

# **MOSEK Command Line Tools**

Release 8.1.0.83

MOSEK ApS

## **CONTENTS**

1	Introduction 1.1 Why the Command Line Tools?	1 2
<b>2</b>	Contact Information	3
3	License Agreement	5
4	Installation 4.1 Testing the installation	<b>7</b>
5	5.4 Solver Parameters	9 9 10 12
6	6.1 Locating the AMPL shell 6.2 An example 6.3 Retrieving solutions 6.4 Optimizer options 6.5 Hot-start 6.6 Infeasibility report 6.7 Sensitivity analysis	15 15 17 18 19 21 21
7	7.1 Linear Optimization       2         7.2 Conic Quadratic Optimization       2         7.3 Semidefinite Optimization       3         7.4 Quadratic and Quadratically Constrained Optimization       3	25 28 30 32 33
8	8.1 Presolve	35 35 37 38 45 49
9	9.1 The Mixed-integer Optimizer Overview	51 51 51 52

	9.4 9.5 9.6	Speeding Up the Solution Process	53
10	Prob	lem Analyzer	55
11	11.1	Example: Primal Infeasibility	59 60 61 61 64 65 65
<b>12</b>	12.1	Sensitivity Analysis for Linear Problems	<b>69</b> 69 75
13	13.1 13.2 13.3		28
14	Supp 14.1 14.2 14.3 14.4 14.5 14.6 14.7 14.8	orted File Formats         The LP File Format       1         The MPS File Format       1         The OPF Format       1         The CBF Format       2         The XML (OSiL) Format       2         The Task Format       2         The JSON Format       2         The Solution File Format       2	.83 .95 .04 .219 .219 .220
<b>15</b>	List	of examples 2	31
	16.1 16.2 16.3		235
Bi	bliog	aphy 2	43
$\mathbf{S}\mathbf{y}$	mbol	Index 2	<b>45</b>
Tn	dov	2	53

## INTRODUCTION

The **MOSEK** Optimization Suite 8.1.0.83 is a powerful software package capable of solving large-scale optimization problems of the following kind:

- linear,
- conic quadratic (also known as second-order cone),
- convex quadratic,
- semidefinite,
- and general convex.

Integer constrained variables are supported for all problem classes except for semidefinite and general convex problems. In order to obtain an overview of features in the **MOSEK** Optimization Suite consult the product introduction guide.

The most widespread class of optimization problems is *linear optimization problems*, where all relations are linear. The tremendous success of both applications and theory of linear optimization can be ascribed to the following factors:

- The required data are simple, i.e. just matrices and vectors.
- Convexity is guaranteed since the problem is convex by construction.
- Linear functions are trivially differentiable.
- There exist very efficient algorithms and software for solving linear problems.
- Duality properties for linear optimization are nice and simple.

Even if the linear optimization model is only an approximation to the true problem at hand, the advantages of linear optimization may outweigh the disadvantages. In some cases, however, the problem formulation is inherently nonlinear and a linear approximation is either intractable or inadequate. *Conic optimization* has proved to be a very expressive and powerful way to introduce nonlinearities, while preserving all the nice properties of linear optimization listed above.

The fundamental expression in linear optimization is a linear expression of the form

$$Ax - b \in \mathcal{K}$$

where  $\mathcal{K} = \{y : y \ge 0\}$ , i.e.,

$$Ax - b = y, y \in \mathcal{K}.$$

In conic optimization a wider class of convex sets  $\mathcal{K}$  is allowed, for example in 3 dimensions  $\mathcal{K}$  may correspond to an ice cream cone. The conic optimizer in **MOSEK** supports three structurally different types of cones  $\mathcal{K}$ , which allows a surprisingly large number of nonlinear relations to be modelled (as described in the **MOSEK** modeling cookbook), while preserving the nice algorithmic and theoretical properties of linear optimization.

## 1.1 Why the Command Line Tools?

The **MOSEK** capabilities can be accessed from the command line without the need to use any programming language. The user can input optimization problems using files in a variety of *formats*, or via the AMPL language shell.

The Command Line Tools provides access to:

- Linear Optimization (LO)
- Conic Quadratic (Second-Order Cone) Optimization (CQO, SOCO)
- Convex Quadratic and Quadratically Constrained Optimization (QCQO)
- Semidefinite Optimization (SDO)
- General Convex Optimization problems (via AMPL).

as well as to additional utilities for:

- problem analysis,
- sensitivity analysis,
- infeasibility diagnostics.

## **TWO**

## **CONTACT INFORMATION**

Phone	$+45\ 7174\ 9373$	
Website	mosek.com	
Email		
	sales@mosek.com	Sales, pricing, and licensing
	support@mosek.com	Technical support, questions and bug reports
	info@mosek.com	Everything else.
Mailing Address		
	MOSEK ApS	
	Fruebjergvej 3	
	Symbion Science Park, Box 16	
	2100 Copenhagen O	
	Denmark	

You can get in touch with  $\mathbf{MOSEK}$  using popular social media as well:

Blogger	http://blog.mosek.com/
Google Group	https://groups.google.com/forum/#!forum/mosek
Twitter	https://twitter.com/mosektw
$\mathbf{Google} +$	$\rm https://plus.google.com/+Mosek/posts$
Linkedin	https://www.linkedin.com/company/mosek-aps

In particular **Twitter** is used for news, updates and release announcements.

## LICENSE AGREEMENT

Before using the MOSEK software, please read the license agreement available in the distribution at MOSEK website https://mosek.com/products/license-agreement.

MOSEK uses some third-party open-source libraries. Their license details follows.

#### zlib

**MOSEK** includes the zlib library obtained from the zlib website. The license agreement for zlib is shown in Listing 3.1.

### Listing 3.1: zlib license.

zlib.h - interface of the 'zlib' general purpose compression library version 1.2.7, May 2nd, 2012

Copyright (C) 1995-2012 Jean-loup Gailly and Mark Adler

This software is provided 'as-is', without any express or implied warranty. In no event will the authors be held liable for any damages arising from the use of this software.

Permission is granted to anyone to use this software for any purpose, including commercial applications, and to alter it and redistribute it freely, subject to the following restrictions:

- The origin of this software must not be misrepresented; you must not claim that you wrote the original software. If you use this software in a product, an acknowledgment in the product documentation would be appreciated but is not required.
- 2. Altered source versions must be plainly marked as such, and must not be misrepresented as being the original software.
- 3. This notice may not be removed or altered from any source distribution.

Jean-loup Gailly Mark Adler

jloup@gzip.org madler@alumni.caltech.edu

### fplib

**MOSEK** includes the floating point formatting library developed by David M. Gay obtained from the netlib website. The license agreement for *fplib* is shown in Listing 3.2.

### Listing 3.2: fplib license.

## **INSTALLATION**

In this section we discuss how to install and setup the MOSEK Command Line Tools.

Important: Before running this MOSEK interface please make sure that you:

- Installed MOSEK correctly. Some operating systems require extra steps. See the Installation guide for instructions and common troubleshooting tips.
- Set up a license. See the Licensing guide for instructions.

## **Locating Files**

The files in Command Line Tools are organized as reported in Table 4.1.

Table 4.1: Relevant files for the Command Line Tools.

Relative Path	Description	Label
<pre><mskhome>/mosek/8/tools/platform/<platform>/bin</platform></mskhome></pre>	Binaries	<bindir></bindir>
<pre><mskhome>/mosek/8/tools/platform/<platform>/bin/mosek</platform></mskhome></pre>	Mosek executable	
<mskhome>/mosek/8/tools/examples/data</mskhome>	Examples	<exdir></exdir>

### where

- <MSKHOME> is the folder in which the MOSEK package has been installed,
- <PLATFORM> is the actual platform among those supported by MOSEK, i.e. win32x86, win64x86, linux64x86 or osx64x86.

## Setting up paths

The executable file is ready for use. It may be convenient to add the directory <BINDIR> to the environment variable PATH, and then MOSEK can simply be used by typing

mosek

in the command line.

## 4.1 Testing the installation

To test that Command Line Tools has been installed correctly go to the examples directory <EXDIR> and run MOSEK on any of the input files, for example lo1.mps:

mosek lo1.mps

Is should produce output similar to:

```
MOSEK Version 8.0.0.53 (Build date: 2017-1-12 22:21:45)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86
Open file 'lo1.mps'
Reading started.
[....]
Optimizer started.
Interior-point optimizer started.
[....]
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
                                               Viol. con: 1e-08
                                                                  var: 0e+00
 Primal. obj: 8.3333333280e+01
                                 nrm: 5e+01
          obj: 8.3333333242e+01 nrm: 4e+00
                                             Viol. con: 2e-10 var: 5e-09
 Dual.
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 8.333333333e+01 nrm: 5e+01 Viol. con: 7e-15 var: 0e+00
 Dual. obj: 8.3333333245e+01 nrm: 4e+00 Viol. con: 2e-10 var: 5e-09
[....]
Open file 'lo1.sol'
Start writing.
done writing. Time: 0.00
Open file 'lo1.bas'
Start writing.
done writing. Time: 0.00
Return code - 0 [MSK_RES_OK]
```

**CHAPTER** 

**FIVE** 

## THE COMMAND LINE TOOL

## 5.1 Introduction

The MOSEK command line tool is used to solve optimization problems from the operating system command line. It is invoked as follows

```
mosek [options] [filename]
```

where both [options] and [filename] are optional arguments:

- [options] consists of command line arguments that modify the behavior of MOSEK. They are listed in Sec. 5.5. In particular, options can be used to set optimizer parameters.
- [filename] is a file describing the optimization problem. The MOSEK command line accepts files in any of the *supported file formats* or in the AMPL .nl format.

If no arguments are given, MOSEK will display a splash screen and exit.

```
user@host:~$ mosek/8/tools/platform/linux64x86/bin/mosek

MOSEK Version 8.0.0.32(BETA) (Build date: 2016-7-12 10:29:26)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86

*** No input file specified. No optimization is performed.
Return code - 0 [MSK_RES_OK]
```

### 5.2 Files

The MOSEK command line tool communicates with the user via files and prints some execution logs and solution summary to the terminal.

### Input files

Optimization problems are read from files. See Sec. 14 for details.

### File format conversion

To convert between two file formats supported by  $\mathbf{MOSEK}$  use the option -x together with -out to specify the target file name. The target file type must support the problem type of the source file, otherwise the conversion will be partial. For instance in case a MPS file must be converted in a more readable OPF format, the following line can be used

```
mosek -x -out lo1.opf lo1.mps
```

With the -x option the solver will not actually solve the problem.

### **Output files**

Solutions are written to files:

- .bas basic solution,
- .sol interior point solution,
- .itg integer solution (the only available solution for mixed-integer problems).

For linear problems both the basic and interior point solution may be present. Infeasibility certificates are stored in the same files. See Sec. 14.8 for details.

## 5.3 Example

To solve a problem stored in file, say lol.mps, write:

```
mosek lo1.mps
```

The solver will

- read lo1.mps from disk,
- solve the problem and display the solution log and
- store the relevant solution files if any solution exists; file content explained in Sec. 14.8.

```
MOSEK Version 8.0.0.34(BETA) (Build date: 2016-8-24 00:51:13)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86
Open file '/home/andrea/mosek/8/tools/examples/data/lo1.mps'
Reading started.
Using 'obj' as objective vector
Read 13 number of A nonzeros in 0.00 seconds.
Using 'rhs' as rhs vector
Using 'bound' as bound vector
Reading terminated. Time: 0.00
Read summary
                  : LO (linear optimization problem)
 Type
 Objective sense : max
 Scalar variables: 4
 Matrix variables : 0
 Constraints : 3
 Cones
                  : 0
 Time
                  : 0.0
Problem
 Name
                        : lo1
  Objective sense
                        : max
                        : LO (linear optimization problem)
 Туре
  Constraints
                        : 3
                        : 0
 Scalar variables
                        : 4
 Matrix variables
                        : 0
  Integer variables
                        : 0
```

```
Optimizer started.
Interior-point optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator: 0
Eliminator terminated.
Eliminator - tries
                                : 1
                                                    time
                                                                         : 0.00
Lin. dep. - tries
                                : 1
                                                    time
                                                                         : 0.00
Lin. dep. - number
Presolve terminated. Time: 0.00
Optimizer - Consec : 2

Optimizer - Constraints : 3

Optimizer - Consec
                                : 0
Optimizer - Cones
Optimizer - Scalar variables : 6
                                                                         : 0
                                                   conic
                                                  scalarized
                                                                        : 0
Optimizer - Semi-definite variables: 0
                                                                        : 0.00
Factor - setup time : 0.00
Factor - ML order time : 0.00
                                                 dense det. time
                                : 0.00
Factor
          - ML order time
                                                 GP order time
                                                                         : 0.00
                                                   after factor
Factor
         - nonzeros before factor : 6
                                                                        : 6
                                                  flops
Factor
          - dense dim.
                          : 0
                                                                         : 1.06e+02
ITE PFEAS DFEAS GFEAS PRSTATUS POBJ
                                                       DOBJ
                                                                      MU
                                                                                TIME
0 8.0e+00 3.2e+00 3.5e+00 1.00e+00 1.000000000e+01 0.00000000e+00 1.0e+00 0.01
  4.2e+00 2.5e+00 4.7e-01 0.00e+00 3.093970927e+01 2.766058702e+01 2.6e+00 0.01
2 4.2e-01 2.5e-01 4.6e-02 -1.82e-02 6.511676243e+01 6.308843559e+01 2.6e-01 0.01
3 3.6e-02 2.1e-02 3.9e-03 5.84e-01 8.096141239e+01 8.061962333e+01 2.2e-02 0.01
4 1.5e-05 9.1e-06 1.7e-06 9.43e-01 8.333280389e+01 8.333241803e+01 9.2e-06 0.01
5 1.5e-09 9.1e-10 1.7e-10 1.00e+00 8.333333328e+01 8.333333324e+01 9.2e-10 0.01
Basis identification started.
Primal basis identification phase started.
TTER.
         TIME.
         0.00
Primal basis identification phase terminated. Time: 0.00
Dual basis identification phase started.
TTER
        TTMF.
         0.00
Dual basis identification phase terminated. Time: 0.00
Basis identification terminated. Time: 0.00
Interior-point optimizer terminated. Time: 0.01.
Optimizer terminated. Time: 0.02
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
  Solution status : OPTIMAL
 Primal. obj: 8.3333333280e+01 nrm: 5e+01
                                              Viol. con: 1e-08 var: 0e+00
          obj: 8.3333333242e+01 nrm: 4e+00 Viol. con: 2e-10 var: 5e-09
 Dual.
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
  Solution status : OPTIMAL
  Primal. obj: 8.3333333333e+01 nrm: 5e+01 Viol. con: 7e-15 var: 0e+00
  Dual. obj: 8.3333333245e+01 nrm: 4e+00 Viol. con: 2e-10 var: 5e-09
Optimizer summary
                                                time: 0.02
 Optimizer
   Interior-point
                                               time: 0.01
                         - iterations : 5
                                               time: 0.00
     Basis identification -
                                               time: 0.00
       Primal - iterations : 0
       Dual
                         - iterations : 0
                                                time: 0.00
       Clean primal
                       - iterations : 0
                                                time: 0.00
```

5.3. Example 11

```
Clean dual
                            - iterations : 0
                                                     time: 0.00
    Simplex
                                                     time: 0.00
     Primal simplex
                                                     time: 0.00
                           - iterations : 0
     Dual simplex
                           - iterations : 0
                                                     time: 0.00
   Mixed integer
                            - relaxations: 0
                                                     time: 0.00
Open file '/home/andrea/mosek/8/tools/examples/data/lo1.sol'
Start writing.
done writing. Time: 0.00
Open file '/home/andrea/mosek/8/tools/examples/data/lo1.bas'
Start writing.
done writing. Time: 0.00
Return code - 0 [MSK_RES_OK]
```

### 5.4 Solver Parameters

**MOSEK** comes with a large number of parameters that allows the user to tune the behavior of the optimizer. The typical settings which can be changed with solver parameters include:

- choice of the optimizer for linear problems,
- choice of primal/dual solver,
- turning presolve on/off,
- turning heuristics in the mixed-integer optimizer on/off,
- level of multi-threading,
- feasibility tolerances,
- solver termination criteria,
- behaviour of the license manager,

and more. All parameters have default settings which will be suitable for most typical users. Each parameter is identified by a unique string name and it can accept either integers or symbolic names, floating point values or symbolic strings. Please refer to Sec. 13.2 for the complete list of available solver parameters.

## 5.4.1 Setting from command line

Setting solver parameters is possible using the command line option -d. If multiple parameters must be specified, option -d must be repeated for each one. For example, the next command will switch off presolve, set a feasibility tolerance and solve the problem from lol.opf:

```
mosek -d MSK_IPAR_PRESOLVE_USE MSK_OFF -d MSK_DPAR_INTPNT_TOL_PFEAS 1.0e-8 lo1.opf
```

## 5.4.2 Using the Parameter File

Solver parameters can also be set using a parameter file, for example:

```
BEGIN MOSEK
% This is a comment.
% The subsequent line tells MOSEK that an optimal
% basis should be computed by the interior-point optimizer.
MSK_IPAR_PRESOLVE_USE MSK_OFF
```

```
MSK_DPAR_INTPNT_TOL_PFEAS 1.0e-9
END MOSEK
```

The syntax of the parameter file must obey a few simple rules:

- The file must begin with BEGIN MOSEK and end with END MOSEK.
- Empty lines and lines starting from a % sign are ignored.
- Each line contains a valid MOSEK parameter name followed by its value.

The parameter file can have any name. Assuming it has been called mosek.par, it can be used using the -p option as follows:

```
mosek -p mosek.par afiro.mps
```

Command-line parameters override those from the parameter file in case of repetition. For instance

```
mosek -p mosek.par -d MSK_DPAR_INTPNT_TOL_PFEAS 1.0e-8 afiro.mps
```

will set  $MSK_DPAR_INTPNT_TOL_PFEAS$  to  $10^{-8}$  using the value provided on the command line.

## 5.5 Command Line Arguments

The following list shows the available command-line arguments for MOSEK:

-anapro

Analyze the problem data.

-anasoli <name>

Analyze the initial solution name e.g. -anasoli bas.

-anasolo <name>

Analyze the final solution name e.g. -anasolo itg.

-a

**MOSEK** is started in AMPL mode.

-basi <name>

Input basic solution file name.

-baso <name>

Output basic solution file name.

-d <name> <value>

Define the value value for the MOSEK parameter name.

-dbgmem <name>

Name of memory debug file.

-f

Complete license information is printed.

-h, -?

Help.

-inti <name>

Input integer solution file name.

-into <name>

Output integer solution file name.

-itri <name>

Input interior point solution file name.

#### -itro <name>

Output interior point solution file name.

### -info <name>

Infeasible subproblem output file name.

### -infrepo <name>

Feasibility reparation output file.

### -1,-L <dir>

dir is the directory where the MOSEK license file mosek.lic is located.

#### -max

The problem is maximized.

#### -min

The problem is minimized.

-n

Ignore errors in subsequent parameter settings.

### -out <name>

Write the task to a data file named name. See Sec. 14.

### -p <name>, -pari <name>

Name of the input parameter file.

### -paro <name>

Name of the output parameter file.

-r

If the option is present, the program returns -1 if an error occurred, otherwise 0.

### -removeitg

Removes all integer constraints after reading the problem.

### -rout <name>

If the option is present, the program writes the return code to file name.

### -q <name>

Name of an optional log file.

### -sen <file>

Perform sensitivity analysis based on file.

### -silent

As little information as possible is send to the terminal.

### -toconic

Translate to conic form after reading.

-v

 $\mathbf{MOSEK}$  version is printed and no optimization is performed.

– w

If this options is on, then MOSEK will wait for a license.

-x

Do not run the optimizer. Useful for converting between file formats.

-=

List all possible solver parameters with default value, lower bound and upper bound (if applicable).

## THE MOSEK-BUNDLED AMPL SHELL

AMPL is a modeling language for specifying linear and nonlinear optimization models in a natural way. AMPL also makes it easy to solve the problem and e.g. display the solution or part of it. We will not discuss the specifics of the AMPL language here but instead refer the reader to [FGK03], http://ampl.com/BOOK/download.html and the AMPL website http://www.ampl.com.

AMPL cannot solve optimization problems by itself but requires a link to an optimizer. The **MOSEK** distribution includes:

- An AMPL link which makes it possible to use **MOSEK** as an optimizer within AMPL. The link can be used from any AMPL shell.
- The full, official AMPL shell repackaged under the name mampl. This is sold as a separate product, and it can be hooked to other optimizers as well.

### Note:

- To use MOSEK from AMPL you need to set up the system path to the MOSEK command line tool.
- It is possible to specify problems in AMPL that cannot be solved by **MOSEK**. The optimization problem must be a smooth convex optimization problem as discussed in Sec. 7.

For the remainder of this section we refer to the **MOSEK**-bundled mampl as the AMPL interpreter of choice. However, the tutorial applies also to any other standard AMPL shell available to the user.

## 6.1 Locating the AMPL shell

Assuming MSKHOME is the folder in which MOSEK has been installed, the AMPL shell is the executable file

{MSKHOME}/mosek/8/tools/platform/{PLATFORM}/bin/mampl

for Linux and OSX users (PLATFORM must be among linux64x86, osx64x86), and under

 ${\tt MSKHOME}\\ \verb|\mosek|8| tools| platform|{\tt PLATFORM}| bin| mample | tools| t$ 

for Windows users (PLATFORM must be among win32x86, win64x86).

## 6.2 An example

In many instances, you can successfully apply **MOSEK** simply by specifying the model and data, setting the solver option to **MOSEK**, and typing solve.

Consider a simple linear optimization problem formulated as an AMPL model in Listing 6.1.

Listing 6.1: An example of an optimization problem in AMPL language.

We can specify the input data using an input file again following the AMPL syntax, as in Listing 6.2.

Listing 6.2: An example of data for an optimization problem using AMPL language.

```
param: FOOD:
                             cost f_min f_max :=
 "Quarter Pounder w/ Cheese"
                            1.84
 "McLean Deluxe w/ Cheese"
                            2.19
 "Big Mac"
                            1.84
 "Filet-O-Fish"
                            1.44
 "McGrilled Chicken"
                           2.29
 "Fries, small"
                             .77
 "Sausage McMuffin"
                             1.29
 "1% Lowfat Milk"
                             .60
 "Orange Juice"
                             .72
param: NUTR: n_min n_max :=
             2000
      Cal
              350
                      375
      Carbo
      Protein 55
       Vit.A
               100
       VitC
              100
       Calc
              100
       Iron
               100
param amt (tr):
                           Cal Carbo Protein VitA VitC Calc Iron :=
 "Quarter Pounder w/ Cheese" 510 34 28 15
                                                    6 30
                                                               20
                           370
                                 35
                                                   10
                                                               20
 "McLean Deluxe w/ Cheese"
                                        24
                                             15
                                                          20
 "Big Mac"
                           500
                                 42
                                        25
                                              6
                                                    2
                                                          25
                                                               20
 "Filet-O-Fish"
                           370
                                  38
                                        14
                                               2
                                                     0
                                                          15
                                                               10
 "McGrilled Chicken"
                          400
                                  42
                                        31
                                               8
                                                    15
                                                          15
                                                                8
 "Fries, small"
                           220
                                  26
                                        3
                                               0
                                                    15
                                                          0
                                                                2
 "Sausage McMuffin"
                                  27
                                        15
                                                          20
                           345
                                               4
                                                     0
                                                               15
 "1% Lowfat Milk"
                           110
                                  12
                                        9
                                              10
                                                     4
                                                          30
```

80 20 1 2 120 2 2;
--------------------

Invoke the AMPL shell:

```
mampl
```

and type in the commands:

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver mosek;
ampl: solve;
```

The resulting output is:

```
MOSEK finished.

Problem status - PRIMAL_AND_DUAL_FEASIBLE

Solution status - OPTIMAL

Primal objective - 14.8557377

Dual objective - 14.8557377

Objective = Total_Cost
```

## 6.3 Retrieving solutions

### 6.3.1 Status codes

The AMPL parameter solve\_result\_num is used to indicate the outcome of the optimization process. It is used as follows

```
ampl: display solve_result_num
```

Please refer to table Table 6.1 for possible values of this parameter.

Table 6.1: Interpretation of solve\_result\_num.

Value	Message
0	the solution is optimal.
100	suboptimal primal solution.
101	superoptimal (dual feasible) solution.
150	the solution is near optimal.
200	primal infeasible problem.
300	dual infeasible problem.
400	too many iterations.
500	solution status is unknown.
501	ill-posed problem, solution status is unknown.
> 501	Mapped MOSEK response code. See note below.

MOSEK response codes are mapped to AMPL return codes greater than 501. In order to get the actual response code the base value 501 must be subtracted. For example: the AMPL return code 502 corresponds to MOSEK response code 1.

### 6.3.2 Which solution is returned

**MOSEK** can produce three types of solutions: basic, interior point and integer. The solution returned to AMPL is determined according to the following rules:

- For problems containing integer variables only the integer solution is available and it is returned.
- For nonlinear problems only the interior point solution is available and it is returned.
- For linear problems, if both basic and interior point solution are available, then the basic solution is returned. Otherwise the only available solution is returned.

## 6.4 Optimizer options

## 6.4.1 The MOSEK parameter database

The **MOSEK** optimizer can be controller using solver parameters, as described in Sec. 5.4. These parameters can be modified within AMPL as shown in the example below:

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver mosek;
ampl: option mosek_options
ampl? 'msk_ipar_optimizer = msk_optimizer_primal_simplex \
ampl? msk_ipar_sim_max_iterations = 100000';
ampl: solve;
```

In the example above a string called mosek\_options is created which contains the parameter settings. Each parameter setting has the format

```
parameter_name = value
```

where parameter\_name is a valid MOSEK parameter name. See Sec. 13.2 for a description of all valid MOSEK parameters.

An alternative way of specifying the parameters is

```
ampl: option mosek_options
ampl? 'msk_ipar_optimizer = msk_optimizer_primal_simplex'
ampl? 'msk_ipar_sim_max_iterations = 1000000';
```

New parameters can also be appended to an existing option string as shown below.

```
ampl: option mosek_options $mosek_options
ampl? ' msk_ipar_sim_print_freq = 0 msk_ipar_sim_max_iterations = 1000';
```

The expression \$mosek\_options expands to the current value of the option. Line two in the example appends an additional value msk\_ipar\_sim\_max\_iterations to the option string.

### 6.4.2 Options

MOSEK recognizes the following AMPL options.

### outlev

Controls the amount of printed output. 0 means no printed output and a higher value means progressively more output. An example of setting outlev is as follows:

```
ampl: option mosek_options 'outlev=2';
```

### wantsol

Controls the solution information generated when run in standalone mode (called without the argument -AMPL). It should be constructed as the sum of

1	to write a .sol file
2	to print the primal variable values
4	to print the dual variable values
8	to suppress printing the solution message

We refer the reader to the AMPL manual [FGK03] for more details.

## 6.4.3 Passing variable names to MOSEK

AMPL assigns meaningful names to all the constraints and variables. Since **MOSEK** uses item names in error and log messages, it may be useful to pass the AMPL names to **MOSEK**. This can be achieved with the command:

```
ampl: option auxfiles rc;
ampl: solve;
```

## 6.5 Hot-start

Frequently, a sequence of optimization problems is solved where each problem differs only slightly from the previous problem. In that case it may be advantageous to use the previous optimal solution to warm-start the optimizer. Such a facility is available in **MOSEK** only when the simplex optimizer is used.

The warm-start facility exploits the AMPL variable suffix sstatus to communicate the optimal basis back to AMPL, and AMPL uses this facility to communicate an initial basis to MOSEK. The following example demonstrates this feature.

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver mosek;
ampl: option mosek_options
ampl? 'msk_ipar_optimizer = msk_optimizer_primal_simplex outlev=2';
ampl: solve;
ampl: display Buy.sstatus;
ampl: solve;
```

The resulting output is:

```
Accepted: msk_ipar_optimizer
                                               = MSK_OPTIMIZER_PRIMAL_SIMPLEX
Accepted: outlev
                                               = 2
Computer
           - Platform
                                     : Linux/64-X86
Computer
           - CPU type
                                     : Intel-P4
MOSEK
           - task name
MOSEK
           - objective sense
                                    : min
MOSEK
                                    : LO (linear optimization problem)

    problem type

MOSEK
                                    : 7
                                                                                 : 9
           - constraints
                                                         variables
MOSEK
           - integer variables
                                     : 0
Optimizer started.
Simplex optimizer started.
Presolve started.
Linear dependency checker started.
```

6.5. Hot-start

```
Linear dependency checker terminated.
Presolve - Stk. size (kb) : 0
Eliminator - tries
                                    : 0
                                                        time
                                                                               : 0.00
Eliminator - elim's
                                   : 0
Lin. dep. - tries
Lin. dep. - number
                                   : 1
                                                        time
                                                                              : 0.00
                                   : 0
Presolve terminated. Time: 0.00
Primal simplex optimizer started.
Primal simplex optimizer setup started.
Primal simplex optimizer setup terminated.
Optimizer - solved problem : the primal
Optimizer - constraints
                                  : 7
                                                       variables
                                                                               : 9
Optimizer - hotstart
                                  : no
        DEGITER(%) PFEAS DFEAS
ITER
                                           POBJ
                                                                    DOBJ
                                                                                          TIME

→ TOTTIME

      0.00
                  1.40e+03 NA
0
                                            1.2586666667e+01
                                                                                          0.00
                                                                    NA
   0.01
        0.00
                    0.00e+00
                               NA
                                             1.4855737705e+01
                                                                                          0.00
3
                                                                   NA
      0.01
\hookrightarrow
Primal simplex optimizer terminated.
Simplex optimizer terminated. Time: 0.00.
Optimizer terminated. Time: 0.01
Return code - 0 [MSK_RES_OK]
MOSEK finished.
Problem status : PRIMAL_A
Solution status : OPTIMAL
                 : PRIMAL_AND_DUAL_FEASIBLE
Primal objective : 14.8557377
Dual objective : 14.8557377
Objective = Total_Cost
Buy.sstatus [*] :=
'Quarter Pounder w/ Cheese' bas
'McLean Deluxe w/ Cheese' low
'Big Mac' low
Filet-O-Fish low
'McGrilled Chicken' low
'Fries, small' bas
'Sausage McMuffin' low
'1% Lowfat Milk' bas
'Orange Juice' low
Accepted: msk_ipar_optimizer
                                             = MSK_OPTIMIZER_PRIMAL_SIMPLEX
Accepted: outlev
                                              = 2
Basic solution
Problem status : UNKNOWN
Solution status : UNKNOWN
Primal - objective: 1.4855737705e+01 eq. infeas.: 3.97e+03 max bound infeas.: 2.00e+03
Dual - objective: 0.00000000000e+00 eq. infeas.: 7.14e-01 max bound infeas.: 0.00e+00
Computer - Platform
                                  : Linux/64-X86
Computer - CPU type
                                   : Intel-P4
MOSEK - task name

MOSEK - objective sense

MOSEK - problem type

MOSEK - constraints

MOSEK - integer variables
                                  : min
                                  : LO (linear optimization problem)
                                  : 7
                                                       variables
                                                                             : 9
                                  : 0
Optimizer started.
Simplex optimizer started.
Presolve started.
Presolve - Stk. size (kb) : 0
Eliminator - tries
                                   : 0
                                                       time
                                                                               : 0.00
                                   : 0
Eliminator - elim's
Lin. dep. - tries
                                    : 0
                                                        time
                                                                               : 0.00
```

```
Lin. dep. - number
Presolve terminated. Time: 0.00
Primal simplex optimizer started.
Primal simplex optimizer setup started.
Primal simplex optimizer setup terminated.
Optimizer - solved problem
                                   : the primal
Optimizer - constraints
                                   : 7
                                                        variables
                                                                               : 9
Optimizer - hotstart
                                   : yes
Optimizer - Num. basic
                                   : 7
                                                        Basis rank
                                                                               : 7
Optimizer - Valid bas. fac.
                                   : no
                                                                    DOBJ
ITER
         DEGITER(%) PFEAS
                                 DFEAS
                                              POBJ
                                                                                          TIME
      TOTTIME
0
                      0.00e+00
                                  NA
                                              1.4855737705e+01
                                                                                          0.00
         0.00
       0.01
                                              1.4855737705e+01
0
         0.00
                      0.00e+00
                                                                    NΑ
                                                                                          0.00
      0.01
Primal simplex optimizer terminated.
Simplex optimizer terminated. Time: 0.00.
Optimizer terminated. Time: 0.01
Return code - 0 [MSK RES OK]
MOSEK finished.
Problem status
                 : PRIMAL_AND_DUAL_FEASIBLE
Solution status
                 : OPTIMAL
Primal objective : 14.8557377
Dual objective
                  : 14.8557377
Objective = Total_Cost
```

Please note that the second solve takes fewer iterations since the previous optimal basis is reused.

## 6.6 Infeasibility report

For linear optimization problems without any integer constrained variables **MOSEK** can generate an infeasibility report automatically. The report provides important information about the infeasibility.

The generation of the infeasibility report is turned on using the parameter setting

```
option auxfiles rc;
option mosek_options 'msk_ipar_infeas_report_auto=msk_on';
```

For further details about infeasibility report see Sec. 11.

## 6.7 Sensitivity analysis

**MOSEK** can calculate sensitivity information for the objective and constraints. To enable sensitivity information set the option:

```
sensitivity = 1
```

Results are returned in variable/constraint suffixes as follows:

- .down Smallest value of objective coefficient/right hand side before the optimal basis changes.
- .up Largest value of objective coefficient/right hand side before the optimal basis changes.
- .current Current value of objective coefficient/right hand side.

For ranged constraints sensitivity information is returned only for the lower bound.

The example below returns sensitivity information on the diet model.

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver mosek;
ampl: option mosek_options 'sensitivity=1';

ampl: solve;
#display sensitivity information and current solution.
ampl: display _var.down,_var.current,_var.up,_var;
#display sensitivity information and optimal dual values.
ampl: display _con.down,_con.current,_con.up,_con;
```

The resulting output is:

```
Return code - 0 [MSK_RES_OK]
MOSEK finished.
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal objective : 14.8557377
Dual objective : 14.8557377
suffix up OUT;
suffix down OUT;
suffix current OUT;
Objective = Total_Cost
  _var.down _var.current
                              _var.up
                                             _var
   1.37385
               1.84
                                1.86075
                                            4.38525
                          Infinity
2
   1.8677
                2.19
                                            0
3
   1.82085
                1.84
                          Infinity
                                            0
                                            0
4
                1.44
   1.35466
                          Infinity
5
  1.57633
                2.29
                          Infinity
                                            0
                0.77
                                 0.794851
6
   0.094
                                            6.14754
7
   1.22759
                1.29
                                            Ω
                          Infinity
8
   0.57559
                 0.6
                                 0.910769
                                            3.42213
9
   0.657279
                 0.72
                          Infinity
ampl: display _con.down,_con.current,_con.up,_con;
     _con.down _con.current _con.up
                                           0
1
    -Infinity
                     2000
                                3965.37
2
         297.6
                       350
                                375
                                           0.0277049
3
   -Infinity
                        55
                                 172.029
                                          0
4
          63.0531
                       100
                                 195.388
                                           0.0267541
5
   -Infinity
                        100
                                 132.213
                                          0
6
   -Infinity
                        100
                                 234.221
                                           0
7
          17.6923
                        100
                                 142.821
                                          0.0248361
```

## 6.8 Using the command line version of the AMPL interface

AMPL can generate a data file containing the optimization problem and all relevant information which can then be read and solved by the **MOSEK** command line tool.

