices. The vectors  $\mathtt{subi}$ ,  $\mathtt{subj}$ , and  $\mathtt{valij}$  contains the row subscripts, column subscripts and values of each element in the symmetric matrix to be appended. Since the matrix that is appended is symmetric, only the lower triangular part should be specified. Moreover, duplicates are not allowed.

Observe the function reports the index (position) of the appended matrix in E. This index should be used for later references to the appended matrix.

#### **Parameters**

- dim (int) Dimension of the symmetric matrix that is appended. (input)
- subi (int[]) Row subscript in the triplets. (input)
- subj (int[]) Column subscripts in the triplets. (input)
- valij (double[]) Values of each triplet. (input)
- idx (long by reference) Unique index assigned to the inputted matrix that can be used for later reference. (output)

**Return** (long) – Unique index assigned to the inputted matrix that can be used for later reference.

Groups Symmetric matrix variable data

Task.appendvars

```
void appendvars (int num)
```

Appends a number of variables to the model. Appended variables will be fixed at zero. Please note that **MOSEK** will automatically expand the problem dimension to accommodate the additional variables.

Parameters num (int) – Number of variables which should be appended. (input)

Groups Scalar variable data

Task.asyncgetresult

```
void asyncgetresult
  (String server,
   String port,
   String token,
   boolean[] respavailable,
   mosek.rescode[] resp,
   mosek.rescode[] trm)
```

```
boolean asyncgetresult
(String server,
String port,
String token,
mosek.rescode[] resp,
mosek.rescode[] trm)
```

Request a response from a remote job. If successful, solver response, termination code and solutions are retrieved.

# Parameters

- server (String) Name or IP address of the solver server. (input)
- port (String) Network port of the solver service. (input)
- token (String) The task token. (input)

- respavailable (boolean by reference) Indicates if a remote response is available. If this is not true, resp and trm should be ignored. (output)
- resp (mosek.rescode by reference) Is the response code from the remote solver. (output)
- trm (mosek.rescode by reference) Is either rescode.ok or a termination response code. (output)

Return (boolean) - Indicates if a remote response is available. If this is not true, resp and trm should be ignored.

### Task.asyncoptimize

```
void asyncoptimize
(String server,
String port,
StringBuffer token)
```

```
String asyncoptimize
(String server,
String port)
```

Offload the optimization task to a solver server defined by server:port. The call will return immediately and not wait for the result.

If the string parameter *sparam.remote\_access\_token* is not blank, it will be passed to the server as authentication.

### Parameters

- server (String) Name or IP address of the solver server (input)
- port (String) Network port of the solver service (input)
- token (StringBuffer) Returns the task token (output)

Return (String) - Returns the task token

# Task.asyncpoll

```
void asyncpoll
  (String server,
   String port,
   String token,
   boolean[] respavailable,
   mosek.rescode[] resp,
   mosek.rescode[] trm)
```

```
boolean asyncpoll
(String server,
String port,
String token,
mosek.rescode[] resp,
mosek.rescode[] trm)
```

Requests information about the status of the remote job.

# **Parameters**

- server (String) Name or IP address of the solver server (input)
- port (String) Network port of the solver service (input)
- token (String) The task token (input)

- respavailable (boolean by reference) Indicates if a remote response is available. If this is not true, resp and trm should be ignored. (output)
- resp (mosek.rescode by reference) Is the response code from the remote solver. (output)
- trm (mosek.rescode by reference) Is either rescode.ok or a termination response code. (output)

Return (boolean) - Indicates if a remote response is available. If this is not true, resp and trm should be ignored.

Task.asyncstop

```
void asyncstop
(String server,
String port,
String token)
```

Request that the job identified by the token is terminated.

#### Parameters

- server (String) Name or IP address of the solver server (input)
- port (String) Network port of the solver service (input)
- token (String) The task token (input)

Task.basiscond

```
void basiscond
(double[] nrmbasis,
double[] nrminvbasis)
```

If a basic solution is available and it defines a nonsingular basis, then this function computes the 1-norm estimate of the basis matrix and a 1-norm estimate for the inverse of the basis matrix. The 1-norm estimates are computed using the method outlined in [Ste98], pp. 388-391.

By definition the 1-norm condition number of a matrix B is defined as

$$\kappa_1(B) := \|B\|_1 \|B^{-1}\|_1.$$

Moreover, the larger the condition number is the harder it is to solve linear equation systems involving B. Given estimates for  $||B||_1$  and  $||B^{-1}||_1$  it is also possible to estimate  $\kappa_1(B)$ .

# Parameters

- nrmbasis (double by reference) An estimate for the 1-norm of the basis. (output)
- nrminvbasis (double by reference) An estimate for the 1-norm of the inverse of the basis. (output)

Groups Basis matrix

Task.checkconvexity

```
void checkconvexity ()
```

This function checks if a quadratic optimization problem is convex. The amount of checking is controlled by <code>iparam.check\_convexity</code>.

The function reports an error if the problem is not convex.

# Groups Task diagnostics

Task.checkmem

```
void checkmem
(String file,
int line)
```

Checks the memory allocated by the task.

### **Parameters**

- file (String) File from which the function is called. (input)
- line (int) Line in the file from which the function is called. (input)

Groups Memory

Task.chgbound Deprecated

```
void chgbound
  (mosek.accmode accmode,
   int i,
   int lower,
   int finite,
   double value)
```

Changes a bound for one constraint or variable. If accmode equals accmode.con, a constraint bound is changed, otherwise a variable bound is changed.

If lower is non-zero, then the lower bound is changed as follows:

$$\text{new lower bound} = \begin{cases} -\infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise.} \end{cases}$$

Otherwise if lower is zero, then

$$\label{eq:new_problem} \text{new upper bound} = \left\{ \begin{array}{ll} \infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Please note that this function automatically updates the bound key for bound, in particular, if the lower and upper bounds are identical, the bound key is changed to fixed.

# Parameters

- accmode (mosek.accmode) Defines if operations are performed row-wise (constraint-oriented) or column-wise (variable-oriented). (input)
- i (int) Index of the constraint or variable for which the bounds should be changed. (input)
- lower (int) If non-zero, then the lower bound is changed, otherwise the upper bound is changed. (input)
- finite (int) If non-zero, then value is assumed to be finite. (input)
- value (double) New value for the bound. (input)

Groups Bound data

Task.chgconbound

```
void chgconbound
  (int i,
   int lower,
```

```
int finite,
double value)
```

Changes a bound for one constraint.

If lower is non-zero, then the lower bound is changed as follows:

$$\text{new lower bound} = \left\{ \begin{array}{ll} -\infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Otherwise if lower is zero, then

$$\label{eq:new_problem} \text{new upper bound} = \left\{ \begin{array}{ll} \infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Please note that this function automatically updates the bound key for the bound, in particular, if the lower and upper bounds are identical, the bound key is changed to fixed.

### Parameters

- i (int) Index of the constraint for which the bounds should be changed. (input)
- lower (int) If non-zero, then the lower bound is changed, otherwise the upper bound is changed. (input)
- finite (int) If non-zero, then value is assumed to be finite. (input)
- value (double) New value for the bound. (input)

Groups Bound data

Task.chgvarbound

```
void chgvarbound
  (int j,
   int lower,
   int finite,
   double value)
```

Changes a bound for one variable.

If lower is non-zero, then the lower bound is changed as follows:

$$\text{new lower bound} = \left\{ \begin{array}{ll} -\infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Otherwise if lower is zero, then

$$\text{new upper bound} = \left\{ \begin{array}{ll} \infty, & \texttt{finite} = 0, \\ \texttt{value} & \text{otherwise}. \end{array} \right.$$

Please note that this function automatically updates the bound key for the bound, in particular, if the lower and upper bounds are identical, the bound key is changed to fixed.

# Parameters

- j (int) Index of the variable for which the bounds should be changed. (input)
- lower (int) If non-zero, then the lower bound is changed, otherwise the upper bound is changed. (input)
- finite (int) If non-zero, then value is assumed to be finite. (input)
- value (double) New value for the bound. (input)

Groups Bound data

Task.commitchanges

```
void commitchanges ()
```

Commits all cached problem changes to the task. It is usually not necessary to call this function explicitly since changes will be committed automatically when required.

Groups Scalar variable data

Task.deletesolution

```
void deletesolution (mosek.soltype whichsol)
```

Undefine a solution and free the memory it uses.

```
Parameters whichsol (mosek.soltype) - Selects a solution. (input)
```

Groups Task management

Task.dispose

```
void dispose()
```

Free the underlying native allocation.

Task.dualsensitivity

```
void dualsensitivity
  (int[] subj,
  double[] leftpricej,
  double[] rightpricej,
  double[] leftrangej,
  double[] rightrangej)
```

Calculates sensitivity information for objective coefficients. The indexes of the coefficients to analyze are

$$\{ \mathtt{subj}[i] \mid i = 0, \dots, \mathtt{numj} - 1 \}$$

The type of sensitivity analysis to perform (basis or optimal partition) is controlled by the parameter  $iparam.sensitivity\_type$ .

For an example, please see Section Example: Sensitivity Analysis.

# **Parameters**

- subj (int[]) Indexes of objective coefficients to analyze. (input)
- leftpricej (double[]) leftpricej[j] is the left shadow price for the coefficient with index subj[j]. (output)
- rightpricej (double[]) rightpricej[j] is the right shadow price for the coefficient with index subj[j]. (output)
- leftrangej (double[]) leftrangej[j] is the left range  $\beta_1$  for the coefficient with index subj[j]. (output)
- rightrangej (double[]) rightrangej[j] is the right range  $\beta_2$  for the coefficient with index subj[j]. (output)

Groups Sensitivity analysis

Task.getacol

```
void getacol
  (int j,
  int[] nzj,
  int[] subj,
  double[] valj)
```

Obtains one column of A in a sparse format.

# **Parameters**

- j (int) Index of the column. (input)
- nzj (int by reference) Number of non-zeros in the column obtained. (output)
- subj (int[]) Row indices of the non-zeros in the column obtained. (output)
- valj (double[]) Numerical values in the column obtained. (output)

Groups Scalar variable data

Task.getacolnumnz

```
void getacolnumnz
  (int i,
   int[] nzj)
```

```
int getacolnumnz (int i)
```

Obtains the number of non-zero elements in one column of A.

#### Parameters

- i (int) Index of the column. (input)
- nzj (int by reference) Number of non-zeros in the j-th column of A. (output)

**Return** (int) – Number of non-zeros in the *j*-th column of A.

Groups Scalar variable data

Task.getacolslicetrip

```
void getacolslicetrip
  (int first,
   int last,
   int[] subi,
   int[] subj,
   double[] val)
```

Obtains a sequence of columns from A in sparse triplet format. The function returns the content of all columns whose index j satisfies first  $\leq j \leq last$ . The triplets corresponding to nonzero entries are stored in the arrays subj, subj and val.

# Parameters

- first (int) Index of the first column in the sequence. (input)
- last (int) Index of the last column in the sequence plus one. (input)
- subi (int[]) Constraint subscripts. (output)
- subj (int[]) Column subscripts. (output)
- val (double[]) Values. (output)

Groups Scalar variable data

Task.getaij

```
void getaij
  (int i,
   int j,
   double[] aij)
```

```
double getaij
  (int i,
  int j)
```

Obtains a single coefficient in A.

# **Parameters**

- i (int) Row index of the coefficient to be returned. (input)
- j (int) Column index of the coefficient to be returned. (input)
- aij (double by reference) The required coefficient  $a_{i,j}$ . (output)

**Return** (double) – The required coefficient  $a_{i,j}$ .

Groups Scalar variable data

Task.getapiecenumnz

```
void getapiecenumnz
  (int firsti,
  int lasti,
  int firstj,
  int lastj,
  int[] numnz)
```

```
int getapiecenumnz
  (int firsti,
   int lasti,
   int firstj,
   int lastj)
```

Obtains the number non-zeros in a rectangular piece of A, i.e. the number of elements in the set

```
\{(i,j) : a_{i,j} \neq 0, \text{ firsti} \leq i \leq \text{lasti} - 1, \text{ firstj} \leq j \leq \text{lastj} - 1\}
```

This function is not an efficient way to obtain the number of non-zeros in one row or column. In that case use the function  $Task.\ getarounumnz$  or  $Task.\ getacolnumnz$ .

# Parameters

- firsti (int) Index of the first row in the rectangular piece. (input)
- lasti (int) Index of the last row plus one in the rectangular piece. (input)
- firstj (int) Index of the first column in the rectangular piece. (input)
- lastj (int) Index of the last column plus one in the rectangular piece. (input)
- numnz (int by reference) Number of non-zero A elements in the rectangular piece. (output)

**Return** (int) – Number of non-zero A elements in the rectangular piece.

Task.getarow

```
void getarow
  (int i,
   int[] nzi,
   int[] subi,
   double[] vali)
```

Obtains one row of A in a sparse format.

# **Parameters**

- i (int) Index of the row. (input)
- nzi (int by reference) Number of non-zeros in the row obtained. (output)
- subi (int[]) Column indices of the non-zeros in the row obtained. (output)
- vali (double[]) Numerical values of the row obtained. (output)

Groups Scalar variable data

Task.getarownumnz

```
void getarownumnz
  (int i,
  int[] nzi)
```

```
int getarownumnz (int i)
```

Obtains the number of non-zero elements in one row of A.

#### Parameters

- i (int) Index of the row. (input)
- nzi (int by reference) Number of non-zeros in the i-th row of A. (output)

**Return** (int) – Number of non-zeros in the i-th row of A.

Groups Scalar variable data

Task.getarowslicetrip

```
void getarowslicetrip
  (int first,
  int last,
  int[] subi,
  int[] subj,
  double[] val)
```

Obtains a sequence of rows from A in sparse triplet format. The function returns the content of all rows whose index i satisfies first  $\leq$  i  $\leq$  last. The triplets corresponding to nonzero entries are stored in the arrays subi, subj and val.

# Parameters

- first (int) Index of the first row in the sequence. (input)
- last (int) Index of the last row in the sequence plus one. (input)
- subi (int[]) Constraint subscripts. (output)
- subj (int[]) Column subscripts. (output)
- val (double[]) Values. (output)

Groups Scalar variable data

### Task.getaslice Deprecated

```
void getaslice
  (mosek.accmode accmode,
   int first,
   int last,
   int[] ptrb,
   int[] ptre,
   int[] sub,
   double[] val)
```

```
void getaslice
  (mosek.accmode accmode,
  int first,
  int last,
  long[] ptrb,
  long[] ptre,
  int[] sub,
  double[] val)
```

Obtains a sequence of rows or columns from A in sparse format.

# Parameters

- accmode (mosek.accmode) Defines whether a column slice or a row slice is requested. (input)
- first (int) Index of the first row or column in the sequence. (input)
- last (int) Index of the last row or column in the sequence plus one. (input)
- ptrb (int[]) ptrb[t] is an index pointing to the first element in the t-th row or column obtained. (output)
- ptrb (long[]) ptrb[t] is an index pointing to the first element in the t-th row or column obtained. (output)
- ptre (int[]) ptre[t] is an index pointing to the last element plus one in the *t*-th row or column obtained. (output)
- ptre (long[]) ptre[t] is an index pointing to the last element plus one in the t-th row or column obtained. (output)
- sub (int[]) Contains the row or column subscripts. (output)
- val (double[]) Contains the coefficient values. (output)

Groups Scalar variable data

# ${\tt Task.getaslicenumnz}\ Deprecated$

```
void getaslicenumnz
  (mosek.accmode accmode,
   int first,
   int last,
   long[] numnz)
```

```
long getaslicenumnz
  (mosek.accmode accmode,
  int first,
  int last)
```

Obtains the number of non-zeros in a slice of rows or columns of A.

### **Parameters**

- accmode (mosek.accmode) Defines whether non-zeros are counted in a column slice or a row slice. (input)
- first (int) Index of the first row or column in the sequence. (input)
- last (int) Index of the last row or column plus one in the sequence. (input)
- numnz (long by reference) Number of non-zeros in the slice. (output)

Return (long) - Number of non-zeros in the slice.

Task.getbarablocktriplet

```
void getbarablocktriplet
  (long[] num,
  int[] subi,
  int[] subj,
  int[] subk,
  int[] subl,
  double[] valijkl)
```

```
long getbarablocktriplet
  (int[] subi,
  int[] subj,
  int[] subk,
  int[] subl,
  double[] valijkl)
```

Obtains  $\overline{A}$  in block triplet form.

### **Parameters**

- num (long by reference) Number of elements in the block triplet form. (output)
- subi (int[]) Constraint index. (output)
- subj (int[]) Symmetric matrix variable index. (output)
- subk (int[]) Block row index. (output)
- subl (int[]) Block column index. (output)
- valijkl (double[]) The numerical value associated with each block triplet. (output)

**Return** (long) – Number of elements in the block triplet form.

Groups Symmetric matrix variable data

Task.getbaraidx

```
void getbaraidx
  (long idx,
   int[] i,
   int[] j,
   long[] num,
   long[] sub,
   double[] weights)
```

```
long getbaraidx
  (long idx,
   int[] i,
   int[] j,
   long[] sub,
   double[] weights)
```

Obtains information about an element in  $\overline{A}$ . Since  $\overline{A}$  is a sparse matrix of symmetric matrices, only the nonzero elements in  $\overline{A}$  are stored in order to save space. Now  $\overline{A}$  is stored vectorized i.e. as one long vector. This function makes it possible to obtain information such as the row index and the column index of a particular element of the vectorized form of  $\overline{A}$ .

Please observe if one element of  $\overline{A}$  is inputted multiple times then it may be stored several times in vectorized form. In that case the element with the highest index is the one that is used.

### **Parameters**

- idx (long) Position of the element in the vectorized form. (input)
- i (int by reference) Row index of the element at position idx. (output)
- j (int by reference) Column index of the element at position idx. (output)
- num (long by reference) Number of terms in weighted sum that forms the element. (output)
- sub (long[]) A list indexes of the elements from symmetric matrix storage that appear in the weighted sum. (output)
- weights (double[]) The weights associated with each term in the weighted sum. (output)

Return (long) – Number of terms in weighted sum that forms the element.

Groups Symmetric matrix variable data

Task.getbaraidxij

```
void getbaraidxij
  (long idx,
  int[] i,
  int[] j)
```

Obtains information about an element in  $\overline{A}$ . Since  $\overline{A}$  is a sparse matrix of symmetric matrices, only the nonzero elements in  $\overline{A}$  are stored in order to save space. Now  $\overline{A}$  is stored vectorized i.e. as one long vector. This function makes it possible to obtain information such as the row index and the column index of a particular element of the vectorized form of  $\overline{A}$ .

Please note that if one element of  $\overline{A}$  is inputted multiple times then it may be stored several times in vectorized form. In that case the element with the highest index is the one that is used.

# Parameters

- idx (long) Position of the element in the vectorized form. (input)
- i (int by reference) Row index of the element at position idx. (output)
- j (int by reference) Column index of the element at position idx. (output)

Groups Symmetric matrix variable data

Task.getbaraidxinfo

```
void getbaraidxinfo
  (long idx,
  long[] num)
```

```
long getbaraidxinfo (long idx)
```

Each nonzero element in  $\overline{A}_{ij}$  is formed as a weighted sum of symmetric matrices. Using this function the number of terms in the weighted sum can be obtained. See description of Task. appends parse symmat for details about the weighted sum.

### **Parameters**

- idx (long) The internal position of the element for which information should be obtained. (input)
- num (long by reference) Number of terms in the weighted sum that form the specified element in  $\overline{A}$ . (output)

**Return** (long) – Number of terms in the weighted sum that form the specified element in  $\overline{A}$ .

Groups Symmetric matrix variable data

Task.getbarasparsity

```
void getbarasparsity
  (long[] numnz,
   long[] idxij)
```

The matrix  $\overline{A}$  is assumed to be a sparse matrix of symmetric matrices. This implies that many of the elements in  $\overline{A}$  are likely to be zero matrices. Therefore, in order to save space, only nonzero elements in  $\overline{A}$  are stored on vectorized form. This function is used to obtain the sparsity pattern of  $\overline{A}$  and the position of each nonzero element in the vectorized form of  $\overline{A}$ . From the index detailed information about each nonzero  $\overline{A}_{i,j}$  can be obtained using Task.getbaraidxinfo and Task.getbaraidx.

### Parameters

- numnz (long by reference) Number of nonzero elements in  $\overline{A}$ . (output)
- idxij (long[]) Position of each nonzero element in the vectorized form of  $\overline{A}$ . (output)

Groups Symmetric matrix variable data

Task.getbarcblocktriplet

```
void getbarcblocktriplet
  (long[] num,
  int[] subj,
  int[] subk,
  int[] subl,
  double[] valjkl)
```

```
long getbarcblocktriplet
  (int[] subj,
   int[] subk,
   int[] subl,
   double[] valjkl)
```

Obtains  $\overline{C}$  in block triplet form.

# Parameters

- num (long by reference) Number of elements in the block triplet form. (output)
- subj (int[]) Symmetric matrix variable index. (output)
- subk (int[]) Block row index. (output)
- subl (int[]) Block column index. (output)
- valjkl (double[]) The numerical value associated with each block triplet. (output)

**Return** (long) – Number of elements in the block triplet form.

# Groups Symmetric matrix variable data

Task.getbarcidx

```
void getbarcidx
  (long idx,
  int[] j,
  long[] num,
  long[] sub,
  double[] weights)
```

Obtains information about an element in  $\overline{C}$ .

### **Parameters**

- idx (long) Index of the element for which information should be obtained. (input)
- j (int by reference) Row index in  $\overline{C}$ . (output)
- num (long by reference) Number of terms in the weighted sum. (output)
- sub (long[]) Elements appearing the weighted sum. (output)
- weights (double[]) Weights of terms in the weighted sum. (output)

Groups Symmetric matrix variable data

Task.getbarcidxinfo

```
void getbarcidxinfo
(long idx,
long[] num)
```

```
long getbarcidxinfo (long idx)
```

Obtains the number of terms in the weighted sum that forms a particular element in  $\overline{C}$ .

# Parameters

- idx (long) Index of the element for which information should be obtained. The value is an index of a symmetric sparse variable. (input)
- num (long by reference) Number of terms that appear in the weighted sum that forms the requested element. (output)

**Return** (long) – Number of terms that appear in the weighted sum that forms the requested element.

Groups Symmetric matrix variable data

Task.getbarcidxj

```
void getbarcidxj
  (long idx,
  int[] j)
```

Obtains the row index of an element in  $\overline{C}$ .

### Parameters

- idx (long) Index of the element for which information should be obtained. (input)
- j (int by reference) Row index in  $\overline{C}$ . (output)

Groups Symmetric matrix variable data

Task.getbarcsparsity

```
void getbarcsparsity
  (long[] numnz,
   long[] idxj)
```

Internally only the nonzero elements of  $\overline{C}$  are stored in a vector. This function is used to obtain the nonzero elements of  $\overline{C}$  and their indexes in the internal vector representation (in idx). From the index detailed information about each nonzero  $\overline{C}_j$  can be obtained using Task.getbarcidxinfo and Task.getbarcidx.

### **Parameters**

- numnz (long by reference) Number of nonzero elements in  $\overline{C}$ . (output)
- idxj (long[]) Internal positions of the nonzeros elements in  $\overline{C}$ . (output)

Groups Symmetric matrix variable data

Task.getbarsj

```
void getbarsj
  (mosek.soltype whichsol,
  int j,
  double[] barsj)
```

Obtains the dual solution for a semidefinite variable. Only the lower triangular part of  $\overline{S}_j$  is returned because the matrix by construction is symmetric. The format is that the columns are stored sequentially in the natural order.

# **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- j (int) Index of the semidefinite variable. (input)
- barsj (double[]) Value of  $\overline{S}_j$ . (output)

Groups Solution (get)

Task.getbarvarname

```
void getbarvarname
(int i,
StringBuffer name)
```

```
String getbarvarname (int i)
```

Obtains the name of a semidefinite variable.

# **Parameters**

- i (int) Index of the variable. (input)
- name (StringBuffer) The requested name is copied to this buffer. (output)

Return (String) - The requested name is copied to this buffer.

Groups Naming

Task.getbarvarnameindex

```
void getbarvarnameindex
  (String somename,
  int[] asgn,
  int[] index)
```

```
int getbarvarnameindex
  (String somename,
  int[] asgn)
```

Obtains the index of semidefinite variable from its name.

### **Parameters**

- somename (String) The name of the variable. (input)
- asgn (int by reference) Non-zero if the name somename is assigned to some semidefinite variable. (output)
- index (int by reference) The index of a semidefinite variable with the name somename (if one exists). (output)

**Return** (int) – The index of a semidefinite variable with the name somename (if one exists).

Groups Naming

Task.getbarvarnamelen

```
void getbarvarnamelen
  (int i,
   int[] len)
```

```
int getbarvarnamelen (int i)
```

Obtains the length of the name of a semidefinite variable.

### **Parameters**

- i (int) Index of the variable. (input)
- len (int by reference) Returns the length of the indicated name. (output)

Return (int) – Returns the length of the indicated name.

Groups Naming

Task.getbarxj

```
void getbarxj
  (mosek.soltype whichsol,
  int j,
  double[] barxj)
```

Obtains the primal solution for a semidefinite variable. Only the lower triangular part of  $\overline{X}_j$  is returned because the matrix by construction is symmetric. The format is that the columns are stored sequentially in the natural order.

# **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- j (int) Index of the semidefinite variable. (input)
- barxj (double[]) Value of  $\overline{X}_i$ . (output)

Groups Solution (get)

Task.getbound Deprecated

```
void getbound
  (mosek.accmode accmode,
   int i,
   mosek.boundkey[] bk,
   double[] bl,
   double[] bu)
```

Obtains bound information for one constraint or variable.

### Parameters

- accmode (mosek.accmode) Defines if operations are performed row-wise (constraint-oriented) or column-wise (variable-oriented). (input)
- i (int) Index of the constraint or variable for which the bound information should be obtained. (input)
- bk (mosek.boundkey by reference) Bound keys. (output)
- bl (double by reference) Values for lower bounds. (output)
- bu (double by reference) Values for upper bounds. (output)

Groups Bound data

 ${\tt Task.getboundslice}\ Deprecated$ 

```
void getboundslice
  (mosek.accmode accmode,
  int first,
  int last,
  mosek.boundkey[] bk,
  double[] bl,
  double[] bu)
```

Obtains bounds information for a slice of variables or constraints.

# Parameters

- accmode (mosek.accmode) Defines if operations are performed row-wise (constraint-oriented) or column-wise (variable-oriented). (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bk (mosek.boundkey[]) Bound keys. (output)
- bl (double[]) Values for lower bounds. (output)
- bu (double[]) Values for upper bounds. (output)

Groups Bound data

Task.getc

```
void getc (double[] c)
```

Obtains all objective coefficients c.

**Parameters** c (double[]) – Linear terms of the objective as a dense vector. The length is the number of variables. (output)

Groups Scalar variable data

Task.getcfix

```
void getcfix (double[] cfix)

double getcfix ()
```

Obtains the fixed term in the objective.

Parameters cfix (double by reference) - Fixed term in the objective. (output)

**Return** (double) – Fixed term in the objective.

Groups Scalar variable data

Task.getcj

```
void getcj
  (int j,
  double[] cj)
```

Obtains one coefficient of c.

### **Parameters**

- j (int) Index of the variable for which the c coefficient should be obtained. (input)
- cj (double by reference) The value of  $c_i$ . (output)

Groups Scalar variable data

Task.getconbound

```
void getconbound
  (int i,
   mosek.boundkey[] bk,
   double[] bl,
   double[] bu)
```

Obtains bound information for one constraint.

# **Parameters**

- i (int) Index of the constraint for which the bound information should be obtained. (input)
- bk (mosek.boundkey by reference) Bound keys. (output)
- bl (double by reference) Values for lower bounds. (output)
- bu (double by reference) Values for upper bounds. (output)

Groups Bound data

 ${\tt Task.get} conbound {\tt slice}$ 

```
void getconboundslice
  (int first,
  int last,
  mosek.boundkey[] bk,
  double[] bl,
  double[] bu)
```

Obtains bounds information for a slice of the constraints.

# Parameters

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bk (mosek.boundkey[]) Bound keys. (output)
- bl (double[]) Values for lower bounds. (output)
- bu (double[]) Values for upper bounds. (output)

Groups Bound data

Task.getcone

```
void getcone
  (int k,
  mosek.conetype[] ct,
  double[] conepar,
  int[] nummem,
  int[] submem)
```

Obtains a cone.

# Parameters

- k (int) Index of the cone. (input)
- ct (mosek.conetype by reference) Specifies the type of the cone. (output)
- conepar (double by reference) This argument is currently not used. It can be set to 0 (output)
- nummem (int by reference) Number of member variables in the cone. (output)
- submem (int[]) Variable subscripts of the members in the cone. (output)

Groups Conic constraint data

Task.getconeinfo

```
void getconeinfo
  (int k,
   mosek.conetype[] ct,
   double[] conepar,
   int[] nummem)
```

Obtains information about a cone.

# Parameters

- k (int) Index of the cone. (input)
- ct (mosek.conetype by reference) Specifies the type of the cone. (output)
- conepar (double by reference) This argument is currently not used. It can be set to 0 (output)
- nummem (int by reference) Number of member variables in the cone. (output)

Groups Conic constraint data

Task.getconename

```
void getconename
(int i,
StringBuffer name)
```

```
String getconename (int i)
```

Obtains the name of a cone.

#### **Parameters**

- i (int) Index of the cone. (input)
- name (StringBuffer) The required name. (output)

Return (String) - The required name.

Groups Naming

Task.getconenameindex

```
void getconenameindex
  (String somename,
   int[] asgn,
   int[] index)
```

```
int getconenameindex
  (String somename,
   int[] asgn)
```

Checks whether the name somename has been assigned to any cone. If it has been assigned to a cone, then the index of the cone is reported.

# Parameters

- somename (String) The name which should be checked. (input)
- asgn (int by reference) Is non-zero if the name somename is assigned to some cone. (output)
- index (int by reference) If the name somename is assigned to some cone, then index is the index of the cone. (output)

Return (int) – If the name somename is assigned to some cone, then index is the index of the cone.

Groups Naming

Task.getconenamelen

```
void getconenamelen
  (int i,
   int[] len)
```

```
int getconenamelen (int i)
```

Obtains the length of the name of a cone.

# Parameters

- i (int) Index of the cone. (input)
- len (int by reference) Returns the length of the indicated name. (output)

**Return** (int) – Returns the length of the indicated name.

Groups Naming

Task.getconname

```
void getconname
(int i,
StringBuffer name)
```

```
String getconname (int i)
```

Obtains the name of a constraint.

# **Parameters**

- i (int) Index of the constraint. (input)
- name (StringBuffer) The required name. (output)

Return (String) - The required name.

Groups Naming

Task.getconnameindex

```
void getconnameindex
  (String somename,
  int[] asgn,
  int[] index)
```

```
int getconnameindex
  (String somename,
   int[] asgn)
```

Checks whether the name somename has been assigned to any constraint. If so, the index of the constraint is reported.

# Parameters

- somename (String) The name which should be checked. (input)
- asgn (int by reference) Is non-zero if the name somename is assigned to some constraint. (output)
- index (int by reference) If the name somename is assigned to a constraint, then index is the index of the constraint. (output)

Return (int) - If the name somename is assigned to a constraint, then index is the index of the constraint.

Groups Naming

Task.getconnamelen

```
void getconnamelen
  (int i,
   int[] len)
```

```
int getconnamelen (int i)
```

Obtains the length of the name of a constraint.

# **Parameters**

- i (int) Index of the constraint. (input)
- len (int by reference) Returns the length of the indicated name. (output)

**Return** (int) – Returns the length of the indicated name.

Groups Naming

Task.getcslice

```
void getcslice
  (int first,
   int last,
   double[] c)
```

Obtains a sequence of elements in c.

### Parameters

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- c (double[]) Linear terms of the requested slice of the objective as a dense vector. The length is last-first. (output)

Groups Scalar variable data

Task.getdimbarvarj

```
void getdimbarvarj
  (int j,
   int[] dimbarvarj)
```

```
int getdimbarvarj (int j)
```

Obtains the dimension of a symmetric matrix variable.

# Parameters

- j (int) Index of the semidefinite variable whose dimension is requested. (input)
- dimbarvarj (int by reference) The dimension of the j-th semidefinite variable. (output)

**Return** (int) – The dimension of the j-th semidefinite variable.

Groups Symmetric matrix variable data

Task.getdouinf

```
void getdouinf
  (mosek.dinfitem whichdinf,
  double[] dvalue)
```

```
double getdouinf (mosek.dinfitem whichdinf)
```

Obtains a double information item from the task information database.

## Parameters

- whichdinf (mosek.dinfitem) Specifies a double information item. (input)
- dvalue (double by reference) The value of the required double information item. (output)

Return (double) - The value of the required double information item.

**Groups** Optimizer statistics

Task.getdouparam

```
void getdouparam
(mosek.dparam param,
double[] parvalue)
```

```
double getdouparam (mosek.dparam param)
```

Obtains the value of a double parameter.

# **Parameters**

- param (mosek.dparam) Which parameter. (input)
- parvalue (double by reference) Parameter value. (output)

Return (double) - Parameter value.

Groups Parameters (get)

Task.getdualobj

```
void getdualobj
  (mosek.soltype whichsol,
  double[] dualobj)
```

Computes the dual objective value associated with the solution. Note that if the solution is a primal infeasibility certificate, then the fixed term in the objective value is not included.

Moreover, since there is no dual solution associated with an integer solution, an error will be reported if the dual objective value is requested for the integer solution.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- dualobj (double by reference) Objective value corresponding to the dual solution. (output)

Groups Solution information

Task.getdualsolutionnorms

```
void getdualsolutionnorms
  (mosek.soltype whichsol,
  double[] nrmy,
  double[] nrmslc,
  double[] nrmsuc,
  double[] nrmslx,
  double[] nrmsux,
  double[] nrmsnx,
  double[] nrmsnx,
```

Compute norms of the dual solution.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- nrmy (double by reference) The norm of the y vector. (output)
- nrmslc (double by reference) The norm of the  $s_l^c$  vector. (output)
- ullet nrmsuc (double by reference) The norm of the  $s_u^c$  vector. (output)
- nrmslx (double by reference) The norm of the  $s_l^x$  vector. (output)

- nrmsux (double by reference) The norm of the  $s_u^x$  vector. (output)
- ullet nrmsnx (double by reference) The norm of the  $s_n^x$  vector. (output)
- nrmbars (double by reference) The norm of the  $\overline{S}$  vector. (output)

**Groups** Solution information

Task.getdviolbarvar

```
void getdviolbarvar
  (mosek.soltype whichsol,
  int[] sub,
  double[] viol)
```

Let  $(\overline{S}_j)^*$  be the value of variable  $\overline{S}_j$  for the specified solution. Then the dual violation of the solution associated with variable  $\overline{S}_j$  is given by

$$\max(-\lambda_{\min}(\overline{S}_j), 0.0).$$

Both when the solution is a certificate of primal infeasibility and when it is dual feasible solution the violation should be small.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of  $\overline{X}$  variables. (input)
- viol (double[]) viol[k] is the violation of the solution for the constraint  $\overline{S}_{\text{sub}[k]} \in \mathcal{S}_+$ . (output)

Groups Solution information

Task.getdviolcon

```
void getdviolcon
  (mosek.soltype whichsol,
  int[] sub,
  double[] viol)
```

The violation of the dual solution associated with the i-th constraint is computed as follows

$$\max(\rho((s_l^c)_i^*, (b_l^c)_i), \ \rho((s_u^c)_i^*, -(b_u^c)_i), \ |-y_i + (s_l^c)_i^* - (s_u^c)_i^*|)$$

where

$$\rho(x,l) = \left\{ \begin{array}{ll} -x, & l > -\infty, \\ |x|, & \text{otherwise.} \end{array} \right.$$

Both when the solution is a certificate of primal infeasibility or it is a dual feasible solution the violation should be small.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of constraints. (input)
- viol (double[]) viol[k] is the violation of dual solution associated with the constraint sub[k]. (output)

Groups Solution information

Task.getdviolcones

```
void getdviolcones
  (mosek.soltype whichsol,
  int[] sub,
  double[] viol)
```

Let  $(s_n^x)^*$  be the value of variable  $(s_n^x)$  for the specified solution. For simplicity let us assume that  $s_n^x$  is a member of a quadratic cone, then the violation is computed as follows

$$\left\{ \begin{array}{ll} \max(0, (\|s_n^x\|_{2:n}^* - (s_n^x)_1^*)/\sqrt{2}, & (s_n^x)^* \geq -\|(s_n^x)_{2:n}^*\|, \\ \|(s_n^x)^*\|, & \text{otherwise.} \end{array} \right.$$

Both when the solution is a certificate of primal infeasibility or when it is a dual feasible solution the violation should be small.

#### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of conic constraints. (input)
- viol (double[]) viol[k] is the violation of the dual solution associated with the conic constraint sub[k]. (output)

Groups Solution information

Task.getdviolvar

```
void getdviolvar
  (mosek.soltype whichsol,
  int[] sub,
  double[] viol)
```

The violation of the dual solution associated with the j-th variable is computed as follows

$$\max \left( \rho((s_l^x)_j^*, (b_l^x)_j), \ \rho((s_u^x)_j^*, -(b_u^x)_j), \ | \sum_{i=0}^{numcon-1} a_{ij} y_i + (s_l^x)_j^* - (s_u^x)_j^* - \tau c_j | \right)$$

where

$$\rho(x,l) = \begin{cases} -x, & l > -\infty, \\ |x|, & \text{otherwise} \end{cases}$$

and  $\tau = 0$  if the solution is a certificate of primal infeasibility and  $\tau = 1$  otherwise. The formula for computing the violation is only shown for the linear case but is generalized appropriately for the more general problems. Both when the solution is a certificate of primal infeasibility or when it is a dual feasible solution the violation should be small.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of x variables. (input)
- viol (double[]) viol[k] is the violation of dual solution associated with the variable sub[k]. (output)

Groups Solution information

Task.getinfeasiblesubproblem

```
void getinfeasiblesubproblem (mosek.soltype whichsol, Task[] inftask)
```

```
Task getinfeasiblesubproblem (mosek.soltype whichsol)
```

Given the solution is a certificate of primal or dual infeasibility then a primal or dual infeasible subproblem is obtained respectively. The subproblem tends to be much smaller than the original problem and hence it is easier to locate the infeasibility inspecting the subproblem than the original problem.

For the procedure to be useful it is important to assign meaningful names to constraints, variables etc. in the original task because those names will be duplicated in the subproblem.

The function is only applicable to linear and conic quadratic optimization problems.

For more information see Section Analyzing Infeasible Problems.

#### Parameters

- whichsol (mosek.soltype) Which solution to use when determining the infeasible subproblem. (input)
- inftask (*Task by reference*) A new task containing the infeasible subproblem. (output)

**Return** (*Task*) – A new task containing the infeasible subproblem.

**Groups** Infeasibility diagnostics

Task.getinfindex

```
void getinfindex
(mosek.inftype inftype,
String infname,
int[] infindex)
```

Obtains the index of a named information item.

### Parameters

- inftype (mosek.inftype) Type of the information item. (input)
- infname (String) Name of the information item. (input)
- infindex (int by reference) The item index. (output)

Groups Optimizer statistics

Task.getinfmax

```
void getinfmax
  (mosek.inftype inftype,
   int[] infmax)
```

Obtains the maximum index of an information item of a given type inftype plus 1.

### **Parameters**

- inftype (mosek.inftype) Type of the information item. (input)
- infmax (int[]) The maximum index (plus 1) requested. (output)

 ${\bf Groups} \ \ {\it Optimizer statistics}$ 

Task.getinfname

```
void getinfname
(mosek.inftype inftype,
int whichinf,
StringBuffer infname)
```

Obtains the name of an information item.

# **Parameters**

- inftype (mosek.inftype) Type of the information item. (input)
- whichinf (int) An information item. (input)
- infname (StringBuffer) Name of the information item. (output)

Groups Optimizer statistics

Task.getintinf

```
void getintinf
  (mosek.iinfitem whichiinf,
  int[] ivalue)
```

```
int getintinf (mosek.iinfitem whichiinf)
```

Obtains an integer information item from the task information database.

### **Parameters**

- whichiinf (mosek. iinfitem) Specifies an integer information item. (input)
- ivalue (int by reference) The value of the required integer information item. (output)

**Return** (int) – The value of the required integer information item.

**Groups** Optimizer statistics

Task.getintparam

```
void getintparam
(mosek.iparam param,
int[] parvalue)
```

```
int getintparam (mosek.iparam param)
```

Obtains the value of an integer parameter.

# **Parameters**

- param (mosek.iparam) Which parameter. (input)
- parvalue (int by reference) Parameter value. (output)

Return (int) - Parameter value.

Groups Parameters (get)

Task.getlenbarvarj

```
void getlenbarvarj
  (int j,
  long[] lenbarvarj)
```

```
long getlenbarvarj (int j)
```

Obtains the length of the j-th semidefinite variable i.e. the number of elements in the lower triangular part.

### **Parameters**

- j (int) Index of the semidefinite variable whose length if requested. (input)
- lenbarvarj (long by reference) Number of scalar elements in the lower triangular part of the semidefinite variable. (output)

**Return** (long) – Number of scalar elements in the lower triangular part of the semidefinite variable.

Groups Scalar variable data

Task.getlintinf

```
void getlintinf
  (mosek.liinfitem whichliinf,
  long[] ivalue)
```

```
long getlintinf (mosek.liinfitem whichliinf)
```

Obtains a long integer information item from the task information database.

#### **Parameters**

- whichliinf (mosek.liinfitem) Specifies a long information item. (input)
- ivalue (long by reference) The value of the required long integer information item. (output)

**Return** (long) – The value of the required long integer information item.

Groups Optimizer statistics

Task.getmaxnumanz

```
void getmaxnumanz (long[] maxnumanz)
```

```
long getmaxnumanz ()
```

Obtains number of preallocated non-zeros in A. When this number of non-zeros is reached **MOSEK** will automatically allocate more space for A.

**Parameters** maxnumanz (long by reference) – Number of preallocated non-zero linear matrix elements. (output)

Return (long) - Number of preallocated non-zero linear matrix elements.

Groups Scalar variable data

Task.getmaxnumbarvar

```
void getmaxnumbarvar (int[] maxnumbarvar)
int getmaxnumbarvar ()
```

Obtains maximum number of symmetric matrix variables for which space is currently preallocated.

**Parameters** maxnumbarvar (int by reference) – Maximum number of symmetric matrix variables for which space is currently preallocated. (output)

**Return** (int) – Maximum number of symmetric matrix variables for which space is currently preallocated.

Groups Symmetric matrix variable data

Task.getmaxnumcon

```
void getmaxnumcon (int[] maxnumcon)
```

Obtains the number of preallocated constraints in the optimization task. When this number of constraints is reached **MOSEK** will automatically allocate more space for constraints.

**Parameters** maxnumcon (int by reference) – Number of preallocated constraints in the optimization task. (output)

Groups Linear constraint data

Task.getmaxnumcone

```
void getmaxnumcone (int[] maxnumcone)
```

Obtains the number of preallocated cones in the optimization task. When this number of cones is reached **MOSEK** will automatically allocate space for more cones.

**Parameters** maxnumcone (int by reference) – Number of preallocated conic constraints in the optimization task. (output)

Groups Task management

Task.getmaxnumqnz

```
void getmaxnumqnz (long[] maxnumqnz)
```

Obtains the number of preallocated non-zeros for Q (both objective and constraints). When this number of non-zeros is reached **MOSEK** will automatically allocate more space for Q.

Parameters maxnumqnz (long by reference) – Number of non-zero elements preallocated in quadratic coefficient matrices. (output)

Groups Scalar variable data

Task.getmaxnumvar

```
void getmaxnumvar (int[] maxnumvar)
```

Obtains the number of preallocated variables in the optimization task. When this number of variables is reached **MOSEK** will automatically allocate more space for variables.

**Parameters** maxnumvar (int by reference) – Number of preallocated variables in the optimization task. (output)

Groups Scalar variable data

Task.getmemusage

```
void getmemusage
  (long[] meminuse,
  long[] maxmemuse)
```

Obtains information about the amount of memory used by a task.

**Parameters** 

- meminuse (long by reference) Amount of memory currently used by the task. (output)
- maxmemuse (long by reference) Maximum amount of memory used by the task until now. (output)

Groups Memory

Task.getnumanz

```
void getnumanz (int[] numanz)
```

int getnumanz ()

Obtains the number of non-zeros in A.

Parameters numanz (int by reference) – Number of non-zero elements in the linear constraint matrix. (output)

Return (int) - Number of non-zero elements in the linear constraint matrix.

Groups Scalar variable data

Task.getnumanz64

```
void getnumanz64 (long[] numanz)
```

long getnumanz64 ()

Obtains the number of non-zeros in A.

Parameters numanz (long by reference) – Number of non-zero elements in the linear constraint matrix. (output)

Return (long) - Number of non-zero elements in the linear constraint matrix.

Groups Scalar variable data

Task.getnumbarablocktriplets

```
void getnumbarablocktriplets (long[] num)
```

```
long getnumbarablocktriplets ()
```

Obtains an upper bound on the number of elements in the block triplet form of  $\overline{A}$ .

**Parameters** num (long by reference) – An upper bound on the number of elements in the block triplet form of  $\overline{A}$ . (output)

**Return** (long) – An upper bound on the number of elements in the block triplet form of  $\overline{A}$ .

Groups Symmetric matrix variable data

Task.getnumbaranz

```
void getnumbaranz (long[] nz)
```

```
long getnumbaranz ()
```

Get the number of nonzero elements in  $\overline{A}$ .

**Parameters** nz (long by reference) – The number of nonzero block elements in  $\overline{A}$  i.e. the number of  $\overline{A}_{ij}$  elements that are nonzero. (output)

**Return** (long) – The number of nonzero block elements in  $\overline{A}$  i.e. the number of  $\overline{A}_{ij}$  elements that are nonzero.

Groups Symmetric matrix variable data

Task.getnumbarcblocktriplets

```
void getnumbarcblocktriplets (long[] num)
```

```
long getnumbarcblocktriplets ()
```

Obtains an upper bound on the number of elements in the block triplet form of  $\overline{C}$ .

**Parameters** num (long by reference) – An upper bound on the number of elements in the block triplet form of  $\overline{C}$ . (output)

**Return** (long) – An upper bound on the number of elements in the block triplet form of  $\overline{C}$ .

Groups Symmetric matrix variable data

Task.getnumbarcnz

```
void getnumbarcnz (long[] nz)
```

```
long getnumbarcnz ()
```

Obtains the number of nonzero elements in  $\overline{C}$ .

**Parameters** nz (long by reference) – The number of nonzeros in  $\overline{C}$  i.e. the number of elements  $\overline{C}_j$  that are nonzero. (output)

**Return** (long) – The number of nonzeros in  $\overline{C}$  i.e. the number of elements  $\overline{C}_j$  that are nonzero.

Groups Symmetric matrix variable data

Task.getnumbarvar

```
void getnumbarvar (int[] numbarvar)
```

```
int getnumbarvar ()
```

Obtains the number of semidefinite variables.

**Parameters** numbarvar (int by reference) – Number of semidefinite variables in the problem. (output)

Return (int) – Number of semidefinite variables in the problem.

Groups Symmetric matrix variable data

Task.getnumcon

```
void getnumcon (int[] numcon)
```

```
int getnumcon ()
```

Obtains the number of constraints.

Parameters numcon (int by reference) – Number of constraints. (output)

Return (int) - Number of constraints.

Groups Linear constraint data

Task.getnumcone

```
void getnumcone (int[] numcone)
```

```
int getnumcone ()
```

Obtains the number of cones.

Parameters numcone (int by reference) – Number of conic constraints. (output)

Return (int) - Number of conic constraints.

Groups Conic constraint data

Task.getnumconemem

```
void getnumconemem
  (int k,
   int[] nummem)
```

Obtains the number of members in a cone.

### **Parameters**

- k (int) Index of the cone. (input)
- nummem (int by reference) Number of member variables in the cone. (output)

Groups Conic constraint data

Task.getnumintvar

```
void getnumintvar (int[] numintvar)
```

Obtains the number of integer-constrained variables.

Parameters numintvar (int by reference) – Number of integer variables. (output)

Groups Scalar variable data

Task.getnumparam

```
void getnumparam
  (mosek.parametertype partype,
  int[] numparam)
```

Obtains the number of parameters of a given type.

### Parameters

- partype (mosek.parametertype) Parameter type. (input)
- numparam (int by reference) The number of parameters of type partype. (output)

Groups Parameter management

Task.getnumqconknz

```
void getnumqconknz
  (int k,
  long[] numqcnz)
```

```
long getnumqconknz (int k)
```

Obtains the number of non-zero quadratic terms in a constraint.

### **Parameters**

- k (int) Index of the constraint for which the number quadratic terms should be obtained. (input)
- numqcnz (long by reference) Number of quadratic terms. (output)

Return (long) – Number of quadratic terms.

Groups Scalar variable data

Task.getnumqobjnz

```
void getnumqobjnz (long[] numqonz)
```

```
long getnumqobjnz ()
```

Obtains the number of non-zero quadratic terms in the objective.

Parameters numqonz (long by reference) - Number of non-zero elements in the quadratic objective terms. (output)

Return (long) - Number of non-zero elements in the quadratic objective terms.

Groups Scalar variable data

Task.getnumsymmat

```
void getnumsymmat (long[] num)
```

Obtains the number of symmetric matrices stored in the vector E.

Parameters num (long by reference) – The number of symmetric sparse matrices. (output)

Groups Scalar variable data

Task.getnumvar

```
void getnumvar (int[] numvar)
```

```
int getnumvar ()
```

Obtains the number of variables.

Parameters numvar (int by reference) - Number of variables. (output)

Return (int) – Number of variables.

Groups Scalar variable data

Task.getobjname

```
void getobjname (StringBuffer objname)
```

```
String getobjname ()
```

Obtains the name assigned to the objective function.

Parameters objname (StringBuffer) - Assigned the objective name. (output)

Return (String) - Assigned the objective name.

Groups Naming

Task.getobjnamelen

```
| void getobjnamelen (int[] len)
```

```
int getobjnamelen ()
```

Obtains the length of the name assigned to the objective function.

Parameters len (int by reference) – Assigned the length of the objective name. (output)

Return (int) - Assigned the length of the objective name.

Groups Naming

Task.getobjsense

```
void getobjsense (mosek.objsense[] sense)
```

```
mosek.objsense getobjsense ()
```

Gets the objective sense of the task.

Parameters sense (mosek.objsense by reference) - The returned objective sense. (output)

Return (mosek.objsense) - The returned objective sense.

Groups Objective data

Task.getparammax

```
void getparammax
  (mosek.parametertype partype,
  int[] parammax)
```

Obtains the maximum index of a parameter of type partype plus 1.

## Parameters

- partype (mosek.parametertype) Parameter type. (input)
- parammax (int by reference) The maximum index (plus 1) of the given parameter type. (output)

Groups Parameter management

Task.getparamname

```
void getparamname
(mosek.parametertype partype,
int param,
StringBuffer parname)
```

Obtains the name for a parameter param of type partype.

#### Parameters

- partype (mosek.parametertype) Parameter type. (input)
- param (int) Which parameter. (input)
- parname (StringBuffer) Parameter name. (output)

**Groups** Parameter management

Task.getprimalobj

```
void getprimalobj
  (mosek.soltype whichsol,
  double[] primalobj)
```

```
double getprimalobj (mosek.soltype whichsol)
```

Computes the primal objective value for the desired solution. Note that if the solution is an infeasibility certificate, then the fixed term in the objective is not included.

## **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- primalobj (double by reference) Objective value corresponding to the primal solution. (output)

Return (double) – Objective value corresponding to the primal solution.

**Groups** Solution information

Task.getprimalsolutionnorms

```
void getprimalsolutionnorms
  (mosek.soltype whichsol,
  double[] nrmxc,
  double[] nrmxx,
  double[] nrmbarx)
```

Compute norms of the primal solution.

## Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- nrmxc (double by reference) The norm of the  $x^c$  vector. (output)
- nrmxx (double by reference) The norm of the x vector. (output)
- nrmbarx (double by reference) The norm of the  $\overline{X}$  vector. (output)

Groups Solution information

 ${\tt Task.getprobtype}$ 

```
void getprobtype (mosek.problemtype[] probtype)
```

```
mosek.problemtype getprobtype ()
```

Obtains the problem type.

Parameters probtype (mosek.problemtype by reference) - The problem type. (output)

Return (mosek.problemtype) - The problem type.

Groups Task diagnostics

Task.getprosta

```
void getprosta
(mosek.soltype whichsol,
mosek.prosta[] prosta)
```

```
mosek.prosta getprosta (mosek.soltype whichsol)
```

Obtains the problem status.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- prosta (mosek.prosta by reference) Problem status. (output)

Return (mosek.prosta) - Problem status.

**Groups** Solution information

Task.getpviolbarvar

```
void getpviolbarvar
  (mosek.soltype whichsol,
  int[] sub,
  double[] viol)
```

Computes the primal solution violation for a set of semidefinite variables. Let  $(\overline{X}_j)^*$  be the value of the variable  $\overline{X}_j$  for the specified solution. Then the primal violation of the solution associated with variable  $\overline{X}_j$  is given by

$$\max(-\lambda_{\min}(\overline{X}_j), 0.0).$$

Both when the solution is a certificate of dual infeasibility or when it is primal feasible the violation should be small.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of  $\overline{X}$  variables. (input)
- viol (double[]) viol[k] is how much the solution violates the constraint  $\overline{X}_{\text{sub}[k]} \in \mathcal{S}_+$ . (output)

**Groups** Solution information

Task.getpviolcon

```
void getpviolcon
  (mosek.soltype whichsol,
  int[] sub,
  double[] viol)
```

Computes the primal solution violation for a set of constraints. The primal violation of the solution associated with the i-th constraint is given by

$$\max(\tau l_i^c - (x_i^c)^*, \ (x_i^c)^* - \tau u_i^c), \ |\sum_{j=0}^{numvar-1} a_{ij} x_j^* - x_i^c|)$$

where  $\tau=0$  if the solution is a certificate of dual infeasibility and  $\tau=1$  otherwise. Both when the solution is a certificate of dual infeasibility and when it is primal feasible the violation should be small. The above formula applies for the linear case but is appropriately generalized in other cases.

#### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of constraints. (input)
- viol (double[]) viol[k] is the violation associated with the solution for the constraint sub[k]. (output)

Groups Solution information

Task.getpviolcones

```
void getpviolcones
  (mosek.soltype whichsol,
  int[] sub,
  double[] viol)
```

Computes the primal solution violation for a set of conic constraints. Let  $x^*$  be the value of the variable x for the specified solution. For simplicity let us assume that x is a member of a quadratic cone, then the violation is computed as follows

$$\begin{cases} \max(0, ||x_{2:n}|| - x_1)/\sqrt{2}, & x_1 \ge -||x_{2:n}||, \\ ||x||, & \text{otherwise.} \end{cases}$$

Both when the solution is a certificate of dual infeasibility or when it is primal feasible the violation should be small.

## Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of conic constraints. (input)
- viol (double[]) viol[k] is the violation of the solution associated with the conic constraint number sub[k]. (output)

Groups Solution information

Task.getpviolvar

```
void getpviolvar
  (mosek.soltype whichsol,
  int[] sub,
  double[] viol)
```

Computes the primal solution violation associated to a set of variables. Let  $x_j^*$  be the value of  $x_j$  for the specified solution. Then the primal violation of the solution associated with variable  $x_j$  is given by

$$\max(\tau l_j^x - x_j^*, \ x_j^* - \tau u_j^x, \ 0).$$

where  $\tau = 0$  if the solution is a certificate of dual infeasibility and  $\tau = 1$  otherwise. Both when the solution is a certificate of dual infeasibility and when it is primal feasible the violation should be small.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of x variables. (input)

• viol (double[]) – viol[k] is the violation associated with the solution for the variable  $x_{\text{sub}[k]}$ . (output)

Groups Solution information

Task.getqconk

```
void getqconk
  (int k,
  long[] numqcnz,
  int[] qcsubi,
  int[] qcsubj,
  double[] qcval)
```

```
long getqconk
  (int k,
  int[] qcsubi,
  int[] qcsubj,
  double[] qcval)
```

Obtains all the quadratic terms in a constraint. The quadratic terms are stored sequentially in qcsubi, qcsubj, and qcval.

### **Parameters**

- k (int) Which constraint. (input)
- numqcnz (long by reference) Number of quadratic terms. (output)
- qcsubi (int[]) Row subscripts for quadratic constraint matrix. (output)
- qcsubj (int[]) Column subscripts for quadratic constraint matrix. (output)
- qcval (double[]) Quadratic constraint coefficient values. (output)

Return (long) – Number of quadratic terms.

Groups Scalar variable data

Task.getqobj

```
void getqobj
  (long[] numqonz,
   int[] qosubi,
   int[] qosubj,
   double[] qoval)
```

Obtains the quadratic terms in the objective. The required quadratic terms are stored sequentially in qosubi, qosubj, and qoval.

## Parameters

- numqonz (long by reference) Number of non-zero elements in the quadratic objective terms. (output)
- qosubi (int[]) Row subscripts for quadratic objective coefficients. (output)
- qosubj (int[]) Column subscripts for quadratic objective coefficients. (output)
- qoval (double[]) Quadratic objective coefficient values. (output)

Groups Scalar variable data

Task.getqobjij

```
void getqobjij
  (int i,
   int j,
   double[] qoij)
```

Obtains one coefficient  $q_{ij}^o$  in the quadratic term of the objective.

## **Parameters**

- i (int) Row index of the coefficient. (input)
- j (int) Column index of coefficient. (input)
- qoij (double by reference) The required coefficient. (output)

Groups Scalar variable data

Task.getreducedcosts

```
void getreducedcosts
  (mosek.soltype whichsol,
   int first,
   int last,
   double[] redcosts)
```

Computes the reduced costs for a slice of variables and returns them in the array redcosts i.e.

$$redcosts[j-first] = (s_l^x)_j - (s_u^x)_j, \ j = first, \dots, last - 1$$
 (16.2)

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) The index of the first variable in the sequence. (input)
- last (int) The index of the last variable in the sequence plus 1. (input)
- redcosts (double[]) The reduced costs for the required slice of variables. (output)

Groups Solution (get)

Task.getskc

```
void getskc
  (mosek.soltype whichsol,
  mosek.stakey[] skc)
```

Obtains the status keys for the constraints.

## Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- skc (mosek.stakey []) Status keys for the constraints. (output)

Groups Solution (get)

Task.getskcslice

```
void getskcslice
  (mosek.soltype whichsol,
  int first,
  int last,
  mosek.stakey[] skc)
```

Obtains the status keys for a slice of the constraints.

#### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- skc (mosek.stakey[]) Status keys for the constraints. (output)

Groups Solution (get)

Task.getskx

```
void getskx
(mosek.soltype whichsol,
mosek.stakey[] skx)
```

Obtains the status keys for the scalar variables.

## **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- skx (mosek.stakey[]) Status keys for the variables. (output)

Groups Solution (get)

Task.getskxslice

```
void getskxslice
  (mosek.soltype whichsol,
  int first,
  int last,
  mosek.stakey[] skx)
```

Obtains the status keys for a slice of the scalar variables.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- skx (mosek.stakey []) Status keys for the variables. (output)

Groups Solution (get)

Task.getslc

```
void getslc
  (mosek.soltype whichsol,
  double[] slc)
```

Obtains the  $s_I^c$  vector for a solution.

## Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (output)

Groups Solution (get)

Task.getslcslice

```
void getslcslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] slc)
```

Obtains a slice of the  $s_l^c$  vector for a solution.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (output)

Groups Solution (get)

Task.getslx

```
void getslx
  (mosek.soltype whichsol,
  double[] slx)
```

Obtains the  $s_l^x$  vector for a solution.

## Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (output)

Groups Solution (get)

Task.getslxslice

```
void getslxslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] slx)
```

Obtains a slice of the  $s_l^x$  vector for a solution.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (output)

Groups Solution (get)

 ${\tt Task.getsnx}$ 

```
void getsnx
  (mosek.soltype whichsol,
  double[] snx)
```

Obtains the  $s_n^x$  vector for a solution.

## **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (output)

Groups Solution (get)

Task.getsnxslice

```
void getsnxslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] snx)
```

Obtains a slice of the  $s_n^x$  vector for a solution.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (output)

Groups Solution (get)

Task.getsolsta

```
void getsolsta
(mosek.soltype whichsol,
mosek.solsta[] solsta)
```

```
mosek.solsta getsolsta (mosek.soltype whichsol)
```

Obtains the solution status.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- solsta (mosek.solsta by reference) Solution status. (output)

Return (mosek.solsta) - Solution status.