When the problem has been loaded into AMPL, the commands

```
ampl: option auxfiles rc;
ampl: write bprob;
```

will make AMPL write the appropriate data files, i.e.

```
prob.nl
prob.col
prob.row
```

Then the problem can be solved using the command line version of MOSEK as follows

```
mosek prob.nl outlev=10 -a
```

The option -a indicates that MOSEK is invoked in AMPL mode. When MOSEK is invoked in AMPL mode the standard MOSEK command line options should appear after the -a option except for the file name which should be the first argument. As the above example demonstrates MOSEK accepts command line options following the AMPL convention. To see which command line arguments MOSEK accepts in AMPL mode write:

```
mosek -= -a
```

For linear, quadratic and quadratically constrained problems a text file representation of the problem can be obtained by performing one of the following conversions:

```
mosek prob.nl -a -x -out prob.mps
mosek prob.nl -a -x -out prob.opf
mosek prob.nl -a -x -out prob.lp
```

## PROBLEM FORMULATION AND SOLUTIONS

In this chapter we will discuss the following issues:

- The formal, mathematical formulations of the problem types that MOSEK can solve and their duals
- The solution information produced by MOSEK.
- The infeasibility certificate produced by **MOSEK** if the problem is infeasible.

## 7.1 Linear Optimization

A linear optimization problem can be written as

where

- $\bullet$  m is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

A primal solution (x) is (primal) feasible if it satisfies all constraints in (7.1). If (7.1) has at least one primal feasible solution, then (7.1) is said to be (primal) feasible.

In case (7.1) does not have a feasible solution, the problem is said to be (primal) infeasible

### 7.1.1 Duality for Linear Optimization

Corresponding to the primal problem (7.1), there is a dual problem

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c} + (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x} + c^{f}$$

$$A^{T} y + s_{l}^{x} - s_{u}^{x} = c,$$
subject to 
$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0.$$

$$(7.2)$$

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. E.g.

$$l_i^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0 \text{ and } l_i^x \cdot (s_l^x)_j = 0.$$

This is equivalent to removing variable  $(s_l^x)_i$  from the dual problem. A solution

$$(y, s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x})$$

to the dual problem is feasible if it satisfies all the constraints in (7.2). If (7.2) has at least one feasible solution, then (7.2) is (dual) feasible, otherwise the problem is (dual) infeasible.

### A Primal-dual Feasible Solution

A solution

$$(x, y, s_l^c, s_u^c, s_l^x, s_u^x)$$

is denoted a *primal-dual feasible solution*, if (x) is a solution to the primal problem (7.1) and  $(y, s_l^c, s_u^c, s_l^x, s_u^x)$  is a solution to the corresponding dual problem (7.2).

### The Duality Gap

Let

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

be a primal-dual feasible solution, and let

$$(x^c)^* := Ax^*.$$

For a primal-dual feasible solution we define the *duality gap* as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] \ge 0$$

$$(7.3)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (7.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

### **An Optimal Solution**

It is well-known that a linear optimization problem has an optimal solution if and only if there exist feasible primal and dual solutions so that the duality gap is zero, or, equivalently, that the *complementarity conditions* 

$$\begin{array}{rclcrcl} (s_{u}^{c})_{i}^{*}((x_{i}^{c})^{*}-l_{i}^{c}) & = & 0, & i=0,\ldots,m-1, \\ (s_{u}^{c})_{i}^{*}(u_{i}^{c}-(x_{i}^{c})^{*}) & = & 0, & i=0,\ldots,m-1, \\ (s_{l}^{x})_{j}^{*}(x_{j}^{*}-l_{j}^{x}) & = & 0, & j=0,\ldots,n-1, \\ (s_{u}^{x})_{j}^{*}(u_{j}^{x}-x_{j}^{*}) & = & 0, & j=0,\ldots,n-1, \end{array}$$

are satisfied.

If (7.1) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

## 7.1.2 Infeasibility for Linear Optimization

### **Primal Infeasible Problems**

If the problem (7.1) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x$$
  
subject to 
$$A^T y + s_l^x - s_u^x = 0,$$

$$-y + s_l^c - s_u^c = 0,$$

$$s_l^c, s_u^c, s_l^x, s_u^x \ge 0,$$
(7.4)

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

to (7.4) so that

$$(l^c)^T(s_l^c)^* - (u^c)^T(s_u^c)^* + (l^x)^T(s_l^x)^* - (u^x)^T(s_u^x)^* > 0.$$

Such a solution implies that (7.4) is unbounded, and that its dual is infeasible. As the constraints to the dual of (7.4) are identical to the constraints of problem (7.1), we thus have that problem (7.1) is also infeasible.

#### **Dual Infeasible Problems**

If the problem (7.2) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

minimize 
$$c^T x$$
  
subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ ,  $\hat{l}^x \leq x \leq \hat{u}^x$ , (7.5)

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{ll} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{ll} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that

$$c^T x < 0.$$

Such a solution implies that (7.5) is unbounded, and that its dual is infeasible. As the constraints to the dual of (7.5) are identical to the constraints of problem (7.2), we thus have that problem (7.2) is also infeasible.

### Primal and Dual Infeasible Case

In case that both the primal problem (7.1) and the dual problem (7.2) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

### Minimalization vs. Maximalization

When the objective sense of problem (7.1) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (7.2). The dual problem thus takes the form

minimize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f$$
 subject to 
$$A^T y + s_l^x - s_u^x = c, \\ -y + s_l^c - s_u^c = 0, \\ s_l^c, s_u^c, s_l^x, s_u^x \leq 0.$$

This means that the duality gap, defined in (7.3) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$A^{T}y + s_{l}^{x} - s_{u}^{x} = 0,$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \leq 0,$$
(7.6)

such that the objective value is strictly negative

$$(l^c)^T(s_l^c)^* - (u^c)^T(s_u^c)^* + (l^x)^T(s_l^x)^* - (u^x)^T(s_u^x)^* < 0.$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (7.5) such that  $c^T x > 0$ .

## 7.2 Conic Quadratic Optimization

Conic quadratic optimization is an extension of linear optimization (see Sec. 7.1) allowing conic domains to be specified for subsets of the problem variables. A conic quadratic optimization problem can be written as

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  
 $l^x \le x \le u^x$ , (7.7)

where set  $\mathcal{K}$  is a Cartesian product of convex cones, namely  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_p$ . Having the domain restriction,  $x \in \mathcal{K}$ , is thus equivalent to

$$x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}$$
,

where  $x = (x^1, ..., x^p)$  is a partition of the problem variables. Please note that the *n*-dimensional Euclidean space  $\mathbb{R}^n$  is a cone itself, so simple linear variables are still allowed.

MOSEK supports only a limited number of cones, specifically:

- The  $\mathbb{R}^n$  set.
- The quadratic cone:

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{1} \ge \sqrt{\sum_{j=2}^{n} x_{j}^{2}} \right\}.$$

• The rotated quadratic cone:

$$Q_r^n = \left\{ x \in \mathbb{R}^n : 2x_1 x_2 \ge \sum_{j=3}^n x_j^2, \quad x_1 \ge 0, \quad x_2 \ge 0 \right\}.$$

Although these cones may seem to provide only limited expressive power they can be used to model a wide range of problems as demonstrated in |MOSEKApS12|.

## 7.2.1 Duality for Conic Quadratic Optimization

The dual problem corresponding to the conic quadratic optimization problem (7.7) is given by

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f$$
 subject to 
$$A^T y + s_l^x - s_u^x + s_n^x = c - y + s_l^c - s_u^c = 0,$$
 
$$s_l^c, s_u^c, s_l^x, s_u^x \ge 0,$$
 
$$s_n^x \in \mathcal{K}^*,$$
 (7.8)

where the dual cone  $\mathcal{K}^*$  is a Cartesian product of the cones

$$\mathcal{K}^* = \mathcal{K}_1^* \times \cdots \times \mathcal{K}_n^*$$

where each  $\mathcal{K}_t^*$  is the dual cone of  $\mathcal{K}_t$ . For the cone types **MOSEK** can handle, the relation between the primal and dual cone is given as follows:

• The  $\mathbb{R}^n$  set:

$$\mathcal{K}_t = \mathbb{R}^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \{ s \in \mathbb{R}^{n_t} : \quad s = 0 \}.$$

• The quadratic cone:

$$\mathcal{K}_t = \mathcal{Q}^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \mathcal{Q}^{n_t} = \left\{ s \in \mathbb{R}^{n_t} : s_1 \ge \sqrt{\sum_{j=2}^{n_t} s_j^2} \right\}.$$

• The rotated quadratic cone:

$$\mathcal{K}_t = \mathcal{Q}_r^{n_t} \quad \Leftrightarrow \quad \mathcal{K}_t^* = \mathcal{Q}_r^{n_t} = \left\{ s \in \mathbb{R}^{n_t} : 2s_1 s_2 \ge \sum_{j=3}^{n_t} s_j^2, \quad s_1 \ge 0, \quad s_2 \ge 0 \right\}.$$

Please note that the dual problem of the dual problem is identical to the original primal problem.

## 7.2.2 Infeasibility for Conic Quadratic Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. This works exactly as for linear problems (see Sec. 7.1.2).

### **Primal Infeasible Problems**

If the problem (7.7) is infeasible, **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the problem

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & &$$

such that the objective value is strictly positive.

### **Dual infeasible problems**

If the problem (7.8) is infeasible, **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{ll} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{ll} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that the objective value is strictly negative.

## 7.3 Semidefinite Optimization

Semidefinite optimization is an extension of conic quadratic optimization (see Sec. 7.2) allowing positive semidefinite matrix variables to be used in addition to the usual scalar variables. A semidefinite optimization problem can be written as

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f$$
subject to  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq u_i^c, \quad i = 0, \dots, m-1$ 

$$l_j^x \leq x_j \leq x_j \leq x_j, \quad j = 0, \dots, n-1$$

$$x \in \mathcal{K}, \overline{X}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, p-1$$

$$(7.9)$$

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_+^{r_j}$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}^{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $U, V \in \mathbb{R}^{m \times n}$  we have

$$\langle U, V \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} U_{ij} V_{ij}.$$

With semidefinite optimization we can model a wide range of problems as demonstrated in [MOSEKApS12].

## 7.3.1 Duality for Semidefinite Optimization

The dual problem corresponding to the semidefinite optimization problem (7.9) is given by

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f$$
 subject to 
$$c - A^T y + s_u^x - s_l^x = s_n^x,$$
 
$$\overline{C}_j - \sum_{i=0}^m y_i \overline{A}_{ij} = \overline{S}_j,$$
  $j = 0, \dots, p-1$  
$$s_l^c - s_u^c = y,$$
 
$$s_l^c, s_u^c, s_l^x, s_u^x \ge 0,$$
 
$$s_n^c \in \mathcal{K}^*, \quad \overline{S}_j \in \mathcal{S}_+^{r_j},$$
  $j = 0, \dots, p-1$ 

where  $A \in \mathbb{R}^{m \times n}$ ,  $A_{ij} = a_{ij}$ , which is similar to the dual problem for conic quadratic optimization (see Sec. 7.2.1), except for the addition of dual constraints

$$\left(\overline{C}_j - \sum_{i=0}^m y_i \overline{A}_{ij}\right) \in \mathcal{S}_+^{r_j}.$$

Note that the dual of the dual problem is identical to the original primal problem.

## 7.3.2 Infeasibility for Semidefinite Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of the infeasibility. This works exactly as for linear problems (see Sec. 7.1.2).

### **Primal Infeasible Problems**

If the problem (7.9) is infeasible, **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is a certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the problem

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & \\ & A^T y + s_l^x - s_u^x + s_n^x = 0, \\ & \sum_{i=0}^{m-1} y_i \overline{A}_{ij} + \overline{S}_j = 0, \\ & -y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\ & s_n^c \in \mathcal{K}^*, \quad \overline{S}_j \in \mathcal{S}_+^{r_j}, & j = 0, \dots, p-1 \\ \end{array}$$

such that the objective value is strictly positive.

### **Dual Infeasible Problems**

If the problem (7.10) is infeasible, **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

$$\begin{array}{lll} \text{minimize} & \sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle \\ \text{subject to} & \hat{l}_i^c & \leq & \sum_{j=1}^n a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle & \leq & \hat{u}_i^c, \quad i = 0, \dots, m-1 \\ & \hat{l}^x & \leq & x & \leq & \hat{u}^x, \\ & x \in \mathcal{K}, & \overline{X}_j \in \mathcal{S}_+^{r_j}, & j = 0, \dots, p-1 \end{array}$$

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c >; -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c <; \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

and

$$\hat{l}_j^x = \left\{ \begin{array}{ll} 0 & \text{if } l_j^x >; -\infty, \\ -\infty & \text{otherwise,} \end{array} \right. \quad \text{and} \quad \hat{u}_j^x := \left\{ \begin{array}{ll} 0 & \text{if } u_j^x <; \infty, \\ \infty & \text{otherwise,} \end{array} \right.$$

such that the objective value is strictly negative.

## 7.4 Quadratic and Quadratically Constrained Optimization

A convex quadratic and quadratically constrained optimization problem has the form

where  $Q^o$  and all  $Q^k$  are symmetric matrices. Moreover, for convexity,  $Q^o$  must be a positive semidefinite matrix and  $Q^k$  must satisfy

$$\begin{array}{rcl} -\infty < l_k^c & \Rightarrow & Q^k \text{ is negative semidefinite,} \\ u_k^c < \infty & \Rightarrow & Q^k \text{ is positive semidefinite,} \\ -\infty < l_k^c \leq u_k^c < \infty & \Rightarrow & Q^k = 0. \end{array}$$

The convexity requirement is very important and MOSEK checks whether it is fulfilled.

### 7.4.1 A Recommendation

Any convex quadratic optimization problem can be reformulated as a conic quadratic optimization problem, see [MOSEKApS12] and in particular [And13]. In fact MOSEK does such conversion internally as a part of the solution process for the following reasons:

- the conic optimizer is numerically more robust than the one for quadratic problems.
- the conic optimizer is usually faster because quadratic cones are simpler than quadratic functions, even though the conic reformulation usually has more constraints and variables than the original quadratic formulation.
- it is easy to dualize the conic formulation if deemed worthwhile potentially leading to (huge) computational savings.

However, instead of relying on the automatic reformulation we recommend to formulate the problem as a conic problem from scratch because:

- it saves the computational overhead of the reformulation including the convexity check. A conic problem is convex by construction and hence no convexity check is needed for conic problems.
- usually the modeller can do a better reformulation than the automatic method because the modeller can exploit the knowledge of the problem at hand.

To summarize we recommend to formulate quadratic problems and in particular quadratically constrained problems directly in conic form.

## 7.4.2 Duality for Quadratic and Quadratically Constrained Optimization

The dual problem corresponding to the quadratic and quadratically constrained optimization problem (7.11) is given by

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + \frac{1}{2} x^T \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x = c, \\ & -y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_u^x, s_u^x \geq 0. \end{array}$$

The dual problem is related to the dual problem for linear optimization (see Sec. 7.1.1), but depends on the variable x which in general can not be eliminated. In the solutions reported by **MOSEK**, the value of x is the same for the primal problem (7.11) and the dual problem (7.12).

## 7.4.3 Infeasibility for Quadratic and Quadratically Constrained Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. This works exactly as for linear problems (see Sec. 7.1.2).

#### **Primal Infeasible Problems**

If the problem (7.11) with all  $Q^k = 0$  is infeasible, **MOSEK** will report a certificate of primal infeasibility. As the constraints are the same as for a linear problem, the certificate of infeasibility is the same as for linear optimization (see Sec. 7.1.2).

#### **Dual Infeasible Problems**

If the problem (7.12) with all  $Q^k = 0$  is dual infeasible, **MOSEK** will report a certificate of dual infeasibility. The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the problem

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{ll} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{ll} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that the objective value is strictly negative.

# 7.5 General Convex Optimization

The general nonlinear optimizer (which may be available for all or some types of nonlinear problems depending on the interface), solves smooth (twice differentiable) convex nonlinear optimization problems of the form

$$\begin{array}{lll} \text{minimize} & & f(x) + c^T x + c^f \\ \text{subject to} & l^c & \leq & g(x) + Ax & \leq & u^c, \\ & l^x & \leq & x & \leq & u^x, \end{array}$$

where

- *m* is the number of constraints.
- n is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part objective function.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.

- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $f: \mathbb{R}^n \to \mathbb{R}$  is a nonlinear function.
- $g: \mathbb{R}^n \to \mathbb{R}^m$  is a nonlinear vector function.

This means that the i-th constraint has the form

$$l_i^c \le g_i(x) + \sum_{j=1}^n a_{ij} x_j \le u_i^c.$$

The linear term Ax is not included in g(x) since it can be handled much more efficiently as a separate entity when optimizing.

The nonlinear functions f and g must be smooth in all  $x \in [l^x; u^x]$ . Moreover, f(x) must be a convex function and  $g_i(x)$  must satisfy

$$\begin{array}{rcl} -\infty < l_i^c & \Rightarrow & g_i(x) \text{ is concave,} \\ u_i^c < \infty & \Rightarrow & g_i(x) \text{ is convex,} \\ -\infty < l_i^c \leq u_i^c < \infty & \Rightarrow & g_i(x) = 0. \end{array}$$

## 7.5.1 Duality for General convex Optimization

Similarly to the linear case, **MOSEK** reports dual information in the general nonlinear case. Indeed in this case the Lagrange function is defined by

$$\begin{array}{lcl} L(x,s_{l}^{c},s_{u}^{c},s_{u}^{x},s_{u}^{x}) & := & f(x)+c^{T}x+c^{f} \\ & -(s_{l}^{c})^{T}(g(x)+Ax-l^{c})-(s_{u}^{c})^{T}(u^{c}-g(x)-Ax) \\ & -(s_{l}^{x})^{T}(x-l^{x})-(s_{u}^{x})^{T}(u^{x}-x), \end{array}$$

and the dual problem is given by

$$\begin{array}{lll} \text{maximize} & L(x, s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x}) \\ \text{subject to} & \nabla_{x} L(x, s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x})^{T} & = & 0, \\ & s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0, \end{array}$$

which is equivalent to

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ & + f(x) - g(x)^T y - (\nabla f(x)^T - \nabla g(x)^T y)^T x \\ \text{subject to} & A^T y + s_l^x - s_u^x - (\nabla f(x)^T - \nabla g(x)^T y) & = & c, \\ & - y + s_l^c - s_u^c & = & 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0. \end{array}$$

In this context we use the following definition for scalar functions

$$\nabla f(x) = \left[\frac{\partial f(x)}{\partial x_1}, \dots, \frac{\partial f(x)}{\partial x_n}\right]$$

and accordingly for vector functions

$$\nabla g(x) = \begin{bmatrix} \nabla g_1(x) \\ \vdots \\ \nabla g_m(x) \end{bmatrix}.$$

## THE OPTIMIZERS FOR CONTINUOUS PROBLEMS

The most essential part of **MOSEK** are the optimizers. This chapter describes the optimizers for the class of *continuous problems* without integer variables, that is:

- linear problems,
- conic problems (quadratic and semidefinite),
- general convex problems.

**MOSEK** offers an interior-point optimizer for each class of problems and also a simplex optimizer for linear problems. The structure of a successful optimization process is roughly:

#### • Presolve

- 1. Elimination: Reduce the size of the problem.
- 2. Dualizer: Choose whether to solve the primal or the dual form of the problem.
- 3. Scaling: Scale the problem for better numerical stability.

## • Optimization

- 1. Optimize: Solve the problem using selected method.
- 2. Terminate: Stop the optimization when specific termination criteria have been met.
- 3. Report: Return the solution or an infeasibility certificate.

The preprocessing stage is transparent to the user, but useful to know about for tuning purposes. The purpose of the preprocessing steps is to make the actual optimization more efficient and robust. We discuss the details of the above steps in the following sections.

## 8.1 Presolve

Before an optimizer actually performs the optimization the problem is preprocessed using the so-called presolve. The purpose of the presolve is to

- 1. remove redundant constraints,
- 2. eliminate fixed variables,
- 3. remove linear dependencies,
- 4. substitute out (implied) free variables, and
- 5. reduce the size of the optimization problem in general.

After the presolved problem has been optimized the solution is automatically postsolved so that the returned solution is valid for the original problem. Hence, the presolve is completely transparent. For further details about the presolve phase, please see [AA95] and [AGMX96].

It is possible to fine-tune the behavior of the presolve or to turn it off entirely. If presolve consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This

is done by setting the parameter  $MSK\_IPAR\_PRESOLVE\_USE$  to  $MSK\_PRESOLVE\_MODE\_OFF$ . The two most time-consuming steps of the presolve are

- the eliminator, and
- the linear dependency check.

Therefore, in some cases it is worthwhile to disable one or both of these.

#### Numerical issues in the presolve

During the presolve the problem is reformulated so that it hopefully solves faster. However, in rare cases the presolved problem may be harder to solve then the original problem. The presolve may also be infeasible although the original problem is not. If it is suspected that presolved problem is much harder to solve than the original, we suggest to first turn the eliminator off by setting the parameter  $MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES$  to 0. If that does not help, then trying to turn entire presolve off may help.

Since all computations are done in finite precision, the presolve employs some tolerances when concluding a variable is fixed or a constraint is redundant. If it happens that  $\mathbf{MOSEK}$  incorrectly concludes a problem is primal or dual infeasible, then it is worthwhile to try to reduce the parameters  $MSK\_DPAR\_PRESOLVE\_TOL\_X$  and  $MSK\_DPAR\_PRESOLVE\_TOL\_S$ . However, if reducing the parameters actually helps then this should be taken as an indication that the problem is badly formulated.

#### **Eliminator**

The purpose of the eliminator is to eliminate free and implied free variables from the problem using substitution. For instance, given the constraints

$$\begin{array}{rcl} y & = & \sum_j x_j, \\ y, x & \geq & 0, \end{array}$$

y is an implied free variable that can be substituted out of the problem, if deemed worthwhile. If the eliminator consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This can be done by setting the parameter  $MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES$  to 0. In rare cases the eliminator may cause that the problem becomes much hard to solve.

#### Linear dependency checker

The purpose of the linear dependency check is to remove linear dependencies among the linear equalities. For instance, the three linear equalities

$$\begin{array}{rcl} x_1 + x_2 + x_3 & = & 1, \\ x_1 + 0.5x_2 & = & 0.5, \\ 0.5x_2 + x_3 & = & 0.5. \end{array}$$

contain exactly one linear dependency. This implies that one of the constraints can be dropped without changing the set of feasible solutions. Removing linear dependencies is in general a good idea since it reduces the size of the problem. Moreover, the linear dependencies are likely to introduce numerical problems in the optimization phase. It is best practice to build models without linear dependencies, but that is not always easy for the user to control. If the linear dependencies are removed at the modelling stage, the linear dependency check can safely be disabled by setting the parameter  $MSK\_IPAR\_PRESOLVE\_LINDEP\_USE$  to  $MSK\_OFF$ .

#### **Dualizer**

All linear, conic, and convex optimization problems have an equivalent dual problem associated with them. **MOSEK** has built-in heuristics to determine if it is more efficient to solve the primal or dual

problem. The form (primal or dual) is displayed in the **MOSEK** log and available as an information item from the solver. Should the internal heuristics not choose the most efficient form of the problem it may be worthwhile to set the dualizer manually by setting the parameters:

- MSK\_IPAR\_INTPNT\_SOLVE\_FORM: In case of the interior-point optimizer.
- MSK\_IPAR\_SIM\_SOLVE\_FORM: In case of the simplex optimizer.

Note that currently only linear and conic quadratic problems may be automatically dualized.

#### **Scaling**

Problems containing data with large and/or small coefficients, say 1.0e+9 or 1.0e-7, are often hard to solve. Significant digits may be truncated in calculations with finite precision, which can result in the optimizer relying on inaccurate data. Since computers work in finite precision, extreme coefficients should be avoided. In general, data around the same *order of magnitude* is preferred, and we will refer to a problem, satisfying this loose property, as being *well-scaled*. If the problem is not well scaled, **MOSEK** will try to scale (multiply) constraints and variables by suitable constants. **MOSEK** solves the scaled problem to improve the numerical properties.

The scaling process is transparent, i.e. the solution to the original problem is reported. It is important to be aware that the optimizer terminates when the termination criterion is met on the scaled problem, therefore significant primal or dual infeasibilities may occur after unscaling for badly scaled problems. The best solution of this issue is to reformulate the problem, making it better scaled.

By default **MOSEK** heuristically chooses a suitable scaling. The scaling for interior-point and simplex optimizers can be controlled with the parameters <code>MSK\_IPAR\_INTPNT\_SCALING</code> and <code>MSK\_IPAR\_SIM\_SCALING</code> respectively.

## 8.2 Using Multiple Threads in an Optimizer

## Multithreading in interior-point optimizers

The interior-point optimizers in **MOSEK** have been parallelized. This means that if you solve linear, quadratic, conic, or general convex optimization problem using the interior-point optimizer, you can take advantage of multiple CPU's. By default **MOSEK** will automatically select the number of threads to be employed when solving the problem. However, the maximum number of threads employed can be changed by setting the parameter  $MSK\_IPAR\_NUM\_THREADS$ . This should never exceed the number of cores on the computer.

The speed-up obtained when using multiple threads is highly problem and hardware dependent, and consequently, it is advisable to compare single threaded and multi threaded performance for the given problem type to determine the optimal settings. For small problems, using multiple threads is not be worthwhile and may even be counter productive because of the additional coordination overhead. Therefore, it may be advantageous to disable multithreading using the parameter MSK\_IPAR\_INTPNT\_MULTI\_THREAD.

The interior-point optimizer parallelizes big tasks such linear algebra computations.

## **Thread Safety**

The MOSEK API is thread-safe provided that a task is only modified or accessed from one thread at any given time. Also accessing two or more separate tasks from threads at the same time is safe. Sharing an environment between threads is safe.

#### **Determinism**

The optimizers are run-to-run deterministic which means if a problem is solved twice on the same computer using the same parameter setting and exactly the same input then exactly the same results is obtained. One restriction is that no time limits must be imposed because the time taken to perform an operation on a computer is dependent on many factors such as the current workload.

## 8.3 Linear Optimization

## 8.3.1 Optimizer Selection

Two different types of optimizers are available for linear problems: The default is an interior-point method, and the alternative is the simplex method (primal or dual). The optimizer can be selected using the parameter  $MSK\_IPAR\_OPTIMIZER$ .

### The Interior-point or the Simplex Optimizer?

Given a linear optimization problem, which optimizer is the best: the simplex or the interior-point optimizer? It is impossible to provide a general answer to this question. However, the interior-point optimizer behaves more predictably: it tends to use between 20 and 100 iterations, almost independently of problem size, but cannot perform warm-start. On the other hand the simplex method can take advantage of an initial solution, but is less predictable from cold-start. The interior-point optimizer is used by default.

#### The Primal or the Dual Simplex Variant?

**MOSEK** provides both a primal and a dual simplex optimizer. Predicting which simplex optimizer is faster is impossible, however, in recent years the dual optimizer has seen several algorithmic and computational improvements, which, in our experience, make it faster on average than the primal version. Still, it depends much on the problem structure and size. Setting the MSK\_IPAR\_OPTIMIZER parameter to MSK\_OPTIMIZER\_FREE\_SIMPLEX instructs **MOSEK** to choose one of the simplex variants automatically.

To summarize, if you want to know which optimizer is faster for a given problem type, it is best to try all the options.

### 8.3.2 The Interior-point Optimizer

The purpose of this section is to provide information about the algorithm employed in the **MOSEK** interior-point optimizer for linear problems and about its termination criteria.

### The homogeneous primal-dual problem

In order to keep the discussion simple it is assumed that  $\mathbf{MOSEK}$  solves linear optimization problems of standard form

$$\begin{array}{lll} \text{minimize} & c^T x \\ \text{subject to} & Ax & = & b, \\ & x \geq 0. & \end{array} \tag{8.1}$$

This is in fact what happens inside MOSEK; for efficiency reasons MOSEK converts the problem to standard form before solving, then converts it back to the input form when reporting the solution.

Since it is not known beforehand whether problem (8.1) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason why **MOSEK** solves the so-called homogeneous model

$$\begin{array}{rcl}
Ax - b\tau & = & 0, \\
A^{T}y + s - c\tau & = & 0, \\
-c^{T}x + b^{T}y - \kappa & = & 0, \\
x, s, \tau, \kappa & \geq & 0,
\end{array}$$
(8.2)

where y and s correspond to the dual variables in (8.1), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (8.2) always has solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (8.2) satisfies

$$x_i^* s_i^* = 0$$
 and  $\tau^* \kappa^* = 0$ .