Groups Solution information

Task.getsolution

```
void getsolution
  (mosek.soltype whichsol,
  mosek.prosta[] prosta,
  mosek.solsta[] solsta,
  mosek.stakey[] skc,
```

```
mosek.stakey[] skx,
mosek.stakey[] skn,
double[] xc,
double[] xx,
double[] y,
double[] slc,
double[] suc,
double[] sux,
double[] slx,
double[] snx)
```

Obtains the complete solution.

Consider the case of linear programming. The primal problem is given by

and the corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x & = c, \\ & -y + s_l^c - s_u^c & = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0. \end{array}$$

A conic optimization problem has the same primal variables as in the linear case. Recall that the dual of a conic optimization problem is given by:

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + s_n^x & = & c, \\ & -y + s_l^c - s_u^c & = & 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x & \geq & 0, \\ & s_n^x \in \mathcal{K}^* & \end{array}$$

The mapping between variables and arguments to the function is as follows:

- xx: Corresponds to variable x (also denoted  $x^x$ ).
- xc : Corresponds to  $x^c := Ax$ .
- y: Corresponds to variable y.
- slc: Corresponds to variable  $s_l^c$ .
- suc: Corresponds to variable  $s_u^c$ .
- slx: Corresponds to variable  $s_l^x$ .
- sux: Corresponds to variable  $s_u^x$ .
- snx: Corresponds to variable  $s_n^x$ .

The meaning of the values returned by this function depend on the *solution status* returned in the argument solsta. The most important possible values of solsta are:

- solsta.optimal: An optimal solution satisfying the optimality criteria for continuous problems is returned.
- solsta.integer\_optimal: An optimal solution satisfying the optimality criteria for integer problems is returned.
- $\bullet \ solsta.prim\_feas$  : A solution satisfying the feasibility criteria.
- solsta.prim\_infeas\_cer: A primal certificate of infeasibility is returned.

• solsta.dual\_infeas\_cer: A dual certificate of infeasibility is returned.

In order to retrieve the primal and dual values of semidefinite variables see *Task.getbarxj* and *Task.getbarxj*.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- prosta (mosek.prosta by reference) Problem status. (output)
- solsta (mosek.solsta by reference) Solution status. (output)
- skc (mosek.stakey []) Status keys for the constraints. (output)
- skx (mosek.stakey []) Status keys for the variables. (output)
- skn (mosek.stakey[]) Status keys for the conic constraints. (output)
- xc (double[]) Primal constraint solution. (output)
- xx (double[]) Primal variable solution. (output)
- y (double[]) Vector of dual variables corresponding to the constraints. (output)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (output)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (output)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (output)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (output)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (output)

Groups Solution (get)

Task.getsolutioni Deprecated

```
void getsolutioni
  (mosek.accmode accmode,
   int i,
   mosek.soltype whichsol,
   mosek.stakey[] sk,
   double[] x,
   double[] sl,
   double[] su,
   double[] su,
```

Obtains the primal and dual solution information for a single constraint or variable.

### **Parameters**

- accmode (mosek.accmode) Defines whether solution information for a constraint or for a variable is retrieved. (input)
- i (int) Index of the constraint or variable. (input)
- whichsol (mosek.soltype) Selects a solution. (input)
- sk (mosek.stakey by reference) Status key of the constraint of variable. (output)
- x (double by reference) Solution value of the primal variable. (output)

- sl (double by reference) Solution value of the dual variable associated with the lower bound. (output)
- su (double by reference) Solution value of the dual variable associated with the upper bound. (output)
- sn (double by reference) Solution value of the dual variable associated with the cone constraint. (output)

Groups Solution (get)

Task.getsolutioninfo

```
void getsolutioninfo
  (mosek.soltype whichsol,
   double[] pobj,
   double[] pviolcon,
   double[] pviolvar,
   double[] pviolbarvar,
   double[] pviolcone,
   double[] pviolitg,
   double[] dobj,
   double[] dviolcon,
   double[] dviolvar,
   double[] dviolvar,
   double[] dviolbarvar,
   double[] dviolcone)
```

Obtains information about a solution.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- pobj (double by reference) The primal objective value as computed by Task. getprimalobj. (output)
- pviolcon (double by reference) Maximal primal violation of the solution associated with the  $x^c$  variables where the violations are computed by Task. getpviolcon. (output)
- pviolvar (double by reference) Maximal primal violation of the solution for the x variables where the violations are computed by Task.getpviolvar. (output)
- pviolbarvar (double by reference) Maximal primal violation of solution for the  $\overline{X}$  variables where the violations are computed by Task.getpviolbarvar. (output)
- pviolcone (double by reference) Maximal primal violation of solution for the conic constraints where the violations are computed by Task.getpviolcones. (output)
- pviolitg (double by reference) Maximal violation in the integer constraints. The violation for an integer variable  $x_j$  is given by  $\min(x_j \lfloor x_j \rfloor, \lceil x_j \rceil x_j)$ . This number is always zero for the interior-point and basic solutions. (output)
- dobj (double by reference) Dual objective value as computed by Task. getdualobj. (output)
- dviolcon (double by reference) Maximal violation of the dual solution associated with the  $x^c$  variable as computed by Task.getdviolcon. (output)
- dviolvar (double by reference) Maximal violation of the dual solution associated with the x variable as computed by Task. getdviolvar. (output)

- dviolbarvar (double by reference) Maximal violation of the dual solution associated with the  $\overline{S}$  variable as computed by Task.getdviolbarvar. (output)
- dviolcone (double by reference) Maximal violation of the dual solution associated with the dual conic constraints as computed by Task. getdviolcones. (output)

Groups Solution information

Task.getsolutionslice

```
void getsolutionslice
  (mosek.soltype whichsol,
  mosek.solitem solitem,
  int first,
  int last,
  double[] values)
```

Obtains a slice of one item from the solution. The format of the solution is exactly as in *Task*. getsolution. The parameter solitem determines which of the solution vectors should be returned.

#### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- solitem (mosek.solitem) Which part of the solution is required. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- values (double[]) The values in the required sequence are stored sequentially in values. (output)

Groups Solution (get)

Task.getsparsesymmat

```
void getsparsesymmat
  (long idx,
  int[] subi,
  int[] subj,
  double[] valij)
```

Get a single symmetric matrix from the matrix store.

# Parameters

- idx (long) Index of the matrix to retrieve. (input)
- subi (int[]) Row subscripts of the matrix non-zero elements. (output)
- subj (int[]) Column subscripts of the matrix non-zero elements. (output)
- valij (double[]) Coefficients of the matrix non-zero elements. (output)

Groups Scalar variable data

Task.getstrparam

```
void getstrparam
(mosek.sparam param,
int[] len,
StringBuffer parvalue)
```

```
String getstrparam
(mosek.sparam param,
int[] len)
```

Obtains the value of a string parameter.

## Parameters

- param (mosek.sparam) Which parameter. (input)
- len (int by reference) The length of the parameter value. (output)
- parvalue (StringBuffer) Parameter value. (output)

Return (String) - Parameter value.

Groups Parameters (get)

 ${\tt Task.getstrparamlen}$ 

```
void getstrparamlen
(mosek.sparam param,
int[] len)
```

```
int getstrparamlen (mosek.sparam param)
```

Obtains the length of a string parameter.

### **Parameters**

- param (mosek.sparam) Which parameter. (input)
- len (int by reference) The length of the parameter value. (output)

Return (int) - The length of the parameter value.

Groups Parameters (get)

Task.getsuc

```
void getsuc
  (mosek.soltype whichsol,
  double[] suc)
```

Obtains the  $s_n^c$  vector for a solution.

## Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (output)

Groups Solution (get)

Task.getsucslice

```
void getsucslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] suc)
```

Obtains a slice of the  $s_u^c$  vector for a solution.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (output)

Groups Solution (get)

Task.getsux

```
void getsux
  (mosek.soltype whichsol,
  double[] sux)
```

Obtains the  $s_u^x$  vector for a solution.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (output)

Groups Solution (get)

Task.getsuxslice

```
void getsuxslice
  (mosek.soltype whichsol,
   int first,
   int last,
   double[] sux)
```

Obtains a slice of the  $s_u^x$  vector for a solution.

## Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (output)

Groups Solution (get)

Task.getsymmatinfo

```
void getsymmatinfo
  (long idx,
  int[] dim,
  long[] nz,
  mosek.symmattype[] type)
```

**MOSEK** maintains a vector denoted by E of symmetric data matrices. This function makes it possible to obtain important information about a single matrix in E.

## Parameters

- idx (long) Index of the matrix for which information is requested. (input)
- dim (int by reference) Returns the dimension of the requested matrix. (output)

- nz (long by reference) Returns the number of non-zeros in the requested matrix. (output)
- type (mosek.symmattype by reference) Returns the type of the requested matrix. (output)

Groups Scalar variable data

Task.gettaskname

```
void gettaskname (StringBuffer taskname)
```

```
String gettaskname ()
```

Obtains the name assigned to the task.

Parameters taskname (StringBuffer) - Returns the task name. (output)

Return (String) - Returns the task name.

Groups Naming

Task.gettasknamelen

```
void gettasknamelen (int[] len)
```

```
int gettasknamelen ()
```

Obtains the length the task name.

Parameters len (int by reference) – Returns the length of the task name. (output)

Return (int) – Returns the length of the task name.

Groups Naming

Task.getvarbound

```
void getvarbound
  (int i,
   mosek.boundkey[] bk,
   double[] bl,
   double[] bu)
```

Obtains bound information for one variable.

# **Parameters**

- i (int) Index of the variable for which the bound information should be obtained. (input)
- bk (mosek.boundkey by reference) Bound keys. (output)
- bl (double by reference) Values for lower bounds. (output)
- bu (double by reference) Values for upper bounds. (output)

Groups Bound data

Task.getvarboundslice

```
void getvarboundslice
  (int first,
  int last,
```

```
mosek.boundkey[] bk,
double[] bl,
double[] bu)
```

Obtains bounds information for a slice of the variables.

### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bk (mosek.boundkey[]) Bound keys. (output)
- bl (double[]) Values for lower bounds. (output)
- bu (double[]) Values for upper bounds. (output)

Groups Bound data

Task.getvarname

```
void getvarname
(int j,
StringBuffer name)
```

```
String getvarname (int j)
```

Obtains the name of a variable.

### Parameters

- j (int) Index of a variable. (input)
- name (StringBuffer) Returns the required name. (output)

**Return** (String) – Returns the required name.

Groups Naming

Task.getvarnameindex

```
void getvarnameindex
  (String somename,
  int[] asgn,
  int[] index)
```

```
int getvarnameindex
  (String somename,
   int[] asgn)
```

Checks whether the name somename has been assigned to any variable. If so, the index of the variable is reported.

## **Parameters**

- somename (String) The name which should be checked. (input)
- asgn (int by reference) Is non-zero if the name somename is assigned to a variable. (output)
- index (int by reference) If the name somename is assigned to a variable, then index is the index of the variable. (output)

Return (int) - If the name somename is assigned to a variable, then index is the index of the variable.

# Groups Naming

Task.getvarnamelen

```
void getvarnamelen
  (int i,
   int[] len)
```

```
int getvarnamelen (int i)
```

Obtains the length of the name of a variable.

### **Parameters**

- i (int) Index of a variable. (input)
- len (int by reference) Returns the length of the indicated name. (output)

Return (int) – Returns the length of the indicated name.

Groups Naming

Task.getvartype

```
void getvartype
  (int j,
   mosek.variabletype[] vartype)
```

```
mosek.variabletype getvartype (int j)
```

Gets the variable type of one variable.

## Parameters

- j (int) Index of the variable. (input)
- vartype (mosek.variabletype by reference) Variable type of the j-th variable. (output)

Return (mosek.variabletype) - Variable type of the j-th variable.

Groups Scalar variable data

Task.getvartypelist

```
void getvartypelist
  (int[] subj,
  mosek.variabletype[] vartype)
```

Obtains the variable type of one or more variables. Upon return vartype[k] is the variable type of variable subj[k].

### Parameters

- subj (int[]) A list of variable indexes. (input)
- vartype (mosek.variabletype[]) The variables types corresponding to the variables specified by subj. (output)

Groups Scalar variable data

Task.getxc

```
void getxc
  (mosek.soltype whichsol,
  double[] xc)
```

Obtains the  $x^c$  vector for a solution.

## Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- xc (double[]) Primal constraint solution. (output)

Groups Solution (get)

Task.getxcslice

```
void getxcslice
  (mosek.soltype whichsol,
   int first,
   int last,
   double[] xc)
```

Obtains a slice of the  $x^c$  vector for a solution.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- xc (double[]) Primal constraint solution. (output)

Groups Solution (get)

Task.getxx

```
void getxx
(mosek.soltype whichsol,
double[] xx)
```

Obtains the  $x^x$  vector for a solution.

## **Parameters**

- $\bullet \ \ which sol \ (\textit{mosek.soltype}) Selects \ a \ solution. \ (input)$
- xx (double[]) Primal variable solution. (output)

Groups Solution (get)

Task.getxxslice

```
void getxxslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] xx)
```

Obtains a slice of the  $x^x$  vector for a solution.

# **Parameters**

• whichsol (mosek.soltype) - Selects a solution. (input)

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- xx (double[]) Primal variable solution. (output)

Groups Solution (get)

Task.gety

```
void gety
  (mosek.soltype whichsol,
  double[] y)
```

Obtains the y vector for a solution.

## **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (output)

Groups Solution (get)

Task.getyslice

```
void getyslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] y)
```

Obtains a slice of the y vector for a solution.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (output)

Groups Solution (get)

Task.initbasissolve

```
void initbasissolve (int[] basis)
```

Prepare a task for use with the Task. solvewithbasis function.

This function should be called

- immediately before the first call to Task. solvewithbasis, and
- immediately before any subsequent call to Task. solvewithbasis if the task has been modified.

If the basis is singular i.e. not invertible, then the error rescode.err\_basis\_singular is reported.

```
Parameters basis (int[]) – The array of basis indexes to use. The array is interpreted as follows: If basis[i] \leq numcon - 1, then x_{basis[i]}^c is in the basis at position i, otherwise x_{basis[i]-numcon} is in the basis at position i. (output)
```

## Groups Basis matrix

### Task.inputdata

```
void inputdata
  (int maxnumcon,
   int maxnumvar,
   double[] c,
   double cfix,
   int[] aptrb,
   int[] aptre,
   int[] asub,
   double[] aval,
   mosek.boundkey[] bkc,
   double[] blc,
   double[] buc,
   mosek.boundkey[] bkx,
   double[] blx,
   double[] blx,
   double[] blx,
```

```
void inputdata
  (int maxnumcon,
   int maxnumvar,
   double[] c,
   double cfix,
   long[] aptrb,
   long[] aptre,
   int[] asub,
   double[] aval,
   mosek.boundkey[] bkc,
   double[] blc,
   double[] blc,
   mosek.boundkey[] bkx,
   double[] blx,
   double[] blx,
```

Input the linear part of an optimization task in one function call.

## **Parameters**

- maxnumcon (int) Number of preallocated constraints in the optimization task. (input)
- maxnumvar (int) Number of preallocated variables in the optimization task. (input)
- c (double[]) Linear terms of the objective as a dense vector. The length is the number of variables. (input)
- cfix (double) Fixed term in the objective. (input)
- aptrb (int[]) Row or column start pointers. (input)
- aptrb (long[]) Row or column start pointers. (input)
- aptre (int[]) Row or column end pointers. (input)
- aptre (long[]) Row or column end pointers. (input)
- asub (int[]) Coefficient subscripts. (input)
- aval (double[]) Coefficient values. (input)
- bkc (mosek.boundkey []) Bound keys for the constraints. (input)
- blc (double[]) Lower bounds for the constraints. (input)

- buc (double[]) Upper bounds for the constraints. (input)
- bkx (mosek.boundkey []) Bound keys for the variables. (input)
- blx (double[]) Lower bounds for the variables. (input)
- bux (double[]) Upper bounds for the variables. (input)

Groups Task management

## Task.isdouparname

```
void isdouparname
(String parname,
mosek.dparam[] param)
```

Checks whether parname is a valid double parameter name.

### **Parameters**

- parname (String) Parameter name. (input)
- param (mosek.dparam by reference) Returns the parameter corresponding to the name, if one exists. (output)

**Groups** Parameter management

## Task.isintparname

```
void isintparname
(String parname,
mosek.iparam[] param)
```

Checks whether parname is a valid integer parameter name.

### Parameters

- parname (String) Parameter name. (input)
- param (mosek.iparam by reference) Returns the parameter corresponding to the name, if one exists. (output)

Groups Parameter management

## Task.isstrparname

```
void isstrparname
(String parname,
mosek.sparam[] param)
```

Checks whether parname is a valid string parameter name.

### **Parameters**

- parname (String) Parameter name. (input)
- param (mosek.sparam by reference) Returns the parameter corresponding to the name, if one exists. (output)

**Groups** Parameter management

Task.linkfiletostream

```
void linkfiletostream
(mosek.streamtype whichstream,
String filename,
int append)
```

Directs all output from a task stream whichstream to a file filename.

## **Parameters**

- whichstream (mosek.streamtype) Index of the stream. (input)
- filename (String) A valid file name. (input)
- append (int) If this argument is 0 the output file will be overwritten, otherwise it will be appended to. (input)

Groups Logging

Task.onesolutionsummary

```
void onesolutionsummary
(mosek.streamtype whichstream,
mosek.soltype whichsol)
```

Prints a short summary of a specified solution.

### Parameters

- whichstream (mosek.streamtype) Index of the stream. (input)
- whichsol (mosek.soltype) Selects a solution. (input)

Groups Task diagnostics

Task.optimize

```
void optimize (mosek.rescode[] trmcode)

mosek.rescode optimize ()
```

Calls the optimizer. Depending on the problem type and the selected optimizer this will call one of the optimizers in **MOSEK**. By default the interior point optimizer will be selected for continuous problems. The optimizer may be selected manually by setting the parameter *iparam.optimizer*.

Parameters trmcode (mosek.rescode by reference) – Is either rescode.ok or a termination response code. (output)

Return (mosek.rescode) - Is either rescode.ok or a termination response code.

Groups Optimization

Task.optimizermt

```
void optimizermt
(String server,
String port,
mosek.rescode[] trmcode)
```

Offload the optimization task to a solver server defined by server:port. The call will block until a result is available or the connection closes.

If the string parameter *sparam.remote\_access\_token* is not blank, it will be passed to the server as authentication.

### Parameters

- server (String) Name or IP address of the solver server. (input)
- port (String) Network port of the solver server. (input)
- trmcode (mosek.rescode by reference) Is either rescode.ok or a termination response code. (output)

Task.optimizersummary

```
void optimizersummary (mosek.streamtype whichstream)
```

Prints a short summary with optimizer statistics from last optimization.

```
Parameters whichstream (mosek.streamtype) - Index of the stream. (input)
Groups Task diagnostics
```

Task.primalrepair

```
void primalrepair
  (double[] wlc,
   double[] wuc,
   double[] wlx,
   double[] wux)
```

The function repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables where the adjustment is computed as the minimal weighted sum of relaxations to the bounds on the constraints and variables. Observe the function only repairs the problem but does not solve it. If an optimal solution is required the problem should be optimized after the repair.

The function is applicable to linear and conic problems possibly with integer variables.

Observe that when computing the minimal weighted relaxation the termination tolerance specified by the parameters of the task is employed. For instance the parameter <code>iparam.mio\_mode</code> can be used to make **MOSEK** ignore the integer constraints during the repair which usually leads to a much faster repair. However, the drawback is of course that the repaired problem may not have an integer feasible solution.

Note the function modifies the task in place. If this is not desired, then apply the function to a cloned task.

# Parameters

- wlc (double[])  $-(w_l^c)_i$  is the weight associated with relaxing the lower bound on constraint i. If the weight is negative, then the lower bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wuc (double[])  $(w_u^c)_i$  is the weight associated with relaxing the upper bound on constraint i. If the weight is negative, then the upper bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wlx (double[])  $(w_l^x)_j$  is the weight associated with relaxing the lower bound on variable j. If the weight is negative, then the lower bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wux (double[])  $(w_l^x)_i$  is the weight associated with relaxing the upper bound on variable j. If the weight is negative, then the upper bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)

## **Groups** Infeasibility diagnostics

Task.primalsensitivity

```
void primalsensitivity
  (int[] subi,
  mosek.mark[] marki,
  int[] subj,
  mosek.mark[] markj,
  double[] leftpricei,
  double[] rightpricei,
  double[] rightrangei,
  double[] leftrangei,
  double[] rightrangei,
  double[] leftpricej,
  double[] rightpricej,
  double[] rightpricej,
  double[] leftrangej,
  double[] rightrangej)
```

Calculates sensitivity information for bounds on variables and constraints. For details on sensitivity analysis, the definitions of *shadow price* and *linearity interval* and an example see Section *Sensitivity Analysis*.

The type of sensitivity analysis to be performed (basis or optimal partition) is controlled by the parameter *iparam.sensitivity\_type*.

### **Parameters**

- subi (int[]) Indexes of constraints to analyze. (input)
- marki (mosek.mark[]) The value of marki[i] indicates for which bound of constraint subi[i] sensitivity analysis is performed. If marki[i] = mark.up the upper bound of constraint subi[i] is analyzed, and if marki[i] = mark. lo the lower bound is analyzed. If subi[i] is an equality constraint, either mark.lo or mark.up can be used to select the constraint for sensitivity analysis. (input)
- subj (int[]) Indexes of variables to analyze. (input)
- markj (mosek.mark[]) The value of markj[j] indicates for which bound of variable subj[j] sensitivity analysis is performed. If markj[j] = mark.up the upper bound of variable subj[j] is analyzed, and if markj[j] = mark.lo the lower bound is analyzed. If subj[j] is a fixed variable, either mark.lo or mark.up can be used to select the bound for sensitivity analysis. (input)
- leftpricei (double[]) leftpricei[i] is the left shadow price for the bound marki[i] of constraint subi[i]. (output)
- rightpricei (double[]) rightpricei[i] is the right shadow price for the bound marki[i] of constraint subi[i]. (output)
- leftrangei (double[]) leftrangei[i] is the left range  $\beta_1$  for the bound marki[i] of constraint subi[i]. (output)
- rightrangei (double[]) rightrangei[i] is the right range  $\beta_2$  for the bound marki[i] of constraint subi[i]. (output)
- leftpricej (double[]) leftpricej[j] is the left shadow price for the bound markj[j] of variable subj[j]. (output)
- rightpricej (double[]) rightpricej[j] is the right shadow price for the bound markj[j] of variable subj[j]. (output)
- leftrangej (double[]) leftrangej[j] is the left range  $\beta_1$  for the bound markj[j] of variable subj[j]. (output)

• rightrangej (double[]) - rightrangej[j] is the right range  $\beta_2$  for the bound markj[j] of variable subj[j]. (output)

Groups Sensitivity analysis

Task.putacol

```
void putacol
  (int j,
   int[] subj,
  double[] valj)
```

Change one column of the linear constraint matrix A. Resets all the elements in column j to zero and then sets

$$a_{\texttt{subj}[k],\texttt{j}} = \texttt{valj}[k], \quad k = 0, \dots, \texttt{nzj} - 1.$$

## **Parameters**

- j (int) Index of a column in A. (input)
- subj (int[]) Row indexes of non-zero values in column j of A. (input)
- valj (double[]) New non-zero values of column j in A. (input)

Groups Scalar variable data

Task.putacollist

```
void putacollist
  (int[] sub,
  int[] ptrb,
  int[] ptre,
  int[] asub,
  double[] aval)
```

Change a set of columns in the linear constraint matrix A with data in sparse triplet format. The requested columns are set to zero and then updated with:

$$\begin{array}{ll} \text{for} & i=0,\dots,num-1 \\ & a_{\texttt{asub}[k],\texttt{sub}[i]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i],\dots,\texttt{ptre}[i]-1. \end{array}$$

## **Parameters**

- sub (int[]) Indexes of columns that should be replaced, no duplicates. (input)
- ptrb (int[]) Array of pointers to the first element in each column. (input)
- ptre (int[]) Array of pointers to the last element plus one in each column. (input)
- asub (int[]) Row indexes of new elements. (input)
- aval (double[]) Coefficient values. (input)

Groups Scalar variable data

Task.putacolslice

```
void putacolslice
  (int first,
   int last,
   int[] ptrb,
   int[] ptre,
   int[] asub,
   double[] aval)
```

```
void putacolslice
  (int first,
   int last,
  long[] ptrb,
  long[] ptre,
  int[] asub,
  double[] aval)
```

Change a slice of columns in the linear constraint matrix A with data in sparse triplet format. The requested columns are set to zero and then updated with:

```
\begin{array}{ll} \text{for} & i = \texttt{first}, \dots, \texttt{last} - 1 \\ & a_{\texttt{asub}[k], i} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{array}
```

### **Parameters**

- first (int) First column in the slice. (input)
- last (int) Last column plus one in the slice. (input)
- ptrb (int[]) Array of pointers to the first element in each column. (input)
- ptrb (long[]) Array of pointers to the first element in each column. (input)
- ptre (int[]) Array of pointers to the last element plus one in each column. (input)
- ptre (long[]) Array of pointers to the last element plus one in each column. (input)
- asub (int[]) Row indexes of new elements. (input)
- aval (double[]) Coefficient values. (input)

Groups Scalar variable data

Task.putaij

```
void putaij
  (int i,
   int j,
   double aij)
```

Changes a coefficient in the linear coefficient matrix A using the method

$$a_{i,j} = \mathtt{aij}.$$

## **Parameters**

- i (int) Constraint (row) index. (input)
- j (int) Variable (column) index. (input)
- aij (double) New coefficient for  $a_{i,j}$ . (input)

Groups Scalar variable data

Task.putaijlist

```
void putaijlist
  (int[] subi,
  int[] subj,
  double[] valij)
```

Changes one or more coefficients in A using the method

$$a_{\mathtt{subi}[\mathtt{k}],\mathtt{subj}[\mathtt{k}]} = \mathtt{valij}[\mathtt{k}], \quad k = 0, \dots, num - 1.$$

Duplicates are not allowed.

### **Parameters**

- subi (int[]) Constraint (row) indices. (input)
- subj (int[]) Variable (column) indices. (input)
- valij (double[]) New coefficient values for  $a_{i,j}$ . (input)

Groups Scalar variable data

Task.putarow

```
void putarow
  (int i,
   int[] subi,
   double[] vali)
```

Change one row of the linear constraint matrix A. Resets all the elements in row i to zero and then sets

$$a_{\mathtt{i},\mathtt{subi}[k]} = \mathtt{vali}[k], \quad k = 0, \dots, \mathtt{nzi} - 1.$$

#### Parameters

- i (int) Index of a row in A. (input)
- subi (int[]) Column indexes of non-zero values in row i of A. (input)
- vali (double[]) New non-zero values of row i in A. (input)

Groups Scalar variable data

Task.putarowlist

```
void putarowlist
  (int[] sub,
  int[] ptrb,
  int[] ptre,
  int[] asub,
  double[] aval)
```

Change a set of rows in the linear constraint matrix A with data in sparse triplet format. The requested rows are set to zero and then updated with:

$$\begin{array}{ll} \text{for} & i=0,\dots,num-1 \\ & a_{\texttt{sub}[i].\texttt{asub}[k]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i],\dots,\texttt{ptre}[i]-1. \end{array}$$

## **Parameters**

- sub (int[]) Indexes of rows that should be replaced, no duplicates. (input)
- ptrb (int[]) Array of pointers to the first element in each row. (input)
- ptre (int[]) Array of pointers to the last element plus one in each row. (input)
- asub (int[]) Column indexes of new elements. (input)
- aval (double[]) Coefficient values. (input)

Groups Scalar variable data

## Task.putarowslice

```
void putarowslice
  (int first,
  int last,
  int[] ptrb,
  int[] ptre,
  int[] asub,
  double[] aval)
```

```
void putarowslice
  (int first,
  int last,
  long[] ptrb,
  long[] ptre,
  int[] asub,
  double[] aval)
```

Change a slice of rows in the linear constraint matrix A with data in sparse triplet format. The requested columns are set to zero and then updated with:

```
\begin{aligned} & \text{for} \quad i = \texttt{first}, \dots, \texttt{last} - 1 \\ & \quad a_{\texttt{sub}[i], \texttt{asub}[k]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{aligned}
```

### **Parameters**

- first (int) First row in the slice. (input)
- last (int) Last row plus one in the slice. (input)
- ptrb (int[]) Array of pointers to the first element in each row. (input)
- ptrb (long[]) Array of pointers to the first element in each row. (input)
- ptre (int[]) Array of pointers to the last element plus one in each row. (input)
- ptre (long[]) Array of pointers to the last element plus one in each row. (input)
- asub (int[]) Column indexes of new elements. (input)
- aval (double[]) Coefficient values. (input)

Groups Scalar variable data

## Task.putbarablocktriplet

```
void putbarablocktriplet
  (long num,
   int[] subi,
   int[] subj,
   int[] subk,
   int[] subl,
   double[] valijkl)
```

Inputs the  $\overline{A}$  matrix in block triplet form.

### **Parameters**

- num (long) Number of elements in the block triplet form. (input)
- subi (int[]) Constraint index. (input)
- subj (int[]) Symmetric matrix variable index. (input)
- subk (int[]) Block row index. (input)

- subl (int[]) Block column index. (input)
- valijkl (double[]) The numerical value associated with each block triplet. (input)

Groups Symmetric matrix variable data

Task.putbaraij

```
void putbaraij
  (int i,
   int j,
   long[] sub,
   double[] weights)
```

This function sets one element in the  $\overline{A}$  matrix.

Each element in the  $\overline{A}$  matrix is a weighted sum of symmetric matrices from the symmetric matrix storage E, so  $\overline{A}_{ij}$  is a symmetric matrix. By default all elements in  $\overline{A}$  are 0, so only non-zero elements need be added. Setting the same element again will overwrite the earlier entry.

The symmetric matrices from E are defined separately using the function Task. appendsparsesymmat.

### **Parameters**

- i (int) Row index of  $\overline{A}$ . (input)
- j (int) Column index of  $\overline{A}$ . (input)
- sub (long[]) Indices in E of the matrices appearing in the weighted sum for  $\overline{A}_{ij}$ . (input)
- weights (double[]) weights [k] is the coefficient of the sub[k]-th element of E in the weighted sum forming  $\overline{A}_{ij}$ . (input)

Groups Symmetric matrix variable data

Task.putbarcblocktriplet

```
void putbarcblocktriplet
  (long num,
  int[] subj,
  int[] subk,
  int[] subl,
  double[] valjkl)
```

Inputs the  $\overline{C}$  matrix in block triplet form.

## **Parameters**

- num (long) Number of elements in the block triplet form. (input)
- subj (int[]) Symmetric matrix variable index. (input)
- subk (int[]) Block row index. (input)
- subl (int[]) Block column index. (input)
- valjkl (double[]) The numerical value associated with each block triplet. (input)

Groups Symmetric matrix variable data

Task.putbarcj

```
void putbarcj
  (int j,
  long[] sub,
  double[] weights)
```

This function sets one entry in the  $\overline{C}$  vector.

Each element in the  $\overline{C}$  vector is a weighted sum of symmetric matrices from the symmetric matrix storage E, so  $\overline{C}_j$  is a symmetric matrix. By default all elements in  $\overline{C}$  are 0, so only non-zero elements need be added. Setting the same element again will overwrite the earlier entry.

The symmetric matrices from E are defined separately using the function Task. appendsparsesymmat.

#### **Parameters**

- j (int) Index of the element in  $\overline{C}$  that should be changed. (input)
- sub (long[]) Indices in E of matrices appearing in the weighted sum for  $\overline{C}_j$  (input)
- weights (double[]) weights [k] is the coefficient of the sub[k]-th element of E in the weighted sum forming  $\overline{C}_i$ . (input)

Groups Symmetric matrix variable data

Task.putbarsj

```
void putbarsj
  (mosek.soltype whichsol,
  int j,
  double[] barsj)
```

Sets the dual solution for a semidefinite variable.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- j (int) Index of the semidefinite variable. (input)
- barsj (double[]) Value of  $\overline{S}_j$ . Format as in Task. getbarsj. (input)

Groups Solution (put)

Task.putbarvarname

```
void putbarvarname
(int j,
String name)
```

Sets the name of a semidefinite variable.

### **Parameters**

- j (int) Index of the variable. (input)
- name (String) The variable name. (input)

Groups Naming

Task.putbarxj

```
void putbarxj
  (mosek.soltype whichsol,
  int j,
  double[] barxj)
```

Sets the primal solution for a semidefinite variable.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- j (int) Index of the semidefinite variable. (input)
- barxj (double[]) Value of  $\overline{X}_i$ . Format as in Task. getbarxj. (input)

Groups Solution (put)

Task.putbound Deprecated

```
void putbound
  (mosek.accmode accmode,
   int i,
   mosek.boundkey bk,
   double bl,
   double bu)
```

Changes the bound for either one constraint or one variable.

### **Parameters**

- accmode (mosek.accmode) Defines whether the bound for a constraint (accmode.con) or variable (accmode.var) is changed. (input)
- i (int) Index of the constraint or variable. (input)
- bk (mosek.boundkey) New bound key. (input)
- bl (double) New lower bound. (input)
- bu (double) New upper bound. (input)

Groups Bound data

Task.putboundlist Deprecated

```
void putboundlist
  (mosek.accmode accmode,
  int[] sub,
  mosek.boundkey[] bk,
  double[] bl,
  double[] bu)
```

Changes the bounds of constraints or variables.

### **Parameters**

- accmode (mosek.accmode) Defines whether bounds for constraints (accmode. con) or variables (accmode.var) are changed. (input)
- sub (int[]) Subscripts of the constraints or variables that should be changed. (input)
- bk (mosek.boundkey[]) Bound keys. (input)
- bl (double[]) Values for lower bounds. (input)
- bu (double[]) Values for upper bounds. (input)

### Groups Bound data

Task.putboundslice Deprecated

```
void putboundslice
  (mosek.accmode con,
   int first,
   int last,
   mosek.boundkey[] bk,
   double[] bl,
   double[] bu)
```

Changes the bounds for a slice of constraints or variables.

#### **Parameters**

- con (mosek.accmode) Defines whether bounds for constraints (accmode.con) or variables (accmode.var) are changed. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bk (mosek.boundkey[]) Bound keys. (input)
- bl (double[]) Values for lower bounds. (input)
- bu (double[]) Values for upper bounds. (input)

Groups Bound data

Task.putcfix

```
void putcfix (double cfix)
```

Replaces the fixed term in the objective by a new one.

Parameters cfix (double) - Fixed term in the objective. (input)

Groups Objective data

Task.putcj

```
void putcj
  (int j,
  double cj)
```

Modifies one coefficient in the linear objective vector c, i.e.

```
c_{j} = c_{j}.
```

If the absolute value exceeds  $dparam.data\_tol\_c\_huge$  an error is generated. If the absolute value exceeds  $dparam.data\_tol\_cj\_large$ , a warning is generated, but the coefficient is inputted as specified.

# Parameters

- $\bullet\,$  j (int) Index of the variable for which c should be changed. (input)
- cj (double) New value of  $c_j$ . (input)

Groups Scalar variable data

Task.putclist

```
void putclist
  (int[] subj,
  double[] val)
```

Modifies the coefficients in the linear term c in the objective using the principle

$$c_{\texttt{subj}[\texttt{t}]} = \texttt{val}[\texttt{t}], \quad t = 0, \dots, num - 1.$$

If a variable index is specified multiple times in subj only the last entry is used. Data checks are performed as in Task.putcj.

### **Parameters**

- subj (int[]) Indices of variables for which the coefficient in c should be changed. (input)
- val (double[]) New numerical values for coefficients in c that should be modified. (input)

Groups Scalar variable data

Task.putconbound

```
void putconbound
  (int i,
   mosek.boundkey bk,
   double bl,
   double bu)
```

Changes the bounds for one constraint.

If the bound value specified is numerically larger than <code>dparam.data\_tol\_bound\_inf</code> it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than <code>dparam.data\_tol\_bound\_wrn</code>, a warning will be displayed, but the bound is inputted as specified.

## **Parameters**

- i (int) Index of the constraint. (input)
- bk (mosek.boundkey) New bound key. (input)
- bl (double) New lower bound. (input)
- bu (double) New upper bound. (input)

Groups Bound data

Task.putconboundlist

```
void putconboundlist
  (int[] sub,
  mosek.boundkey[] bk,
  double[] bl,
  double[] bu)
```

Changes the bounds for a list of constraints. If multiple bound changes are specified for a constraint, then only the last change takes effect. Data checks are performed as in *Task.putconbound*.

#### **Parameters**

- sub (int[]) List of constraint indexes. (input)
- bk (mosek.boundkey []) Bound keys. (input)
- bl (double[]) Values for lower bounds. (input)

• bu (double[]) - Values for upper bounds. (input)

### Groups Bound data

Task.putconboundslice

```
void putconboundslice
  (int first,
  int last,
  mosek.boundkey[] bk,
  double[] bl,
  double[] bu)
```

Changes the bounds for a slice of the constraints. Data checks are performed as in Task. putconbound.

#### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bk (mosek.boundkey[]) Bound keys. (input)
- bl (double[]) Values for lower bounds. (input)
- bu (double[]) Values for upper bounds. (input)

Groups Linear constraint data, Bound data

Task.putcone

```
void putcone
  (int k,
  mosek.conetype ct,
  double conepar,
  int[] submem)
```

Replaces a conic constraint.

# **Parameters**

- k (int) Index of the cone. (input)
- ct (mosek.conetype) Specifies the type of the cone. (input)
- conepar (double) This argument is currently not used. It can be set to 0 (input)
- submem (int[]) Variable subscripts of the members in the cone. (input)

Groups Conic constraint data

Task.putconename

```
void putconename
(int j,
String name)
```

Sets the name of a cone.

# Parameters

- j (int) Index of the cone. (input)
- name (String) The name of the cone. (input)

### Groups Naming

Task.putconname

```
void putconname
(int i,
String name)
```

Sets the name of a constraint.

#### Parameters

- i (int) Index of the constraint. (input)
- name (String) The name of the constraint. (input)

Groups Naming

Task.putcslice

```
void putcslice
  (int first,
   int last,
   double[] slice)
```

Modifies a slice in the linear term c in the objective using the principle

$$c_{j} = \mathtt{slice}[\mathtt{j-first}], \quad j = first, .., last - 1$$

Data checks are performed as in Task.putcj.

### **Parameters**

- first (int) First element in the slice of c. (input)
- last (int) Last element plus 1 of the slice in c to be changed. (input)
- $\bullet$  slice (double[]) New numerical values for coefficients in c that should be modified. (input)

Groups Scalar variable data

Task.putdouparam

```
void putdouparam
(mosek.dparam param,
double parvalue)
```

Sets the value of a double parameter.

# Parameters

- param (mosek.dparam) Which parameter. (input)
- parvalue (double) Parameter value. (input)

Groups Parameters (put)

Task.putintparam

```
void putintparam
(mosek.iparam param,
int parvalue)
```

Sets the value of an integer parameter.

Please notice that some parameters take values that are defined in Enum classes. This function accepts only integer values, so to use e.g. the value onoffkey.on, is necessary to use the member .value. For example:

```
task.putintparam(mosek.iparam.opf_write_problem, mosek.onoffkey.on.value)
```

#### **Parameters**

- param (mosek.iparam) Which parameter. (input)
- parvalue (int) Parameter value. (input)

Groups Parameters (put)

Task.putmaxnumanz

```
void putmaxnumanz (long maxnumanz)
```

Sets the number of preallocated non-zero entries in A.

**MOSEK** stores only the non-zero elements in the linear coefficient matrix A and it cannot predict how much storage is required to store A. Using this function it is possible to specify the number of non-zeros to preallocate for storing A.

If the number of non-zeros in the problem is known, it is a good idea to set maxnumanz slightly larger than this number, otherwise a rough estimate can be used. In general, if A is inputted in many small chunks, setting this value may speed up the data input phase.

It is not mandatory to call this function, since **MOSEK** will reallocate internal structures whenever it is necessary.

The function call has no effect if both maxnumcon and maxnumvar are zero.

Parameters maxnumanz (long) – Number of preallocated non-zeros in A. (input)

Groups Scalar variable data

Task.putmaxnumbarvar

```
void putmaxnumbarvar (int maxnumbarvar)
```

Sets the number of preallocated symmetric matrix variables in the optimization task. When this number of variables is reached **MOSEK** will automatically allocate more space for variables.

It is not mandatory to call this function. It only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that maxnumbarvar must be larger than the current number of symmetric matrix variables in the task.

Parameters maxnumbarvar (int) – Number of preallocated symmetric matrix variables. (input)

Groups Symmetric matrix variable data

Task.putmaxnumcon

```
void putmaxnumcon (int maxnumcon)
```

Sets the number of preallocated constraints in the optimization task. When this number of constraints is reached  $\mathbf{MOSEK}$  will automatically allocate more space for constraints.

It is never mandatory to call this function, since MOSEK will reallocate any internal structures whenever it is required.

Please note that maxnumcon must be larger than the current number of constraints in the task.

Parameters maxnumcon (int) – Number of preallocated constraints in the optimization task. (input)

Groups Task management

Task.putmaxnumcone

```
void putmaxnumcone (int maxnumcone)
```

Sets the number of preallocated conic constraints in the optimization task. When this number of conic constraints is reached **MOSEK** will automatically allocate more space for conic constraints.

It is not mandatory to call this function, since **MOSEK** will reallocate any internal structures whenever it is required.

Please note that maxnumcon must be larger than the current number of conic constraints in the task.

Parameters maxnumcone (int) – Number of preallocated conic constraints in the optimization task. (input)

Groups Task management

Task.putmaxnumqnz

```
void putmaxnumqnz (long maxnumqnz)
```

Sets the number of preallocated non-zero entries in quadratic terms.

**MOSEK** stores only the non-zero elements in Q. Therefore, **MOSEK** cannot predict how much storage is required to store Q. Using this function it is possible to specify the number non-zeros to preallocate for storing Q (both objective and constraints).

It may be advantageous to reserve more non-zeros for Q than actually needed since it may improve the internal efficiency of **MOSEK**, however, it is never worthwhile to specify more than the double of the anticipated number of non-zeros in Q.

It is not mandatory to call this function, since **MOSEK** will reallocate internal structures whenever it is necessary.

Parameters maxnumqnz (long) - Number of non-zero elements preallocated in quadratic coefficient matrices. (input)

Groups Scalar variable data

Task.putmaxnumvar

```
void putmaxnumvar (int maxnumvar)
```

Sets the number of preallocated variables in the optimization task. When this number of variables is reached **MOSEK** will automatically allocate more space for variables.

It is not mandatory to call this function. It only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that maxnumvar must be larger than the current number of variables in the task.

Parameters maxnumvar (int) - Number of preallocated variables in the optimization task. (input)

### Groups Scalar variable data

Task.putnadouparam

```
void putnadouparam
(String paramname,
double parvalue)
```

Sets the value of a named double parameter.

#### **Parameters**

- paramname (String) Name of a parameter. (input)
- parvalue (double) Parameter value. (input)

Groups Parameters (put)

Task.putnaintparam

```
void putnaintparam
(String paramname,
int parvalue)
```

Sets the value of a named integer parameter.

#### **Parameters**

- paramname (String) Name of a parameter. (input)
- parvalue (int) Parameter value. (input)

Groups Parameters (put)

Task.putnastrparam

```
void putnastrparam
(String paramname,
String parvalue)
```

Sets the value of a named string parameter.

# Parameters

- paramname (String) Name of a parameter. (input)
- parvalue (String) Parameter value. (input)

Groups Parameters (put)

Task.putobjname

```
void putobjname (String objname)
```

Assigns a new name to the objective.

```
{\bf Parameters\ objname\ (String)-Name\ of\ the\ objective.\ (input)}
```

Groups Naming

Task.putobjsense

```
void putobjsense (mosek.objsense sense)
```

Sets the objective sense of the task.

**Parameters sense** (mosek.objsense) – The objective sense of the task. The values objsense.maximize and objsense.minimize mean that the problem is maximized or minimized respectively. (input)

Groups Objective data

Task.putparam

```
void putparam
(String parname,
String parvalue)
```

Checks if parname is valid parameter name. If it is, the parameter is assigned the value specified by parvalue.

### **Parameters**

- parname (String) Parameter name. (input)
- parvalue (String) Parameter value. (input)

Groups Parameters (put)

Task.putqcon

```
void putqcon
  (int[] qcsubk,
   int[] qcsubi,
   int[] qcsubj,
   double[] qcval)
```

Replace all quadratic entries in the constraints. The list of constraints has the form

$$l_k^c \leq \frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^k x_i x_j + \sum_{j=0}^{numvar-1} a_{kj} x_j \leq u_k^c, \quad k = 0, \dots, m-1.$$

This function sets all the quadratic terms to zero and then performs the update:

$$q_{\mathtt{qcsubh}[\mathtt{t}],\mathtt{qcsubj}[\mathtt{t}]}^{\mathtt{qcsubk}[\mathtt{t}]} = q_{\mathtt{qcsubh}[\mathtt{t}],\mathtt{qcsubi}[\mathtt{t}]}^{\mathtt{qcsubk}[\mathtt{t}]} = q_{\mathtt{qcsubh}[\mathtt{t}],\mathtt{qcsubi}[\mathtt{t}]}^{\mathtt{qcsubk}[\mathtt{t}]} + \mathtt{qcval}[\mathtt{t}],$$

for  $t = 0, \dots, numqcnz - 1$ .

Please note that:

- For large problems it is essential for the efficiency that the function *Task.putmaxnumqnz* is employed to pre-allocate space.
- Only the lower triangular parts should be specified because the Q matrices are symmetric. Specifying entries where i < j will result in an error.
- Only non-zero elements should be specified.
- The order in which the non-zero elements are specified is insignificant.
- Duplicate elements are added together as shown above. Hence, it is usually not recommended to specify the same entry multiple times.

For a code example see Section Quadratic Optimization

#### **Parameters**

- qcsubk (int[]) Constraint subscripts for quadratic coefficients. (input)
- qcsubi (int[]) Row subscripts for quadratic constraint matrix. (input)
- qcsubj (int[]) Column subscripts for quadratic constraint matrix. (input)

• qcval (double[]) - Quadratic constraint coefficient values. (input)

Groups Scalar variable data

Task.putqconk

```
void putqconk
  (int k,
   int[] qcsubi,
   int[] qcsubj,
   double[] qcval)
```

Replaces all the quadratic entries in one constraint. This function performs the same operations as *Task.putqcon* but only with respect to constraint number **k** and it does not modify the other constraints. See the description of *Task.putqcon* for definitions and important remarks.

### **Parameters**

- k (int) The constraint in which the new Q elements are inserted. (input)
- $\bullet$ qcsubi (int[]) Row subscripts for quadratic constraint matrix. (input)
- qcsubj (int[]) Column subscripts for quadratic constraint matrix. (input)
- qcval (double[]) Quadratic constraint coefficient values. (input)

Groups Scalar variable data

Task.putqobj

```
void putqobj
  (int[] qosubi,
  int[] qosubj,
  double[] qoval)
```

Replace all quadratic terms in the objective. If the objective has the form

$$\frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^o x_i x_j + \sum_{j=0}^{numvar-1} c_j x_j + c^f$$

then this function sets all the quadratic terms to zero and then performs the update:

$$q^o_{\texttt{qosubi[t]},\texttt{qosubj[t]}} = q^o_{\texttt{qosubj[t]},\texttt{qosubi[t]}} = q^o_{\texttt{qosubj[t]},\texttt{qosubi[t]}} + \texttt{qoval[t]},$$

for  $t = 0, \ldots, numgon z - 1$ .

See the description of Task. putgeon for important remarks and example.

#### Parameters

- qosubi (int[]) Row subscripts for quadratic objective coefficients. (input)
- qosubj (int[]) Column subscripts for quadratic objective coefficients. (input)
- qoval (double[]) Quadratic objective coefficient values. (input)

Groups Scalar variable data

Task.putqobjij

```
void putqobjij
  (int i,
   int j,
   double qoij)
```

Replaces one coefficient in the quadratic term in the objective. The function performs the assignment

$$q_{ij}^o=q_{ji}^o= ext{qoij}.$$

Only the elements in the lower triangular part are accepted. Setting  $q_{ij}$  with j > i will cause an error.

Please note that replacing all quadratic elements one by one is more computationally expensive than replacing them all at once. Use Task. putqobj instead whenever possible.

#### **Parameters**

- i (int) Row index for the coefficient to be replaced. (input)
- j (int) Column index for the coefficient to be replaced. (input)
- qoij (double) The new value for  $q_{ij}^o$ . (input)

Groups Scalar variable data

Task.putskc

```
void putskc
  (mosek.soltype whichsol,
  mosek.stakey[] skc)
```

Sets the status keys for the constraints.

#### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- skc (mosek.stakey []) Status keys for the constraints. (input)

Groups Solution (put)

Task.putskcslice

```
void putskcslice
  (mosek.soltype whichsol,
  int first,
  int last,
  mosek.stakey[] skc)
```

Sets the status keys for a slice of the constraints.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- skc (mosek.stakey[]) Status keys for the constraints. (input)

Groups Solution (put)

Task.putskx

```
void putskx
(mosek.soltype whichsol,
mosek.stakey[] skx)
```

Sets the status keys for the scalar variables.

#### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- skx (mosek.stakey[]) Status keys for the variables. (input)

Groups Solution (put)

Task.putskxslice

```
void putskxslice
  (mosek.soltype whichsol,
  int first,
  int last,
  mosek.stakey[] skx)
```

Sets the status keys for a slice of the variables.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- skx (mosek.stakey[]) Status keys for the variables. (input)

Groups Solution (put)

Task.putslc

```
void putslc
  (mosek.soltype whichsol,
  double[] slc)
```

Sets the  $s_l^c$  vector for a solution.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (input)

Groups Solution (put)

Task.putslcslice

```
void putslcslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] slc)
```

Sets a slice of the  $s_l^c$  vector for a solution.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (input)

```
Groups Solution (put)
```

Task.putslx

```
void putslx
  (mosek.soltype whichsol,
  double[] slx)
```

Sets the  $s_l^x$  vector for a solution.

#### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (input)

Groups Solution (put)

Task.putslxslice

```
void putslxslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] slx)
```

Sets a slice of the  $s_l^x$  vector for a solution.

#### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (input)

Groups Solution (put)

Task.putsnx

```
void putsnx
  (mosek.soltype whichsol,
  double[] sux)
```

Sets the  $s_n^x$  vector for a solution.

# **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (input)

Groups Solution (put)

Task.putsnxslice

```
void putsnxslice
  (mosek.soltype whichsol,
  int first,
```

```
int last,
double[] snx)
```

Sets a slice of the  $s_n^x$  vector for a solution.

## **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (input)

Groups Solution (put)

Task.putsolution

```
void putsolution
  (mosek.soltype whichsol,
  mosek.stakey[] skc,
  mosek.stakey[] skx,
  mosek.stakey[] skn,
  double[] xc,
  double[] xx,
  double[] y,
  double[] slc,
  double[] suc,
  double[] sux,
  double[] slx,
  double[] slx,
  double[] slx,
  double[] snx)
```

Inserts a solution into the task.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- skc (mosek.stakey []) Status keys for the constraints. (input)
- skx (mosek.stakey[]) Status keys for the variables. (input)
- skn (mosek.stakey[]) Status keys for the conic constraints. (input)
- xc (double[]) Primal constraint solution. (input)
- xx (double[]) Primal variable solution. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (input)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (input)

Groups Solution (put)

Task.putsolutioni Deprecated

```
void putsolutioni
  (mosek.accmode accmode,
   int i,
   mosek.soltype whichsol,
   mosek.stakey sk,
   double x,
   double sl,
   double su,
   double sn)
```

Sets the primal and dual solution information for a single constraint or variable.

### **Parameters**

- accmode (mosek.accmode) Defines whether solution information for a constraint (accmode.con) or for a variable (accmode.var) is modified. (input)
- i (int) Index of the constraint or variable. (input)
- whichsol (mosek.soltype) Selects a solution. (input)
- sk (mosek.stakey) Status key of the constraint or variable. (input)
- x (double) Solution value of the primal constraint or variable. (input)
- sl (double) Solution value of the dual variable associated with the lower bound. (input)
- su (double) Solution value of the dual variable associated with the upper bound. (input)
- sn (double) Solution value of the dual variable associated with the conic constraint. (input)

Groups Solution (put)

Task.putsolutionyi

```
void putsolutionyi
  (int i,
   mosek.soltype whichsol,
  double y)
```

Inputs the dual variable of a solution.

# Parameters

- i (int) Index of the dual variable. (input)
- whichsol (mosek.soltype) Selects a solution. (input)
- y (double) Solution value of the dual variable. (input)

Task.putstrparam

```
void putstrparam
(mosek.sparam param,
String parvalue)
```

Sets the value of a string parameter.

#### **Parameters**

• param (mosek.sparam) - Which parameter. (input)

• parvalue (String) - Parameter value. (input)

Groups Parameters (put)

Task.putsuc

```
void putsuc
  (mosek.soltype whichsol,
  double[] suc)
```

Sets the  $s_u^c$  vector for a solution.

#### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (input)

Groups Solution (put)

Task.putsucslice

```
void putsucslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] suc)
```

Sets a slice of the  $s_u^c$  vector for a solution.

### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (input)

Groups Solution (put)

Task.putsux

```
void putsux
  (mosek.soltype whichsol,
  double[] sux)
```

Sets the  $s_u^x$  vector for a solution.

# **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (input)

Groups Solution (put)

Task.putsuxslice

```
void putsuxslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] sux)
```

Sets a slice of the  $s_u^x$  vector for a solution.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (input)

Groups Solution (put)

Task.puttaskname

```
void puttaskname (String taskname)
```

Assigns a new name to the task.

```
Parameters taskname (String) - Name assigned to the task. (input)
```

Groups Naming

Task.putvarbound

```
void putvarbound
  (int j,
   mosek.boundkey bk,
   double bl,
   double bu)
```

Changes the bounds for one variable.

If the bound value specified is numerically larger than  $dparam.data\_tol\_bound\_inf$  it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than  $dparam.data\_tol\_bound\_wrn$ , a warning will be displayed, but the bound is inputted as specified.

### **Parameters**

- j (int) Index of the variable. (input)
- bk (mosek.boundkey) New bound key. (input)
- bl (double) New lower bound. (input)
- $\bullet$  bu (double) New upper bound. (input)

Groups Bound data

Task.putvarboundlist

```
void putvarboundlist
  (int[] sub,
  mosek.boundkey[] bkx,
  double[] blx,
  double[] bux)
```

Changes the bounds for one or more variables. If multiple bound changes are specified for a variable, then only the last change takes effect. Data checks are performed as in *Task.putvarbound*.

#### **Parameters**

- sub (int[]) List of variable indexes. (input)
- bkx (mosek.boundkey []) Bound keys for the variables. (input)
- blx (double[]) Lower bounds for the variables. (input)
- $\bullet$  bux (double[]) Upper bounds for the variables. (input)

Groups Bound data

Task.putvarboundslice

```
void putvarboundslice
  (int first,
   int last,
   mosek.boundkey[] bk,
   double[] bl,
   double[] bu)
```

Changes the bounds for a slice of the variables. Data checks are performed as in Task. putvarbound.

#### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bk (mosek.boundkey[]) Bound keys. (input)
- bl (double[]) Values for lower bounds. (input)
- bu (double[]) Values for upper bounds. (input)

Groups Scalar variable data

Task.putvarname

```
void putvarname
(int j,
String name)
```

Sets the name of a variable.

### Parameters

- j (int) Index of the variable. (input)
- name (String) The variable name. (input)

Groups Naming

Task.putvartype

```
void putvartype
  (int j,
  mosek.variabletype vartype)
```

Sets the variable type of one variable.

#### Parameters

• j (int) – Index of the variable. (input)

• vartype (mosek.variabletype) - The new variable type. (input)

Groups Scalar variable data

Task.putvartypelist

```
void putvartypelist
  (int[] subj,
  mosek.variabletype[] vartype)
```

Sets the variable type for one or more variables. If the same index is specified multiple times in subj only the last entry takes effect.

# Parameters

- subj (int[]) A list of variable indexes for which the variable type should be changed. (input)
- vartype (mosek.variabletype[]) A list of variable types that should be assigned to the variables specified by subj. (input)

Groups Scalar variable data

Task.putxc

```
void putxc
  (mosek.soltype whichsol,
  double[] xc)
```

Sets the  $x^c$  vector for a solution.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- xc (double[]) Primal constraint solution. (output)

Groups Solution (put)

Task.putxcslice

```
void putxcslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] xc)
```

Sets a slice of the  $x^c$  vector for a solution.

# Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- xc (double[]) Primal constraint solution. (input)

Groups Solution (put)

Task.putxx

```
void putxx
  (mosek.soltype whichsol,
  double[] xx)
```

Sets the  $x^x$  vector for a solution.

### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- xx (double[]) Primal variable solution. (input)

Groups Solution (put)

Task.putxxslice

```
void putxxslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] xx)
```

Obtains a slice of the  $x^x$  vector for a solution.

#### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- xx (double[]) Primal variable solution. (input)

Groups Solution (put)

Task.puty

```
void puty
  (mosek.soltype whichsol,
  double[] y)
```

Sets the y vector for a solution.

# **Parameters**

- $\bullet \ \ which sol \ (\textit{mosek.soltype}) Selects \ a \ solution. \ (input)$
- y (double[]) Vector of dual variables corresponding to the constraints. (input)

Groups Solution (put)

Task.putyslice

```
void putyslice
  (mosek.soltype whichsol,
  int first,
  int last,
  double[] y)
```

Sets a slice of the y vector for a solution.

# **Parameters**

• whichsol (mosek.soltype) - Selects a solution. (input)

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (input)

Groups Solution (put)

Task.readdata

```
void readdata (String filename)
```

Reads an optimization problem and associated data from a file.

```
Parameters filename (String) - A valid file name. (input)
```

Groups Data file

Task.readdataformat

```
void readdataformat
(String filename,
mosek.dataformat format,
mosek.compresstype compress)
```

Reads an optimization problem and associated data from a file.

#### Parameters

- filename (String) A valid file name. (input)
- format (mosek.dataformat) File data format. (input)
- compress (mosek.compresstype) File compression type. (input)

Groups Data file

Task.readparamfile

```
void readparamfile (String filename)
```

Reads **MOSEK** parameters from a file. Data is read from the file **filename** if it is a nonempty string. Otherwise data is read from the file specified by <code>sparam.param\_read\_file\_name</code>.

```
Parameters filename (String) - A valid file name. (input)
```

Groups Data file

Task.readsolution

```
void readsolution
(mosek.soltype whichsol,
String filename)
```

Reads a solution file and inserts it as a specified solution in the task. Data is read from the file filename if it is a nonempty string. Otherwise data is read from one of the files specified by sparam. bas\_sol\_file\_name, sparam.itr\_sol\_file\_name or sparam.int\_sol\_file\_name depending on which solution is chosen.

# **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- filename (String) A valid file name. (input)

Groups Data file

#### Task.readsummary

```
void readsummary (mosek.streamtype whichstream)
```

Prints a short summary of last file that was read.

Parameters whichstream (mosek.streamtype) - Index of the stream. (input)

Groups Task diagnostics

Task.readtask

```
void readtask (String filename)
```

Load task data from a file, replacing any data that already exists in the task object. All problem data, parameters and other settings are resorted, but if the file contains solutions, the solution status after loading a file is set to unknown, even if it was optimal or otherwise well-defined when the file was dumped.

See section The Task Format for a description of the Task format.

Parameters filename (String) - A valid file name. (input)

Task.removebarvars

```
void removebarvars (int[] subset)
```

The function removes a subset of the symmetric matrices from the optimization task. This implies that the remaining symmetric matrices are renumbered.

Parameters subset (int[]) – Indexes of symmetric matrices which should be removed. (input)

Groups Symmetric matrix variable data

Task.removecones

```
void removecones (int[] subset)
```

Removes a number of conic constraints from the problem. This implies that the remaining conic constraints are renumbered. In general, it is much more efficient to remove a cone with a high index than a low index.

Parameters subset (int[]) - Indexes of cones which should be removed. (input)

Groups Conic constraint data

Task.removecons

```
void removecons (int[] subset)
```

The function removes a subset of the constraints from the optimization task. This implies that the remaining constraints are renumbered.

Parameters subset (int[]) - Indexes of constraints which should be removed. (input)

Groups Linear constraint data

Task.removevars

```
void removevars (int[] subset)
```

The function removes a subset of the variables from the optimization task. This implies that the remaining variables are renumbered.