Moreover, there is always a solution that has the property  $\tau^* + \kappa^* > 0$ .

First, assume that  $\tau^* > 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left\{ \frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*} \right\}$$

is a primal-dual optimal solution (see Sec. 7.1 for the mathematical background on duality and optimality).

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This implies that at least one of

$$c^T x^* < 0 (8.3)$$

or

$$b^T y^* > 0 (8.4)$$

is satisfied. If (8.3) is satisfied then  $x^*$  is a certificate of dual infeasibility, whereas if (8.4) is satisfied then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

#### **Interior-point Termination Criterion**

For efficiency reasons it is not practical to solve the homogeneous model exactly. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In the k-th iteration of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to homogeneous model is generated, where

$$x^k, s^k, \tau^k, \kappa^k > 0.$$

#### **Optimal** case

Whenever the trial solution satisfies the criterion

$$\left\| A \frac{x^{k}}{\tau^{k}} - b \right\|_{\infty} \leq \epsilon_{p} (1 + \|b\|_{\infty}),$$

$$\left\| A^{T} \frac{y^{k}}{\tau^{k}} + \frac{s^{k}}{\tau^{k}} - c \right\|_{\infty} \leq \epsilon_{d} (1 + \|c\|_{\infty}), \text{ and}$$

$$\min \left( \frac{(x^{k})^{T} s^{k}}{(\tau^{k})^{2}}, \left| \frac{c^{T} x^{k}}{\tau^{k}} - \frac{b^{T} y^{k}}{\tau^{k}} \right| \right) \leq \epsilon_{g} \max \left( 1, \frac{\min(\left| c^{T} x^{k} \right|, \left| b^{T} y^{k} \right|)}{\tau^{k}} \right), \tag{8.5}$$

the interior-point optimizer is terminated and

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is reported as the primal-dual optimal solution. The interpretation of (8.5) is that the optimizer is terminated if

- $\frac{x^k}{\tau^k}$  is approximately primal feasible,
- $\left\{\frac{y^k}{\tau^k}, \frac{s^k}{\tau^k}\right\}$  is approximately dual feasible, and
- the duality gap is almost zero.

## **Dual infeasibility certificate**

On the other hand, if the trial solution satisfies

$$-\epsilon_{i}c^{T}x^{k} > \frac{\left\|c\right\|_{\infty}}{\max\left(1, \left\|b\right\|_{\infty}\right)} \left\|Ax^{k}\right\|_{\infty}$$

then the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows: First assume that  $\|Ax^k\|_{\infty} = 0$ ; then  $x^k$  is an exact certificate of dual infeasibility. Next assume that this is not the case, i.e.

$$||Ax^k||_{\infty} > 0,$$

and define

$$\bar{x} := \epsilon_i \frac{\max(1, \|b\|_{\infty})}{\|Ax^k\|_{\infty} \|c\|_{\infty}} x^k.$$

It is easy to verify that

$$\|A\bar{x}\|_{\infty} = \epsilon_i \frac{\max\left(1, \|b\|_{\infty}\right)}{\|c\|_{\infty}} \text{ and } -c^T\bar{x} > 1,$$

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation. A smaller value means a better approximation.

#### Primal infeasibility certificate

Finally, if

$$\epsilon_i b^T y^k > \frac{\|b\|_{\infty}}{\max\left(1, \|c\|_{\infty}\right)} \left\| A^T y^k + s^k \right\|_{\infty}$$

then  $y^k$  is reported as a certificate of primal infeasibility.

#### Adjusting optimality criteria and near optimality

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$  using parameters; see table for details.

Table 8.1: Parameters employed in termination criterion

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (8.5) reveals that the quality of the solution depends on  $\|b\|_{\infty}$  and  $\|c\|_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

In some cases the interior-point method terminates having found a solution not too far from meeting the optimality condition (8.5). A solution is defined as near optimal if scaling the termination tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_g$  by the same factor  $\varepsilon_n \in [1.0, +\infty]$  makes the condition (8.5) satisfied. A near optimal solution is therefore of lower quality but still potentially valuable. If for instance the solver stalls, i.e. it can make no more significant progress towards the optimal solution, a near optimal solution could be available and be good enough for the user. Near infeasibility certificates are defined similarly. The value of  $\varepsilon_n$  can be adjusted with the parameter  $MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL$ .

The basis identification discussed in Sec. 8.3.2 requires an optimal solution to work well; hence basis identification should be turned off if the termination criterion is relaxed.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

#### **Basis Identification**

An interior-point optimizer does not return an optimal basic solution unless the problem has a unique primal and dual optimal solution. Therefore, the interior-point optimizer has an optimal post-processing step that computes an optimal basic solution starting from the optimal interior-point solution. More information about the basis identification procedure may be found in [AY96]. In the following we provide an overall idea of the procedure.

There are some cases in which a basic solution could be more valuable:

- a basic solution is often more accurate than an interior-point solution,
- a basic solution can be used to warm-start the simplex algorithm in case of reoptimization,
- a basic solution is in general more sparse, i.e. more variables are fixed to zero. This is particularly appealing when solving continuous relaxations of mixed integer problems, as well as in all applications in which sparser solutions are preferred.

To illustrate how the basis identification routine works, we use the following trivial example:

$$\begin{array}{lll} \text{minimize} & x+y \\ \text{subject to} & x+y & = & 1, \\ & x,y \geq 0. & \end{array}$$

It is easy to see that all feasible solutions are also optimal. In particular, there are two basic solutions, namely

$$\begin{array}{rcl} (x_1^*,y_1^*) & = & (1,0), \\ (x_2^*,y_2^*) & = & (0,1). \end{array}$$

The interior point algorithm will actually converge to the center of the optimal set, i.e. to  $(x^*, y^*) = (1/2, 1/2)$  (to see this in **MOSEK** deactivate *Presolve*).

In practice, when the algorithm gets close to the optimal solution, it is possible to construct in polynomial time an initial basis for the simplex algorithm from the current interior point solution. This basis is used to warm-start the simplex algorithm that will provide the optimal basic solution. In most cases the constructed basis is optimal, or very few iterations are required by the simplex algorithm to make it optimal and hence the final *clean-up* phase be short. However, for some cases of ill-conditioned problems the additional simplex clean up phase may take of lot a time.

By default **MOSEK** performs a basis identification. However, if a basic solution is not needed, the basis identification procedure can be turned off. The parameters

- MSK\_IPAR\_INTPNT\_BASIS,
- MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER, and
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR

control when basis identification is performed.

The type of simplex algorithm to be used (primal/dual) can be tuned with the parameter  $MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER$ , and the maximum number of iterations can be set with  $MSK\_IPAR\_BI\_MAX\_ITERATIONS$ .

Finally, it should be mentioned that there is no guarantee on which basic solution will be returned.

## The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer - threads
                                  : 1
Optimizer - solved problem
                                  : the dual
Optimizer - Constraints
                                  : 2
Optimizer - Cones
                                  : 0
Optimizer - Scalar variables
                                 : 6
                                                                             : 0
                                                      conic
Optimizer - Semi-definite variables: 0
                                                      scalarized
                                                                             : 0
Factor
          - setup time
                                  : 0.00
                                                      dense det. time
                                                                             : 0.00
Factor
          - ML order time
                                  : 0.00
                                                      GP order time
                                                                             : 0.00
          - nonzeros before factor : 3
                                                      after factor
                                                                             : 3
Factor
Factor
          - dense dim.
                                   : 0
                                                      flops
                                                                            : 7.00e+001
ITE PFEAS
            DFEAS GFEAS
                             PRSTATUS
                                        POBJ
                                                          DOBJ
                                                                           MU
  1.0e+000 8.6e+000 6.1e+000 1.00e+000 0.000000000e+000 -2.208000000e+003 1.0e+000 0.00
   1.1e+000 2.5e+000 1.6e-001 0.00e+000
                                        -7.901380925e+003 -7.394611417e+003 2.5e+000 0.00
   1.4e-001 3.4e-001 2.1e-002 8.36e-001
                                        -8.113031650e+003 -8.055866001e+003 3.3e-001 0.00
   2.4e-002 5.8e-002 3.6e-003 1.27e+000 -7.777530698e+003 -7.766471080e+003 5.7e-002 0.01
   1.3e-004 3.2e-004 2.0e-005 1.08e+000 -7.668323435e+003 -7.668207177e+003 3.2e-004 0.01
   1.3e-008 3.2e-008 2.0e-009 1.00e+000
                                        -7.668000027e+003 -7.668000015e+003 3.2e-008 0.01
   1.3e-012 3.2e-012 2.0e-013 1.00e+000
                                        -7.667999994e+003 -7.667999994e+003 3.2e-012 0.01
```

The first line displays the number of threads used by the optimizer and the second line tells that the optimizer chose to solve the dual problem rather than the primal problem. The next line displays the

problem dimensions as seen by the optimizer, and the Factor... lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 8.3.2 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $\|A^Ty^k + s^k c\tau^k\|_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to -1 if that is not the case.
- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- $\bullet$  DOBJ:  $b^Ty^k/\tau^k.$  An estimate for the dual objective value.
- MU:  $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$  . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started.

## 8.3.3 The Simplex Optimizer

An alternative to the interior-point optimizer is the simplex optimizer. The simplex optimizer uses a different method that allows exploiting an initial guess for the optimal solution to reduce the solution time. Depending on the problem it may be faster or slower to use an initial guess; see Sec. 8.3.1 for a discussion. **MOSEK** provides both a primal and a dual variant of the simplex optimizer.

#### **Simplex Termination Criterion**

The simplex optimizer terminates when it finds an optimal basic solution or an infeasibility certificate. A basic solution is optimal when it is primal and dual feasible; see Sec. 7.1 for a definition of the primal and dual problem. Due to the fact that computations are performed in finite precision **MOSEK** allows violations of primal and dual feasibility within certain tolerances. The user can control the allowed primal and dual tolerances with the parameters  $MSK_DPAR_BASIS_TOL_X$  and  $MSK_DPAR_BASIS_TOL_S$ .

Setting the parameter  $MSK\_IPAR\_OPTIMIZER$  to  $MSK\_OPTIMIZER\_FREE\_SIMPLEX$  instructs  $\mathbf{MOSEK}$  to select automatically between the primal and the dual simplex optimizers. Hence,  $\mathbf{MOSEK}$  tries to choose the best optimizer for the given problem and the available solution. The same parameter can also be used to force one of the variants.

#### Starting From an Existing Solution

When using the simplex optimizer it may be possible to reuse an existing solution and thereby reduce the solution time significantly. When a simplex optimizer starts from an existing solution it is said to perform a *warm-start*. If the user is solving a sequence of optimization problems by solving the problem, making modifications, and solving again, **MOSEK** will warm-start automatically.

By default **MOSEK** uses presolve when performing a warm-start. If the optimizer only needs very few iterations to find the optimal solution it may be better to turn off the presolve.

#### Numerical Difficulties in the Simplex Optimizers

Though MOSEK is designed to minimize numerical instability, completely avoiding it is impossible when working in finite precision. MOSEK treats a "numerically unexpected behavior" event inside the optimizer as a *set-back*. The user can define how many set-backs the optimizer accepts; if that number is exceeded, the optimization will be aborted. Set-backs are a way to escape long sequences where the optimizer tries to recover from an unstable situation.

Examples of set-backs are: repeated singularities when factorizing the basis matrix, repeated loss of feasibility, degeneracy problems (no progress in objective) and other events indicating numerical difficulties. If the simplex optimizer encounters a lot of set-backs the problem is usually badly scaled; in such a situation try to reformulate it into a better scaled problem. Then, if a lot of set-backs still occur, trying one or more of the following suggestions may be worthwhile:

- Raise tolerances for allowed primal or dual feasibility: increase the value of
  - MSK\_DPAR\_BASIS\_TOL\_X, and
  - MSK\_DPAR\_BASIS\_TOL\_S.
- Raise or lower pivot tolerance: Change the MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV parameter.
- Switch optimizer: Try another optimizer.
- Switch off crash: Set both MSK\_IPAR\_SIM\_PRIMAL\_CRASH and MSK\_IPAR\_SIM\_DUAL\_CRASH to 0.
- Experiment with other pricing strategies: Try different values for the parameters
  - MSK\_IPAR\_SIM\_PRIMAL\_SELECTION and
  - MSK\_IPAR\_SIM\_DUAL\_SELECTION.
- If you are using warm-starts, in rare cases switching off this feature may improve stability. This is controlled by the  $MSK\_IPAR\_SIM\_HOTSTART$  parameter.
- Increase maximum number of set-backs allowed controlled by MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS.
- If the problem repeatedly becomes infeasible try switching off the special degeneracy handling. See the parameter  $MSK\_IPAR\_SIM\_DEGEN$  for details.

#### The Simplex Log

Below is a typical log output from the simplex optimizer:

Optimizer - solved problem Optimizer - Constraints		: the pr	rimal				
Optimi	Optimizer - Scalar variables		: 1424	conic		: 0	
Optimizer - hotstart		: no					
ITER	DEGITER(%)	PFEAS	DFEAS	POBJ	DOBJ		TIME
$\hookrightarrow$	TOTTIME						
0	0.00	1.43e+05	NA	6.5584140832e+03	NA		0.00 <sub>L</sub>
$\hookrightarrow$	0.02						
1000	1.10	0.00e+00	NA	1.4588289726e+04	NA		0.13 <mark>⊔</mark>
$\hookrightarrow$	0.14						
2000	0.75	0.00e+00	NA	7.3705564855e+03	NA		0.21 <sub>L</sub>
$\hookrightarrow$	0.22						
3000	0.67	0.00e+00	NA	6.0509727712e+03	NA		0.29 <sub>L</sub>
$\hookrightarrow$	0.31						
4000	0.52	0.00e+00	NA	5.5771203906e+03	NA		0.38 <sub>L</sub>
$\hookrightarrow$	0.39						
4533	0.49	0.00e+00	NA	5.5018458883e+03	NA		0.42 <mark>u</mark>
$\hookrightarrow$	0.44						

The first lines summarize the problem the optimizer is solving. This is followed by the iteration log, with the following meaning:

- ITER: Number of iterations.
- DEGITER(%): Ratio of degenerate iterations.
- PFEAS: Primal feasibility measure reported by the simplex optimizer. The numbers should be 0 if the problem is primal feasible (when the primal variant is used).
- DFEAS: Dual feasibility measure reported by the simplex optimizer. The number should be 0 if the problem is dual feasible (when the dual variant is used).
- POBJ: An estimate for the primal objective value (when the primal variant is used).
- DOBJ: An estimate for the dual objective value (when the dual variant is used).
- TIME: Time spent since this instance of the simplex optimizer was invoked (in seconds).
- TOTTIME: Time spent since optimization started (in seconds).

## 8.4 Conic Optimization

For conic optimization problems only an interior-point type optimizer is available.

## 8.4.1 The Interior-point optimizer

#### The homogeneous primal-dual problem

The interior-point optimizer is an implementation of the so-called homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [ART03]. In order to keep our discussion simple we will assume that **MOSEK** solves a conic optimization problem of the form:

minimize 
$$c^T x$$
  
subject to  $Ax = b$ ,  $x \in \mathcal{K}$  (8.6)

where K is a convex cone. The corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & b^T y \\ \text{subject to} & A^T y + s & = & c, \\ & x \in \mathcal{K}^* \end{array} \tag{8.7}$$

where  $K^*$  is the dual cone of K. See Sec. 7.2 for definitions.

Since it is not known beforehand whether problem (8.6) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason that **MOSEK** solves the so-called homogeneous model

$$Ax - b\tau = 0,$$

$$A^{T}y + s - c\tau = 0,$$

$$-c^{T}x + b^{T}y - \kappa = 0,$$

$$x \in \mathcal{K},$$

$$s \in \mathcal{K}^{*},$$

$$\tau, \kappa \geq 0,$$

$$(8.8)$$

where y and s correspond to the dual variables in (8.6), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (8.8) always has a solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (8.8) satisfies

$$(x^*)^T s^* + \tau^* \kappa^* = 0$$

i.e. complementarity. Observe that  $x^* \in \mathcal{K}$  and  $s^* \in \mathcal{K}^*$  implies

$$(x^*)^T s^* \ge 0$$

and therefore

$$\tau^* \kappa^* = 0$$

since  $\tau^*, \kappa^* \geq 0$ . Hence, at least one of  $\tau^*$  and  $\kappa^*$  is zero.

First, assume that  $\tau^* > 0$  and hence  $\kappa^* = 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*/\tau^* & \in & \mathcal{K}, \\ s^*/\tau^* & \in & \mathcal{K}^*. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left(\frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*}\right)$$

is a primal-dual optimal solution.

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^* & \in & \mathcal{K}, \\ s^* & \in & \mathcal{K}^*. \end{array}$$

This implies that at least one of

$$c^T x^* < 0 (8.9)$$

or

$$b^T y^* > 0 (8.10)$$

holds. If (8.9) is satisfied, then  $x^*$  is a certificate of dual infeasibility, whereas if (8.10) holds then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

## **Interior-point Termination Criterion**

Since computations are performed in finite precision, and for efficiency reasons, it is not possible to solve the homogeneous model exactly in general. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In every iteration k of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to the homogeneous model is generated, where

$$x^k \in \mathcal{K}, s^k \in \mathcal{K}^*, \tau^k, \kappa^k > 0.$$

Therefore, it is possible to compute the values:

$$\begin{array}{lll} \rho_p^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \leq \rho \varepsilon_p (1 + \|b\|_{\infty}) \right\}, \\ \rho_d^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T \frac{y^k}{\tau^k} + \frac{s^k}{\tau^k} - c \right\|_{\infty} \leq \rho \varepsilon_d (1 + \|c\|_{\infty}) \right\}, \\ \rho_g^k &=& \arg\min_{\rho} \left\{ \rho \mid \left( \frac{(x^k)^T s^k}{(\tau^k)^2}, \left| \frac{c^T x^k}{\tau^k} - \frac{b^T y^k}{\tau^k} \right| \right) \leq \rho \varepsilon_g \max \left( 1, \frac{\min\left(\left| c^T x^k \right|, \left| b^T y^k \right| \right)}{\tau^k} \right) \right\}, \\ \rho_{pi}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T y^k + s^k \right\|_{\infty} \leq \rho \varepsilon_i b^T y^k, \, b^T y^k > 0 \right\} \text{ and } \\ \rho_{di}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A x^k \right\|_{\infty} \leq -\rho \varepsilon_i c^T x^k, \, c^T x^k < 0 \right\}. \end{array}$$

Note  $\varepsilon_p, \varepsilon_d, \varepsilon_q$  and  $\varepsilon_i$  are nonnegative user specified tolerances.

#### **Optimal Case**

Observe  $\rho_p^k$  measures how far  $x^k/\tau^k$  is from being a good approximate primal feasible solution. Indeed if  $\rho_p^k \leq 1$ , then

$$\left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \le \varepsilon_p (1 + \|b\|_{\infty}). \tag{8.11}$$

This shows the violations in the primal equality constraints for the solution  $x^k/\tau^k$  is small compared to the size of b given  $\varepsilon_p$  is small.

Similarly, if  $\rho_d^k \leq 1$ , then  $(y^k, s^k)/\tau^k$  is an approximate dual feasible solution. If in addition  $\rho_g \leq 1$ , then the solution  $(x^k, y^k, s^k)/\tau^k$  is approximate optimal because the associated primal and dual objective values are almost identical.

In other words if  $\max(\rho_p^k, \rho_d^k, \rho_q^k) \leq 1$ , then

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is an approximate optimal solution.

#### **Dual Infeasibility Certificate**

Next assume that  $\rho_{di}^k \leq 1$  and hence

$$||Ax^k||_{\infty} \le -\varepsilon_i c^T x^k$$
 and  $-c^T x^k > 0$ 

holds. Now in this case the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{x} := \frac{x^k}{-c^T x^k}$$

and it is easy to verify that

$$||A\bar{x}||_{\infty} \leq \varepsilon_i \text{ and } c^T\bar{x} = -1$$

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

#### **Primal Infeasiblity Certificate**

Next assume that  $\rho_{pi}^k \leq 1$  and hence

$$||A^T y^k + s^k||_{\infty} \le \varepsilon_i b^T y^k$$
 and  $b^T y^k > 0$ 

holds. Now in this case the problem is declared primal infeasible and  $(y^k, s^k)$  is reported as a certificate of primal infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{y} := \frac{y^k}{b^T y^k}$$
 and  $\bar{s} := \frac{s^k}{b^T y^k}$ 

and it is easy to verify that

$$||A^T \bar{y} + \bar{s}||_{\infty} \le \varepsilon_i \text{ and } b^T \bar{y} = 1$$

which shows  $(y^k, s^k)$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

#### Adjusting optimality criteria and near optimality

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_q$  and  $\varepsilon_i$  using parameters; see table for details.

1 0							
ToleranceParameter	name						
$\varepsilon_p$	MSK_DPAR_INTPNT_CO_TOL_PFEAS						
$arepsilon_d$	MSK_DPAR_INTPNT_CO_TOL_DFEAS						
â	אמע החום דוומדווד מו דוו הגו מוח						

Table 8.2: Parameters employed in termination criterion

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (8.11) reveals that the quality of the solution depends on  $||b||_{\infty}$  and  $||c||_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

In some cases the interior-point method terminates having found a solution not too far from meeting the optimality condition (8.11). A solution is defined as near optimal if scaling the termination tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_g$  by the same factor  $\varepsilon_n \in [1.0, +\infty]$  makes the condition (8.11) satisfied. A near optimal solution is therefore of lower quality but still potentially valuable. If for instance the solver stalls, i.e. it can make no more significant progress towards the optimal solution, a near optimal solution could be available and be good enough for the user. Near infeasibility certificates are defined similarly. The value of  $\varepsilon_n$  can be adjusted with the parameter MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

## The Interior-point Log

Below is a typical log output from the interior-point optimizer:

Optimizer	- threads	: 20
Optimizer	- solved problem	: the primal
Optimizer	- Constraints	: 1
Optimizer	- Cones	: 2

```
Optimizer - Scalar variables
                                   : 6
                                                       conic
                                                                              : 6
          - Semi-definite variables: 0
Optimizer
                                                       scalarized
                                                                              : 0
                                                       dense det. time
          - setup time
                                   : 0.00
                                                                              : 0.00
Factor
Factor
          - ML order time
                                   : 0.00
                                                       GP order time
                                                                              : 0.00
Factor
          - nonzeros before factor : 1
                                                       after factor
                                                                              : 1
                                                                              : 1.70e+01
Factor
          - dense dim.
                                    : 0
                                                       flops
ITE PFEAS
            DFEAS
                   GFEAS
                              PRSTATUS
                                         POBJ
                                                           DOBJ
                                                                             MU
                                                                                      TIME
   1.0e+00 2.9e-01 3.4e+00 0.00e+00
                                         2.414213562e+00
                                                           0.000000000e+00
                                                                             1.0e+00
                                                                                      0.01
1
   2.7e-01 7.9e-02 2.2e+00 8.83e-01
                                         6.969257574e-01
                                                           -9.685901771e-03 2.7e-01
   6.5e-02 1.9e-02 1.2e+00 1.16e+00
                                         7.606090061e-01
                                                           6.046141322e-01
                                                                             6.5e-02
   1.7e-03 5.0e-04 2.2e-01
                              1.12e+00
                                         7.084385672e-01
                                                           7.045122560e-01
                                                                             1.7e-03
           4.2e-09
                     4.9e-08
                             1.00e+00
                                         7.071067941e-01
                                                           7.071067599e-01
                                                                             1.4e-08
```

The first line displays the number of threads used by the optimizer and the second line tells that the optimizer chose to solve the dual problem rather than the primal problem. The next line displays the problem dimensions as seen by the optimizer, and the Factor... lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 8.4.1 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $\|A^Ty^k + s^k c\tau^k\|_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to −1 if that is not the case.
- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- MU:  $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$ . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started (in seconds).

# 8.5 Nonlinear Convex Optimization

## 8.5.1 The Interior-point Optimizer

For general convex optimization problems an interior-point type optimizer is available. The interior-point optimizer is an implementation of the homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [AY98], [AY99].

### The Convexity Requirement

Continuous nonlinear problems are required to be convex. For quadratic problems **MOSEK** tests this requirement before optimizing. Specifying a non-convex problem results in an error message.

The following parameters are available to control the convexity check:

- MSK\_IPAR\_CHECK\_CONVEXITY: Turn convexity check on/off.
- MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL: Tolerance for convexity check.
- MSK\_IPAR\_LOG\_CHECK\_CONVEXITY: Turn on more log information for debugging.

#### The Differentiability Requirement

The nonlinear optimizer in **MOSEK** requires both first order and second order derivatives. This of course implies care should be taken when solving problems involving non-differentiable functions.

For instance, the function

$$f(x) = x^2$$

is differentiable everywhere whereas the function

$$f(x) = \sqrt{x}$$

is only differentiable for x>0. In order to make sure that  $\mathbf{MOSEK}$  evaluates the functions at points where they are differentiable, the function domains must be defined by setting appropriate variable bounds.

In general, if a variable is not ranged  $\mathbf{MOSEK}$  will only evaluate that variable at points strictly within the bounds. Hence, imposing the bound

$$x \ge 0$$

in the case of  $\sqrt{x}$  is sufficient to guarantee that the function will only be evaluated in points where it is differentiable.

However, if a function is defined on a closed range, specifying the variable bounds is not sufficient. Consider the function

$$f(x) = \frac{1}{x} + \frac{1}{1 - x}. ag{8.12}$$

In this case the bounds

$$0 \le x \le 1$$

will not guarantee that  $\mathbf{MOSEK}$  only evaluates the function for x strictly between 0 and 1. To force  $\mathbf{MOSEK}$  to strictly satisfy both bounds on ranged variables set the parameter  $\mathbf{MSK\_IPAR\_INTPNT\_STARTING\_POINT}$  to  $\mathbf{MSK\_STARTING\_POINT\_SATISFY\_BOUNDS}$ .

For efficiency reasons it may be better to reformulate the problem than to force **MOSEK** to observe ranged bounds strictly. For instance, (8.12) can be reformulated as follows

$$\begin{array}{rcl} f(x) & = & \frac{1}{x} + \frac{1}{y} \\ 0 & = & 1 - x - y \\ 0 & \leq & x \\ 0 & \leq & y. \end{array}$$

### Interior-point Termination Criteria

The parameters controlling when the general convex interior-point optimizer terminates are shown in Table 8.3.

Table 8.3: Parameters employed in termination criteria.

Parameter name	Purpose
MSK_DPAR_INTPNT_NL_TOL_PFEAS	Controls primal feasibility
MSK_DPAR_INTPNT_NL_TOL_DFEAS	Controls dual feasibility
MSK_DPAR_INTPNT_NL_TOL_REL_GAP	Controls relative gap
MSK_DPAR_INTPNT_TOL_INFEAS	Controls when the problem is declared infeasible
MSK_DPAR_INTPNT_NL_TOL_MU_RED	Controls when the complementarity is reduced enough

## THE OPTIMIZER FOR MIXED-INTEGER PROBLEMS

A problem is a mixed-integer optimization problem when one or more of the variables are constrained to be integer valued. Readers unfamiliar with integer optimization are recommended to consult some relevant literature, e.g. the book [Wol98] by Wolsey.

## 9.1 The Mixed-integer Optimizer Overview

MOSEK can solve mixed-integer

- linear,
- quadratic and quadratically constrained, and
- conic quadratic

problems, at least as long as they do not contain both quadratic objective or constraints and conic constraints at the same time. The mixed-integer optimizer is specialized for solving linear and conic optimization problems. Pure quadratic and quadratically constrained problems are automatically converted to conic form.

By default the mixed-integer optimizer is run-to-run deterministic. This means that if a problem is solved twice on the same computer with identical parameter settings and no time limit then the obtained solutions will be identical. If a time limit is set then this may not be case since the time taken to solve a problem is not deterministic. The mixed-integer optimizer is parallelized i.e. it can exploit multiple cores during the optimization.

The solution process can be split into these phases:

- 1. Presolve: See Sec. 8.1.
- 2. Cut generation: Valid inequalities (cuts) are added to improve the lower bound.
- 3. **Heuristic:** Using heuristics the optimizer tries to guess a good feasible solution. Heuristics can be controlled by the parameter MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL.
- 4. **Search:** The optimal solution is located by branching on integer variables.

## 9.2 Relaxations and bounds

It is important to understand that, in a worst-case scenario, the time required to solve integer optimization problems grows exponentially with the size of the problem (solving mixed-integer problems is NP-hard). For instance, a problem with n binary variables, may require time proportional to  $2^n$ . The value of  $2^n$  is huge even for moderate values of n.

In practice this implies that the focus should be on computing a near-optimal solution quickly rather than on locating an optimal solution. Even if the problem is only solved approximately, it is important to know how far the approximate solution is from an optimal one. In order to say something about the quality of an approximate solution the concept of *relaxation* is important.

Consider for example a mixed-integer optimization problem

$$z^* = \underset{\text{subject to}}{\text{minimize}} c^T x$$

$$subject to \quad Ax = b,$$

$$x \ge 0$$

$$x_j \in \mathbb{Z}, \qquad \forall j \in \mathcal{J}.$$

$$(9.1)$$

It has the continuous relaxation

$$\underline{z} = \underset{\text{subject to}}{\text{minimize}} \quad c^T x \\ \text{subject to} \quad Ax = b, \\ x \ge 0$$
 (9.2)

obtained simply by ignoring the integrality restrictions. The relaxation is a continuous problem, and therefore much faster to solve to optimality with a linear (or, in the general case, conic) optimizer. We call the optimal value  $\underline{z}$  the *objective bound*. The objective bound  $\underline{z}$  normally increases during the solution search process when the continuous relaxation is gradually refined.

Moreover, if  $\hat{x}$  is any feasible solution to (9.1) and

$$\bar{z} := c^T \hat{x}$$

then

$$z < z^* < \bar{z}.$$

These two inequalities allow us to estimate the quality of the integer solution: it is no further away from the optimum than  $\bar{z} - \underline{z}$  in terms of the objective value. Whenever a mixed-integer problem is solved **MOSEK** reports this lower bound so that the quality of the reported solution can be evaluated.

## 9.3 Termination Criterion

In general, it is time consuming to find an exact feasible and optimal solution to an integer optimization problem, though in many practical cases it may be possible to find a sufficiently good solution. The issue of terminating the mixed-integer optimizer is rather delicate and the user has numerous possibilities of influencing it with various parameters. The mixed-integer optimizer employs a relaxed feasibility and optimality criterion to determine when a satisfactory solution is located.

A candidate solution that is feasible for the continuous relaxation is said to be an *integer feasible solution* if the criterion

$$\min(x_i - |x_i|, \lceil x_i \rceil - x_i) \le \delta_1 \quad \forall j \in \mathcal{J}$$

is satisfied, meaning that  $x_i$  is at most  $\delta_1$  from the nearest integer.

Whenever the integer optimizer locates an integer feasible solution it will check if the criterion

$$\bar{z} - \underline{z} \le \max(\delta_2, \delta_3 \max(10^{-10}, |\bar{z}|))$$

is satisfied. If this is the case, the integer optimizer terminates and reports the integer feasible solution as an optimal solution. If an optimal solution cannot be located after the time specified by the parameter  $MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME$  (in seconds), it may be advantageous to relax the termination criteria, and they become replaced with

$$\bar{z} - \underline{z} \le \max(\delta_4, \delta_5 \max(10^{-10}, |\bar{z}|)).$$

Any solution satisfying those will now be reported as **near optimal** and the solver will be terminated (note that since this criterion depends on timing, the optimizer will not be run to run deterministic).

All the  $\delta$  tolerances discussed above can be adjusted using suitable parameters — see Table 9.1.

Table 9.1: Tolerances for the mixed-integer optimizer.

Tolerance	Parameter name
$\delta_1$	MSK_DPAR_MIO_TOL_ABS_RELAX_INT
$\delta_2$	MSK_DPAR_MIO_TOL_ABS_GAP
$\delta_3$	MSK_DPAR_MIO_TOL_REL_GAP
$\delta_4$	MSK_DPAR_MIO_NEAR_TOL_ABS_GAP
$\delta_5$	MSK_DPAR_MIO_NEAR_TOL_REL_GAP

In Table 9.2 some other common parameters affecting the integer optimizer termination criterion are shown. Please note that if the effect of a parameter is delayed, the associated termination criterion is applied only after some time, specified by the MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME parameter.

Table 9.2: Other parameters affecting the integer optimizer termination criterion.

Parameter name	De-	Explanation
	layed	
MSK_IPAR_MIO_MAX_NUM_BRANCHES	Yes	Maximum number of branches allowed.
MSK_IPAR_MIO_MAX_NUM_RELAXS	Yes	Maximum number of relaxations allowed.
MSK_IPAR_MIO_MAX_NUM_SOLUTIONS	Yes	Maximum number of feasible integer solutions al-
		lowed.

## 9.4 Speeding Up the Solution Process

As mentioned previously, in many cases it is not possible to find an optimal solution to an integer optimization problem in a reasonable amount of time. Some suggestions to reduce the solution time are:

- Relax the termination criterion: In case the run time is not acceptable, the first thing to do is to relax the termination criterion see Sec. 9.3 for details.
- Specify a good initial solution: In many cases a good feasible solution is either known or easily computed using problem-specific knowledge. If a good feasible solution is known, it is usually worthwhile to use this as a starting point for the integer optimizer.
- Improve the formulation: A mixed-integer optimization problem may be impossible to solve in one form and quite easy in another form. However, it is beyond the scope of this manual to discuss good formulations for mixed-integer problems. For discussions on this topic see for example [Wol98].

# 9.5 Understanding Solution Quality

To determine the quality of the solution one should check the following:

- The problem status and solution status returned by MOSEK, as well as constraint violations in case of suboptimal solutions.
- ullet The optimality gap defined as

$$\epsilon = |(\text{objective value of feasible solution}) - (\text{objective bound})| = |\bar{z} - \underline{z}|.$$

which measures how much the located solution can deviate from the optimal solution to the problem. The optimality gap can be retrieved through the information item  $MSK\_DINF\_MIO\_OBJ\_ABS\_GAP$ . Often it is more meaningful to look at the relative optimality gap normalized against the magnitude of the solution.

$$\epsilon_{\rm rel} = \frac{|\bar{z} - \underline{z}|}{\max(10^{-10}, |\bar{z}|)}.$$

The relative optimality gap is available in  $MSK\_DINF\_MIO\_OBJ\_REL\_GAP$ .

## 9.6 The Optimizer Log

Below is a typical log output from the mixed-integer optimizer:

	-			, 35728 constraints,					
	_	-	eral inte	ger, 4294 binary, 2279	9 continuous				
-	e table si								
BRANCHES RELAXS ACT_NDS DEPTH			S DEPTH	BEST_INT_OBJ	BEST_RELAX_OBJ	REL_GAP(%)	TIME		
0	1	0	0	NA	1.8218819866e+07	NA	1.6		
0	1	0	0	1.8331557950e+07	1.8218819866e+07	0.61	3.5		
0	1	0	0	1.8300507546e+07	1.8218819866e+07	0.45	4.3		
Cut ge	eneration	started.							
0	2	0	0	1.8300507546e+07	1.8218819866e+07	0.45	5.3		
Cut ge	eneration <sup>e</sup>	terminate	d. Time =	1.43					
0	3	0	0	1.8286893047e+07	1.8231580587e+07	0.30	7.5		
15	18	1	0	1.8286893047e+07	1.8231580587e+07	0.30	10.5		
31	34	1	0	1.8286893047e+07	1.8231580587e+07	0.30	11.1		
51	54	1	0	1.8286893047e+07	1.8231580587e+07	0.30	11.6		
91	94	1	0	1.8286893047e+07	1.8231580587e+07	0.30	12.4		
171	174	1	0	1.8286893047e+07	1.8231580587e+07	0.30	14.3		
331	334	1	0	1.8286893047e+07	1.8231580587e+07	0.30	17.9		
[	-								
Object	tive of be	st intege	r solutio	n : 1.825846762609e+07	7				
Best objective bound				: 1.823311032986e+0	7				
Const	ruct solut	ion objec	tive	: Not employed					
Const	ruct solut	ion # row	ndings	: 0					
User o	objective	cut value		: 0					
Number of cuts generated				: 117					
Number of Gomory cuts				: 108					
Number of CMIR cuts				: 9					
Number	r of branc	hes		: 4425					
Number	r of relax	ations so	lved	: 4410					
Number	r of inter	ior point	iteratio	ns: 25					
Number	r of simpl	ex iterat:	ions	: 221131					

The first lines contain a summary of the problem as seen by the optimizer. This is followed by the iteration log. The columns have the following meaning:

- $\bullet$  BRANCHES: Number of branches generated.
- RELAXS: Number of relaxations solved.
- ACT\_NDS: Number of active branch bound nodes.
- DEPTH: Depth of the recently solved node.
- $\bullet$  BEST\_INT\_OBJ: The best integer objective value,  $\bar{z}.$
- BEST\_RELAX\_OBJ: The best objective bound,  $\underline{z}$ .
- REL\_GAP(%): Relative optimality gap,  $100\% \cdot \epsilon_{\rm rel}$
- TIME: Time (in seconds) from the start of optimization.

Following that a summary of the optimization process is printed.

## PROBLEM ANALYZER

The problem analyzer prints a detailed survey of the

- linear constraints and objective
- quadratic constraints
- conic constraints
- variables

of the model.

In the initial stages of model formulation the problem analyzer may be used as a quick way of verifying that the model has been built or imported correctly. In later stages it can help revealing special structures within the model that may be used to tune the optimizer's performance or to identify the causes of numerical difficulties.

The problem analyzer is run from the command line using the -anapro argument and produces something similar to the following (this is the problem analyzer's survey of the aflow30a problem from the MIPLIB 2003 collection.)

```
Analyzing the problem
Constraints
                         Bounds
                                                   Variables
upper bd:
                421
                          ranged : all
                                                    cont:
                                                                421
fixed
                                                    bin :
                                                                421
Objective, min cx
  range: min |c|: 0.00000 min |c|>0: 11.0000
                                                  max |c|: 500.000
distrib:
                |c|
                           vars
                            421
                 0
          [11, 100)
                            150
          [100, 500]
                            271
Constraint matrix A has
      479 rows (constraints)
      842 columns (variables)
     2091 (0.518449%) nonzero entries (coefficients)
Row nonzeros, A_i
                                  max A_i: 34 (4.038%)
  range: min A_i: 2 (0.23753%)
                A_i rows
distrib:
                                      rows%
                                                   acc%
                            421
                                      87.89
                                                  87.89
                  2
             [8, 15]
                             20
                                       4.18
                                                  92.07
            [16, 31]
                             30
                                       6.26
                                                  98.33
            [32, 34]
                              8
                                       1.67
                                                 100.00
```

```
Column nonzeros, Alj
  range: min A|j: 2 (0.417537%)
                                    max A|j: 3 (0.626305%)
                                       cols%
                 Alj
                   2
                             435
                                       51.66
                                                    51.66
                                       48.34
                   3
                             407
                                                   100.00
A nonzeros, A(ij)
  range: min |A(ij)|: 1.00000
                                   max |A(ij)|: 100.000
distrib:
              A(ij)
                         coeffs
             [1, 10)
                            1670
           [10, 100]
                             421
Constraint bounds, lb <= Ax <= ub
           |b|
distrib:
                                                  ubs
                  0
                                                  421
             [1, 10]
                                  58
                                                   58
Variable bounds, lb <= x <= ub
distrib:
                 |b|
                                 lbs
                                                  ubs
                   0
                                 842
             [1, 10)
                                                  421
           [10, 100]
                                                  421
```

The survey is divided into six different sections, each described below. To keep the presentation short with focus on key elements. The analyzer generally attempts to display information on issues relevant for the current model only: e.g., if the model does not have any conic constraints (this is the case in the example above) or any integer variables, those parts of the analysis will not appear.

## **General Characteristics**

The first part of the survey consists of a brief summary of the model's linear and quadratic constraints (indexed by i) and variables (indexed by j). The summary is divided into three subsections:

## Constraints

- upper bd The number of upper bounded constraints,  $\sum_{j=0}^{n-1} a_{ij}x_j \leq u_i^c$
- lower bd The number of lower bounded constraints,  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j$
- ranged The number of ranged constraints,  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j \leq u_i^c$
- $\bullet$  fixed The number of fixed constraints,  $l_i^c = \sum_{j=0}^{n-1} a_{ij} x_j = u_i^c$
- free The number of free constraints

#### Bounds

- upper bd The number of upper bounded variables,  $x_j \leq u_i^x$
- lower bd The number of lower bounded variables,  $l_k^x \leq x_i$
- ranged The number of ranged variables,  $l_k^x \leq x_j \leq u_j^x$
- fixed The number of fixed variables,  $l_k^x = x_j = u_i^x$
- free The number of free variables

#### Variables

- cont The number of continuous variables,  $x_j \in \mathbb{R}$
- bin The number of binary variables,  $x_j \in \{0,1\}$
- int The number of general integer variables,  $x_i \in \mathbb{Z}$

Only constraints, bounds and domains actually in the model will be reported on; if all entities in a section turn out to be of the same kind, the number will be replaced by all for brevity.

### **Objective**

The second part of the survey focuses on (the linear part of) the objective, summarizing the optimization sense and the coefficients' absolute value range and distribution. The number of 0 (zero) coefficients is singled out (if any such variables are in the problem).

The range is displayed using three terms:

- min |c| The minimum absolute value among all coeffecients
- min |c|>0 The minimum absolute value among the nonzero coefficients
- max |c| The maximum absolute value among the coefficients

If some of these extrema turn out to be equal, the display is shortened accordingly:

- If min |c| is greater than zero, the min |c|>0 term is obsolete and will not be displayed
- If only one or two different coefficients occur this will be displayed using all and an explicit listing of the coefficients

The absolute value distribution is displayed as a table summarizing the numbers by orders of magnitude (with a ratio of 10). Again, the number of variables with a coefficient of 0 (if any) is singled out. Each line of the table is headed by an interval (half-open intervals including their lower bounds), and is followed by the number of variables with their objective coefficient in this interval. Intervals with no elements are skipped.

#### **Linear Constraints**

The third part of the survey displays information on the nonzero coefficients of the linear constraint matrix.

Following a brief summary of the matrix dimensions and the number of nonzero coefficients in total, three sections provide further details on how the nonzero coefficients are distributed by row-wise count (A\_i), by column-wise count (A|j), and by absolute value (|A(ij)|). Each section is headed by a brief display of the distribution's range (min and max), and for the row/column-wise counts the corresponding densities are displayed too (in parentheses).

The distribution tables single out three particularly interesting counts: zero, one, and two nonzeros per row/column; the remaining row/column nonzeros are displayed by orders of magnitude (ratio 2). For each interval the relative and accumulated relative counts are also displayed.

Note that constraints may have both linear and quadratic terms, but the empty rows and columns reported in this part of the survey relate to the linear terms only. If empty rows and/or columns are found in the linear constraint matrix, the problem is analyzed further in order to determine if the corresponding constraints have any quadratic terms or the corresponding variables are used in conic or quadratic constraints.

The distribution of the absolute values, |A(ij)|, is displayed just as for the objective coefficients described above.

#### **Constraint and Variable Bounds**

The fourth part of the survey displays distributions for the absolute values of the finite lower and upper bounds for both constraints and variables. The number of bounds at 0 is singled out and, otherwise, displayed by orders of magnitude (with a ratio of 10).

#### **Quadratic Constraints**

The fifth part of the survey displays distributions for the nonzero elements in the gradient of the quadratic constraints, i.e. the nonzero row counts for the column vectors Qx. The table is similar to the tables for the linear constraints' nonzero row and column counts described in the survey's third part.

Quadratic constraints may also have a linear part, but that will be included in the linear constraints survey; this means that if a problem has one or more pure quadratic constraints, part three of the survey will report the number of linear constraint rows with 0 (zero) nonzeros. Likewise, variables that appear in quadratic terms only will be reported as empty columns (0 nonzeros) in the linear constraint report.

## **Conic Constraints**

The last part of the survey summarizes the model's conic constraints. For each of the two types of cones, quadratic and rotated quadratic, the total number of cones are reported, and the distribution of the cones' dimensions are displayed using intervals. Cones dimensions of 2, 3, and 4 are singled out.

## ANALYZING INFEASIBLE PROBLEMS

When developing and implementing a new optimization model, the first attempts will often be either infeasible, due to specification of inconsistent constraints, or unbounded, if important constraints have been left out.

In this section we will

- go over an example demonstrating how to locate infeasible constraints using the **MOSEK** infeasibility report tool,
- discuss in more general terms which properties may cause infeasibilities, and
- present the more formal theory of infeasible and unbounded problems.

## 11.1 Example: Primal Infeasibility

A problem is said to be *primal infeasible* if no solution exists that satisfies all the constraints of the problem.

As an example of a primal infeasible problem consider the problem of minimizing the cost of transportation between a number of production plants and stores: Each plant produces a fixed number of goods, and each store has a fixed demand that must be met. Supply, demand and cost of transportation per unit are given in Fig. 11.1.

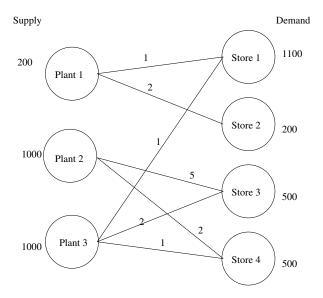


Fig. 11.1: Supply, demand and cost of transportation.

The problem represented in Fig. 11.1 is infeasible, since the total demand

$$2300 = 1100 + 200 + 500 + 500$$

exceeds the total supply

$$2200 = 200 + 1000 + 1000$$

If we denote the number of transported goods from plant i to store j by  $x_{ij}$ , the problem can be formulated as the LP:

minimize 
$$x_{11}$$
 +  $2x_{12}$  +  $5x_{23}$  +  $2x_{24}$  +  $x_{31}$  +  $2x_{33}$  +  $x_{34}$  subject to  $x_{11}$  +  $x_{12}$   $\leq 200$ ,  $\leq 1000$ ,  $x_{23}$  +  $x_{24}$   $\leq 1000$ ,  $x_{31}$  +  $x_{33}$  +  $x_{34}$   $\leq 1000$ ,  $x_{11}$   $= 1100$ ,  $x_{12}$   $= 200$ ,  $x_{23}$  +  $x_{24}$  +  $x_{31}$   $= 200$ ,  $x_{23}$  +  $x_{24}$  +  $x_{31}$   $= 500$ ,  $x_{34}$  =  $x_{31}$  =  $x_{32}$  =  $x_{33}$  =  $x_{33}$  =  $x_{33}$  =  $x_{34}$  =  $x_$ 

Solving problem (11.1) using **MOSEK** will result in a solution, a solution status and a problem status. Among the log output from the execution of **MOSEK** on the above problem are the lines:

```
Basic solution
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
```

The first line indicates that the problem status is primal infeasible. The second line says that a *certificate* of the infeasibility was found. The certificate is returned in place of the solution to the problem.

## 11.2 Locating the cause of Primal Infeasibility

Usually a primal infeasible problem status is caused by a mistake in formulating the problem and therefore the question arises: What is the cause of the infeasible status? When trying to answer this question, it is often advantageous to follow these steps:

- Remove the objective function. This does not change the infeasibility status but simplifies the problem, eliminating any possibility of issues related to the objective function.
- Consider whether your problem has some necessary conditions for feasibility and examine if these are satisfied, e.g. total supply should be greater than or equal to total demand.
- Verify that coefficients and bounds are reasonably sized in your problem.

If the problem is still primal infeasible, some of the constraints must be relaxed or removed completely. The **MOSEK** infeasibility report (Sec. 11.4) may assist you in finding the constraints causing the infeasibility.

Possible ways of relaxing your problem nclude:

- Increasing (decreasing) upper (lower) bounds on variables and constraints.
- Removing suspected constraints from the problem.

Returning to the transportation example, we discover that removing the fifth constraint

$$x_{12} = 200$$

makes the problem feasible.

## 11.3 Locating the Cause of Dual Infeasibility

A problem may also be *dual infeasible*. In this case the primal problem is often unbounded, meaning that feasible solutions exists such that the objective tends towards infinity. An example of a dual infeasible and primal unbounded problem is:

```
minimize x_1 subject to x_1 \le 5.
```

To resolve a dual infeasibility the primal problem must be made more restricted by

- Adding upper or lower bounds on variables or constraints.
- Removing variables.
- Changing the objective.

## 11.3.1 A cautionary note

The problem

minimize 
$$0$$
  
subject to  $0 \le x_1$ ,  
 $x_j \le x_{j+1}$ ,  $j = 1, \dots, n-1$ ,  
 $x_n \le -1$ 

is clearly infeasible. Moreover, if any one of the constraints is dropped, then the problem becomes feasible.

This illustrates the worst case scenario where all, or at least a significant portion of the constraints are involved in causing infeasibility. Hence, it may not always be easy or possible to pinpoint a few constraints responsible for infeasibility.

# 11.4 The Infeasibility Report

**MOSEK** includes functionality for diagnosing the cause of a primal or a dual infeasibility. It can be turned on by setting the MSK\_IPAR\_INFEAS\_REPORT\_AUTO to MSK\_ON. This causes **MOSEK** to print a report on variables and constraints involved in the infeasibility.

The MSK\_IPAR\_INFEAS\_REPORT\_LEVEL parameter controls the amount of information presented in the infeasibility report. The default value is 1.

## 11.4.1 Example: Primal Infeasibility

We will keep working with the problem (11.1) written in LP format:

Listing 11.1: The code for problem (11.1).

```
d1: + x11 + x31 = 1100
d2: + x12 = 200
d3: + x23 + x33 = 500
d4: + x24 + x34 = 500
bounds
end
```

Using the command line (please remeber it accepts options following the C API format)

```
mosek -d MSK_IPAR_INFEAS_REPORT_AUTO MSK_ON infeas.lp
```

MOSEK produces the following infeasibility report:

```
MOSEK PRIMAL INFEASIBILITY REPORT.
Problem status: The problem is primal infeasible
The following constraints are involved in the primal infeasibility.
Index
                   Lower bound
                                    Upper bound
                                                     Dual lower
         Name
                                                                       Dual upper
                                                                       1.000000e+000
         s0
                   NONE.
                                    2.000000e+002
                                                      0.000000e+000
2
         s2
                   NONE
                                    1.000000e+003
                                                      0.000000e+000
                                                                       1.000000e+000
3
         d1
                   1.100000e+003
                                    1.100000e+003
                                                      1.000000e+000
                                                                       0.000000e+000
4
         d2
                   2.000000e+002
                                    2.000000e+002
                                                      1.000000e+000
                                                                       0.000000e+000
The following bound constraints are involved in the infeasibility.
Index
                                                                       Dual upper
                   Lower bound
                                    Upper bound
                                                     Dual lower
         Name
                   0.000000e+000
                                    NONE
                                                      1.000000e+000
                                                                       0.000000e+000
         x33
10
         x34
                   0.000000e+000
                                    NONE
                                                      1.000000e+000
                                                                       0.000000e+000
```

The infeasibility report is divided into two sections corresponding to constraints and variables. It is a selection of those lines from the problem solution (in this case the file infeas.sol), which are important in understanding primal infeasibility. In this case the constraints s0, s2, d1, d2 and variables x33, x34 are of importance.

The columns Dual lower and Dual upper contain the values of dual variables  $s_l^c$ ,  $s_u^c$ ,  $s_l^x$  and  $s_u^x$  in the primal infeasibility certificate (see Sec. 7.1.2). Only the non-zero ones, which contribute to the optimization objective and thus are important for infeasibility, are shown.

It is also possible to obtain the infeasible subproblem. The command line

```
mosek -d MSK_IPAR_INFEAS_REPORT_AUTO MSK_ON infeas.lp -info rinfeas.lp
```

produces the files rinfeas.bas.inf.lp. In this case the content of the file rinfeas.bas.inf.lp is

```
minimize
obj: + 0 x11 + 0 x12 + 0 x13 + 0 x21 + 0 x22 + 0 x23
      + 0 x31 + 0 x32 + 0 x33 + 0 x24 + 0 x34
subject to
s0: + x11 + x12 \le 2e+02
s2: + x31 + x33 + x34 \le 1e+03
    + x11 + x31 = 1.1e+03
d2: + x12 = 2e+02
bounds
x11 free
x12 free
x13 free
x21 free
x22 free
x23 free
x31 free
```

```
x32 free

0 <= x33 <= +infinity

x24 free

0 <= x34 <= +infinity

end
```

which is an optimization problem. This problem is identical to (11.1), except that the objective and some of the constraints and bounds have been removed. Executing the command

```
mosek -d MSK_IPAR_INFEAS_REPORT_AUTO MSK_ON rinfeas.bas.inf.lp
```

demonstrates that the reduced problem is **primal infeasible**. Since the reduced problem is usually smaller than original problem, it should be easier to locate the cause of infeasibility in this rather than in the original (11.1).

## 11.4.2 Example: Dual Infeasibility

The following problem is dual to (11.1) and therefore it is dual infeasible.

Listing 11.2: The dual of problem (11.1).

```
maximize + 200 y1 + 1000 y2 + 1000 y3 + 1100 y4 + 200 y5 + 500 y6 + 500 y7
subject to
  x11: y1+y4 < 1
   x12: y1+y5 < 2
   x23: y2+y6 < 5
   x24: y2+y7 < 2
   x31: y3+y4 < 1
   x33: y3+y6 < 2
   x34: y3+y7 < 1
bounds
   -inf <= y1 < 0
   -\inf <= y2 < 0
   -\inf <= y3 < 0
  y4 free
  y5 free
  y6 free
  y7 free
end
```

This can be verified by proving that

$$(y_1,\ldots,y_7)=(-1,0,-1,1,1,0,0)$$

is a certificate of dual infeasibility (see Sec. 7.1.2) as we can see from this report:

```
MOSEK DUAL INFEASIBILITY REPORT.
Problem status: The problem is dual infeasible
The following constraints are involved in the infeasibility.
Index
         Name
                           Activity
                                            Objective
                                                              Lower bound
                                                                                Upper bound
                           -1.000000e+00
                                                                                2.000000e+00
         x33
                                                              NONE
                                                                                1.000000e+00
                           -1.000000e+00
                                                              NONE
         x34
The following variables are involved in the infeasibility.
Index
                                                              Lower bound
         Name
                                                                                Upper bound
                           Activity
                                            Objective
                           -1.000000e+00
                                            2.000000e+02
                                                              NONE
                                                                                0.000000e+00
         у1
2
         уЗ
                           -1.000000e+00
                                            1.000000e+03
                                                              NONE
                                                                                0.000000e+00
```

3	y4	1.000000e	+00	1.10000	0e+03	NC	NE		NONE	
4	у5	1.000000e	+00	2.00000	0e+02	NC	NE		NONE	
	rior-point sol oblem status	ution summary : DUAL_INFEASIBLE								
So	lution status	: DUAL_INFEASIBLE	_CER							
Pr	imal. obj: 1.	0000000000e+02	nrm: 1	e+00	Viol.	con:	0e+00	var:	0e+00	

Let  $y^*$  denote the reported primal solution. **MOSEK** states

- that the problem is *dual infeasible*,
- that the reported solution is a certificate of dual infeasibility, and
- that the infeasibility measure for  $y^*$  is approximately zero.

Since the original objective was maximization, we have that  $c^Ty^* > 0$ . See Sec. 7.1.2 for how to interpret the parameter values in the infeasibility report for a linear program. We see that the variables y1, y3, y4, y5 and the constraints x33 and x34 contribute to infeasibility with non-zero values in the Activity column.

One possible strategy to fix the infeasibility is to modify the problem so that the certificate of infeasibility becomes invalid. In this case we could do one the following things:

- Add a lower bound on y3. This will directly invalidate the certificate of dual infeasibility.
- Increase the object coefficient of y3. Changing the coefficients sufficiently will invalidate the inequality  $c^T y^* > 0$  and thus the certificate.
- Add lower bounds on x11 or x31. This will directly invalidate the certificate of infeasibility.

Please note that modifying the problem to invalidate the reported certificate does *not* imply that the problem becomes dual feasible — the reason for infeasibility may simply *move*, resulting a problem that is still infeasible, but for a different reason.

More often, the reported certificate can be used to give a hint about errors or inconsistencies in the model that produced the problem.

# 11.5 Theory Concerning Infeasible Problems

This section discusses the theory of infeasibility certificates and how MOSEK uses a certificate to produce an infeasibility report. In general, MOSEK solves the problem

where the corresponding dual problem is

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c}$$

$$+ (l^{x})^{T} s_{l}^{r} - (u^{x})^{T} s_{u}^{x} + c^{f}$$
subject to 
$$A^{T} y + s_{l}^{x} - s_{u}^{x} = c,$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \leq 0.$$

$$(11.3)$$

We use the convension that for any bound that is not finite, the corresponding dual variable is fixed at zero (and thus will have no influence on the dual problem). For example

$$l_j^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0$$

## 11.6 The Certificate of Primal Infeasibility

A certificate of primal infeasibility is any solution to the homogenized dual problem

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & A^T y + s_l^x - s_u^x & = & 0, \\ & -y + s_l^c - s_u^c & = & 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \leq 0. \end{array}$$

with a positive objective value. That is,  $(s_l^{c*}, s_u^{c*}, s_l^{x*}, s_u^{x*})$  is a certificate of primal infeasibility if

$$(l^c)^T s_l^{c*} - (u^c)^T s_u^{c*} + (l^x)^T s_l^{x*} - (u^x)^T s_u^{x*} > 0$$

and

$$\begin{array}{lcl} A^Ty + s_l^{x*} - s_u^{x*} & = & 0, \\ -y + s_l^{c*} - s_u^{c*} & = & 0, \\ s_l^{c*}, s_u^{c*}, s_l^{x*}, s_u^{x*} \leq 0. \end{array}$$

The well-known  $Farkas\ Lemma$  tells us that (11.2) is infeasible if and only if a certificate of primal infeasibility exists.

Let  $(s_l^{c*}, s_u^{c*}, s_l^{c*}, s_u^{a*}, s_u^{a*})$  be a certificate of primal infeasibility then

$$(s_l^{c*})_i > 0((s_u^{c*})_i > 0)$$

implies that the lower (upper) bound on the i th constraint is important for the infeasibility. Furthermore,

$$(s_l^{x*})_i > 0((s_u^{x*})_i > 0)$$

implies that the lower (upper) bound on the j th variable is important for the infeasibility.

## 11.7 The certificate of dual infeasibility

A certificate of dual infeasibility is any solution to the problem

with negative objective value, where we use the definitions

$$\bar{l}_i^c := \left\{ \begin{array}{ll} 0, & l_i^c > -\infty, \\ -\infty, & \text{otherwise,} \end{array} \right\}, \ \bar{u}_i^c := \left\{ \begin{array}{ll} 0, & u_i^c < \infty, \\ \infty, & \text{otherwise,} \end{array} \right\}$$

and

$$\bar{l}_i^x := \left\{ \begin{array}{ll} 0, & l_i^x > -\infty, \\ -\infty, & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \bar{u}_i^x := \left\{ \begin{array}{ll} 0, & u_i^x < \infty, \\ \infty, & \text{otherwise.} \end{array} \right\}$$

Stated differently, a certificate of dual infeasibility is any  $x^*$  such that

$$c^{T}x^{*} < 0,$$

$$\bar{l}^{c} \leq Ax^{*} \leq \bar{u}^{c},$$

$$\bar{l}^{x} \leq x^{*} \leq \bar{u}^{x}$$

$$(11.4)$$

The well-known Farkas Lemma tells us that (11.3) is infeasible if and only if a certificate of dual infeasibility exists.

Note that if  $x^*$  is a certificate of dual infeasibility then for any j such that

$$x_j^* \le 0,$$

variable j is involved in the dual infeasibility.

Given the assumption that all weights are 1 then the command

```
mosek -primalrepair -d MSK_IPAR_LOG_FEAS_REPAIR 3 feasrepair.lp
```

will form the repaired problem and solve it. The parameter MSK\_IPAR\_LOG\_FEAS\_REPAIR controls the amount of log output from the repair. A value of 2 causes the optimal repair to printed out.

The output from running the above command is:

```
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Open file 'feasrepair.lp'
Read summary
Туре
                                     : LO (linear optimization problem)
Objective sense : min
Constraints
Scalar variables : 2
Matrix variables : 0
Time
                           : 0.0
Computer
Platform
                                              : Windows/64-X86
Cores
                                                 : 4
Problem
Name
Objective sense : min
Туре
                                              : LO (linear optimization problem)
                                              : 4
Constraints
                                               : 0
Cones
                                              : 2
Scalar variables
Matrix variables
Integer variables
Primal feasibility repair started.
Optimizer started.
Interior-point optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Total number of eliminations : 2
Eliminator terminated.
Eliminator - tries
                                                                            : 1
                                                                                                                                                                            : 0.00
                                                                                                                          time
Eliminator - elim's
                                                                            : 2
Lin. dep. - tries
Lin. dep. - number
                                                                             : 1
                                                                                                                          time
                                                                                                                                                                            : 0.00
                                                                            : 0
Presolve terminated. Time: 0.00
Optimizer - threads : 1
Optimizer - solved problem : the primal
Optimizer - Constraints : 2
Optimizer - Cones
                                                                             : 0
Optimizer - Scalar variables : 6
                                                                                                                         conic
                                                                                                                                                                           : 0
                                                                                                                                                                          : 0
                                                                                                                         scalarized
Optimizer - Semi-definite variables: 0
Factor - setup time : 0.00
Factor - ML order time : 0.00
                                                                                                                        dense det. time
                                                                                                                                                                           : 0.00
Factor - setup time : 0.00 dense det. time : 0.0
Factor - ML order time : 0.00 GP order time : 0.0
Factor - nonzeros before factor : 3 after factor : 3
Factor - dense dim. : 0 flops : 5.4
ITE PFEAS DFEAS GFEAS PRSTATUS POBJ DOBJ MU
                                                                                                                                                                          : 0.00
                                                                                                                                                                         : 5.40e+001
                                                                                                                                                                                             TIME
2.4 e + 001 \ 8.6 e - 001 \ 1.5 e + 000 \ 0.00 e + 000 \ 1.227497414 e + 001 \ 1.504971820 e + 001 \ 2.6 e + 000 \ 0.00 e + 
2 2.6e+000 9.7e-002 1.7e-001 -6.19e-001 4.363064729e+001 4.648523094e+001 3.0e-001 0.00
3 4.7e-001 1.7e-002 3.1e-002 1.24e+000 4.256803136e+001 4.298540657e+001 5.2e-002 0.00
4 8.7e-004 3.2e-005 5.7e-005 1.08e+000 4.249989892e+001 4.250078747e+001 9.7e-005 0.00
```