Parameters subset (int[]) - Indexes of variables which should be removed. (input)

Groups Scalar variable data

Task.resizetask

```
void resizetask
(int maxnumcon,
int maxnumvar,
int maxnumcone,
long maxnumanz,
long maxnumqnz)
```

Sets the amount of preallocated space assigned for each type of data in an optimization task.

It is never mandatory to call this function, since it only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that the procedure is **destructive** in the sense that all existing data stored in the task is destroyed.

#### **Parameters**

- maxnumcon (int) New maximum number of constraints. (input)
- maxnumvar (int) New maximum number of variables. (input)
- maxnumcone (int) New maximum number of cones. (input)
- maxnumanz (long) New maximum number of non-zeros in A. (input)
- maxnumqnz (long) New maximum number of non-zeros in all Q matrices. (input)

Task.sensitivityreport

```
void sensitivityreport (mosek.streamtype whichstream)
```

Reads a sensitivity format file from a location given by <code>sparam.sensitivity\_file\_name</code> and writes the result to the stream <code>whichstream</code>. If <code>sparam.sensitivity\_res\_file\_name</code> is set to a non-empty string, then the sensitivity report is also written to a file of this name.

```
\textbf{Parameters which stream } (\textit{mosek.streamtype}) - Index \ of the stream. \ (input)
```

Groups Sensitivity analysis

 ${\tt Task.set\_InfoCallback}$ 

```
void set_InfoCallback (mosek.DataCallback callback)
```

Receive callbacks with solver status and information during optimization.

For example:

```
return 0;
} );
```

Parameters callback (DataCallback) - The callback object. (input)

Task.set\_ItgSolutionCallback

```
void set_ItgSolutionCallback (mosek.ItgSolutionCallback callback)
```

Receive callbacks with solution updates from the mixed-integer optimizer.

For example:

```
task.set_ItgSolutionCallback(
  new mosek.ItgSolutionCallback() {
    void callback(double[] xx) {
        System.out.print("New integer solution: ");
        for (double v : xx) System.out.print("" + v + " ");
        System.out.println("");
    } });
```

Parameters callback (ItgSolutionCallback) - The callback object. (input)

Task.set\_Progress

```
void set_Progress (mosek.Progress callback)
```

Receive callbacks about current status of the solver during optimization.

For example:

```
task.set_Progress(new mosek.Progress() { int progress(mosek.callbackcode code) { System. 
→println("Callback "+code); return 0; } });
```

Parameters callback (*Progress*) - The callback object. (input)

Task.set\_Stream

```
void set_Stream(
mosek.streamtype whichstream,
mosek.Stream callback)
```

Directs all output from a task stream to a callback object.

Can for example be called as:

```
task.set_Stream(mosek.streamtype.log, new Stream() { public void stream(String s) {⊔ 
→System.out.print(s); } });
```

### Parameters

- whichstream (streamtype) Index of the stream. (input)
- callback (Stream) The callback object. (input)

Task.setdefaults

```
void setdefaults ()
```

Resets all the parameters to their default values.

**Groups** Parameter management

Task.solutiondef

```
void solutiondef
  (mosek.soltype whichsol,
  boolean[] isdef)
```

```
boolean solutiondef (mosek.soltype whichsol)
```

Checks whether a solution is defined.

#### Parameters

- whichsol (mosek.soltype) Selects a solution. (input)
- isdef (boolean by reference) Is non-zero if the requested solution is defined. (output)

Return (boolean) – Is non-zero if the requested solution is defined.

**Groups** Solution information

Task.solutionsummary

```
void solutionsummary (mosek.streamtype whichstream)
```

Prints a short summary of the current solutions.

Parameters whichstream (mosek.streamtype) - Index of the stream. (input)

Groups Task diagnostics

Task.solvewithbasis

```
void solvewithbasis
  (int transp,
   int[] numnz,
   int[] sub,
   double[] val)
```

```
int solvewithbasis
  (int transp,
   int numnz,
   int[] sub,
   double[] val)
```

If a basic solution is available, then exactly numcon basis variables are defined. These numcon basis variables are denoted the basis. Associated with the basis is a basis matrix denoted B. This function solves either the linear equation system

$$B\overline{X} = b \tag{16.3}$$

or the system

$$B^T \overline{X} = b \tag{16.4}$$

for the unknowns  $\overline{X}$ , with b being a user-defined vector. In order to make sense of the solution  $\overline{X}$  it is important to know the ordering of the variables in the basis because the ordering specifies how B is constructed. When calling Task.initbasissolve an ordering of the basis variables is obtained, which can be used to deduce how MOSEK has constructed B. Indeed if the k-th basis variable is variable  $x_j$  it implies that

$$B_{i,k} = A_{i,j}, i = 0, \dots, numcon - 1.$$

Otherwise if the k-th basis variable is variable  $x_i^c$  it implies that

$$B_{i,k} = \begin{cases} -1, & i = j, \\ 0, & i \neq j. \end{cases}$$

The function Task.initbasissolve must be called before a call to this function. Please note that this function exploits the sparsity in the vector b to speed up the computations.

#### **Parameters**

- transp (int) If this argument is zero, then (16.3) is solved, if non-zero then (16.4) is solved. (input)
- numnz (int by reference) As input it is the number of non-zeros in b. As output it is the number of non-zeros in  $\overline{X}$ . (input/output)
- numnz (int) As input it is the number of non-zeros in b. As output it is the number of non-zeros in  $\overline{X}$ . (input/output)
- sub (int[]) As input it contains the positions of non-zeros in b. As output it contains the positions of the non-zeros in  $\overline{X}$ . It must have room for numcon elements. (input/output)
- val (double[]) As input it is the vector b as a dense vector (although the positions of non-zeros are specified in sub it is required that val[i] = 0 when b[i] = 0). As output val is the vector  $\overline{X}$  as a dense vector. It must have length numcon. (input/output)

**Return** (int) – As input it is the number of non-zeros in b. As output it is the number of non-zeros in  $\overline{X}$ .

Groups Basis matrix

Task.strtoconetype

```
void strtoconetype
(String str,
  mosek.conetype[] conetype)
```

Obtains cone type code corresponding to a cone type string.

#### **Parameters**

- str (String) String corresponding to the cone type code conetype. (input)
- conetype (mosek.conetype by reference) The cone type corresponding to the string str. (output)

Task.strtosk

```
void strtosk
(String str,
int[] sk)
```

Obtains the status key corresponding to an explanatory string.

#### **Parameters**

- str (String) Status key string. (input)
- sk (int by reference) Status key corresponding to the string. (output)

Task.toconic

void toconic ()

This function tries to reformulate a given Quadratically Constrained Quadratic Optimization problem (QCQP) as a Conic Quadratic Optimization problem (CQO). The first step of the reformulation is to convert the quadratic term of the objective function, if any, into a constraint. Then the following steps are repeated for each quadratic constraint:

- a conic constraint is added along with a suitable number of auxiliary variables and constraints;
- the original quadratic constraint is not removed, but all its coefficients are zeroed out.

Note that the reformulation preserves all the original variables.

The conversion is performed in-place, i.e. the task passed as argument is modified on exit. That also means that if the reformulation fails, i.e. the given QCQP is not representable as a CQO, then the task has an undefined state. In some cases, users may want to clone the task to ensure a clean copy is preserved.

Task.unset\_Progress

void unset\_Progress ()

Deactivates all user callback functions.

Task.updatesolutioninfo

void updatesolutioninfo (mosek.soltype whichsol)

Update the information items related to the solution.

Parameters whichsol (mosek.soltype) - Selects a solution. (input)

Groups Task diagnostics

Task.writedata

void writedata (String filename)

Writes problem data associated with the optimization task to a file in one of the supported formats. See Section Supported File Formats for the complete list.

By default the data file format is determined by the file name extension. This behaviour can be overridden by setting the *iparam.write\_data\_format* parameter. To write in compressed format append the extension .gz. E.g to write a gzip compressed MPS file use the extension mps.gz.

Please note that MPS, LP and OPF files require all variables to have unique names. If a task contains no names, it is possible to write the file with automatically generated anonymous names by setting the <code>iparam.write\_generic\_names</code> parameter to <code>onoffkey.on</code>.

Data is written to the file filename if it is a nonempty string. Otherwise data is written to the file specified by  $sparam.data_file_name$ .

Please note that if a general nonlinear function appears in the problem then such function *cannot* be written to file and **MOSEK** will issue a warning.

Parameters filename (String) - A valid file name. (input)

Groups Data file

Task.writejsonsol

```
void writejsonsol (String filename)
```

Saves the current solutions and solver information items in a JSON file.

Parameters filename (String) - A valid file name. (input)

Groups Data file

Task.writeparamfile

```
void writeparamfile (String filename)
```

Writes all the parameters to a parameter file.

Parameters filename (String) - A valid file name. (input)

Groups Data file

Task.writesolution

```
void writesolution
  (mosek.soltype whichsol,
   String filename)
```

Saves the current basic, interior-point, or integer solution to a file.

#### **Parameters**

- whichsol (mosek.soltype) Selects a solution. (input)
- filename (String) A valid file name. (input)

Groups Data file

Task.writetask

```
void writetask (String filename)
```

Write a binary dump of the task data. This format saves all problem data, coefficients and parameter settings but does not save callback functions and general non-linear terms.

See section  $The\ Task\ Format$  for a description of the Task format.

Parameters filename (String) - A valid file name. (input)

# 16.5 Exceptions

# MosekException

The base class for all exceptions in MOSEK.

#### Exception

Base class for exceptions that correspond to MOSEK response codes.

Implements MosekException

Error

Exception class used for all error response codes from MOSEK.

Implements Exception

16.5. Exceptions 275

### Warning

Exception class used for all warning response codes from MOSEK.

# Implements Exception

# NullArrayException

Exception thrown when null was passed to a method that expected non-null array argument.

### Implements MosekException

## ArrayLengthException

Exception thrown the length of an array was smaller than required. This will happen, for example, if requesting a list of N values, but the array passed to the method is less than N elements long.

Implements MosekException

# 16.6 Parameters grouped by topic

# **Analysis**

- dparam.ana\_sol\_infeas\_tol
- ullet iparam.ana\_sol\_basis
- $\bullet \ \ iparam. \ ana\_sol\_print\_violated$
- iparam.log\_ana\_pro

### **Basis identification**

- dparam.sim\_lu\_tol\_rel\_piv
- iparam.bi\_clean\_optimizer
- iparam.bi\_ignore\_max\_iter
- iparam.bi\_ignore\_num\_error
- iparam.bi\_max\_iterations
- iparam.intpnt\_basis
- iparam.log\_bi
- iparam.log\_bi\_freq

# Conic interior-point method

- $\bullet$  dparam.intpnt\_co\_tol\_dfeas
- $\bullet \ \ dparam. intpnt\_co\_tol\_infeas$
- dparam.intpnt\_co\_tol\_mu\_red
- $\bullet \ dparam. intpnt\_co\_tol\_near\_rel$
- dparam.intpnt\_co\_tol\_pfeas
- dparam.intpnt\_co\_tol\_rel\_gap

#### Data check

- dparam.data\_sym\_mat\_tol
- $\bullet \ dparam.\, data\_sym\_mat\_tol\_huge$
- dparam.data\_sym\_mat\_tol\_large
- dparam.data\_tol\_aij
- $\bullet$  dparam.data\_tol\_aij\_huge
- dparam.data\_tol\_aij\_large
- dparam.data\_tol\_bound\_inf
- dparam.data\_tol\_bound\_wrn
- $\bullet \ dparam.\, data\_tol\_c\_huge$
- dparam.data\_tol\_cj\_large
- dparam.data\_tol\_qij
- $\bullet$  dparam.data\_tol\_x
- $\bullet \ \ dparam.semidefinite\_tol\_approx$
- iparam.check\_convexity
- iparam.log\_check\_convexity

# Data input/output

- iparam.infeas\_report\_auto
- $\bullet \ iparam.log\_file$
- iparam.opf\_max\_terms\_per\_line
- iparam.opf\_write\_header
- iparam.opf\_write\_hints
- iparam.opf\_write\_parameters
- $\bullet$  iparam.opf\_write\_problem
- iparam.opf\_write\_sol\_bas
- $\bullet \ \ iparam.opf\_write\_sol\_itg$
- iparam.opf\_write\_sol\_itr
- $ullet \ iparam.opf\_write\_solutions$
- $\bullet \ iparam.param\_read\_case\_name$
- $\bullet \ iparam.param\_read\_ign\_error$
- $ullet iparam.read\_data\_compressed$
- $\bullet \ iparam.read\_data\_format$
- iparam.read\_debug
- $\bullet$  iparam.read\_keep\_free\_con
- $\bullet \ iparam.read\_lp\_drop\_new\_vars\_in\_bou$
- iparam.read\_lp\_quoted\_names
- $\bullet \ iparam.read\_mps\_format$

- $\bullet$   $iparam.read\_mps\_width$
- $\bullet$  iparam.read\_task\_ignore\_param
- $\bullet \ iparam.sol\_read\_name\_width$
- $\bullet$   $iparam.sol\_read\_width$
- iparam.write\_bas\_constraints
- iparam.write\_bas\_head
- $\bullet \ \ iparam.write\_bas\_variables$
- iparam.write\_data\_compressed
- $\bullet \ \ iparam.write\_data\_format$
- $\bullet$  iparam.write\_data\_param
- iparam.write\_free\_con
- $\bullet \ iparam.write\_generic\_names$
- iparam.write\_generic\_names\_io
- iparam.write\_ignore\_incompatible\_items
- $\bullet \ iparam.write\_int\_constraints$
- iparam.write\_int\_head
- iparam.write\_int\_variables
- iparam.write\_lp\_full\_obj
- iparam.write\_lp\_line\_width
- iparam.write\_lp\_quoted\_names
- iparam.write\_lp\_strict\_format
- iparam.write\_lp\_terms\_per\_line
- iparam.write\_mps\_format
- iparam.write\_mps\_int
- ullet iparam.write\_precision
- iparam.write\_sol\_barvariables
- iparam.write\_sol\_constraints
- $\bullet \ \ iparam.write\_sol\_head$
- iparam.write\_sol\_ignore\_invalid\_names
- iparam.write\_sol\_variables
- $\bullet \ \ iparam.write\_task\_inc\_sol$
- iparam.write\_xml\_mode
- ullet sparam.bas\_sol\_file\_name
- $\bullet$  sparam.data\_file\_name
- sparam.debug\_file\_name
- sparam.int\_sol\_file\_name
- $\bullet$  sparam.itr\_sol\_file\_name
- sparam.mio\_debug\_string
- sparam.param\_comment\_sign

- sparam.param\_read\_file\_name
- sparam.param\_write\_file\_name
- sparam.read\_mps\_bou\_name
- $\bullet \ \textit{sparam.read\_mps\_obj\_name}$
- sparam.read\_mps\_ran\_name
- sparam.read\_mps\_rhs\_name
- $\bullet \ \textit{sparam.sensitivity\_file\_name}$
- sparam.sensitivity\_res\_file\_name
- $\bullet \ sparam.sol\_filter\_xc\_low$
- $\bullet$  sparam.sol\_filter\_xc\_upr
- sparam.sol\_filter\_xx\_low
- $\bullet \ sparam.sol\_filter\_xx\_upr$
- $\bullet$  sparam.stat\_file\_name
- sparam.stat\_key
- sparam.stat\_name
- sparam.write\_lp\_gen\_var\_name

# Debugging

 $\bullet \ \ iparam.\, auto\_sort\_a\_before\_opt$ 

# **Dual simplex**

- $\bullet \ iparam.sim\_dual\_crash$
- $\bullet \ iparam.sim\_dual\_restrict\_selection$
- $\bullet$  iparam.sim\_dual\_selection

# Infeasibility report

- ullet  $iparam.infeas\_generic\_names$
- iparam.infeas\_report\_level
- iparam.log\_infeas\_ana

# Interior-point method

- dparam.check\_convexity\_rel\_tol
- $\bullet \ \ dparam. intpnt\_co\_tol\_dfeas$
- $\bullet$  dparam.intpnt\_co\_tol\_infeas
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_mu\_red$
- $\bullet \ \ dparam. intpnt\_co\_tol\_near\_rel$
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_pfeas$
- dparam.intpnt\_co\_tol\_rel\_gap

- $\bullet \ \ dparam.intpnt\_nl\_merit\_bal$
- ullet dparam.intpnt\_nl\_tol\_dfeas
- $\bullet \ \ dparam.intpnt\_nl\_tol\_mu\_red$
- dparam.intpnt\_nl\_tol\_near\_rel
- dparam.intpnt\_nl\_tol\_pfeas
- $\bullet \ dparam. intpnt\_nl\_tol\_rel\_gap$
- $\bullet \ \ dparam.intpnt\_nl\_tol\_rel\_step$
- dparam.intpnt\_qo\_tol\_dfeas
- $\bullet \ \ dparam. \ intpnt\_qo\_tol\_infeas$
- $\bullet$  dparam.intpnt\_qo\_tol\_mu\_red
- dparam.intpnt\_qo\_tol\_near\_rel
- $\bullet \ dparam. intpnt\_qo\_tol\_pfeas$
- $\bullet$  dparam.intpnt\_qo\_tol\_rel\_gap
- $\bullet$  dparam.intpnt\_tol\_dfeas
- $\bullet$  dparam.intpnt\_tol\_dsafe
- dparam.intpnt\_tol\_infeas
- $\bullet$  dparam.intpnt\_tol\_mu\_red
- dparam.intpnt\_tol\_path
- dparam.intpnt\_tol\_pfeas
- $\bullet$  dparam.intpnt\_tol\_psafe
- dparam.intpnt\_tol\_rel\_gap
- dparam.intpnt\_tol\_rel\_step
- $\bullet$  dparam.intpnt\_tol\_step\_size
- dparam.qcqo\_reformulate\_rel\_drop\_tol
- iparam.bi\_ignore\_max\_iter
- ullet  $iparam.bi\_ignore\_num\_error$
- iparam.intpnt\_basis
- $\bullet$  iparam.intpnt\_diff\_step
- $\bullet \ \ iparam. \ intpnt\_hotstart$
- $\bullet \ iparam. \ intpnt\_max\_iterations$
- iparam.intpnt\_max\_num\_cor
- iparam.intpnt\_max\_num\_refinement\_steps
- $\bullet \ \ iparam. \ intpnt\_off\_col\_trh$
- $ullet iparam.intpnt\_order\_method$
- $\bullet \ iparam. intpnt\_regularization\_use$
- iparam.intpnt\_scaling
- iparam.intpnt\_solve\_form
- $\bullet$  iparam.intpnt\_starting\_point
- iparam.log\_intpnt

### License manager

- iparam.cache\_license
- $\bullet \ \ iparam. \ license\_debug$
- iparam.license\_pause\_time
- iparam.license\_suppress\_expire\_wrns
- $\bullet$  iparam.license\_trh\_expiry\_wrn
- iparam.license\_wait

# Logging

- iparam.log
- iparam.log\_ana\_pro
- iparam.log\_bi
- $\bullet \ iparam. log\_bi\_freq$
- iparam.log\_cut\_second\_opt
- iparam.log\_expand
- iparam.log\_feas\_repair
- iparam.log\_file
- iparam.log\_infeas\_ana
- iparam.log\_intpnt
- iparam.log\_mio
- iparam.log\_mio\_freq
- iparam.log\_order
- iparam.log\_presolve
- iparam.log\_response
- $\bullet$  iparam.log\_sensitivity
- iparam.log\_sensitivity\_opt
- $\bullet$   $iparam.log\_sim$
- iparam.log\_sim\_freq
- $\bullet$  iparam.log\_storage

# Mixed-integer optimization

- dparam.mio\_disable\_term\_time
- dparam.mio\_max\_time
- dparam.mio\_near\_tol\_abs\_gap
- $\bullet \ dparam.mio\_near\_tol\_rel\_gap$
- dparam.mio\_rel\_gap\_const
- dparam.mio\_tol\_abs\_gap
- dparam.mio\_tol\_abs\_relax\_int

- dparam.mio\_tol\_feas
- $\bullet \ \ dparam.mio\_tol\_rel\_dual\_bound\_improvement$
- $\bullet$  dparam.mio\_tol\_rel\_gap
- iparam.log\_mio
- iparam.log\_mio\_freq
- iparam.mio\_branch\_dir
- $\bullet$   $iparam.mio\_construct\_sol$
- iparam.mio\_cut\_clique
- iparam.mio\_cut\_cmir
- iparam.mio\_cut\_gmi
- iparam.mio\_cut\_implied\_bound
- iparam.mio\_cut\_knapsack\_cover
- iparam.mio\_cut\_selection\_level
- iparam.mio\_heuristic\_level
- iparam.mio\_max\_num\_branches
- iparam.mio\_max\_num\_relaxs
- $\bullet \ \ iparam.mio\_max\_num\_solutions$
- iparam.mio\_node\_optimizer
- iparam.mio\_node\_selection
- iparam.mio\_perspective\_reformulate
- iparam.mio\_probing\_level
- iparam.mio\_rins\_max\_nodes
- iparam.mio\_root\_optimizer
- iparam.mio\_root\_repeat\_presolve\_level
- iparam.mio\_vb\_detection\_level

# Nonlinear convex method

- dparam.intpnt\_nl\_merit\_bal
- dparam.intpnt\_nl\_tol\_dfeas
- $\bullet \ dparam.intpnt\_nl\_tol\_mu\_red$
- dparam.intpnt\_nl\_tol\_near\_rel
- dparam.intpnt\_nl\_tol\_pfeas
- $\bullet$  dparam.intpnt\_nl\_tol\_rel\_gap
- $\bullet \ \ dparam.intpnt\_nl\_tol\_rel\_step$
- dparam.intpnt\_tol\_infeas
- iparam.check\_convexity
- iparam.log\_check\_convexity

## **Output information**

- iparam.infeas\_report\_level
- $\bullet \ iparam.\ license\_suppress\_expire\_wrns$
- iparam.license\_trh\_expiry\_wrn
- iparam.log
- iparam.log\_bi
- iparam.log\_bi\_freq
- iparam.log\_cut\_second\_opt
- $\bullet$  iparam.log\_expand
- $\bullet \ iparam. log\_feas\_repair$
- iparam.log\_file
- iparam.log\_infeas\_ana
- iparam.log\_intpnt
- iparam.log\_mio
- iparam.log\_mio\_freq
- iparam.log\_order
- iparam.log\_response
- iparam.log\_sensitivity
- iparam.log\_sensitivity\_opt
- $\bullet$  iparam.log\_sim
- iparam.log\_sim\_freq
- $\bullet \ iparam. log\_sim\_minor$
- $\bullet$  iparam.log\_storage
- iparam.max\_num\_warnings

## Overall solver

- $\bullet \ \ iparam.bi\_clean\_optimizer$
- iparam.infeas\_prefer\_primal
- ullet  $iparam.license\_wait$
- iparam.mio\_mode
- $\bullet \ \ iparam.optimizer$
- iparam.presolve\_level
- $\bullet \quad iparam.presolve\_max\_num\_reductions$
- iparam.presolve\_use
- iparam.primal\_repair\_optimizer
- $\bullet \ \ iparam.sensitivity\_all$
- iparam.sensitivity\_optimizer
- ullet iparam.sensitivity\_type

 $\bullet \ \ iparam.solution\_callback$ 

## Overall system

- iparam.auto\_update\_sol\_info
- iparam.intpnt\_multi\_thread
- iparam.license\_wait
- iparam.log\_storage
- iparam.mio\_mt\_user\_cb
- iparam.mt\_spincount
- iparam.num\_threads
- iparam.remove\_unused\_solutions
- iparam.timing\_level
- sparam.remote\_access\_token

#### **Presolve**

- dparam.presolve\_tol\_abs\_lindep
- dparam.presolve\_tol\_aij
- $\bullet \ \ dparam.presolve\_tol\_rel\_lindep$
- dparam.presolve\_tol\_s
- $\bullet$  dparam.presolve\_tol\_x
- $\bullet \ \ iparam.presolve\_eliminator\_max\_fill$
- iparam.presolve\_eliminator\_max\_num\_tries
- $\bullet \ \ iparam.presolve\_level$
- $\bullet$  iparam.presolve\_lindep\_abs\_work\_trh
- $\bullet \ iparam.presolve\_lindep\_rel\_work\_trh$
- iparam.presolve\_lindep\_use
- iparam.presolve\_max\_num\_reductions
- iparam.presolve\_use

## **Primal simplex**

- $\bullet \ iparam.sim\_primal\_crash$
- $\bullet \ iparam.sim\_primal\_restrict\_selection$
- $\bullet$  iparam.sim\_primal\_selection

## Progress callback

 $\bullet$  iparam.solution\_callback

## Simplex optimizer

- dparam.basis\_rel\_tol\_s
- dparam.basis\_tol\_s
- $\bullet$  dparam.basis\_tol\_x
- $\bullet \ \ dparam.sim\_lu\_tol\_rel\_piv$
- $\bullet \ \ dparam.simplex\_abs\_tol\_piv$
- iparam.basis\_solve\_use\_plus\_one
- iparam.log\_sim
- iparam.log\_sim\_freq
- iparam.log\_sim\_minor
- iparam.sensitivity\_optimizer
- iparam.sim\_basis\_factor\_use
- iparam.sim\_degen
- iparam.sim\_dual\_phaseone\_method
- iparam.sim\_exploit\_dupvec
- iparam.sim\_hotstart
- $\bullet$  iparam.sim\_hotstart\_lu
- ullet iparam.sim\_max\_iterations
- iparam.sim\_max\_num\_setbacks
- $\bullet \ iparam.sim\_non\_singular$
- $\bullet \ \ iparam.sim\_primal\_phase one\_method$
- $\bullet \ iparam.sim\_refactor\_freq$
- $\bullet$  iparam.sim\_reformulation
- ullet  $iparam.sim\_save\_lu$
- $\bullet$  iparam.sim\_scaling
- iparam.sim\_scaling\_method
- iparam.sim\_solve\_form
- $\bullet \ iparam.sim\_stability\_priority$
- $\bullet \ \ iparam.sim\_switch\_optimizer$

## Solution input/output

- $\bullet$  iparam.infeas\_report\_auto
- $\bullet \ iparam.sol\_filter\_keep\_basic$
- iparam.sol\_filter\_keep\_ranged
- $\bullet \ iparam.sol\_read\_name\_width$
- $\bullet$   $iparam.sol\_read\_width$
- iparam.write\_bas\_constraints
- ullet  $iparam.write\_bas\_head$

- $\bullet \ \ iparam.write\_bas\_variables$
- iparam.write\_int\_constraints
- iparam.write\_int\_head
- ullet iparam.write\_int\_variables
- iparam.write\_sol\_barvariables
- iparam.write\_sol\_constraints
- iparam.write\_sol\_head
- iparam.write\_sol\_ignore\_invalid\_names
- $\bullet \ iparam.write\_sol\_variables$
- sparam.bas\_sol\_file\_name
- sparam.int\_sol\_file\_name
- $\bullet$  sparam.itr\_sol\_file\_name
- sparam.sol\_filter\_xc\_low
- $\bullet$  sparam.sol\_filter\_xc\_upr
- $\bullet$  sparam.sol\_filter\_xx\_low
- sparam.sol\_filter\_xx\_upr

#### Termination criteria

- $\bullet \ dparam.basis\_rel\_tol\_s$
- dparam.basis\_tol\_s
- $\bullet$  dparam.basis\_tol\_x
- $\bullet$  dparam.intpnt\_co\_tol\_dfeas
- dparam.intpnt\_co\_tol\_infeas
- $\bullet \ \ dparam.intpnt\_co\_tol\_mu\_red$
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_near\_rel$
- dparam.intpnt\_co\_tol\_pfeas
- $\bullet \ dparam.intpnt\_co\_tol\_rel\_gap$
- dparam.intpnt\_nl\_tol\_dfeas
- dparam.intpnt\_nl\_tol\_mu\_red
- $\bullet \ \ dparam. \ intpnt\_nl\_tol\_near\_rel$
- dparam.intpnt\_nl\_tol\_pfeas
- dparam.intpnt\_nl\_tol\_rel\_gap
- $\bullet$  dparam.intpnt\_qo\_tol\_dfeas
- $\bullet \ \ dparam. \ intpnt\_qo\_tol\_infeas$
- dparam.intpnt\_qo\_tol\_mu\_red
- $\bullet \ \ dparam. \ intpnt\_qo\_tol\_near\_rel$
- dparam.intpnt\_qo\_tol\_pfeas
- $\bullet \ dparam.intpnt\_qo\_tol\_rel\_gap$
- dparam.intpnt\_tol\_dfeas

- dparam.intpnt\_tol\_infeas
- $\bullet \ dparam.intpnt\_tol\_mu\_red$
- dparam.intpnt\_tol\_pfeas
- dparam.intpnt\_tol\_rel\_gap
- dparam.lower\_obj\_cut
- dparam.lower\_obj\_cut\_finite\_trh
- dparam.mio\_disable\_term\_time
- dparam.mio\_max\_time
- dparam.mio\_near\_tol\_rel\_gap
- $\bullet$  dparam.mio\_rel\_gap\_const
- dparam.mio\_tol\_rel\_gap
- dparam.optimizer\_max\_time
- dparam.upper\_obj\_cut
- dparam.upper\_obj\_cut\_finite\_trh
- iparam.bi\_max\_iterations
- iparam.intpnt\_max\_iterations
- iparam.mio\_max\_num\_branches
- iparam.mio\_max\_num\_solutions
- ullet iparam.sim\_max\_iterations

## Other

• iparam.compress\_statfile

# 16.7 Parameters (alphabetical list sorted by type)

- Double parameters
- Integer parameters
- String parameters

## 16.7.1 Double parameters

## dparam

The enumeration type containing all double parameters.

## dparam.ana\_sol\_infeas\_tol

If a constraint violates its bound with an amount larger than this value, the constraint name, index and violation will be printed by the solution analyzer.

Default 1e-6

Accepted [0.0; +inf]

Groups Analysis

## dparam.basis\_rel\_tol\_s

Maximum relative dual bound violation allowed in an optimal basic solution.

**Default** 1.0e-12

Accepted [0.0; +inf]

Groups Simplex optimizer, Termination criteria

dparam.basis\_tol\_s

Maximum absolute dual bound violation in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Groups Simplex optimizer, Termination criteria

dparam.basis\_tol\_x

Maximum absolute primal bound violation allowed in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Groups Simplex optimizer, Termination criteria

## dparam.check\_convexity\_rel\_tol

This parameter controls when the full convexity check declares a problem to be non-convex. Increasing this tolerance relaxes the criteria for declaring the problem non-convex.

A problem is declared non-convex if negative (positive) pivot elements are detected in the Cholesky factor of a matrix which is required to be PSD (NSD). This parameter controls how much this non-negativity requirement may be violated.

If  $d_i$  is the pivot element for column i, then the matrix Q is considered to not be PSD if:

$$d_i \leq -|Q_{ii}|$$
check\_convexity\_rel\_tol

Default 1e-10

Accepted [0; +inf]

Groups Interior-point method

### dparam.data\_sym\_mat\_tol

Absolute zero tolerance for elements in in suymmetric matrixes. If any value in a symmetric matrix is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

**Default** 1.0e-12

**Accepted** [1.0e-16; 1.0e-6]

Groups Data check

#### dparam.data\_sym\_mat\_tol\_huge

An element in a symmetric matrix which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Groups Data check

## ${\tt dparam.data\_sym\_mat\_tol\_large}$

An element in a symmetric matrix which is larger than this value in absolute size causes a warning message to be printed.

Default 1.0e10

Accepted [0.0; +inf]

Groups Data check

#### dparam.data\_tol\_aij

Absolute zero tolerance for elements in A. If any value  $A_{ij}$  is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

**Default** 1.0e-12

**Accepted** [1.0e-16; 1.0e-6]

Groups Data check

## dparam.data\_tol\_aij\_huge

An element in A which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Groups Data check

## dparam.data\_tol\_aij\_large

An element in A which is larger than this value in absolute size causes a warning message to be printed.

**Default** 1.0e10

Accepted [0.0; +inf]

Groups Data check

#### dparam.data\_tol\_bound\_inf

Any bound which in absolute value is greater than this parameter is considered infinite.

**Default** 1.0e16

Accepted [0.0; +inf]

Groups Data check

#### dparam.data\_tol\_bound\_wrn

If a bound value is larger than this value in absolute size, then a warning message is issued.

Default 1.0e8

Accepted [0.0; +inf]

Groups Data check

## dparam.data\_tol\_c\_huge

An element in c which is larger than the value of this parameter in absolute terms is considered to be huge and generates an error.

**Default** 1.0e16

Accepted [0.0; +inf]

Groups Data check

## dparam.data\_tol\_cj\_large

An element in c which is larger than this value in absolute terms causes a warning message to be printed.

Default 1.0e8

Accepted [0.0; +inf]

Groups Data check

#### dparam.data\_tol\_qij

Absolute zero tolerance for elements in Q matrices.

**Default** 1.0e-16

Accepted [0.0; +inf]

```
Groups Data check
```

```
dparam.data_tol_x
```

Zero tolerance for constraints and variables i.e. if the distance between the lower and upper bound is less than this value, then the lower and upper bound is considered identical.

Default 1.0e-8

Accepted [0.0; +inf]

Groups Data check

#### dparam.intpnt\_co\_tol\_dfeas

Dual feasibility tolerance used by the conic interior-point optimizer.

Default 1.0e-8

**Accepted** [0.0; 1.0]

**Groups** Interior-point method, Termination criteria, Conic interior-point method

See also dparam.intpnt\_co\_tol\_near\_rel

#### dparam.intpnt\_co\_tol\_infeas

Controls when the conic interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Conic interior-point method

#### dparam.intpnt\_co\_tol\_mu\_red

Relative complementarity gap feasibility tolerance used by the conic interior-point optimizer.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Conic interior-point method

#### dparam.intpnt\_co\_tol\_near\_rel

If **MOSEK** cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

Default 1000

Accepted [1.0; +inf]

Groups Interior-point method, Termination criteria, Conic interior-point method

## dparam.intpnt\_co\_tol\_pfeas

Primal feasibility tolerance used by the conic interior-point optimizer.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Conic interior-point method

See also dparam.intpnt\_co\_tol\_near\_rel

## dparam.intpnt\_co\_tol\_rel\_gap

Relative gap termination tolerance used by the conic interior-point optimizer.

Default 1.0e-7

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Conic interior-point method

See also dparam.intpnt\_co\_tol\_near\_rel

dparam.intpnt\_nl\_merit\_bal

Controls if the complementarity and infeasibility is converging to zero at about equal rates.

Default 1.0e-4

Accepted [0.0; 0.99]

Groups Interior-point method, Nonlinear convex method

dparam.intpnt\_nl\_tol\_dfeas

Dual feasibility tolerance used when a nonlinear model is solved.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Nonlinear convex method

dparam.intpnt\_nl\_tol\_mu\_red

Relative complementarity gap tolerance for the nonlinear solver.

**Default** 1.0e-12

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Nonlinear convex method

dparam.intpnt\_nl\_tol\_near\_rel

If the MOSEK nonlinear interior-point optimizer cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

**Default** 1000.0

Accepted [1.0; +inf]

Groups Interior-point method, Termination criteria, Nonlinear convex method

 ${\tt dparam.intpnt\_nl\_tol\_pfeas}$ 

Primal feasibility tolerance used when a nonlinear model is solved.

**Default** 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Nonlinear convex method

dparam.intpnt\_nl\_tol\_rel\_gap

Relative gap termination tolerance for nonlinear problems.

Default 1.0e-6

Accepted [1.0e-14; +inf]

Groups Termination criteria, Interior-point method, Nonlinear convex method

dparam.intpnt\_nl\_tol\_rel\_step

Relative step size to the boundary for general nonlinear optimization problems.

Default 0.995

**Accepted** [1.0e-4; 0.9999999]

Groups Interior-point method, Nonlinear convex method

dparam.intpnt\_qo\_tol\_dfeas

Dual feasibility tolerance used when the interior-point optimizer is applied to a quadratic optimization problem..

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

See also dparam.intpnt\_qo\_tol\_near\_rel

#### dparam.intpnt\_qo\_tol\_infeas

Controls when the conic interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

Accepted [0.0; 1.0]

Groups Interior-point method, Termination criteria

#### dparam.intpnt\_qo\_tol\_mu\_red

Relative complementarity gap feasibility tolerance used when interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

#### dparam.intpnt\_qo\_tol\_near\_rel

If **MOSEK** cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

Default 1000

Accepted [1.0; +inf]

Groups Interior-point method, Termination criteria

## dparam.intpnt\_qo\_tol\_pfeas

Primal feasibility tolerance used when the interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

Accepted [0.0; 1.0]

Groups Interior-point method, Termination criteria

See also dparam.intpnt\_qo\_tol\_near\_rel

## ${\tt dparam.intpnt\_qo\_tol\_rel\_gap}$

Relative gap termination tolerance used when the interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

Accepted [0.0; 1.0]

Groups Interior-point method, Termination criteria

See also dparam.intpnt\_go\_tol\_near\_rel

## dparam.intpnt\_tol\_dfeas

Dual feasibility tolerance used for linear optimization problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Groups Interior-point method, Termination criteria

#### dparam.intpnt\_tol\_dsafe

Controls the initial dual starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it might be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Groups Interior-point method

#### dparam.intpnt\_tol\_infeas

Controls when the optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

**Accepted** [0.0; 1.0]

**Groups** Interior-point method, Termination criteria, Nonlinear convex method

#### dparam.intpnt\_tol\_mu\_red

Relative complementarity gap tolerance for linear problems.

**Default** 1.0e-16

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

## dparam.intpnt\_tol\_path

Controls how close the interior-point optimizer follows the central path. A large value of this parameter means the central is followed very closely. On numerical unstable problems it may be worthwhile to increase this parameter.

Default 1.0e-8

**Accepted** [0.0; 0.9999]

Groups Interior-point method

#### dparam.intpnt\_tol\_pfeas

Primal feasibility tolerance used for linear optimization problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

#### dparam.intpnt\_tol\_psafe

Controls the initial primal starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it may be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Groups Interior-point method

## dparam.intpnt\_tol\_rel\_gap

Relative gap termination tolerance for linear problems.

Default 1.0e-8

Accepted [1.0e-14; +inf]

 ${\bf Groups} \ \ {\it Termination \ criteria}, \ {\it Interior-point \ method}$ 

## dparam.intpnt\_tol\_rel\_step

Relative step size to the boundary for linear and quadratic optimization problems.

**Default** 0.9999

Accepted [1.0e-4; 0.999999]

Groups Interior-point method

#### dparam.intpnt\_tol\_step\_size

Minimal step size tolerance. If the step size falls below the value of this parameter, then the interior-point optimizer assumes that it is stalled. In other words the interior-point optimizer does not make any progress and therefore it is better stop.

Default 1.0e-6

**Accepted** [0.0; 1.0]

Groups Interior-point method

#### dparam.lower\_obj\_cut

If either a primal or dual feasible solution is found proving that the optimal objective value is outside, the interval [ <code>dparam.lower\_obj\_cut</code>, <code>dparam.upper\_obj\_cut</code>], then MOSEK is terminated.

**Default** -1.0e30

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

See also dparam.lower\_obj\_cut\_finite\_trh

## dparam.lower\_obj\_cut\_finite\_trh

If the lower objective cut is less than the value of this parameter value, then the lower objective cut i.e.  $dparam.lower\_obj\_cut$  is treated as  $-\infty$ .

**Default** -0.5e30

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

#### dparam.mio\_disable\_term\_time

This parameter specifies the number of seconds n during which the termination criteria governed by

- iparam.mio\_max\_num\_relaxs
- iparam.mio\_max\_num\_branches
- dparam.mio\_near\_tol\_abs\_gap
- $\bullet$  dparam.mio\_near\_tol\_rel\_gap

is disabled since the beginning of the optimization.

A negative value is identical to infinity i.e. the termination criteria are never checked.

Default -1.0

Accepted  $[-\inf; +\inf]$ 

Groups Mixed-integer optimization, Termination criteria

See also iparam.mio\_max\_num\_relaxs, iparam.mio\_max\_num\_branches, dparam.mio\_near\_tol\_abs\_gap, dparam.mio\_near\_tol\_rel\_gap

#### dparam.mio\_max\_time

This parameter limits the maximum time spent by the mixed-integer optimizer. A negative number means infinity.

**Default** -1.0

Accepted  $[-\inf; +\inf]$ 

 ${\bf Groups}\ {\it Mixed-integer}\ optimization,\ Termination\ criteria$ 

#### dparam.mio\_near\_tol\_abs\_gap

Relaxed absolute optimality tolerance employed by the mixed-integer optimizer. This termination criteria is delayed. See <code>dparam.mio\_disable\_term\_time</code> for details.

Default 0.0

Accepted [0.0; +inf]

Groups Mixed-integer optimization

See also dparam.mio\_disable\_term\_time

### dparam.mio\_near\_tol\_rel\_gap

The mixed-integer optimizer is terminated when this tolerance is satisfied. This termination criteria is delayed. See <code>dparam.mio\_disable\_term\_time</code> for details.

Default 1.0e-3

Accepted [0.0; +inf]

Groups Mixed-integer optimization, Termination criteria

See also dparam.mio\_disable\_term\_time

#### dparam.mio\_rel\_gap\_const

This value is used to compute the relative gap for the solution to an integer optimization problem.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Groups Mixed-integer optimization, Termination criteria

#### dparam.mio\_tol\_abs\_gap

Absolute optimality tolerance employed by the mixed-integer optimizer.

Default 0.0

Accepted [0.0; +inf]

**Groups** Mixed-integer optimization

## dparam.mio\_tol\_abs\_relax\_int

Absolute integer feasibility tolerance. If the distance to the nearest integer is less than this tolerance then an integer constraint is assumed to be satisfied.

Default 1.0e-5

Accepted [1e-9; +inf]

**Groups** Mixed-integer optimization

#### dparam.mio\_tol\_feas

Feasibility tolerance for mixed integer solver.

Default 1.0e-6

Accepted [1e-9; 1e-3]

**Groups** Mixed-integer optimization

## ${\tt dparam.mio\_tol\_rel\_dual\_bound\_improvement}$

If the relative improvement of the dual bound is smaller than this value, the solver will terminate the root cut generation. A value of 0.0 means that the value is selected automatically.

Default 0.0

Accepted [0.0; 1.0]

**Groups** Mixed-integer optimization

## dparam.mio\_tol\_rel\_gap

Relative optimality tolerance employed by the mixed-integer optimizer.

Default 1.0e-4

Accepted [0.0; +inf]

Groups Mixed-integer optimization, Termination criteria

#### dparam.optimizer\_max\_time

Maximum amount of time the optimizer is allowed to spent on the optimization. A negative number means infinity.

**Default** -1.0

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

## dparam.presolve\_tol\_abs\_lindep

Absolute tolerance employed by the linear dependency checker.

Default 1.0e-6

Accepted [0.0; +inf]

Groups Presolve

## dparam.presolve\_tol\_aij

Absolute zero tolerance employed for  $a_{ij}$  in the presolve.

**Default** 1.0e-12

Accepted [1.0e-15; +inf]

Groups Presolve

## dparam.presolve\_tol\_rel\_lindep

Relative tolerance employed by the linear dependency checker.

 $\textbf{Default} \ \ 1.0\text{e-}10$ 

Accepted [0.0; +inf]

 ${\bf Groups}\ {\it Presolve}$ 

## dparam.presolve\_tol\_s

Absolute zero tolerance employed for  $s_i$  in the presolve.

**Default** 1.0e-8

Accepted [0.0; +inf]

Groups Presolve

## dparam.presolve\_tol\_x

Absolute zero tolerance employed for  $x_j$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Groups Presolve

## dparam.qcqo\_reformulate\_rel\_drop\_tol

This parameter determines when columns are dropped in incomplete Cholesky factorization during reformulation of quadratic problems.

Default 1e-15

Accepted [0; +inf]

Groups Interior-point method

## dparam.semidefinite\_tol\_approx

Tolerance to define a matrix to be positive semidefinite.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Groups Data check

## dparam.sim\_lu\_tol\_rel\_piv

Relative pivot tolerance employed when computing the LU factorization of the basis in the simplex optimizers and in the basis identification procedure.

A value closer to 1.0 generally improves numerical stability but typically also implies an increase in the computational work.

Default 0.01

**Accepted** [1.0e-6; 0.999999]

Groups Basis identification, Simplex optimizer

## dparam.simplex\_abs\_tol\_piv

Absolute pivot tolerance employed by the simplex optimizers.

Default 1.0e-7

Accepted [1.0e-12; +inf]

Groups Simplex optimizer

### dparam.upper\_obj\_cut

If either a primal or dual feasible solution is found proving that the optimal objective value is outside, the interval [ dparam.lower\_obj\_cut, dparam.upper\_obj\_cut], then MOSEK is terminated.

**Default** 1.0e30

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

See also  $dparam.upper\_obj\_cut\_finite\_trh$ 

#### dparam.upper\_obj\_cut\_finite\_trh

If the upper objective cut is greater than the value of this parameter, then the upper objective cut  $dparam.upper\_obj\_cut$  is treated as  $\infty$ .

Default 0.5e30

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

## 16.7.2 Integer parameters

iparam

The enumeration type containing all integer parameters.

## iparam.ana\_sol\_basis

Controls whether the basis matrix is analyzed in solution analyzer.

Default on

Accepted on, off (see onoffkey)

Groups Analysis

## iparam.ana\_sol\_print\_violated

 $Controls \ whether \ a \ list \ of \ violated \ constraints \ is \ printed \ when \ calling \ \textit{Task.analyzesolution}.$ 

All constraints violated by more than the value set by the parameter dparam.ana\_sol\_infeas\_tol will be printed.

```
Default off
Accepted on, off (see onoffkey)
Groups Analysis
```

#### iparam.auto\_sort\_a\_before\_opt

Controls whether the elements in each column of A are sorted before an optimization is performed. This is not required but makes the optimization more deterministic.

```
Default off
Accepted on, off (see onoffkey)
Groups Debugging
```

## iparam.auto\_update\_sol\_info

Controls whether the solution information items are automatically updated after an optimization is performed.

```
Default off
Accepted on, off (see onoffkey)
Groups Overall system
```

## iparam.basis\_solve\_use\_plus\_one

If a slack variable is in the basis, then the corresponding column in the basis is a unit vector with -1 in the right position. However, if this parameter is set to <code>onoffkey.on</code>, -1 is replaced by 1.

This has significance for the results returned by the Task. solvewithbasis function.

```
Default off
Accepted on, off (see onoffkey)
Groups Simplex optimizer
```

#### iparam.bi\_clean\_optimizer

Controls which simplex optimizer is used in the clean-up phase.

```
Default free

Accepted free, intpnt, conic, primal_simplex, dual_simplex, free_simplex,
    mixed_int (see optimizertype)

Groups Basis identification, Overall solver
```

#### iparam.bi\_ignore\_max\_iter

If the parameter  $iparam.intpnt\_basis$  has the value  $basindtype.no\_error$  and the interior-point optimizer has terminated due to maximum number of iterations, then basis identification is performed if this parameter has the value onoffkey.on.

```
Default off
Accepted on, off (see onoffkey)
Groups Interior-point method, Basis identification
```

## iparam.bi\_ignore\_num\_error

If the parameter <code>iparam.intpnt\_basis</code> has the value <code>basindtype.no\_error</code> and the interior-point optimizer has terminated due to a numerical problem, then basis identification is performed if this parameter has the value <code>onoffkey.on</code>.

```
Default off
Accepted on, off (see onoffkey)
Groups Interior-point method, Basis identification
```

#### iparam.bi\_max\_iterations

Controls the maximum number of simplex iterations allowed to optimize a basis after the basis identification.

**Default** 1000000

Accepted [0; +inf]

Groups Basis identification, Termination criteria

#### iparam.cache\_license

Specifies if the license is kept checked out for the lifetime of the mosek environment (onoffkey.on) or returned to the server immediately after the optimization (onoffkey.off).

By default the license is checked out for the lifetime of the  $\mathbf{MOSEK}$  environment by the first call to Task.optimize.

Check-in and check-out of licenses have an overhead. Frequent communication with the license server should be avoided.

Default on

Accepted on, off (see onoffkey)

Groups License manager

## iparam.check\_convexity

Specify the level of convexity check on quadratic problems.

Default full

Accepted none, simple, full (see checkconvexitytype)

Groups Data check, Nonlinear convex method

## iparam.compress\_statfile

Control compression of stat files.

Default on

Accepted on, off (see onoffkey)

## iparam.infeas\_generic\_names

Controls whether generic names are used when an infeasible subproblem is created.

Default off

Accepted on, off (see onoffkey)

Groups Infeasibility report

## iparam.infeas\_prefer\_primal

If both certificates of primal and dual infeasibility are supplied then only the primal is used when this option is turned on.

Default on

Accepted on, off (see onoffkey)

Groups Overall solver

## iparam.infeas\_report\_auto

Controls whether an infeasibility report is automatically produced after the optimization if the problem is primal or dual infeasible.

Default off

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

#### iparam.infeas\_report\_level

Controls the amount of information presented in an infeasibility report. Higher values imply more information.

#### Default 1

Accepted [0; +inf]

Groups Infeasibility report, Output information

#### iparam.intpnt\_basis

Controls whether the interior-point optimizer also computes an optimal basis.

```
Default always
```

Accepted never, always, no\_error, if\_feasible, reservered (see basindtype)

Groups Interior-point method, Basis identification

See also iparam.bi\_ignore\_max\_iter, iparam.bi\_ignore\_num\_error, iparam.bi\_max\_iterations, iparam.bi\_clean\_optimizer

## iparam.intpnt\_diff\_step

Controls whether different step sizes are allowed in the primal and dual space.

#### Default on

#### Accepted

- on: Different step sizes are allowed.
- off: Different step sizes are not allowed.

Groups Interior-point method

## iparam.intpnt\_hotstart

Currently not in use.

Default none

Accepted none, primal, dual, primal\_dual (see intpnthotstart)

Groups Interior-point method

## iparam.intpnt\_max\_iterations

Controls the maximum number of iterations allowed in the interior-point optimizer.

Default 400

Accepted [0; +inf]

Groups Interior-point method, Termination criteria

## iparam.intpnt\_max\_num\_cor

Controls the maximum number of correctors allowed by the multiple corrector procedure. A negative value means that **MOSEK** is making the choice.

Default -1

Accepted [-1; +inf]

Groups Interior-point method

#### iparam.intpnt\_max\_num\_refinement\_steps

Maximum number of steps to be used by the iterative refinement of the search direction. A negative value implies that the optimizer chooses the maximum number of iterative refinement steps.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Interior-point method

#### iparam.intpnt\_multi\_thread

Controls whether the interior-point optimizers are allowed to employ multiple threads if more threads is available.

Default on

Accepted on, off (see onoffkey)

Groups Overall system

#### iparam.intpnt\_off\_col\_trh

Controls how many offending columns are detected in the Jacobian of the constraint matrix.

0	no detection
1	aggressive detection
> 1	higher values mean less aggressive detection

Default 40

Accepted [0; +inf]

Groups Interior-point method

### iparam.intpnt\_order\_method

Controls the ordering strategy used by the interior-point optimizer when factorizing the Newton equation system.

Default free

**Accepted** free, appminloc, experimental, try\_graphpar, force\_graphpar, none (see orderingtype)

Groups Interior-point method

## iparam.intpnt\_regularization\_use

Controls whether regularization is allowed.

Default on

Accepted on, off (see onoffkey)

Groups Interior-point method

## iparam.intpnt\_scaling

Controls how the problem is scaled before the interior-point optimizer is used.

Default free

Accepted free, none, moderate, aggressive (see scalingtype)

**Groups** Interior-point method

## ${\tt iparam.intpnt\_solve\_form}$

Controls whether the primal or the dual problem is solved.

Default free

Accepted free, primal, dual (see solveform)

Groups Interior-point method

## iparam.intpnt\_starting\_point

Starting point used by the interior-point optimizer.

Default free

Accepted free, guess, constant, satisfy\_bounds (see startpointtype)

Groups Interior-point method

```
iparam.license_debug
```

This option is used to turn on debugging of the license manager.

```
Default off
```

Accepted on, off (see onoffkey)

Groups License manager

#### iparam.license\_pause\_time

If  $iparam.license\_wait = onoffkey.on$  and no license is available, then **MOSEK** sleeps a number of milliseconds between each check of whether a license has become free.

Default 100

**Accepted** [0; 1000000]

Groups License manager

## iparam.license\_suppress\_expire\_wrns

Controls whether license features expire warnings are suppressed.

Default off

Accepted on, off (see onoffkey)

Groups License manager, Output information

### iparam.license\_trh\_expiry\_wrn

If a license feature expires in a numbers days less than the value of this parameter then a warning will be issued.

Default 7

Accepted [0; +inf]

Groups License manager, Output information

#### iparam.license\_wait

If all licenses are in use **MOSEK** returns with an error code. However, by turning on this parameter **MOSEK** will wait for an available license.

Default off

Accepted on, off (see onoffkey)

Groups Overall solver, Overall system, License manager

#### iparam.log

Controls the amount of log information. The value 0 implies that all log information is suppressed. A higher level implies that more information is logged.

Please note that if a task is employed to solve a sequence of optimization problems the value of this parameter is reduced by the value of  $iparam.log\_cut\_second\_opt$  for the second and any subsequent optimizations.

Default 10

Accepted [0; +inf]

Groups Output information, Logging

See also iparam.log\_cut\_second\_opt

## iparam.log\_ana\_pro

Controls amount of output from the problem analyzer.

Default 1

Accepted [0; +inf]

Groups Analysis, Logging

#### iparam.log\_bi

Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.

#### Default 1

Accepted [0; +inf]

Groups Basis identification, Output information, Logging

#### iparam.log\_bi\_freq

Controls how frequent the optimizer outputs information about the basis identification and how frequent the user-defined callback function is called.

Default 2500

Accepted [0; +inf]

**Groups** Basis identification, Output information, Logging

## iparam.log\_check\_convexity

Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on. If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

#### Default 0

Accepted  $[0; +\inf]$ 

Groups Data check, Nonlinear convex method

#### iparam.log\_cut\_second\_opt

If a task is employed to solve a sequence of optimization problems, then the value of the log levels is reduced by the value of this parameter. E.g *iparam.log* and *iparam.log\_sim* are reduced by the value of this parameter for the second and any subsequent optimizations.

## Default 1

Accepted [0; +inf]

Groups Output information, Logging

See also iparam.log, iparam.log\_intpnt, iparam.log\_mio, iparam.log\_sim

#### iparam.log\_expand

Controls the amount of logging when a data item such as the maximum number constrains is expanded.

#### Default 0

Accepted [0; +inf]

Groups Output information, Logging

## iparam.log\_feas\_repair

Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.

#### Default 1

Accepted  $[0; +\inf]$ 

Groups Output information, Logging

## iparam.log\_file

If turned on, then some log info is printed when a file is written or read.

## Default 1

Accepted [0; +inf]

Groups Data input/output, Output information, Logging

#### iparam.log\_infeas\_ana

Controls amount of output printed by the infeasibility analyzer procedures. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Groups Infeasibility report, Output information, Logging

#### iparam.log\_intpnt

Controls amount of output printed by the interior-point optimizer. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Groups Interior-point method, Output information, Logging

#### iparam.log\_mio

Controls the log level for the mixed-integer optimizer. A higher level implies that more information is logged.

Default 4

Accepted [0; +inf]

Groups Mixed-integer optimization, Output information, Logging

#### iparam.log\_mio\_freq

Controls how frequent the mixed-integer optimizer prints the log line. It will print line every time <code>iparam.log\_mio\_freq</code> relaxations have been solved.

Default 10

Accepted  $[-\inf; +\inf]$ 

Groups Mixed-integer optimization, Output information, Logging

## iparam.log\_order

If turned on, then factor lines are added to the log.

Default 1

Accepted [0; +inf]

Groups Output information, Logging

## iparam.log\_presolve

Controls amount of output printed by the presolve procedure. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Groups Logging

## iparam.log\_response

Controls amount of output printed when response codes are reported. A higher level implies that more information is logged.

**Default** 0

Accepted [0; +inf]

Groups Output information, Logging

## iparam.log\_sensitivity

Controls the amount of logging during the sensitivity analysis.

- 0. Means no logging information is produced.
- 1. Timing information is printed.
- 2. Sensitivity results are printed.

#### Default 1

Accepted [0; +inf]

Groups Output information, Logging

#### iparam.log\_sensitivity\_opt

Controls the amount of logging from the optimizers employed during the sensitivity analysis. 0 means no logging information is produced.

Default 0

Accepted  $[0; +\inf]$ 

**Groups** Output information, Logging

#### iparam.log\_sim

Controls amount of output printed by the simplex optimizer. A higher level implies that more information is logged.

Default 4

Accepted [0; +inf]

Groups Simplex optimizer, Output information, Logging

#### iparam.log\_sim\_freq

Controls how frequent the simplex optimizer outputs information about the optimization and how frequent the user-defined callback function is called.

Default 1000

Accepted [0; +inf]

Groups Simplex optimizer, Output information, Logging

## iparam.log\_sim\_minor

Currently not in use.

Default 1

Accepted [0; +inf]

Groups Simplex optimizer, Output information

#### iparam.log\_storage

When turned on, MOSEK prints messages regarding the storage usage and allocation.

Default 0

Accepted [0; +inf]

Groups Output information, Overall system, Logging

## iparam.max\_num\_warnings

Each warning is shown a limit number times controlled by this parameter. A negative value is identical to infinite number of times.

Default 10

Accepted [-inf; +inf]

Groups Output information

## iparam.mio\_branch\_dir

Controls whether the mixed-integer optimizer is branching up or down by default.

```
Default free
```

Accepted free, up, down, near, far, root\_lp, guided, pseudocost (see branchdir)

**Groups** Mixed-integer optimization

#### iparam.mio\_construct\_sol

If set to *onoffkey.on* and all integer variables have been given a value for which a feasible mixed integer solution exists, then **MOSEK** generates an initial solution to the mixed integer problem by fixing all integer values and solving the remaining problem.

Default off

Accepted on, off (see onoffkey)

**Groups** Mixed-integer optimization

iparam.mio\_cut\_clique

Controls whether clique cuts should be generated.

Default on

## Accepted

- on: Turns generation of this cut class on.
- off: Turns generation of this cut class off.

**Groups** Mixed-integer optimization

iparam.mio\_cut\_cmir

Controls whether mixed integer rounding cuts should be generated.

Default on

#### Accepted

- on: Turns generation of this cut class on.
- off: Turns generation of this cut class off.

**Groups** Mixed-integer optimization

 ${\tt iparam.mio\_cut\_gmi}$ 

Controls whether GMI cuts should be generated.

Default on

## Accepted

- $\bullet$  on: Turns generation of this cut class on.
- off: Turns generation of this cut class off.

**Groups** Mixed-integer optimization

iparam.mio\_cut\_implied\_bound

Controls whether implied bound cuts should be generated.

Default off

## Accepted

- $\bullet$  on: Turns generation of this cut class on.
- off: Turns generation of this cut class off.

 ${\bf Groups}\ \textit{Mixed-integer optimization}$ 

## iparam.mio\_cut\_knapsack\_cover

Controls whether knapsack cover cuts should be generated.

Default off

## Accepted

- on: Turns generation of this cut class on.
- off: Turns generation of this cut class off.

**Groups** Mixed-integer optimization

#### iparam.mio\_cut\_selection\_level

Controls how aggressively generated cuts are selected to be included in the relaxation.

- -1. The optimizer chooses the level of cut selection
  - 0. Generated cuts less likely to be added to the relaxation
  - 1. Cuts are more aggressively selected to be included in the relaxation

#### Default -1

Accepted [-1; +1]

 ${\bf Groups}\ \textit{Mixed-integer optimization}$ 

## iparam.mio\_heuristic\_level

Controls the heuristic employed by the mixed-integer optimizer to locate an initial good integer feasible solution. A value of zero means the heuristic is not used at all. A larger value than 0 means that a gradually more sophisticated heuristic is used which is computationally more expensive. A negative value implies that the optimizer chooses the heuristic. Normally a value around 3 to 5 should be optimal.

#### Default -1

Accepted  $[-\inf; +\inf]$ 

**Groups** Mixed-integer optimization

## iparam.mio\_max\_num\_branches

Maximum number of branches allowed during the branch and bound search. A negative value means infinite.

## Default -1

Accepted [-inf; +inf]

Groups Mixed-integer optimization, Termination criteria

See also dparam.mio\_disable\_term\_time

### iparam.mio\_max\_num\_relaxs

Maximum number of relaxations allowed during the branch and bound search. A negative value means infinite.