```
8.7e-008 3.2e-009 5.7e-009 1.00e+000 4.249999999e+001 4.250000008e+001 9.7e-009 0.00
  8.7e-012 3.2e-013 5.7e-013 1.00e+000 4.250000000e+001 4.250000000e+001 9.7e-013 0.00
Basis identification started.
Primal basis identification phase started.
         TIME
ITER
         0.00
0
Primal basis identification phase terminated. Time: 0.00
Dual basis identification phase started.
         TIME
Dual basis identification phase terminated. Time: 0.00
Basis identification terminated. Time: 0.00
Interior-point optimizer terminated. Time: 0.00.
Optimizer terminated. Time: 0.03
Basic solution summary
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal. obj: 4.2500000000e+001 Viol. con: 1e-013 var: 0e+000
        obj: 4.2500000000e+001 Viol. con: 0e+000 var: 5e-013
Optimal objective value of the penalty problem: 4.250000000000e+001
Repairing bounds.
Increasing the upper bound -2.25e+001 on constraint 'c4' (3) with 1.35e+002.
Decreasing the lower bound 6.50e+002 on variable 'x2' (4) with 2.00e+001.
Primal feasibility repair terminated.
Optimizer started.
Interior-point optimizer started.
Presolve started.
Presolve terminated. Time: 0.00
Interior-point optimizer terminated. Time: 0.00.
Optimizer terminated. Time: 0.00
Interior-point solution summary
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal. obj: -5.6700000000e+003 Viol. con: 0e+000 var: 0e+000
        obj: -5.6700000000e+003 Viol. con: 0e+000 var: 0e+000
Dual.
Basic solution summary
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal. obj: -5.6700000000e+003 Viol. con: 0e+000
        obj: -5.6700000000e+003 Viol. con: 0e+000 var: 0e+000
Optimizer summary
Optimizer
                                                 time: 0.00
Interior-point
                       - iterations : 0
                                              time: 0.00
Basis identification -
                                            time: 0.00
Primal
                 - iterations : 0
                                         time: 0.00
Dual
                  - iterations : 0
                                         time: 0.00
Clean primal
                  - iterations : 0
                                         time: 0.00
                                        time: 0.00
Clean dual
                  - iterations : 0
Clean primal-dual - iterations : 0
                                         time: 0.00
Simplex
                                              time: 0.00
                    - iterations : 0
Primal simplex
                                            time: 0.00
Dual simplex
                    - iterations : 0
                                            time: 0.00
                                             time: 0.00
Primal-dual simplex - iterations : 0
                     - relaxations: 0
                                             time: 0.00
Mixed integer
```

reports the optimal repair. In this case it is to increase the upper bound on constraint c4 by 1.35e2 and

decrease the lower bound on variable  $\mathtt{x2}$  by 20.

# SENSITIVITY ANALYSIS

Given an optimization problem it is often useful to obtain information about how the optimal objective value changes when the problem parameters are perturbed. E.g, assume that a bound represents the capacity of a machine. Now, it may be possible to expand the capacity for a certain cost and hence it is worthwhile knowing what the value of additional capacity is. This is precisely the type of questions the sensitivity analysis deals with.

Analyzing how the optimal objective value changes when the problem data is changed is called *sensitivity* analysis.

#### References

The book [Chv83] discusses the classical sensitivity analysis in Chapter 10 whereas the book [RTV97] presents a modern introduction to sensitivity analysis. Finally, it is recommended to read the short paper [Wal00] to avoid some of the pitfalls associated with sensitivity analysis.

Warning: Currently, sensitivity analysis is only available for continuous linear optimization problems. Moreover, MOSEK can only deal with perturbations of bounds and objective function coefficients.

# 12.1 Sensitivity Analysis for Linear Problems

# 12.1.1 The Optimal Objective Value Function

Assume that we are given the problem

$$z(l^c, u^c, l^x, u^x, c) = \underset{\text{subject to}}{\text{minimize}} c^T x$$

$$subject to \quad l^c \leq Ax \leq u^c,$$

$$l^x \leq x \leq u^x,$$

$$(12.1)$$

and we want to know how the optimal objective value changes as  $l_i^c$  is perturbed. To answer this question we define the perturbed problem for  $l_i^c$  as follows

$$\begin{array}{lll} f_{l_i^c}(\beta) & = & \text{minimize} & & c^T x \\ & & \text{subject to} & l^c + \beta e_i & \leq & Ax & \leq u^c, \\ & & l^x & \leq & x \leq & u^x, \end{array}$$

where  $e_i$  is the *i*-th column of the identity matrix. The function

$$f_{l_s^c}(\beta) \tag{12.2}$$

shows the optimal objective value as a function of  $\beta$ . Please note that a change in  $\beta$  corresponds to a perturbation in  $l_i^c$  and hence (12.2) shows the optimal objective value as a function of varying  $l_i^c$  with the other bounds fixed.

It is possible to prove that the function (12.2) is a piecewise linear and convex function, i.e. its graph may look like in Fig. 12.1 and Fig. 12.2.

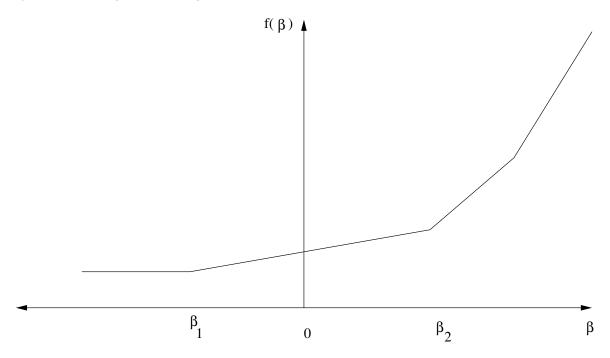


Fig. 12.1:  $\beta = 0$  is in the interior of linearity interval.

Clearly, if the function  $f_{l_i^c}(\beta)$  does not change much when  $\beta$  is changed, then we can conclude that the optimal objective value is insensitive to changes in  $l_i^c$ . Therefore, we are interested in the rate of change in  $f_{l_i^c}(\beta)$  for small changes in  $\beta$  — specifically the gradient

$$f'_{l^c}(0),$$

which is called the *shadow price* related to  $l_i^c$ . The shadow price specifies how the objective value changes for small changes of  $\beta$  around zero. Moreover, we are interested in the *linearity interval* 

$$\beta \in [\beta_1, \beta_2]$$

for which

$$f'_{l_i^c}(\beta) = f'_{l_i^c}(0).$$

Since  $f_{l_i^c}$  is not a smooth function  $f'_{l_i^c}$  may not be defined at 0, as illustrated in Fig. 12.2. In this case we can define a left and a right shadow price and a left and a right linearity interval.

The function  $f_{l_i^c}$  considered only changes in  $l_i^c$ . We can define similar functions for the remaining parameters of the z defined in (12.1) as well:

$$f_{l_i^c}(\beta) = z(l^c + \beta e_i, u^c, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{u_i^c}(\beta) = z(l^c, u^c + \beta e_i, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{l_j^x}(\beta) = z(l^c, u^c, l^x + \beta e_j, u^x, c), \quad j = 1, \dots, n,$$

$$f_{u_j^x}(\beta) = z(l^c, u^c, l^x, u^x + \beta e_j, c), \quad j = 1, \dots, n,$$

$$f_{c_j}(\beta) = z(l^c, u^c, l^x, u^x, c + \beta e_j), \quad j = 1, \dots, n.$$

Given these definitions it should be clear how linearity intervals and shadow prices are defined for the parameters  $u_i^c$  etc.

### **Equality Constraints**

In **MOSEK** a constraint can be specified as either an equality constraint or a ranged constraint. If some constraint  $e_i^c$  is an equality constraint, we define the optimal value function for this constraint as

$$f_{e_i^c}(\beta) = z(l^c + \beta e_i, u^c + \beta e_i, l^x, u^x, c)$$

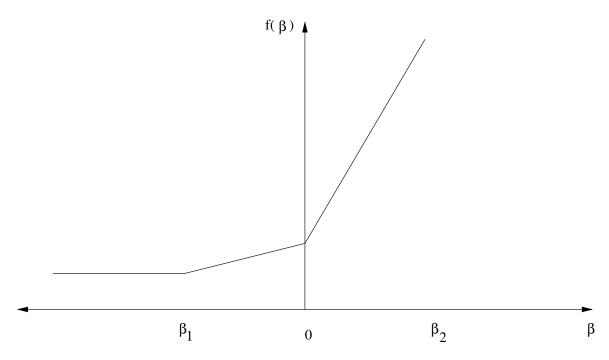


Fig. 12.2:  $\beta = 0$  is a breakpoint.

Thus for an equality constraint the upper and the lower bounds (which are equal) are perturbed simultaneously. Therefore, **MOSEK** will handle sensitivity analysis differently for a ranged constraint with  $l_i^c = u_i^c$  and for an equality constraint.

# 12.1.2 The Basis Type Sensitivity Analysis

The classical sensitivity analysis discussed in most textbooks about linear optimization, e.g. [Chv83], is based on an optimal basic solution or, equivalently, on an optimal basis. This method may produce misleading results [RTV97] but is **computationally cheap**. Therefore, and for historical reasons, this method is available in **MOSEK**.

We will now briefly discuss the basis type sensitivity analysis. Given an optimal basic solution which provides a partition of variables into basic and non-basic variables, the basis type sensitivity analysis computes the linearity interval  $[\beta_1, \beta_2]$  so that the basis remains optimal for the perturbed problem. A shadow price associated with the linearity interval is also computed. However, it is well-known that an optimal basic solution may not be unique and therefore the result depends on the optimal basic solution employed in the sensitivity analysis. This implies that the computed interval is only a subset of the largest interval for which the shadow price is constant. Furthermore, the optimal objective value function might have a breakpoint for  $\beta = 0$ . In this case the basis type sensitivity method will only provide a subset of either the left or the right linearity interval.

In summary, the basis type sensitivity analysis is computationally cheap but does not provide complete information. Hence, the results of the basis type sensitivity analysis should be used with care.

# 12.1.3 The Optimal Partition Type Sensitivity Analysis

Another method for computing the complete linearity interval is called the *optimal partition type sensitivity analysis*. The main drawback of the optimal partition type sensitivity analysis is that it is computationally expensive compared to the basis type analysis. This type of sensitivity analysis is currently provided as an experimental feature in **MOSEK**.

Given the optimal primal and dual solutions to (12.1), i.e.  $x^*$  and  $((s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^x)^*, (s_u^x)^*)$  the optimal

objective value is given by

$$z^* := c^T x^*.$$

The left and right shadow prices  $\sigma_1$  and  $\sigma_2$  for  $l_i^c$  are given by this pair of optimization problems:

$$\begin{array}{lll} \sigma_1 & = & \text{minimize} & e_i^T s_l^c \\ & \text{subject to} & A^T (s_l^c - s_u^c) + s_l^x - s_u^x & = & c, \\ & & (l^c)^T (s_l^c) - (u^c)^T (s_u^c) + (l^x)^T (s_l^x) - (u^x)^T (s_u^x) & = & z^*, \\ & & s_l^c, s_u^c, s_l^c, s_u^s \geq 0 \end{array}$$

and

$$\begin{array}{lll} \sigma_2 & = & \text{maximize} & & e_i^T s_l^c \\ & \text{subject to} & & A^T (s_l^c - s_u^c) + s_l^x - s_u^x & = & c, \\ & & & (l^c)^T (s_l^c) - (u^c)^T (s_u^c) + (l^x)^T (s_l^x) - (u^x)^T (s_u^x) & = & z^*, \\ & & & s_l^c, s_u^c, s_l^c, s_u^x \geq 0. \end{array}$$

These two optimization problems make it easy to interpret the shadow price. Indeed, if  $((s_l^c)^*, (s_u^c)^*, (s_l^c)^*, (s_u^c)^*, (s_u^c)^*)$  is an arbitrary optimal solution then

$$(s_l^c)_i^* \in [\sigma_1, \sigma_2].$$

Next, the linearity interval  $[\beta_1, \beta_2]$  for  $l_i^c$  is computed by solving the two optimization problems

$$\beta_1 = \underset{\text{subject to}}{\text{minimize}} \qquad \beta \\ \text{subject to} \quad l^c + \beta e_i \leq \underset{c}{Ax} \leq u^c, \\ c^T x - \sigma_1 \beta = z^*, \\ l^x \leq x \leq u^x,$$

and

$$\beta_2 = \underset{\text{subject to}}{\text{maximize}} \qquad \beta \\ \text{subject to} \quad l^c + \beta e_i \leq \underset{c^T x - \sigma_2 \beta}{Ax} \leq \underset{c}{u^c}, \\ l^x \leq \underset{x}{x} \leq u^x.$$

The linearity intervals and shadow prices for  $u_i^c$ ,  $l_i^x$ , and  $u_i^x$  are computed similarly to  $l_i^c$ .

The left and right shadow prices for  $c_i$  denoted  $\sigma_1$  and  $\sigma_2$  respectively are computed as follows:

$$\sigma_1 = \underset{\text{subject to}}{\text{minimize}} \qquad e_j^T x \\ \text{subject to} \quad l^c + \beta e_i \leq Ax \leq u^c, \\ c^T x = z^*, \\ l^x \leq x \leq u^x,$$

and

$$\begin{array}{llll} \sigma_2 & = & \text{maximize} & & e_j^T x \\ & \text{subject to} & l^c + \beta e_i & \leq & Ax & \leq & u^c, \\ & & & c^T x & = & z^*, \\ & l^x & \leq & x & \leq & u^x. \end{array}$$

Once again the above two optimization problems make it easy to interpret the shadow prices. Indeed, if  $x^*$  is an arbitrary primal optimal solution, then

$$x_i^* \in [\sigma_1, \sigma_2].$$

The linearity interval  $[\beta_1, \beta_2]$  for a  $c_j$  is computed as follows:

$$\begin{array}{lll} \beta_1 & = & \text{minimize} & \beta \\ & \text{subject to} & A^T(s_l^c - s_u^c) + s_l^x - s_u^x & = & c + \beta e_j, \\ & & (l^c)^T(s_l^c) - (u^c)^T(s_u^c) + (l^x)^T(s_l^x) - (u^x)^T(s_u^x) - \sigma_1 \beta & \leq & z^*, \\ & & s_l^c, s_u^c, s_l^c, s_u^x \geq 0 \end{array}$$

and

$$\begin{array}{lll} \beta_2 & = & \text{maximize} & \beta \\ & \text{subject to} & A^T(s_l^c - s_u^c) + s_l^x - s_u^x & = & c + \beta e_j, \\ & & (l^c)^T(s_l^c) - (u^c)^T(s_u^c) + (l^x)^T(s_l^x) - (u^x)^T(s_u^x) - \sigma_2\beta & \leq & z^*, \\ & & s_l^c, s_u^c, s_l^c, s_u^x \geq 0. \end{array}$$

# 12.1.4 Example: Sensitivity Analysis

As an example we will use the following transportation problem. Consider the problem of minimizing the transportation cost between a number of production plants and stores. Each plant supplies a number of goods and each store has a given demand that must be met. Supply, demand and cost of transportation per unit are shown in Fig. 12.3.

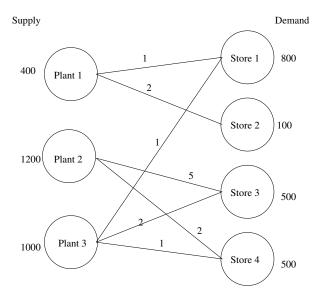


Fig. 12.3: Supply, demand and cost of transportation.

If we denote the number of transported goods from location i to location j by  $x_{ij}$ , problem can be formulated as the linear optimization problem of minimizing

$$1x_{11} + 2x_{12} + 5x_{23} + 2x_{24} + 1x_{31} + 2x_{33} + 1x_{34}$$

subject to

The sensitivity parameters are shown in Table 12.1 and Table 12.2 for the basis type analysis and in Table 12.3 and Table 12.4 for the optimal partition type analysis.

Table 12.1: Ranges and shadow prices related to bounds on constraints and variables: results for the basis type sensitivity analysis.

Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
1	-300.00	0.00	3.00	3.00
2	-700.00	$+\infty$	0.00	0.00
3	-500.00	0.00	3.00	3.00
4	-0.00	500.00	4.00	4.00
5	-0.00	300.00	5.00	5.00
6	-0.00	700.00	5.00	5.00
7	-500.00	700.00	2.00	2.00
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$x_{11}$	$-\infty$	300.00	0.00	0.00
$x_{12}$	$-\infty$	100.00	0.00	0.00
$x_{23}$	$-\infty$	0.00	0.00	0.00
$x_{24}$	$-\infty$	500.00	0.00	0.00
$x_{31}$	$-\infty$	500.00	0.00	0.00
$x_{33}$	$-\infty$	500.00	0.00	0.00
$x_{34}$	-0.000000	500.00	2.00	2.00

Table 12.2: Ranges and shadow prices related to bounds on constraints and variables: results for the optimal partition type sensitivity analysis.

Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
1	-300.00	500.00	3.00	1.00
2	-700.00	$+\infty$	-0.00	-0.00
3	-500.00	500.00	3.00	1.00
4	-500.00	500.00	2.00	4.00
5	-100.00	300.00	3.00	5.00
6	-500.00	700.00	3.00	5.00
7	-500.00	700.00	2.00	2.00
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$x_{11}$	$-\infty$	300.00	0.00	0.00
$x_{12}$	$-\infty$	100.00	0.00	0.00
$x_{23}$	$-\infty$	500.00	0.00	2.00
$x_{24}$	$-\infty$	500.00	0.00	0.00
$x_{31}$	$-\infty$	500.00	0.00	0.00
$x_{33}$	$-\infty$	500.00	0.00	0.00
$x_{34}$	$-\infty$	500.00	0.00	2.00

Table 12.3: Ranges and shadow prices related to the objective coefficients: results for the basis type sensitivity analysis.

Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Table 12.4: Ranges and shadow prices related to the objective coefficients: results for the optimal partition type sensitivity analysis.

Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Examining the results from the optimal partition type sensitivity analysis we see that for constraint number 1 we have  $\sigma_1 = 3$ ,  $\sigma_2 = 1$  and  $\beta_1 = -300$ ,  $\beta_2 = 500$ . Therefore, we have a left linearity interval of [-300, 0] and a right interval of [0, 500]. The corresponding left and right shadow prices are 3 and 1 respectively. This implies that if the upper bound on constraint 1 increases by

$$\beta \in [0, \beta_1] = [0, 500]$$

then the optimal objective value will decrease by the value

$$\sigma_2\beta = 1\beta$$
.

Correspondingly, if the upper bound on constraint 1 is decreased by

$$\beta \in [0, 300]$$

then the optimal objective value will increase by the value

$$\sigma_1 \beta = 3\beta$$
.

# 12.2 Sensitivity Analysis with MOSEK

A sensitivity analysis can be performed with the **MOSEK** command line tool specifying the option -sen, e.g.

```
mosek myproblem.mps -sen sensitivity.ssp
```

where sensitivity.ssp is a file in the format described in the next section. The ssp file describes which parts of the problem the sensitivity analysis should be performed on, see Sec. 12.2.1.

By default results are written to a file named myproblem.sen. If necessary, this file name can be changed by setting the MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME parameter. By default a basis type sensitivity analysis is performed. However, the type of sensitivity analysis (basis or optimal partition) can be changed by setting the parameter MSK\_IPAR\_SENSITIVITY\_TYPE appropriately. Following values are accepted for this parameter:

- MSK\_SENSITIVITY\_TYPE\_BASIS
- MSK\_SENSITIVITY\_TYPE\_OPTIMAL\_PARTITION

## 12.2.1 Sensitivity Analysis Specification File

**MOSEK** employs an MPS-like file format to specify on which model parameters the sensitivity analysis should be performed. As the optimal partition type sensitivity analysis can be computationally expensive it is important to limit the sensitivity analysis.

The format of the sensitivity specification file is shown in Listing 12.1, where capitalized names are keywords, and names in brackets are names of the constraints and variables to be included in the analysis.

Listing 12.1: Sensitivity analysis file specification.

```
BOUNDS CONSTRAINTS
U|L|LU [cname1]
U|L|LU [cname2]-[cname3]
BOUNDS VARIABLES
U|L|LU [vname1]
U|L|LU [vname2]-[vname3]
OBJECTIVE VARIABLES
[vname1]
[vname2]-[vname3]
```

The sensitivity specification file has three sections, i.e.

- BOUNDS CONSTRAINTS: Specifies on which bounds on constraints the sensitivity analysis should be performed.
- BOUNDS VARIABLES: Specifies on which bounds on variables the sensitivity analysis should be performed
- OBJECTIVE VARIABLES: Specifies on which objective coefficients the sensitivity analysis should be performed.

A line in the body of a section must begin with a whitespace. In the BOUNDS sections one of the keys L, U, and LU must appear next. These keys specify whether the sensitivity analysis is performed on the lower bound, on the upper bound, or on both the lower and the upper bound respectively. Next, a single constraint (variable) or range of constraints (variables) is specified.

Recall from Sec. 12.1.1 that equality constraints are handled in a special way. Sensitivity analysis of an equality constraint can be specified with either L, U, or LU, all indicating the same, namely that upper and lower bounds (which are equal) are perturbed simultaneously.

As an example consider

```
BOUNDS CONSTRAINTS
L "cons1"
U "cons2"
LU "cons3"-"cons6"
```

which requests that sensitivity analysis is performed on the lower bound of the constraint named cons1, on the upper bound of the constraint named cons2, and on both lower and upper bound on the constraints named cons3 to cons6.

It is allowed to use indexes instead of names, for instance

```
BOUNDS CONSTRAINTS
L "cons1"
U 2
LU 3 - 6
```

The character \* indicates that the line contains a comment and is ignored.

# 12.2.2 Example: Sensitivity Analysis from Command Line

As an example consider problem (12.3): the sensitivity file shown below (included in the distribution among the examples).

Listing 12.2: Sensitivity file for problem (12.3).

# The command