#### Default -1

Accepted  $[-\inf; +\inf]$ 

**Groups** Mixed-integer optimization

See also dparam.mio\_disable\_term\_time

## iparam.mio\_max\_num\_solutions

The mixed-integer optimizer can be terminated after a certain number of different feasible solutions has been located. If this parameter has the value n > 0, then the mixed-integer optimizer will be terminated when n feasible solutions have been located.

## Default -1

Accepted  $[-\inf; +\inf]$ 

 ${\bf Groups}\ {\it Mixed-integer}\ optimization,\ Termination\ criteria$ 

See also dparam.mio\_disable\_term\_time

#### iparam.mio\_mode

Controls whether the optimizer includes the integer restrictions when solving a (mixed) integer optimization problem.

Default satisfied

Accepted ignored, satisfied (see miomode)

Groups Overall solver

#### iparam.mio\_mt\_user\_cb

If true user callbacks are called from each thread used by mixed-integer optimizer. Otherwise it is only called from a single thread.

Default off

Accepted on, off (see onoffkey)

Groups Overall system

#### iparam.mio\_node\_optimizer

Controls which optimizer is employed at the non-root nodes in the mixed-integer optimizer.

Default free

Accepted free, intpnt, conic, primal\_simplex, dual\_simplex, free\_simplex, mixed\_int (see optimizertype)

Groups Mixed-integer optimization

#### iparam.mio\_node\_selection

Controls the node selection strategy employed by the mixed-integer optimizer.

Default free

Accepted free, first, best, worst, hybrid, pseudo (see mionodeseltype)

**Groups** Mixed-integer optimization

## iparam.mio\_perspective\_reformulate

Enables or disables perspective reformulation in presolve.

Default on

Accepted on, off (see onoffkey)

**Groups** Mixed-integer optimization

#### iparam.mio\_probing\_level

Controls the amount of probing employed by the mixed-integer optimizer in presolve.

- -1. The optimizer chooses the level of probing employed
  - 0. Probing is disabled
  - 1. A low amount of probing is employed
  - 2. A medium amount of probing is employed
  - 3. A high amount of probing is employed

Default -1

Accepted [-1; 3]

**Groups** Mixed-integer optimization

## iparam.mio\_rins\_max\_nodes

Controls the maximum number of nodes allowed in each call to the RINS heuristic. The default value of -1 means that the value is determined automatically. A value of zero turns off the heuristic.

Default -1

```
Accepted [-1; +inf]
```

**Groups** Mixed-integer optimization

#### iparam.mio\_root\_optimizer

Controls which optimizer is employed at the root node in the mixed-integer optimizer.

Default free

Accepted free, intpnt, conic, primal\_simplex, dual\_simplex, free\_simplex, mixed\_int (see optimizertype)

**Groups** Mixed-integer optimization

## iparam.mio\_root\_repeat\_presolve\_level

Controls whether presolve can be repeated at root node.

- -1 The optimizer chooses whether presolve is repeated
- 0 Never repeat presolve
- 1 Always repeat presolve

Default -1

Accepted [-1; 1]

**Groups** Mixed-integer optimization

#### iparam.mio\_vb\_detection\_level

Controls how much effort is put into detecting variable bounds.

- -1. The optimizer chooses
  - 0. No variable bounds are detected
  - 1. Only detect variable bounds that are directly represented in the problem
  - 2. Detect variable bounds in probing

Default -1

Accepted [-1; +2]

**Groups** Mixed-integer optimization

### iparam.mt\_spincount

Set the number of iterations to spin before sleeping.

Default 0

**Accepted** [0; 1000000000]

Groups Overall system

## iparam.num\_threads

Controls the number of threads employed by the optimizer. If set to 0 the number of threads used will be equal to the number of cores detected on the machine.

Default 0

**Accepted**  $[0; +\inf]$ 

Groups Overall system

## iparam.opf\_max\_terms\_per\_line

The maximum number of terms (linear and quadratic) per line when an OPF file is written.

Default 5

Accepted [0; +inf]

```
Groups Data input/output
iparam.opf_write_header
     Write a text header with date and MOSEK version in an OPF file.
         Accepted on, off (see onoffkey)
         Groups Data input/output
iparam.opf_write_hints
     Write a hint section with problem dimensions in the beginning of an OPF file.
         Default on
         Accepted on, off (see onoffkey)
         Groups Data input/output
iparam.opf_write_parameters
     Write a parameter section in an OPF file.
         Default off
         Accepted on, off (see onoffkey)
         Groups Data input/output
iparam.opf_write_problem
     Write objective, constraints, bounds etc. to an OPF file.
         Default on
          Accepted on, off (see onoffkey)
         Groups Data input/output
iparam.opf_write_sol_bas
     If iparam. opf_write_solutions is onoffkey. on and a basic solution is defined, include the basic
     solution in OPF files.
         Default on
         Accepted on, off (see onoffkey)
         Groups Data input/output
iparam.opf_write_sol_itg
     If iparam.opf_write_solutions is onoffkey.on and an integer solution is defined, write the
     integer solution in OPF files.
         Default on
         Accepted on, off (see onoffkey)
         Groups Data input/output
iparam.opf_write_sol_itr
     If iparam.opf_write_solutions is onoffkey.on and an interior solution is defined, write the
     interior solution in OPF files.
         Default on
         Accepted on, off (see onoffkey)
         Groups Data input/output
iparam.opf_write_solutions
     Enable inclusion of solutions in the OPF files.
         Default off
          Accepted on, off (see onoffkey)
```

```
Groups Data input/output
iparam.optimizer
     The parameter controls which optimizer is used to optimize the task.
         Default free
          Accepted free, intpnt, conic, primal_simplex, dual_simplex, free_simplex,
              mixed_int (see optimizertype)
          Groups Overall solver
iparam.param_read_case_name
     If turned on, then names in the parameter file are case sensitive.
         Default on
          Accepted on, off (see onoffkey)
          Groups Data input/output
iparam.param_read_ign_error
     If turned on, then errors in parameter settings is ignored.
         Default off
          Accepted on, off (see onoffkey)
          Groups Data input/output
iparam.presolve_eliminator_max_fill
     Controls the maximum amount of fill-in that can be created by one pivot in the elimination phase
     of the presolve. A negative value means the parameter value is selected automatically.
          Default -1
          Accepted [-\inf; +\inf]
          Groups Presolve
iparam.presolve_eliminator_max_num_tries
     Control the maximum number of times the eliminator is tried. A negative value implies MOSEK
     decides.
         Default -1
          Accepted [-\inf; +\inf]
          Groups Presolve
iparam.presolve_level
     Currently not used.
          Default -1
          Accepted [-\inf; +\inf]
          Groups Overall solver, Presolve
iparam.presolve_lindep_abs_work_trh
     The linear dependency check is potentially computationally expensive.
          Default 100
          Accepted [-\inf; +\inf]
          Groups Presolve
iparam.presolve_lindep_rel_work_trh
     The linear dependency check is potentially computationally expensive.
         Default 100
          Accepted [-\inf; +\inf]
```

### Groups Presolve

#### iparam.presolve\_lindep\_use

Controls whether the linear constraints are checked for linear dependencies.

#### Default on

## Accepted

- on: Turns the linear dependency check on.
- off: Turns the linear dependency check off.

#### Groups Presolve

#### iparam.presolve\_max\_num\_reductions

Controls the maximum number of reductions performed by the presolve. The value of the parameter is normally only changed in connection with debugging. A negative value implies that an infinite number of reductions are allowed.

```
Default -1
```

```
Accepted [-\inf; +\inf]
```

Groups Overall solver, Presolve

## iparam.presolve\_use

Controls whether the presolve is applied to a problem before it is optimized.

```
Default free
```

```
Accepted off, on, free (see presolvemode)
```

Groups Overall solver, Presolve

## iparam.primal\_repair\_optimizer

Controls which optimizer that is used to find the optimal repair.

```
Default free
```

```
Accepted free, intpnt, conic, primal_simplex, dual_simplex, free_simplex, mixed_int (see optimizertype)
```

Groups Overall solver

## iparam.read\_data\_compressed

If this option is turned on, it is assumed that the data file is compressed.

```
Default free
```

```
Accepted none, free, gzip (see compresstype)
```

Groups Data input/output

### iparam.read\_data\_format

Format of the data file to be read.

```
Default extension
```

```
Accepted extension, mps, lp, op, xml, free_mps, task, cb, json_task (see dataformat)
```

Groups Data input/output

## iparam.read\_debug

Turns on additional debugging information when reading files.

```
Default off
```

```
Accepted on, off (see onoffkey)
```

Groups Data input/output

#### iparam.read\_keep\_free\_con

Controls whether the free constraints are included in the problem.

Default off

## Accepted

- on: The free constraints are kept.
- off: The free constraints are discarded.

Groups Data input/output

## iparam.read\_lp\_drop\_new\_vars\_in\_bou

If this option is turned on, **MOSEK** will drop variables that are defined for the first time in the bounds section.

Default off

Accepted on, off (see onoffkey)

Groups Data input/output

#### iparam.read\_lp\_quoted\_names

If a name is in quotes when reading an LP file, the quotes will be removed.

Default on

Accepted on, off (see onoffkey)

Groups Data input/output

## iparam.read\_mps\_format

Controls how strictly the MPS file reader interprets the MPS format.

Default free

Accepted strict, relaxed, free, cplex (see mpsformat)

Groups Data input/output

#### iparam.read\_mps\_width

Controls the maximal number of characters allowed in one line of the MPS file.

Default 1024

Accepted [80; +inf]

Groups Data input/output

## iparam.read\_task\_ignore\_param

Controls whether **MOSEK** should ignore the parameter setting defined in the task file and use the default parameter setting instead.

Default off

Accepted on, off (see onoffkey)

Groups Data input/output

## iparam.remove\_unused\_solutions

Removes unsued solutions before the optimization is performed.

Default off

Accepted on, off (see onoffkey)

Groups Overall system

## ${\tt iparam.sensitivity\_all}$

If set to <code>onoffkey.on</code>, then <code>Task.sensitivityreport</code> analyzes all bounds and variables instead of reading a specification from the file.

Default off

```
Accepted on, off (see onoffkey)
```

Groups Overall solver

## iparam.sensitivity\_optimizer

Controls which optimizer is used for optimal partition sensitivity analysis.

Default free\_simplex

Accepted free, intpnt, conic, primal\_simplex, dual\_simplex, free\_simplex, mixed\_int (see optimizertype)

Groups Overall solver, Simplex optimizer

#### iparam.sensitivity\_type

Controls which type of sensitivity analysis is to be performed.

Default basis

Accepted basis, optimal\_partition (see sensitivitytype)

Groups Overall solver

#### iparam.sim\_basis\_factor\_use

Controls whether an LU factorization of the basis is used in a hot-start. Forcing a refactorization sometimes improves the stability of the simplex optimizers, but in most cases there is a performance penalty.

Default on

Accepted on, off (see onoffkey)

Groups Simplex optimizer

#### iparam.sim\_degen

Controls how aggressively degeneration is handled.

Default free

Accepted none, free, aggressive, moderate, minimum (see simdegen)

Groups Simplex optimizer

## iparam.sim\_dual\_crash

Controls whether crashing is performed in the dual simplex optimizer.

If this parameter is set to x, then a crash will be performed if a basis consists of more than (100-x) mod  $f_v$  entries, where  $f_v$  is the number of fixed variables.

**Default** 90

Accepted [0; +inf]

Groups Dual simplex

## $\verb"iparam.sim_dual_phase one_method"$

An experimental feature.

Default 0

Accepted [0; 10]

Groups Simplex optimizer

## iparam.sim\_dual\_restrict\_selection

The dual simplex optimizer can use a so-called restricted selection/pricing strategy to chooses the outgoing variable. Hence, if restricted selection is applied, then the dual simplex optimizer first choose a subset of all the potential outgoing variables. Next, for some time it will choose the outgoing variable only among the subset. From time to time the subset is redefined.

A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

```
Default 50
```

**Accepted** [0; 100]

Groups Dual simplex

#### iparam.sim\_dual\_selection

Controls the choice of the incoming variable, known as the selection strategy, in the dual simplex optimizer.

Default free

Accepted free, full, ase, devex, se, partial (see simseltype)

Groups Dual simplex

## iparam.sim\_exploit\_dupvec

Controls if the simplex optimizers are allowed to exploit duplicated columns.

Default off

Accepted on, off, free (see simdupvec)

Groups Simplex optimizer

### iparam.sim\_hotstart

Controls the type of hot-start that the simplex optimizer perform.

Default free

Accepted none, free, status\_keys (see simhotstart)

 ${\bf Groups} \ \, Simplex \ \, optimizer$ 

#### iparam.sim\_hotstart\_lu

Determines if the simplex optimizer should exploit the initial factorization.

Default on

## Accepted

- on: Factorization is reused if possible.
- *off*: Factorization is recomputed.

Groups Simplex optimizer

## iparam.sim\_max\_iterations

Maximum number of iterations that can be used by a simplex optimizer.

 $\mathbf{Default}\ 10000000$ 

Accepted [0; +inf]

Groups Simplex optimizer, Termination criteria

#### iparam.sim\_max\_num\_setbacks

Controls how many set-backs are allowed within a simplex optimizer. A set-back is an event where the optimizer moves in the wrong direction. This is impossible in theory but may happen due to numerical problems.

Default 250

**Accepted**  $[0; +\inf]$ 

Groups Simplex optimizer

## iparam.sim\_non\_singular

Controls if the simplex optimizer ensures a non-singular basis, if possible.

Default on

Accepted on, off (see onoffkey)

## Groups Simplex optimizer

#### iparam.sim\_primal\_crash

Controls whether crashing is performed in the primal simplex optimizer.

In general, if a basis consists of more than (100-this parameter value)% fixed variables, then a crash will be performed.

Default 90

Accepted [0; +inf]

Groups Primal simplex

#### iparam.sim\_primal\_phaseone\_method

An experimental feature.

Default 0

**Accepted** [0; 10]

Groups Simplex optimizer

#### iparam.sim\_primal\_restrict\_selection

The primal simplex optimizer can use a so-called restricted selection/pricing strategy to chooses the outgoing variable. Hence, if restricted selection is applied, then the primal simplex optimizer first choose a subset of all the potential incoming variables. Next, for some time it will choose the incoming variable only among the subset. From time to time the subset is redefined.

A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

Default 50

**Accepted** [0; 100]

Groups Primal simplex

## iparam.sim\_primal\_selection

Controls the choice of the incoming variable, known as the selection strategy, in the primal simplex optimizer.

Default free

Accepted free, full, ase, devex, se, partial (see simseltype)

Groups Primal simplex

#### iparam.sim\_refactor\_freq

Controls how frequent the basis is refactorized. The value 0 means that the optimizer determines the best point of refactorization.

It is strongly recommended NOT to change this parameter.

**Default** 0

Accepted [0; +inf]

Groups Simplex optimizer

## iparam.sim\_reformulation

Controls if the simplex optimizers are allowed to reformulate the problem.

Default off

Accepted on, off, free, aggressive (see simreform)

Groups Simplex optimizer

#### iparam.sim\_save\_lu

Controls if the LU factorization stored should be replaced with the LU factorization corresponding to the initial basis.

```
Default off
          Accepted on, off (see onoffkey)
          Groups Simplex optimizer
iparam.sim_scaling
     Controls how much effort is used in scaling the problem before a simplex optimizer is used.
          Default free
          Accepted free, none, moderate, aggressive (see scalingtype)
          Groups Simplex optimizer
iparam.sim_scaling_method
     Controls how the problem is scaled before a simplex optimizer is used.
          Default pow2
          Accepted pow2, free (see scalingmethod)
          Groups Simplex optimizer
iparam.sim_solve_form
     Controls whether the primal or the dual problem is solved by the primal-/dual-simplex optimizer.
          Default free
          Accepted free, primal, dual (see solveform)
          Groups Simplex optimizer
iparam.sim_stability_priority
     Controls how high priority the numerical stability should be given.
          Default 50
          Accepted [0; 100]
          Groups Simplex optimizer
iparam.sim_switch_optimizer
     The simplex optimizer sometimes chooses to solve the dual problem instead of the primal problem.
     This implies that if you have chosen to use the dual simplex optimizer and the problem is dualized,
     then it actually makes sense to use the primal simplex optimizer instead. If this parameter is on
     and the problem is dualized and furthermore the simplex optimizer is chosen to be the primal
     (dual) one, then it is switched to the dual (primal).
          Default off
          Accepted on, off (see onoffkey)
          Groups Simplex optimizer
iparam.sol_filter_keep_basic
     If turned on, then basic and super basic constraints and variables are written to the solution file
     independent of the filter setting.
          Default off
          Accepted on, off (see onoffkey)
          Groups Solution input/output
iparam.sol_filter_keep_ranged
     If turned on, then ranged constraints and variables are written to the solution file independent of
     the filter setting.
```

Accepted on, off (see onoffkey)

Default off

```
Groups Solution input/output
```

#### iparam.sol\_read\_name\_width

When a solution is read by **MOSEK** and some constraint, variable or cone names contain blanks, then a maximum name width much be specified. A negative value implies that no name contain blanks.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Data input/output, Solution input/output

#### iparam.sol\_read\_width

Controls the maximal acceptable width of line in the solutions when read by MOSEK.

Default 1024

Accepted [80; +inf]

Groups Data input/output, Solution input/output

#### iparam.solution\_callback

Indicates whether solution callbacks will be performed during the optimization.

Default off

Accepted on, off (see onoffkey)

Groups Progress callback, Overall solver

#### iparam.timing\_level

Controls the amount of timing performed inside **MOSEK**.

Default 1

Accepted [0; +inf]

Groups Overall system

## iparam.write\_bas\_constraints

Controls whether the constraint section is written to the basic solution file.

Default on

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

#### iparam.write\_bas\_head

Controls whether the header section is written to the basic solution file.

Default on

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

#### iparam.write\_bas\_variables

Controls whether the variables section is written to the basic solution file.

Default on

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

## iparam.write\_data\_compressed

Controls whether the data file is compressed while it is written. 0 means no compression while higher values mean more compression.

Default 0

**Accepted**  $[0; +\inf]$ 

# Groups Data input/output

#### iparam.write\_data\_format

Controls the data format when a task is written using Task.writedata.

### Default extension

Accepted extension, mps, lp, op, xml, free\_mps, task, cb, json\_task (see dataformat)

 ${\bf Groups}\ \, {\it Data\ input/output}$ 

#### iparam.write\_data\_param

If this option is turned on the parameter settings are written to the data file as parameters.

Default off

Accepted on, off (see onoffkey)

Groups Data input/output

### iparam.write\_free\_con

Controls whether the free constraints are written to the data file.

Default on

# Accepted

- on: The free constraints are written.
- off: The free constraints are discarded.

Groups Data input/output

# iparam.write\_generic\_names

Controls whether the generic names or user-defined names are used in the data file.

Default off

# Accepted

- on: Generic names are used.
- off: Generic names are not used.

Groups Data input/output

# iparam.write\_generic\_names\_io

Index origin used in generic names.

**Default** 1

Accepted [0; +inf]

Groups Data input/output

# iparam.write\_ignore\_incompatible\_items

Controls if the writer ignores incompatible problem items when writing files.

Default off

# Accepted

- on: Ignore items that cannot be written to the current output file format.
- off: Produce an error if the problem contains items that cannot the written to the current output file format.

Groups Data input/output

### iparam.write\_int\_constraints

Controls whether the constraint section is written to the integer solution file.

Default on

```
Accepted on, off (see onoffkey)
          Groups Data input/output, Solution input/output
iparam.write_int_head
     Controls whether the header section is written to the integer solution file.
         Default on
          Accepted on, off (see onoffkey)
          Groups Data input/output, Solution input/output
iparam.write_int_variables
     Controls whether the variables section is written to the integer solution file.
         Default on
          Accepted on, off (see onoffkey)
          Groups Data input/output, Solution input/output
iparam.write_lp_full_obj
     Write all variables, including the ones with 0-coefficients, in the objective.
         Default on
          Accepted on, off (see onoffkey)
          Groups Data input/output
iparam.write_lp_line_width
     Maximum width of line in an LP file written by MOSEK.
          Default 80
          Accepted [40; +inf]
          Groups Data input/output
iparam.write_lp_quoted_names
     If this option is turned on, then MOSEK will quote invalid LP names when writing an LP file.
         Default on
          Accepted on, off (see onoffkey)
          Groups Data input/output
iparam.write_lp_strict_format
     Controls whether LP output files satisfy the LP format strictly.
         Default off
          Accepted on, off (see onoffkey)
          Groups Data input/output
iparam.write_lp_terms_per_line
     Maximum number of terms on a single line in an LP file written by MOSEK. 0 means unlimited.
          Default 10
          Accepted [0; +\inf]
          {\bf Groups}\ {\it Data\ input/output}
iparam.write_mps_format
     Controls in which format the MPS is written.
         Default free
          Accepted strict, relaxed, free, cplex (see mpsformat)
          Groups Data input/output
```

### iparam.write\_mps\_int

Controls if marker records are written to the MPS file to indicate whether variables are integer restricted.

#### Default on

# Accepted

- on: Marker records are written.
- off: Marker records are not written.

```
Groups Data input/output
```

#### iparam.write\_precision

Controls the precision with which double numbers are printed in the MPS data file. In general it is not worthwhile to use a value higher than 15.

```
Default 15
```

Accepted [0; +inf]

Groups Data input/output

### iparam.write\_sol\_barvariables

Controls whether the symmetric matrix variables section is written to the solution file.

```
Default on
```

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

#### iparam.write\_sol\_constraints

Controls whether the constraint section is written to the solution file.

```
Default on
```

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

### iparam.write\_sol\_head

Controls whether the header section is written to the solution file.

```
Default on
```

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

# iparam.write\_sol\_ignore\_invalid\_names

Even if the names are invalid MPS names, then they are employed when writing the solution file.

```
Default off
```

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

# iparam.write\_sol\_variables

Controls whether the variables section is written to the solution file.

Default on

Accepted on, off (see onoffkey)

Groups Data input/output, Solution input/output

### iparam.write\_task\_inc\_sol

Controls whether the solutions are stored in the task file too.

Default on

```
Accepted on, off (see onoffkey)
```

Groups Data input/output

iparam.write\_xml\_mode

Controls if linear coefficients should be written by row or column when writing in the XML file format.

Default row

Accepted row, col (see xmlwriteroutputtype)

Groups Data input/output

# 16.7.3 String parameters

sparam

The enumeration type containing all string parameters.

sparam.bas\_sol\_file\_name

Name of the bas solution file.

**Accepted** Any valid file name.

Groups Data input/output, Solution input/output

sparam.data\_file\_name

Data are read and written to this file.

Accepted Any valid file name.

Groups Data input/output

sparam.debug\_file\_name

MOSEK debug file.

Accepted Any valid file name.

Groups Data input/output

 ${\tt sparam.int\_sol\_file\_name}$ 

Name of the int solution file.

Accepted Any valid file name.

Groups Data input/output, Solution input/output

sparam.itr\_sol\_file\_name

Name of the itr solution file.

Accepted Any valid file name.

Groups Data input/output, Solution input/output

sparam.mio\_debug\_string

For internal debugging purposes.

Accepted Any valid string.

Groups Data input/output

sparam.param\_comment\_sign

Only the first character in this string is used. It is considered as a start of comment sign in the **MOSEK** parameter file. Spaces are ignored in the string.

Default

%%

Accepted Any valid string.

Groups Data input/output

#### sparam.param\_read\_file\_name

Modifications to the parameter database is read from this file.

Accepted Any valid file name.

Groups Data input/output

#### sparam.param\_write\_file\_name

The parameter database is written to this file.

**Accepted** Any valid file name.

Groups Data input/output

# sparam.read\_mps\_bou\_name

Name of the BOUNDS vector used. An empty name means that the first BOUNDS vector is used.

Accepted Any valid MPS name.

Groups Data input/output

#### sparam.read\_mps\_obj\_name

Name of the free constraint used as objective function. An empty name means that the first constraint is used as objective function.

Accepted Any valid MPS name.

Groups Data input/output

#### sparam.read\_mps\_ran\_name

Name of the RANGE vector used. An empty name means that the first RANGE vector is used.

Accepted Any valid MPS name.

Groups Data input/output

#### sparam.read\_mps\_rhs\_name

Name of the RHS used. An empty name means that the first RHS vector is used.

Accepted Any valid MPS name.

Groups Data input/output

#### sparam.remote\_access\_token

An access token used to submit tasks to a remote **MOSEK** server. An access token is a random 32-byte string encoded in base64, i.e. it is a 44 character ASCII string.

Accepted Any valid string.

Groups Overall system

# sparam.sensitivity\_file\_name

If defined *Task.sensitivityreport* reads this file as a sensitivity analysis data file specifying the type of analysis to be done.

Accepted Any valid string.

Groups Data input/output

# sparam.sensitivity\_res\_file\_name

If this is a nonempty string, then Task. sensitivity report writes results to this file.

Accepted Any valid string.

Groups Data input/output

### sparam.sol\_filter\_xc\_low

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]>0.5 should be listed, whereas +0.5 means that all constraints having xc[i]>=blc[i]+0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

Groups Data input/output, Solution input/output

# sparam.sol\_filter\_xc\_upr

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]<0.5 should be listed, whereas -0.5 means all constraints having xc[i]<-buc[i]-0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

Groups Data input/output, Solution input/output

### sparam.sol\_filter\_xx\_low

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j] >= 0.5 should be listed, whereas "+0.5" means that all constraints having xx[j] >= blx[j] + 0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid filter.

Groups Data input/output, Solution input/output

#### sparam.sol\_filter\_xx\_upr

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j]<0.5 should be printed, whereas "-0.5" means all constraints having xx[j]<-bux[j]-0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid file name.

Groups Data input/output, Solution input/output

sparam.stat\_file\_name

Statistics file name.

Accepted Any valid file name.

Groups Data input/output

sparam.stat\_key

Key used when writing the summary file.

Accepted Any valid string.

Groups Data input/output

sparam.stat\_name

Name used when writing the statistics file.

Accepted Any valid XML string.

Groups Data input/output

# sparam.write\_lp\_gen\_var\_name

Sometimes when an LP file is written additional variables must be inserted. They will have the prefix denoted by this parameter.

Default xmskgen

Accepted Any valid string.

Groups Data input/output

# 16.8 Response codes

- Termination
- Warnings
- Errors

#### rescode

The enumeration type containing all response codes.

## 16.8.1 Termination

#### rescode.ok

No error occurred.

### rescode.trm\_max\_iterations

The optimizer terminated at the maximum number of iterations.

#### rescode.trm\_max\_time

The optimizer terminated at the maximum amount of time.

#### rescode.trm\_objective\_range

The optimizer terminated with an objective value outside the objective range.

#### rescode.trm\_mio\_near\_rel\_gap

The mixed-integer optimizer terminated as the delayed near optimal relative gap tolerance was satisfied.

### rescode.trm\_mio\_near\_abs\_gap

The mixed-integer optimizer terminated as the delayed near optimal absolute gap tolerance was satisfied.

# rescode.trm\_mio\_num\_relaxs

The mixed-integer optimizer terminated as the maximum number of relaxations was reached.

#### rescode.trm\_mio\_num\_branches

The mixed-integer optimizer terminated as the maximum number of branches was reached.

#### rescode.trm\_num\_max\_num\_int\_solutions

The mixed-integer optimizer terminated as the maximum number of feasible solutions was reached.

# rescode.trm\_stall

The optimizer is terminated due to slow progress.

Stalling means that numerical problems prevent the optimizer from making reasonable progress and that it make no sense to continue. In many cases this happens if the problem is badly scaled or otherwise ill-conditioned. There is no guarantee that the solution will be (near) feasible or near optimal. However, often stalling happens near the optimum, and the returned solution may be of good quality. Therefore, it is recommended to check the status of then solution. If the solution near optimal the solution is most likely good enough for most practical purposes.

Please note that if a linear optimization problem is solved using the interior-point optimizer with basis identification turned on, the returned basic solution likely to have high accuracy, even though the optimizer stalled.

Some common causes of stalling are a) badly scaled models, b) near feasible or near infeasible problems and c) a non-convex problems. Case c) is only relevant for general non-linear problems. It is not possible in general for **MOSEK** to check if a specific problems is convex since such a check would be NP hard in itself. This implies that care should be taken when solving problems involving general user defined functions.

# rescode.trm\_user\_callback

The optimizer terminated due to the return of the user-defined callback function.

#### rescode.trm\_max\_num\_setbacks

The optimizer terminated as the maximum number of set-backs was reached. This indicates serious numerical problems and a possibly badly formulated problem.

# rescode.trm\_numerical\_problem

The optimizer terminated due to numerical problems.

#### rescode.trm\_internal

The optimizer terminated due to some internal reason. Please contact MOSEK support.

#### rescode.trm\_internal\_stop

The optimizer terminated for internal reasons. Please contact MOSEK support.

# 16.8.2 Warnings

### rescode.wrn\_open\_param\_file

The parameter file could not be opened.

#### rescode.wrn\_large\_bound

A numerically large bound value is specified.

# rescode.wrn\_large\_lo\_bound

A numerically large lower bound value is specified.

### rescode.wrn\_large\_up\_bound

A numerically large upper bound value is specified.

### rescode.wrn\_large\_con\_fx

An equality constraint is fixed to a numerically large value. This can cause numerical problems.

# rescode.wrn\_large\_cj

A numerically large value is specified for one  $c_j$ .

#### rescode.wrn\_large\_aij

A numerically large value is specified for an  $a_{i,j}$  element in A. The parameter dparam.  $data\_tol\_aij\_large$  controls when an  $a_{i,j}$  is considered large.

#### rescode.wrn\_zero\_aij

One or more zero elements are specified in A.

# ${\tt rescode.wrn\_name\_max\_len}$

A name is longer than the buffer that is supposed to hold it.

#### rescode.wrn\_spar\_max\_len

A value for a string parameter is longer than the buffer that is supposed to hold it.

#### rescode.wrn\_mps\_split\_rhs\_vector

An RHS vector is split into several nonadjacent parts in an MPS file.

### rescode.wrn\_mps\_split\_ran\_vector

A RANGE vector is split into several nonadjacent parts in an MPS file.

#### rescode.wrn\_mps\_split\_bou\_vector

A BOUNDS vector is split into several nonadjacent parts in an MPS file.

### rescode.wrn\_lp\_old\_quad\_format

Missing '/2' after quadratic expressions in bound or objective.

## rescode.wrn\_lp\_drop\_variable

Ignored a variable because the variable was not previously defined. Usually this implies that a variable appears in the bound section but not in the objective or the constraints.

### rescode.wrn\_nz\_in\_upr\_tri

Non-zero elements specified in the upper triangle of a matrix were ignored.

### rescode.wrn\_dropped\_nz\_qobj

One or more non-zero elements were dropped in the Q matrix in the objective.

## rescode.wrn\_ignore\_integer

Ignored integer constraints.

# rescode.wrn\_no\_global\_optimizer

No global optimizer is available.

### rescode.wrn\_mio\_infeasible\_final

The final mixed-integer problem with all the integer variables fixed at their optimal values is infeasible.

#### rescode.wrn\_sol\_filter

Invalid solution filter is specified.

#### rescode.wrn\_undef\_sol\_file\_name

Undefined name occurred in a solution.

### rescode.wrn\_sol\_file\_ignored\_con

One or more lines in the constraint section were ignored when reading a solution file.

#### rescode.wrn\_sol\_file\_ignored\_var

One or more lines in the variable section were ignored when reading a solution file.

### rescode.wrn\_too\_few\_basis\_vars

An incomplete basis has been specified. Too few basis variables are specified.

### rescode.wrn\_too\_many\_basis\_vars

A basis with too many variables has been specified.

# rescode.wrn\_no\_nonlinear\_function\_write

The problem contains a general nonlinear function in either the objective or the constraints. Such a nonlinear function cannot be written to a disk file. Note that quadratic terms when inputted explicitly can be written to disk.

### rescode.wrn\_license\_expire

The license expires.

#### rescode.wrn\_license\_server

The license server is not responding.

# ${\tt rescode.wrn\_empty\_name}$

A variable or constraint name is empty. The output file may be invalid.

### rescode.wrn\_using\_generic\_names

Generic names are used because a name is not valid. For instance when writing an LP file the names must not contain blanks or start with a digit.

# rescode.wrn\_license\_feature\_expire

The license expires.

#### rescode.wrn\_param\_name\_dou

The parameter name is not recognized as a double parameter.

#### rescode.wrn\_param\_name\_int

The parameter name is not recognized as a integer parameter.

#### rescode.wrn\_param\_name\_str

The parameter name is not recognized as a string parameter.

#### rescode.wrn\_param\_str\_value

The string is not recognized as a symbolic value for the parameter.

## rescode.wrn\_param\_ignored\_cmio

A parameter was ignored by the conic mixed integer optimizer.

#### rescode.wrn\_zeros\_in\_sparse\_row

One or more (near) zero elements are specified in a sparse row of a matrix. Since, it is redundant to specify zero elements then it may indicate an error.

### rescode.wrn\_zeros\_in\_sparse\_col

One or more (near) zero elements are specified in a sparse column of a matrix. It is redundant to specify zero elements. Hence, it may indicate an error.

# ${\tt rescode.wrn\_incomplete\_linear\_dependency\_check}$

The linear dependency check(s) is incomplete. Normally this is not an important warning unless

the optimization problem has been formulated with linear dependencies. Linear dependencies may prevent **MOSEK** from solving the problem.

#### rescode.wrn\_eliminator\_space

The eliminator is skipped at least once due to lack of space.

### rescode.wrn\_presolve\_outofspace

The presolve is incomplete due to lack of space.

### rescode.wrn\_write\_changed\_names

Some names were changed because they were invalid for the output file format.

#### rescode.wrn\_write\_discarded\_cfix

The fixed objective term could not be converted to a variable and was discarded in the output file.

### rescode.wrn\_construct\_solution\_infeas

After fixing the integer variables at the suggested values then the problem is infeasible.

#### rescode.wrn\_construct\_invalid\_sol\_itg

The initial value for one or more of the integer variables is not feasible.

## rescode.wrn\_construct\_no\_sol\_itg

The construct solution requires an integer solution.

### rescode.wrn\_duplicate\_constraint\_names

Two constraint names are identical.

## rescode.wrn\_duplicate\_variable\_names

Two variable names are identical.

### rescode.wrn\_duplicate\_barvariable\_names

Two barvariable names are identical.

### rescode.wrn\_duplicate\_cone\_names

Two cone names are identical.

#### rescode.wrn\_ana\_large\_bounds

This warning is issued by the problem analyzer, if one or more constraint or variable bounds are very large. One should consider omitting these bounds entirely by setting them to +inf or -inf.

## rescode.wrn\_ana\_c\_zero

This warning is issued by the problem analyzer, if the coefficients in the linear part of the objective are all zero.

# rescode.wrn\_ana\_empty\_cols

This warning is issued by the problem analyzer, if columns, in which all coefficients are zero, are found.

### rescode.wrn\_ana\_close\_bounds

This warning is issued by problem analyzer, if ranged constraints or variables with very close upper and lower bounds are detected. One should consider treating such constraints as equalities and such variables as constants.

# ${\tt rescode.wrn\_ana\_almost\_int\_bounds}$

This warning is issued by the problem analyzer if a constraint is bound nearly integral.

### rescode.wrn\_quad\_cones\_with\_root\_fixed\_at\_zero

For at least one quadratic cone the root is fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problems, or to fix all the variables in the cone to 0.

### rescode.wrn\_rquad\_cones\_with\_root\_fixed\_at\_zero

For at least one rotated quadratic cone at least one of the root variables are fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problems, or to fix all the variables in the cone to 0.

## rescode.wrn\_no\_dualizer

No automatic dualizer is available for the specified problem. The primal problem is solved.

### rescode.wrn\_sym\_mat\_large

A numerically large value is specified for an  $e_{i,j}$  element in E. The parameter dparam.  $data_sym_mat_tol_large$  controls when an  $e_{i,j}$  is considered large.

### 16.8.3 Errors

### rescode.err\_license

Invalid license.

### rescode.err\_license\_expired

The license has expired.

#### rescode.err\_license\_version

The license is valid for another version of **MOSEK**.

#### rescode.err\_size\_license

The problem is bigger than the license.

#### rescode.err\_prob\_license

The software is not licensed to solve the problem.

### rescode.err\_file\_license

Invalid license file.

### rescode.err\_missing\_license\_file

MOSEK cannot license file or a token server. See the MOSEK installation manual for details.

### rescode.err\_size\_license\_con

The problem has too many constraints to be solved with the available license.

#### rescode.err\_size\_license\_var

The problem has too many variables to be solved with the available license.

# rescode.err\_size\_license\_intvar

The problem contains too many integer variables to be solved with the available license.

### rescode.err\_optimizer\_license

The optimizer required is not licensed.

#### rescode.err\_flexlm

The FLEXIm license manager reported an error.

# rescode.err\_license\_server

The license server is not responding.

#### rescode.err\_license\_max

Maximum number of licenses is reached.

## rescode.err\_license\_moseklm\_daemon

The MOSEKLM license manager daemon is not up and running.

#### rescode.err\_license\_feature

A requested feature is not available in the license file(s). Most likely due to an incorrect license system setup.

# rescode.err\_platform\_not\_licensed

A requested license feature is not available for the required platform.

### rescode.err\_license\_cannot\_allocate

The license system cannot allocate the memory required.

## rescode.err\_license\_cannot\_connect

MOSEK cannot connect to the license server. Most likely the license server is not up and running.

#### rescode.err\_license\_invalid\_hostid

The host ID specified in the license file does not match the host ID of the computer.

#### rescode.err\_license\_server\_version

The version specified in the checkout request is greater than the highest version number the daemon supports.

# rescode.err\_license\_no\_server\_support

The license server does not support the requested feature. Possible reasons for this error include:

- The feature has expired.
- The feature's start date is later than today's date.
- The version requested is higher than feature's the highest supported version.
- A corrupted license file.

Try restarting the license and inspect the license server debug file, usually called lmgrd.log.

#### rescode.err\_license\_no\_server\_line

There is no SERVER line in the license file. All non-zero license count features need at least one SERVER line.

#### rescode.err\_open\_dl

A dynamic link library could not be opened.

### rescode.err\_older\_dll

The dynamic link library is older than the specified version.

#### rescode.err\_newer\_dll

The dynamic link library is newer than the specified version.

#### rescode.err\_link\_file\_dll

A file cannot be linked to a stream in the DLL version.

#### rescode.err\_thread\_mutex\_init

Could not initialize a mutex.

## rescode.err\_thread\_mutex\_lock

Could not lock a mutex.

# rescode.err\_thread\_mutex\_unlock

Could not unlock a mutex.

# ${\tt rescode.err\_thread\_create}$

Could not create a thread. This error may occur if a large number of environments are created and not deleted again. In any case it is a good practice to minimize the number of environments created.

# rescode.err\_thread\_cond\_init

Could not initialize a condition.

# rescode.err\_unknown

Unknown error.

#### rescode.err\_space

Out of space.

#### rescode.err\_file\_open

Error while opening a file.

#### rescode.err\_file\_read

File read error.

# rescode.err\_file\_write

File write error.

# rescode.err\_data\_file\_ext

The data file format cannot be determined from the file name.

## rescode.err\_invalid\_file\_name

An invalid file name has been specified.

### rescode.err\_invalid\_sol\_file\_name

An invalid file name has been specified.

#### rescode.err\_end\_of\_file

End of file reached.

# rescode.err\_null\_env

env is a NULL pointer.

### rescode.err\_null\_task

task is a NULL pointer.

#### rescode.err\_invalid\_stream

An invalid stream is referenced.

### rescode.err\_no\_init\_env

env is not initialized.

### rescode.err\_invalid\_task

The task is invalid.

### rescode.err\_null\_pointer

An argument to a function is unexpectedly a NULL pointer.

### rescode.err\_living\_tasks

All tasks associated with an environment must be deleted before the environment is deleted. There are still some undeleted tasks.

#### rescode.err\_blank\_name

An all blank name has been specified.

# rescode.err\_dup\_name

The same name was used multiple times for the same problem item type.

# rescode.err\_invalid\_obj\_name

An invalid objective name is specified.

# ${\tt rescode.err\_invalid\_con\_name}$

An invalid constraint name is used.

## rescode.err\_invalid\_var\_name

An invalid variable name is used.

# rescode.err\_invalid\_cone\_name

An invalid cone name is used.

#### rescode.err\_invalid\_barvar\_name

An invalid symmetric matrix variable name is used.

#### rescode.err\_space\_leaking

MOSEK is leaking memory. This can be due to either an incorrect use of MOSEK or a bug.

#### rescode.err\_space\_no\_info

No available information about the space usage.

### rescode.err\_read\_format

The specified format cannot be read.

# rescode.err\_mps\_file

An error occurred while reading an MPS file.

# rescode.err\_mps\_inv\_field

A field in the MPS file is invalid. Probably it is too wide.

# ${\tt rescode.err\_mps\_inv\_marker}$

An invalid marker has been specified in the MPS file.

# rescode.err\_mps\_null\_con\_name

An empty constraint name is used in an MPS file.

### rescode.err\_mps\_null\_var\_name

An empty variable name is used in an MPS file.

# rescode.err\_mps\_undef\_con\_name

An undefined constraint name occurred in an MPS file.

# rescode.err\_mps\_undef\_var\_name

An undefined variable name occurred in an MPS file.

### rescode.err\_mps\_inv\_con\_key

An invalid constraint key occurred in an MPS file.

#### rescode.err\_mps\_inv\_bound\_key

An invalid bound key occurred in an MPS file.

### rescode.err\_mps\_inv\_sec\_name

An invalid section name occurred in an MPS file.

# rescode.err\_mps\_no\_objective

No objective is defined in an MPS file.

### rescode.err\_mps\_splitted\_var

All elements in a column of the A matrix must be specified consecutively. Hence, it is illegal to specify non-zero elements in A for variable 1, then for variable 2 and then variable 1 again.

#### rescode.err\_mps\_mul\_con\_name

A constraint name was specified multiple times in the ROWS section.

# rescode.err\_mps\_mul\_qsec

Multiple QSECTIONs are specified for a constraint in the MPS data file.

#### rescode.err\_mps\_mul\_qobj

The Q term in the objective is specified multiple times in the MPS data file.

# rescode.err\_mps\_inv\_sec\_order

The sections in the MPS data file are not in the correct order.

# ${\tt rescode.err\_mps\_mul\_csec}$

Multiple  ${\tt CSECTIONs}$  are given the same name.

## rescode.err\_mps\_cone\_type

Invalid cone type specified in a CSECTION.

### rescode.err\_mps\_cone\_overlap

A variable is specified to be a member of several cones.

### rescode.err\_mps\_cone\_repeat

A variable is repeated within the CSECTION.

#### rescode.err\_mps\_non\_symmetric\_q

A non symmetric matrice has been speciefied.

### rescode.err\_mps\_duplicate\_q\_element

Duplicate elements is specified in a Q matrix.

### rescode.err\_mps\_invalid\_objsense

An invalid objective sense is specified.

# ${\tt rescode.err\_mps\_tab\_in\_field2}$

A tab char occurred in field 2.

# ${\tt rescode.err\_mps\_tab\_in\_field3}$

A tab char occurred in field 3.

# ${\tt rescode.err\_mps\_tab\_in\_field5}$

A tab char occurred in field 5.

# rescode.err\_mps\_invalid\_obj\_name

An invalid objective name is specified.

### rescode.err\_lp\_incompatible

The problem cannot be written to an LP formatted file.

#### rescode.err\_lp\_empty

The problem cannot be written to an LP formatted file.

#### rescode.err\_lp\_dup\_slack\_name

The name of the slack variable added to a ranged constraint already exists.

### rescode.err\_write\_mps\_invalid\_name

An invalid name is created while writing an MPS file. Usually this will make the MPS file unreadable

#### rescode.err\_lp\_invalid\_var\_name

A variable name is invalid when used in an LP formatted file.

# ${\tt rescode.err\_lp\_free\_constraint}$

Free constraints cannot be written in LP file format.

### rescode.err\_write\_opf\_invalid\_var\_name

Empty variable names cannot be written to OPF files.

# rescode.err\_lp\_file\_format

Syntax error in an LP file.

### rescode.err\_write\_lp\_format

Problem cannot be written as an LP file.

# rescode.err\_read\_lp\_missing\_end\_tag

Syntax error in LP file. Possibly missing End tag.

#### rescode.err\_lp\_format

Syntax error in an LP file.

## rescode.err\_write\_lp\_non\_unique\_name

An auto-generated name is not unique.

# ${\tt rescode.err\_read\_lp\_nonexisting\_name}$

A variable never occurred in objective or constraints.

## rescode.err\_lp\_write\_conic\_problem

The problem contains cones that cannot be written to an LP formatted file.

### rescode.err\_lp\_write\_geco\_problem

The problem contains general convex terms that cannot be written to an LP formatted file.

### rescode.err\_writing\_file

An error occurred while writing file

#### rescode.err\_opf\_format

Syntax error in an OPF file

### rescode.err\_opf\_new\_variable

Introducing new variables is now allowed. When a [variables] section is present, it is not allowed to introduce new variables later in the problem.

## rescode.err\_invalid\_name\_in\_sol\_file

An invalid name occurred in a solution file.

### rescode.err\_lp\_invalid\_con\_name

A constraint name is invalid when used in an LP formatted file.

#### rescode.err\_opf\_premature\_eof

Premature end of file in an OPF file.

# ${\tt rescode.err\_json\_syntax}$

Syntax error in an JSON data

# rescode.err\_json\_string

Error in JSON string.

### rescode.err\_json\_number\_overflow

Invalid number entry - wrong type or value overflow.

### rescode.err\_json\_format

Error in an JSON Task file

### rescode.err\_json\_data

Inconsistent data in JSON Task file

#### rescode.err\_json\_missing\_data

Missing data section in JSON task file.

# rescode.err\_argument\_lenneq

Incorrect length of arguments.

### rescode.err\_argument\_type

Incorrect argument type.

### rescode.err\_nr\_arguments

Incorrect number of function arguments.

### rescode.err\_in\_argument

A function argument is incorrect.

## rescode.err\_argument\_dimension

A function argument is of incorrect dimension.

### rescode.err\_index\_is\_too\_small

An index in an argument is too small.

### rescode.err\_index\_is\_too\_large

An index in an argument is too large.

#### rescode.err\_param\_name

The parameter name is not correct.

#### rescode.err\_param\_name\_dou

The parameter name is not correct for a double parameter.

# rescode.err\_param\_name\_int

The parameter name is not correct for an integer parameter.

# ${\tt rescode.err\_param\_name\_str}$

The parameter name is not correct for a string parameter.

# rescode.err\_param\_index

Parameter index is out of range.

# ${\tt rescode.err\_param\_is\_too\_large}$

The parameter value is too large.

# rescode.err\_param\_is\_too\_small

The parameter value is too small.

### rescode.err\_param\_value\_str

The parameter value string is incorrect.

# rescode.err\_param\_type

The parameter type is invalid.

### rescode.err\_inf\_dou\_index

A double information index is out of range for the specified type.

### rescode.err\_inf\_int\_index

An integer information index is out of range for the specified type.

# rescode.err\_index\_arr\_is\_too\_small

An index in an array argument is too small.

#### rescode.err\_index\_arr\_is\_too\_large

An index in an array argument is too large.

### rescode.err\_inf\_lint\_index

A long integer information index is out of range for the specified type.

### rescode.err\_arg\_is\_too\_small

The value of a argument is too small.

#### rescode.err\_arg\_is\_too\_large

The value of a argument is too small.

### rescode.err\_invalid\_whichsol

whichsol is invalid.

### rescode.err\_inf\_dou\_name

A double information name is invalid.

### rescode.err\_inf\_int\_name

An integer information name is invalid.

#### rescode.err\_inf\_type

The information type is invalid.

#### rescode.err\_inf\_lint\_name

A long integer information name is invalid.

#### rescode.err\_index

An index is out of range.

#### rescode.err\_whichsol

The solution defined by which sol does not exists.

#### rescode.err\_solitem

The solution item number solitem is invalid. Please note that *solitem.snx* is invalid for the basic solution.

## rescode.err\_whichitem\_not\_allowed

whichitem is unacceptable.

### rescode.err\_maxnumcon

The maximum number of constraints specified is smaller than the number of constraints in the task.

## rescode.err\_maxnumvar

The maximum number of variables specified is smaller than the number of variables in the task.

# rescode.err\_maxnumbarvar

The maximum number of semidefinite variables specified is smaller than the number of semidefinite variables in the task.

### rescode.err\_maxnumqnz

The maximum number of non-zeros specified for the Q matrices is smaller than the number of non-zeros in the current Q matrices.

#### rescode.err\_too\_small\_max\_num\_nz

The maximum number of non-zeros specified is too small.

## rescode.err\_invalid\_idx

A specified index is invalid.

# ${\tt rescode.err\_invalid\_max\_num}$

A specified index is invalid.

## rescode.err\_numconlim

Maximum number of constraints limit is exceeded.

#### rescode.err\_numvarlim

Maximum number of variables limit is exceeded.

#### rescode.err\_too\_small\_maxnumanz

The maximum number of non-zeros specified for A is smaller than the number of non-zeros in the current A.

#### rescode.err\_inv\_aptre

aptre[j] is strictly smaller than aptrb[j] for some j.

## rescode.err\_mul\_a\_element

An element in A is defined multiple times.

#### rescode.err\_inv\_bk

Invalid bound key.

### rescode.err\_inv\_bkc

Invalid bound key is specified for a constraint.

#### rescode.err\_inv\_bkx

An invalid bound key is specified for a variable.

### rescode.err\_inv\_var\_type

An invalid variable type is specified for a variable.

### rescode.err\_solver\_probtype

Problem type does not match the chosen optimizer.

### rescode.err\_objective\_range

Empty objective range.

#### rescode.err\_first

Invalid first.

### rescode.err\_last

Invalid index last. A given index was out of expected range.

# rescode.err\_negative\_surplus

Negative surplus.

## rescode.err\_negative\_append

Cannot append a negative number.

### rescode.err\_undef\_solution

MOSEK has the following solution types:

- an interior-point solution,
- an basic solution,
- and an integer solution.

Each optimizer may set one or more of these solutions; e.g by default a successful optimization with the interior-point optimizer defines the interior-point solution, and, for linear problems, also the basic solution. This error occurs when asking for a solution or for information about a solution that is not defined.

### rescode.err\_basis

An invalid basis is specified. Either too many or too few basis variables are specified.

# rescode.err\_inv\_skc

Invalid value in skc.

# rescode.err\_inv\_skx

Invalid value in skx.

#### rescode.err\_inv\_skn

Invalid value in skn.

#### rescode.err\_inv\_sk\_str

Invalid status key string encountered.

#### rescode.err\_inv\_sk

Invalid status key code.

### rescode.err\_inv\_cone\_type\_str

Invalid cone type string encountered.

### rescode.err\_inv\_cone\_type

Invalid cone type code is encountered.

#### rescode.err\_invalid\_surplus

Invalid surplus.

### rescode.err\_inv\_name\_item

An invalid name item code is used.

### rescode.err\_pro\_item

An invalid problem is used.

# ${\tt rescode.err\_invalid\_format\_type}$

Invalid format type.

## rescode.err\_firsti

Invalid firsti.

# rescode.err\_lasti

Invalid lasti.

### rescode.err\_firstj

Invalid firstj.

# rescode.err\_lastj

Invalid lastj.

#### rescode.err\_max\_len\_is\_too\_small

An maximum length that is too small has been specified.

#### rescode.err\_nonlinear\_equality

The model contains a nonlinear equality which defines a nonconvex set.

### rescode.err\_nonconvex

The optimization problem is nonconvex.

# ${\tt rescode.err\_nonlinear\_ranged}$

Nonlinear constraints with finite lower and upper bound always define a nonconvex feasible set.

# rescode.err\_con\_q\_not\_psd

The quadratic constraint matrix is not positive semidefinite as expected for a constraint with finite upper bound. This results in a nonconvex problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

# rescode.err\_con\_q\_not\_nsd

The quadratic constraint matrix is not negative semidefinite as expected for a constraint with finite lower bound. This results in a nonconvex problem. The parameter *dparam*. *check\_convexity\_rel\_tol* can be used to relax the convexity check.

#### rescode.err\_obj\_q\_not\_psd

The quadratic coefficient matrix in the objective is not positive semidefinite as expected for a minimization problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

# rescode.err\_obj\_q\_not\_nsd

The quadratic coefficient matrix in the objective is not negative semidefinite as expected for a maximization problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

### rescode.err\_argument\_perm\_array

An invalid permutation array is specified.

#### rescode.err\_cone\_index

An index of a non-existing cone has been specified.

### rescode.err\_cone\_size

A cone with too few members is specified.

### rescode.err\_cone\_overlap

One or more of the variables in the cone to be added is already member of another cone. Now assume the variable is  $x_i$  then add a new variable say  $x_k$  and the constraint

$$x_j = x_k$$

and then let  $x_k$  be member of the cone to be appended.

### rescode.err\_cone\_rep\_var

A variable is included multiple times in the cone.

#### rescode.err\_maxnumcone

The value specified for maxnumcone is too small.

#### rescode.err\_cone\_type

Invalid cone type specified.

## rescode.err\_cone\_type\_str

Invalid cone type specified.

# rescode.err\_cone\_overlap\_append

The cone to be appended has one variable which is already member of another cone.

#### rescode.err\_remove\_cone\_variable

A variable cannot be removed because it will make a cone invalid.

## rescode.err\_sol\_file\_invalid\_number

An invalid number is specified in a solution file.

# rescode.err\_huge\_c

A huge value in absolute size is specified for one  $c_j$ .

## rescode.err\_huge\_aij

A numerically huge value is specified for an  $a_{i,j}$  element in A. The parameter dparam.  $data\_tol\_aij\_huge$  controls when an  $a_{i,j}$  is considered huge.

## rescode.err\_duplicate\_aij

An element in the A matrix is specified twice.

### rescode.err\_lower\_bound\_is\_a\_nan

The lower bound specified is not a number (nan).

## rescode.err\_upper\_bound\_is\_a\_nan

The upper bound specified is not a number (nan).

### rescode.err\_infinite\_bound

A numerically huge bound value is specified.

# rescode.err\_inv\_qobj\_subi

Invalid value in qosubi.

### rescode.err\_inv\_qobj\_subj

Invalid value in qosubj.

# rescode.err\_inv\_qobj\_val

Invalid value in qoval.

# rescode.err\_inv\_qcon\_subk

Invalid value in qcsubk.

### rescode.err\_inv\_qcon\_subi

Invalid value in qcsubi.

#### rescode.err\_inv\_qcon\_subj

Invalid value in qcsubj.

# rescode.err\_inv\_qcon\_val

Invalid value in qcval.

# rescode.err\_qcon\_subi\_too\_small

Invalid value in qcsubi.

#### rescode.err\_qcon\_subi\_too\_large

Invalid value in qcsubi.

### rescode.err\_qobj\_upper\_triangle

An element in the upper triangle of  $Q^o$  is specified. Only elements in the lower triangle should be specified.

# rescode.err\_qcon\_upper\_triangle

An element in the upper triangle of a  $Q^k$  is specified. Only elements in the lower triangle should be specified.

### rescode.err\_fixed\_bound\_values

A fixed constraint/variable has been specified using the bound keys but the numerical value of the lower and upper bound is different.

### rescode.err\_nonlinear\_functions\_not\_allowed

An operation that is invalid for problems with nonlinear functions defined has been attempted.

#### rescode.err\_user\_func\_ret

An user function reported an error.

#### rescode.err\_user\_func\_ret\_data

An user function returned invalid data.

# rescode.err\_user\_nlo\_func

The user-defined nonlinear function reported an error.

#### rescode.err\_user\_nlo\_eval

The user-defined nonlinear function reported an error.

# rescode.err\_user\_nlo\_eval\_hessubi

The user-defined nonlinear function reported an invalid subscript in the Hessian.

#### rescode.err\_user\_nlo\_eval\_hessubj

The user-defined nonlinear function reported an invalid subscript in the Hessian.

# rescode.err\_invalid\_objective\_sense

An invalid objective sense is specified.

# rescode.err\_undefined\_objective\_sense

The objective sense has not been specified before the optimization.

# rescode.err\_y\_is\_undefined

The solution item y is undefined.

# rescode.err\_nan\_in\_double\_data

An invalid floating point value was used in some double data.

# rescode.err\_nan\_in\_blc

 $l^c$  contains an invalid floating point value, i.e. a NaN.

# rescode.err\_nan\_in\_buc

 $u^c$  contains an invalid floating point value, i.e. a NaN.

# rescode.err\_nan\_in\_c

c contains an invalid floating point value, i.e. a NaN.

#### rescode.err\_nan\_in\_blx

 $l^x$  contains an invalid floating point value, i.e. a NaN.

#### rescode.err\_nan\_in\_bux

 $u^x$  contains an invalid floating point value, i.e. a NaN.

### rescode.err\_invalid\_aij

 $a_{i,j}$  contains an invalid floating point value, i.e. a NaN or an infinite value.

### rescode.err\_sym\_mat\_invalid

A symmetric matrix contains an invalid floating point value, i.e. a NaN or an infinite value.

#### rescode.err\_sym\_mat\_huge

A symmetric matrix contains a huge value in absolute size. The parameter dparam.  $data\_sym\_mat\_tol\_huge$  controls when an  $e_{i,j}$  is considered huge.

#### rescode.err\_inv\_problem

Invalid problem type. Probably a nonconvex problem has been specified.

### rescode.err\_mixed\_conic\_and\_nl

The problem contains nonlinear terms conic constraints. The requested operation cannot be applied to this type of problem.

# rescode.err\_global\_inv\_conic\_problem

The global optimizer can only be applied to problems without semidefinite variables.

#### rescode.err\_inv\_optimizer

An invalid optimizer has been chosen for the problem. This means that the simplex or the conic optimizer is chosen to optimize a nonlinear problem.

#### rescode.err\_mio\_no\_optimizer

No optimizer is available for the current class of integer optimization problems.

#### rescode.err\_no\_optimizer\_var\_type

No optimizer is available for this class of optimization problems.

### rescode.err\_final\_solution

An error occurred during the solution finalization.

#### rescode.err\_postsolve

An error occurred during the postsolve. Please contact **MOSEK** support.

### rescode.err\_overflow

A computation produced an overflow i.e. a very large number.

#### rescode.err\_no\_basis\_sol

No basic solution is defined.

#### rescode.err\_basis\_factor

The factorization of the basis is invalid.

# rescode.err\_basis\_singular

The basis is singular and hence cannot be factored.

### rescode.err\_factor

An error occurred while factorizing a matrix.

# ${\tt rescode.err\_feasrepair\_cannot\_relax}$

An optimization problem cannot be relaxed. This is the case e.g. for general nonlinear optimization problems.

## rescode.err\_feasrepair\_solving\_relaxed

The relaxed problem could not be solved to optimality. Please consult the log file for further details.

# rescode.err\_feasrepair\_inconsistent\_bound

The upper bound is less than the lower bound for a variable or a constraint. Please correct this before running the feasibility repair.

### rescode.err\_repair\_invalid\_problem

The feasibility repair does not support the specified problem type.

### rescode.err\_repair\_optimization\_failed

Computation the optimal relaxation failed. The cause may have been numerical problems.

#### rescode.err\_name\_max\_len

A name is longer than the buffer that is supposed to hold it.

### rescode.err\_name\_is\_null

The name buffer is a NULL pointer.

#### rescode.err\_invalid\_compression

Invalid compression type.

### rescode.err\_invalid\_iomode

Invalid io mode.

#### rescode.err\_no\_primal\_infeas\_cer

A certificate of primal infeasibility is not available.

### rescode.err\_no\_dual\_infeas\_cer

A certificate of infeasibility is not available.

### rescode.err\_no\_solution\_in\_callback

The required solution is not available.

### rescode.err\_inv\_marki

Invalid value in marki.

### rescode.err\_inv\_markj

Invalid value in markj.

#### rescode.err\_inv\_numi

Invalid numi.

#### rescode.err\_inv\_numj

Invalid numj.

#### rescode.err\_cannot\_clone\_nl

A task with a nonlinear function callback cannot be cloned.

# rescode.err\_cannot\_handle\_nl

A function cannot handle a task with nonlinear function callbacks.

### rescode.err\_invalid\_accmode

An invalid access mode is specified.

# rescode.err\_task\_incompatible

The Task file is incompatible with this platform. This results from reading a file on a 32 bit platform generated on a 64 bit platform.

#### rescode.err\_task\_invalid

The Task file is invalid.

### rescode.err\_task\_write

Failed to write the task file.

# rescode.err\_lu\_max\_num\_tries

Could not compute the LU factors of the matrix within the maximum number of allowed tries.

# rescode.err\_invalid\_utf8

An invalid UTF8 string is encountered.

# ${\tt rescode.err\_invalid\_wchar}$

An invalid wchar string is encountered.

# rescode.err\_no\_dual\_for\_itg\_sol

No dual information is available for the integer solution.

### rescode.err\_no\_snx\_for\_bas\_sol

 $s_n^x$  is not available for the basis solution.

#### rescode.err\_internal

An internal error occurred. Please report this problem.

#### rescode.err\_api\_array\_too\_small

An input array was too short.

### rescode.err\_api\_cb\_connect

Failed to connect a callback object.

#### rescode.err\_api\_fatal\_error

An internal error occurred in the API. Please report this problem.

### rescode.err\_api\_internal

An internal fatal error occurred in an interface function.

### rescode.err\_sen\_format

Syntax error in sensitivity analysis file.

### rescode.err\_sen\_undef\_name

An undefined name was encountered in the sensitivity analysis file.

#### rescode.err\_sen\_index\_range

Index out of range in the sensitivity analysis file.

## rescode.err\_sen\_bound\_invalid\_up

Analysis of upper bound requested for an index, where no upper bound exists.

#### rescode.err\_sen\_bound\_invalid\_lo

Analysis of lower bound requested for an index, where no lower bound exists.

#### rescode.err\_sen\_index\_invalid

Invalid range given in the sensitivity file.

#### rescode.err\_sen\_invalid\_regexp

Syntax error in regexp or regexp longer than 1024.

#### rescode.err\_sen\_solution\_status

No optimal solution found to the original problem given for sensitivity analysis.

# rescode.err\_sen\_numerical

Numerical difficulties encountered performing the sensitivity analysis.

# ${\tt rescode.err\_sen\_unhandled\_problem\_type}$

Sensitivity analysis cannot be performed for the specified problem. Sensitivity analysis is only possible for linear problems.

#### rescode.err\_unb\_step\_size

A step size in an optimizer was unexpectedly unbounded. For instance, if the step-size becomes unbounded in phase 1 of the simplex algorithm then an error occurs. Normally this will happen only if the problem is badly formulated. Please contact **MOSEK** support if this error occurs.

### rescode.err\_identical\_tasks

Some tasks related to this function call were identical. Unique tasks were expected.

## rescode.err\_ad\_invalid\_codelist

The code list data was invalid.

# rescode.err\_internal\_test\_failed

An internal unit test function failed.

# rescode.err\_xml\_invalid\_problem\_type

The problem type is not supported by the XML format.

# rescode.err\_invalid\_ampl\_stub

Invalid AMPL stub.

### rescode.err\_int64\_to\_int32\_cast

An 32 bit integer could not cast to a 64 bit integer.

#### rescode.err\_size\_license\_numcores

The computer contains more cpu cores than the license allows for.

### rescode.err\_infeas\_undefined

The requested value is not defined for this solution type.

### rescode.err\_no\_barx\_for\_solution

There is no  $\overline{X}$  available for the solution specified. In particular note there are no  $\overline{X}$  defined for the basic and integer solutions.

#### rescode.err\_no\_bars\_for\_solution

There is no  $\bar{s}$  available for the solution specified. In particular note there are no  $\bar{s}$  defined for the basic and integer solutions.

#### rescode.err\_bar\_var\_dim

The dimension of a symmetric matrix variable has to greater than 0.

#### rescode.err\_sym\_mat\_invalid\_row\_index

A row index specified for sparse symmetric matrix is invalid.

### rescode.err\_sym\_mat\_invalid\_col\_index

A column index specified for sparse symmetric matrix is invalid.

#### rescode.err\_sym\_mat\_not\_lower\_tringular

Only the lower triangular part of sparse symmetric matrix should be specified.

#### rescode.err\_sym\_mat\_invalid\_value

The numerical value specified in a sparse symmetric matrix is not a value floating value.

### rescode.err\_sym\_mat\_duplicate

A value in a symmetric matric as been specified more than once.

## rescode.err\_invalid\_sym\_mat\_dim

A sparse symmetric matrix of invalid dimension is specified.

# rescode.err\_invalid\_file\_format\_for\_sym\_mat

The file format does not support a problem with symmetric matrix variables.

## rescode.err\_invalid\_file\_format\_for\_cones

The file format does not support a problem with conic constraints.

# rescode.err\_invalid\_file\_format\_for\_general\_nl

The file format does not support a problem with general nonlinear terms.

# rescode.err\_duplicate\_constraint\_names

Two constraint names are identical.

# rescode.err\_duplicate\_variable\_names

Two variable names are identical.

### rescode.err\_duplicate\_barvariable\_names

Two barvariable names are identical.

## rescode.err\_duplicate\_cone\_names

Two cone names are identical.

### rescode.err\_non\_unique\_array

An array does not contain unique elements.

# rescode.err\_argument\_is\_too\_large

The value of a function argument is too large.

#### rescode.err\_mio\_internal

A fatal error occurred in the mixed integer optimizer. Please contact MOSEK support.

### rescode.err\_invalid\_problem\_type

An invalid problem type.

#### rescode.err\_unhandled\_solution\_status

Unhandled solution status.

### rescode.err\_upper\_triangle

An element in the upper triangle of a lower triangular matrix is specified.

# rescode.err\_lau\_singular\_matrix

A matrix is singular.

#### rescode.err\_lau\_not\_positive\_definite

A matrix is not positive definite.

# rescode.err\_lau\_invalid\_lower\_triangular\_matrix

An invalid lower triangular matrix.

### rescode.err\_lau\_unknown

An unknown error.

### rescode.err\_lau\_arg\_m

Invalid argument m.

### rescode.err\_lau\_arg\_n

Invalid argument n.

### rescode.err\_lau\_arg\_k

Invalid argument k.

#### rescode.err\_lau\_arg\_transa

Invalid argument transa.

### rescode.err\_lau\_arg\_transb

Invalid argument transb.

### rescode.err\_lau\_arg\_uplo

Invalid argument uplo.

# ${\tt rescode.err\_lau\_arg\_trans}$

Invalid argument trans.

# rescode.err\_lau\_invalid\_sparse\_symmetric\_matrix

An invalid sparse symmetric matrix is specified. Note only the lower triangular part with no duplicates is specified.

### rescode.err\_cbf\_parse

An error occurred while parsing an CBF file.

#### rescode.err\_cbf\_obj\_sense

An invalid objective sense is specified.

### rescode.err\_cbf\_no\_variables

No variables are specified.

### rescode.err\_cbf\_too\_many\_constraints

Too many constraints specified.

# ${\tt rescode.err\_cbf\_too\_many\_variables}$

Too many variables specified.

# ${\tt rescode.err\_cbf\_no\_version\_specified}$

No version specified.

# ${\tt rescode.err\_cbf\_syntax}$

Invalid syntax.

# rescode.err\_cbf\_duplicate\_obj

Duplicate OBJ keyword.

- rescode.err\_cbf\_duplicate\_con Duplicate CON keyword.
- rescode.err\_cbf\_duplicate\_var Duplicate VAR keyword.
- rescode.err\_cbf\_duplicate\_int Duplicate INT keyword.
- rescode.err\_cbf\_invalid\_var\_type
  Invalid variable type.
- rescode.err\_cbf\_invalid\_con\_type Invalid constraint type.
- rescode.err\_cbf\_invalid\_domain\_dimension Invalid domain dimension.
- rescode.err\_cbf\_duplicate\_objacoord Duplicate index in OBJCOORD.
- rescode.err\_cbf\_duplicate\_bcoord Duplicate index in BCOORD.
- rescode.err\_cbf\_duplicate\_acoord Duplicate index in ACOORD.
- rescode.err\_cbf\_too\_few\_variables
  Too few variables defined.
- rescode.err\_cbf\_too\_few\_constraints
  Too few constraints defined.
- rescode.err\_cbf\_too\_few\_ints
  Too few ints are specified.
- rescode.err\_cbf\_too\_many\_ints
  Too many ints are specified.
- rescode.err\_cbf\_invalid\_int\_index Invalid INT index.
- ${\bf rescode.err\_cbf\_unsupported} \\ {\bf Unsupported\ feature\ is\ present.}$
- rescode.err\_cbf\_duplicate\_psdvar Duplicate PSDVAR keyword.
- rescode.err\_cbf\_invalid\_psdvar\_dimension Invalid PSDVAR dimmension.
- rescode.err\_cbf\_too\_few\_psdvar
  Too few variables defined.
- rescode.err\_mio\_invalid\_root\_optimizer

  An invalid root optimizer was selected for the problem type.
- rescode.err\_mio\_invalid\_node\_optimizer

  An invalid node optimizer was selected for the problem type.
- rescode.err\_toconic\_constr\_q\_not\_psd

  The matrix defining the quadratric part of constraint is not positive semidefinite.
- rescode.err\_toconic\_constraint\_fx

  The quadratic constraint is an equality, thus not convex.
- rescode.err\_toconic\_constraint\_ra

  The quadratic constraint has finite lower and upper bound, and therefore it is not convex.

#### 16.8. Response codes

### rescode.err\_toconic\_constr\_not\_conic

The constraint is not conic representable.

### rescode.err\_toconic\_objective\_not\_psd

The matrix defining the quadratric part of the objective function is not positive semidefinite.

### rescode.err\_server\_connect

Failed to connect to remote solver server. The server string or the port string were invalid, or the server did not accept connection.

### rescode.err\_server\_protocol

Unexpected message or data from solver server.

#### rescode.err\_server\_status

Server returned non-ok HTTP status code

#### rescode.err\_server\_token

The job ID specified is incorrect or invalid

# 16.9 Enumerations

### language

Language selection constants

### language.eng

English language selection

#### language.dan

Danish language selection

#### accmode

Constraint or variable access modes. All functions using this enum are deprecated. Use separate functions for rows/columns instead.

#### accmode.var

Access data by columns (variable oriented)

#### accmode.con

Access data by rows (constraint oriented)

#### basindtype

Basis identification

# basindtype.never

Never do basis identification.

### basindtype.always

Basis identification is always performed even if the interior-point optimizer terminates abnormally.

#### basindtype.no\_error

Basis identification is performed if the interior-point optimizer terminates without an error.

#### basindtype.if\_feasible

Basis identification is not performed if the interior-point optimizer terminates with a problem status saying that the problem is primal or dual infeasible.

## basindtype.reservered

Not currently in use.

### boundkey

Bound keys

#### boundkey.lo

The constraint or variable has a finite lower bound and an infinite upper bound.

```
boundkey.up
          The constraint or variable has an infinite lower bound and an finite upper bound.
     boundkey.fx
          The constraint or variable is fixed.
     boundkey.fr
          The constraint or variable is free.
     boundkey.ra
          The constraint or variable is ranged.
mark
     Mark
     mark.lo
          The lower bound is selected for sensitivity analysis.
          The upper bound is selected for sensitivity analysis.
simdegen
     Degeneracy strategies
     simdegen.none
          The simplex optimizer should use no degeneration strategy.
     simdegen.free
          The simplex optimizer chooses the degeneration strategy.
     simdegen.aggressive
          The simplex optimizer should use an aggressive degeneration strategy.
     simdegen.moderate
          The simplex optimizer should use a moderate degeneration strategy.
     simdegen.minimum
          The simplex optimizer should use a minimum degeneration strategy.
transpose
     Transposed matrix.
     transpose.no
          No transpose is applied.
     transpose.yes
          A transpose is applied.
uplo
     Triangular part of a symmetric matrix.
     uplo.lo
          Lower part.
     uplo.up
          Upper part
simreform
     Problem reformulation.
     simreform.on
          Allow the simplex optimizer to reformulate the problem.
     simreform.off
          Disallow the simplex optimizer to reformulate the problem.
     simreform.free
          The simplex optimizer can choose freely.
```

16.9. Enumerations 347

### simreform.aggressive

The simplex optimizer should use an aggressive reformulation strategy.

#### simdupvec

Exploit duplicate columns.

# simdupvec.on

Allow the simplex optimizer to exploit duplicated columns.

#### simdupvec.off

Disallow the simplex optimizer to exploit duplicated columns.

#### simdupvec.free

The simplex optimizer can choose freely.

#### simhotstart

Hot-start type employed by the simplex optimizer

### simhotstart.none

The simplex optimizer performs a coldstart.

### simhotstart.free

The simplex optimize chooses the hot-start type.

### simhotstart.status\_keys

Only the status keys of the constraints and variables are used to choose the type of hot-start.

# intpnthotstart

Hot-start type employed by the interior-point optimizers.

### intpnthotstart.none

The interior-point optimizer performs a coldstart.

# intpnthotstart.primal

The interior-point optimizer exploits the primal solution only.

#### intpnthotstart.dual

The interior-point optimizer exploits the dual solution only.

#### intpnthotstart.primal\_dual

The interior-point optimizer exploits both the primal and dual solution.

#### callbackcode

Progress callback codes

### callbackcode.begin\_bi

The basis identification procedure has been started.

# callbackcode.begin\_conic

The callback function is called when the conic optimizer is started.

# $\verb|callbackcode.begin_dual_bi|\\$

The callback function is called from within the basis identification procedure when the dual phase is started.

# callbackcode.begin\_dual\_sensitivity

Dual sensitivity analysis is started.

# callbackcode.begin\_dual\_setup\_bi

The callback function is called when the dual BI phase is started.

# callbackcode.begin\_dual\_simplex

The callback function is called when the dual simplex optimizer started.

# callbackcode.begin\_dual\_simplex\_bi

The callback function is called from within the basis identification procedure when the dual simplex clean-up phase is started.

### callbackcode.begin\_full\_convexity\_check

Begin full convexity check.

#### callbackcode.begin\_infeas\_ana

The callback function is called when the infeasibility analyzer is started.

# callbackcode.begin\_intpnt

The callback function is called when the interior-point optimizer is started.

### callbackcode.begin\_license\_wait

Begin waiting for license.

#### callbackcode.begin\_mio

The callback function is called when the mixed-integer optimizer is started.

### callbackcode.begin\_optimizer

The callback function is called when the optimizer is started.

#### callbackcode.begin\_presolve

The callback function is called when the presolve is started.

### callbackcode.begin\_primal\_bi

The callback function is called from within the basis identification procedure when the primal phase is started.

## callbackcode.begin\_primal\_repair

Begin primal feasibility repair.

## callbackcode.begin\_primal\_sensitivity

Primal sensitivity analysis is started.

#### callbackcode.begin\_primal\_setup\_bi

The callback function is called when the primal BI setup is started.

# callbackcode.begin\_primal\_simplex

The callback function is called when the primal simplex optimizer is started.

# $\verb|callbackcode.begin_primal_simplex_bi|\\$

The callback function is called from within the basis identification procedure when the primal simplex clean-up phase is started.

# $\verb|callbackcode.begin_qcqo_reformulate|\\$

Begin QCQO reformulation.

# callbackcode.begin\_read

MOSEK has started reading a problem file.

# callbackcode.begin\_root\_cutgen

The callback function is called when root cut generation is started.

# callbackcode.begin\_simplex

The callback function is called when the simplex optimizer is started.

### callbackcode.begin\_simplex\_bi

The callback function is called from within the basis identification procedure when the simplex clean-up phase is started.

# callbackcode.begin\_to\_conic

Begin conic reformulation.

# callbackcode.begin\_write

MOSEK has started writing a problem file.

### callbackcode.conic

The callback function is called from within the conic optimizer after the information database has been updated.

## callbackcode.dual\_simplex

The callback function is called from within the dual simplex optimizer.

16.9. Enumerations 349

#### callbackcode.end\_bi

The callback function is called when the basis identification procedure is terminated.

#### callbackcode.end\_conic

The callback function is called when the conic optimizer is terminated.

# callbackcode.end\_dual\_bi

The callback function is called from within the basis identification procedure when the dual phase is terminated.

### callbackcode.end\_dual\_sensitivity

Dual sensitivity analysis is terminated.

#### callbackcode.end\_dual\_setup\_bi

The callback function is called when the dual BI phase is terminated.

### callbackcode.end\_dual\_simplex

The callback function is called when the dual simplex optimizer is terminated.

#### callbackcode.end\_dual\_simplex\_bi

The callback function is called from within the basis identification procedure when the dual clean-up phase is terminated.

### callbackcode.end\_full\_convexity\_check

End full convexity check.

#### callbackcode.end\_infeas\_ana

The callback function is called when the infeasibility analyzer is terminated.

#### callbackcode.end\_intpnt

The callback function is called when the interior-point optimizer is terminated.

#### callbackcode.end\_license\_wait

End waiting for license.

## callbackcode.end\_mio

The callback function is called when the mixed-integer optimizer is terminated.

# callbackcode.end\_optimizer

The callback function is called when the optimizer is terminated.

# ${\tt callbackcode.end\_presolve}$

The callback function is called when the presolve is completed.

# callbackcode.end\_primal\_bi

The callback function is called from within the basis identification procedure when the primal phase is terminated.

# callbackcode.end\_primal\_repair

End primal feasibility repair.

# callbackcode.end\_primal\_sensitivity

Primal sensitivity analysis is terminated.

### callbackcode.end\_primal\_setup\_bi

The callback function is called when the primal BI setup is terminated.

# ${\tt callbackcode.end\_primal\_simplex}$

The callback function is called when the primal simplex optimizer is terminated.

# callbackcode.end\_primal\_simplex\_bi

The callback function is called from within the basis identification procedure when the primal clean-up phase is terminated.

# callbackcode.end\_qcqo\_reformulate

End QCQO reformulation.

## callbackcode.end\_read

**MOSEK** has finished reading a problem file.

#### callbackcode.end\_root\_cutgen

The callback function is called when root cut generation is is terminated.

#### callbackcode.end\_simplex

The callback function is called when the simplex optimizer is terminated.

# callbackcode.end\_simplex\_bi

The callback function is called from within the basis identification procedure when the simplex clean-up phase is terminated.

## callbackcode.end\_to\_conic

End conic reformulation.

#### callbackcode.end\_write

**MOSEK** has finished writing a problem file.

#### callbackcode.im\_bi

The callback function is called from within the basis identification procedure at an intermediate point.

#### callbackcode.im\_conic

The callback function is called at an intermediate stage within the conic optimizer where the information database has not been updated.

#### callbackcode.im\_dual\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

# callbackcode.im\_dual\_sensivity

The callback function is called at an intermediate stage of the dual sensitivity analysis.

#### callbackcode.im\_dual\_simplex

The callback function is called at an intermediate point in the dual simplex optimizer.

# $\verb|callbackcode.im_full_convexity_check| \\$

The callback function is called at an intermediate stage of the full convexity check.

# callbackcode.im\_intpnt

The callback function is called at an intermediate stage within the interior-point optimizer where the information database has not been updated.

# callbackcode.im\_license\_wait

MOSEK is waiting for a license.

# callbackcode.im\_lu

The callback function is called from within the LU factorization procedure at an intermediate point.

#### callbackcode.im\_mio

The callback function is called at an intermediate point in the mixed-integer optimizer.

### callbackcode.im\_mio\_dual\_simplex

The callback function is called at an intermediate point in the mixed-integer optimizer while running the dual simplex optimizer.

#### callbackcode.im\_mio\_intpnt

The callback function is called at an intermediate point in the mixed-integer optimizer while running the interior-point optimizer.

# callbackcode.im\_mio\_primal\_simplex

The callback function is called at an intermediate point in the mixed-integer optimizer while running the primal simplex optimizer.

### callbackcode.im\_order

The callback function is called from within the matrix ordering procedure at an intermediate point.

16.9. Enumerations 351

### callbackcode.im\_presolve

The callback function is called from within the presolve procedure at an intermediate stage.

# callbackcode.im\_primal\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

#### callbackcode.im\_primal\_sensivity

The callback function is called at an intermediate stage of the primal sensitivity analysis.

### callbackcode.im\_primal\_simplex

The callback function is called at an intermediate point in the primal simplex optimizer.

#### callbackcode.im\_qo\_reformulate

The callback function is called at an intermediate stage of the conic quadratic reformulation.

#### callbackcode.im\_read

Intermediate stage in reading.

#### callbackcode.im\_root\_cutgen

The callback is called from within root cut generation at an intermediate stage.

# callbackcode.im\_simplex

The callback function is called from within the simplex optimizer at an intermediate point.

### callbackcode.im\_simplex\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the simplex clean-up phase. The frequency of the callbacks is controlled by the *iparam.log\_sim\_freq* parameter.

#### callbackcode.intpnt

The callback function is called from within the interior-point optimizer after the information database has been updated.

### callbackcode.new\_int\_mio

The callback function is called after a new integer solution has been located by the mixed-integer optimizer.

#### callbackcode.primal\_simplex

The callback function is called from within the primal simplex optimizer.

# callbackcode.read\_opf

The callback function is called from the OPF reader.

### callbackcode.read\_opf\_section

A chunk of Q non-zeros has been read from a problem file.

# callbackcode.solving\_remote

The callback function is called while the task is being solved on a remote server.

### callbackcode.update\_dual\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

### callbackcode.update\_dual\_simplex

The callback function is called in the dual simplex optimizer.

# callbackcode.update\_dual\_simplex\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the dual simplex clean-up phase. The frequency of the callbacks is controlled by the <code>iparam.log\_sim\_freq</code> parameter.

# callbackcode.update\_presolve

The callback function is called from within the presolve procedure.

## callbackcode.update\_primal\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

```
callbackcode.update_primal_simplex
          The callback function is called in the primal simplex optimizer.
     callbackcode.update_primal_simplex_bi
          The callback function is called from within the basis identification procedure at an interme-
          diate point in the primal simplex clean-up phase. The frequency of the callbacks is controlled
          by the iparam.log_sim_freq parameter.
     callbackcode.write_opf
          The callback function is called from the OPF writer.
checkconvexitytype
     Types of convexity checks.
     checkconvexitytype.none
          No convexity check.
     checkconvexitytype.simple
          Perform simple and fast convexity check.
     checkconvexitytype.full
          Perform a full convexity check.
compresstype
     Compression types
     compresstype.none
          No compression is used.
     compresstype.free
          The type of compression used is chosen automatically.
     compresstype.gzip
          The type of compression used is gzip compatible.
conetype
     Cone types
     conetype.quad
          The cone is a quadratic cone.
     conetype.rquad
          The cone is a rotated quadratic cone.
nametype
     Name types
     nametype.gen
          General names. However, no duplicate and blank names are allowed.
     nametype.mps
          MPS type names.
     nametype.lp
          LP type names.
symmattype
     Cone types
     symmattype.sparse
          Sparse symmetric matrix.
dataformat
     Data format types
     dataformat.extension
          The file extension is used to determine the data file format.
```

16.9. Enumerations 353

#### dataformat.mps

The data file is MPS formatted.

#### dataformat.lp

The data file is LP formatted.

## dataformat.op

The data file is an optimization problem formatted file.

### dataformat.xml

The data file is an XML formatted file.

#### dataformat.free\_mps

The data a free MPS formatted file.

#### dataformat.task

Generic task dump file.

### dataformat.cb

Conic benchmark format,

### dataformat.json\_task

JSON based task format.

#### dinfitem

Double information items

#### dinfitem.bi\_clean\_dual\_time

Time spent within the dual clean-up optimizer of the basis identification procedure since its invocation.

#### dinfitem.bi\_clean\_primal\_time

Time spent within the primal clean-up optimizer of the basis identification procedure since its invocation.

#### dinfitem.bi\_clean\_time

Time spent within the clean-up phase of the basis identification procedure since its invocation.

# dinfitem.bi\_dual\_time

Time spent within the dual phase basis identification procedure since its invocation.

## dinfitem.bi\_primal\_time

Time spent within the primal phase of the basis identification procedure since its invocation.

# dinfitem.bi\_time

Time spent within the basis identification procedure since its invocation.

# dinfitem.intpnt\_dual\_feas

Dual feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed.)

#### dinfitem.intpnt\_dual\_obj

Dual objective value reported by the interior-point optimizer.

## dinfitem.intpnt\_factor\_num\_flops

An estimate of the number of flops used in the factorization.

#### dinfitem.intpnt\_opt\_status

A measure of optimality of the solution. It should converge to +1 if the problem has a primal-dual optimal solution, and converge to -1 if the problem is (strictly) primal or dual infeasible. If the measure converges to another constant, or fails to settle, the problem is usually ill-posed.

# dinfitem.intpnt\_order\_time

Order time (in seconds).

# dinfitem.intpnt\_primal\_feas

Primal feasibility measure reported by the interior-point optimizer. (For the interior-point

optimizer this measure is not directly related to the original problem because a homogeneous model is employed).

## dinfitem.intpnt\_primal\_obj

Primal objective value reported by the interior-point optimizer.

## dinfitem.intpnt\_time

Time spent within the interior-point optimizer since its invocation.

### dinfitem.mio\_clique\_separation\_time

Separation time for clique cuts.

### dinfitem.mio\_cmir\_separation\_time

Separation time for CMIR cuts.

### dinfitem.mio\_construct\_solution\_obj

If **MOSEK** has successfully constructed an integer feasible solution, then this item contains the optimal objective value corresponding to the feasible solution.

## ${\tt dinfitem.mio\_dual\_bound\_after\_presolve}$

Value of the dual bound after presolve but before cut generation.

## dinfitem.mio\_gmi\_separation\_time

Separation time for GMI cuts.

### dinfitem.mio\_heuristic\_time

Total time spent in the optimizer.

### dinfitem.mio\_implied\_bound\_time

Separation time for implied bound cuts.

### dinfitem.mio\_knapsack\_cover\_separation\_time

Separation time for knapsack cover.

## dinfitem.mio\_obj\_abs\_gap

Given the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the absolute gap defined by

|(objective value of feasible solution) – (objective bound)|.

Otherwise it has the value -1.0.

### dinfitem.mio\_obj\_bound

The best known bound on the objective function. This value is undefined until at least one relaxation has been solved: To see if this is the case check that  $iinfitem.mio\_num\_relax$  is strictly positive.

## dinfitem.mio\_obj\_int

The primal objective value corresponding to the best integer feasible solution. Please note that at least one integer feasible solution must have been located i.e. check iinfitem.  $mio\_num\_int\_solutions$ .

### dinfitem.mio\_obj\_rel\_gap

Given that the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the relative gap defined by

```
\frac{|(\text{objective value of feasible solution}) - (\text{objective bound})|}{\max(\delta, |(\text{objective value of feasible solution})|)}
```

where  $\delta$  is given by the parameter  $dparam.mio\_rel\_gap\_const$ . Otherwise it has the value -1.0.

### dinfitem.mio\_optimizer\_time

Total time spent in the optimizer.

### dinfitem.mio\_probing\_time

Total time for probing.

### dinfitem.mio\_root\_cutgen\_time

Total time for cut generation.

### dinfitem.mio\_root\_optimizer\_time

Time spent in the optimizer while solving the root relaxation.

### dinfitem.mio\_root\_presolve\_time

Time spent in while presolving the root relaxation.

### dinfitem.mio\_time

Time spent in the mixed-integer optimizer.

### dinfitem.mio\_user\_obj\_cut

If the objective cut is used, then this information item has the value of the cut.

### dinfitem.optimizer\_time

Total time spent in the optimizer since it was invoked.

### dinfitem.presolve\_eli\_time

Total time spent in the eliminator since the presolve was invoked.

### dinfitem.presolve\_lindep\_time

Total time spent in the linear dependency checker since the presolve was invoked.

### dinfitem.presolve\_time

Total time (in seconds) spent in the presolve since it was invoked.

### dinfitem.primal\_repair\_penalty\_obj

The optimal objective value of the penalty function.

### dinfitem.qcqo\_reformulate\_max\_perturbation

Maximum absolute diagonal perturbation occurring during the QCQO reformulation.

### dinfitem.qcqo\_reformulate\_time

Time spent with conic quadratic reformulation.

### dinfitem.qcqo\_reformulate\_worst\_cholesky\_column\_scaling

Worst Cholesky column scaling.

### dinfitem.qcqo\_reformulate\_worst\_cholesky\_diag\_scaling

Worst Cholesky diagonal scaling.

### dinfitem.rd\_time

Time spent reading the data file.

### dinfitem.sim\_dual\_time

Time spent in the dual simplex optimizer since invoking it.

### dinfitem.sim\_feas

Feasibility measure reported by the simplex optimizer.

## dinfitem.sim\_obj

Objective value reported by the simplex optimizer.

## dinfitem.sim\_primal\_time

Time spent in the primal simplex optimizer since invoking it.

### dinfitem.sim\_time

Time spent in the simplex optimizer since invoking it.

### dinfitem.sol\_bas\_dual\_obj

Dual objective value of the basic solution.

### dinfitem.sol\_bas\_dviolcon

Maximal dual bound violation for  $x^c$  in the basic solution.

## dinfitem.sol\_bas\_dviolvar

Maximal dual bound violation for  $x^x$  in the basic solution.

### dinfitem.sol\_bas\_nrm\_barx

Infinity norm of  $\overline{X}$  in the basic solution.

### dinfitem.sol\_bas\_nrm\_slc

Infinity norm of  $s_l^c$  in the basic solution.

### dinfitem.sol\_bas\_nrm\_slx

Infinity norm of  $s_l^x$  in the basic solution.

### dinfitem.sol\_bas\_nrm\_suc

Infinity norm of  $s_u^c$  in the basic solution.

### dinfitem.sol\_bas\_nrm\_sux

Infinity norm of  $s_u^X$  in the basic solution.

### dinfitem.sol\_bas\_nrm\_xc

Infinity norm of  $x^c$  in the basic solution.

### dinfitem.sol\_bas\_nrm\_xx

Infinity norm of  $x^x$  in the basic solution.

### dinfitem.sol\_bas\_nrm\_y

Infinity norm of y in the basic solution.

### dinfitem.sol\_bas\_primal\_obj

Primal objective value of the basic solution.

### dinfitem.sol\_bas\_pviolcon

Maximal primal bound violation for  $x^c$  in the basic solution.

### dinfitem.sol\_bas\_pviolvar

Maximal primal bound violation for  $x^x$  in the basic solution.

### dinfitem.sol\_itg\_nrm\_barx

Infinity norm of  $\overline{X}$  in the integer solution.

### dinfitem.sol\_itg\_nrm\_xc

Infinity norm of  $x^c$  in the integer solution.

### dinfitem.sol\_itg\_nrm\_xx

Infinity norm of  $x^x$  in the integer solution.

## dinfitem.sol\_itg\_primal\_obj

Primal objective value of the integer solution.

## dinfitem.sol\_itg\_pviolbarvar

Maximal primal bound violation for  $\overline{X}$  in the integer solution.

### dinfitem.sol\_itg\_pviolcon

Maximal primal bound violation for  $x^c$  in the integer solution.

## ${\tt dinfitem.sol\_itg\_pviolcones}$

Maximal primal violation for primal conic constraints in the integer solution.

### dinfitem.sol\_itg\_pviolitg

Maximal violation for the integer constraints in the integer solution.

### dinfitem.sol\_itg\_pviolvar

Maximal primal bound violation for  $x^x$  in the integer solution.

### dinfitem.sol\_itr\_dual\_obj

Dual objective value of the interior-point solution.

### dinfitem.sol\_itr\_dviolbarvar

Maximal dual bound violation for  $\overline{X}$  in the interior-point solution.

### dinfitem.sol\_itr\_dviolcon

Maximal dual bound violation for  $x^c$  in the interior-point solution.

# dinfitem.sol\_itr\_dviolcones Maximal dual violation for dual conic constraints in the interior-point solution. dinfitem.sol\_itr\_dviolvar

Maximal dual bound violation for  $x^x$  in the interior-point solution.

# dinfitem.sol\_itr\_nrm\_bars

Infinity norm of  $\overline{S}$  in the interior-point solution.

## dinfitem.sol\_itr\_nrm\_barx

Infinity norm of  $\overline{X}$  in the interior-point solution.

### dinfitem.sol\_itr\_nrm\_slc

Infinity norm of  $s_l^c$  in the interior-point solution.

### dinfitem.sol\_itr\_nrm\_slx

Infinity norm of  $s_I^x$  in the interior-point solution.

### dinfitem.sol\_itr\_nrm\_snx

Infinity norm of  $s_n^x$  in the interior-point solution.

### dinfitem.sol\_itr\_nrm\_suc

Infinity norm of  $s_u^c$  in the interior-point solution.

## dinfitem.sol\_itr\_nrm\_sux

Infinity norm of  $s_u^X$  in the interior-point solution.

### dinfitem.sol\_itr\_nrm\_xc

Infinity norm of  $x^c$  in the interior-point solution.

### dinfitem.sol\_itr\_nrm\_xx

Infinity norm of  $x^x$  in the interior-point solution.

### dinfitem.sol\_itr\_nrm\_y

Infinity norm of y in the interior-point solution.

### dinfitem.sol\_itr\_primal\_obj

Primal objective value of the interior-point solution.

### dinfitem.sol\_itr\_pviolbarvar

Maximal primal bound violation for  $\overline{X}$  in the interior-point solution.

## dinfitem.sol\_itr\_pviolcon

Maximal primal bound violation for  $x^c$  in the interior-point solution.

### dinfitem.sol\_itr\_pviolcones

Maximal primal violation for primal conic constraints in the interior-point solution.

### dinfitem.sol\_itr\_pviolvar

Maximal primal bound violation for  $x^x$  in the interior-point solution.

### dinfitem.to\_conic\_time

Time spent in the last to conic reformulation.

## feature

License feature

### feature.pts

Base system.

### feature.pton

Nonlinear extension.

## liinfitem

Long integer information items.

### liinfitem.bi\_clean\_dual\_deg\_iter

Number of dual degenerate clean iterations performed in the basis identification.

### liinfitem.bi\_clean\_dual\_iter

Number of dual clean iterations performed in the basis identification.

### liinfitem.bi\_clean\_primal\_deg\_iter

Number of primal degenerate clean iterations performed in the basis identification.

### liinfitem.bi\_clean\_primal\_iter

Number of primal clean iterations performed in the basis identification.

### liinfitem.bi\_dual\_iter

Number of dual pivots performed in the basis identification.

### liinfitem.bi\_primal\_iter

Number of primal pivots performed in the basis identification.

## liinfitem.intpnt\_factor\_num\_nz

Number of non-zeros in factorization.

### liinfitem.mio\_intpnt\_iter

Number of interior-point iterations performed by the mixed-integer optimizer.

### liinfitem.mio\_presolved\_anz

Number of non-zero entries in the constraint matrix of presolved problem.

### liinfitem.mio\_sim\_maxiter\_setbacks

Number of times the simplex optimizer has hit the maximum iteration limit when reoptimizing.

### liinfitem.mio\_simplex\_iter

Number of simplex iterations performed by the mixed-integer optimizer.

### liinfitem.rd\_numanz

Number of non-zeros in A that is read.

### liinfitem.rd\_numqnz

Number of Q non-zeros.

## iinfitem

Integer information items.

### iinfitem.ana\_pro\_num\_con

Number of constraints in the problem.

This value is set by Task. analyzeproblem.

## iinfitem.ana\_pro\_num\_con\_eq

Number of equality constraints.

This value is set by Task. analyzeproblem.

### iinfitem.ana\_pro\_num\_con\_fr

Number of unbounded constraints.

This value is set by Task. analyzeproblem.

### iinfitem.ana\_pro\_num\_con\_lo

Number of constraints with a lower bound and an infinite upper bound.

This value is set by Task.analyzeproblem.

### iinfitem.ana\_pro\_num\_con\_ra

Number of constraints with finite lower and upper bounds.

This value is set by Task. analyzeproblem.

## ${\tt iinfitem.ana\_pro\_num\_con\_up}$

Number of constraints with an upper bound and an infinite lower bound.

This value is set by Task.analyzeproblem.

### iinfitem.ana\_pro\_num\_var

Number of variables in the problem.

This value is set by Task. analyzeproblem.

### iinfitem.ana\_pro\_num\_var\_bin

Number of binary (0-1) variables.

This value is set by Task. analyzeproblem.

### iinfitem.ana\_pro\_num\_var\_cont

Number of continuous variables.

This value is set by Task. analyzeproblem.

### iinfitem.ana\_pro\_num\_var\_eq

Number of fixed variables.

This value is set by Task. analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_fr

Number of free variables.

This value is set by Task. analyzeproblem.

### iinfitem.ana\_pro\_num\_var\_int

Number of general integer variables.

This value is set by Task. analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_lo

Number of variables with a lower bound and an infinite upper bound.

This value is set by Task. analyzeproblem.

### iinfitem.ana\_pro\_num\_var\_ra

Number of variables with finite lower and upper bounds.

This value is set by Task. analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_up

Number of variables with an upper bound and an infinite lower bound. This value is set by

This value is set by Task.analyzeproblem.

## iinfitem.intpnt\_factor\_dim\_dense

Dimension of the dense sub system in factorization.

### iinfitem.intpnt\_iter

Number of interior-point iterations since invoking the interior-point optimizer.

### iinfitem.intpnt\_num\_threads

Number of threads that the interior-point optimizer is using.

### iinfitem.intpnt\_solve\_dual

Non-zero if the interior-point optimizer is solving the dual problem.

### iinfitem.mio\_absgap\_satisfied

Non-zero if absolute gap is within tolerances.

### iinfitem.mio\_clique\_table\_size

Size of the clique table.

## $\verb|iinfitem.mio_construct_num_roundings|\\$

Number of values in the integer solution that is rounded to an integer value.

### iinfitem.mio\_construct\_solution

If this item has the value 0, then **MOSEK** did not try to construct an initial integer feasible solution. If the item has a positive value, then **MOSEK** successfully constructed an initial integer feasible solution.

### iinfitem.mio\_initial\_solution

Is non-zero if an initial integer solution is specified.

### iinfitem.mio\_near\_absgap\_satisfied

Non-zero if absolute gap is within relaxed tolerances.

### iinfitem.mio\_near\_relgap\_satisfied

Non-zero if relative gap is within relaxed tolerances.

### iinfitem.mio\_node\_depth

Depth of the last node solved.

### iinfitem.mio\_num\_active\_nodes

Number of active branch bound nodes.

### iinfitem.mio\_num\_branch

Number of branches performed during the optimization.

### iinfitem.mio\_num\_clique\_cuts

Number of clique cuts.

### iinfitem.mio\_num\_cmir\_cuts

Number of Complemented Mixed Integer Rounding (CMIR) cuts.

### iinfitem.mio\_num\_gomory\_cuts

Number of Gomory cuts.

### iinfitem.mio\_num\_implied\_bound\_cuts

Number of implied bound cuts.

### iinfitem.mio\_num\_int\_solutions

Number of integer feasible solutions that has been found.

### iinfitem.mio\_num\_knapsack\_cover\_cuts

Number of clique cuts.

### iinfitem.mio\_num\_relax

Number of relaxations solved during the optimization.

### iinfitem.mio\_num\_repeated\_presolve

Number of times presolve was repeated at root.

### iinfitem.mio\_numcon

Number of constraints in the problem solved by the mixed-integer optimizer.

### iinfitem.mio\_numint

Number of integer variables in the problem solved be the mixed-integer optimizer.

### iinfitem.mio\_numvar

Number of variables in the problem solved by the mixed-integer optimizer.

## $\verb|iinfitem.mio_obj_bound_defined|\\$

Non-zero if a valid objective bound has been found, otherwise zero.

## iinfitem.mio\_presolved\_numbin

Number of binary variables in the problem solved be the mixed-integer optimizer.

### iinfitem.mio\_presolved\_numcon

Number of constraints in the presolved problem.

### iinfitem.mio\_presolved\_numcont

Number of continuous variables in the problem solved be the mixed-integer optimizer.

### iinfitem.mio\_presolved\_numint

Number of integer variables in the presolved problem.

### iinfitem.mio\_presolved\_numvar

Number of variables in the presolved problem.

### iinfitem.mio\_relgap\_satisfied

Non-zero if relative gap is within tolerances.

## iinfitem.mio\_total\_num\_cuts

Total number of cuts generated by the mixed-integer optimizer.

## iinfitem.mio\_user\_obj\_cut

If it is non-zero, then the objective cut is used.

### iinfitem.opt\_numcon

Number of constraints in the problem solved when the optimizer is called.

### iinfitem.opt\_numvar

Number of variables in the problem solved when the optimizer is called

### iinfitem.optimize\_response

The response code returned by optimize.

### iinfitem.rd\_numbarvar

Number of variables read.

### iinfitem.rd\_numcon

Number of constraints read.

### iinfitem.rd\_numcone

Number of conic constraints read.

### iinfitem.rd\_numintvar

Number of integer-constrained variables read.

### iinfitem.rd\_numq

Number of nonempty Q matrices read.

### iinfitem.rd\_numvar

Number of variables read.

### iinfitem.rd\_protype

Problem type.

### iinfitem.sim\_dual\_deg\_iter

The number of dual degenerate iterations.

## iinfitem.sim\_dual\_hotstart

If 1 then the dual simplex algorithm is solving from an advanced basis.

### iinfitem.sim\_dual\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the dual simplex algorithm.

### iinfitem.sim\_dual\_inf\_iter

The number of iterations taken with dual infeasibility.

### iinfitem.sim\_dual\_iter

Number of dual simplex iterations during the last optimization.

### iinfitem.sim\_numcon

Number of constraints in the problem solved by the simplex optimizer.

## iinfitem.sim\_numvar

Number of variables in the problem solved by the simplex optimizer.

## ${\tt iinfitem.sim\_primal\_deg\_iter}$

The number of primal degenerate iterations.

## $\verb|iinfitem.sim_primal_hotstart|\\$

If 1 then the primal simplex algorithm is solving from an advanced basis.

### iinfitem.sim\_primal\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the primal simplex algorithm.

## iinfitem.sim\_primal\_inf\_iter

The number of iterations taken with primal infeasibility.

## iinfitem.sim\_primal\_iter

Number of primal simplex iterations during the last optimization.

### iinfitem.sim\_solve\_dual

Is non-zero if dual problem is solved.

### iinfitem.sol\_bas\_prosta

Problem status of the basic solution. Updated after each optimization.

### iinfitem.sol\_bas\_solsta

Solution status of the basic solution. Updated after each optimization.

### iinfitem.sol\_itg\_prosta

Problem status of the integer solution. Updated after each optimization.

### iinfitem.sol\_itg\_solsta

Solution status of the integer solution. Updated after each optimization.

### iinfitem.sol\_itr\_prosta

Problem status of the interior-point solution. Updated after each optimization.

### iinfitem.sol\_itr\_solsta

Solution status of the interior-point solution. Updated after each optimization.

### iinfitem.sto\_num\_a\_realloc

Number of times the storage for storing A has been changed. A large value may indicates that memory fragmentation may occur.

## $\verb"inftype"$

Information item types

## inftype.dou\_type

Is a double information type.

## inftype.int\_type

Is an integer.

## inftype.lint\_type

Is a long integer.

### iomode

Input/output modes

## iomode.read

The file is read-only.

## iomode.write

The file is write-only. If the file exists then it is truncated when it is opened. Otherwise it is created when it is opened.

### iomode.readwrite

The file is to read and written.

### branchdir

Specifies the branching direction.

## branchdir.free

The mixed-integer optimizer decides which branch to choose.

## branchdir.up

The mixed-integer optimizer always chooses the up branch first.

#### branchdir.down

The mixed-integer optimizer always chooses the down branch first.

#### branchdir.near

Branch in direction nearest to selected fractional variable.

### branchdir.far

Branch in direction farthest from selected fractional variable.

### branchdir.root\_lp

Chose direction based on root lp value of selected variable.

#### branchdir.guided

Branch in direction of current incumbent.

## branchdir.pseudocost

Branch based on the pseudocost of the variable.

### miocontsoltype

Continuous mixed-integer solution type

### miocontsoltype.none

No interior-point or basic solution are reported when the mixed-integer optimizer is used.

### miocontsoltype.root

The reported interior-point and basic solutions are a solution to the root node problem when mixed-integer optimizer is used.

### miocontsoltype.itg

The reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. A solution is only reported in case the problem has a primal feasible solution.

### miocontsoltype.itg\_rel

In case the problem is primal feasible then the reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. If the problem is primal infeasible, then the solution to the root node problem is reported.

## miomode

Integer restrictions

### miomode.ignored

The integer constraints are ignored and the problem is solved as a continuous problem.

### miomode.satisfied

Integer restrictions should be satisfied.

### mionodeseltype

Mixed-integer node selection types

### mionodeseltype.free

The optimizer decides the node selection strategy.

### mionodeseltype.first

The optimizer employs a depth first node selection strategy.

## mionodeseltype.best

The optimizer employs a best bound node selection strategy.

## ${\tt mionodeseltype.worst}$

The optimizer employs a worst bound node selection strategy.

## mionodeseltype.hybrid

The optimizer employs a hybrid strategy.

## mionodeseltype.pseudo

The optimizer employs selects the node based on a pseudo cost estimate.

```
mpsformat
     MPS file format type
     mpsformat.strict
          It is assumed that the input file satisfies the MPS format strictly.
     mpsformat.relaxed
          It is assumed that the input file satisfies a slightly relaxed version of the MPS format.
     mpsformat.free
          It is assumed that the input file satisfies the free MPS format. This implies that spaces are
          not allowed in names. Otherwise the format is free.
     mpsformat.cplex
          The CPLEX compatible version of the MPS format is employed.
objsense
     Objective sense types
     objsense.minimize
          The problem should be minimized.
     objsense.maximize
          The problem should be maximized.
onoffkey
     On/off
     onoffkey.on
          Switch the option on.
     onoffkey.off
          Switch the option off.
optimizertype
     Optimizer types
     optimizertype.conic
          The optimizer for problems having conic constraints.
     optimizertype.dual_simplex
          The dual simplex optimizer is used.
     optimizertype.free
          The optimizer is chosen automatically.
     optimizertype.free_simplex
          One of the simplex optimizers is used.
     optimizertype.intpnt
          The interior-point optimizer is used.
     optimizertype.mixed_int
          The mixed-integer optimizer.
     optimizertype.primal_simplex
          The primal simplex optimizer is used.
orderingtype
     Ordering strategies
     orderingtype.free
          The ordering method is chosen automatically.
     orderingtype.appminloc
          Approximate minimum local fill-in ordering is employed.
     orderingtype.experimental
          This option should not be used.
```

```
orderingtype.try_graphpar
          Always try the graph partitioning based ordering.
     orderingtype.force_graphpar
          Always use the graph partitioning based ordering even if it is worse than the approximate
          minimum local fill ordering.
     orderingtype.none
          No ordering is used.
presolvemode
     Presolve method.
     presolvemode.off
          The problem is not presolved before it is optimized.
     presolvemode.on
          The problem is presolved before it is optimized.
     presolvemode.free
          It is decided automatically whether to presolve before the problem is optimized.
parametertype
     Parameter type
     {\tt parametertype.invalid\_type}
          Not a valid parameter.
     parametertype.dou_type
          Is a double parameter.
     parametertype.int_type
          Is an integer parameter.
     parametertype.str_type
          Is a string parameter.
problemitem
     Problem data items
     problemitem.var
          Item is a variable.
     problemitem.con
          Item is a constraint.
     problemitem.cone
          Item is a cone.
problemtype
     Problem types
     problemtype.lo
          The problem is a linear optimization problem.
     problemtype.qo
          The problem is a quadratic optimization problem.
     problemtype.qcqo
          The problem is a quadratically constrained optimization problem.
     problemtype.geco
          General convex optimization.
     problemtype.conic
          A conic optimization.
```

### problemtype.mixed

General nonlinear constraints and conic constraints. This combination can not be solved by  $\mathbf{MOSEK}$ .

### prosta

Problem status keys

#### prosta.unknown

Unknown problem status.

### prosta.prim\_and\_dual\_feas

The problem is primal and dual feasible.

### prosta.prim\_feas

The problem is primal feasible.

### prosta.dual\_feas

The problem is dual feasible.

## prosta.near\_prim\_and\_dual\_feas

The problem is at least nearly primal and dual feasible.

### prosta.near\_prim\_feas

The problem is at least nearly primal feasible.

### prosta.near\_dual\_feas

The problem is at least nearly dual feasible.

### prosta.prim\_infeas

The problem is primal infeasible.

### prosta.dual\_infeas

The problem is dual infeasible.

## prosta.prim\_and\_dual\_infeas

The problem is primal and dual infeasible.

## prosta.ill\_posed

The problem is ill-posed. For example, it may be primal and dual feasible but have a positive duality gap.

### prosta.prim\_infeas\_or\_unbounded

The problem is either primal infeasible or unbounded. This may occur for mixed-integer problems.

### xmlwriteroutputtype

XML writer output mode

## xmlwriteroutputtype.row

Write in row order.

## xmlwriteroutputtype.col

Write in column order.

### rescodetype

Response code type

## rescodetype.ok

The response code is OK.

## rescodetype.wrn

The response code is a warning.

## rescodetype.trm

The response code is an optimizer termination status.

## rescodetype.err

The response code is an error.

### rescodetype.unk

The response code does not belong to any class.

### scalingtype

Scaling type

## scalingtype.free

The optimizer chooses the scaling heuristic.

## scalingtype.none

No scaling is performed.

### scalingtype.moderate

A conservative scaling is performed.

## scalingtype.aggressive

A very aggressive scaling is performed.

### scalingmethod

Scaling method

### scalingmethod.pow2

Scales only with power of 2 leaving the mantissa untouched.

## scalingmethod.free

The optimizer chooses the scaling heuristic.

### sensitivitytype

Sensitivity types

### sensitivitytype.basis

Basis sensitivity analysis is performed.

## sensitivitytype.optimal\_partition

Optimal partition sensitivity analysis is performed.

### simseltype

Simplex selection strategy

### simseltype.free

The optimizer chooses the pricing strategy.

## simseltype.full

The optimizer uses full pricing.

## ${\tt simseltype.ase}$

The optimizer uses approximate steepest-edge pricing.

### simseltype.devex

The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steepedge selection).

## simseltype.se

The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

## simseltype.partial

The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.

### solitem

Solution items

## solitem.xc

Solution for the constraints.

### solitem.xx

Variable solution.

### solitem.y

Lagrange multipliers for equations.

#### solitem.slc

Lagrange multipliers for lower bounds on the constraints.

#### solitem.suc

Lagrange multipliers for upper bounds on the constraints.

### solitem.slx

Lagrange multipliers for lower bounds on the variables.

#### solitem.sux

Lagrange multipliers for upper bounds on the variables.

#### solitem.snx

Lagrange multipliers corresponding to the conic constraints on the variables.

### solsta

Solution status keys

### solsta.unknown

Status of the solution is unknown.

### solsta.optimal

The solution is optimal.

### solsta.prim\_feas

The solution is primal feasible.

### solsta.dual\_feas

The solution is dual feasible.

### solsta.prim\_and\_dual\_feas

The solution is both primal and dual feasible.

### solsta.near\_optimal

The solution is nearly optimal.

### solsta.near\_prim\_feas

The solution is nearly primal feasible.

### solsta.near\_dual\_feas

The solution is nearly dual feasible.

## solsta.near\_prim\_and\_dual\_feas

The solution is nearly both primal and dual feasible.

## solsta.prim\_infeas\_cer

The solution is a certificate of primal infeasibility.

## solsta.dual\_infeas\_cer

The solution is a certificate of dual infeasibility.

## solsta.near\_prim\_infeas\_cer

The solution is almost a certificate of primal infeasibility.

### solsta.near\_dual\_infeas\_cer

The solution is almost a certificate of dual infeasibility.

### solsta.prim\_illposed\_cer

The solution is a certificate that the primal problem is illposed.

### solsta.dual\_illposed\_cer

The solution is a certificate that the dual problem is illposed.

### solsta.integer\_optimal

The primal solution is integer optimal.

```
solsta.near_integer_optimal
          The primal solution is near integer optimal.
soltype
     Solution types
     soltype.bas
          The basic solution.
     soltype.itr
          The interior solution.
     soltype.itg
          The integer solution.
solveform
     Solve primal or dual form
     solveform.free
          The optimizer is free to solve either the primal or the dual problem.
     solveform.primal
          The optimizer should solve the primal problem.
     solveform.dual
          The optimizer should solve the dual problem.
stakey
     Status keys
     stakey.unk
          The status for the constraint or variable is unknown.
     stakey.bas
          The constraint or variable is in the basis.
     stakey.supbas
          The constraint or variable is super basic.
     stakey.low
          The constraint or variable is at its lower bound.
     stakey.upr
          The constraint or variable is at its upper bound.
     stakey.fix
          The constraint or variable is fixed.
     stakey.inf
          The constraint or variable is infeasible in the bounds.
startpointtype
     Starting point types
     startpointtype.free
          The starting point is chosen automatically.
     startpointtype.guess
          The optimizer guesses a starting point.
     startpointtype.constant
```

### startpointtype.constant

The optimizer constructs a starting point by assigning a constant value to all primal and dual variables. This starting point is normally robust.

### startpointtype.satisfy\_bounds

The starting point is chosen to satisfy all the simple bounds on nonlinear variables. If this starting point is employed, then more care than usual should employed when choosing the bounds on the nonlinear variables. In particular very tight bounds should be avoided.

### streamtype

Stream types

### streamtype.log

Log stream. Contains the aggregated contents of all other streams. This means that a message written to any other stream will also be written to this stream.

#### streamtype.msg

Message stream. Log information relating to performance and progress of the optimization is written to this stream.

### streamtype.err

Error stream. Error messages are written to this stream.

### streamtype.wrn

Warning stream. Warning messages are written to this stream.

### value

Integer values

### value.max\_str\_len

Maximum string length allowed in **MOSEK**.

### value.license\_buffer\_length

The length of a license key buffer.

### variabletype

Variable types

### variabletype.type\_cont

Is a continuous variable.

### variabletype.type\_int

Is an integer variable.

# 16.10 Class types

### mosek.ItgSolutionCallback

A handler class for integer solution callbacks.

ItgSolutionCallback.callback

```
void callback (double[] xx)
```

The integer solution callback is a user-defined function which will be called by **MOSEK** when it improves the best mixed-integer solution.

The user must not call any MOSEK function directly or indirectly from the callback function.

**Parameters** xx (double[]) – An array with the values of all variables in the currently best solution. (input)

### mosek.Stream

A stream handler class.

Stream.stream

```
void stream (String msg)
```

The message-stream callback function is a user-defined function which can be linked to any of the MOSEK streams. Doing so, the function is called whenever MOSEK sends a message to the stream.

16.10. Class types 371

The user must not call any MOSEK function directly or indirectly from the callback function.

Parameters msg (String) - A message passed to the handler. (input)

### mosek.Progress

A handler class for progress callbacks.

Progress.progress

```
int progress (mosek.callbackcode code)
```

The progress callback is a user-defined function which will be called by **MOSEK** occasionally during the optimization process. In particular, the callback function is called at the beginning of each iteration in the interior-point optimizer. For the simplex optimizers <code>iparam.log\_sim\_freq</code> controls how frequently the callback is called.

The user must not call any MOSEK function directly or indirectly from the callback function.

Parameters code (callbackcode) - Callback code indicating current operation of the solver. (input)

**Return** (int) – Non-zero if the optimizer should be stopped; zero otherwise.

#### mosek.DataCallback

A handler class for data callbacks.

DataCallback.callback

```
int callback
  (mosek.callbackcode code,
  double[] dinf,
  int[] iinf,
  long[] liinf)
```

The data callback is a user-defined function which will be called by **MOSEK** occasionally during the optimization process. In particular, the callback function is called at the beginning of each iteration in the interior-point optimizer. For the simplex optimizers <code>iparam.log\_sim\_freq</code> controls how frequently the callback is called.

The user *must not* call any **MOSEK** function directly or indirectly from the callback function. The only exception is the possibility to retrieve an integer solution, see *Progress and data callback*.

### Parameters

- code (callbackcode) Callback code indicating current operation of the solver. (input)
- dinf (double[]) Array of double information items. (input)
- iinf (int[]) Array of integer information items. (input)
- liinf (long[]) Array of long integer information items. (input)

Return (int) - Non-zero if the optimizer should be stopped; zero otherwise.

## 16.11 Nonlinear extensions

## 16.11.1 Separable Convex Optimization (SCopt)

SCopt is an easy-to-use interface to the nonlinear optimizer when solving separable convex problems. See Sec. 8.1 for a tutorial and example code. As currently implemented, SCopt can handle only the nonlinear expressions  $x \ln(x)$ ,  $e^x$ ,  $\ln(x)$ , and  $x^g$ . However, it should be fairly easy to extend the interface to other nonlinear function of a single variable if needed.

All the linear data of the problem, such as c and A, is inputted to  $\mathbf{MOSEK}$  as usual, i.e. using the relevant functions in the  $\mathbf{MOSEK}$  API. Every nonlinear expression added to the objective should be specified by a 5-tuple of parameters:

opro[k]	oprjo[k]	oprfo[k]	oprgo[k]	oprho[k]	Expression added in objective
scopr.ent	j	f	g	h	$fx_j \ln(x_j)$
scopr.exp	j	f	g	h	$\int fe^{gx_j+h}$
scopr.log	j	f	g	h	$\int \ln(gx_j+h)$
scopr.pow	j	f	g	h	$f(x_j+h)^g$

Every nonlinear expression added to the constraints should be specified by a 6-tuple of parameters:

oprc[k]	opric[k]	oprjc[k]	oprfc[k]	oprgc[k]	oprhc[k]	Expression added to constraint
						i
scopr. ent	i	j	f	g	h	$fx_j \ln(x_j)$
scopr. exp	i	j	f	g	h	$\int e^{gx_j+h}$
scopr. log	i	j	$\int f$	g	h	$\int \ln(gx_j + h)$
scopr. pow	i	j	f	g	h	$f(x_j+h)^g$

In each case opr specifies the kind of expression to be added, oprf, oprg and oprh are the parameters and opri, oprj determine the variable and/or constraint to be considered. The concrete API specification follows.

### scopr

Type of nonlinear term in the SCopt interface.

```
scopr.ent Entropy function fx \ln(x) scopr.exp Exponential function fe^{gx+h} scopr.log Logarithm f \ln(gx+h) scopr.pow Power function f(x+h)^g
```

Task.putSCeval

```
void putSCeval(
 scopr[] opro,
 int[]
           oprjo,
 double[] oprfo,
 double[] oprgo,
 double[] oprho,
 scopr[]
          oprc,
 int[]
           opric,
 int[]
           oprjc,
 double[] oprfc,
 double[] oprgc,
 double[] oprhc)
```

Define the nonlinear part of the problem in the format specified by the SCopt interface. The first five arguments describe the nonlinear terms added to the objective, and should have the same length. The remaining six arguments describe the nonlinear terms added to the constraints

and should have the same length. Multiple terms involving the same variable and constraint are possible, they will be added up.

### **Parameters**

- opro (scopr[]) List of function indicators defining the objective terms. (input)
- oprjo (int[]) List of variable indexes for the objective terms. (input)
- oprfo (double[]) List of f values for the objective terms. (input)
- oprgo (double[]) List of g values for the objective terms. (input)
- oprho (double[]) List of h values for the objective terms. (input)
- oprc (scopr[]) List of function indicators defining the constraint terms. (input)
- opric (int[]) List of constraint indexes for the constraint terms. (input)
- oprjc (int[]) List of variable indexes for the constraint terms. (input)
- oprfc (double[]) List of f values for the constraint terms. (input)
- oprgc (double[]) List of g values for the constraint terms. (input)
- oprhc (double[]) List of h values for the constraint terms. (input)

### Task.clearSCeval

```
void clearSCeval ()
```

Remove all non-linear separable terms from the task.

### Task.writeSC

```
void writeSC
(String scfilename,
String taskfilename)
```

Write problem to an SCopt file and a normal problem file.

### **Parameters**

- scfilename (String) Name of SCopt terms file. (input)
- taskfilename (String) Name of problem file. (input)

## SUPPORTED FILE FORMATS

MOSEK supports a range of problem and solution formats listed in Table 17.1 and Table 17.2. The **Task** format is MOSEK's native binary format and it supports all features that MOSEK supports. The **OPF** format is MOSEK's human-readable alternative that supports nearly all features (everything except semidefinite problems). In general, text formats are significantly slower to read, but can be examined and edited directly in any text editor.

## **Problem formats**

See Table 17.1.

Table 17.1: List of supported file formats for optimization problems.

Format Type	Ext.	Binary/Text	LP	QO	CQO	SDP
LP	lp	plain text X		X		
MPS	mps	plain text	X	X		
OPF	opf	plain text	X	X	X	
CBF	cbf	plain text	X		X	X
OSiL	xml	xml text	X	X		
Task format	task	binary	X	X	X	X
Jtask format	jtask	text	X	X	X	X

## **Solution formats**

See Table 17.2.

Table 17.2: List of supported solution formats.

Format Type	Ext.	Binary/Text	Description
SOL	sol	plain text	Interior Solution
	bas	plain text	Basic Solution
	$_{ m int}$	plain text	Integer
Jsol format	jsol	text	Solution

## Compression

MOSEK supports GZIP compression of files. Problem files with an additional .gz extension are assumed to be compressed when read, and are automatically compressed when written. For example, a file called

problem.mps.gz

will be considered as a GZIP compressed MPS file.