```
mosek transport.lp -sen sensitivity.ssp -d MSK_IPAR_SENSITIVITY_TYPE sensitivitytype.basis
```

produces the output file as follow

Listing 12.3: Results of sensitivity analysis

BOUNDS CONSTRAINTS INDEX NAME  RIGHTPRICE	BOUND	LEFTRANGE	RIGHTRANGE	LEFTPRICE	Ш
0 c1 →000000e+00	UP	-6.574875e-18	5.000000e+02	1.000000e+00	1.
2 c3 →000000e+00	UP	-6.574875e-18	5.000000e+02	1.000000e+00	1.
3 c4 →000000e+00	FIX	-5.000000e+02	6.574875e-18	2.000000e+00	2.
4 c5 →000000e+00	FIX	-1.000000e+02	6.574875e-18	3.000000e+00	3.
	FIX	-5.000000e+02	6.574875e-18	3.000000e+00	3.
BOUNDS VARIABLES					
INDEX NAME  RIGHTPRICE	BOUND	LEFTRANGE	RIGHTRANGE	LEFTPRICE	Ц
2 x23	LO	-6.574875e-18	5.000000e+02	2.000000e+00	2.
→000000e+00 3 x24 →000000e+00	LO	-inf	5.000000e+02	0.000000e+00	0.
→ VVVVVVVETVV					

4 x	:31	LO	-inf	5.000000e+02	0.000000e+00	0.
→000000e+	-00					
	:11	LO	-inf	3.000000e+02	0.000000e+00	0.
→000000e+	-00					
OBJECTIVE	VARIABLES					
INDEX N	IAME		LEFTRANGE	RIGHTRANGE	LEFTPRICE	ш
→RIGHTPRI	CE					
	:11		-inf	1.000000e+00	3.000000e+02	3.
→000000e+	-02					
	:23		-2.000000e+00	+inf	0.000000e+00	0.
→000000e+	-00					

# 12.2.3 Controlling Log Output

Setting the parameter  $MSK\_IPAR\_LOG\_SENSITIVITY$  to 1 or 0 (default) controls whether or not the results from sensitivity calculations are printed to the message stream.

The parameter  $MSK\_IPAR\_LOG\_SENSITIVITY\_OPT$  controls the amount of debug information on internal calculations from the sensitivity analysis.

## **CHAPTER**

# **THIRTEEN**

# **API REFERENCE**

## • Optimizer parameters:

- Double, Integer, String
- Full list
- Browse by topic
- Optimizer response codes
- Constants

# 13.1 Parameters grouped by topic

# **Analysis**

- MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL
- MSK\_IPAR\_ANA\_SOL\_BASIS
- MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED
- MSK\_IPAR\_LOG\_ANA\_PRO

## **Basis identification**

- MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV
- MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER
- MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR
- MSK\_IPAR\_BI\_MAX\_ITERATIONS
- MSK\_IPAR\_INTPNT\_BASIS
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ

# Conic interior-point method

- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED

- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP

#### Data check

- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL
- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE
- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE
- MSK\_DPAR\_DATA\_TOL\_AIJ
- MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE
- MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE
- MSK\_DPAR\_DATA\_TOL\_BOUND\_INF
- MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN
- MSK\_DPAR\_DATA\_TOL\_C\_HUGE
- MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE
- MSK\_DPAR\_DATA\_TOL\_QIJ
- MSK\_DPAR\_DATA\_TOL\_X
- MSK\_DPAR\_SEMIDEFINITE\_TOL\_APPROX
- MSK\_IPAR\_CHECK\_CONVEXITY
- MSK\_IPAR\_LOG\_CHECK\_CONVEXITY

#### Data input/output

- MSK\_IPAR\_INFEAS\_REPORT\_AUTO
- MSK\_IPAR\_LOG\_FILE
- MSK\_IPAR\_OPF\_MAX\_TERMS\_PER\_LINE
- MSK\_IPAR\_OPF\_WRITE\_HEADER
- MSK\_IPAR\_OPF\_WRITE\_HINTS
- MSK\_IPAR\_OPF\_WRITE\_PARAMETERS
- MSK\_IPAR\_OPF\_WRITE\_PROBLEM
- MSK\_IPAR\_OPF\_WRITE\_SOL\_BAS
- $\bullet \ \textit{MSK\_IPAR\_OPF\_WRITE\_SOL\_ITG}$
- MSK\_IPAR\_OPF\_WRITE\_SOL\_ITR
- MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS
- MSK\_IPAR\_PARAM\_READ\_CASE\_NAME
- MSK\_IPAR\_PARAM\_READ\_IGN\_ERROR
- MSK\_IPAR\_READ\_DATA\_COMPRESSED
- MSK\_IPAR\_READ\_DATA\_FORMAT
- MSK\_IPAR\_READ\_DEBUG

- MSK\_IPAR\_READ\_KEEP\_FREE\_CON
- MSK\_IPAR\_READ\_LP\_DROP\_NEW\_VARS\_IN\_BOU
- MSK\_IPAR\_READ\_LP\_QUOTED\_NAMES
- MSK\_IPAR\_READ\_MPS\_FORMAT
- MSK\_IPAR\_READ\_MPS\_WIDTH
- MSK\_IPAR\_READ\_TASK\_IGNORE\_PARAM
- MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH
- MSK\_IPAR\_SOL\_READ\_WIDTH
- MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_BAS\_HEAD
- MSK\_IPAR\_WRITE\_BAS\_VARIABLES
- MSK\_IPAR\_WRITE\_DATA\_COMPRESSED
- MSK\_IPAR\_WRITE\_DATA\_FORMAT
- MSK\_IPAR\_WRITE\_DATA\_PARAM
- MSK\_IPAR\_WRITE\_FREE\_CON
- MSK\_IPAR\_WRITE\_GENERIC\_NAMES
- MSK\_IPAR\_WRITE\_GENERIC\_NAMES\_IO
- MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_ITEMS
- MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_INT\_HEAD
- MSK\_IPAR\_WRITE\_INT\_VARIABLES
- MSK\_IPAR\_WRITE\_LP\_FULL\_OBJ
- MSK\_IPAR\_WRITE\_LP\_LINE\_WIDTH
- MSK\_IPAR\_WRITE\_LP\_QUOTED\_NAMES
- MSK\_IPAR\_WRITE\_LP\_STRICT\_FORMAT
- MSK\_IPAR\_WRITE\_LP\_TERMS\_PER\_LINE
- MSK\_IPAR\_WRITE\_MPS\_FORMAT
- MSK\_IPAR\_WRITE\_MPS\_INT
- MSK\_IPAR\_WRITE\_PRECISION
- MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES
- MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_SOL\_HEAD
- MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES
- MSK\_IPAR\_WRITE\_SOL\_VARIABLES
- MSK\_IPAR\_WRITE\_TASK\_INC\_SOL
- MSK\_IPAR\_WRITE\_XML\_MODE
- MSK\_SPAR\_BAS\_SOL\_FILE\_NAME
- MSK\_SPAR\_DATA\_FILE\_NAME
- MSK\_SPAR\_DEBUG\_FILE\_NAME

- MSK\_SPAR\_INT\_SOL\_FILE\_NAME
- MSK\_SPAR\_ITR\_SOL\_FILE\_NAME
- MSK\_SPAR\_MIO\_DEBUG\_STRING
- MSK\_SPAR\_PARAM\_COMMENT\_SIGN
- MSK\_SPAR\_PARAM\_READ\_FILE\_NAME
- MSK\_SPAR\_PARAM\_WRITE\_FILE\_NAME
- MSK\_SPAR\_READ\_MPS\_BOU\_NAME
- MSK\_SPAR\_READ\_MPS\_OBJ\_NAME
- MSK\_SPAR\_READ\_MPS\_RAN\_NAME
- MSK\_SPAR\_READ\_MPS\_RHS\_NAME
- MSK\_SPAR\_SENSITIVITY\_FILE\_NAME
- MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME
- MSK\_SPAR\_SOL\_FILTER\_XC\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XC\_UPR
- MSK\_SPAR\_SOL\_FILTER\_XX\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XX\_UPR
- MSK\_SPAR\_STAT\_FILE\_NAME
- MSK\_SPAR\_STAT\_KEY
- MSK\_SPAR\_STAT\_NAME
- MSK\_SPAR\_WRITE\_LP\_GEN\_VAR\_NAME

## **Debugging**

• MSK\_IPAR\_AUTO\_SORT\_A\_BEFORE\_OPT

#### **Dual simplex**

- MSK\_IPAR\_SIM\_DUAL\_CRASH
- MSK\_IPAR\_SIM\_DUAL\_RESTRICT\_SELECTION
- MSK\_IPAR\_SIM\_DUAL\_SELECTION

# Infeasibility report

- MSK\_IPAR\_INFEAS\_GENERIC\_NAMES
- $\bullet \ \textit{MSK\_IPAR\_INFEAS\_REPORT\_LEVEL}$
- MSK\_IPAR\_LOG\_INFEAS\_ANA

#### Interior-point method

- MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_NL\_MERIT\_BAL
- MSK\_DPAR\_INTPNT\_NL\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_NL\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_NL\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_NL\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_STEP
- MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_TOL\_DSAFE
- MSK\_DPAR\_INTPNT\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_TOL\_PATH
- MSK\_DPAR\_INTPNT\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_TOL\_PSAFE
- MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP
- MSK\_DPAR\_INTPNT\_TOL\_STEP\_SIZE
- MSK\_DPAR\_QCQO\_REFORMULATE\_REL\_DROP\_TOL
- $\bullet \quad \mathit{MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER}$
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR
- MSK\_IPAR\_INTPNT\_BASIS
- MSK\_IPAR\_INTPNT\_DIFF\_STEP
- MSK\_IPAR\_INTPNT\_HOTSTART
- MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS

- MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR
- MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS
- MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH
- MSK\_IPAR\_INTPNT\_ORDER\_METHOD
- MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE
- MSK\_IPAR\_INTPNT\_SCALING
- MSK\_IPAR\_INTPNT\_SOLVE\_FORM
- MSK\_IPAR\_INTPNT\_STARTING\_POINT
- MSK\_IPAR\_LOG\_INTPNT

#### License manager

- MSK\_IPAR\_CACHE\_LICENSE
- MSK\_IPAR\_LICENSE\_DEBUG
- MSK\_IPAR\_LICENSE\_PAUSE\_TIME
- MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS
- MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN
- MSK\_IPAR\_LICENSE\_WAIT

## Logging

- MSK\_IPAR\_LOG
- MSK\_IPAR\_LOG\_ANA\_PRO
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ
- MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT
- MSK\_IPAR\_LOG\_EXPAND
- MSK\_IPAR\_LOG\_FEAS\_REPAIR
- MSK\_IPAR\_LOG\_FILE
- MSK\_IPAR\_LOG\_INFEAS\_ANA
- MSK\_IPAR\_LOG\_INTPNT
- MSK\_IPAR\_LOG\_MIO
- MSK\_IPAR\_LOG\_MIO\_FREQ
- MSK\_IPAR\_LOG\_ORDER
- MSK\_IPAR\_LOG\_PRESOLVE
- $\bullet \ \textit{MSK\_IPAR\_LOG\_RESPONSE}$
- MSK\_IPAR\_LOG\_SENSITIVITY
- MSK\_IPAR\_LOG\_SENSITIVITY\_OPT
- MSK\_IPAR\_LOG\_SIM
- MSK\_IPAR\_LOG\_SIM\_FREQ

• MSK\_IPAR\_LOG\_STORAGE

#### Mixed-integer optimization

- MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME
- MSK\_DPAR\_MIO\_MAX\_TIME
- MSK\_DPAR\_MIO\_NEAR\_TOL\_ABS\_GAP
- MSK\_DPAR\_MIO\_NEAR\_TOL\_REL\_GAP
- MSK\_DPAR\_MIO\_REL\_GAP\_CONST
- MSK\_DPAR\_MIO\_TOL\_ABS\_GAP
- MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT
- MSK\_DPAR\_MIO\_TOL\_FEAS
- MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT
- MSK\_DPAR\_MIO\_TOL\_REL\_GAP
- MSK\_IPAR\_LOG\_MIO
- MSK\_IPAR\_LOG\_MIO\_FREQ
- MSK\_IPAR\_MIO\_BRANCH\_DIR
- MSK\_IPAR\_MIO\_CONSTRUCT\_SOL
- MSK\_IPAR\_MIO\_CUT\_CLIQUE
- MSK\_IPAR\_MIO\_CUT\_CMIR
- MSK\_IPAR\_MIO\_CUT\_GMI
- MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND
- MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER
- MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL
- MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL
- MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES
- MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS
- MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS
- MSK\_IPAR\_MIO\_NODE\_OPTIMIZER
- MSK\_IPAR\_MIO\_NODE\_SELECTION
- MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE
- MSK\_IPAR\_MIO\_PROBING\_LEVEL
- MSK\_IPAR\_MIO\_RINS\_MAX\_NODES
- MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER
- MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL
- MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL

#### Nonlinear convex method

- MSK\_DPAR\_INTPNT\_NL\_MERIT\_BAL
- MSK\_DPAR\_INTPNT\_NL\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_NL\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_NL\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_NL\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_STEP
- MSK\_DPAR\_INTPNT\_TOL\_INFEAS
- MSK\_IPAR\_CHECK\_CONVEXITY
- MSK\_IPAR\_LOG\_CHECK\_CONVEXITY

## **Output information**

- MSK\_IPAR\_INFEAS\_REPORT\_LEVEL
- MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS
- MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN
- MSK\_IPAR\_LOG
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ
- MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT
- MSK\_IPAR\_LOG\_EXPAND
- MSK\_IPAR\_LOG\_FEAS\_REPAIR
- MSK\_IPAR\_LOG\_FILE
- MSK\_IPAR\_LOG\_INFEAS\_ANA
- MSK\_IPAR\_LOG\_INTPNT
- MSK\_IPAR\_LOG\_MIO
- MSK\_IPAR\_LOG\_MIO\_FREQ
- MSK\_IPAR\_LOG\_ORDER
- $\bullet \ \textit{MSK\_IPAR\_LOG\_RESPONSE}$
- MSK\_IPAR\_LOG\_SENSITIVITY
- $\bullet \ \textit{MSK\_IPAR\_LOG\_SENSITIVITY\_OPT}$
- MSK\_IPAR\_LOG\_SIM
- MSK\_IPAR\_LOG\_SIM\_FREQ
- MSK\_IPAR\_LOG\_SIM\_MINOR
- MSK\_IPAR\_LOG\_STORAGE
- MSK\_IPAR\_MAX\_NUM\_WARNINGS

#### Overall solver

- MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER
- MSK\_IPAR\_INFEAS\_PREFER\_PRIMAL
- MSK\_IPAR\_LICENSE\_WAIT
- MSK\_IPAR\_MIO\_MODE
- MSK\_IPAR\_OPTIMIZER
- MSK\_IPAR\_PRESOLVE\_LEVEL
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS
- MSK\_IPAR\_PRESOLVE\_USE
- MSK\_IPAR\_PRIMAL\_REPAIR\_OPTIMIZER
- MSK\_IPAR\_SENSITIVITY\_ALL
- MSK\_IPAR\_SENSITIVITY\_OPTIMIZER
- MSK\_IPAR\_SENSITIVITY\_TYPE
- MSK\_IPAR\_SOLUTION\_CALLBACK

#### Overall system

- MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO
- MSK\_IPAR\_INTPNT\_MULTI\_THREAD
- MSK\_IPAR\_LICENSE\_WAIT
- MSK\_IPAR\_LOG\_STORAGE
- MSK\_IPAR\_MIO\_MT\_USER\_CB
- MSK\_IPAR\_MT\_SPINCOUNT
- MSK\_IPAR\_NUM\_THREADS
- MSK\_IPAR\_REMOVE\_UNUSED\_SOLUTIONS
- MSK\_IPAR\_TIMING\_LEVEL
- MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN

# Presolve

- MSK\_DPAR\_PRESOLVE\_TOL\_ABS\_LINDEP
- MSK\_DPAR\_PRESOLVE\_TOL\_AIJ
- MSK\_DPAR\_PRESOLVE\_TOL\_REL\_LINDEP
- MSK\_DPAR\_PRESOLVE\_TOL\_S
- MSK\_DPAR\_PRESOLVE\_TOL\_X
- MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_FILL
- MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES
- MSK\_IPAR\_PRESOLVE\_LEVEL
- MSK\_IPAR\_PRESOLVE\_LINDEP\_ABS\_WORK\_TRH
- MSK\_IPAR\_PRESOLVE\_LINDEP\_REL\_WORK\_TRH

- MSK\_IPAR\_PRESOLVE\_LINDEP\_USE
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS
- MSK\_IPAR\_PRESOLVE\_USE

#### **Primal simplex**

- MSK\_IPAR\_SIM\_PRIMAL\_CRASH
- MSK\_IPAR\_SIM\_PRIMAL\_RESTRICT\_SELECTION
- MSK\_IPAR\_SIM\_PRIMAL\_SELECTION

## Progress callback

• MSK\_IPAR\_SOLUTION\_CALLBACK

# Simplex optimizer

- MSK\_DPAR\_BASIS\_REL\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_X
- MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV
- MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV
- MSK\_IPAR\_BASIS\_SOLVE\_USE\_PLUS\_ONE
- MSK\_IPAR\_LOG\_SIM
- MSK\_IPAR\_LOG\_SIM\_FREQ
- MSK\_IPAR\_LOG\_SIM\_MINOR
- MSK\_IPAR\_SENSITIVITY\_OPTIMIZER
- MSK\_IPAR\_SIM\_BASIS\_FACTOR\_USE
- MSK\_IPAR\_SIM\_DEGEN
- MSK\_IPAR\_SIM\_DUAL\_PHASEONE\_METHOD
- MSK\_IPAR\_SIM\_EXPLOIT\_DUPVEC
- MSK\_IPAR\_SIM\_HOTSTART
- MSK\_IPAR\_SIM\_HOTSTART\_LU
- MSK\_IPAR\_SIM\_MAX\_ITERATIONS
- MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS
- MSK\_IPAR\_SIM\_NON\_SINGULAR
- MSK\_IPAR\_SIM\_PRIMAL\_PHASEONE\_METHOD
- MSK\_IPAR\_SIM\_REFACTOR\_FREQ
- MSK\_IPAR\_SIM\_REFORMULATION
- MSK\_IPAR\_SIM\_SAVE\_LU
- MSK\_IPAR\_SIM\_SCALING
- MSK\_IPAR\_SIM\_SCALING\_METHOD

- MSK\_IPAR\_SIM\_SOLVE\_FORM
- MSK\_IPAR\_SIM\_STABILITY\_PRIORITY
- MSK\_IPAR\_SIM\_SWITCH\_OPTIMIZER

## Solution input/output

- MSK\_IPAR\_INFEAS\_REPORT\_AUTO
- MSK\_IPAR\_SOL\_FILTER\_KEEP\_BASIC
- MSK\_IPAR\_SOL\_FILTER\_KEEP\_RANGED
- MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH
- MSK\_IPAR\_SOL\_READ\_WIDTH
- MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_BAS\_HEAD
- MSK\_IPAR\_WRITE\_BAS\_VARIABLES
- MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_INT\_HEAD
- MSK\_IPAR\_WRITE\_INT\_VARIABLES
- MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES
- MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_SOL\_HEAD
- MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES
- MSK\_IPAR\_WRITE\_SOL\_VARIABLES
- MSK\_SPAR\_BAS\_SOL\_FILE\_NAME
- MSK\_SPAR\_INT\_SOL\_FILE\_NAME
- MSK\_SPAR\_ITR\_SOL\_FILE\_NAME
- MSK\_SPAR\_SOL\_FILTER\_XC\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XC\_UPR
- MSK\_SPAR\_SOL\_FILTER\_XX\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XX\_UPR

### Termination criteria

- MSK\_DPAR\_BASIS\_REL\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_X
- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS

- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_NL\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_NL\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_NL\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_NL\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP
- MSK\_DPAR\_LOWER\_OBJ\_CUT
- MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH
- MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME
- MSK\_DPAR\_MIO\_MAX\_TIME
- MSK\_DPAR\_MIO\_NEAR\_TOL\_REL\_GAP
- MSK\_DPAR\_MIO\_REL\_GAP\_CONST
- MSK\_DPAR\_MIO\_TOL\_REL\_GAP
- MSK\_DPAR\_OPTIMIZER\_MAX\_TIME
- MSK\_DPAR\_UPPER\_OBJ\_CUT
- MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH
- MSK\_IPAR\_BI\_MAX\_ITERATIONS
- MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS
- MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES
- MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS
- MSK\_IPAR\_SIM\_MAX\_ITERATIONS

## Other

• MSK\_IPAR\_COMPRESS\_STATFILE

# 13.2 Parameters (alphabetical list sorted by type)

- Double parameters
- Integer parameters
- String parameters

## 13.2.1 Double parameters

#### MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL

If a constraint violates its bound with an amount larger than this value, the constraint name, index and violation will be printed by the solution analyzer.

Default 1e-6

Accepted [0.0; +inf]

Groups Analysis

#### MSK\_DPAR\_BASIS\_REL\_TOL\_S

Maximum relative dual bound violation allowed in an optimal basic solution.

**Default** 1.0e-12

Accepted [0.0; +inf]

Groups Simplex optimizer, Termination criteria

#### MSK\_DPAR\_BASIS\_TOL\_S

Maximum absolute dual bound violation in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Groups Simplex optimizer, Termination criteria

#### MSK\_DPAR\_BASIS\_TOL\_X

Maximum absolute primal bound violation allowed in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Groups Simplex optimizer, Termination criteria

# MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL

This parameter controls when the full convexity check declares a problem to be non-convex. Increasing this tolerance relaxes the criteria for declaring the problem non-convex.

A problem is declared non-convex if negative (positive) pivot elements are detected in the Cholesky factor of a matrix which is required to be PSD (NSD). This parameter controls how much this non-negativity requirement may be violated.

If  $d_i$  is the pivot element for column i, then the matrix Q is considered to not be PSD if:

$$d_i \leq -|Q_{ii}|$$
check\_convexity\_rel\_tol

**Default** 1e-10

Accepted [0; +inf]

Groups Interior-point method

#### MSK\_DPAR\_DATA\_SYM\_MAT\_TOL

Absolute zero tolerance for elements in in suymmetric matrixes. If any value in a symmetric matrix is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

```
Default 1.0e-12
```

**Accepted** [1.0e-16; 1.0e-6]

Groups Data check

#### MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE

An element in a symmetric matrix which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Groups Data check

#### MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE

An element in a symmetric matrix which is larger than this value in absolute size causes a warning message to be printed.

**Default** 1.0e10

Accepted [0.0; +inf]

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_AIJ

Absolute zero tolerance for elements in A. If any value  $A_{ij}$  is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

**Default** 1.0e-12

**Accepted** [1.0e-16; 1.0e-6]

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE

An element in A which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Groups Data check

# MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE

An element in A which is larger than this value in absolute size causes a warning message to be printed.

**Default** 1.0e10

Accepted [0.0; +inf]

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_BOUND\_INF

Any bound which in absolute value is greater than this parameter is considered infinite.

**Default** 1.0e16

Accepted [0.0; +inf]

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN

If a bound value is larger than this value in absolute size, then a warning message is issued.

Default 1.0e8

Accepted [0.0; +inf]

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_C\_HUGE

An element in c which is larger than the value of this parameter in absolute terms is considered to be huge and generates an error.

Default 1.0e16

Accepted [0.0; +inf]

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE

An element in c which is larger than this value in absolute terms causes a warning message to be printed.

Default 1.0e8

Accepted [0.0; +inf]

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_QIJ

Absolute zero tolerance for elements in Q matrices.

**Default** 1.0e-16

Accepted [0.0; +inf]

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_X

Zero tolerance for constraints and variables i.e. if the distance between the lower and upper bound is less than this value, then the lower and upper bound is considered identical.

Default 1.0e-8

Accepted [0.0; +inf]

Groups Data check

## MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS

Dual feasibility tolerance used by the conic interior-point optimizer.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Conic interior-point method

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS

Controls when the conic interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

Accepted [0.0; 1.0]

Groups Interior-point method, Termination criteria, Conic interior-point method

## MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED

Relative complementarity gap feasibility tolerance used by the conic interior-point optimizer.

**Default** 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Conic interior-point method

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

If MOSEK cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

Default 1000

Accepted [1.0; +inf]

Groups Interior-point method, Termination criteria, Conic interior-point method

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS

Primal feasibility tolerance used by the conic interior-point optimizer.

Default 1.0e-8

**Accepted** [0.0; 1.0]

**Groups** Interior-point method, Termination criteria, Conic interior-point method

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

## MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP

Relative gap termination tolerance used by the conic interior-point optimizer.

Default 1.0e-7

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Conic interior-point method

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

#### MSK\_DPAR\_INTPNT\_NL\_MERIT\_BAL

Controls if the complementarity and infeasibility is converging to zero at about equal rates.

Default 1.0e-4

**Accepted** [0.0; 0.99]

Groups Interior-point method, Nonlinear convex method

#### MSK\_DPAR\_INTPNT\_NL\_TOL\_DFEAS

Dual feasibility tolerance used when a nonlinear model is solved.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Nonlinear convex method

#### MSK\_DPAR\_INTPNT\_NL\_TOL\_MU\_RED

Relative complementarity gap tolerance for the nonlinear solver.

**Default** 1.0e-12

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Nonlinear convex method

#### MSK\_DPAR\_INTPNT\_NL\_TOL\_NEAR\_REL

If the MOSEK nonlinear interior-point optimizer cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

**Default** 1000.0

Accepted [1.0; +inf]

Groups Interior-point method, Termination criteria, Nonlinear convex method

# MSK\_DPAR\_INTPNT\_NL\_TOL\_PFEAS

Primal feasibility tolerance used when a nonlinear model is solved.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria, Nonlinear convex method

#### MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_GAP

Relative gap termination tolerance for nonlinear problems.

Default 1.0e-6

Accepted [1.0e-14; +inf]

Groups Termination criteria, Interior-point method, Nonlinear convex method

## MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_STEP

Relative step size to the boundary for general nonlinear optimization problems.

Default 0.995

**Accepted** [1.0e-4; 0.9999999]

Groups Interior-point method, Nonlinear convex method

#### MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS

Dual feasibility tolerance used when the interior-point optimizer is applied to a quadratic optimization problem..

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

## MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS

Controls when the conic interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED

Relative complementarity gap feasibility tolerance used when interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

If **MOSEK** cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

Default 1000

Accepted [1.0; +inf]

Groups Interior-point method, Termination criteria

## MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS

Primal feasibility tolerance used when the interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

#### MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP

Relative gap termination tolerance used when the interior-point optimizer is applied to a quadratic optimization problem.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

#### MSK\_DPAR\_INTPNT\_TOL\_DFEAS

Dual feasibility tolerance used for linear optimization problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

## MSK\_DPAR\_INTPNT\_TOL\_DSAFE

Controls the initial dual starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it might be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Groups Interior-point method

#### MSK\_DPAR\_INTPNT\_TOL\_INFEAS

Controls when the optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

**Accepted** [0.0; 1.0]

**Groups** Interior-point method, Termination criteria, Nonlinear convex method

## MSK\_DPAR\_INTPNT\_TOL\_MU\_RED

Relative complementarity gap tolerance for linear problems.

**Default** 1.0e-16

Accepted [0.0; 1.0]

Groups Interior-point method, Termination criteria

## MSK\_DPAR\_INTPNT\_TOL\_PATH

Controls how close the interior-point optimizer follows the central path. A large value of this parameter means the central is followed very closely. On numerical unstable problems it may be worthwhile to increase this parameter.

Default 1.0e-8

**Accepted** [0.0; 0.9999]

Groups Interior-point method

#### MSK\_DPAR\_INTPNT\_TOL\_PFEAS

Primal feasibility tolerance used for linear optimization problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_TOL\_PSAFE

Controls the initial primal starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it may be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Groups Interior-point method

#### MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP

Relative gap termination tolerance for linear problems.

Default 1.0e-8

Accepted [1.0e-14; +inf]

Groups Termination criteria, Interior-point method

#### MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP

Relative step size to the boundary for linear and quadratic optimization problems.

**Default** 0.9999

**Accepted** [1.0e-4; 0.999999]

Groups Interior-point method

#### MSK\_DPAR\_INTPNT\_TOL\_STEP\_SIZE

Minimal step size tolerance. If the step size falls below the value of this parameter, then the interior-point optimizer assumes that it is stalled. In other words the interior-point optimizer does not make any progress and therefore it is better stop.

Default 1.0e-6

**Accepted** [0.0; 1.0]

Groups Interior-point method

#### MSK\_DPAR\_LOWER\_OBJ\_CUT

If either a primal or dual feasible solution is found proving that the optimal objective value is outside, the interval [  $MSK\_DPAR\_LOWER\_OBJ\_CUT$ ,  $MSK\_DPAR\_UPPER\_OBJ\_CUT$ ], then **MOSEK** is terminated.

**Default** -1.0e30

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

See also MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH

#### MSK DPAR LOWER OBJ CUT FINITE TRH

If the lower objective cut is less than the value of this parameter value, then the lower objective cut i.e.  $MSK\_DPAR\_LOWER\_OBJ\_CUT$  is treated as  $-\infty$ .

Default -0.5e30

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

# MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME

This parameter specifies the number of seconds n during which the termination criteria governed by

- MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS
- MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES

- MSK\_DPAR\_MIO\_NEAR\_TOL\_ABS\_GAP
- MSK\_DPAR\_MIO\_NEAR\_TOL\_REL\_GAP

is disabled since the beginning of the optimization.

A negative value is identical to infinity i.e. the termination criteria are never checked.

Default -1.0

Accepted  $[-\inf; +\inf]$ 

Groups Mixed-integer optimization, Termination criteria

See also MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS, MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES, MSK\_DPAR\_MIO\_NEAR\_TOL\_ABS\_GAP, MSK\_DPAR\_MIO\_NEAR\_TOL\_REL\_GAP

#### MSK\_DPAR\_MIO\_MAX\_TIME

This parameter limits the maximum time spent by the mixed-integer optimizer. A negative number means infinity.

Default -1.0

Accepted  $[-\inf; +\inf]$ 

Groups Mixed-integer optimization, Termination criteria

#### MSK\_DPAR\_MIO\_NEAR\_TOL\_ABS\_GAP

Relaxed absolute optimality tolerance employed by the mixed-integer optimizer. This termination criteria is delayed. See MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME for details.

Default 0.0

Accepted [0.0; +inf]

Groups Mixed-integer optimization

See also MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME

#### MSK\_DPAR\_MIO\_NEAR\_TOL\_REL\_GAP

The mixed-integer optimizer is terminated when this tolerance is satisfied. This termination criteria is delayed. See  $MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME$  for details.

Default 1.0e-3

Accepted [0.0; +inf]

Groups Mixed-integer optimization, Termination criteria

See also MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME

### MSK\_DPAR\_MIO\_REL\_GAP\_CONST

This value is used to compute the relative gap for the solution to an integer optimization problem.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Groups Mixed-integer optimization, Termination criteria

#### MSK\_DPAR\_MIO\_TOL\_ABS\_GAP

Absolute optimality tolerance employed by the mixed-integer optimizer.

Default 0.0

Accepted [0.0; +inf]

**Groups** Mixed-integer optimization

# MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT

Absolute integer feasibility tolerance. If the distance to the nearest integer is less than this tolerance then an integer constraint is assumed to be satisfied.

Default 1.0e-5

Accepted [1e-9; +inf]

**Groups** Mixed-integer optimization

#### MSK\_DPAR\_MIO\_TOL\_FEAS

Feasibility tolerance for mixed integer solver.

Default 1.0e-6

**Accepted** [1e-9; 1e-3]

**Groups** Mixed-integer optimization

#### MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT

If the relative improvement of the dual bound is smaller than this value, the solver will terminate the root cut generation. A value of 0.0 means that the value is selected automatically.

Default 0.0

**Accepted** [0.0; 1.0]

**Groups** Mixed-integer optimization

#### MSK\_DPAR\_MIO\_TOL\_REL\_GAP

Relative optimality tolerance employed by the mixed-integer optimizer.

Default 1.0e-4

Accepted [0.0; +inf]

Groups Mixed-integer optimization, Termination criteria

#### MSK\_DPAR\_OPTIMIZER\_MAX\_TIME

Maximum amount of time the optimizer is allowed to spent on the optimization. A negative number means infinity.

Default -1.0

Accepted [-inf; +inf]

Groups Termination criteria

#### MSK\_DPAR\_PRESOLVE\_TOL\_ABS\_LINDEP

Absolute tolerance employed by the linear dependency checker.

Default 1.0e-6

Accepted [0.0; +inf]

Groups Presolve

# MSK\_DPAR\_PRESOLVE\_TOL\_AIJ

Absolute zero tolerance employed for  $a_{ij}$  in the presolve.

**Default** 1.0e-12

Accepted [1.0e-15; +inf]

Groups Presolve

### MSK\_DPAR\_PRESOLVE\_TOL\_REL\_LINDEP

Relative tolerance employed by the linear dependency checker.

**Default** 1.0e-10

Accepted [0.0; +inf]

Groups Presolve

# MSK\_DPAR\_PRESOLVE\_TOL\_S

Absolute zero tolerance employed for  $s_i$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Groups Presolve

#### MSK\_DPAR\_PRESOLVE\_TOL\_X

Absolute zero tolerance employed for  $x_j$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Groups Presolve

#### MSK\_DPAR\_QCQO\_REFORMULATE\_REL\_DROP\_TOL

This parameter determines when columns are dropped in incomplete Cholesky factorization during reformulation of quadratic problems.

Default 1e-15

Accepted [0; +inf]

Groups Interior-point method

## MSK\_DPAR\_SEMIDEFINITE\_TOL\_APPROX

Tolerance to define a matrix to be positive semidefinite.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Groups Data check

#### MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV

Relative pivot tolerance employed when computing the LU factorization of the basis in the simplex optimizers and in the basis identification procedure.

A value closer to 1.0 generally improves numerical stability but typically also implies an increase in the computational work.

Default 0.01

Accepted [1.0e-6; 0.999999]

Groups Basis identification, Simplex optimizer

## MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV

Absolute pivot tolerance employed by the simplex optimizers.

Default 1.0e-7

Accepted [1.0e-12; +inf]

Groups Simplex optimizer

#### MSK\_DPAR\_UPPER\_OBJ\_CUT

If either a primal or dual feasible solution is found proving that the optimal objective value is outside, the interval [  $MSK\_DPAR\_LOWER\_OBJ\_CUT$ ,  $MSK\_DPAR\_UPPER\_OBJ\_CUT$ ], then **MOSEK** is terminated.

Default 1.0e30

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

See also MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH

## MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH

If the upper objective cut is greater than the value of this parameter, then the upper objective cut  $MSK\_DPAR\_UPPER\_OBJ\_CUT$  is treated as  $\infty$ .

Default 0.5e30

Accepted  $[-\inf; +\inf]$ 

Groups Termination criteria

# 13.2.2 Integer parameters

#### MSK\_IPAR\_ANA\_SOL\_BASIS

Controls whether the basis matrix is analyzed in solution analyzer.

Default ON

Accepted ON, OFF

Groups Analysis

# MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED

Controls whether a list of violated constraints is printed.

All constraints violated by more than the value set by the parameter <code>MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL</code> will be printed.

Default OFF

Accepted ON, OFF

Groups Analysis

#### MSK\_IPAR\_AUTO\_SORT\_A\_BEFORE\_OPT

Controls whether the elements in each column of A are sorted before an optimization is performed. This is not required but makes the optimization more deterministic.

Default OFF

Accepted ON, OFF

Groups Debugging

# MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO

Controls whether the solution information items are automatically updated after an optimization is performed.

Default OFF

Accepted ON, OFF

Groups Overall system

## MSK\_IPAR\_BASIS\_SOLVE\_USE\_PLUS\_ONE

If a slack variable is in the basis, then the corresponding column in the basis is a unit vector with -1 in the right position. However, if this parameter is set to  $MSK_ON$ , -1 is replaced by 1.

Default OFF

Accepted ON, OFF

Groups Simplex optimizer

## MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER

Controls which simplex optimizer is used in the clean-up phase.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

**Groups** Basis identification, Overall solver

#### MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER

If the parameter  $MSK\_IPAR\_INTPNT\_BASIS$  has the value  $MSK\_BI\_NO\_ERROR$  and the interior-point optimizer has terminated due to maximum number of iterations, then basis identification is performed if this parameter has the value  $MSK\_ON$ .

Default OFF

Accepted ON, OFF

Groups Interior-point method, Basis identification

## MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR

If the parameter  $MSK\_IPAR\_INTPNT\_BASIS$  has the value  $MSK\_BI\_NO\_ERROR$  and the interior-point optimizer has terminated due to a numerical problem, then basis identification is performed if this parameter has the value  $MSK\_ON$ .

Default OFF

Accepted ON, OFF

Groups Interior-point method, Basis identification

#### MSK\_IPAR\_BI\_MAX\_ITERATIONS

Controls the maximum number of simplex iterations allowed to optimize a basis after the basis identification.

**Default** 1000000

Accepted  $[0; +\inf]$ 

Groups Basis identification, Termination criteria

#### MSK\_IPAR\_CACHE\_LICENSE

Specifies if the license is kept checked out for the lifetime of the mosek environment  $(MSK\_ON)$  or returned to the server immediately after the optimization  $(MSK\_OFF)$ .

By default the license is checked out for the lifetime of the  $\mathbf{MOSEK}$  environment by the first call to the optimizer.

Check-in and check-out of licenses have an overhead. Frequent communication with the license server should be avoided.

Default ON

Accepted ON, OFF

Groups License manager

## MSK\_IPAR\_CHECK\_CONVEXITY

Specify the level of convexity check on quadratic problems.

Default FULL

Accepted NONE, SIMPLE, FULL

Groups Data check, Nonlinear convex method

#### MSK\_IPAR\_COMPRESS\_STATFILE

Control compression of stat files.

Default ON

Accepted ON, OFF

## MSK\_IPAR\_INFEAS\_GENERIC\_NAMES

Controls whether generic names are used when an infeasible subproblem is created.

Default OFF

Accepted ON, OFF

Groups Infeasibility report

#### MSK\_IPAR\_INFEAS\_PREFER\_PRIMAL

If both certificates of primal and dual infeasibility are supplied then only the primal is used when this option is turned on.

Default ON

Accepted ON, OFF

Groups Overall solver

#### MSK\_IPAR\_INFEAS\_REPORT\_AUTO

Controls whether an infeasibility report is automatically produced after the optimization if the problem is primal or dual infeasible.

Default OFF

Accepted ON, OFF

Groups Data input/output, Solution input/output

#### MSK\_IPAR\_INFEAS\_REPORT\_LEVEL

Controls the amount of information presented in an infeasibility report. Higher values imply more information.

Default 1

Accepted [0; +inf]

**Groups** Infeasibility report, Output information

#### MSK\_IPAR\_INTPNT\_BASIS

Controls whether the interior-point optimizer also computes an optimal basis.

Default ALWAYS

Accepted NEVER, ALWAYS, NO\_ERROR, IF\_FEASIBLE, RESERVERED

Groups Interior-point method, Basis identification

See also MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER, MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR, MSK\_IPAR\_BI\_MAX\_ITERATIONS, MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER

# MSK\_IPAR\_INTPNT\_DIFF\_STEP

Controls whether different step sizes are allowed in the primal and dual space.

Default ON

#### Accepted

- $\bullet$  ON: Different step sizes are allowed.
- OFF: Different step sizes are not allowed.

Groups Interior-point method

#### MSK\_IPAR\_INTPNT\_HOTSTART

Currently not in use.

Default NONE

Accepted NONE, PRIMAL, DUAL, PRIMAL\_DUAL

Groups Interior-point method

## MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS

Controls the maximum number of iterations allowed in the interior-point optimizer.

Default 400

**Accepted**  $[0; +\inf]$ 

Groups Interior-point method, Termination criteria

#### MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR

Controls the maximum number of correctors allowed by the multiple corrector procedure. A negative value means that **MOSEK** is making the choice.

Default -1

Accepted [-1; +inf]

Groups Interior-point method

#### MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS

Maximum number of steps to be used by the iterative refinement of the search direction. A negative value implies that the optimizer chooses the maximum number of iterative refinement steps.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Interior-point method

#### MSK\_IPAR\_INTPNT\_MULTI\_THREAD

Controls whether the interior-point optimizers are allowed to employ multiple threads if more threads is available.

Default ON

Accepted ON, OFF

Groups Overall system

# MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH

Controls how many offending columns are detected in the Jacobian of the constraint matrix.

0	no detection
1	aggressive detection
> 1	higher values mean less aggressive detection

Default 40

Accepted [0; +inf]

Groups Interior-point method

#### MSK\_IPAR\_INTPNT\_ORDER\_METHOD

Controls the ordering strategy used by the interior-point optimizer when factorizing the Newton equation system.

Default FREE

Accepted FREE, APPMINLOC, EXPERIMENTAL, TRY\_GRAPHPAR, FORCE\_GRAPHPAR, NONE

Groups Interior-point method

#### MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE

Controls whether regularization is allowed.

Default ON

Accepted ON, OFF

Groups Interior-point method

# MSK\_IPAR\_INTPNT\_SCALING

Controls how the problem is scaled before the interior-point optimizer is used.

Default FREE

Accepted FREE, NONE, MODERATE, AGGRESSIVE

Groups Interior-point method

### MSK\_IPAR\_INTPNT\_SOLVE\_FORM

Controls whether the primal or the dual problem is solved.

Default FREE

Accepted FREE, PRIMAL, DUAL

Groups Interior-point method

#### MSK\_IPAR\_INTPNT\_STARTING\_POINT

Starting point used by the interior-point optimizer.

Default FREE

Accepted FREE, GUESS, CONSTANT, SATISFY\_BOUNDS

Groups Interior-point method

### MSK\_IPAR\_LICENSE\_DEBUG

This option is used to turn on debugging of the license manager.

Default OFF

Accepted ON, OFF

Groups License manager

#### MSK\_IPAR\_LICENSE\_PAUSE\_TIME

If  $MSK\_IPAR\_LICENSE\_WAIT = MSK\_ON$  and no license is available, then MOSEK sleeps a number of milliseconds between each check of whether a license has become free.

Default 100

**Accepted** [0; 1000000]

Groups License manager

### MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS

Controls whether license features expire warnings are suppressed.

Default OFF

Accepted ON, OFF

Groups License manager, Output information

# MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN

If a license feature expires in a numbers days less than the value of this parameter then a warning will be issued.

Default 7

**Accepted**  $[0; +\inf]$ 

Groups License manager, Output information

### MSK\_IPAR\_LICENSE\_WAIT

If all licenses are in use **MOSEK** returns with an error code. However, by turning on this parameter **MOSEK** will wait for an available license.

Default OFF

Accepted ON, OFF

Groups Overall solver, Overall system, License manager

# MSK\_IPAR\_LOG

Controls the amount of log information. The value 0 implies that all log information is suppressed. A higher level implies that more information is logged.

Please note that if a task is employed to solve a sequence of optimization problems the value of this parameter is reduced by the value of  $MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT$  for the second and any subsequent optimizations.

Default 10

Accepted [0; +inf]

Groups Output information, Logging

See also MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT

### MSK\_IPAR\_LOG\_ANA\_PRO

Controls amount of output from the problem analyzer.

Default 1

Accepted [0; +inf]

Groups Analysis, Logging

### MSK\_IPAR\_LOG\_BI

Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Groups Basis identification, Output information, Logging

#### MSK\_IPAR\_LOG\_BI\_FREQ

Controls how frequent the optimizer outputs information about the basis identification and how frequent the user-defined callback function is called.

Default 2500

Accepted [0; +inf]

**Groups** Basis identification, Output information, Logging

### MSK\_IPAR\_LOG\_CHECK\_CONVEXITY

Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on. If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

Default 0

Accepted [0; +inf]

 ${\bf Groups}\ \textit{Data check, Nonlinear convex method}$ 

# MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT

If a task is employed to solve a sequence of optimization problems, then the value of the log levels is reduced by the value of this parameter. E.g  $MSK\_IPAR\_LOG$  and  $MSK\_IPAR\_LOG\_SIM$  are reduced by the value of this parameter for the second and any subsequent optimizations.

Default 1

Accepted [0; +inf]

Groups Output information, Logging

### MSK\_IPAR\_LOG\_EXPAND

Controls the amount of logging when a data item such as the maximum number constrains is expanded.

Default 0

Accepted [0; +inf]

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_FEAS\_REPAIR

Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.

Default 1

Accepted [0; +inf]

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_FILE

If turned on, then some log info is printed when a file is written or read.

Default 1

Accepted [0; +inf]

Groups Data input/output, Output information, Logging

### MSK\_IPAR\_LOG\_INFEAS\_ANA

Controls amount of output printed by the infeasibility analyzer procedures. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

**Groups** Infeasibility report, Output information, Logging

#### MSK\_IPAR\_LOG\_INTPNT

Controls amount of output printed by the interior-point optimizer. A higher level implies that more information is logged.

Default 1

**Accepted**  $[0; +\inf]$ 

Groups Interior-point method, Output information, Logging

### MSK\_IPAR\_LOG\_MIO

Controls the log level for the mixed-integer optimizer. A higher level implies that more information is logged.

Default 4

Accepted [0; +inf]

Groups Mixed-integer optimization, Output information, Logging

### MSK\_IPAR\_LOG\_MIO\_FREQ

Controls how frequent the mixed-integer optimizer prints the log line. It will print line every time  $MSK\_IPAR\_LOG\_MIO\_FREQ$  relaxations have been solved.

Default 10

Accepted  $[-\inf; +\inf]$ 

Groups Mixed-integer optimization, Output information, Logging

# MSK\_IPAR\_LOG\_ORDER

If turned on, then factor lines are added to the log.

Default 1

Accepted [0; +inf]

Groups Output information, Logging

### MSK\_IPAR\_LOG\_PRESOLVE

Controls amount of output printed by the presolve procedure. A higher level implies that more information is logged.

Default 1