## 17.1 The LP File Format

MOSEK supports the LP file format with some extensions. The LP format is not a completely well-defined standard and hence different optimization packages may interpret the same LP file in slightly different ways. MOSEK tries to emulate as closely as possible CPLEX's behavior, but tries to stay backward compatible.

The LP file format can specify problems on the form

$$\begin{array}{lll} \text{minimize/maximize} & & c^Tx + \frac{1}{2}q^o(x) \\ \text{subject to} & l^c & \leq & Ax + \frac{1}{2}q(x) & \leq & u^c, \\ l^x & \leq & x & \leq & u^x, \\ & & & x_{\mathcal{J}} \text{ integer,} \end{array}$$

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear term in the objective.
- $q^o :\in \mathbb{R}^n \to \mathbb{R}$  is the quadratic term in the objective where

$$q^o(x) = x^T Q^o x$$

and it is assumed that

$$Q^o = (Q^o)^T$$
.

- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T$$
.

•  $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer constrained variables.

## 17.1.1 File Sections

An LP formatted file contains a number of sections specifying the objective, constraints, variable bounds, and variable types. The section keywords may be any mix of upper and lower case letters.

## **Objective Function**

The first section beginning with one of the keywords

max
maximum
maximize
min
minimum
minimize

defines the objective sense and the objective function, i.e.

$$c^T x + \frac{1}{2} x^T Q^o x.$$

The objective may be given a name by writing

### myname:

before the expressions. If no name is given, then the objective is named obj.

The objective function contains linear and quadratic terms. The linear terms are written as:

```
4 x1 + x2 - 0.1 x3
```

and so forth. The quadratic terms are written in square brackets ([]) and are either squared or multiplied as in the examples

```
x1^2
```

and

```
x1 * x2
```

There may be zero or more pairs of brackets containing quadratic expressions.

An example of an objective section is

```
minimize
myobj: 4 x1 + x2 - 0.1 x3 + [ x1^2 + 2.1 x1 * x2 ]/2
```

Please note that the quadratic expressions are multiplied with  $\frac{1}{2}$ , so that the above expression means

minimize 
$$4x_1 + x_2 - 0.1 \cdot x_3 + \frac{1}{2}(x_1^2 + 2.1 \cdot x_1 \cdot x_2)$$

If the same variable occurs more than once in the linear part, the coefficients are added, so that  $4 \times 1 + 2 \times 1$  is equivalent to  $6 \times 1$ . In the quadratic expressions  $\times 1 \times 2$  is equivalent to  $\times 2 \times 1$  and, as in the linear part, if the same variables multiplied or squared occur several times their coefficients are added.

## **Constraints**

The second section beginning with one of the keywords

```
subj to
subject to
s.t.
st
```

defines the linear constraint matrix A and the quadratic matrices  $Q^i$ .

A constraint contains a name (optional), expressions adhering to the same rules as in the objective and a bound:

```
subject to con1: x1 + x2 + [ x3^2 ]/2 <= 5.1
```

The bound type (here <=) may be any of <, <=, =, >, >= (< and <= mean the same), and the bound may be any number.

In the standard LP format it is not possible to define more than one bound, but **MOSEK** supports defining ranged constraints by using double-colon (::) instead of a single-colon (:) after the constraint name, i.e.

$$-5 \le x_1 + x_2 \le 5 \tag{17.1}$$

may be written as

```
con:: -5 < x_1 + x_2 < 5
```

By default MOSEK writes ranged constraints this way.

If the files must adhere to the LP standard, ranged constraints must either be split into upper bounded and lower bounded constraints or be written as an equality with a slack variable. For example the expression (17.1) may be written as

$$x_1 + x_2 - sl_1 = 0, -5 \le sl_1 \le 5.$$

### **Bounds**

Bounds on the variables can be specified in the bound section beginning with one of the keywords

```
bound bounds
```

The bounds section is optional but should, if present, follow the subject to section. All variables listed in the bounds section must occur in either the objective or a constraint.

The default lower and upper bounds are 0 and  $+\infty$ . A variable may be declared free with the keyword free, which means that the lower bound is  $-\infty$  and the upper bound is  $+\infty$ . Furthermore it may be assigned a finite lower and upper bound. The bound definitions for a given variable may be written in one or two lines, and bounds can be any number or  $\pm\infty$  (written as  $+\inf/-\inf/+\inf\inf$ ) as in the example

```
bounds

x1 free

x2 <= 5

0.1 <= x2

x3 = 42

2 <= x4 < +inf
```

## Variable Types

The final two sections are optional and must begin with one of the keywords

```
bin
binaries
binary
```

and

```
gen
general
```

Under general all integer variables are listed, and under binary all binary (integer variables with bounds 0 and 1) are listed:

```
general
x1 x2
binary
x3 x4
```

Again, all variables listed in the binary or general sections must occur in either the objective or a constraint.

## **Terminating Section**

Finally, an LP formatted file must be terminated with the keyword

end

## 17.1.2 LP File Examples

## Linear example lo1.lp

```
\ File: lo1.lp
maximize
obj: 3 x1 + x2 + 5 x3 + x4
subject to
c1: 3 x1 + x2 + 2 x3 = 30
c2: 2 x1 + x2 + 3 x3 + x4 >= 15
c3: 2 x2 + 3 x4 <= 25
bounds
0 <= x1 <= +infinity
0 <= x2 <= 10
0 <= x3 <= +infinity
0 <= x4 <= +infinity
end</pre>
```

## Mixed integer example milo1.lp

```
maximize
obj: x1 + 6.4e-01 x2
subject to
c1: 5e+01 x1 + 3.1e+01 x2 <= 2.5e+02
c2: 3e+00 x1 - 2e+00 x2 >= -4e+00
bounds
0 <= x1 <= +infinity
0 <= x2 <= +infinity
general
x1 x2
end
```

# 17.1.3 LP Format peculiarities

### Comments

Anything on a line after a \ is ignored and is treated as a comment.

### **Names**

A name for an objective, a constraint or a variable may contain the letters a-z, A-Z, the digits  $\theta$ - $\theta$  and the characters

```
!"#$%&()/,.;?@_'`|~
```

The first character in a name must not be a number, a period or the letter e or E. Keywords must not be used as names.

**MOSEK** accepts any character as valid for names, except \0. A name that is not allowed in LP file will be changed and a warning will be issued.

The algorithm for making names LP valid works as follows: The name is interpreted as an  $\mathtt{utf-8}$  string. For a unicode character  $\mathtt{c}$ :

- If c==\_ (underscore), the output is \_\_ (two underscores).
- If c is a valid LP name character, the output is just c.
- If c is another character in the ASCII range, the output is \_XX, where XX is the hexadecimal code for the character.
- If c is a character in the range 127-65535, the output is \_uXXXX, where XXXX is the hexadecimal code for the character.
- If c is a character above 65535, the output is \_UXXXXXXXX, where XXXXXXXX is the hexadecimal code for the character.

Invalid  $\mathtt{utf-8}$  substrings are escaped as  $\mathtt{XX'}$ , and if a name starts with a period, e or E, that character is escaped as  $\mathtt{XX}$ .

### Variable Bounds

Specifying several upper or lower bounds on one variable is possible but **MOSEK** uses only the tightest bounds. If a variable is fixed (with =), then it is considered the tightest bound.

### MOSEK Extensions to the LP Format

Some optimization software packages employ a more strict definition of the LP format than the one used by **MOSEK**. The limitations imposed by the strict LP format are the following:

- Quadratic terms in the constraints are not allowed.
- Names can be only 16 characters long.
- Lines must not exceed 255 characters in length.

If an LP formatted file created by MOSEK should satisfy the strict definition, then the parameter

• iparam.write\_lp\_strict\_format

should be set; note, however, that some problems cannot be written correctly as a strict LP formatted file. For instance, all names are truncated to 16 characters and hence they may loose their uniqueness and change the problem.

To get around some of the inconveniences converting from other problem formats,  $\mathbf{MOSEK}$  allows lines to contain 1024 characters and names may have any length (shorter than the 1024 characters).

Internally in MOSEK names may contain any (printable) character, many of which cannot be used in LP names. Setting the parameters

- iparam.read\_lp\_quoted\_names and
- iparam.write\_lp\_quoted\_names

allows MOSEK to use quoted names. The first parameter tells MOSEK to remove quotes from quoted names e.g, "x1", when reading LP formatted files. The second parameter tells MOSEK to put quotes around any semi-illegal name (names beginning with a number or a period) and fully illegal name (containing illegal characters). As double quote is a legal character in the LP format, quoting semi-illegal names makes them legal in the pure LP format as long as they are still shorter than 16 characters. Fully illegal names are still illegal in a pure LP file.

## 17.1.4 The strict LP format

The LP format is not a formal standard and different vendors have slightly different interpretations of the LP format. To make **MOSEK**'s definition of the LP format more compatible with the definitions of other vendors, use the parameter setting

• iparam.write\_lp\_strict\_format = onoffkey.on

This setting may lead to truncation of some names and hence to an invalid LP file. The simple solution to this problem is to use the parameter setting

• iparam.write\_generic\_names = onoffkey.on

which will cause all names to be renamed systematically in the output file.

## 17.1.5 Formatting of an LP File

A few parameters control the visual formatting of LP files written by **MOSEK** in order to make it easier to read the files. These parameters are

- iparam.write\_lp\_line\_width
- iparam.write\_lp\_terms\_per\_line

The first parameter sets the maximum number of characters on a single line. The default value is 80 corresponding roughly to the width of a standard text document.

The second parameter sets the maximum number of terms per line; a term means a sign, a coefficient, and a name (for example + 42 elephants). The default value is 0, meaning that there is no maximum.

### **Unnamed Constraints**

Reading and writing an LP file with **MOSEK** may change it superficially. If an LP file contains unnamed constraints or objective these are given their generic names when the file is read (however unnamed constraints in **MOSEK** are written without names).

## 17.2 The MPS File Format

**MOSEK** supports the standard MPS format with some extensions. For a detailed description of the MPS format see the book by Nazareth [Naz87].

## 17.2.1 MPS File Structure

The version of the MPS format supported by  $\mathbf{MOSEK}$  allows specification of an optimization problem of the form

$$l^{c} \leq Ax + q(x) \leq u^{c},$$

$$l^{x} \leq x \leq u^{x},$$

$$x \in \mathcal{K},$$

$$x_{\mathcal{J}} \text{ integer},$$

$$(17.2)$$

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = \frac{1}{2}x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T.$$

Please note the explicit  $\frac{1}{2}$  in the quadratic term and that  $Q^i$  is required to be symmetric.

- K is a convex cone.
- $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer-constrained variables.

An MPS file with one row and one column can be illustrated like this:

```
*23456789012345678901234567890123456789012345678901234567890
NAME
OBJSENSE
[objsense]
OBJNAME
[objname]
ROWS
? [cname1]
COLUMNS
[vname1]
          [cname1]
                        [value1]
                                      [vname3]
                                                 [value2]
RHS
           [cname1]
                        [value1]
                                      [cname2]
                                                 [value2]
[name]
RANGES
[name]
           [cname1]
                        [value1]
                                      [cname2]
                                                 [value2]
QSECTION
               [cname1]
                                      [vname3]
                                                 [value2]
[vname1]
           [vname2]
                        [value1]
QMATRIX
                        [value1]
[vname1]
           [vname2]
QUADOBJ
           [vname2]
                        [value1]
[vname1]
QCMATRIX
               [cname1]
           [vname2]
[vname1]
                        [value1]
BOUNDS
?? [name]
              [vname1]
                           [value1]
CSECTION
               [kname1]
                            [value1]
                                          [ktype]
[vname1]
ENDATA
```

Here the names in capitals are keywords of the MPS format and names in brackets are custom defined names or values. A couple of notes on the structure:

• Fields: All items surrounded by brackets appear in *fields*. The fields named "valueN" are numerical values. Hence, they must have the format

```
[+|-]XXXXXXX.XXXXXX[[e|E][+|-]XXX]
where
```

```
.. code-block:: text
X = [0|1|2|3|4|5|6|7|8|9].
```

- Sections: The MPS file consists of several sections where the names in capitals indicate the beginning of a new section. For example, COLUMNS denotes the beginning of the columns section.
- Comments: Lines starting with an \* are comment lines and are ignored by MOSEK.
- Keys: The question marks represent keys to be specified later.
- Extensions: The sections QSECTION and CSECTION are specific MOSEK extensions of the MPS format. The sections QMATRIX, QUADOBJ and QCMATRIX are included for sake of compatibility with other vendors extensions to the MPS format.

The standard MPS format is a fixed format, i.e. everything in the MPS file must be within certain fixed positions. **MOSEK** also supports a *free format*. See Sec. 17.2.9 for details.

### Linear example lo1.mps

A concrete example of a MPS file is presented below:

```
* File: lo1.mps
NAME
               101
OBJSENSE
    MAX
ROWS
 N obj
 E c1
 G c2
 L c3
COLUMNS
                          3
    x1
               obj
                          3
    x1
               c1
               c2
                          2
    x1
               obj
    x2
                          1
    x2
               c1
                          1
    x2
               c2
                          1
    x2
               сЗ
                          2
    xЗ
               obj
                          5
    xЗ
               c1
                          2
    хЗ
               c2
                          3
    x4
               obj
                          1
    x4
               c2
                          1
    x4
               сЗ
                          3
RHS
                          30
    rhs
               c1
               c2
                          15
    rhs
               сЗ
                          25
    rhs
RANGES
BOUNDS
UP bound
               x2
                          10
ENDATA
```

Subsequently each individual section in the MPS format is discussed.

## Section NAME

In this section a name ([name]) is assigned to the problem.

## OBJSENSE (optional)

This is an optional section that can be used to specify the sense of the objective function. The OBJSENSE section contains one line at most which can be one of the following

MIN
MINIMIZE
MAX
MAXIMIZE

It should be obvious what the implication is of each of these four lines.

## OBJNAME (optional)

This is an optional section that can be used to specify the name of the row that is used as objective function. The OBJNAME section contains one line at most which has the form

objname

objname should be a valid row name.

### ROWS

A record in the ROWS section has the form

# ? [cname1]

where the requirements for the fields are as follows:

Field	Starting Position	Max Width	required	Description
?	2	1	Yes	Constraint key
[cname1]	5	8	Yes	Constraint name

Hence, in this section each constraint is assigned an unique name denoted by [cname1]. Please note that [cname1] starts in position 5 and the field can be at most 8 characters wide. An initial key ? must be present to specify the type of the constraint. The key can have the values E, G, L, or N with the following interpretation:

Constraint type	$l_i^c$	$u_i^c$
E	finite	$l_i^c$
G	finite	$\infty$
L	$-\infty$	finite
N	$-\infty$	$\infty$

In the MPS format an objective vector is not specified explicitly, but one of the constraints having the key N will be used as the objective vector c. In general, if multiple N type constraints are specified, then the first will be used as the objective vector c.

### COLUMNS

In this section the elements of A are specified using one or more records having the form:

[vname1]	[cname1]	[value1]	[cname2]	[value2]
----------	----------	----------	----------	----------

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

Hence, a record specifies one or two elements  $a_{ij}$  of A using the principle that [vname1] and [cname1] determines j and i respectively. Please note that [cname1] must be a constraint name specified in the ROWS section. Finally, [value1] denotes the numerical value of  $a_{ij}$ . Another optional element is specified by [cname2], and [value2] for the variable specified by [vname1]. Some important comments are:

- All elements belonging to one variable must be grouped together.
- Zero elements of A should not be specified.
- At least one element for each variable should be specified.

### RHS (optional)

A record in this section has the format

|--|--|--|

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RHS vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The interpretation of a record is that [name] is the name of the RHS vector to be specified. In general, several vectors can be specified. [cname1] denotes a constraint name previously specified in the ROWS section. Now, assume that this name has been assigned to the i th constraint and  $v_1$  denotes the value specified by [value1], then the interpretation of  $v_1$  is:

Constraint	$l_i^c$	$u_i^c$
type		
E	$v_1$	$v_1$
G	$v_1$	
L		$v_1$
N		

An optional second element is specified by [cname2] and [value2] and is interpreted in the same way. Please note that it is not necessary to specify zero elements, because elements are assumed to be zero.

## RANGES (optional)

A record in this section has the form

value2]	[value1] [cname2]	[cname1]	[name]
---------	-------------------	----------	--------

where the requirements for each fields are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RANGE vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The records in this section are used to modify the bound vectors for the constraints, i.e. the values in  $l^c$  and  $u^c$ . A record has the following interpretation: [name] is the name of the RANGE vector and [cname1] is a valid constraint name. Assume that [cname1] is assigned to the i th constraint and let  $v_1$  be the value specified by [value1], then a record has the interpretation:

Constraint type	Sign of $v_1$	$l_i^c$	$u_i^c$
E	_	$u_i^c + v_1$	
E	+		$l_i^c + v_1$
G	- or +	$l_i^c +  v_1 $	
L	- or +	$u_i^c -  v_1 $	
N			

## QSECTION (optional)

Within the QSECTION the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

lue2]	] [	vname3]	[	[value1]		[vname2]	vname1]
-------	-----	---------	---	----------	--	----------	---------

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value
[vname3]	40	8	No	Variable name
[value2]	50	12	No	Numerical value

A record specifies one or two elements in the lower triangular part of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k th and j th variable, then  $Q^i_{kj}$  is assigned the value given by [value1] An optional second element is specified in the same way by the fields [vname1], [vname3], and [value2].

The example

$$\begin{array}{ll} \text{minimize} & -x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & x \geq 0 & \end{array}$$

has the following MPS file representation

```
* File: qo1.mps
NAME qo1
ROWS
N obj
G c1
COLUMNS
```

x1	c1	
x2	obj	
x2	c1	
x3	c1	
RHS		
rhs	c1	
QSECTION		0
x1	x1	
x1	xЗ	
x2	x2	
х3	xЗ	
ENDATA		

Regarding the QSECTIONs please note that:

- Only one QSECTION is allowed for each constraint.
- The QSECTIONs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- ullet All entries specified in a QSECTION are assumed to belong to the lower triangular part of the quadratic term of Q.

### QMATRIX/QUADOBJ (optional)

The QMATRIX and QUADOBJ sections allow to define the quadratic term of the objective function. They differ in how the quadratic term of the objective function is stored:

- $\bullet$  QMATRIX It stores all the nonzeros coefficients, without taking advantage of the symmetry of the Q matrix.
- ullet QUADOBJ It only store the upper diagonal nonzero elements of the Q matrix.

A record in both sections has the form:

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies one elements of the Q matrix in the objective function. Hence, if the names [vname1] and [vname2] have been assigned to the k th and j th variable, then  $Q_{kj}$  is assigned the value given by [value1]. Note that a line must apper for each off-diagonal coefficient if using a QMATRIX section, while only one entry is required in a QUADOBJ section. The quadratic part of the objective function will be evaluated as  $1/2x^TQx$ .

The example

minimize 
$$\begin{array}{ccc} -x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & x > 0 & \end{array}$$

has the following MPS file representation using QMATRIX

```
* File: qo1_matrix.mps
NAME qo1_qmatrix
ROWS
```

```
N obj
 G c1
COLUMNS
    x1
              c1
                         1.0
                         -1.0
    x2
              obj
                         1.0
    x2
              c1
    xЗ
                         1.0
              c1
RHS
              c1
                         1.0
    rhs
QMATRIX
                         2.0
    x1
              x1
                         -1.0
    x1
              хЗ
    xЗ
              x1
                         -1.0
    x2
              x2
                         0.2
    xЗ
              хЗ
                         2.0
ENDATA
```

or the following using QUADOBJ

```
* File: qo1_quadobj.mps
NAME
              qo1_quadobj
ROWS
N obj
G c1
COLUMNS
    x1
              c1
                         1.0
    x2
              obj
                        -1.0
    x2
              c1
                         1.0
    xЗ
              c1
                         1.0
RHS
                         1.0
   rhs
              c1
QUADOBJ
                         2.0
              x1
    x1
              хЗ
                         -1.0
    x1
              x2
                         0.2
    x2
              хЗ
                         2.0
    xЗ
ENDATA
```

Please also note that:

- ullet A QMATRIX/QUADOBJ section can appear in an arbitrary order after the COLUMNS section.
- ullet All variable names occurring in the QMATRIX/QUADOBJ section must already be specified in the COLUMNS section.

## 17.2.2 QCMATRIX (optional)

A QCMATRIX section allows to specify the quadratic part of a given constraints. Within the QCMATRIX the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

|--|--|

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies an entry of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k th and j th variable, then  $Q^i_{kj}$  is assigned the value given by [value1]. Moreover, the quadratic term is represented as  $1/2x^TQx$ .

The example

minimize 
$$x_2$$
 subject to  $x_1 + x_2 + x_3 \ge 1$ ,  $\frac{1}{2}(-2x_1x_3 + 0.2x_2^2 + 2x_3^2) \le 10$ ,  $x \ge 0$ 

has the following MPS file representation

```
* File: qo1.mps
NAME
ROWS
N obj
G c1
L q1
COLUMNS
                         1.0
    x1
              c1
              obj
    x2
                         -1.0
                         1.0
    x2
              c1
                         1.0
RHS
                         1.0
    rhs
              c1
    rhs
              q1
                         10.0
QCMATRIX
              q1
                         2.0
    x1
              x1
                         -1.0
    x1
              xЗ
    xЗ
              x1
                         -1.0
    x2
              x2
                         0.2
    хЗ
              xЗ
                         2.0
ENDATA
```

Regarding the QCMATRIXs please note that:

- Only one QCMATRIX is allowed for each constraint.
- The QCMATRIXs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- A QCMATRIX does not exploit the symmetry of Q: an off-diagonal entry (i,j) should appear twice.

## 17.2.3 BOUNDS (optional)

In the BOUNDS section changes to the default bounds vectors  $l^x$  and  $u^x$  are specified. The default bounds vectors are  $l^x=0$  and  $u^x=\infty$ . Moreover, it is possible to specify several sets of bound vectors. A record in this section has the form

where the requirements for each field are:

Field	Starting Position	Max Width	Required	Description
??	2	2	Yes	Bound key
[name]	5	8	Yes	Name of the BOUNDS vector
[vname1]	15	8	Yes	Variable name
[value1]	25	12	No	Numerical value

Hence, a record in the BOUNDS section has the following interpretation: [name] is the name of the bound vector and [vname1] is the name of the variable which bounds are modified by the record. ?? and [value1] are used to modify the bound vectors according to the following table:

??	$l_j^x$	$u_j^x$	Made integer (added to ${\mathcal J}$ )
FR	$-\infty$	$\infty$	No
FX	$v_1$	$v_1$	No
LO	$v_1$	unchanged	No
MI	$-\infty$	unchanged	No
PL	unchanged	$\infty$	No
UP	unchanged	$v_1$	No
BV	0	1	Yes
LI	$\lceil v_1 \rceil$	unchanged	Yes
UI	unchanged	$\lfloor v_1 \rfloor$	Yes

 $v_1$  is the value specified by [value1].

# 17.2.4 CSECTION (optional)

The purpose of the CSECTION is to specify the constraint

$$x \in \mathcal{K}$$
.

in (17.2). It is assumed that K satisfies the following requirements. Let

$$x^t \in \mathbb{R}^{n^t}, \quad t = 1, \dots, k$$

be vectors comprised of parts of the decision variables x so that each decision variable is a member of exactly **one** vector  $x^t$ , for example

$$x^1 = \begin{bmatrix} x_1 \\ x_4 \\ x_7 \end{bmatrix}$$
 and  $x^2 = \begin{bmatrix} x_6 \\ x_5 \\ x_3 \\ x_2 \end{bmatrix}$ .

Next define

$$\mathcal{K} := \left\{ x \in \mathbb{R}^n : \quad x^t \in \mathcal{K}_t, \quad t = 1, \dots, k \right\}$$

where  $\mathcal{K}_t$  must have one of the following forms

• R set:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} \right\}.$$

• Quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : x_1 \ge \sqrt{\sum_{j=2}^{n^t} x_j^2} \right\}. \tag{17.3}$$

• Rotated quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : 2x_1 x_2 \ge \sum_{j=3}^{n^t} x_j^2, \quad x_1, x_2 \ge 0 \right\}.$$
 (17.4)

In general, only quadratic and rotated quadratic cones are specified in the MPS file whereas membership of the  $\mathbb{R}$  set is not. If a variable is not a member of any other cone then it is assumed to be a member of an  $\mathbb{R}$  cone.

Next, let us study an example. Assume that the quadratic cone

$$x_4 \ge \sqrt{x_5^2 + x_8^2}$$

and the rotated quadratic cone

$$x_3x_7 \ge x_1^2 + x_0^2, \quad x_3, x_7 \ge 0,$$

should be specified in the MPS file. One CSECTION is required for each cone and they are specified as follows:

*	1	2	3	4	5	6
*2345678	90123456	7890123	45678901234	567890123456	78901234	567890
CSECTION	ko	nea	0.0	QUAD		
x4						
x5						
x8						
CSECTION	ko	neb	0.0	RQUAD		
x7						
x3						
x1						
x0						

This first CSECTION specifies the cone (17.3) which is given the name konea. This is a quadratic cone which is specified by the keyword QUAD in the CSECTION header. The 0.0 value in the CSECTION header is not used by the QUAD cone.

The second CSECTION specifies the rotated quadratic cone (17.4). Please note the keyword RQUAD in the CSECTION which is used to specify that the cone is a rotated quadratic cone instead of a quadratic cone. The 0.0 value in the CSECTION header is not used by the RQUAD cone.

In general, a CSECTION header has the format

|--|--|

where the requirement for each field are as follows:

Field	Starting Position	Max Width	Required	Description	
[kname1]	5	8	Yes	Name of the cone	
[value1]	15	12	No	Cone parameter	
[ktype]	25		Yes	Type of the cone.	

The possible cone type keys are:

Cone type key	Members	Interpretation.
QUAD	$\leq 1$	Quadratic cone i.e. (17.3).
RQUAD	$\leq 2$	Rotated quadratic cone i.e. (17.4).

Please note that a quadratic cone must have at least one member whereas a rotated quadratic cone must have at least two members. A record in the CSECTION has the format

[vname1]	
----------	--

where the requirements for each field are

Field	Starting Position	Max Width	required	Description
[vname1]	2	8	Yes	A valid variable name

The most important restriction with respect to the CSECTION is that a variable must occur in only one CSECTION.

#### 17.2.5 ENDATA

This keyword denotes the end of the MPS file.

# 17.2.6 Integer Variables

Using special bound keys in the BOUNDS section it is possible to specify that some or all of the variables should be integer-constrained i.e. be members of  $\mathcal{J}$ . However, an alternative method is available.

This method is available only for backward compatibility and we recommend that it is not used. This method requires that markers are placed in the COLUMNS section as in the example:

```
COLUMNS
x1
           obj
                      -10.0
                                       c1
                                                   0.7
x1
           c2
                      0.5
                                       с3
                                                   1.0
x1
           c4
                      0.1
* Start of integer-constrained variables.
MARK000
                                       'INTORG'
           'MARKER'
                                                   1.0
                      -9.0
x2
           obj
                                       c1
                                                   0.6666667
                      0.8333333333
x2
           c2
                                       с3
x2
                      0.25
           c4
x3
                      1.0
                                       с6
                                                   2.0
           obj
MARKO01
           'MARKER'
                                       'INTEND'
```

• End of integer-constrained variables.

Please note that special marker lines are used to indicate the start and the end of the integer variables. Furthermore be aware of the following

- IMPORTANT: All variables between the markers are assigned a default lower bound of 0 and a default upper bound of 1. **This may not be what is intended.** If it is not intended, the correct bounds should be defined in the BOUNDS section of the MPS formatted file.
- MOSEK ignores field 1, i.e. MARKO001 and MARKO01, however, other optimization systems require them
- Field 2, i.e. MARKER, must be specified including the single quotes. This implies that no row can be assigned the name MARKER.
- Field 3 is ignored and should be left blank.
- Field 4, i.e. INTORG and INTEND, must be specified.
- It is possible to specify several such integer marker sections within the COLUMNS section.

### 17.2.7 General Limitations

• An MPS file should be an ASCII file.

# 17.2.8 Interpretation of the MPS Format

Several issues related to the MPS format are not well-defined by the industry standard. However, **MOSEK** uses the following interpretation:

• If a matrix element in the COLUMNS section is specified multiple times, then the multiple entries are added together.

• If a matrix element in a QSECTION section is specified multiple times, then the multiple entries are added together.

# 17.2.9 The Free MPS Format

**MOSEK** supports a free format variation of the MPS format. The free format is similar to the MPS file format but less restrictive, e.g. it allows longer names. However, it also presents two main limitations:

- A name must not contain any blanks.
- By default a line in the MPS file must not contain more than 1024 characters. However, by modifying the parameter <code>iparam.read\_mps\_width</code> an arbitrary large line width will be accepted.

To use the free MPS format instead of the default MPS format the **MOSEK** parameter *iparam*.  $read\_mps\_format$  should be changed.

# 17.3 The OPF Format

The Optimization Problem Format (OPF) is an alternative to LP and MPS files for specifying optimization problems. It is row-oriented, inspired by the CPLEX LP format.

Apart from containing objective, constraints, bounds etc. it may contain complete or partial solutions, comments and extra information relevant for solving the problem. It is designed to be easily read and modified by hand and to be forward compatible with possible future extensions.

#### Intended use

The OPF file format is meant to replace several other files:

- The LP file format: Any problem that can be written as an LP file can be written as an OPF file too; furthermore it naturally accommodates ranged constraints and variables as well as arbitrary characters in names, fixed expressions in the objective, empty constraints, and conic constraints.
- Parameter files: It is possible to specify integer, double and string parameters along with the problem (or in a separate OPF file).
- Solution files: It is possible to store a full or a partial solution in an OPF file and later reload it.

### 17.3.1 The File Format

The format uses tags to structure data. A simple example with the basic sections may look like this:

```
[comment]
This is a comment. You may write almost anything here...
[/comment]
# This is a single-line comment.

[objective min 'myobj']
x + 3 y + x^2 + 3 y^2 + z + 1
[/objective]

[constraints]
[con 'con01'] 4 <= x + y [/con]
[/constraints]

[bounds]
[b] -10 <= x,y <= 10 [/b]</pre>
```

```
[cone quad] x,y,z [/cone] [/bounds]
```

A scope is opened by a tag of the form [tag] and closed by a tag of the form [/tag]. An opening tag may accept a list of unnamed and named arguments, for examples:

```
[tag value] tag with one unnamed argument [/tag]
[tag arg=value] tag with one named argument in quotes [/tag]
```

Unnamed arguments are identified by their order, while named arguments may appear in any order, but never before an unnamed argument. The value can be a quoted, single-quoted or double-quoted text string, i.e.

```
[tag 'value'] single-quoted value [/tag]
[tag arg='value'] single-quoted value [/tag]
[tag "value"] double-quoted value [/tag]
[tag arg="value"] double-quoted value [/tag]
```

#### **Sections**

The recognized tags are

#### [comment]

A comment section. This can contain *almost* any text: Between single quotes (') or double quotes (") any text may appear. Outside quotes the markup characters ([ and ]) must be prefixed by backslashes. Both single and double quotes may appear alone or inside a pair of quotes if it is prefixed by a backslash.

### [objective]

The objective function: This accepts one or two parameters, where the first one (in the above example min) is either min or max (regardless of case) and defines the objective sense, and the second one (above myobj), if present, is the objective name. The section may contain linear and quadratic expressions. If several objectives are specified, all but the last are ignored.

### [constraints]

This does not directly contain any data, but may contain the subsection con defining a linear constraint.

[con] defines a single constraint; if an argument is present ([con NAME]) this is used as the name of the constraint, otherwise it is given a null-name. The section contains a constraint definition written as linear and quadratic expressions with a lower bound, an upper bound, with both or with an equality. Examples:

Constraint names are unique. If a constraint is specified which has the same name as a previously defined constraint, the new constraint replaces the existing one.

#### [bounds]

This does not directly contain any data, but may contain the subsections b (linear bounds on variables) and cone (quadratic cone).

[b]. Bound definition on one or several variables separated by comma (,). An upper or lower bound on a variable replaces any earlier defined bound on that variable. If only one bound (upper or lower) is given only this bound is replaced. This means that upper and lower bounds can be specified separately. So the OPF bound definition:

```
[b] x,y \ge -10 [/b]
[b] x,y \le 10 [/b]
```

results in the bound  $-10 \le x, y \le 10$ .

[cone]. currently supports the quadratic cone and the rotated quadratic cone.

A conic constraint is defined as a set of variables which belong to a single unique cone.

• A quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^2 \ge \sum_{i=2}^n x_i^2, \quad x_1 \ge 0.$$

• A rotated quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$2x_1x_2 \ge \sum_{i=3}^n x_i^2, \quad x_1, x_2 \ge 0.$$

A [bounds]-section example:

By default all variables are free.

### [variables]

This defines an ordering of variables as they should appear in the problem. This is simply a space-separated list of variable names. Optionally, an attribute can be added [variables disallow\_new\_variables] indicating that if any variable not listed here occurs later in the file it is an error.

### [integer]

This contains a space-separated list of variables and defines the constraint that the listed variables must be integer values.

#### [hints]

This may contain only non-essential data; for example estimates of the number of variables, constraints and non-zeros. Placed before all other sections containing data this may reduce the time spent reading the file.

In the hints section, any subsection which is not recognized by MOSEK is simply ignored. In this section a hint in a subsection is defined as follows:

```
[hint ITEM] value [/hint]
```

where ITEM may be replaced by numvar (number of variables), numcon (number of linear/quadratic constraints), numanz (number of linear non-zeros in constraints) and numqnz (number of quadratic non-zeros in constraints).

#### [solutions]

This section can contain a set of full or partial solutions to a problem. Each solution must be specified using a [solution]-section, i.e.

```
[solutions]
[solution]...[/solution] #solution 1
[solution]...[/solution] #solution 2
#other solutions....
[solution]...[/solution] #solution n
[/solutions]
```

Note that a [solution]-section must be always specified inside a [solutions]-section. The syntax of a [solution]-section is the following:

```
[solution SOLTYPE status=STATUS]...[/solution]
```

where SOLTYPE is one of the strings

- interior, a non-basic solution,
- basic, a basic solution,
- integer, an integer solution,

and STATUS is one of the strings

- UNKNOWN,
- OPTIMAL,
- INTEGER\_OPTIMAL,
- PRIM\_FEAS,
- DUAL\_FEAS,
- PRIM\_AND\_DUAL\_FEAS,
- NEAR\_OPTIMAL,
- NEAR\_PRIM\_FEAS,
- NEAR\_DUAL\_FEAS,
- NEAR\_PRIM\_AND\_DUAL\_FEAS,
- PRIM\_INFEAS\_CER,
- DUAL\_INFEAS\_CER,
- NEAR\_PRIM\_INFEAS\_CER,

- NEAR\_DUAL\_INFEAS\_CER,
- NEAR\_INTEGER\_OPTIMAL.

Most of these values are irrelevant for input solutions; when constructing a solution for simplex hot-start or an initial solution for a mixed integer problem the safe setting is UNKNOWN.

A [solution]-section contains [con] and [var] sections. Each [con] and [var] section defines solution information for a single variable or constraint, specified as list of KEYWORD/value pairs, in any order, written as

#### KEYWORD=value

Allowed keywords are as follows:

- sk. The status of the item, where the value is one of the following strings:
  - LOW, the item is on its lower bound.
  - UPR, the item is on its upper bound.
  - FIX, it is a fixed item.
  - BAS, the item is in the basis.
  - SUPBAS, the item is super basic.
  - UNK, the status is unknown.
  - INF, the item is outside its bounds (infeasible).
- 1vl Defines the level of the item.
- s1 Defines the level of the dual variable associated with its lower bound.
- su Defines the level of the dual variable associated with its upper bound.
- sn Defines the level of the variable associated with its cone.
- y Defines the level of the corresponding dual variable (for constraints only).

A [var] section should always contain the items sk, lvl, sl and su. Items sl and su are not required for integer solutions.

A [con] section should always contain sk, lvl, sl, su and y.

An example of a solution section

• [vendor] This contains solver/vendor specific data. It accepts one argument, which is a vendor ID – for MOSEK the ID is simply mosek – and the section contains the subsection parameters defining solver parameters. When reading a vendor section, any unknown vendor can be safely ignored. This is described later.

Comments using the # may appear anywhere in the file. Between the # and the following line-break any text may be written, including markup characters.

#### **Numbers**

Numbers, when used for parameter values or coefficients, are written in the usual way by the printf function. That is, they may be prefixed by a sign (+ or -) and may contain an integer part, decimal part and an exponent. The decimal point is always . (a dot). Some examples are

```
1
1.0
.0
.0
1.
1e10
1e+10
1e-10
```

Some invalid examples are

```
e10 # invalid, must contain either integer or decimal part
. # invalid
.e10 # invalid
```

More formally, the following standard regular expression describes numbers as used:

```
[+|-]?([0-9]+[.][0-9]*|[.][0-9]+)([eE][+|-]?[0-9]+)?
```

#### **Names**

Variable names, constraint names and objective name may contain arbitrary characters, which in some cases must be enclosed by quotes (single or double) that in turn must be preceded by a backslash. Unquoted names must begin with a letter (a-z or A-Z) and contain only the following characters: the letters a-z and A-Z, the digits 0-9, braces ({ and }) and underscore (\_).

Some examples of legal names:

```
an_unquoted_name
another_name{123}
'single quoted name'
"double quoted name"
"name with \\"quote\\" in it"
"name with []s in it"
```

### 17.3.2 Parameters Section

In the vendor section solver parameters are defined inside the parameters subsection. Each parameter is written as

```
[p PARAMETER_NAME] value [/p]
```

where PARAMETER\_NAME is replaced by a MOSEK parameter name, usually of the form MSK\_IPAR\_..., MSK\_DPAR\_... or MSK\_SPAR\_..., and the value is replaced by the value of that parameter; both integer values and named values may be used. Some simple examples are

# 17.3.3 Writing OPF Files from MOSEK

To write an OPF file set the parameter <code>iparam.write\_data\_format</code> to <code>dataformat.op</code> as this ensures that OPF format is used.

Then modify the following parameters to define what the file should contain:

$iparam.opf\_write\_sol\_bas$	Include basic solution, if defined.
$iparam.opf\_write\_sol\_itg$	Include integer solution, if defined.
$iparam.opf\_write\_sol\_itr$	Include interior solution, if defined.
iparam.	Include solutions if they are defined. If this is off, no solutions are
$opf\_write\_solutions$	included.
$iparam.opf\_write\_header$	Include a small header with comments.
$iparam.opf\_write\_problem$	Include the problem itself — objective, constraints and bounds.
iparam.	Include all parameter settings.
$opf\_write\_parameters$	
$iparam.opf\_write\_hints$	Include hints about the size of the problem.

# 17.3.4 Examples

This section contains a set of small examples written in OPF and describing how to formulate linear, quadratic and conic problems.

### Linear Example 101.opf

Consider the example:

having the bounds

$$\begin{array}{ccccccc}
0 & \leq & x_0 & \leq & \infty, \\
0 & \leq & x_1 & \leq & 10, \\
0 & \leq & x_2 & \leq & \infty, \\
0 & \leq & x_3 & \leq & \infty.
\end{array}$$

In the OPF format the example is displayed as shown in Listing 17.1.

Listing 17.1: Example of an OPF file for a linear problem.

```
[comment]
  The lo1 example in OPF format
[/comment]

[hints]
  [hint NUMVAR] 4 [/hint]
  [hint NUMCON] 3 [/hint]
  [hint NUMANZ] 9 [/hint]
[/hints]

[variables disallow_new_variables]
  x1 x2 x3 x4
[/variables]

[objective maximize 'obj']
  3 x1 + x2 + 5 x3 + x4
```

```
[/objective]

[constraints]
[con 'c1'] 3 x1 + x2 + 2 x3 = 30 [/con]
[con 'c2'] 2 x1 + x2 + 3 x3 + x4 >= 15 [/con]
[con 'c3'] 2 x2 + 3 x4 <= 25 [/con]
[/constraints]

[bounds]
[b] 0 <= * [/b]
[b] 0 <= x2 <= 10 [/b]
[/bounds]
```

### Quadratic Example qo1.opf

An example of a quadratic optimization problem is

minimize 
$$x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2$$
  
subject to  $1 \le x_1 + x_2 + x_3, x \ge 0.$ 

This can be formulated in opf as shown below.

Listing 17.2: Example of an OPF file for a quadratic problem.

```
[comment]
 The qo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 3 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
  [hint NUMQNZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3
[/variables]
[objective minimize 'obj']
 # The quadratic terms are often written with a factor of 1/2 as here,
 # but this is not required.
  - x2 + 0.5 ( 2.0 x1 ^ 2 - 2.0 x3 * x1 + 0.2 x2 ^ 2 + 2.0 x3 ^ 2 )
[/objective]
[constraints]
 [con 'c1'] 1.0 \le x1 + x2 + x3 [/con]
[/constraints]
[bounds]
 [b] 0 <= * [/b]
[/bounds]
```

### Conic Quadratic Example cqo1.opf

Consider the example:

$$\begin{array}{lll} \text{minimize} & x_3 + x_4 + x_5 \\ \text{subject to} & x_0 + x_1 + 2x_2 & = & 1, \\ & x_0, x_1, x_2 & \geq & 0, \\ & x_3 \geq \sqrt{x_0^2 + x_1^2}, \\ & 2x_4x_5 \geq x_2^2. \end{array}$$

Please note that the type of the cones is defined by the parameter to [cone ...]; the content of the cone-section is the names of variables that belong to the cone. The resulting OPF file is in Listing 17.3.

Listing 17.3: Example of an OPF file for a conic quadratic problem.

```
[comment]
 The cqo1 example in OPF format.
[/comment]
[hints]
  [hint NUMVAR] 6 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4 x5 x6
[/variables]
[objective minimize 'obj']
  x4 + x5 + x6
[/objective]
[constraints]
  [con 'c1'] x1 + x2 + 2e+00 x3 = 1e+00 [/con]
[/constraints]
[bounds]
  # We let all variables default to the positive orthant
  [b] 0 \ll * [/b]
  # ...and change those that differ from the default
  [b] x4,x5,x6 free [/b]
  # Define quadratic cone: x4 \ge sqrt(x1^2 + x2^2)
  [cone quad 'k1'] x4, x1, x2 [/cone]
  # Define rotated quadratic cone: 2 x5 x6 >= x3^2
  [cone rquad 'k2'] x5, x6, x3 [/cone]
[/bounds]
```

### Mixed Integer Example milo1.opf

Consider the mixed integer problem:

This can be implemented in OPF with the file in Listing 17.4.

Listing 17.4: Example of an OPF file for a mixed-integer linear problem.

```
[comment]
 The milo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 2 [/hint]
  [hint NUMCON] 2 [/hint]
  [hint NUMANZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2
[/variables]
[objective maximize 'obj']
  x1 + 6.4e-1 x2
[/objective]
[constraints]
  [con 'c1'] 5e+1 x1 + 3.1e+1 x2 \le 2.5e+2 [/con]
  [con 'c2'] -4 \le 3 x1 - 2 x2 [/con]
[/constraints]
[bounds]
  [b] 0 \ll * [/b]
[/bounds]
[integer]
 x1 x2
[/integer]
```

# 17.4 The CBF Format

This document constitutes the technical reference manual of the *Conic Benchmark Format* with file extension: .cbf or .CBF. It unifies linear, second-order cone (also known as conic quadratic) and semidefinite optimization with mixed-integer variables. The format has been designed with benchmark libraries in mind, and therefore focuses on compact and easily parsable representations. The problem structure is separated from the problem data, and the format moreover facilitates benchmarking of hotstart capability through sequences of changes.

#### 17.4.1 How Instances Are Specified

This section defines the spectrum of conic optimization problems that can be formulated in terms of the keywords of the CBF format.

In the CBF format, conic optimization problems are considered in the following form:

min / max 
$$g^{obj}$$
  
 $g_i \in \mathcal{K}_i, \quad i \in \mathcal{I},$   
s.t.  $G_i \in \mathcal{K}_i, \quad i \in \mathcal{I}^{PSD},$   
 $x_j \in \mathcal{K}_j, \quad j \in \mathcal{J},$   
 $\overline{X}_j \in \mathcal{K}_j, \quad j \in \mathcal{J}^{PSD}.$  (17.5)

• Variables are either scalar variables,  $x_j$  for  $j \in \mathcal{J}$ , or variables,  $\overline{X}_j$  for  $j \in \mathcal{J}^{PSD}$ . Scalar variables can also be declared as integer.

• Constraints are affine expressions of the variables, either scalar-valued  $g_i$  for  $i \in \mathcal{I}$ , or matrix-valued  $G_i$  for  $i \in \mathcal{I}^{PSD}$ 

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$
$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i.$$

• The **objective function** is a scalar-valued affine expression of the variables, either to be minimized or maximized. We refer to this expression as  $g^{obj}$ 

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj}.$$

CBF format can represent the following cones  $\mathcal{K}$ :

• Free domain - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n\}$$
, for  $n \ge 1$ .

• Positive orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_j \ge 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \ge 1.$$

• Negative orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i \leq 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \geq 1.$$

• Fixpoint zero - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i = 0 \text{ for } j = 1, \dots, n\}, \text{ for } n > 1.$$

• Quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \left( \begin{array}{c} p \\ x \end{array} \right) \in \mathbb{R} \times \mathbb{R}^{n-1}, \ p^2 \ge x^T x, \ p \ge 0 \right\}, \ \text{for } n \ge 2.$$

• Rotated quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \begin{pmatrix} p \\ q \\ x \end{pmatrix} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{n-2}, \ 2pq \ge x^T x, \ p \ge 0, \ q \ge 0 \right\}, \text{ for } n \ge 3.$$

# 17.4.2 The Structure of CBF Files

This section defines how information is written in the CBF format, without being specific about the type of information being communicated.

All information items belong to exactly one of the three groups of information. These information groups, and the order they must appear in, are:

17.4. The CBF Format

- 1. File format.
- 2. Problem structure.
- 3. Problem data.

The first group, file format, provides information on how to interpret the file. The second group, problem structure, provides the information needed to deduce the type and size of the problem instance. Finally, the third group, problem data, specifies the coefficients and constants of the problem instance.

#### Information items

The format is composed as a list of information items. The first line of an information item is the KEYWORD, revealing the type of information provided. The second line - of some keywords only - is the HEADER, typically revealing the size of information that follows. The remaining lines are the BODY holding the actual information to be specified.

```
KEYWORD
BODY
KEYWORD
HEADER
BODY
```

The KEYWORD determines how each line in the HEADER and BODY is structured. Moreover, the number of lines in the BODY follows either from the KEYWORD, the HEADER, or from another information item required to precede it.

#### **Embedded hotstart-sequences**

A sequence of problem instances, based on the same problem structure, is within a single file. This is facilitated via the CHANGE within the problem data information group, as a separator between the information items of each instance. The information items following a CHANGE keyword are appending to, or changing (e.g., setting coefficients back to their default value of zero), the problem data of the preceding instance.

The sequence is intended for benchmarking of hotstart capability, where the solvers can reuse their internal state and solution (subject to the achieved accuracy) as warmpoint for the succeeding instance. Whenever this feature is unsupported or undesired, the keyword CHANGE should be interpreted as the end of file.

#### File encoding and line width restrictions

The format is based on the US-ASCII printable character set with two extensions as listed below. Note, by definition, that none of these extensions can be misinterpreted as printable US-ASCII characters:

- A line feed marks the end of a line, carriage returns are ignored.
- Comment-lines may contain unicode characters in UTF-8 encoding.

The line width is restricted to 512 bytes, with 3 bytes reserved for the potential carriage return, line feed and null-terminator.

Integers and floating point numbers must follow the ISO C decimal string representation in the standard C locale. The format does not impose restrictions on the magnitude of, or number of significant digits in numeric data, but the use of 64-bit integers and 64-bit IEEE 754 floating point numbers should be sufficient to avoid loss of precision.

### Comment-line and whitespace rules

The format allows single-line comments respecting the following rule:

• Lines having first byte equal to '#' (US-ASCII 35) are comments, and should be ignored. Comments are only allowed between information items.

Given that a line is not a comment-line, whitespace characters should be handled according to the following rules:

- Leading and trailing whitespace characters should be ignored.
  - The seperator between multiple pieces of information on one line, is either one or more whitespace characters.
- Lines containing only whitespace characters are empty, and should be ignored. Empty lines are only allowed between information items.

# 17.4.3 Problem Specification

#### The problem structure

The problem structure defines the objective sense, whether it is minimization and maximization. It also defines the index sets,  $\mathcal{J}$ ,  $\mathcal{J}^{PSD}$ ,  $\mathcal{I}$  and  $\mathcal{I}^{PSD}$ , which are all numbered from zero,  $\{0, 1, \ldots\}$ , and empty until explicitly constructed.

• Scalar variables are constructed in vectors restricted to a conic domain, such as  $(x_0, x_1) \in \mathbb{R}^2_+$ ,  $(x_2, x_3, x_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$x \in \mathcal{K}_1^{n_1} \times \mathcal{K}_2^{n_2} \times \dots \times \mathcal{K}_k^{n_k}$$

which in the CBF format becomes:

```
VAR
n k
K1 n1
K2 n2
...
Kk nk
```

where  $\sum_{i} n_{i} = n$  is the total number of scalar variables. The list of supported cones is found in Table 17.3. Integrality of scalar variables can be specified afterwards.

• **PSD variables** are constructed one-by-one. That is,  $X_j \succeq \mathbf{0}^{n_j \times n_j}$  for  $j \in \mathcal{J}^{PSD}$ , constructs a matrix-valued variable of size  $n_j \times n_j$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes:

```
PSDVAR
N
n1
n2
...
nN
```

where N is the total number of PSD variables.

• Scalar constraints are constructed in vectors restricted to a conic domain, such as  $(g_0, g_1) \in \mathbb{R}^2_+$ ,  $(g_2, g_3, g_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$g \in \mathcal{K}_1^{m_1} \times \mathcal{K}_2^{m_2} \times \cdots \times \mathcal{K}_k^{m_k}$$

which in the CBF format becomes:

17.4. The CBF Format

CON
m k
K1 m1
K2 m2
..
Kk mk

where  $\sum_{i} m_{i} = m$  is the total number of scalar constraints. The list of supported cones is found in Table 17.3.

• **PSD constraints** are constructed one-by-one. That is,  $G_i \succeq \mathbf{0}^{m_i \times m_i}$  for  $i \in \mathcal{I}^{PSD}$ , constructs a matrix-valued affine expressions of size  $m_i \times m_i$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes

```
PSDCON
M m1
m2
..
mM
```

where M is the total number of PSD constraints.

With the objective sense, variables (with integer indications) and constraints, the definitions of the many affine expressions follow in problem data.

#### Problem data

The problem data defines the coefficients and constants of the affine expressions of the problem instance. These are considered zero until explicitly defined, implying that instances with no keywords from this information group are, in fact, valid. Duplicating or conflicting information is a failure to comply with the standard. Consequently, two coefficients written to the same position in a matrix (or to transposed positions in a symmetric matrix) is an error.

The affine expressions of the objective,  $g^{obj}$ , of the scalar constraints,  $g_i$ , and of the PSD constraints,  $G_i$ , are defined separately. The following notation uses the standard trace inner product for matrices,  $\langle X, Y \rangle = \sum_{i,j} X_{ij} Y_{ij}$ .

• The affine expression of the objective is defined as

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj},$$

in terms of the symmetric matrices,  $F_j^{obj}$ , and scalars,  $a_j^{obj}$  and  $b^{obj}$ .

• The affine expressions of the scalar constraints are defined, for  $i \in \mathcal{I}$ , as

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$

in terms of the symmetric matrices,  $F_{ij}$ , and scalars,  $a_{ij}$  and  $b_i$ .

• The affine expressions of the PSD constraints are defined, for  $i \in \mathcal{I}^{PSD}$ , as

$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i,$$

in terms of the symmetric matrices,  $H_{ij}$  and  $D_i$ .

#### List of cones

The format uses an explicit syntax for symmetric positive semidefinite cones as shown above. For scalar variables and constraints, constructed in vectors, the supported conic domains and their minimum sizes are given as follows.

Table 17.3: Cones available in the CBF format

Name	CBF keyword	Cone family	
Free domain	F	linear	
Positive orthant	L+	linear	
Negative orthant	L-	linear	
Fixpoint zero	L=	linear	
Quadratic cone	Q	second-order	
Rotated quadratic cone	QR	second-order	

# 17.4.4 File Format Keywords

#### **VER**

Description: The version of the Conic Benchmark Format used to write the file.

**HEADER:** None

BODY: One line formatted as:

INT

This is the version number.

Must appear exactly once in a file, as the first keyword.

### **OBJSENSE**

Description: Define the objective sense.

HEADER: None

 ${\tt BODY:}$  One line formatted as:

STR

having MIN indicates minimize, and MAX indicates maximize. Capital letters are required.

Must appear exactly once in a file.

# **PSDVAR**

Description: Construct the PSD variables.

**HEADER**: One line formatted as:

INT

This is the number of PSD variables in the problem.

BODY: A list of lines formatted as:

INT

This indicates the number of rows (equal to the number of columns) in the matrix-valued PSD variable. The number of lines should match the number stated in the header.

#### **VAR**

Description: Construct the scalar variables.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar variables, followed by the number of conic domains they are restricted to.

BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 17.3), and the number of scalar variables restricted to this cone. These numbers should add up to the number of scalar variables stated first in the header. The number of lines should match the second number stated in the header.

#### INT

Description: Declare integer requirements on a selected subset of scalar variables.

**HEADER**: one line formatted as:

INT

This is the number of integer scalar variables in the problem.

BODY: a list of lines formatted as:

INT

This indicates the scalar variable index  $j \in \mathcal{J}$ . The number of lines should match the number stated in the header.

Can only be used after the keyword VAR.

### **PSDCON**

Description: Construct the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of PSD constraints in the problem.

BODY: A list of lines formatted as:

TNT

This indicates the number of rows (equal to the number of columns) in the matrix-valued affine expression of the PSD constraint. The number of lines should match the number stated in the header.

Can only be used after these keywords: PSDVAR, VAR.

#### CON

Description: Construct the scalar constraints.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar constraints, followed by the number of conic domains they restrict to.

BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 17.3), and the number of affine expressions restricted to this cone. These numbers should add up to the number of scalar constraints stated first in the header. The number of lines should match the second number stated in the header.

Can only be used after these keywords: PSDVAR, VAR

### **OBJFCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices  $F_j^{obj}$ , as used in the objective.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT REAL

This indicates the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

### **OBJACOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $a_i^{obj}$ , as used in the objective.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

### **OBJBCOORD**

Description: Input the scalar,  $b^{obj}$ , as used in the objective.

HEADER: None.

BODY: One line formatted as:

#### REAL

This indicates the coefficient value.

### **FCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $F_{ij}$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **ACOORD**

Description: Input sparse coordinates (triplets) to define the scalars,  $a_{ij}$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

#### **BCOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $b_i$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$  and the coefficient value. The number of lines should match the number stated in the header.

### **HCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $H_{ij}$ , as used in the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as

INT INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the scalar variable index  $j \in \mathcal{J}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **DCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices,  $D_i$ , as used in the PSD constraints.

**HEADER**: One line formatted as

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **CHANGE**

Start of a new instance specification based on changes to the previous. Can be interpreted as the end of file when the hotstart-sequence is unsupported or undesired.

BODY: None Header: None

# 17.4.5 CBF Format Examples

# Minimal Working Example

The conic optimization problem (17.6), has three variables in a quadratic cone - first one is integer - and an affine expression in domain 0 (equality constraint).

minimize 
$$5.1 x_0$$
  
subject to  $6.2 x_1 + 7.3 x_2 - 8.4 \in \{0\}$   
 $x \in \mathcal{Q}^3, x_0 \in \mathbb{Z}.$  (17.6)

Its formulation in the Conic Benchmark Format begins with the version of the CBF format used, to safeguard against later revisions.

```
VER 1
```

Next follows the problem structure, consisting of the objective sense, the number and domain of variables, the indices of integer variables, and the number and domain of scalar-valued affine expressions (i.e., the equality constraint).

```
OBJSENSE
MIN

VAR
3 1
Q 3

INT
1
0

CON
1 1
L= 1
```

Finally follows the problem data, consisting of the coefficients of the objective, the coefficients of the constraints, and the constant terms of the constraints. All data is specified on a sparse coordinate form.

```
OBJACOORD

1
0 5.1

ACOORD
2
0 1 6.2
0 2 7.3

BCOORD
1
0 -8.4
```

This concludes the example.

#### Mixing Linear, Second-order and Semidefinite Cones

The conic optimization problem (17.7), has a semidefinite cone, a quadratic cone over unordered subindices, and two equality constraints.

The equality constraints are easily rewritten to the conic form,  $(g_0, g_1) \in \{0\}^2$ , by moving constants such that the right-hand-side becomes zero. The quadratic cone does not fit under the VAR keyword in this variable permutation. Instead, it takes a scalar constraint  $(g_2, g_3, g_4) = (x_1, x_0, x_2) \in \mathcal{Q}^3$ , with scalar

variables constructed as  $(x_0, x_1, x_2) \in \mathbb{R}^3$ . Its formulation in the CBF format is reported in the following list

```
\mbox{\tt\#} File written using this version of the Conic Benchmark Format:
#
     | Version 1.
VER
1
# The sense of the objective is:
    | Minimize.
OBJSENSE
MIN
# One PSD variable of this size:
# | Three times three.
PSDVAR
1
# Three scalar variables in this one conic domain:
      | Three are free.
VAR
3 1
F 3
\ensuremath{\mathtt{\#}} Five scalar constraints with affine expressions in two conic domains:
# | Two are fixed to zero.
      | Three are in conic quadratic domain.
CON
5 2
L= 2
Q3
# Five coordinates in F^{obj}_j coefficients:
# | F^{obj}[0][0,0] = 2.0
     | F^{obj}[0][1,0] = 1.0
     and more...
OBJFCOORD
0 0 0 2.0
0 1 0 1.0
0 1 1 2.0
0 2 1 1.0
0 2 2 2.0
# One coordinate in a^{obj}_j coefficients:
\# | a^{obj}[1] = 1.0
OBJACOORD
1 1.0
# Nine coordinates in F_ij coefficients:
     | F[0,0][0,0] = 1.0
     | F[0,0][1,1] = 1.0
#
     and more...
FCOORD
0 0 0 0 1.0
0 0 1 1 1.0
0 0 2 2 1.0
1 0 0 0 1.0
1 0 1 0 1.0
1 0 2 0 1.0
```

```
1 0 1 1 1.0
1 0 2 1 1.0
1 0 2 2 1.0
# Six coordinates in a_ij coefficients:
     | a[0,1] = 1.0
      | a[1,0] = 1.0
      | and more...
ACOORD
0 1 1.0
1 0 1.0
1 2 1.0
2 1 1.0
3 0 1.0
4 2 1.0
# Two coordinates in b_i coefficients:
   | b[0] = -1.0
      | b[1] = -0.5
BCOORD
0 -1.0
1 -0.5
```

### Mixing Semidefinite Variables and Linear Matrix Inequalities

The standard forms in semidefinite optimization are usually based either on semidefinite variables or linear matrix inequalities. In the CBF format, both forms are supported and can even be mixed as shown in.

minimize 
$$\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle + x_1 + x_2 + 1$$
  
subject to  $\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle - x_1 - x_2 \qquad \geq 0.0,$ 

$$x_1 \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} + x_2 \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \succeq \mathbf{0},$$

$$X_1 \succeq \mathbf{0}.$$

$$(17.8)$$

Its formulation in the CBF format is written in what follows

```
# File written using this version of the Conic Benchmark Format:
#
      | Version 1.
VER
1
# The sense of the objective is:
    | Minimize.
OBJSENSE
# One PSD variable of this size:
# | Two times two.
PSDVAR
1
2
# Two scalar variables in this one conic domain:
      | Two are free.
VAR
2 1
```

```
# One PSD constraint of this size:
# | Two times two.
PSDCON
1
2
\mbox{\tt\#} One scalar constraint with an affine expression in this one conic domain:
     | One is greater than or equal to zero.
CON
1 1
L+ 1
# Two coordinates in F^{obj}_j coefficients:
# | F^{obj}[0][0,0] = 1.0
    | F^{obj}[0][1,1] = 1.0
#
OBJFCOORD
0 0 0 1.0
0 1 1 1.0
# Two coordinates in a^{obj}_j coefficients:
# | a^{obj}[0] = 1.0
#
     | a^{obj}[1] = 1.0
OBJACOORD
0 1.0
1 1.0
# One coordinate in b^{obj} coefficient:
# | b^{obj} = 1.0
OBJBCOORD
1.0
# One coordinate in F_ij coefficients:
# | F[0,0][1,0] = 1.0
FCOORD
1
0 0 1 0 1.0
# Two coordinates in a_ij coefficients:
     | a[0,0] = -1.0
     | a[0,1] = -1.0
#
ACOORD
0 0 -1.0
0 1 -1.0
# Four coordinates in H_ij coefficients:
    | H[0,0][1,0] = 1.0
     | H[0,0][1,1] = 3.0
     and more...
HCOORD
0 0 1 0 1.0
0 0 1 1 3.0
0 1 0 0 3.0
0 1 1 0 1.0
# Two coordinates in D_i coefficients:
     | D[0][0,0] = -1.0
     | D[0][1,1] = -1.0
```

```
DCOORD
2
0 0 0 -1.0
0 1 1 -1.0
```

### Optimization Over a Sequence of Objectives

The linear optimization problem (17.9), is defined for a sequence of objectives such that hotstarting from one to the next might be advantages.

$$\begin{array}{llll} \text{maximize}_k & g_k^{obj} \\ \text{subject to} & 50 \, x_0 + 31 & \leq & 250 \,, \\ & 3 \, x_0 - 2 x_1 & \geq & -4 \,, \\ & x \in \mathbb{R}_+^2, \end{array} \tag{17.9}$$

given,

```
1. g_0^{obj} = x_0 + 0.64x_1.

2. g_1^{obj} = 1.11x_0 + 0.76x_1.

3. g_2^{obj} = 1.11x_0 + 0.85x_1.
```

Its formulation in the CBF format is reported in Listing 17.5.

Listing 17.5: Problem (17.9) in CBF format.

```
# File written using this version of the Conic Benchmark Format:
#
      | Version 1.
VER
1
# The sense of the objective is:
     | Maximize.
OBJSENSE
MAX
# Two scalar variables in this one conic domain:
     | Two are nonnegative.
VAR
2 1
L+ 2
# Two scalar constraints with affine expressions in these two conic domains:
     | One is in the nonpositive domain.
      | One is in the nonnegative domain.
CON
2 2
L- 1
L+ 1
# Two coordinates in a^{obj}_j coefficients:
     | a^{obj}[0] = 1.0
      | a^{obj}[1] = 0.64
OBJACOORD
0 1.0
1 0.64
# Four coordinates in a_ij coefficients:
      | a[0,0] = 50.0
      | a[1,0] = 3.0
```

```
and more...
ACOORD
0 0 50.0
1 0 3.0
0 1 31.0
1 1 -2.0
# Two coordinates in b_i coefficients:
      | b[0] = -250.0
      | b[1] = 4.0
BCOORD
0 -250.0
1 4.0
# New problem instance defined in terms of changes.
CHANGE
# Two coordinate changes in a^{obj}_j coefficients. Now it is:
      | a^{obj}[0] = 1.11
      | a^{obj}[1] = 0.76
OBJACOORD
0 1.11
1 0.76
# New problem instance defined in terms of changes.
# One coordinate change in a^{obj}_j coefficients. Now it is:
      | a^{obj}[0] = 1.11
      | a^{obj}[1] = 0.85
OBJACOORD
1 0.85
```

# 17.5 The XML (OSiL) Format

 $\mathbf{MOSEK}$  can write data in the standard OSiL xml format. For a definition of the OSiL format please see  $\mathbf{http://www.optimizationservices.org/.}$ 

Only linear constraints (possibly with integer variables) are supported. By default output files with the extension .xml are written in the OSiL format.

The parameter  $iparam.write\_xml\_mode$  controls if the linear coefficients in the A matrix are written in row or column order.

# 17.6 The Task Format

The Task format is MOSEK's native binary format. It contains a complete image of a MOSEK task, i.e.

- Problem data: Linear, conic quadratic, semidefinite and quadratic data
- Problem item names: Variable names, constraints names, cone names etc.
- Parameter settings
- Solutions

There are a few things to be aware of:

- The task format *does not* support General Convex problems since these are defined by arbitrary user-defined functions.
- Status of a solution read from a file will always be unknown.
- Parameter settings in a task file *always override* any parameters set on the command line or in a parameter file.

The format is based on the TAR (USTar) file format. This means that the individual pieces of data in a .task file can be examined by unpacking it as a TAR file. Please note that the inverse may not work: Creating a file using TAR will most probably not create a valid **MOSEK** Task file since the order of the entries is important.

# 17.7 The JSON Format

MOSEK provides the possibility to read/write problems in valid JSON format.

JSON (JavaScript Object Notation) is a lightweight data-interchange format. It is easy for humans to read and write. It is easy for machines to parse and generate. It is based on a subset of the JavaScript Programming Language, Standard ECMA-262 3rd Edition - December 1999. JSON is a text format that is completely language independent but uses conventions that are familiar to programmers of the C-family of languages, including C, C++, C#, Java, JavaScript, Perl, Python, and many others. These properties make JSON an ideal data-interchange language.

The official JSON website http://www.json.org provides plenty of information along with the format definition.

MOSEK defines two JSON-like formats:

- jtask
- jsol

Warning: Despite being text-based human-readable formats, *jtask* and *jsol* files will include no indentation and no new-lines, in order to keep the files as compact as possible. We therefore strongly advise to use JSON viewer tools to inspect *jtask* and *jsol* files.

### 17.7.1 jtask format

It stores a problem instance. The *jtask* format contains the same information as a *task format*.

Even though a jtask file is human-readable, we do not recommend users to create it by hand, but to rely on **MOSEK**.

# 17.7.2 jsol format

It stores a problem solution. The jsol format contains all solutions and information items.

You can write a jsol file using Task.writejsonsol. You can not read a jsol file into MOSEK.

# 17.7.3 A jtask example

In Listing 17.6 we present a file in the jtask format that corresponds to the sample problem from lollp. The listing has been formatted for readability.

Listing 17.6: A formatted *jtask* file for the lol.lp example.

```
{
    "$schema": "http://mosek.com/json/schema#",
    "Task/INFO":{
        "taskname": "lo1",
        "numvar":4,
        "numcon":3,
        "numcone":0,
        "numbarvar":0,
        "numanz":9,
        "numsymmat":0,
        "mosekver":[
            8,
            0,
            0,
            9
    },
    "Task/data":{
        "var":{
            "name":[
                 "x1",
                 "x2",
                 "x3",
                 "x4"
            ],
             "bk":[
                 "lo",
                 "ra",
                 "lo",
                 "lo"
            ],
             "bl":[
                0.0,
                 0.0,
                 0.0,
                 0.0
            ],
             "bu":[
                1e+30,
                1e+1,
                 1e+30,
                 1e+30
            ],
             "type":[
                 "cont",
                 "cont",
                 "cont",
                 "cont"
            ]
        },
        "con":{
             "name":[
                 "c1",
                 "c2",
                 "c3"
            ],
             "bk":[
                 "fx",
                 "lo",
                 "up"
```

```
],
    "bl":[
        3e+1,
        1.5e+1,
            -1e+30
    ],
    "bu":[
        3e+1,
        1e+30,
        2.5e+1
},
"objective":{
    "sense":"max",
    "name":"obj",
    "c":{
        "subj":[
           0,
            1,
            2,
            3
        ],
        "val":[
            3e+0,
            1e+0,
            5e+0,
            1e+0
        ]
    },
    "cfix":0.0
},
"A":{
    "subi":[
       0,
        0,
        Ο,
        1,
        1,
        1,
        1,
        2,
        2
    ],
    "subj":[
       0,
        1,
        2,
        Ο,
        1,
        2,
        3,
        1,
        3
    ],
"val":[
        3e+0,
        1e+0,
        2e+0,
        2e+0,
        1e+0,
        3e+0,
        1e+0,
        2e+0,
```

```
]
    }
"Task/parameters":{
    "iparam":{
        "ANA_SOL_BASIS":"ON",
        "ANA_SOL_PRINT_VIOLATED": "OFF",
        "AUTO_SORT_A_BEFORE_OPT": "OFF",
        "AUTO_UPDATE_SOL_INFO": "OFF",
        "BASIS_SOLVE_USE_PLUS_ONE": "OFF",
        "BI_CLEAN_OPTIMIZER": "OPTIMIZER_FREE",
        "BI_IGNORE_MAX_ITER":"OFF",
        "BI_IGNORE_NUM_ERROR": "OFF",
        "BI_MAX_ITERATIONS":1000000,
        "CACHE_LICENSE": "ON",
        "CHECK_CONVEXITY": "CHECK_CONVEXITY_FULL",
        "COMPRESS_STATFILE": "ON",
        "CONCURRENT_NUM_OPTIMIZERS":2,
        "CONCURRENT_PRIORITY_DUAL_SIMPLEX":2,
        "CONCURRENT_PRIORITY_FREE_SIMPLEX":3,
        "CONCURRENT_PRIORITY_INTPNT":4,
        "CONCURRENT_PRIORITY_PRIMAL_SIMPLEX":1,
        "FEASREPAIR_OPTIMIZE": "FEASREPAIR_OPTIMIZE_NONE",
        "INFEAS_GENERIC_NAMES":"OFF",
        "INFEAS_PREFER_PRIMAL":"ON",
        "INFEAS_REPORT_AUTO":"OFF",
        "INFEAS_REPORT_LEVEL":1,
        "INTPNT_BASIS": "BI_ALWAYS",
        "INTPNT_DIFF_STEP": "ON",
        "INTPNT_FACTOR_DEBUG_LVL":0,
        "INTPNT_FACTOR_METHOD":0,
        "INTPNT_HOTSTART": "INTPNT_HOTSTART_NONE",
        "INTPNT_MAX_ITERATIONS":400,
        "INTPNT_MAX_NUM_COR":-1,
        "INTPNT_MAX_NUM_REFINEMENT_STEPS":-1,
        "INTPNT_OFF_COL_TRH":40,
        "INTPNT_ORDER_METHOD": "ORDER_METHOD_FREE",
        "INTPNT_REGULARIZATION_USE":"ON",
        "INTPNT_SCALING": "SCALING_FREE",
        "INTPNT_SOLVE_FORM": "SOLVE_FREE",
        "INTPNT_STARTING_POINT": "STARTING_POINT_FREE",
        "LIC_TRH_EXPIRY_WRN":7,
        "LICENSE_DEBUG": "OFF",
        "LICENSE_PAUSE_TIME":0,
        "LICENSE_SUPPRESS_EXPIRE_WRNS": "OFF",
        "LICENSE_WAIT": "OFF",
        "LOG":10,
        "LOG_ANA_PRO":1,
        "LOG_BI":4,
        "LOG_BI_FREQ":2500,
        "LOG_CHECK_CONVEXITY":0,
        "LOG_CONCURRENT":1,
        "LOG_CUT_SECOND_OPT":1,
        "LOG_EXPAND":0,
        "LOG_FACTOR":1,
        "LOG_FEAS_REPAIR":1,
        "LOG_FILE":1,
        "LOG_HEAD":1,
        "LOG_INFEAS_ANA":1,
        "LOG_INTPNT":4,
        "LOG MIO":4.
        "LOG_MIO_FREQ":1000,
```

```
"LOG_OPTIMIZER":1,
"LOG_ORDER":1,
"LOG_PRESOLVE":1,
"LOG_RESPONSE":0,
"LOG_SENSITIVITY":1,
"LOG_SENSITIVITY_OPT":0,
"LOG_SIM":4,
"LOG_SIM_FREQ":1000,
"LOG SIM MINOR":1.
"LOG_STORAGE":1,
"MAX_NUM_WARNINGS":10,
"MIO_BRANCH_DIR": "BRANCH_DIR_FREE",
"MIO_CONSTRUCT_SOL": "OFF",
"MIO_CUT_CLIQUE": "ON",
"MIO_CUT_CMIR": "ON",
"MIO_CUT_GMI": "ON",
"MIO_CUT_KNAPSACK_COVER": "OFF",
"MIO_HEURISTIC_LEVEL":-1,
"MIO_MAX_NUM_BRANCHES":-1,
"MIO_MAX_NUM_RELAXS":-1,
"MIO_MAX_NUM_SOLUTIONS":-1,
"MIO_MODE": "MIO_MODE_SATISFIED",
"MIO_MT_USER_CB":"ON",
"MIO_NODE_OPTIMIZER": "OPTIMIZER_FREE",
"MIO_NODE_SELECTION": "MIO_NODE_SELECTION_FREE",
"MIO_PERSPECTIVE_REFORMULATE":"ON",
"MIO_PROBING_LEVEL":-1,
"MIO_RINS_MAX_NODES":-1,
"MIO_ROOT_OPTIMIZER": "OPTIMIZER_FREE",
"MIO_ROOT_REPEAT_PRESOLVE_LEVEL":-1,
"MT_SPINCOUNT":0,
"NUM_THREADS":0,
"OPF_MAX_TERMS_PER_LINE":5,
"OPF_WRITE_HEADER": "ON",
"OPF_WRITE_HINTS": "ON",
"OPF_WRITE_PARAMETERS": "OFF",
"OPF_WRITE_PROBLEM": "ON",
"OPF_WRITE_SOL_BAS":"ON",
"OPF_WRITE_SOL_ITG":"ON",
"OPF_WRITE_SOL_ITR":"ON",
"OPF_WRITE_SOLUTIONS": "OFF",
"OPTIMIZER": "OPTIMIZER_FREE",
"PARAM_READ_CASE_NAME": "ON",
"PARAM_READ_IGN_ERROR":"OFF"
"PRESOLVE_ELIMINATOR_MAX_FILL":-1,
"PRESOLVE_ELIMINATOR_MAX_NUM_TRIES":-1,
"PRESOLVE_LEVEL":-1,
"PRESOLVE_LINDEP_ABS_WORK_TRH":100,
"PRESOLVE_LINDEP_REL_WORK_TRH":100,
"PRESOLVE_LINDEP_USE": "ON",
"PRESOLVE_MAX_NUM_REDUCTIONS":-1,
"PRESOLVE_USE": "PRESOLVE_MODE_FREE",
"PRIMAL_REPAIR_OPTIMIZER": "OPTIMIZER_FREE",
"QO_SEPARABLE_REFORMULATION": "OFF",
"READ_DATA_COMPRESSED": "COMPRESS_FREE",
"READ_DATA_FORMAT": "DATA_FORMAT_EXTENSION",
"READ_DEBUG": "OFF",
"READ_KEEP_FREE_CON":"OFF",
"READ_LP_DROP_NEW_VARS_IN_BOU":"OFF",
"READ_LP_QUOTED_NAMES":"ON",
"READ_MPS_FORMAT": "MPS_FORMAT_FREE",
"READ_MPS_WIDTH": 1024,
"READ_TASK_IGNORE_PARAM":"OFF"
```

```
"SENSITIVITY_ALL": "OFF",
"SENSITIVITY_OPTIMIZER": "OPTIMIZER_FREE_SIMPLEX",
"SENSITIVITY_TYPE": "SENSITIVITY_TYPE_BASIS",
"SIM_BASIS_FACTOR_USE":"ON",
"SIM_DEGEN": "SIM_DEGEN_FREE",
"SIM_DUAL_CRASH":90,
"SIM_DUAL_PHASEONE_METHOD":0,
"SIM_DUAL_RESTRICT_SELECTION":50,
"SIM_DUAL_SELECTION": "SIM_SELECTION_FREE",
"SIM_EXPLOIT_DUPVEC": "SIM_EXPLOIT_DUPVEC_OFF",
"SIM_HOTSTART": "SIM_HOTSTART_FREE",
"SIM_HOTSTART_LU": "ON",
"SIM_INTEGER":0,
"SIM_MAX_ITERATIONS":10000000,
"SIM_MAX_NUM_SETBACKS":250,
"SIM_NON_SINGULAR": "ON",
"SIM_PRIMAL_CRASH":90,
"SIM_PRIMAL_PHASEONE_METHOD":0,
"SIM_PRIMAL_RESTRICT_SELECTION":50,
"SIM_PRIMAL_SELECTION": "SIM_SELECTION_FREE",
"SIM_REFACTOR_FREQ":0,
"SIM_REFORMULATION": "SIM_REFORMULATION_OFF",
"SIM_SAVE_LU":"OFF",
"SIM_SCALING": "SCALING_FREE",
"SIM_SCALING_METHOD": "SCALING_METHOD_POW2",
"SIM_SOLVE_FORM": "SOLVE_FREE",
"SIM_STABILITY_PRIORITY":50,
"SIM_SWITCH_OPTIMIZER":"OFF",
"SOL_FILTER_KEEP_BASIC": "OFF",
"SOL_FILTER_KEEP_RANGED": "OFF",
"SOL_READ_NAME_WIDTH":-1,
"SOL_READ_WIDTH": 1024,
"SOLUTION_CALLBACK": "OFF",
"TIMING_LEVEL":1,
"WRITE_BAS_CONSTRAINTS": "ON",
"WRITE_BAS_HEAD":"ON",
"WRITE_BAS_VARIABLES": "ON",
"WRITE_DATA_COMPRESSED":0,
"WRITE_DATA_FORMAT": "DATA_FORMAT_EXTENSION",
"WRITE_DATA_PARAM": "OFF",
"WRITE_FREE_CON": "OFF",
"WRITE_GENERIC_NAMES": "OFF",
"WRITE_GENERIC_NAMES_IO":1,
"WRITE_IGNORE_INCOMPATIBLE_CONIC_ITEMS": "OFF",
"WRITE_IGNORE_INCOMPATIBLE_ITEMS": "OFF",
"WRITE_IGNORE_INCOMPATIBLE_NL_ITEMS": "OFF"
"WRITE_IGNORE_INCOMPATIBLE_PSD_ITEMS":"OFF",
"WRITE_INT_CONSTRAINTS":"ON",
"WRITE_INT_HEAD":"ON",
"WRITE_INT_VARIABLES": "ON",
"WRITE_LP_FULL_OBJ": "ON",
"WRITE_LP_LINE_WIDTH":80,
"WRITE_LP_QUOTED_NAMES": "ON",
"WRITE_LP_STRICT_FORMAT": "OFF",
"WRITE_LP_TERMS_PER_LINE":10,
"WRITE_MPS_FORMAT": "MPS_FORMAT_FREE",
"WRITE_MPS_INT":"ON",
"WRITE_PRECISION":15,
"WRITE_SOL_BARVARIABLES": "ON",
"WRITE_SOL_CONSTRAINTS": "ON",
"WRITE_SOL_HEAD": "ON",
"WRITE_SOL_IGNORE_INVALID_NAMES": "OFF",
"WRITE_SOL_VARIABLES": "ON",
```

```
"WRITE_TASK_INC_SOL": "ON",
    "WRITE_XML_MODE": "WRITE_XML_MODE_ROW"
},
"dparam":{
    "ANA_SOL_INFEAS_TOL":1e-6,
    "BASIS_REL_TOL_S":1e-12,
    "BASIS_TOL_S":1e-6,
    "BASIS_TOL_X":1e-6,
    "CHECK_CONVEXITY_REL_TOL":1e-10,
    "DATA_TOL_AIJ":1e-12,
    "DATA_TOL_AIJ_HUGE":1e+20,
    "DATA_TOL_AIJ_LARGE":1e+10,
    "DATA_TOL_BOUND_INF":1e+16,
    "DATA_TOL_BOUND_WRN":1e+8,
    "DATA_TOL_C_HUGE":1e+16,
    "DATA_TOL_CJ_LARGE":1e+8,
    "DATA_TOL_QIJ":1e-16,
    "DATA_TOL_X":1e-8,
    "FEASREPAIR_TOL":1e-10,
    "INTPNT_CO_TOL_DFEAS":1e-8,
    "INTPNT_CO_TOL_INFEAS":1e-10,
    "INTPNT_CO_TOL_MU_RED":1e-8,
    "INTPNT_CO_TOL_NEAR_REL":1e+3,
    "INTPNT_CO_TOL_PFEAS":1e-8,
    "INTPNT_CO_TOL_REL_GAP":1e-7,
    "INTPNT_NL_MERIT_BAL":1e-4,
    "INTPNT_NL_TOL_DFEAS":1e-8,
    "INTPNT_NL_TOL_MU_RED":1e-12,
    "INTPNT_NL_TOL_NEAR_REL":1e+3,
    "INTPNT_NL_TOL_PFEAS":1e-8,
    "INTPNT_NL_TOL_REL_GAP":1e-6,
    "INTPNT_NL_TOL_REL_STEP":9.95e-1,
    "INTPNT_QO_TOL_DFEAS":1e-8,
    "INTPNT_QO_TOL_INFEAS":1e-10,
    "INTPNT_QO_TOL_MU_RED":1e-8,
    "INTPNT_QO_TOL_NEAR_REL":1e+3,
    "INTPNT_QO_TOL_PFEAS":1e-8,
    "INTPNT_QO_TOL_REL_GAP":1e-8,
    "INTPNT_TOL_DFEAS":1e-8,
    "INTPNT_TOL_DSAFE":1e+0,
    "INTPNT_TOL_INFEAS":1e-10,
    "INTPNT_TOL_MU_RED":1e-16,
    "INTPNT_TOL_PATH":1e-8,
    "INTPNT_TOL_PFEAS":1e-8,
    "INTPNT_TOL_PSAFE":1e+0,
    "INTPNT_TOL_REL_GAP":1e-8,
    "INTPNT_TOL_REL_STEP":9.999e-1,
    "INTPNT_TOL_STEP_SIZE":1e-6,
    "LOWER_OBJ_CUT":-1e+30,
    "LOWER_OBJ_CUT_FINITE_TRH":-5e+29,
    "MIO_DISABLE_TERM_TIME":-1e+0,
    "MIO_MAX_TIME":-1e+0,
    "MIO_MAX_TIME_APRX_OPT":6e+1,
    "MIO_NEAR_TOL_ABS_GAP":0.0,
    "MIO_NEAR_TOL_REL_GAP":1e-3,
    "MIO_REL_GAP_CONST":1e-10,
    "MIO_TOL_ABS_GAP":0.0,
    "MIO_TOL_ABS_RELAX_INT":1e-5,
    "MIO_TOL_FEAS":1e-6,
    "MIO_TOL_REL_DUAL_BOUND_IMPROVEMENT":0.0,
    "MIO_TOL_REL_GAP":1e-4,
    "MIO_TOL_X":1e-6,
    "OPTIMIZER_MAX_TIME":-1e+0,
```

```
"PRESOLVE_TOL_ABS_LINDEP": 1e-6,
            "PRESOLVE_TOL_AIJ":1e-12,
            "PRESOLVE_TOL_REL_LINDEP":1e-10,
            "PRESOLVE_TOL_S":1e-8,
            "PRESOLVE_TOL_X":1e-8,
            "QCQO_REFORMULATE_REL_DROP_TOL":1e-15,
            "SEMIDEFINITE_TOL_APPROX":1e-10,
            "SIM_LU_TOL_REL_PIV":1e-2,
            "SIMPLEX_ABS_TOL_PIV":1e-7,
            "UPPER_OBJ_CUT":1e+30,
            "UPPER_OBJ_CUT_FINITE_TRH":5e+29
        "sparam":{
            "BAS_SOL_FILE_NAME":"",
            "DATA_FILE_NAME": "examples/tools/data/lo1.mps",
            "DEBUG_FILE_NAME":"",
            "INT_SOL_FILE_NAME":""
            "ITR_SOL_FILE_NAME":"",
            "MIO_DEBUG_STRING":"",
            "PARAM_COMMENT_SIGN":"%%",
            "PARAM_READ_FILE_NAME":"",
            "PARAM_WRITE_FILE_NAME":"",
            "READ_MPS_BOU_NAME":"",
            "READ_MPS_OBJ_NAME":"",
            "READ_MPS_RAN_NAME":"",
            "READ_MPS_RHS_NAME":"",
            "SENSITIVITY_FILE_NAME":"",
            "SENSITIVITY_RES_FILE_NAME":"",
            "SOL_FILTER_XC_LOW":"",
            "SOL_FILTER_XC_UPR":"",
            "SOL_FILTER_XX_LOW":"",
            "SOL_FILTER_XX_UPR":"",
            "STAT_FILE_NAME":"",
            "STAT_KEY":"",
            "STAT_NAME":""
            "WRITE_LP_GEN_VAR_NAME": "XMSKGEN"
        }
   }
}
```

# 17.8 The Solution File Format

MOSEK provides several solution files depending on the problem type and the optimizer used:

- basis solution file (extension .bas) if the problem is optimized using the simplex optimizer or basis identification is performed,
- interior solution file (extension .sol) if a problem is optimized using the interior-point optimizer and no basis identification is required,
- integer solution file (extension .int) if the problem contains integer constrained variables.

All solution files have the format:

INDEX ?	NAME <name></name>	AT ACTIVITY ?? <a value=""></a>	LOWER LIMIT <a value=""></a>	UPPER LIMIT <a value=""></a>	DUAL LOWER <a value=""></a>	DUAL UPPER <a value=""></a>	
VARIAB INDEX → DUAL	NAME	AT ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL LOWER	DUAL UPPER	CONIC
?	<name></name>	?? <a value=""></a>	<a value=""></a>	<a value=""></a>	<a value=""></a>	<a value=""></a>	<a value=""></a>

In the example the fields ? and <> will be filled with problem and solution specific information. As can be observed a solution report consists of three sections, i.e.

- HEADER In this section, first the name of the problem is listed and afterwards the problem and solution status are shown. Next the primal and dual objective values are displayed.
- ullet CONSTRAINTS For each constraint i of the form

$$l_i^c \le \sum_{j=1}^n a_{ij} x_j \le u_i^c, \tag{17.10}$$

the following information is listed:

- INDEX: A sequential index assigned to the constraint by MOSEK
- NAME: The name of the constraint assigned by the user.
- AT: The status of the constraint. In Table 17.4 the possible values of the status keys and their interpretation are shown.

	Table 11.1. Status Reys.
Status key	Interpretation
UN	Unknown status
BS	Is basic
SB	Is superbasic
LL	Is at the lower limit (bound)
UL	Is at the upper limit (bound)
EQ	Lower limit is identical to upper limit
**	Is infeasible i.e. the lower limit is greater than the upper limit.

Table 17.4: Status keys.

- ACTIVITY: the quantity  $\sum_{j=1}^{n} a_{ij}x_{j}^{*}$ , where  $x^{*}$  is the value of the primal solution.
- LOWER LIMIT: the quantity  $l_i^c$  (see (17.10).)
- UPPER LIMIT: the quantity  $u_i^c$  (see (17.10).)
- DUAL LOWER: the dual multiplier corresponding to the lower limit on the constraint.
- DUAL UPPER: the dual multiplier corresponding to the upper limit on the constraint.
- VARIABLES The last section of the solution report lists information about the variables. This information has a similar interpretation as for the constraints. However, the column with the header CONIC DUAL is included for problems having one or more conic constraints. This column shows the dual variables corresponding to the conic constraints.

# Example: lo1.sol

In Listing 17.7 we show the solution file for the lol.opf problem.

Listing 17.7: An example of .sol file.

NAME	:
PROBLEM STATUS	: PRIMAL_AND_DUAL_FEASIBLE
SOLUTION STATUS	: OPTIMAL
OBJECTIVE NAME	: obj

8.33333333e+01			
8.33333332e+01			
AT ACTIVITY	LOWER LIMIT	UPPER LIMIT	Ш
DUAL UPPER			
EQ 3.0000000000000e+01	3.00000000e+01	3.00000000e+01	-0.
-2.4999999741654e+00			
SB 5.3333333349188e+01	1.50000000e+01	NONE	2.
-0.0000000000000e+00			
UL 2.49999999842049e+01	NONE	2.50000000e+01	-0.
-3.33333332895110e-01			
AT ACTIVITY	LOWER LIMIT	UPPER LIMIT	ш
DUAL UPPER			
LL 1.67020427073508e-09	0.0000000e+00	NONE	-4.
-0.0000000000000e+00			
LL 2.93510446280504e-09	0.0000000e+00	1.00000000e+01	-2.
6.20863861687316e-10			
SB 1.49999999899425e+01	0.00000000e+00	NONE	-8.
-0.0000000000000e+00			
SB 8.33333332273116e+00	0.00000000e+00	NONE	-1.
-0.000000000000e+00			
	AT ACTIVITY DUAL UPPER EQ 3.000000000000000000000000000000000000	AT ACTIVITY LOWER LIMIT  DUAL UPPER  EQ 3.000000000000000000000000000000000000	AT ACTIVITY LOWER LIMIT UPPER LIMIT  DUAL UPPER  EQ 3.000000000000000000000000000000000000

# **EIGHTEEN**

# LIST OF EXAMPLES

List of examples shipped in the distribution of Optimizer API for Java:

Table 18.1: List of distributed examples

File	Description
blas_lapack.	Demonstrates the MOSEK interface to BLAS/LAPACK linear algebra routines
java	
callback.java	An example of data/progress callback
case_portfolio_1	. Implements a basic portfolio optimization model
java	
case_portfolio_2	. Implements a basic portfolio optimization model with efficient frontier
java	
case_portfolio_3	. Implements a basic portfolio optimization model with market impact costs
java	
cqo1.java	A simple conic quadratic problem
feasrepairex1.	A simple example of how to repair an infeasible problem
java	
lo1.java	A simple linear problem
lo2.java	A simple linear problem
milo1.java	A simple mixed-integer linear problem
mioinitsol.	A simple mixed-integer linear problem with an initial guess
java	
opt_server_async	. Uses MOSEK OptServer to solve an optimization problem asynchronously
java	
opt_server_sync.	Uses MOSEK OptServer to solve an optimization problem synchronously
java	
parameters.	Shows how to set optimizer parameters and read information items
java	
production.	Demonstrate how to modify and re-optimize a linear problem
java	
qcqo1.java	A simple quadratically constrained quadratic problem
qo1.java	A simple quadratic problem
response.java	Demonstrates proper response handling
scopt1.java	Shows how to solve a simple non-linear separable problem using the SCopt in-
	terface
sdo1.java	A simple semidefinite optimization problem
sensitivity.	Sensitivity analysis performed on a small linear problem
java	
simple.java	A simple I/O example: read problem from a file, solve and write solutions
solutionquality.	Demonstrates how to examine the quality of a solution
java	
solvebasis.	Demonstrates solving a linear system with the basis matrix
java	
solvelinear.	Demonstrates solving a general linear system
java	

Additional examples can be found on the $\mathbf{MOSEK}$ website and in other $\mathbf{MOSEK}$ publications.	

## **NINETEEN**

## **INTERFACE CHANGES**

The section show interface-specific changes to the **MOSEK** Optimizer API for Java in version 8. See the release notes for general changes and new features of the **MOSEK** Optimization Suite.

# 19.1 Compatibility

• Compatibility guarantees for this interface has been updated. See the new state of compatibility.

## 19.2 Functions

#### **Added**

### Changed

#### Removed

- Env.init
- Env.putdllpath
- Env.putkeepdlls
- Env.set\_stream
- Task.getdbi
- Task.getdcni
- Task.getdeqi
- Task.getinti
- Task.getnumqconknz64
- Task.getpbi
- Task.getpcni
- Task.getpeqi
- Task.getqobj64
- Task.getsolutioninf
- Task.getvarbranchdir
- Task.getvarbranchorder
- Task.getvarbranchpri

- Task.progress
- Task.putvarbranchorder
- Task.readbranchpriorities
- Task.relaxprimal
- Task.set\_stream
- Task.writebranchpriorities

### 19.3 Parameters

### Added

- $\bullet \ \ dparam. \ data\_sym\_mat\_tol$
- $\bullet \ dparam.\, data\_sym\_mat\_tol\_huge$
- $\bullet \ dparam.\, data\_sym\_mat\_tol\_large$
- $\bullet$  dparam.intpnt\_qo\_tol\_dfeas
- $\bullet \ \ dparam. \ intpnt\_qo\_tol\_infeas$
- $\bullet$  dparam.intpnt\_qo\_tol\_mu\_red
- dparam.intpnt\_qo\_tol\_near\_rel
- dparam.intpnt\_qo\_tol\_pfeas
- $\bullet \ dparam.intpnt\_qo\_tol\_rel\_gap$
- $\bullet \ \ dparam.semidefinite\_tol\_approx$
- iparam.intpnt\_multi\_thread
- $\bullet \ \ iparam. \ license\_trh\_expiry\_wrn$
- iparam.log\_ana\_pro
- iparam.mio\_cut\_clique
- iparam.mio\_cut\_gmi
- iparam.mio\_cut\_implied\_bound
- iparam.mio\_cut\_knapsack\_cover
- iparam.mio\_cut\_selection\_level
- $\bullet \ iparam.mio\_perspective\_reformulate$
- $\bullet \ iparam.mio\_root\_repeat\_presolve\_level$
- iparam.mio\_vb\_detection\_level
- iparam.presolve\_eliminator\_max\_fill
- iparam.remove\_unused\_solutions
- iparam.write\_lp\_full\_obj
- $\bullet \ iparam.write\_mps\_format$
- sparam.remote\_access\_token

#### Removed

- dparam.feasrepair\_tol
- dparam.mio\_heuristic\_time
- dparam.mio\_max\_time\_aprx\_opt
- dparam.mio\_rel\_add\_cut\_limited
- dparam.mio\_tol\_max\_cut\_frac\_rhs
- dparam.mio\_tol\_min\_cut\_frac\_rhs
- dparam.mio\_tol\_rel\_relax\_int
- dparam.mio\_tol\_x
- dparam.nonconvex\_tol\_feas
- dparam.nonconvex\_tol\_opt
- iparam.alloc\_add\_qnz
- iparam.concurrent\_num\_optimizers
- iparam.concurrent\_priority\_dual\_simplex
- iparam.concurrent\_priority\_free\_simplex
- iparam.concurrent\_priority\_intpnt
- iparam.concurrent\_priority\_primal\_simplex
- iparam.feasrepair\_optimize
- iparam.intpnt\_factor\_debug\_lvl
- iparam.intpnt\_factor\_method
- iparam.lic\_trh\_expiry\_wrn
- iparam.log\_concurrent
- iparam.log\_factor
- iparam.log\_head
- iparam.log\_nonconvex
- iparam.log\_optimizer
- iparam.log\_param
- iparam.log\_sim\_network\_freq
- iparam.mio\_branch\_priorities\_use
- iparam.mio\_cont\_sol
- iparam.mio\_cut\_cg
- iparam.mio\_cut\_level\_root
- iparam.mio\_cut\_level\_tree
- iparam.mio\_feaspump\_level
- iparam.mio\_hotstart
- iparam.mio\_keep\_basis
- iparam.mio\_local\_branch\_number
- iparam.mio\_optimizer\_mode

19.3. Parameters 433

- iparam.mio\_presolve\_aggregate
- iparam.mio\_presolve\_probing
- iparam.mio\_presolve\_use
- iparam.mio\_strong\_branch
- iparam.mio\_use\_multithreaded\_optimizer
- iparam.nonconvex\_max\_iterations
- iparam.presolve\_elim\_fill
- iparam.presolve\_eliminator\_use
- $\bullet \ \, iparam.qo\_separable\_reformulation \\$
- iparam.read\_anz
- iparam.read\_con
- iparam.read\_cone
- iparam.read\_mps\_keep\_int
- iparam.read\_mps\_obj\_sense
- iparam.read\_mps\_relax
- iparam.read\_qnz
- iparam.read\_var
- iparam.sim\_integer
- iparam.warning\_level
- iparam.write\_ignore\_incompatible\_conic\_items
- iparam.write\_ignore\_incompatible\_nl\_items
- $\bullet \ \, \texttt{iparam.write\_ignore\_incompatible\_psd\_items} \\$
- sparam.feasrepair\_name\_prefix
- $\bullet \ \mathtt{sparam.feasrepair\_name\_separator}$
- sparam.feasrepair\_name\_wsumviol

### 19.4 Constants

### Added

- branchdir.far
- branchdir.guided
- branchdir.near
- $\bullet \ \textit{branchdir.pseudocost}$
- branchdir.root\_lp
- $\bullet \ \ callbackcode.begin\_root\_cutgen$
- $\bullet \ \ callbackcode.begin\_to\_conic$
- $\bullet \ \ callbackcode.\,end\_root\_cutgen$
- callbackcode.end\_to\_conic

- callbackcode.im\_root\_cutgen
- callbackcode.solving\_remote
- dataformat.json\_task
- $\bullet \ \ dinfitem.mio\_clique\_separation\_time$
- dinfitem.mio\_cmir\_separation\_time
- $\bullet \ \ dinfitem.mio\_gmi\_separation\_time$
- dinfitem.mio\_implied\_bound\_time
- dinfitem.mio\_knapsack\_cover\_separation\_time
- $\bullet \ \ dinfitem.\ qcqo\_reformulate\_max\_perturbation$
- dinfitem.qcqo\_reformulate\_worst\_cholesky\_column\_scaling
- dinfitem.qcqo\_reformulate\_worst\_cholesky\_diaq\_scaling
- ullet dinfitem.sol\_bas\_nrm\_barx
- dinfitem.sol\_bas\_nrm\_slc
- dinfitem.sol\_bas\_nrm\_slx
- dinfitem.sol\_bas\_nrm\_suc
- dinfitem.sol\_bas\_nrm\_sux
- $\bullet$  dinfitem.sol\_bas\_nrm\_xc
- dinfitem.sol\_bas\_nrm\_xx
- dinfitem.sol\_bas\_nrm\_y
- $\bullet \ \ dinfitem.sol\_itg\_nrm\_barx$
- dinfitem.sol\_itq\_nrm\_xc
- $\bullet \ \ dinfitem.sol\_itg\_nrm\_xx$
- $\bullet \ \ dinfitem.sol\_itr\_nrm\_bars$
- dinfitem.sol\_itr\_nrm\_barx
- dinfitem.sol\_itr\_nrm\_slc
- $\bullet \ \ dinfitem.sol\_itr\_nrm\_slx$
- $\bullet \ \ dinfitem.sol\_itr\_nrm\_snx$
- $\bullet \ \ dinfitem.sol\_itr\_nrm\_suc$
- $\bullet \ \ dinfitem.sol\_itr\_nrm\_sux$
- $\bullet \ \ dinfitem.sol\_itr\_nrm\_xc$
- $\bullet \ \ dinfitem.sol\_itr\_nrm\_xx$
- $\bullet \ \ dinfitem.sol\_itr\_nrm\_y$
- $\bullet \ \ dinfitem. \ to\_conic\_time$
- $\bullet \ \ iinfitem.mio\_absgap\_satisfied$
- iinfitem.mio\_clique\_table\_size
- $ullet \ iinfitem.mio\_near\_absgap\_satisfied$
- $\bullet \ \ iinfitem.mio\_near\_relgap\_satisfied$
- iinfitem.mio\_node\_depth
- iinfitem.mio\_num\_cmir\_cuts

19.4. Constants 435

- $\bullet \ \ iinfitem.mio\_num\_implied\_bound\_cuts$
- iinfitem.mio\_num\_knapsack\_cover\_cuts
- $\bullet \ \ iinfitem.mio\_num\_repeated\_presolve$
- $\bullet \ \ iinfitem.mio\_presolved\_numbin$
- iinfitem.mio\_presolved\_numcon
- iinfitem.mio\_presolved\_numcont
- iinfitem.mio\_presolved\_numint
- iinfitem.mio\_presolved\_numvar
- $\bullet \ \ iinfitem.mio\_relgap\_satisfied$
- liinfitem.mio\_presolved\_anz
- liinfitem.mio\_sim\_maxiter\_setbacks
- mpsformat.cplex
- solsta.dual\_illposed\_cer
- solsta.prim\_illposed\_cer

### Changed

- solsta.integer\_optimal
- solsta.near\_dual\_feas
- $\bullet \ \ solsta.near\_dual\_infeas\_cer$
- solsta.near\_integer\_optimal
- $\bullet$  solsta.near\_optimal
- $\bullet \ \ \textit{solsta.near\_prim\_and\_dual\_feas}$
- solsta.near\_prim\_feas
- solsta.near\_prim\_infeas\_cer
- value.license\_buffer\_length

#### Removed

- constant.callbackcode.begin\_concurrent
- constant.callbackcode.begin\_network\_dual\_simplex
- constant.callbackcode.begin\_network\_primal\_simplex
- constant.callbackcode.begin\_network\_simplex
- constant.callbackcode.begin\_nonconvex
- constant.callbackcode.begin\_primal\_dual\_simplex
- constant.callbackcode.begin\_primal\_dual\_simplex\_bi
- constant.callbackcode.begin\_simplex\_network\_detect
- constant.callbackcode.end\_concurrent
- constant.callbackcode.end\_network\_dual\_simplex
- constant.callbackcode.end\_network\_primal\_simplex

- constant.callbackcode.end\_network\_simplex
- constant.callbackcode.end\_nonconvex
- constant.callbackcode.end\_primal\_dual\_simplex
- constant.callbackcode.end\_primal\_dual\_simplex\_bi
- constant.callbackcode.end\_simplex\_network\_detect
- constant.callbackcode.im\_mio\_presolve
- constant.callbackcode.im\_network\_dual\_simplex
- constant.callbackcode.im\_network\_primal\_simplex
- constant.callbackcode.im\_nonconvex
- constant.callbackcode.im\_primal\_dual\_simplex
- constant.callbackcode.noncovex
- constant.callbackcode.update\_network\_dual\_simplex
- constant.callbackcode.update\_network\_primal\_simplex
- constant.callbackcode.update\_nonconvex
- constant.callbackcode.update\_primal\_dual\_simplex
- constant.callbackcode.update\_primal\_dual\_simplex\_bi
- constant.dinfitem.bi\_clean\_primal\_dual\_time
- constant.dinfitem.concurrent\_time
- constant.dinfitem.mio\_cg\_seperation\_time
- constant.dinfitem.mio\_cmir\_seperation\_time
- constant.dinfitem.sim\_network\_dual\_time
- constant.dinfitem.sim\_network\_primal\_time
- constant.dinfitem.sim\_network\_time
- constant.dinfitem.sim\_primal\_dual\_time
- constant.feature.ptom
- constant.feature.ptox
- constant.iinfitem.concurrent\_fastest\_optimizer
- constant.iinfitem.mio\_num\_basis\_cuts
- constant.iinfitem.mio\_num\_cardgub\_cuts
- constant.iinfitem.mio\_num\_coef\_redc\_cuts
- constant.iinfitem.mio\_num\_contra\_cuts
- constant.iinfitem.mio\_num\_disagg\_cuts
- constant.iinfitem.mio\_num\_flow\_cover\_cuts
- constant.iinfitem.mio\_num\_gcd\_cuts
- constant.iinfitem.mio\_num\_gub\_cover\_cuts
- constant.iinfitem.mio\_num\_knapsur\_cover\_cuts
- constant.iinfitem.mio\_num\_lattice\_cuts
- constant.iinfitem.mio\_num\_lift\_cuts
- constant.iinfitem.mio\_num\_obj\_cuts

19.4. Constants 437

- constant.iinfitem.mio\_num\_plan\_loc\_cuts
- constant.iinfitem.sim\_network\_dual\_deg\_iter
- constant.iinfitem.sim\_network\_dual\_hotstart
- constant.iinfitem.sim\_network\_dual\_hotstart\_lu
- constant.iinfitem.sim\_network\_dual\_inf\_iter
- constant.iinfitem.sim\_network\_dual\_iter
- constant.iinfitem.sim\_network\_primal\_deg\_iter
- constant.iinfitem.sim\_network\_primal\_hotstart
- constant.iinfitem.sim\_network\_primal\_hotstart\_lu
- constant.iinfitem.sim\_network\_primal\_inf\_iter
- constant.iinfitem.sim\_network\_primal\_iter
- constant.iinfitem.sim\_primal\_dual\_deg\_iter
- constant.iinfitem.sim\_primal\_dual\_hotstart
- constant.iinfitem.sim\_primal\_dual\_hotstart\_lu
- constant.iinfitem.sim\_primal\_dual\_inf\_iter
- constant.iinfitem.sim\_primal\_dual\_iter
- constant.iinfitem.sol\_int\_prosta
- constant.iinfitem.sol\_int\_solsta
- constant.iinfitem.sto\_num\_a\_cache\_flushes
- constant.iinfitem.sto\_num\_a\_transposes
- constant.liinfitem.bi\_clean\_primal\_dual\_deg\_iter
- $\bullet \ {\tt constant.liinfitem.bi\_clean\_primal\_dual\_iter}$
- constant.liinfitem.bi\_clean\_primal\_dual\_sub\_iter
- $\bullet \ {\tt constant.miomode.lazy}$
- constant.optimizertype.concurrent
- constant.optimizertype.mixed\_int\_conic
- constant.optimizertype.network\_primal\_simplex
- constant.optimizertype.nonconvex
- constant.optimizertype.primal\_dual\_simplex

## 19.5 Response Codes

## **Added**

- $\bullet \ \ rescode. err\_cbf\_duplicate\_psdvar$
- $\bullet \ \ rescode. \ err\_cbf\_invalid\_psdvar\_dimension$
- rescode.err\_cbf\_too\_few\_psdvar
- rescode.err\_duplicate\_aij
- rescode.err\_final\_solution

- $\bullet \ \ rescode. \ err\_json\_data$
- rescode.err\_json\_format
- rescode.err\_json\_missing\_data
- $\bullet \ \textit{rescode.err\_json\_number\_overflow}$
- rescode.err\_json\_string
- rescode.err\_json\_syntax
- rescode.err\_lau\_invalid\_lower\_triangular\_matrix
- rescode.err\_lau\_invalid\_sparse\_symmetric\_matrix
- rescode.err\_lau\_not\_positive\_definite
- rescode.err\_mixed\_conic\_and\_nl
- rescode.err\_server\_connect
- rescode.err\_server\_protocol
- rescode.err\_server\_status
- rescode.err\_server\_token
- rescode.err\_sym\_mat\_huge
- rescode.err\_sym\_mat\_invalid
- rescode.err\_task\_write
- rescode.err\_toconic\_constr\_not\_conic
- rescode.err\_toconic\_constr\_q\_not\_psd
- $\bullet \ \ rescode. \ err\_toconic\_constraint\_fx$
- rescode.err\_toconic\_constraint\_ra
- rescode.err\_toconic\_objective\_not\_psd
- rescode.wrn\_sym\_mat\_large

#### Removed

- rescode.err\_ad\_invalid\_operand
- rescode.err\_ad\_invalid\_operator
- rescode.err\_ad\_missing\_operand
- $\bullet \ \texttt{rescode.err\_ad\_missing\_return}$
- rescode.err\_concurrent\_optimizer
- rescode.err\_inv\_conic\_problem
- rescode.err\_invalid\_branch\_direction
- rescode.err\_invalid\_branch\_priority
- rescode.err\_invalid\_network\_problem
- rescode.err\_mbt\_incompatible
- rescode.err\_mbt\_invalid
- rescode.err\_mio\_not\_loaded
- rescode.err\_mixed\_problem
- rescode.err\_no\_dual\_info\_for\_itg\_sol

- $\bullet \ {\tt rescode.err\_ord\_invalid}$
- rescode.err\_ord\_invalid\_branch\_dir
- rescode.err\_toconic\_conversion\_fail
- $\bullet \ \texttt{rescode.err\_too\_many\_concurrent\_tasks} \\$
- rescode.wrn\_too\_many\_threads\_concurrent

## **BIBLIOGRAPHY**

- [AA95] E. D. Andersen and K. D. Andersen. Presolving in linear programming. *Math. Programming*, 71(2):221–245, 1995.
- [AGMX96] E. D. Andersen, J. Gondzio, Cs. Mészáros, and X. Xu. Implementation of interior point methods for large scale linear programming. In T. Terlaky, editor, *Interior-point methods of mathematical programming*, pages 189–252. Kluwer Academic Publishers, 1996.
- [ART03] E. D. Andersen, C. Roos, and T. Terlaky. On implementing a primal-dual interior-point method for conic quadratic optimization. *Math. Programming*, February 2003.
- [AY96] E. D. Andersen and Y. Ye. Combining interior-point and pivoting algorithms. *Management Sci.*, 42(12):1719–1731, December 1996.
- [AY98] E. D. Andersen and Y. Ye. A computational study of the homogeneous algorithm for large-scale convex optimization. *Computational Optimization and Applications*, 10:243–269, 1998.
- [AY99] E. D. Andersen and Y. Ye. On a homogeneous algorithm for the monotone complementarity problem. *Math. Programming*, 84(2):375–399, February 1999.
- [And09] Erling D. Andersen. The homogeneous and self-dual model and algorithm for linear optimization. Technical Report TR-1-2009, MOSEK ApS, 2009. URL: http://docs.mosek.com/whitepapers/homolo.pdf.
- [And13] Erling D. Andersen. On formulating quadratic functions in optimization models. Technical Report TR-1-2013, MOSEK ApS, 2013. Last revised 23-feb-2016. URL: http://docs.mosek.com/whitepapers/qmodel.pdf.
- [Chv83] V. Chvátal. Linear programming. W.H. Freeman and Company, 1983.
- [CT07] Gerard Cornuejols and Reha Tütüncü. Optimization methods in finance. Cambridge University Press, New York, 2007.
- [GK00] Richard C. Grinold and Ronald N. Kahn. *Active portfolio management*. McGraw-Hill, New York, 2 edition, 2000.
- [Naz87] J. L. Nazareth. Computer Solution of Linear Programs. Oxford University Press, New York, 1987.
- [RTV97] C. Roos, T. Terlaky, and J. -Ph. Vial. Theory and algorithms for linear optimization: an interior point approach. John Wiley and Sons, New York, 1997.
- [Ste98] G. W. Stewart. Matrix Algorithms. Volume 1: Basic decompositions. SIAM, 1998.
- [Wal00] S. W. Wallace. Decision making under uncertainty: is sensitivity of any use. *Oper. Res.*, 48(1):20–25, January 2000.
- [Wol98] L. A. Wolsey. Integer programming. John Wiley and Sons, 1998.
- [MOSEKApS12] MOSEK ApS. *The MOSEK Modeling Cookbook*. MOSEK ApS, Fruebjergvej 3, Boks 16, 2100 Copenhagen O, 2012. URL: https://docs.mosek.com/modeling-cookbook/index.html.

442 Bibliography

## **SYMBOL INDEX**

Classes	Task.strtosk, 273
	Task.strtoconetype, 273
DataCallback, 372	Task.solvewithbasis, 272
DataCallback.callback, 372	Task.solutionsummary, 272
Env, 170	Task.solutiondef, 272
Env.syrk, 179	Task.setdefaults, 271
Env. syevd, 179	Task.set_Stream, 271
Env. speig, 178	Task.set_Progress, 271
Env. sparsetriangularsolvedense, 178	Task.set_ItgSolutionCallback, 271
Env.set_Stream, 177 Env.putlicensewait, 177	Task.set_InfoCallback, 270
Env.putlicensewalt, 177	Task.sensitivityreport, 270
Env.putlicensedebug, 177	Task.resizetask, 270
Env.putlicensecode, 176	Task.removevars, 269
Env.potrf, 176	Task.removecons, 269
Env.linkfiletostream, 176	Task.removecones, 269
Env.licensecleanup, 176	Task.removebarvars, 269
Env.getversion, 175	Task.readtask, 269
Env.getcodedesc, 175	Task.readsummary, $269$
Env.gemv, 174	Task.readsolution, $268$
Env.gemm, 174	${\tt Task.readparamfile},268$
Env.Env, 170	${\tt Task.readdataformat},268$
Env.echointro, 173	Task.readdata, $268$
Env.dot, 173	Task.putyslice, $267$
Env.dispose, 173	${\tt Task.puty},267$
Env.computesparsecholesky, 172	Task.putxxslice, $267$
Env.checkoutlicense, 171	Task.putxx, $266$
Env.checkinlicense, 171	Task.putxcslice, $266$
Env.checkinall, 171	Task.putxc, 266
Env.axpy, 170	Task.putvartypelist, 266
${\tt ItgSolutionCallback},371$	Task.putvartype, 265
ItgSolutionCallback.callback, 371	Task.putvarname, 265
Progress, 372	Task.putvarboundslice, 265
Progress.progress, 372	Task.putvarboundlist, 264
Stream, $371$	Task.putvarbound, 264
${\tt Stream.stream},371$	Task.puttaskname, 264
Task,180	Task.putsuxslice, 263
${\tt Task.writetask},275$	Task.putsux, 263
Task.writesolution, $275$	Task.putsucslice, 263
Task.writeSC, 374	Task.putsuc, 263
Task.writeparamfile, $275$	Task.putstrparam, 262
Task.writejsonsol, $274$	Task.putsolutionyi, 262
Task.writedata, 274	Task.putsolutioni, 261
Task.updatesolutioninfo, 274	Task putspyelice 260
Task.unset_Progress, 274	Task putsnx 260
Task.toconic, 274	Task putslyslice 260
Task.Task, 180	Task.putslxslice, 260

T1	T1+i 020
Task.putslx, 260	Task.optimize, 239
Task.putslcslice, 259	Task onesolutionsummary, 239
Task.putslc, 259	Task.linkfiletostream, 238
Task.putskxslice, 259	Task isstrparname, 238
Task.putskx, 258	Task.isintparname, 238
Task.putskcslice, 258	Task.isdouparname, 238
Task.putskc, 258	Task.inputdata, 237
Task.putSCeval, 373	Task.initbasissolve, 236
Task.putqobjij, 257	Task.getyslice, 236
Task.putqobj, 257	Task.gety, 236
Task.putqconk, 257	Task.getxxslice, 235
Task.putqcon, 256	Task.getxx, 235
Task.putparam, 256	Task getxcslice, 235
Task.putobjsense, 255	Task.getxc, 234
Task.putobjname, 255	Task getvartypelist, 234
Task.putnastrparam, 255	Task.getvartype, 234
Task.putnaintparam, 255	Task getvarnamelen, 234
Task.putnadouparam, 255	Task.getvarnameindex, 233
Task.putmaxnumvar, 254	Task.getvarname, 233
Task.putmaxnumqnz, 254	Task getvarboundslice, 232
Task.putmaxnumcone, 254	Task.getvarbound, 232
Task.putmaxnumcon, 253	Task.gettasknamelen, 232
Task.putmaxnumbarvar, 253	Task.gettaskname, 232
Task.putmaxnumanz, 253	Task.getsymmatinfo, 231
Task.putintparam, 252	Task.getsuxslice, 231
Task.putdouparam, 252	Task.getsux, 231
Task.putcslice, 252	Task.getsucslice, 230
Task.putconname, 252	Task.getsuc, 230
Task.putconename, 251	Task.getstrparamlen, 230
Task.putcone, 251	Task getstrparam, 229
Task.putconboundslice, 251	Task getsparsesymmat, 229
Task.putconboundlist, 250	Task getsolutionslice, 229
Task.putconbound, 250	Task getsolutioninfo, 228
Task.putclist, 249	Task getsolution, 227
Task.putcj, 249	Task getsolution, 225
Task.putcfix, 249	Task getsolsta, 225
Task.putboundslice, 249	Task getsnxslice, 225
Task.putboundlist, 248	Task.getsnx, 224 Task.getslxslice, 224
Task.putbound, 248	,
Task.putbarxj, 247	Task getslx, 224
Task.putbarvarname, 247 Task.putbarsj, 247	Task.getslcslice, 223 Task.getslc, 223
Task.putbarsj, 247	Task.getskxslice, 223
Task.putbarchlocktriplet, 246	Task.getskx, 223
Task.putbaraij, 246	Task.getskcslice, 222
Task.putbaralj, 240 Task.putbarablocktriplet, 245	Task.getskcs, 222
Task.putarowslice, 244	Task.getreducedcosts, 222
Task.putarowlist, 244	Task.getqobjij, 221
Task.putarow, 244	Task.getqobjij, 221
Task.putaijlist, 243	Task.getqconk, 221
Task.putaij, 243	Task.getpviolvar, 220
Task.putacolslice, 242	Task.getpviolcones, 220
Task.putacollist, 242	Task.getpviolcones, 220
Task.putacol, 242	Task.getpviolbarvar, 219
Task.primalsensitivity, 241	Task.getprosta, 219
Task.primalrepair, 240	Task.getprosta, 219
Task.optimizersummary, 240	Task.getprimalsolutionnorms, 218
Task.optimizermt, 239	Task.getprimalobj, $218$