```
Accepted [0; +inf]
```

Groups Logging

### MSK\_IPAR\_LOG\_RESPONSE

Controls amount of output printed when response codes are reported. A higher level implies that more information is logged.

Default 0

Accepted [0; +inf]

Groups Output information, Logging

### MSK\_IPAR\_LOG\_SENSITIVITY

Controls the amount of logging during the sensitivity analysis.

- 0. Means no logging information is produced.
- 1. Timing information is printed.
- 2. Sensitivity results are printed.

Default 1

Accepted [0; +inf]

Groups Output information, Logging

### MSK\_IPAR\_LOG\_SENSITIVITY\_OPT

Controls the amount of logging from the optimizers employed during the sensitivity analysis. 0 means no logging information is produced.

Default 0

Accepted [0; +inf]

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_SIM

Controls amount of output printed by the simplex optimizer. A higher level implies that more information is logged.

Default 4

Accepted [0; +inf]

Groups Simplex optimizer, Output information, Logging

### MSK\_IPAR\_LOG\_SIM\_FREQ

Controls how frequent the simplex optimizer outputs information about the optimization and how frequent the user-defined callback function is called.

Default 1000

Accepted [0; +inf]

Groups Simplex optimizer, Output information, Logging

### MSK\_IPAR\_LOG\_SIM\_MINOR

Currently not in use.

Default 1

Accepted [0; +inf]

Groups Simplex optimizer, Output information

### MSK\_IPAR\_LOG\_STORAGE

When turned on, MOSEK prints messages regarding the storage usage and allocation.

Default 0

Accepted [0; +inf]

Groups Output information, Overall system, Logging

### MSK\_IPAR\_MAX\_NUM\_WARNINGS

Each warning is shown a limit number times controlled by this parameter. A negative value is identical to infinite number of times.

Default 10

Accepted  $[-\inf; +\inf]$ 

Groups Output information

#### MSK\_IPAR\_MIO\_BRANCH\_DIR

Controls whether the mixed-integer optimizer is branching up or down by default.

Default FREE

Accepted FREE, UP, DOWN, NEAR, FAR, ROOT\_LP, GUIDED, PSEUDOCOST

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_CONSTRUCT\_SOL

If set to  $MSK_ON$  and all integer variables have been given a value for which a feasible mixed integer solution exists, then MOSEK generates an initial solution to the mixed integer problem by fixing all integer values and solving the remaining problem.

Default OFF

Accepted ON, OFF

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_CUT\_CLIQUE

Controls whether clique cuts should be generated.

Default ON

# Accepted

- ON: Turns generation of this cut class on.
- OFF: Turns generation of this cut class off.

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_CUT\_CMIR

Controls whether mixed integer rounding cuts should be generated.

Default ON

### Accepted

- ON: Turns generation of this cut class on.
- OFF: Turns generation of this cut class off.

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_CUT\_GMI

Controls whether GMI cuts should be generated.

Default ON

#### Accepted

- $\bullet$  ON: Turns generation of this cut class on.
- $\bullet~\ensuremath{\mathit{OFF}}$  : Turns generation of this cut class off.

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND

Controls whether implied bound cuts should be generated.

# Default OFF

# Accepted

- ON: Turns generation of this cut class on.
- OFF: Turns generation of this cut class off.

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER

Controls whether knapsack cover cuts should be generated.

 $\begin{array}{ccc} \textbf{Default} & \textit{OFF} \\ \textbf{Accepted} & \end{array}$ 

- $\bullet$  ON: Turns generation of this cut class on.
- OFF: Turns generation of this cut class off.

Groups Mixed-integer optimization

### MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL

Controls how aggressively generated cuts are selected to be included in the relaxation.

- -1. The optimizer chooses the level of cut selection
  - 0. Generated cuts less likely to be added to the relaxation
  - 1. Cuts are more aggressively selected to be included in the relaxation

Default -1

Accepted [-1; +1]

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL

Controls the heuristic employed by the mixed-integer optimizer to locate an initial good integer feasible solution. A value of zero means the heuristic is not used at all. A larger value than 0 means that a gradually more sophisticated heuristic is used which is computationally more expensive. A negative value implies that the optimizer chooses the heuristic. Normally a value around 3 to 5 should be optimal.

Default -1

Accepted  $[-\inf; +\inf]$ 

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES

Maximum number of branches allowed during the branch and bound search. A negative value means infinite.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Mixed-integer optimization, Termination criteria

See also MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME

### MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS

Maximum number of relaxations allowed during the branch and bound search. A negative value means infinite.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Mixed-integer optimization

See also MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME

#### MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS

The mixed-integer optimizer can be terminated after a certain number of different feasible solutions has been located. If this parameter has the value n > 0, then the mixed-integer optimizer will be terminated when n feasible solutions have been located.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Mixed-integer optimization, Termination criteria

See also MSK\_DPAR\_MIO\_DISABLE\_TERM\_TIME

### MSK\_IPAR\_MIO\_MODE

Controls whether the optimizer includes the integer restrictions when solving a (mixed) integer optimization problem.

Default SATISFIED

Accepted IGNORED, SATISFIED

Groups Overall solver

#### MSK\_IPAR\_MIO\_MT\_USER\_CB

If true user callbacks are called from each thread used by mixed-integer optimizer. Otherwise it is only called from a single thread.

Default OFF

Accepted ON, OFF

Groups Overall system

### MSK\_IPAR\_MIO\_NODE\_OPTIMIZER

Controls which optimizer is employed at the non-root nodes in the mixed-integer optimizer.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_NODE\_SELECTION

Controls the node selection strategy employed by the mixed-integer optimizer.

Default FREE

Accepted FREE, FIRST, BEST, WORST, HYBRID, PSEUDO

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE

Enables or disables perspective reformulation in presolve.

Default ON

Accepted ON, OFF

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_PROBING\_LEVEL

Controls the amount of probing employed by the mixed-integer optimizer in presolve.

- -1. The optimizer chooses the level of probing employed
  - 0. Probing is disabled

- 1. A low amount of probing is employed
- 2. A medium amount of probing is employed
- 3. A high amount of probing is employed

#### Default -1

Accepted [-1; 3]

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_RINS\_MAX\_NODES

Controls the maximum number of nodes allowed in each call to the RINS heuristic. The default value of -1 means that the value is determined automatically. A value of zero turns off the heuristic.

### Default -1

Accepted  $[-1; +\inf]$ 

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER

Controls which optimizer is employed at the root node in the mixed-integer optimizer.

```
Default FREE
```

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL

Controls whether presolve can be repeated at root node.

- -1 The optimizer chooses whether presolve is repeated
- 0 Never repeat presolve
- 1 Always repeat presolve

### Default -1

Accepted [-1; 1]

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL

Controls how much effort is put into detecting variable bounds.

- -1. The optimizer chooses
  - 0. No variable bounds are detected
  - 1. Only detect variable bounds that are directly represented in the problem
  - 2. Detect variable bounds in probing

### Default -1

Accepted [-1; +2]

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MT\_SPINCOUNT

Set the number of iterations to spin before sleeping.

### **Default** 0

Accepted [0; 1000000000]

### Groups Overall system

### MSK\_IPAR\_NUM\_THREADS

Controls the number of threads employed by the optimizer. If set to 0 the number of threads used will be equal to the number of cores detected on the machine.

Default 0

Accepted [0; +inf]

Groups Overall system

### MSK\_IPAR\_OPF\_MAX\_TERMS\_PER\_LINE

The maximum number of terms (linear and quadratic) per line when an OPF file is written.

Default 5

**Accepted**  $[0; +\inf]$ 

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_HEADER

Write a text header with date and MOSEK version in an OPF file.

Default ON

Accepted ON, OFF

Groups Data input/output

# MSK\_IPAR\_OPF\_WRITE\_HINTS

Write a hint section with problem dimensions in the beginning of an OPF file.

Default ON

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_PARAMETERS

Write a parameter section in an OPF file.

Default OFF

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_PROBLEM

Write objective, constraints, bounds etc. to an OPF file.

Default ON

Accepted ON, OFF

Groups Data input/output

# MSK\_IPAR\_OPF\_WRITE\_SOL\_BAS

If  $MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS$  is  $MSK\_ON$  and a basic solution is defined, include the basic solution in OPF files.

Default ON

Accepted ON, OFF

 ${\bf Groups}\ {\it Data\ input/output}$ 

### MSK\_IPAR\_OPF\_WRITE\_SOL\_ITG

If  $MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS$  is  $MSK\_ON$  and an integer solution is defined, write the integer solution in OPF files.

Default ON

Accepted ON, OFF

```
Groups Data input/output
```

### MSK\_IPAR\_OPF\_WRITE\_SOL\_ITR

If  $MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS$  is  $MSK\_ON$  and an interior solution is defined, write the interior solution in OPF files.

Default ON

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS

Enable inclusion of solutions in the OPF files.

Default OFF

Accepted ON, OFF

Groups Data input/output

# MSK\_IPAR\_OPTIMIZER

The parameter controls which optimizer is used to optimize the task.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

Groups Overall solver

### MSK\_IPAR\_PARAM\_READ\_CASE\_NAME

If turned on, then names in the parameter file are case sensitive.

Default ON

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_PARAM\_READ\_IGN\_ERROR

If turned on, then errors in parameter settings is ignored.

Default OFF

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_FILL

Controls the maximum amount of fill-in that can be created by one pivot in the elimination phase of the presolve. A negative value means the parameter value is selected automatically.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Presolve

# MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES

Control the maximum number of times the eliminator is tried. A negative value implies **MOSEK** decides.

Default -1

Accepted [-inf; +inf]

 ${\bf Groups}\ {\it Presolve}$ 

### MSK\_IPAR\_PRESOLVE\_LEVEL

Currently not used.

Default -1

```
Accepted [-\inf; +\inf]
```

Groups Overall solver, Presolve

### MSK\_IPAR\_PRESOLVE\_LINDEP\_ABS\_WORK\_TRH

The linear dependency check is potentially computationally expensive.

Default 100

Accepted  $[-\inf; +\inf]$ 

Groups Presolve

#### MSK\_IPAR\_PRESOLVE\_LINDEP\_REL\_WORK\_TRH

The linear dependency check is potentially computationally expensive.

Default 100

Accepted  $[-\inf; +\inf]$ 

Groups Presolve

# MSK\_IPAR\_PRESOLVE\_LINDEP\_USE

Controls whether the linear constraints are checked for linear dependencies.

Default ON

### Accepted

- ON: Turns the linear dependency check on.
- OFF: Turns the linear dependency check off.

Groups Presolve

### MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS

Controls the maximum number of reductions performed by the presolve. The value of the parameter is normally only changed in connection with debugging. A negative value implies that an infinite number of reductions are allowed.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Overall solver, Presolve

# MSK\_IPAR\_PRESOLVE\_USE

Controls whether the presolve is applied to a problem before it is optimized.

Default FREE

Accepted OFF, ON, FREE

Groups Overall solver, Presolve

### MSK\_IPAR\_PRIMAL\_REPAIR\_OPTIMIZER

Controls which optimizer that is used to find the optimal repair.

Default FREE

 $\begin{array}{llll} \mathbf{Accepted} & \mathit{FREE}, & \mathit{INTPNT}, & \mathit{CONIC}, & \mathit{PRIMAL\_SIMPLEX}, & \mathit{DUAL\_SIMPLEX}, & \mathit{FREE\_SIMPLEX}, \\ & & \mathit{MIXED\_INT} \end{array}$ 

Groups Overall solver

### MSK\_IPAR\_READ\_DATA\_COMPRESSED

If this option is turned on, it is assumed that the data file is compressed.

Default FREE

Accepted NONE, FREE, GZIP

Groups Data input/output

### MSK\_IPAR\_READ\_DATA\_FORMAT

Format of the data file to be read.

Default EXTENSION

Accepted EXTENSION, MPS, LP, OP, XML, FREE\_MPS, TASK, CB, JSON\_TASK

Groups Data input/output

#### MSK\_IPAR\_READ\_DEBUG

Turns on additional debugging information when reading files.

Default OFF

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_READ\_KEEP\_FREE\_CON

Controls whether the free constraints are included in the problem.

Default OFF

#### Accepted

- ON: The free constraints are kept.
- OFF: The free constraints are discarded.

Groups Data input/output

### MSK\_IPAR\_READ\_LP\_DROP\_NEW\_VARS\_IN\_BOU

If this option is turned on, **MOSEK** will drop variables that are defined for the first time in the bounds section.

Default OFF

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_READ\_LP\_QUOTED\_NAMES

If a name is in quotes when reading an LP file, the quotes will be removed.

Default ON

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_READ\_MPS\_FORMAT

Controls how strictly the MPS file reader interprets the MPS format.

Default FREE

Accepted STRICT, RELAXED, FREE, CPLEX

Groups Data input/output

### MSK\_IPAR\_READ\_MPS\_WIDTH

Controls the maximal number of characters allowed in one line of the MPS file.

Default 1024

Accepted [80; +inf]

Groups Data input/output

### MSK\_IPAR\_READ\_TASK\_IGNORE\_PARAM

Controls whether **MOSEK** should ignore the parameter setting defined in the task file and use the default parameter setting instead.

Default OFF

```
Accepted ON, OFF
```

Groups Data input/output

### MSK\_IPAR\_REMOVE\_UNUSED\_SOLUTIONS

Removes unsued solutions before the optimization is performed.

Default OFF

Accepted ON, OFF

Groups Overall system

#### MSK\_IPAR\_SENSITIVITY\_ALL

Not applicable.

Default OFF

Accepted ON, OFF

Groups Overall solver

### MSK\_IPAR\_SENSITIVITY\_OPTIMIZER

Controls which optimizer is used for optimal partition sensitivity analysis.

Default FREE\_SIMPLEX

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

Groups Overall solver, Simplex optimizer

### MSK\_IPAR\_SENSITIVITY\_TYPE

Controls which type of sensitivity analysis is to be performed.

Default BASIS

Accepted BASIS, OPTIMAL\_PARTITION

Groups Overall solver

### MSK\_IPAR\_SIM\_BASIS\_FACTOR\_USE

Controls whether an LU factorization of the basis is used in a hot-start. Forcing a refactorization sometimes improves the stability of the simplex optimizers, but in most cases there is a performance penalty.

Default ON

Accepted ON, OFF

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_DEGEN

Controls how aggressively degeneration is handled.

Default FREE

Accepted NONE, FREE, AGGRESSIVE, MODERATE, MINIMUM

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_DUAL\_CRASH

Controls whether crashing is performed in the dual simplex optimizer.

If this parameter is set to x, then a crash will be performed if a basis consists of more than (100-x) mod  $f_v$  entries, where  $f_v$  is the number of fixed variables.

Default 90

Accepted [0; +inf]

Groups Dual simplex

### MSK\_IPAR\_SIM\_DUAL\_PHASEONE\_METHOD

An experimental feature.

Default 0

**Accepted** [0; 10]

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_DUAL\_RESTRICT\_SELECTION

The dual simplex optimizer can use a so-called restricted selection/pricing strategy to chooses the outgoing variable. Hence, if restricted selection is applied, then the dual simplex optimizer first choose a subset of all the potential outgoing variables. Next, for some time it will choose the outgoing variable only among the subset. From time to time the subset is redefined.

A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

Default 50

**Accepted** [0; 100]

Groups Dual simplex

### MSK\_IPAR\_SIM\_DUAL\_SELECTION

Controls the choice of the incoming variable, known as the selection strategy, in the dual simplex optimizer.

Default FREE

Accepted FREE, FULL, ASE, DEVEX, SE, PARTIAL

Groups Dual simplex

### MSK\_IPAR\_SIM\_EXPLOIT\_DUPVEC

Controls if the simplex optimizers are allowed to exploit duplicated columns.

Default OFF

Accepted ON, OFF, FREE

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_HOTSTART

Controls the type of hot-start that the simplex optimizer perform.

Default FREE

Accepted NONE, FREE, STATUS\_KEYS

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_HOTSTART\_LU

Determines if the simplex optimizer should exploit the initial factorization.

Default ON

### Accepted

- $\bullet$  ON: Factorization is reused if possible.
- *OFF*: Factorization is recomputed.

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_MAX\_ITERATIONS

Maximum number of iterations that can be used by a simplex optimizer.

**Default** 10000000

Accepted [0; +inf]

Groups Simplex optimizer, Termination criteria

### MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS

Controls how many set-backs are allowed within a simplex optimizer. A set-back is an event where the optimizer moves in the wrong direction. This is impossible in theory but may happen due to numerical problems.

Default 250

Accepted [0; +inf]

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_NON\_SINGULAR

Controls if the simplex optimizer ensures a non-singular basis, if possible.

Default ON

Accepted ON, OFF

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_PRIMAL\_CRASH

Controls whether crashing is performed in the primal simplex optimizer.

In general, if a basis consists of more than (100-this parameter value)% fixed variables, then a crash will be performed.

Default 90

Accepted [0; +inf]

Groups Primal simplex

#### MSK\_IPAR\_SIM\_PRIMAL\_PHASEONE\_METHOD

An experimental feature.

Default 0

**Accepted** [0; 10]

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_PRIMAL\_RESTRICT\_SELECTION

The primal simplex optimizer can use a so-called restricted selection/pricing strategy to chooses the outgoing variable. Hence, if restricted selection is applied, then the primal simplex optimizer first choose a subset of all the potential incoming variables. Next, for some time it will choose the incoming variable only among the subset. From time to time the subset is redefined.

A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

Default 50

**Accepted** [0; 100]

Groups Primal simplex

# MSK\_IPAR\_SIM\_PRIMAL\_SELECTION

Controls the choice of the incoming variable, known as the selection strategy, in the primal simplex optimizer.

Default FREE

Accepted FREE, FULL, ASE, DEVEX, SE, PARTIAL

Groups Primal simplex

# MSK\_IPAR\_SIM\_REFACTOR\_FREQ

Controls how frequent the basis is refactorized. The value 0 means that the optimizer determines the best point of refactorization.

It is strongly recommended NOT to change this parameter.

```
Default 0
```

Accepted [0; +inf]

Groups Simplex optimizer

# MSK\_IPAR\_SIM\_REFORMULATION

Controls if the simplex optimizers are allowed to reformulate the problem.

Default OFF

Accepted ON, OFF, FREE, AGGRESSIVE

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_SAVE\_LU

Controls if the LU factorization stored should be replaced with the LU factorization corresponding to the initial basis.

Default OFF

Accepted ON, OFF

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_SCALING

Controls how much effort is used in scaling the problem before a simplex optimizer is used.

Default FREE

Accepted FREE, NONE, MODERATE, AGGRESSIVE

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_SCALING\_METHOD

Controls how the problem is scaled before a simplex optimizer is used.

Default POW2

Accepted POW2, FREE

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_SOLVE\_FORM

Controls whether the primal or the dual problem is solved by the primal-/dual-simplex optimizer.

Default FREE

Accepted FREE, PRIMAL, DUAL

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_STABILITY\_PRIORITY

Controls how high priority the numerical stability should be given.

Default 50

Accepted [0; 100]

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_SWITCH\_OPTIMIZER

The simplex optimizer sometimes chooses to solve the dual problem instead of the primal problem. This implies that if you have chosen to use the dual simplex optimizer and the problem is dualized, then it actually makes sense to use the primal simplex optimizer instead. If this parameter is on and the problem is dualized and furthermore the simplex optimizer is chosen to be the primal (dual) one, then it is switched to the dual (primal).

Default OFF

Accepted ON, OFF

Groups Simplex optimizer

### MSK\_IPAR\_SOL\_FILTER\_KEEP\_BASIC

If turned on, then basic and super basic constraints and variables are written to the solution file independent of the filter setting.

Default OFF

Accepted ON, OFF

Groups Solution input/output

### MSK\_IPAR\_SOL\_FILTER\_KEEP\_RANGED

If turned on, then ranged constraints and variables are written to the solution file independent of the filter setting.

Default OFF

Accepted ON, OFF

Groups Solution input/output

### MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH

When a solution is read by **MOSEK** and some constraint, variable or cone names contain blanks, then a maximum name width much be specified. A negative value implies that no name contain blanks

Default -1

Accepted [-inf; +inf]

Groups Data input/output, Solution input/output

### MSK\_IPAR\_SOL\_READ\_WIDTH

Controls the maximal acceptable width of line in the solutions when read by MOSEK.

Default 1024

Accepted [80; +inf]

Groups Data input/output, Solution input/output

### MSK\_IPAR\_SOLUTION\_CALLBACK

Indicates whether solution callbacks will be performed during the optimization.

Default OFF

Accepted ON, OFF

Groups Progress callback, Overall solver

### MSK\_IPAR\_TIMING\_LEVEL

Controls the amount of timing performed inside MOSEK.

Default 1

Accepted [0; +inf]

Groups Overall system

### MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS

Controls whether the constraint section is written to the basic solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_BAS\_HEAD

Controls whether the header section is written to the basic solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_BAS\_VARIABLES

Controls whether the variables section is written to the basic solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

# MSK\_IPAR\_WRITE\_DATA\_COMPRESSED

Controls whether the data file is compressed while it is written. 0 means no compression while higher values mean more compression.

Default 0

**Accepted**  $[0; +\inf]$ 

Groups Data input/output

### MSK\_IPAR\_WRITE\_DATA\_FORMAT

Controls the file format when writing task data to a file.

Default EXTENSION

Accepted EXTENSION, MPS, LP, OP, XML, FREE\_MPS, TASK, CB, JSON\_TASK

Groups Data input/output

### MSK\_IPAR\_WRITE\_DATA\_PARAM

If this option is turned on the parameter settings are written to the data file as parameters.

Default OFF

Accepted ON, OFF

Groups Data input/output

# MSK\_IPAR\_WRITE\_FREE\_CON

Controls whether the free constraints are written to the data file.

Default ON

#### Accepted

- $\bullet$  ON: The free constraints are written.
- OFF: The free constraints are discarded.

Groups Data input/output

### MSK\_IPAR\_WRITE\_GENERIC\_NAMES

Controls whether the generic names or user-defined names are used in the data file.

Default OFF

### Accepted

- ON: Generic names are used.
- *OFF*: Generic names are not used.

Groups Data input/output

### MSK\_IPAR\_WRITE\_GENERIC\_NAMES\_IO

Index origin used in generic names.

Default 1

**Accepted**  $[0; +\inf]$ 

Groups Data input/output

### MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_ITEMS

Controls if the writer ignores incompatible problem items when writing files.

Default OFF

### Accepted

- ON: Ignore items that cannot be written to the current output file format.
- OFF: Produce an error if the problem contains items that cannot the written to the current output file format.

Groups Data input/output

### MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS

Controls whether the constraint section is written to the integer solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_INT\_HEAD

Controls whether the header section is written to the integer solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_INT\_VARIABLES

Controls whether the variables section is written to the integer solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_LP\_FULL\_OBJ

Write all variables, including the ones with 0-coefficients, in the objective.

Default ON

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_WRITE\_LP\_LINE\_WIDTH

Maximum width of line in an LP file written by MOSEK.

Default 80

Accepted [40; +inf]

Groups Data input/output

### MSK\_IPAR\_WRITE\_LP\_QUOTED\_NAMES

If this option is turned on, then MOSEK will quote invalid LP names when writing an LP file.

Default ON

Accepted ON, OFF

Groups Data input/output

# MSK\_IPAR\_WRITE\_LP\_STRICT\_FORMAT

Controls whether LP output files satisfy the LP format strictly.

Default OFF

Accepted ON, OFF