Task.getparamname, 217	Task.getc, 200
Task.getparammax, 217	Task.getboundslice, 200
Task.getobjsense, 217	Task.getbound, 200
Task.getobjnamelen, 217	Task.getbarxj, 199
Task.getobjname, 216	Task.getbarvarnamelen, 199
Task.getnumvar, 216	Task.getbarvarnameindex, 198
Task.getnumsymmat, 216	Task.getbarvarname, 198
Task.getnumqobjnz, 216	Task.getbarsj, 198
Task.getnumqconknz, 215	Task.getbarcsparsity, $198$
${\tt Task.getnumparam},215$	Task.getbarcidxj, 197
Task.getnumintvar, 215	${\tt Task.getbarcidxinfo},197$
Task.getnumconemem, 215	Task.getbarcidx, 197
Task.getnumcone, 215	Task.getbarcblocktriplet, 196
Task.getnumcon, 214	Task.getbarasparsity, 196
Task.getnumbarvar, 214	Task.getbaraidxinfo, $195$
Task.getnumbarcnz, 214	Task.getbaraidxij, 195
Task.getnumbarcblocktriplets, 214	Task.getbaraidx, 194
Task.getnumbaranz, 213	Task.getbarablocktriplet, 194
Task.getnumbarablocktriplets, 213	Task.getaslicenumnz, 193
Task.getnumanz64, 213	Task.getaslice, 192
Task.getnumanz, 213	Task.getarowslicetrip, 192
Task.getmemusage, 212	Task.getarownumnz, 192
Task.getmaxnumvar, 212	Task.getarow, 191
Task.getmaxnumqnz, 212	Task.getapiecenumnz, 191
Task.getmaxnumcone, 212	Task.getaij, 190
Task.getmaxnumcon, 212	Task.getacolslicetrip, 190
Task.getmaxnumbarvar, 211	Task.getacolnumnz, 190
Task.getmaxnumanz, 211	Task.getacol, 189
Task.getlintinf, 211	Task.dualsensitivity, 189
Task.getlenbarvarj, 210	Task.dispose, 189
Task.getintparam, 210	Task.deletesolution, 189
Task.getintinf, 210	Task.commitchanges, 188
Task.getinfname, 209	Task.clearSCeval, 374
Task.getinfmax, 209	Task.chgvarbound, 188
Task.getinfindex, 209	Task.chgconbound, 187
Task.getinfeasiblesubproblem, 208	Task.chgbound, 187
Task.getdviolvar, 208	Task.checkmem, 187
Task.getdviolcones, 207	Task.checkconvexity, 186
Task.getdviolcon, 207	Task.basiscond, 186
Task.getdviolbarvar, 207	Task.asyncstop, 186
Task.getdualsolutionnorms, 206	Task.asyncpoll, 185
Task.getdualobj, 206	Task.asyncoptimize, 185
Task.getduarobj, 200	Task.asyncgetresult, 184
Task.getdouinf, 205	Task.appendvars, 184
Task.getdimbarvarj, 205	Task.appendsparsesymmat, 183
Task.getcslice, 205	Task.appendcons, 183
	Task.appendconesseq, 183
Task getconnamelen, 204	Task.appendconesseq, 183
Task.getconnameindex, 204	± ± ,
Task.getconname, 203	Task.appendcone, 181
Task.getconenamelen, 203	Task.appendbarvars, 181
Task.getconenameindex, 203	Task.analyzesolution, 181
Task.getconename, 202	Task.analyzeproblem, 181
Task.getconeinfo, 202	Task.analyzenames, $180$
Task.getcone, 202	Enumerations
Task.getconboundslice, 201	
Task.getconbound, 201	accmode, 346
Task.getcj, 201	accmode.var, 346
Task.getcfix, 200	accmode.con, 346

basindtype, 346	callbackcode.im_bi, 351
basindtype.reservered, 346	callbackcode.end_write, 351
basindtype.no_error, 346	callbackcode.end_to_conic, 351
basindtype.never, 346	callbackcode.end_simplex_bi, 351
basindtype.if_feasible, 346	${\tt callbackcode.end\_simplex},351$
basindtype.always, 346	${\tt callbackcode.end\_root\_cutgen},351$
boundkey, 346	callbackcode.end_read, 350
boundkey.up, 346	callbackcode.end_qcqo_reformulate, 350
boundkey.ra, 347	callbackcode.end_primal_simplex_bi, 350
boundkey.lo, 346	callbackcode.end_primal_simplex, 350
boundkey.fx, 347	callbackcode.end_primal_setup_bi, 350
boundkey.fr, 347	callbackcode.end_primal_sensitivity, 350
branchdir, 363	callbackcode.end_primal_repair, 350
branchdir.up, 363	callbackcode.end_primal_bi, 350
branchdir.root_lp, 364	callbackcode.end_presolve, 350
branchdir.pseudocost, 364	callbackcode.end_optimizer, 350
branchdir.near, 364	callbackcode.end_mio, 350
branchdir.guided, 364	callbackcode.end_license_wait, 350
branchdir.free, 363	callbackcode.end_intpnt, 350
branchdir.far, 364	callbackcode.end_infeas_ana, 350
branchdir.down, 363	callbackcode.end_full_convexity_check, 350
callbackcode, 348	callbackcode.end_dual_simplex_bi, 350
callbackcode.write_opf, 353	callbackcode.end_dual_simplex, 350
callbackcode.update_primal_simplex_bi, 353	callbackcode.end_dual_setup_bi, 350
callbackcode.update_primal_simplex, 353	callbackcode.end_dual_sensitivity, 350
callbackcode.update_primal_bi, 352	callbackcode.end_dual_bi, 350
callbackcode.update_presolve, 352	callbackcode.end_conic, 350
callbackcode.update_dual_simplex_bi, 352	callbackcode.end_bi, 350
callbackcode.update_dual_simplex, 352	callbackcode.dual_simplex, 349
callbackcode.update_dual_bi, 352	callbackcode.conic, 349
callbackcode.solving_remote, 352	callbackcode.begin_write, 349
callbackcode.read_opf_section, 352	callbackcode.begin_to_conic, 349
callbackcode.read_opf, 352	callbackcode.begin_simplex_bi, 349
callbackcode.primal_simplex, 352	callbackcode.begin_simplex, 349
callbackcode.new_int_mio, 352	callbackcode.begin_root_cutgen, 349
callbackcode.intpnt, 352	callbackcode.begin_read, 349
callbackcode.im_simplex_bi, 352	callbackcode.begin_qcqo_reformulate, 349
callbackcode.im_simplex, 352	callbackcode.begin_primal_simplex_bi, 349
callbackcode.im_root_cutgen, 352	callbackcode.begin_primal_simplex, 349
callbackcode.im_read, 352	callbackcode.begin_primal_setup_bi, 349
callbackcode.im_qo_reformulate, 352	callbackcode.begin_primal_sensitivity, 349
callbackcode.im_primal_simplex, 352	callbackcode.begin_primal_repair, 349
callbackcode.im_primal_sensivity, 352	callbackcode.begin_primal_bi, 349
callbackcode.im_primal_bi, 352	callbackcode.begin_presolve, 349
callbackcode.im_presolve, 351	callbackcode.begin_optimizer, 349
callbackcode.im_order, 351	callbackcode.begin_mio, 349
callbackcode.im_mio_primal_simplex, 351	callbackcode.begin_license_wait, 349
callbackcode.im_mio_intpnt, 351	callbackcode.begin_intpnt, 349
callbackcode.im_mio_dual_simplex, 351	callbackcode.begin_infeas_ana, 349
callbackcode.im_mio, 351	callbackcode.begin_full_convexity_check,
callbackcode.im_lu, 351	348
callbackcode.im_license_wait, 351	callbackcode.begin_dual_simplex_bi, 348
callbackcode.im_intpnt, 351	callbackcode.begin_dual_simplex, 348
callbackcode.im_full_convexity_check, 351	callbackcode.begin_dual_setup_bi, 348
callbackcode.im_dual_simplex, 351	callbackcode.begin_dual_sensitivity, 348
callbackcode.im_dual_sensivity, 351	callbackcode.begin_dual_bi, 348
callbackcode.im_dual_bi, 351	callbackcode.begin_conic, 348
callbackcode.im_conic, 351	callbackcode.begin_bi, 348

```
checkconvexitytype, 353
                                              dinfitem.sol_bas_nrm_sux, 357
checkconvexitytype.simple, 353
                                              dinfitem.sol_bas_nrm_suc, 357
checkconvexitytype.none, 353
                                              dinfitem.sol_bas_nrm_slx, 357
checkconvexitytype.full, 353
                                              dinfitem.sol_bas_nrm_slc, 357
compresstype, 353
                                              dinfitem.sol_bas_nrm_barx, 356
compresstype.none, 353
                                              dinfitem.sol_bas_dviolvar, 356
compresstype.gzip, 353
                                              dinfitem.sol_bas_dviolcon, 356
compresstype.free, 353
                                              dinfitem.sol_bas_dual_obj, 356
conetype, 353
                                              dinfitem.sim\_time, 356
conetype.rquad, 353
                                              dinfitem.sim_primal_time, 356
conetype.quad, 353
                                              dinfitem.sim_obj, 356
dataformat, 353
                                              dinfitem.sim_feas, 356
dataformat.xml, 354
                                              dinfitem.sim_dual_time, 356
{\tt dataformat.task},\,354
                                              {\tt dinfitem.rd\_time},\,356
dataformat.op, 354
                                              dinfitem.qcqo_reformulate_worst_cholesky_diag_scaling,
dataformat.mps, 353
dataformat.lp, 354
                                              dinfitem.qcqo_reformulate_worst_cholesky_column_scaling,
dataformat.json_task, 354
dataformat.free_mps, 354
                                              dinfitem.qcqo_reformulate_time, 356
dataformat.extension, 353
                                              dinfitem.qcqo_reformulate_max_perturbation,
dataformat.cb, 354
                                                      356
                                              dinfitem.primal_repair_penalty_obj, 356
dinfitem, 354
dinfitem.to_conic_time, 358
                                              dinfitem.presolve_time, 356
dinfitem.sol_itr_pviolvar, 358
                                              dinfitem.presolve_lindep_time, 356
dinfitem.sol_itr_pviolcones, 358
                                              dinfitem.presolve_eli_time, 356
dinfitem.sol_itr_pviolcon, 358
                                              dinfitem.optimizer_time, 356
dinfitem.sol_itr_pviolbarvar, 358
                                              dinfitem.mio_user_obj_cut, 356
dinfitem.sol_itr_primal_obj, 358
                                              dinfitem.mio_time, 356
dinfitem.sol_itr_nrm_y, 358
                                              dinfitem.mio_root_presolve_time, 356
dinfitem.sol_itr_nrm_xx, 358
                                              dinfitem.mio_root_optimizer_time, 356
dinfitem.sol_itr_nrm_xc, 358
                                              dinfitem.mio_root_cutgen_time, 355
dinfitem.sol_itr_nrm_sux, 358
                                              dinfitem.mio_probing_time, 355
dinfitem.sol_itr_nrm_suc, 358
                                              dinfitem.mio_optimizer_time, 355
dinfitem.sol_itr_nrm_snx, 358
                                              dinfitem.mio_obj_rel_gap, 355
dinfitem.sol_itr_nrm_slx, 358
                                              dinfitem.mio_obj_int, 355
dinfitem.sol_itr_nrm_slc, 358
                                              dinfitem.mio_obj_bound, 355
dinfitem.sol_itr_nrm_barx, 358
                                              dinfitem.mio_obj_abs_gap, 355
dinfitem.sol_itr_nrm_bars, 358
                                              dinfitem.mio_knapsack_cover_separation_time,
dinfitem.sol_itr_dviolvar, 358
dinfitem.sol_itr_dviolcones, 357
                                              dinfitem.mio_implied_bound_time, 355
dinfitem.sol_itr_dviolcon, 357
                                              dinfitem.mio_heuristic_time, 355
dinfitem.sol_itr_dviolbarvar, 357
                                              dinfitem.mio_gmi_separation_time, 355
                                              dinfitem.mio_dual_bound_after_presolve, 355
dinfitem.sol_itr_dual_obj, 357
dinfitem.sol_itg_pviolvar, 357
                                              dinfitem.mio_construct_solution_obj, 355
dinfitem.sol_itg_pviolitg, 357
                                              dinfitem.mio_cmir_separation_time, 355
dinfitem.sol_itg_pviolcones, 357
                                              dinfitem.mio_clique_separation_time, 355
dinfitem.sol_itg_pviolcon, 357
                                              dinfitem.intpnt_time, 355
dinfitem.sol_itg_pviolbarvar, 357
                                              dinfitem.intpnt_primal_obj, 355
dinfitem.sol_itg_primal_obj, 357
                                              dinfitem.intpnt_primal_feas, 354
dinfitem.sol_itg_nrm_xx, 357
                                              dinfitem.intpnt_order_time, 354
dinfitem.sol_itg_nrm_xc, 357
                                              dinfitem.intpnt_opt_status, 354
dinfitem.sol_itg_nrm_barx, 357
                                              dinfitem.intpnt_factor_num_flops, 354
dinfitem.sol_bas_pviolvar, 357
                                              dinfitem.intpnt_dual_obj, 354
dinfitem.sol_bas_pviolcon, 357
                                              dinfitem.intpnt_dual_feas, 354
dinfitem.sol_bas_primal_obj, 357
                                              dinfitem.bi_time, 354
dinfitem.sol_bas_nrm_y, 357
                                              dinfitem.bi_primal_time, 354
dinfitem.sol_bas_nrm_xx, 357
                                              dinfitem.bi_dual_time, 354
dinfitem.sol_bas_nrm_xc, 357
                                              dinfitem.bi_clean_time, 354
```

1. 6 1. 7	
dinfitem.bi_clean_primal_time, 354	iinfitem.mio_num_active_nodes, 361
dinfitem.bi_clean_dual_time, 354	iinfitem.mio_node_depth, 361
dparam, 287	iinfitem.mio_near_relgap_satisfied, 361
feature, 358	iinfitem.mio_near_absgap_satisfied, 361
feature.pts, 358	iinfitem.mio_initial_solution, 360
feature.pton, 358	iinfitem.mio_construct_solution, 360
iinfitem, 359	iinfitem.mio_construct_num_roundings, 360
iinfitem.sto_num_a_realloc, 363	iinfitem.mio_clique_table_size, 360
iinfitem.sol_itr_solsta, 363	iinfitem.mio_absgap_satisfied, 360
iinfitem.sol_itr_prosta, 363	iinfitem.intpnt_solve_dual, 360
iinfitem.sol_itg_solsta, 363	iinfitem.intpnt_num_threads, 360
iinfitem.sol_itg_prosta, 363	iinfitem.intpnt_iter, 360
iinfitem.sol_bas_solsta, 363	iinfitem.intpnt_factor_dim_dense, 360
iinfitem.sol_bas_prosta, 363	iinfitem.ana_pro_num_var_up, 360
iinfitem.sim_solve_dual, 363	iinfitem.ana_pro_num_var_ra, 360
iinfitem.sim_primal_iter, 363	iinfitem.ana_pro_num_var_lo, 360
iinfitem.sim_primal_inf_iter, 363	iinfitem.ana_pro_num_var_int, 360
iinfitem.sim_primal_hotstart_lu, 362	${\tt iinfitem.ana\_pro\_num\_var\_fr},360$
${\tt iinfitem.sim\_primal\_hotstart},362$	${\tt iinfitem.ana\_pro\_num\_var\_eq},360$
${\tt iinfitem.sim\_primal\_deg\_iter},362$	${\tt iinfitem.ana\_pro\_num\_var\_cont},360$
$iinfitem.sim_numvar, 362$	${\tt iinfitem.ana\_pro\_num\_var\_bin},360$
$iinfitem.sim_numcon, 362$	$\verb iinfitem.ana_pro_num_var , 359$
$iinfitem.sim\_dual\_iter, 362$	$\verb iinfitem.ana_pro_num_con_up , 359$
$iinfitem.sim_dual_inf_iter, 362$	$\verb iinfitem.ana_pro_num_con_ra , 359$
$iinfitem.sim_dual_hotstart_lu, 362$	${\tt iinfitem.ana\_pro\_num\_con\_lo},359$
$iinfitem.sim\_dual\_hotstart, 362$	${\tt iinfitem.ana\_pro\_num\_con\_fr},359$
$iinfitem.sim\_dual\_deg\_iter, 362$	${\tt iinfitem.ana\_pro\_num\_con\_eq},359$
$iinfitem.rd_protype, 362$	$iinfitem.ana\_pro\_num\_con, 359$
iinfitem.rd_numvar, 362	inftype, $363$
iinfitem.rd_numq, 362	inftype.lint_type, 363
iinfitem.rd_numintvar, 362	inftype.int_type, 363
iinfitem.rd_numcone, 362	inftype.dou_type, 363
iinfitem.rd_numcon, 362	intpnthotstart, 348
iinfitem.rd_numbarvar, 362	intpnthotstart.primal_dual, 348
iinfitem.optimize_response, 362	intpnthotstart.primal, 348
iinfitem.opt_numvar, 362	intpnthotstart.none, 348
iinfitem.opt_numcon, 362	intpnthotstart.dual, 348
iinfitem.mio_user_obj_cut, 362	iomode, 363
iinfitem.mio_total_num_cuts, 362	iomode.write, 363
iinfitem.mio_relgap_satisfied, 361	iomode.readwrite, 363
iinfitem.mio_presolved_numvar, 361	iomode.read, 363
iinfitem.mio_presolved_numint, 361	iparam, 297
iinfitem.mio_presolved_numcont, 361	language, 346
iinfitem.mio_presolved_numcon, 361	language.eng, 346
iinfitem.mio_presolved_numbin, 361	language.dan, 346
iinfitem.mio_obj_bound_defined, 361	liinfitem, 358
iinfitem.mio_numvar, 361	liinfitem.rd_numqnz, 359
iinfitem.mio_numint, 361	liinfitem.rd_numanz, 359
iinfitem.mio_numcon, 361	liinfitem.mio_simplex_iter, 359
iinfitem.mio_num_repeated_presolve, 361	liinfitem.mio_sim_maxiter_setbacks, 359
iinfitem.mio_num_relax, 361	liinfitem.mio_presolved_anz, 359
iinfitem.mio_num_knapsack_cover_cuts, 361	liinfitem.mio_presorved_anz, 359
iinfitem.mio_num_int_solutions, 361	liinfitem.intpnt_factor_num_nz, 359
	<del>-</del>
iinfitem.mio_num_implied_bound_cuts, 361	liinfitem.bi_primal_iter, 359
iinfitem.mio_num_gomory_cuts, 361	liinfitem.bi_dual_iter, 359
iinfitem.mio_num_cmir_cuts, 361	liinfitem.bi_clean_primal_iter, 359
iinfitem.mio_num_clique_cuts, 361	liinfitem.bi_clean_primal_deg_iter, 359
$iinfitem.mio_num_branch, 361$	$liinfitem.bi\_clean\_dual\_iter, 358$

liinfitem.bi_clean_dual_deg_iter, 358	problemitem, 366
mark, 347	problemitem.var, 366
mark.up, 347	problemitem.cone, 366
mark.lo, 347	problemitem.con, 366
miocontsoltype, 364	problemtype, 366
miocontsoltype.root, 364	problemtype.qo, 366
miocontsoltype.none, 364	problemtype.qcqo, 366
miocontsoltype.itg_rel, 364	problemtype.mixed, 366
miocontsoltype.itg, 364	problemtype.lo, 366
miomode, 364	problemtype.geco, 366
miomode.satisfied, 364	problemtype.conic, 366
miomode.ignored, 364 mionodeseltype, 364	prosta, 367
mionodeseltype.worst, 364	prosta.unknown, 367 prosta.prim_infeas_or_unbounded, 367
mionodeseltype.pseudo, 364	prosta.prim_infeas, 367
mionodeseltype.hybrid, 364 mionodeseltype.free, 364	prosta.prim_feas, 367 prosta.prim_and_dual_infeas, 367
mionodeseltype.first, 364	prosta.prim_and_dual_feas, 367
mionodeseltype.best, 364	prosta.near_prim_feas, 367
mpsformat, 364	prosta.near_prim_neas, 307 prosta.near_prim_and_dual_feas, 367
mpsformat.strict, 365	prosta.near_dual_feas, 367
mpsformat.relaxed, 365	prosta.ill_posed, 367
mpsformat.free, 365	prosta.dual_infeas, 367
mpsformat.cplex, 365	prosta.dual_feas, 367
nametype, 353	rescode, 324
nametype.mps, 353	rescodetype, 367
nametype.lp, 353	rescodetype.wrn, 367
nametype.gen, 353	rescodetype.unk, 367
objsense, $365$	rescodetype.trm, 367
objsense.minimize, 365	rescodetype.ok, 367
objsense.maximize, 365	rescodetype.err, 367
onoffkey, 365	scalingmethod, 368
onoffkey.on, 365	scalingmethod.pow2, 368
onoffkey.off, 365	scalingmethod.free, 368
optimizertype, 365	scalingtype, 368
optimizertype.primal_simplex, 365	scalingtype.none, 368
optimizertype.mixed_int, 365	scalingtype.moderate, 368
optimizertype.intpnt, 365	scalingtype.free, 368
optimizertype.free_simplex, 365	scalingtype.aggressive, 368
optimizertype.free, 365	scopr, 373
optimizertype.dual_simplex, 365	scopr.pow, 373
optimizertype.conic, 365	scopr.log, 373
orderingtype, 365	scopr.exp, 373
orderingtype.try_graphpar, 365	scopr.ent, 373
orderingtype.none, 366	sensitivitytype, 368
orderingtype.free, 365	sensitivitytype.optimal_partition, 368
orderingtype.force_graphpar, 366	sensitivitytype.basis, 368
orderingtype.experimental, 365	simdegen, 347
orderingtype.appminloc, 365	simdegen.none, 347
parametertype, 366	simdegen.moderate, 347
parametertype.str_type, 366	simdegen.minimum, 347
parametertype.invalid_type, 366	simdegen.free, 347
parametertype.int_type, 366	simdegen.aggressive, 347
parametertype.dou_type, 366	simdupvec, 348
presolvemode, 366	simdupvec.on, 348
presolvemode.on, 366	simdupvec.off, 348
presolvemode.off, 366	simdupvec.free, 348
presolvemode.free, 366	simhotstart, 348

simhotstart.status_keys, 348	stakey.bas, 370
simhotstart.none, 348	startpointtype, 370
simhotstart.free, 348	startpointtype.satisfy_bounds, 370
simreform, 347	startpointtype.guess, 370
simreform.on, 347	startpointtype.free, 370
simreform.off, 347	startpointtype.constant, 370
simreform.free, 347	streamtype, 370
simreform.aggressive, 347	streamtype.wrn, 371
simseltype, 368	streamtype.msg, 371
simseltype.se, 368	streamtype.log, 371
simseltype.partial, 368	streamtype.err, 371
simseltype.full, 368	symmattype, 353
simseltype.free, 368	symmattype.sparse, 353
simseltype.devex, 368	transpose, 347
simseltype.ase, 368	transpose.yes, 347
solitem, 368	transpose.no, 347
solitem.y, 368	uplo, 347
solitem.xx, 368	uplo.up, 347
solitem.xc, 368	uplo.lo, 347
solitem.sux, 369	value, 371
solitem.suc, 369	value.max_str_len, 371
solitem.snx, 369	value.license_buffer_length, 371
solitem.slx, 369	variabletype, 371
solitem.slc, 369	variabletype.type_int, 371
solsta, 369	variabletype.type_cont, 371
solsta.unknown, 369	xmlwriteroutputtype, 367
solsta.prim_infeas_cer, 369	xmlwriteroutputtype.row, 367
solsta.prim_illposed_cer, 369	xmlwriteroutputtype.col, 367
solsta.prim_feas, 369	mmrurrustudopususper. dor.
solsta.prim_and_dual_feas, 369	Exceptions
solsta.optimal, 369	•
solsta.near_prim_infeas_cer, 369	ArrayLengthException, 276
solsta.near_prim_feas, 369	Error, 275
solsta.near_prim_and_dual_feas, 369	Exception, 275 MosekException, 275
solsta.near_optimal, 369	- · · · · · · · · · · · · · · · · · · ·
solsta.near_integer_optimal, 369	NullArrayException, 276 Warning, 275
solsta.near_dual_infeas_cer, 369	warning, 210
solsta.near_dual_feas, 369	Parameters
solsta.integer_optimal, 369	
solsta.dual_infeas_cer, 369	Double parameters, 287
solsta.dual_illposed_cer, 369	dparam.ana_sol_infeas_tol, 287
solsta.dual_feas, 369	dparam.basis_rel_tol_s, 287
soltype, $370$	dparam.basis_tol_s, 288
soltype.itr, 370	dparam.basis_tol_x, 288
soltype.itg, 370	dparam.check_convexity_rel_tol, 288
soltype.bas, 370	dparam.data_sym_mat_tol, 288
solveform, 370	dparam.data_sym_mat_tol_huge, 288
solveform.primal, 370	dparam.data_sym_mat_tol_large, 288
solveform.free, 370	dparam.data_tol_aij, 288
solveform.dual, 370	dparam.data_tol_aij_huge, 289
sparam, 322	dparam.data_tol_aij_large, 289
stakey, 370	dparam.data_tol_bound_inf, 289
stakey.upr, 370	dparam.data_tol_bound_wrn, 289
stakey.upf, 370 stakey.unk, 370	dparam.data_tol_c_huge, 289
stakey.supbas, 370	dparam.data_tol_cj_large, 289
stakey.low, 370	dparam.data_tol_qij, 289
	dparam.data_tol_x, 290
stakey.inf, 370	dparam.intpnt_co_tol_dfeas, 290
stakey.fix, 370	dparam.intpnt_co_tol_infeas, 290

```
dparam.intpnt_co_tol_mu_red, 290
                                               iparam.bi_clean_optimizer, 298
dparam.intpnt_co_tol_near_rel, 290
                                               iparam.bi_ignore_max_iter, 298
dparam.intpnt_co_tol_pfeas, 290
                                              iparam.bi_ignore_num_error, 298
dparam.intpnt_co_tol_rel_gap, 290
                                              iparam.bi_max_iterations, 298
dparam.intpnt_nl_merit_bal, 291
                                              iparam.cache_license, 299
dparam.intpnt_nl_tol_dfeas, 291
                                              iparam.check_convexity, 299
                                              iparam.compress\_statfile, 299
dparam.intpnt_nl_tol_mu_red, 291
dparam.intpnt_nl_tol_near_rel, 291
                                               iparam.infeas_generic_names, 299
dparam.intpnt_nl_tol_pfeas, 291
                                              iparam.infeas_prefer_primal, 299
dparam.intpnt_nl_tol_rel_gap, 291
                                              iparam.infeas_report_auto, 299
dparam.intpnt_nl_tol_rel_step, 291
                                              iparam.infeas_report_level, 299
dparam.intpnt_qo_tol_dfeas, 291
                                              iparam.intpnt_basis, 300
dparam.intpnt_qo_tol_infeas, 292
                                              iparam.intpnt_diff_step, 300
dparam.intpnt_qo_tol_mu_red, 292
                                              iparam.intpnt_hotstart, 300
dparam.intpnt_qo_tol_near_rel, 292
                                               iparam.intpnt_max_iterations, 300
dparam.intpnt_qo_tol_pfeas, 292
                                               iparam.intpnt_max_num_cor, 300
{\tt dparam.intpnt\_qo\_tol\_rel\_gap},\ 292
                                               iparam.intpnt_max_num_refinement_steps, 300
dparam.intpnt_tol_dfeas, 292
                                               iparam.intpnt_multi_thread, 300
dparam.intpnt_tol_dsafe, 292
                                               iparam.intpnt_off_col_trh, 301
dparam.intpnt_tol_infeas, 293
                                              iparam.intpnt_order_method, 301
dparam.intpnt_tol_mu_red, 293
                                               iparam.intpnt_regularization_use, 301
dparam.intpnt_tol_path, 293
                                              iparam.intpnt_scaling, 301
dparam.intpnt_tol_pfeas, 293
                                              iparam.intpnt_solve_form, 301
dparam.intpnt_tol_psafe, 293
                                              iparam.intpnt_starting_point, 301
dparam.intpnt_tol_rel_gap, 293
                                              {\tt iparam.license\_debug},\,301
dparam.intpnt_tol_rel_step, 293
                                              iparam.license_pause_time, 302
dparam.intpnt_tol_step_size, 293
                                              iparam.license_suppress_expire_wrns, 302
dparam.lower_obj_cut, 294
                                              iparam.license_trh_expiry_wrn, 302
dparam.lower_obj_cut_finite_trh, 294
                                              iparam.license_wait, 302
                                              iparam.log, 302
dparam.mio_disable_term_time, 294
dparam.mio_max_time, 294
                                               iparam.log_ana_pro, 302
dparam.mio_near_tol_abs_gap, 294
                                               iparam.log_bi, 302
dparam.mio_near_tol_rel_gap, 295
                                               iparam.log_bi_freq, 303
dparam.mio_rel_gap_const, 295
                                               iparam.log_check_convexity, 303
dparam.mio_tol_abs_gap, 295
                                               iparam.log_cut_second_opt, 303
dparam.mio_tol_abs_relax_int, 295
                                              iparam.log_expand, 303
dparam.mio_tol_feas, 295
                                               iparam.log_feas_repair, 303
dparam.mio_tol_rel_dual_bound_improvement,
                                               iparam.log_file, 303
                                               iparam.log_infeas_ana, 304
                                               iparam.log_intpnt, 304
dparam.mio_tol_rel_gap, 295
dparam.optimizer_max_time, 296
                                               iparam.log_mio, 304
dparam.presolve_tol_abs_lindep, 296
                                               iparam.log_mio_freq, 304
dparam.presolve_tol_aij, 296
                                               iparam.log_order, 304
{\tt dparam.presolve\_tol\_rel\_lindep},\,296
                                              iparam.log_presolve, 304
dparam.presolve_tol_s, 296
                                              iparam.log_response, 304
dparam.presolve_tol_x, 296
                                               iparam.log_sensitivity, 304
dparam.qcqo_reformulate_rel_drop_tol, 296
                                               iparam.log_sensitivity_opt, 305
dparam.semidefinite_tol_approx, 296
                                              iparam.log_sim, 305
dparam.sim_lu_tol_rel_piv, 297
                                               iparam.log_sim_freq, 305
dparam.simplex_abs_tol_piv, 297
                                               iparam.log_sim_minor, 305
dparam.upper_obj_cut, 297
                                               iparam.log_storage, 305
dparam.upper_obj_cut_finite_trh, 297
                                               iparam.max_num_warnings, 305
Integer parameters, 297
                                               iparam.mio_branch_dir, 305
iparam.ana_sol_basis, 297
                                               iparam.mio_construct_sol, 306
iparam.ana_sol_print_violated, 297
                                               iparam.mio_cut_clique, 306
iparam.auto_sort_a_before_opt, 298
                                               iparam.mio_cut_cmir, 306
iparam.auto_update_sol_info, 298
                                               iparam.mio_cut_gmi, 306
iparam.basis_solve_use_plus_one, 298
                                              iparam.mio_cut_implied_bound, 306
```

inamam mia aut Irnangaal aarran 206	increm sim duel selection 215
iparam.mio_cut_knapsack_cover, 306	iparam.sim_dual_selection, 315
iparam.mio_cut_selection_level, 307	iparam.sim_exploit_dupvec, 315
iparam.mio_heuristic_level, 307	iparam.sim_hotstart, 315
iparam.mio_max_num_branches, 307	iparam.sim_hotstart_lu, 315
iparam.mio_max_num_relaxs, 307	iparam.sim_max_iterations, 315
iparam.mio_max_num_solutions, 307	iparam.sim_max_num_setbacks, 315
iparam.mio_mode, 308	iparam.sim_non_singular, 315
iparam.mio_mt_user_cb, 308	iparam.sim_primal_crash, 316
iparam.mio_node_optimizer, 308	iparam.sim_primal_phaseone_method, 316
iparam.mio_node_selection, 308	iparam.sim_primal_restrict_selection, 316
iparam.mio_perspective_reformulate, 308	iparam.sim_primal_selection, 316
iparam.mio_probing_level, 308	iparam.sim_refactor_freq, 316
iparam.mio_rins_max_nodes, 308	iparam.sim_reformulation, 316
iparam.mio_root_optimizer, 309	iparam.sim_save_lu, 316
iparam.mio_root_repeat_presolve_level, 309	iparam.sim_scaling, 317
iparam.mio_vb_detection_level, 309	iparam.sim_scaling_method, 317
iparam.mt_spincount, 309	iparam.sim_solve_form, 317
iparam.num_threads, 309	$iparam.sim\_stability\_priority, 317$
iparam.opf_max_terms_per_line, 309	$iparam.sim\_switch\_optimizer, 317$
${\tt iparam.opf\_write\_header},310$	$iparam.sol\_filter\_keep\_basic, 317$
$iparam.opf\_write\_hints, 310$	$iparam.sol\_filter\_keep\_ranged, 317$
${\tt iparam.opf\_write\_parameters},310$	$iparam.sol\_read\_name\_width, 318$
$iparam.opf\_write\_problem, 310$	$iparam.sol\_read\_width, 318$
$iparam.opf\_write\_sol\_bas, 310$	iparam.solution_callback, 318
$iparam.opf\_write\_sol\_itg, 310$	$iparam.timing\_level, 318$
iparam.opf_write_sol_itr, 310	$iparam.write\_bas\_constraints, 318$
$iparam.opf\_write\_solutions, 310$	$iparam.write\_bas\_head, 318$
iparam.optimizer, 311	$iparam.write\_bas\_variables, 318$
${\tt iparam.param\_read\_case\_name},311$	$\verb"iparam.write_data_compressed", 318$
$iparam.param_read_ign_error, 311$	$iparam.write\_data\_format, 319$
iparam.presolve_eliminator_max_fill, 311	$\verb"iparam.write_data_param", 319$
<pre>iparam.presolve_eliminator_max_num_tries,</pre>	$iparam.write\_free\_con, 319$
311	$iparam.write\_generic\_names, 319$
$iparam.presolve\_level, 311$	iparam.write_generic_names_io, 319
iparam.presolve_lindep_abs_work_trh, 311	iparam.write_ignore_incompatible_items, 319
iparam.presolve_lindep_rel_work_trh, 311	iparam.write_int_constraints, 319
iparam.presolve_lindep_use, 312	$iparam.write_int_head, 320$
iparam.presolve_max_num_reductions, 312	$iparam.write_int_variables, 320$
iparam.presolve_use, 312	iparam.write_lp_full_obj, 320
iparam.primal_repair_optimizer, 312	iparam.write_lp_line_width, 320
iparam.read_data_compressed, 312	iparam.write_lp_quoted_names, 320
iparam.read_data_format, 312	iparam.write_lp_strict_format, 320
iparam.read_debug, 312	iparam.write_lp_terms_per_line, 320
iparam.read_keep_free_con, 312	iparam.write_mps_format, 320
iparam.read_lp_drop_new_vars_in_bou, 313	iparam.write_mps_int, 321
iparam.read_lp_quoted_names, 313	iparam.write_precision, 321
iparam.read_mps_format, 313	iparam.write_sol_barvariables, 321
iparam.read_mps_width, 313	iparam.write_sol_constraints, 321
iparam.read_task_ignore_param, 313	iparam.write_sol_head, 321
iparam.remove_unused_solutions, 313	iparam.write_sol_ignore_invalid_names, 321
iparam.sensitivity_all, 313	iparam.write_sol_variables, 321
iparam.sensitivity_optimizer, 314	iparam.write_task_inc_sol, 321
iparam.sensitivity_type, 314	iparam.write_xml_mode, 322
iparam.sim_basis_factor_use, 314	String parameters, 322
iparam.sim_degen, 314	sparam.bas_sol_file_name, 322
iparam.sim_dual_crash, 314	sparam.data_file_name, 322
iparam.sim_dual_phaseone_method, 314	sparam.debug_file_name, 322
iparam.sim_dual_restrict_selection, 314	sparam.int_sol_file_name, 322
r	1

```
{\tt rescode.wrn\_large\_aij},\,326
sparam.itr_sol_file_name, 322
sparam.mio_debug_string, 322
                                              rescode.wrn_large_bound, 326
sparam.param_comment_sign, 322
                                              rescode.wrn_large_cj, 326
sparam.param_read_file_name, 323
                                              rescode.wrn_large_con_fx, 326
sparam.param_write_file_name, 323
                                              rescode.wrn_large_lo_bound, 326
sparam.read_mps_bou_name, 323
                                              rescode.wrn_large_up_bound, 326
sparam.read_mps_obj_name, 323
                                              rescode.wrn_license_expire, 327
sparam.read_mps_ran_name, 323
                                              rescode.wrn_license_feature_expire, 327
sparam.read_mps_rhs_name, 323
                                              rescode.wrn_license_server, 327
sparam.remote_access_token, 323
                                              rescode.wrn_lp_drop_variable, 326
sparam.sensitivity_file_name, 323
                                              rescode.wrn_lp_old_quad_format, 326
sparam.sensitivity_res_file_name, 323
                                              rescode.wrn_mio_infeasible_final, 326
sparam.sol_filter_xc_low, 323
                                              rescode.wrn_mps_split_bou_vector, 326
sparam.sol_filter_xc_upr, 324
                                              rescode.wrn_mps_split_ran_vector, 326
sparam.sol_filter_xx_low, 324
                                              rescode.wrn_mps_split_rhs_vector, 326
sparam.sol_filter_xx_upr, 324
                                              rescode.wrn_name_max_len, 326
sparam.stat_file_name, 324
                                              rescode.wrn_no_dualizer, 328
sparam.stat_key, 324
                                              rescode.wrn_no_global_optimizer, 326
sparam.stat_name, 324
                                              rescode.wrn_no_nonlinear_function_write,
sparam.write_lp_gen_var_name, 324
                                              rescode.wrn_nz_in_upr_tri, 326
Response codes
                                              rescode.wrn_open_param_file, 326
                                              rescode.wrn_param_ignored_cmio, 327
Termination, 325
                                              rescode.wrn_param_name_dou, 327
rescode.ok, 325
                                              rescode.wrn_param_name_int, 327
rescode.trm_internal, 325
                                              rescode.wrn_param_name_str, 327
rescode.trm_internal_stop, 326
                                              rescode.wrn_param_str_value, 327
rescode.trm_max_iterations, 325
                                              rescode.wrn_presolve_outofspace, 328
rescode.trm_max_num_setbacks, 325
                                              rescode.wrn_quad_cones_with_root_fixed_at_zero,
rescode.trm_max_time, 325
                                                      328
rescode.trm_mio_near_abs_gap, 325
                                              rescode.wrn_rquad_cones_with_root_fixed_at_zero,
rescode.trm_mio_near_rel_gap, 325
rescode.trm_mio_num_branches, 325
                                              rescode.wrn_sol_file_ignored_con, 327
rescode.trm_mio_num_relaxs, 325
                                              rescode.wrn_sol_file_ignored_var, 327
rescode.trm_num_max_num_int_solutions, 325
                                              rescode.wrn_sol_filter, 327
rescode.trm_numerical_problem, 325
                                              rescode.wrn_spar_max_len, 326
rescode.trm_objective_range, 325
                                              rescode.wrn_sym_mat_large, 329
rescode.trm_stall, 325
                                              rescode.wrn_too_few_basis_vars, 327
rescode.trm_user_callback, 325
                                              rescode.wrn_too_many_basis_vars, 327
Warnings, 326
                                              rescode.wrn_undef_sol_file_name, 327
rescode.wrn_ana_almost_int_bounds, 328
                                              rescode.wrn_using_generic_names, 327
rescode.wrn_ana_c_zero, 328
                                              rescode.wrn_write_changed_names, 328
rescode.wrn_ana_close_bounds, 328
                                              rescode.wrn_write_discarded_cfix, 328
rescode.wrn_ana_empty_cols, 328
                                              rescode.wrn_zero_aij, 326
{\tt rescode.wrn\_ana\_large\_bounds},\,328
                                              rescode.wrn_zeros_in_sparse_col, 327
rescode.wrn_construct_invalid_sol_itg, 328
                                              rescode.wrn_zeros_in_sparse_row, 327
rescode.wrn_construct_no_sol_itg, 328
                                              Errors, 329
rescode.wrn_construct_solution_infeas, 328
                                              rescode.err_ad_invalid_codelist, 342
rescode.wrn_dropped_nz_qobj, 326
                                              rescode.err_api_array_too_small, 342
rescode.wrn_duplicate_barvariable_names,
                                              rescode.err_api_cb_connect, 342
                                              rescode.err_api_fatal_error, 342
rescode.wrn_duplicate_cone_names, 328
                                              rescode.err_api_internal, 342
rescode.wrn_duplicate_constraint_names, 328
                                              rescode.err_arg_is_too_large, 335
rescode.wrn_duplicate_variable_names, 328
                                              rescode.err_arg_is_too_small, 335
rescode.wrn_eliminator_space, 328
                                              rescode.err_argument_dimension, 334
rescode.wrn_empty_name, 327
                                              \verb|rescode.err_argument_is_too_large|, 343|
rescode.wrn_ignore_integer, 326
{\tt rescode.wrn\_incomplete\_linear\_dependency\_che} {\tt FR}; {\tt scode.err\_argument\_lenneq}, 334
                                              rescode.err_argument_perm_array, 337
```

```
rescode.err_argument_type, 334
                                              rescode.err_feasrepair_solving_relaxed, 340
rescode.err_bar_var_dim, 343
                                              rescode.err_file_license, 329
rescode.err_basis, 336
                                              rescode.err_file_open, 330
rescode.err_basis_factor, 340
                                              rescode.err_file_read, 330
rescode.err_basis_singular, 340
                                              rescode.err_file_write, 330
                                              rescode.err_final_solution, 340
rescode.err_blank_name, 331
rescode.err_cannot_clone_nl, 341
                                              rescode.err_first, 336
rescode.err_cannot_handle_nl, 341
                                              rescode.err_firsti, 337
rescode.err_cbf_duplicate_acoord, 345
                                              rescode.err_firstj, 337
rescode.err_cbf_duplicate_bcoord, 345
                                              rescode.err_fixed_bound_values, 339
rescode.err_cbf_duplicate_con, 344
                                              rescode.err_flexlm, 329
                                              {\tt rescode.err\_global\_inv\_conic\_problem},\,340
rescode.err_cbf_duplicate_int, 345
                                              {\tt rescode.err\_huge\_aij},\,338
rescode.err_cbf_duplicate_obj, 344
rescode.err_cbf_duplicate_objacoord, 345
                                              rescode.err_huge_c, 338
rescode.err_cbf_duplicate_psdvar, 345
                                              rescode.err_identical_tasks, 342
rescode.err_cbf_duplicate_var, 345
                                              rescode.err_in_argument, 334
rescode.err_cbf_invalid_con_type, 345
                                              rescode.err_index, 335
                                              rescode.err_index_arr_is_too_large, 335
rescode.err_cbf_invalid_domain_dimension,
        345
                                              rescode.err_index_arr_is_too_small, 334
rescode.err_cbf_invalid_int_index, 345
                                              rescode.err_index_is_too_large, 334
rescode.err_cbf_invalid_psdvar_dimension,
                                              rescode.err_index_is_too_small, 334
                                              rescode.err_inf_dou_index, 334
rescode.err_cbf_invalid_var_type, 345
                                              rescode.err_inf_dou_name, 335
rescode.err_cbf_no_variables, 344
                                              rescode.err_inf_int_index, 334
{\tt rescode.err\_cbf\_no\_version\_specified}, 344
                                              {\tt rescode.err\_inf\_int\_name},\,335
rescode.err_cbf_obj_sense, 344
                                              rescode.err_inf_lint_index, 335
rescode.err_cbf_parse, 344
                                              rescode.err_inf_lint_name, 335
rescode.err_cbf_syntax, 344
                                              rescode.err_inf_type, 335
rescode.err_cbf_too_few_constraints, 345
                                              rescode.err_infeas_undefined, 343
rescode.err_cbf_too_few_ints, 345
                                              rescode.err_infinite_bound, 338
rescode.err_cbf_too_few_psdvar, 345
                                              rescode.err_int64_to_int32_cast, 342
rescode.err_cbf_too_few_variables, 345
                                              rescode.err_internal, 342
rescode.err_cbf_too_many_constraints, 344
                                              rescode.err_internal_test_failed, 342
rescode.err_cbf_too_many_ints, 345
                                              rescode.err_inv_aptre, 336
rescode.err_cbf_too_many_variables, 344
                                              rescode.err_inv_bk, 336
rescode.err_cbf_unsupported, 345
                                              rescode.err_inv_bkc, 336
                                              rescode.err_inv_bkx, 336
rescode.err_con_q_not_nsd, 337
rescode.err_con_q_not_psd, 337
                                              rescode.err_inv_cone_type, 337
rescode.err_cone_index, 338
                                              rescode.err_inv_cone_type_str, 337
rescode.err_cone_overlap, 338
                                              rescode.err_inv_marki, 341
rescode.err_cone_overlap_append, 338
                                              rescode.err_inv_markj, 341
rescode.err_cone_rep_var, 338
                                              rescode.err_inv_name_item, 337
rescode.err_cone_size, 338
                                              rescode.err_inv_numi, 341
rescode.err_cone_type, 338
                                              rescode.err_inv_numj, 341
rescode.err_cone_type_str, 338
                                              rescode.err_inv_optimizer, 340
rescode.err_data_file_ext, 330
                                              rescode.err_inv_problem, 340
                                              {\tt rescode.err\_inv\_qcon\_subi},\,338
rescode.err_dup_name, 331
rescode.err_duplicate_aij, 338
                                              rescode.err_inv_qcon_subj, 339
rescode.err_duplicate_barvariable_names,
                                              rescode.err_inv_qcon_subk, 338
                                              rescode.err_inv_qcon_val, 339
rescode.err_duplicate_cone_names, 343
                                              rescode.err_inv_qobj_subi, 338
rescode.err_duplicate_constraint_names, 343
                                              rescode.err_inv_qobj_subj, 338
rescode.err_duplicate_variable_names, 343
                                              rescode.err_inv_qobj_val, 338
rescode.err_end_of_file, 331
                                              rescode.err_inv_sk, 337
rescode.err_factor, 340
                                              rescode.err_inv_sk_str, 336
rescode.err_feasrepair_cannot_relax, 340
                                              rescode.err_inv_skc, 336
rescode.err_feasrepair_inconsistent_bound,
                                              rescode.err_inv_skn, 336
        340
                                              rescode.err_inv_skx, 336
```

```
rescode.err_inv_var_type, 336
                                              rescode.err_license_expired, 329
rescode.err_invalid_accmode, 341
                                              rescode.err_license_feature, 329
rescode.err_invalid_aij, 340
                                              rescode.err_license_invalid_hostid, 329
rescode.err_invalid_ampl_stub, 342
                                              rescode.err_license_max, 329
rescode.err_invalid_barvar_name, 331
                                              rescode.err_license_moseklm_daemon, 329
rescode.err_invalid_compression, 341
                                              rescode.err_license_no_server_line, 330
rescode.err_invalid_con_name, 331
                                              rescode.err_license_no_server_support, 330
                                              rescode.err_license_server, 329
rescode.err_invalid_cone_name, 331
rescode.err_invalid_file_format_for_cones,
                                              rescode.err_license_server_version, 329
                                              rescode.err_license_version, 329
rescode.err_invalid_file_format_for_general_mbscode.err_link_file_dll, 330
                                              rescode.err_living_tasks, 331
\verb|rescode.err_invalid_file_format_for_sym_mat|, \verb|rescode.err_lower_bound_is_a_nan|, 338|
        343
                                              rescode.err_lp_dup_slack_name, 333
rescode.err_invalid_file_name, 330
                                              rescode.err_lp_empty, 333
rescode.err_invalid_format_type, 337
                                              rescode.err_lp_file_format, 333
rescode.err_invalid_idx, 335
                                              rescode.err_lp_format, 333
rescode.err_invalid_iomode, 341
                                              rescode.err_lp_free_constraint, 333
rescode.err_invalid_max_num, 335
                                              rescode.err_lp_incompatible, 332
rescode.err_invalid_name_in_sol_file, 333
                                              rescode.err_lp_invalid_con_name, 333
rescode.err_invalid_obj_name, 331
                                              rescode.err_lp_invalid_var_name, 333
rescode.err_invalid_objective_sense, 339
                                              rescode.err_lp_write_conic_problem, 333
rescode.err_invalid_problem_type, 343
                                              rescode.err_lp_write_geco_problem, 333
rescode.err_invalid_sol_file_name, 331
                                              rescode.err_lu_max_num_tries, 341
{\tt rescode.err\_invalid\_stream},\,331
                                              {\tt rescode.err\_max\_len\_is\_too\_small},\,337
rescode.err_invalid_surplus, 337
                                              rescode.err_maxnumbarvar, 335
rescode.err_invalid_sym_mat_dim, 343
                                              rescode.err_maxnumcon, 335
rescode.err_invalid_task, 331
                                              rescode.err_maxnumcone, 338
rescode.err_invalid_utf8, 341
                                              rescode.err_maxnumqnz, 335
rescode.err_invalid_var_name, 331
                                              rescode.err_maxnumvar, 335
rescode.err_invalid_wchar, 341
                                              rescode.err_mio_internal, 343
rescode.err_invalid_whichsol, 335
                                              rescode.err_mio_invalid_node_optimizer, 345
                                              rescode.err_mio_invalid_root_optimizer, 345
rescode.err_json_data, 334
rescode.err_json_format, 334
                                              rescode.err_mio_no_optimizer, 340
rescode.err_json_missing_data, 334
                                              rescode.err_missing_license_file, 329
                                              {\tt rescode.err\_mixed\_conic\_and\_nl},\,340
rescode.err_json_number_overflow, 334
rescode.err_json_string, 333
                                              rescode.err_mps_cone_overlap, 332
rescode.err_json_syntax, 333
                                              rescode.err_mps_cone_repeat, 332
rescode.err_last, 336
                                              rescode.err_mps_cone_type, 332
rescode.err_lasti, 337
                                              rescode.err_mps_duplicate_q_element, 332
rescode.err_lastj, 337
                                              rescode.err_mps_file, 331
rescode.err_lau_arg_k, 344
                                              rescode.err_mps_inv_bound_key, 332
rescode.err_lau_arg_m, 344
                                              rescode.err_mps_inv_con_key, 332
rescode.err_lau_arg_n, 344
                                              rescode.err_mps_inv_field, 331
rescode.err_lau_arg_trans, 344
                                              rescode.err_mps_inv_marker, 331
rescode.err_lau_arg_transa, 344
                                              rescode.err_mps_inv_sec_name, 332
rescode.err_lau_arg_transb, 344
                                              rescode.err_mps_inv_sec_order, 332
                                              rescode.err_mps_invalid_obj_name, 332
rescode.err_lau_arg_uplo, 344
rescode.err_lau_invalid_lower_triangular_matrescode.err_mps_invalid_objsense, 332
                                              rescode.err_mps_mul_con_name, 332
rescode.err_lau_invalid_sparse_symmetric_matrexcode.err_mps_mul_csec, 332
                                              rescode.err_mps_mul_qobj, 332
rescode.err_lau_not_positive_definite, 344
                                              rescode.err_mps_mul_qsec, 332
                                              rescode.err_mps_no_objective, 332
rescode.err_lau_singular_matrix, 344
rescode.err_lau_unknown, 344
                                              rescode.err_mps_non_symmetric_q, 332
rescode.err_license, 329
                                              rescode.err_mps_null_con_name, 331
rescode.err_license_cannot_allocate, 329
                                              rescode.err_mps_null_var_name, 331
rescode.err_license_cannot_connect, 329
                                              rescode.err_mps_splitted_var, 332
```

```
rescode.err_mps_tab_in_field2, 332
                                              rescode.err_platform_not_licensed, 329
rescode.err_mps_tab_in_field3, 332
                                              rescode.err_postsolve, 340
rescode.err_mps_tab_in_field5, 332
                                              rescode.err_pro_item, 337
rescode.err_mps_undef_con_name, 332
                                              rescode.err_prob_license, 329
rescode.err_mps_undef_var_name, 332
                                              rescode.err_qcon_subi_too_large, 339
rescode.err_mul_a_element, 336
                                              rescode.err_qcon_subi_too_small, 339
rescode.err_name_is_null, 341
                                              rescode.err_qcon_upper_triangle, 339
                                              rescode.err_qobj_upper_triangle, 339
rescode.err_name_max_len, 341
rescode.err_nan_in_blc, 339
                                              rescode.err_read_format, 331
rescode.err_nan_in_blx, 339
                                              rescode.err_read_lp_missing_end_tag, 333
rescode.err_nan_in_buc, 339
                                              rescode.err_read_lp_nonexisting_name, 333
rescode.err_nan_in_bux, 340
                                              rescode.err_remove_cone_variable, 338
rescode.err_nan_in_c, 339
                                              rescode.err_repair_invalid_problem, 340
rescode.err_nan_in_double_data, 339
                                              rescode.err_repair_optimization_failed, 341
rescode.err_negative_append, 336
                                              rescode.err_sen_bound_invalid_lo, 342
rescode.err_negative_surplus, 336
                                              rescode.err_sen_bound_invalid_up, 342
rescode.err_newer_dll, 330
                                              rescode.err_sen_format, 342
rescode.err_no_bars_for_solution, 343
                                              rescode.err_sen_index_invalid, 342
rescode.err_no_barx_for_solution, 343
                                              rescode.err_sen_index_range, 342
rescode.err_no_basis_sol, 340
                                              rescode.err_sen_invalid_regexp, 342
rescode.err_no_dual_for_itg_sol, 341
                                              rescode.err_sen_numerical, 342
                                              rescode.err_sen_solution_status, 342
rescode.err_no_dual_infeas_cer, 341
rescode.err_no_init_env, 331
                                              rescode.err_sen_undef_name, 342
rescode.err_no_optimizer_var_type, 340
                                              rescode.err_sen_unhandled_problem_type, 342
{\tt rescode.err\_no\_primal\_infeas\_cer},\,341
                                              {\tt rescode.err\_server\_connect},\,346
rescode.err_no_snx_for_bas_sol, 341
                                              rescode.err_server_protocol, 346
rescode.err_no_solution_in_callback, 341
                                              rescode.err_server_status, 346
rescode.err_non_unique_array, 343
                                              rescode.err_server_token, 346
rescode.err_nonconvex, 337
                                              rescode.err_size_license, 329
rescode.err_nonlinear_equality, 337
                                              rescode.err_size_license_con, 329
rescode.err_nonlinear_functions_not_allowed, rescode.err_size_license_intvar, 329
                                              rescode.err_size_license_numcores, 343
{\tt rescode.err\_nonlinear\_ranged}, \, 337
                                              rescode.err_size_license_var, 329
rescode.err_nr_arguments, 334
                                              rescode.err_sol_file_invalid_number, 338
rescode.err_null_env, 331
                                              rescode.err_solitem, 335
rescode.err_null_pointer, 331
                                              rescode.err_solver_probtype, 336
rescode.err_null_task, 331
                                              rescode.err_space, 330
rescode.err_numconlim, 335
                                              rescode.err_space_leaking, 331
rescode.err_numvarlim, 335
                                              rescode.err_space_no_info, 331
rescode.err_obj_q_not_nsd, 337
                                              rescode.err_sym_mat_duplicate, 343
rescode.err_obj_q_not_psd, 337
                                              rescode.err_sym_mat_huge, 340
rescode.err_objective_range, 336
                                              rescode.err_sym_mat_invalid, 340
rescode.err_older_dll, 330
                                              rescode.err_sym_mat_invalid_col_index, 343
rescode.err_open_dl, 330
                                              rescode.err_sym_mat_invalid_row_index, 343
rescode.err_opf_format, 333
                                              rescode.err_sym_mat_invalid_value, 343
rescode.err_opf_new_variable, 333
                                              rescode.err_sym_mat_not_lower_tringular,
rescode.err_opf_premature_eof, 333
                                                      343
rescode.err_optimizer_license, 329
                                              rescode.err_task_incompatible, 341
rescode.err_overflow, 340
                                              rescode.err_task_invalid, 341
rescode.err_param_index, 334
                                              rescode.err_task_write, 341
rescode.err_param_is_too_large, 334
                                              rescode.err_thread_cond_init, 330
rescode.err_param_is_too_small, 334
                                              rescode.err_thread_create, 330
rescode.err_param_name, 334
                                              rescode.err_thread_mutex_init, 330
                                              rescode.err_thread_mutex_lock, 330
rescode.err_param_name_dou, 334
                                              rescode.err_thread_mutex_unlock, 330
rescode.err_param_name_int, 334
rescode.err_param_name_str, 334
                                              rescode.err_toconic_constr_not_conic, 345
rescode.err_param_type, 334
                                              rescode.err_toconic_constr_q_not_psd, 345
rescode.err_param_value_str, 334
                                              rescode.err_toconic_constraint_fx, 345
```

```
rescode.err_toconic_constraint_ra, 345
rescode.err_toconic_objective_not_psd, 346
rescode.err_too_small_max_num_nz, 335
rescode.err_too_small_maxnumanz, 336
rescode.err_unb_step_size, 342
rescode.err_undef_solution, 336
rescode.err_undefined_objective_sense, 339
rescode.err_unhandled_solution_status, 344
rescode.err_unknown, 330
rescode.err_upper_bound_is_a_nan, 338
rescode.err_upper_triangle, 344
rescode.err_user_func_ret, 339
rescode.err_user_func_ret_data, 339
rescode.err_user_nlo_eval, 339
rescode.err_user_nlo_eval_hessubi, 339
rescode.err_user_nlo_eval_hessubj, 339
rescode.err_user_nlo_func, 339
rescode.err_whichitem_not_allowed, 335
rescode.err_whichsol, 335
rescode.err_write_lp_format, 333
rescode.err_write_lp_non_unique_name, 333
rescode.err_write_mps_invalid_name, 333
rescode.err_write_opf_invalid_var_name, 333
rescode.err_writing_file, 333
rescode.err_xml_invalid_problem_type, 342
rescode.err_y_is_undefined, 339
```

# **INDEX**

A	lower limit, 117
attaching	upper limit, 117
streams, 17	convex interior-point
Streams, 17	optimizers, 133
В	cqo1
basic	example, 27
solution, 49	cut, 135
basis identification, 69, 125	D
basis type	D
sensitivity analysis, 150	decision
BLAS, 77	variables, 117
bound	defining
constraint, 13, 109	objective, 17
linear optimization, 13	determinism, 86, 122
variable, 13, 109	dual
	certificate, 111, 114, 115, 117
C	cone, 113
callback, 57	feasible, 110
CBF format, 402	infeasible, 110, 111, 114, 115, 117
certificate, 50	problem, 109, 113, 114
dual, 111, 114, 115, 117	solution, 51
primal, 111, 113, 115	variable, 110, 113
Cholesky factorization, 78, 99	duality
CLASSPATH, 7	conic, 113
column ordered	gap, 110
matrix format, 162	linear, 109
complementarity, 110	semidefinite, 114
cone	dualizer, 120
dual, 113	Г
quadratic, 26, 112	E
rotated quadratic, 26, 112	eliminator, 120
semidefinite, 31, 114	environment variable
conic optimization, 26, 112	CLASSPATH, 7
infeasibility, 113	error
interior-point, 129	optimization, 49
termination criteria, 130	errors, 53
conic problem	example
example, 27	conic problem, 27
conic quadratic optimization, 26	cqo1, 27
Conic quadratic reformulation, 81	lo1, 17
constraint	qo1, 20
bound, 13, 109	quadratic objective, 20
linear optimization, 13	exceptions, 53
matrix, 13, 109, 117	Г
quadratic, 116	F
constraints	factor model, 99

feasible	conic optimization, 129
dual, 110	linear optimization, 122
primal, 109, 123, 130	logging, 126, 132
problem, 109	optimizer, 122, 129
format, 55	solution, 49
CBF, 402	termination criteria, 124, 130
json, 418	interior-point optimizer, 133
LP, 376	
MPS, 381	J
OPF, 393	json format, 418
OSiL, 417	•
sol, 425	L
task, 417	LAPACK, 77
full	license, 87
vector format, 161	linear
	objective, 17
G	linear constraint matrix, 13
gap	linear dependency, 120
duality, 110	linear optimization, 13, 109
duality, 110	bound, 13
H	
	constraint, 13
hot-start, 127	infeasibility, 111
	interior-point, 122
	objective, 13
I/O, 55	simplex, 127
infeasibility, 50, 111, 113, 115	termination criteria, 124, 127
conic optimization, 113	variable, 13
linear optimization, 111	linearity interval, 150
semidefinite, 115	lo1
infeasible, 142	example, 17
dual, 110, 111, 114, 115, 117	logging, 54
primal, 109, 111, 113, 115, 123, 130	integer optimizer, 137
problem, 109, 111, 113, 115	interior-point, 126, 132
infeasible problems, 142	optimizer, 126, 128, 132
information item, 56, 58	simplex, 128
installation, 6	lower limit
nmake (command), 8, 9	constraints, 117
requirements, 6	variables, 118
troubleshooting, 6	LP format, 376
integer	
optimizer, 134	M
solution, 49	market impact cost, 100
variable, 35	Markowitz
integer feasible	model, 89
solution, 136	Markowitz model
integer optimization, 35, 134	portfolio optimization, 89
cut, 135	matrix
delayed termination criteria, 136	constraint, 13, 109, 117
initial solution, 38	semidefinite, 31
objective bound, 135	symmetric, 31 matrix format
optimality gap, 137	
parameter, 35	column ordered, 162
relaxation, 135	row ordered, 162
termination criteria, 136	triplets, 162
tolerance, 136	memory management, 85
integer optimizer	MIP, see integer optimization
logging, 137	mixed-integer, see integer
interior-point	

460 Index

mixed-integer optimization, see integer optimiza-	slippage cost, 100
tion	positive semidefinite, 20
model	presolve, 119
Markowitz, 89	eliminator, 120
portfolio optimization, 89	linear dependency check, 120
modelling	numerical issues, 120
design, 9	primal
MPS format, 381	certificate, 111, 113, 115
free, 393	feasible, 109, 123, 130
N.I.	infeasible, 109, 111, 113, 115, 123, 130
N	problem, 109, 113, 114
near-optimal	solution, 51, 109
solution, 50, 125, 132, 136	primal-dual
numerical issues	problem, 122, 129
presolve, 120	solution, 110
scaling, 121	problem
simplex, 128	dual, 109, 113, 114
	feasible, 109
0	infeasible, 109, 111, 113, 115
objective, 109	load, 55
defining, 17	primal, 109, 113, 114
linear, 17	primal-dual, 122, 129
	save, 55
linear optimization, 13	status, 49
objective bound, 135	unbounded, 111
objective vector, 117	unbounded, 111
OPF format, 393	Q
optimal	
solution, 50, 110	qo1
optimality gap, 137	example, 20
optimization	quadratic
conic quadratic, 112	constraint, 116
error, 49	quadratic cone, 26, 112
linear, 13, 109	quadratic objective
semidefinite, 114	example, 20
optimizer	quadratic optimization, 116
determinism, 86, 122	quality
integer, 134	solution, 137
interior-point, 122, 129	D
interrupt, 57	R
logging, 126, 128, 132	relaxation, 135
parallelization, 121	response code, 53
selection, 120, 122	rotated quadratic cone, 26, 112
simplex, 127	row ordered
optimizers	matrix format, 162
convex interior-point, 133	6
OSiL format, 417	S
D	scaling, 121
P	scopt, 372
pair sensitivity analysis	semidefinite
optimal partition type, 150	cone, 31, 114
parallelization, 86, 121	infeasibility, 115
parameter, 55	matrix, 31
integer optimization, 35	variable, 31, 114
simplex, 128	semidefinite optimization, 31, 114
portfolio optimization	sensitivity analysis, 148
factor model, 99	basis type, 150
market impact cost, 100	separable convex optimization, 65
model, 89	shadow price, 150
,	. r r

Index 461

simplex	U
linear optimization, 127	unbounded
logging, 128	problem, 111
numerical issues, 128	upper limit
optimizer, 127	constraints, 117
parameter, 128	variables, 118
termination criteria, 127	user callback, see callback
slippage cost, 100	
sol format, 425	V
solution	variable, 109
basic, 49	bound, 13, 109
dual, 51	dual, 110, 113
file format, 425	integer, 35
integer, 49	linear optimization, 13
integer feasible, 136	semidefinite, 31, 114
interior-point, 49	variables
near-optimal, 50, 125, 132, 136	decision, 117
optimal, 50, 110	lower limit, 118
primal, 51, 109	upper limit, 118
primal-dual, 110	vector format
quality, 137	full, 161
retrieve, 49	sparse, 162
status, 16, 50	
solution summary, 44, 47 solving linear system, 73	
sparse	
vector format, 162	
sparse vector, 162	
status	
problem, 49	
solution, 16, 50	
streams	
attaching, 17	
symmetric	
matrix, 31	
,	
T	
task format, 417	
termination, 49	
termination criteria, 57	
conic optimization, 130	
delayed, 136	
integer optimization, 136	
interior-point, 124, 130	
linear optimization, 124, 127	
simplex, 127	
tolerance, 125, 132, 136	
thread, 86, 121	
time limit, 57	
tolerance	
integer optimization, 136	
termination criteria, 125, 132, 136	
triplets	
matrix format, 162	
troubleshooting	
installation, 6	

462 Index