```
Groups Data input/output
```

### MSK\_IPAR\_WRITE\_LP\_TERMS\_PER\_LINE

Maximum number of terms on a single line in an LP file written by MOSEK. 0 means unlimited.

Default 10

Accepted [0; +inf]

Groups Data input/output

### MSK\_IPAR\_WRITE\_MPS\_FORMAT

Controls in which format the MPS is written.

Default FREE

Accepted STRICT, RELAXED, FREE, CPLEX

Groups Data input/output

### MSK\_IPAR\_WRITE\_MPS\_INT

Controls if marker records are written to the MPS file to indicate whether variables are integer restricted.

Default ON

### Accepted

- ON: Marker records are written.
- OFF: Marker records are not written.

Groups Data input/output

#### MSK\_IPAR\_WRITE\_PRECISION

Controls the precision with which double numbers are printed in the MPS data file. In general it is not worthwhile to use a value higher than 15.

Default 15

Accepted [0; +inf]

Groups Data input/output

#### MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES

Controls whether the symmetric matrix variables section is written to the solution file.

 $\mathbf{Default} \ \textit{ON}$ 

Accepted ON, OFF

Groups Data input/output, Solution input/output

# MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS

Controls whether the constraint section is written to the solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_SOL\_HEAD

Controls whether the header section is written to the solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

# MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES

Even if the names are invalid MPS names, then they are employed when writing the solution file.

Default OFF

Accepted ON, OFF

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_SOL\_VARIABLES

Controls whether the variables section is written to the solution file.

Default ON

Accepted ON, OFF

Groups Data input/output, Solution input/output

#### MSK\_IPAR\_WRITE\_TASK\_INC\_SOL

Controls whether the solutions are stored in the task file too.

Default ON

Accepted ON, OFF

Groups Data input/output

### MSK\_IPAR\_WRITE\_XML\_MODE

Controls if linear coefficients should be written by row or column when writing in the XML file format

Default ROW

Accepted ROW, COL

Groups Data input/output

# 13.2.3 String parameters

MSK\_SPAR\_BAS\_SOL\_FILE\_NAME

Name of the bas solution file.

Accepted Any valid file name.

Groups Data input/output, Solution input/output

MSK\_SPAR\_DATA\_FILE\_NAME

Data are read and written to this file.

Accepted Any valid file name.

Groups Data input/output

MSK\_SPAR\_DEBUG\_FILE\_NAME

MOSEK debug file.

Accepted Any valid file name.

Groups Data input/output

MSK\_SPAR\_INT\_SOL\_FILE\_NAME

Name of the int solution file.

Accepted Any valid file name.

Groups Data input/output, Solution input/output

MSK\_SPAR\_ITR\_SOL\_FILE\_NAME

Name of the itr solution file.

Accepted Any valid file name.

Groups Data input/output, Solution input/output

### MSK\_SPAR\_MIO\_DEBUG\_STRING

For internal debugging purposes.

**Accepted** Any valid string.

Groups Data input/output

### MSK\_SPAR\_PARAM\_COMMENT\_SIGN

Only the first character in this string is used. It is considered as a start of comment sign in the **MOSEK** parameter file. Spaces are ignored in the string.

#### Default

%%

**Accepted** Any valid string.

Groups Data input/output

### MSK\_SPAR\_PARAM\_READ\_FILE\_NAME

Modifications to the parameter database is read from this file.

Accepted Any valid file name.

Groups Data input/output

### MSK\_SPAR\_PARAM\_WRITE\_FILE\_NAME

The parameter database is written to this file.

**Accepted** Any valid file name.

Groups Data input/output

### MSK\_SPAR\_READ\_MPS\_BOU\_NAME

Name of the BOUNDS vector used. An empty name means that the first BOUNDS vector is used.

Accepted Any valid MPS name.

Groups Data input/output

#### MSK\_SPAR\_READ\_MPS\_OBJ\_NAME

Name of the free constraint used as objective function. An empty name means that the first constraint is used as objective function.

Accepted Any valid MPS name.

Groups Data input/output

### MSK\_SPAR\_READ\_MPS\_RAN\_NAME

Name of the RANGE vector used. An empty name means that the first RANGE vector is used.

Accepted Any valid MPS name.

Groups Data input/output

### MSK\_SPAR\_READ\_MPS\_RHS\_NAME

Name of the RHS used. An empty name means that the first RHS vector is used.

Accepted Any valid MPS name.

Groups Data input/output

### MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN

An access token used to submit tasks to a remote **MOSEK** server. An access token is a random 32-byte string encoded in base64, i.e. it is a 44 character ASCII string.

Accepted Any valid string.

Groups Overall system

# MSK\_SPAR\_SENSITIVITY\_FILE\_NAME

Not applicable.

Accepted Any valid string.

Groups Data input/output

MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME

Not applicable.

**Accepted** Any valid string.

Groups Data input/output

### MSK\_SPAR\_SOL\_FILTER\_XC\_LOW

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]>0.5 should be listed, whereas +0.5 means that all constraints having xc[i]>=blc[i]+0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

Groups Data input/output, Solution input/output

#### MSK\_SPAR\_SOL\_FILTER\_XC\_UPR

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]<0.5 should be listed, whereas -0.5 means all constraints having xc[i]<=buc[i]-0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

Groups Data input/output, Solution input/output

#### MSK SPAR SOL FILTER XX LOW

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j] >= 0.5 should be listed, whereas "+0.5" means that all constraints having xx[j] >= blx[j] + 0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid filter.

Groups Data input/output, Solution input/output

# MSK\_SPAR\_SOL\_FILTER\_XX\_UPR

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j]<0.5 should be printed, whereas "-0.5" means all constraints having xx[j]<=bux[j]-0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid file name.

Groups Data input/output, Solution input/output

### MSK\_SPAR\_STAT\_FILE\_NAME

Statistics file name.

**Accepted** Any valid file name.

 ${\bf Groups}\ {\it Data\ input/output}$ 

# MSK\_SPAR\_STAT\_KEY

Key used when writing the summary file.

Accepted Any valid string.

Groups Data input/output

### MSK\_SPAR\_STAT\_NAME

Name used when writing the statistics file.

Accepted Any valid XML string.

Groups Data input/output

### MSK\_SPAR\_WRITE\_LP\_GEN\_VAR\_NAME

Sometimes when an LP file is written additional variables must be inserted. They will have the prefix denoted by this parameter.

Default xmskgen

Accepted Any valid string.

Groups Data input/output

# 13.3 Response codes

- Termination
- Warnings
- Errors

### 13.3.1 Termination

### MSK\_RES\_OK

No error occurred.

#### MSK RES TRM MAX ITERATIONS

The optimizer terminated at the maximum number of iterations.

#### MSK\_RES\_TRM\_MAX\_TIME

The optimizer terminated at the maximum amount of time.

### MSK\_RES\_TRM\_OBJECTIVE\_RANGE

The optimizer terminated with an objective value outside the objective range.

#### MSK\_RES\_TRM\_MIO\_NEAR\_REL\_GAP

The mixed-integer optimizer terminated as the delayed near optimal relative gap tolerance was satisfied.

#### MSK\_RES\_TRM\_MIO\_NEAR\_ABS\_GAP

The mixed-integer optimizer terminated as the delayed near optimal absolute gap tolerance was satisfied.

### MSK\_RES\_TRM\_MIO\_NUM\_RELAXS

The mixed-integer optimizer terminated as the maximum number of relaxations was reached.

### MSK\_RES\_TRM\_MIO\_NUM\_BRANCHES

The mixed-integer optimizer terminated as the maximum number of branches was reached.

### MSK\_RES\_TRM\_NUM\_MAX\_NUM\_INT\_SOLUTIONS

The mixed-integer optimizer terminated as the maximum number of feasible solutions was reached.

#### MSK\_RES\_TRM\_STALL

The optimizer is terminated due to slow progress.

Stalling means that numerical problems prevent the optimizer from making reasonable progress and that it make no sense to continue. In many cases this happens if the problem is badly scaled or otherwise ill-conditioned. There is no guarantee that the solution will be (near) feasible or near optimal. However, often stalling happens near the optimum, and the returned solution may be of good quality. Therefore, it is recommended to check the status of then solution. If the solution near optimal the solution is most likely good enough for most practical purposes.

Please note that if a linear optimization problem is solved using the interior-point optimizer with basis identification turned on, the returned basic solution likely to have high accuracy, even though the optimizer stalled.

Some common causes of stalling are a) badly scaled models, b) near feasible or near infeasible problems and c) a non-convex problems. Case c) is only relevant for general non-linear problems. It is not possible in general for **MOSEK** to check if a specific problems is convex since such a check would be NP hard in itself. This implies that care should be taken when solving problems involving general user defined functions.

#### MSK\_RES\_TRM\_USER\_CALLBACK

The optimizer terminated due to the return of the user-defined callback function.

#### MSK\_RES\_TRM\_MAX\_NUM\_SETBACKS

The optimizer terminated as the maximum number of set-backs was reached. This indicates serious numerical problems and a possibly badly formulated problem.

#### MSK\_RES\_TRM\_NUMERICAL\_PROBLEM

The optimizer terminated due to numerical problems.

### MSK\_RES\_TRM\_INTERNAL

The optimizer terminated due to some internal reason. Please contact MOSEK support.

#### MSK\_RES\_TRM\_INTERNAL\_STOP

The optimizer terminated for internal reasons. Please contact MOSEK support.

# 13.3.2 Warnings

#### MSK\_RES\_WRN\_OPEN\_PARAM\_FILE

The parameter file could not be opened.

### MSK\_RES\_WRN\_LARGE\_BOUND

A numerically large bound value is specified.

#### MSK\_RES\_WRN\_LARGE\_LO\_BOUND

A numerically large lower bound value is specified.

#### MSK\_RES\_WRN\_LARGE\_UP\_BOUND

A numerically large upper bound value is specified.

#### MSK\_RES\_WRN\_LARGE\_CON\_FX

An equality constraint is fixed to a numerically large value. This can cause numerical problems.

# MSK\_RES\_WRN\_LARGE\_CJ

A numerically large value is specified for one  $c_i$ .

# MSK\_RES\_WRN\_LARGE\_AIJ

A numerically large value is specified for an  $a_{i,j}$  element in A. The parameter  $\textit{MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE}$  controls when an  $a_{i,j}$  is considered large.

### MSK\_RES\_WRN\_ZERO\_AIJ

One or more zero elements are specified in A.

# MSK\_RES\_WRN\_NAME\_MAX\_LEN

A name is longer than the buffer that is supposed to hold it.

### MSK\_RES\_WRN\_SPAR\_MAX\_LEN

A value for a string parameter is longer than the buffer that is supposed to hold it.

#### MSK\_RES\_WRN\_MPS\_SPLIT\_RHS\_VECTOR

An RHS vector is split into several nonadjacent parts in an MPS file.

### MSK\_RES\_WRN\_MPS\_SPLIT\_RAN\_VECTOR

A RANGE vector is split into several nonadjacent parts in an MPS file.

### MSK\_RES\_WRN\_MPS\_SPLIT\_BOU\_VECTOR

A BOUNDS vector is split into several nonadjacent parts in an MPS file.

# ${\tt MSK\_RES\_WRN\_LP\_OLD\_QUAD\_FORMAT}$

Missing '/2' after quadratic expressions in bound or objective.

#### MSK\_RES\_WRN\_LP\_DROP\_VARIABLE

Ignored a variable because the variable was not previously defined. Usually this implies that a variable appears in the bound section but not in the objective or the constraints.

### MSK\_RES\_WRN\_NZ\_IN\_UPR\_TRI

Non-zero elements specified in the upper triangle of a matrix were ignored.

#### MSK\_RES\_WRN\_DROPPED\_NZ\_QOBJ

One or more non-zero elements were dropped in the Q matrix in the objective.

### MSK\_RES\_WRN\_IGNORE\_INTEGER

Ignored integer constraints.

### MSK\_RES\_WRN\_NO\_GLOBAL\_OPTIMIZER

No global optimizer is available.

### MSK\_RES\_WRN\_MIO\_INFEASIBLE\_FINAL

The final mixed-integer problem with all the integer variables fixed at their optimal values is infeasible.

#### MSK\_RES\_WRN\_SOL\_FILTER

Invalid solution filter is specified.

### MSK\_RES\_WRN\_UNDEF\_SOL\_FILE\_NAME

Undefined name occurred in a solution.

### MSK\_RES\_WRN\_SOL\_FILE\_IGNORED\_CON

One or more lines in the constraint section were ignored when reading a solution file.

### MSK\_RES\_WRN\_SOL\_FILE\_IGNORED\_VAR

One or more lines in the variable section were ignored when reading a solution file.

### MSK\_RES\_WRN\_TOO\_FEW\_BASIS\_VARS

An incomplete basis has been specified. Too few basis variables are specified.

### MSK\_RES\_WRN\_TOO\_MANY\_BASIS\_VARS

A basis with too many variables has been specified.

#### MSK\_RES\_WRN\_NO\_NONLINEAR\_FUNCTION\_WRITE

The problem contains a general nonlinear function in either the objective or the constraints. Such a nonlinear function cannot be written to a disk file. Note that quadratic terms when inputted explicitly can be written to disk.

### MSK\_RES\_WRN\_LICENSE\_EXPIRE

The license expires.

### MSK\_RES\_WRN\_LICENSE\_SERVER

The license server is not responding.

#### MSK\_RES\_WRN\_EMPTY\_NAME

A variable or constraint name is empty. The output file may be invalid.

### MSK\_RES\_WRN\_USING\_GENERIC\_NAMES

Generic names are used because a name is not valid. For instance when writing an LP file the names must not contain blanks or start with a digit.

### MSK\_RES\_WRN\_LICENSE\_FEATURE\_EXPIRE

The license expires.

#### MSK\_RES\_WRN\_PARAM\_NAME\_DOU

The parameter name is not recognized as a double parameter.

#### MSK\_RES\_WRN\_PARAM\_NAME\_INT

The parameter name is not recognized as a integer parameter.

#### MSK\_RES\_WRN\_PARAM\_NAME\_STR

The parameter name is not recognized as a string parameter.

### MSK\_RES\_WRN\_PARAM\_STR\_VALUE

The string is not recognized as a symbolic value for the parameter.

### MSK\_RES\_WRN\_PARAM\_IGNORED\_CMIO

A parameter was ignored by the conic mixed integer optimizer.

#### MSK\_RES\_WRN\_ZEROS\_IN\_SPARSE\_ROW

One or more (near) zero elements are specified in a sparse row of a matrix. Since, it is redundant to specify zero elements then it may indicate an error.

### MSK\_RES\_WRN\_ZEROS\_IN\_SPARSE\_COL

One or more (near) zero elements are specified in a sparse column of a matrix. It is redundant to specify zero elements. Hence, it may indicate an error.

#### MSK\_RES\_WRN\_INCOMPLETE\_LINEAR\_DEPENDENCY\_CHECK

The linear dependency check(s) is incomplete. Normally this is not an important warning unless the optimization problem has been formulated with linear dependencies. Linear dependencies may prevent **MOSEK** from solving the problem.

#### MSK\_RES\_WRN\_ELIMINATOR\_SPACE

The eliminator is skipped at least once due to lack of space.

### MSK\_RES\_WRN\_PRESOLVE\_OUTOFSPACE

The presolve is incomplete due to lack of space.

### MSK\_RES\_WRN\_WRITE\_CHANGED\_NAMES

Some names were changed because they were invalid for the output file format.

#### MSK\_RES\_WRN\_WRITE\_DISCARDED\_CFIX

The fixed objective term could not be converted to a variable and was discarded in the output file.

#### MSK\_RES\_WRN\_CONSTRUCT\_SOLUTION\_INFEAS

After fixing the integer variables at the suggested values then the problem is infeasible.

### MSK\_RES\_WRN\_CONSTRUCT\_INVALID\_SOL\_ITG

The initial value for one or more of the integer variables is not feasible.

#### MSK\_RES\_WRN\_CONSTRUCT\_NO\_SOL\_ITG

The construct solution requires an integer solution.

### MSK\_RES\_WRN\_DUPLICATE\_CONSTRAINT\_NAMES

Two constraint names are identical.

### MSK\_RES\_WRN\_DUPLICATE\_VARIABLE\_NAMES

Two variable names are identical.

### MSK\_RES\_WRN\_DUPLICATE\_BARVARIABLE\_NAMES

Two barvariable names are identical.

### MSK\_RES\_WRN\_DUPLICATE\_CONE\_NAMES

Two cone names are identical.

### MSK\_RES\_WRN\_ANA\_LARGE\_BOUNDS

This warning is issued by the problem analyzer, if one or more constraint or variable bounds are very large. One should consider omitting these bounds entirely by setting them to +inf or -inf.

#### MSK RES WRN ANA C ZERO

This warning is issued by the problem analyzer, if the coefficients in the linear part of the objective are all zero.

#### MSK\_RES\_WRN\_ANA\_EMPTY\_COLS

This warning is issued by the problem analyzer, if columns, in which all coefficients are zero, are found.

### MSK\_RES\_WRN\_ANA\_CLOSE\_BOUNDS

This warning is issued by problem analyzer, if ranged constraints or variables with very close upper and lower bounds are detected. One should consider treating such constraints as equalities and such variables as constants.

### MSK\_RES\_WRN\_ANA\_ALMOST\_INT\_BOUNDS

This warning is issued by the problem analyzer if a constraint is bound nearly integral.

### MSK\_RES\_WRN\_QUAD\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO

For at least one quadratic cone the root is fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problems, or to fix all the variables in the cone to 0.

### MSK\_RES\_WRN\_RQUAD\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO

For at least one rotated quadratic cone at least one of the root variables are fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problems, or to fix all the variables in the cone to 0.

### MSK\_RES\_WRN\_NO\_DUALIZER

No automatic dualizer is available for the specified problem. The primal problem is solved.

#### MSK\_RES\_WRN\_SYM\_MAT\_LARGE

A numerically large value is specified for an  $e_{i,j}$  element in E. The parameter  $MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE$  controls when an  $e_{i,j}$  is considered large.

### 13.3.3 Errors

### MSK\_RES\_ERR\_LICENSE

Invalid license.

#### MSK\_RES\_ERR\_LICENSE\_EXPIRED

The license has expired.

#### MSK\_RES\_ERR\_LICENSE\_VERSION

The license is valid for another version of **MOSEK**.

### MSK\_RES\_ERR\_SIZE\_LICENSE

The problem is bigger than the license.

#### MSK\_RES\_ERR\_PROB\_LICENSE

The software is not licensed to solve the problem.

#### MSK\_RES\_ERR\_FILE\_LICENSE

Invalid license file.

### MSK\_RES\_ERR\_MISSING\_LICENSE\_FILE

MOSEK cannot license file or a token server. See the MOSEK installation manual for details.

### MSK\_RES\_ERR\_SIZE\_LICENSE\_CON

The problem has too many constraints to be solved with the available license.

### MSK\_RES\_ERR\_SIZE\_LICENSE\_VAR

The problem has too many variables to be solved with the available license.

### MSK\_RES\_ERR\_SIZE\_LICENSE\_INTVAR

The problem contains too many integer variables to be solved with the available license.

### MSK\_RES\_ERR\_OPTIMIZER\_LICENSE

The optimizer required is not licensed.

# MSK\_RES\_ERR\_FLEXLM

The FLEXIm license manager reported an error.

# MSK\_RES\_ERR\_LICENSE\_SERVER

The license server is not responding.

### MSK\_RES\_ERR\_LICENSE\_MAX

Maximum number of licenses is reached.

### MSK\_RES\_ERR\_LICENSE\_MOSEKLM\_DAEMON

The MOSEKLM license manager daemon is not up and running.

#### MSK\_RES\_ERR\_LICENSE\_FEATURE

A requested feature is not available in the license file(s). Most likely due to an incorrect license system setup.

### MSK\_RES\_ERR\_PLATFORM\_NOT\_LICENSED

A requested license feature is not available for the required platform.

#### MSK\_RES\_ERR\_LICENSE\_CANNOT\_ALLOCATE

The license system cannot allocate the memory required.

### MSK\_RES\_ERR\_LICENSE\_CANNOT\_CONNECT

MOSEK cannot connect to the license server. Most likely the license server is not up and running.

#### MSK\_RES\_ERR\_LICENSE\_INVALID\_HOSTID

The host ID specified in the license file does not match the host ID of the computer.

### MSK\_RES\_ERR\_LICENSE\_SERVER\_VERSION

The version specified in the checkout request is greater than the highest version number the daemon supports.

### MSK\_RES\_ERR\_LICENSE\_NO\_SERVER\_SUPPORT

The license server does not support the requested feature. Possible reasons for this error include:

- The feature has expired.
- The feature's start date is later than today's date.
- The version requested is higher than feature's the highest supported version.
- A corrupted license file.

Try restarting the license and inspect the license server debug file, usually called lmgrd.log.

### MSK\_RES\_ERR\_LICENSE\_NO\_SERVER\_LINE

There is no SERVER line in the license file. All non-zero license count features need at least one SERVER line.

### MSK\_RES\_ERR\_OPEN\_DL

A dynamic link library could not be opened.

### MSK\_RES\_ERR\_OLDER\_DLL

The dynamic link library is older than the specified version.

# MSK\_RES\_ERR\_NEWER\_DLL

The dynamic link library is newer than the specified version.

### MSK\_RES\_ERR\_LINK\_FILE\_DLL

A file cannot be linked to a stream in the DLL version.

### MSK\_RES\_ERR\_THREAD\_MUTEX\_INIT

Could not initialize a mutex.

# ${\tt MSK\_RES\_ERR\_THREAD\_MUTEX\_LOCK}$

Could not lock a mutex.

### MSK\_RES\_ERR\_THREAD\_MUTEX\_UNLOCK

Could not unlock a mutex.

# MSK\_RES\_ERR\_THREAD\_CREATE

Could not create a thread. This error may occur if a large number of environments are created and not deleted again. In any case it is a good practice to minimize the number of environments created.

### MSK\_RES\_ERR\_THREAD\_COND\_INIT

Could not initialize a condition.

### MSK\_RES\_ERR\_UNKNOWN

Unknown error.

#### MSK\_RES\_ERR\_SPACE

Out of space.

#### MSK\_RES\_ERR\_FILE\_OPEN

Error while opening a file.

### MSK\_RES\_ERR\_FILE\_READ

File read error.

### MSK\_RES\_ERR\_FILE\_WRITE

File write error.

#### MSK\_RES\_ERR\_DATA\_FILE\_EXT

The data file format cannot be determined from the file name.

### MSK\_RES\_ERR\_INVALID\_FILE\_NAME

An invalid file name has been specified.

### MSK\_RES\_ERR\_INVALID\_SOL\_FILE\_NAME

An invalid file name has been specified.

### MSK\_RES\_ERR\_END\_OF\_FILE

End of file reached.

### MSK\_RES\_ERR\_NULL\_ENV

env is a NULL pointer.

#### MSK\_RES\_ERR\_NULL\_TASK

task is a NULL pointer.

### MSK\_RES\_ERR\_INVALID\_STREAM

An invalid stream is referenced.

### MSK\_RES\_ERR\_NO\_INIT\_ENV

env is not initialized.

#### MSK\_RES\_ERR\_INVALID\_TASK

The task is invalid.

### MSK\_RES\_ERR\_NULL\_POINTER

An argument to a function is unexpectedly a NULL pointer.

### MSK\_RES\_ERR\_LIVING\_TASKS

All tasks associated with an environment must be deleted before the environment is deleted. There are still some undeleted tasks.

#### MSK\_RES\_ERR\_BLANK\_NAME

An all blank name has been specified.

### MSK\_RES\_ERR\_DUP\_NAME

The same name was used multiple times for the same problem item type.

#### MSK\_RES\_ERR\_INVALID\_OBJ\_NAME

An invalid objective name is specified.

### MSK\_RES\_ERR\_INVALID\_CON\_NAME

An invalid constraint name is used.

### MSK\_RES\_ERR\_INVALID\_VAR\_NAME

An invalid variable name is used.

### MSK\_RES\_ERR\_INVALID\_CONE\_NAME

An invalid cone name is used.

### MSK\_RES\_ERR\_INVALID\_BARVAR\_NAME

An invalid symmetric matrix variable name is used.

### MSK\_RES\_ERR\_SPACE\_LEAKING

MOSEK is leaking memory. This can be due to either an incorrect use of MOSEK or a bug.

#### MSK\_RES\_ERR\_SPACE\_NO\_INFO

No available information about the space usage.

#### MSK\_RES\_ERR\_READ\_FORMAT

The specified format cannot be read.

### MSK\_RES\_ERR\_MPS\_FILE

An error occurred while reading an MPS file.

### MSK\_RES\_ERR\_MPS\_INV\_FIELD

A field in the MPS file is invalid. Probably it is too wide.

#### MSK\_RES\_ERR\_MPS\_INV\_MARKER

An invalid marker has been specified in the MPS file.

### MSK\_RES\_ERR\_MPS\_NULL\_CON\_NAME

An empty constraint name is used in an MPS file.

### MSK\_RES\_ERR\_MPS\_NULL\_VAR\_NAME

An empty variable name is used in an MPS file.

#### MSK\_RES\_ERR\_MPS\_UNDEF\_CON\_NAME

An undefined constraint name occurred in an MPS file.

### MSK\_RES\_ERR\_MPS\_UNDEF\_VAR\_NAME

An undefined variable name occurred in an MPS file.

### MSK\_RES\_ERR\_MPS\_INV\_CON\_KEY

An invalid constraint key occurred in an MPS file.

### MSK\_RES\_ERR\_MPS\_INV\_BOUND\_KEY

An invalid bound key occurred in an MPS file.

### MSK\_RES\_ERR\_MPS\_INV\_SEC\_NAME

An invalid section name occurred in an MPS file.

#### MSK\_RES\_ERR\_MPS\_NO\_OBJECTIVE

No objective is defined in an MPS file.

#### MSK\_RES\_ERR\_MPS\_SPLITTED\_VAR

All elements in a column of the A matrix must be specified consecutively. Hence, it is illegal to specify non-zero elements in A for variable 1, then for variable 2 and then variable 1 again.

# MSK\_RES\_ERR\_MPS\_MUL\_CON\_NAME

A constraint name was specified multiple times in the ROWS section.

#### MSK\_RES\_ERR\_MPS\_MUL\_QSEC

Multiple QSECTIONs are specified for a constraint in the MPS data file.

#### MSK\_RES\_ERR\_MPS\_MUL\_QOBJ

The Q term in the objective is specified multiple times in the MPS data file.

#### MSK\_RES\_ERR\_MPS\_INV\_SEC\_ORDER

The sections in the MPS data file are not in the correct order.

### MSK\_RES\_ERR\_MPS\_MUL\_CSEC

Multiple CSECTIONs are given the same name.

### MSK\_RES\_ERR\_MPS\_CONE\_TYPE

Invalid cone type specified in a CSECTION.

### MSK\_RES\_ERR\_MPS\_CONE\_OVERLAP

A variable is specified to be a member of several cones.

### MSK\_RES\_ERR\_MPS\_CONE\_REPEAT

A variable is repeated within the CSECTION.

# MSK\_RES\_ERR\_MPS\_NON\_SYMMETRIC\_Q

A non symmetric matrice has been speciefied.

#### MSK\_RES\_ERR\_MPS\_DUPLICATE\_Q\_ELEMENT

Duplicate elements is specified in a Q matrix.

#### MSK\_RES\_ERR\_MPS\_INVALID\_OBJSENSE

An invalid objective sense is specified.

### MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD2

A tab char occurred in field 2.

### MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD3

A tab char occurred in field 3.

#### MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD5

A tab char occurred in field 5.

### MSK\_RES\_ERR\_MPS\_INVALID\_OBJ\_NAME

An invalid objective name is specified.

### MSK\_RES\_ERR\_LP\_INCOMPATIBLE

The problem cannot be written to an LP formatted file.

### MSK\_RES\_ERR\_LP\_EMPTY

The problem cannot be written to an LP formatted file.

### MSK\_RES\_ERR\_LP\_DUP\_SLACK\_NAME

The name of the slack variable added to a ranged constraint already exists.

#### MSK\_RES\_ERR\_WRITE\_MPS\_INVALID\_NAME

An invalid name is created while writing an MPS file. Usually this will make the MPS file unreadable.

#### MSK\_RES\_ERR\_LP\_INVALID\_VAR\_NAME

A variable name is invalid when used in an LP formatted file.

### ${\tt MSK\_RES\_ERR\_LP\_FREE\_CONSTRAINT}$

Free constraints cannot be written in LP file format.

# MSK\_RES\_ERR\_WRITE\_OPF\_INVALID\_VAR\_NAME

Empty variable names cannot be written to OPF files.

# MSK\_RES\_ERR\_LP\_FILE\_FORMAT

Syntax error in an LP file.

### MSK\_RES\_ERR\_WRITE\_LP\_FORMAT

Problem cannot be written as an LP file.

### MSK\_RES\_ERR\_READ\_LP\_MISSING\_END\_TAG

Syntax error in LP file. Possibly missing End tag.

#### MSK\_RES\_ERR\_LP\_FORMAT

Syntax error in an LP file.

### MSK\_RES\_ERR\_WRITE\_LP\_NON\_UNIQUE\_NAME

An auto-generated name is not unique.

### MSK\_RES\_ERR\_READ\_LP\_NONEXISTING\_NAME

A variable never occurred in objective or constraints.

# MSK\_RES\_ERR\_LP\_WRITE\_CONIC\_PROBLEM

The problem contains cones that cannot be written to an LP formatted file.

### MSK\_RES\_ERR\_LP\_WRITE\_GECO\_PROBLEM

The problem contains general convex terms that cannot be written to an LP formatted file.

### MSK\_RES\_ERR\_WRITING\_FILE

An error occurred while writing file

# MSK\_RES\_ERR\_OPF\_FORMAT

Syntax error in an OPF file

#### MSK\_RES\_ERR\_OPF\_NEW\_VARIABLE

Introducing new variables is now allowed. When a [variables] section is present, it is not allowed to introduce new variables later in the problem.

#### MSK\_RES\_ERR\_INVALID\_NAME\_IN\_SOL\_FILE

An invalid name occurred in a solution file.

### MSK\_RES\_ERR\_LP\_INVALID\_CON\_NAME

A constraint name is invalid when used in an LP formatted file.

### MSK\_RES\_ERR\_OPF\_PREMATURE\_EOF

Premature end of file in an OPF file.

#### MSK\_RES\_ERR\_JSON\_SYNTAX

Syntax error in an JSON data

#### MSK\_RES\_ERR\_JSON\_STRING

Error in JSON string.

### MSK\_RES\_ERR\_JSON\_NUMBER\_OVERFLOW

Invalid number entry - wrong type or value overflow.

### MSK\_RES\_ERR\_JSON\_FORMAT

Error in an JSON Task file

### MSK\_RES\_ERR\_JSON\_DATA

Inconsistent data in JSON Task file

### MSK\_RES\_ERR\_JSON\_MISSING\_DATA

Missing data section in JSON task file.

#### MSK\_RES\_ERR\_ARGUMENT\_LENNEQ

Incorrect length of arguments.

### MSK\_RES\_ERR\_ARGUMENT\_TYPE

Incorrect argument type.

### MSK\_RES\_ERR\_NR\_ARGUMENTS

Incorrect number of function arguments.

# MSK\_RES\_ERR\_IN\_ARGUMENT

A function argument is incorrect.

# MSK\_RES\_ERR\_ARGUMENT\_DIMENSION

A function argument is of incorrect dimension.

### MSK\_RES\_ERR\_INDEX\_IS\_TOO\_SMALL

An index in an argument is too small.

#### MSK\_RES\_ERR\_INDEX\_IS\_TOO\_LARGE

An index in an argument is too large.

### MSK\_RES\_ERR\_PARAM\_NAME

The parameter name is not correct.

### MSK\_RES\_ERR\_PARAM\_NAME\_DOU

The parameter name is not correct for a double parameter.

### MSK\_RES\_ERR\_PARAM\_NAME\_INT

The parameter name is not correct for an integer parameter.

### MSK\_RES\_ERR\_PARAM\_NAME\_STR

The parameter name is not correct for a string parameter.

### MSK\_RES\_ERR\_PARAM\_INDEX

Parameter index is out of range.

# MSK\_RES\_ERR\_PARAM\_IS\_TOO\_LARGE

The parameter value is too large.

#### MSK\_RES\_ERR\_PARAM\_IS\_TOO\_SMALL

The parameter value is too small.

### MSK\_RES\_ERR\_PARAM\_VALUE\_STR

The parameter value string is incorrect.

#### MSK\_RES\_ERR\_PARAM\_TYPE

The parameter type is invalid.

### MSK\_RES\_ERR\_INF\_DOU\_INDEX

A double information index is out of range for the specified type.

#### MSK\_RES\_ERR\_INF\_INT\_INDEX

An integer information index is out of range for the specified type.

### MSK\_RES\_ERR\_INDEX\_ARR\_IS\_TOO\_SMALL

An index in an array argument is too small.

### MSK\_RES\_ERR\_INDEX\_ARR\_IS\_TOO\_LARGE

An index in an array argument is too large.

#### MSK\_RES\_ERR\_INF\_LINT\_INDEX

A long integer information index is out of range for the specified type.

### MSK\_RES\_ERR\_ARG\_IS\_TOO\_SMALL

The value of a argument is too small.

### MSK\_RES\_ERR\_ARG\_IS\_TOO\_LARGE

The value of a argument is too small.

### MSK\_RES\_ERR\_INVALID\_WHICHSOL

which sol is invalid.

### MSK\_RES\_ERR\_INF\_DOU\_NAME

A double information name is invalid.

#### MSK\_RES\_ERR\_INF\_INT\_NAME

An integer information name is invalid.

### MSK\_RES\_ERR\_INF\_TYPE

The information type is invalid.

### MSK\_RES\_ERR\_INF\_LINT\_NAME

A long integer information name is invalid.

### MSK\_RES\_ERR\_INDEX

An index is out of range.

### MSK\_RES\_ERR\_WHICHSOL

The solution defined by which sol does not exists.

#### MSK RES ERR SOLITEM

The solution item number solitem is invalid. Please note that MSK\_SOL\_ITEM\_SNX is invalid for the basic solution.

# MSK\_RES\_ERR\_WHICHITEM\_NOT\_ALLOWED

whichitem is unacceptable.

### MSK\_RES\_ERR\_MAXNUMCON

The maximum number of constraints specified is smaller than the number of constraints in the task.

#### MSK\_RES\_ERR\_MAXNUMVAR

The maximum number of variables specified is smaller than the number of variables in the task.

### MSK\_RES\_ERR\_MAXNUMBARVAR

The maximum number of semidefinite variables specified is smaller than the number of semidefinite variables in the task.

#### MSK\_RES\_ERR\_MAXNUMQNZ

The maximum number of non-zeros specified for the Q matrices is smaller than the number of non-zeros in the current Q matrices.

#### MSK\_RES\_ERR\_TOO\_SMALL\_MAX\_NUM\_NZ

The maximum number of non-zeros specified is too small.

#### MSK\_RES\_ERR\_INVALID\_IDX

A specified index is invalid.

### MSK\_RES\_ERR\_INVALID\_MAX\_NUM

A specified index is invalid.

#### MSK\_RES\_ERR\_NUMCONLIM

Maximum number of constraints limit is exceeded.

### MSK\_RES\_ERR\_NUMVARLIM

Maximum number of variables limit is exceeded.

### MSK\_RES\_ERR\_TOO\_SMALL\_MAXNUMANZ

The maximum number of non-zeros specified for A is smaller than the number of non-zeros in the current A.

### MSK\_RES\_ERR\_INV\_APTRE

aptre[j] is strictly smaller than aptrb[j] for some j.

### MSK\_RES\_ERR\_MUL\_A\_ELEMENT

An element in A is defined multiple times.

#### MSK\_RES\_ERR\_INV\_BK

Invalid bound key.

#### MSK\_RES\_ERR\_INV\_BKC

Invalid bound key is specified for a constraint.

### MSK\_RES\_ERR\_INV\_BKX

An invalid bound key is specified for a variable.

### MSK\_RES\_ERR\_INV\_VAR\_TYPE

An invalid variable type is specified for a variable.

#### MSK\_RES\_ERR\_SOLVER\_PROBTYPE

Problem type does not match the chosen optimizer.

### MSK\_RES\_ERR\_OBJECTIVE\_RANGE

Empty objective range.

### MSK\_RES\_ERR\_FIRST

Invalid first.

### MSK\_RES\_ERR\_LAST

Invalid index last. A given index was out of expected range.

### MSK\_RES\_ERR\_NEGATIVE\_SURPLUS

Negative surplus.

### MSK\_RES\_ERR\_NEGATIVE\_APPEND

Cannot append a negative number.

### MSK\_RES\_ERR\_UNDEF\_SOLUTION

 $\mathbf{MOSEK}$  has the following solution types:

- an interior-point solution,
- an basic solution,
- and an integer solution.

Each optimizer may set one or more of these solutions; e.g by default a successful optimization with the interior-point optimizer defines the interior-point solution, and, for linear problems, also the basic solution. This error occurs when asking for a solution or for information about a solution that is not defined.

#### MSK\_RES\_ERR\_BASIS

An invalid basis is specified. Either too many or too few basis variables are specified.

#### MSK\_RES\_ERR\_INV\_SKC

Invalid value in skc.

#### MSK\_RES\_ERR\_INV\_SKX

Invalid value in skx.

### MSK\_RES\_ERR\_INV\_SKN

Invalid value in skn.

#### MSK\_RES\_ERR\_INV\_SK\_STR

Invalid status key string encountered.

#### MSK\_RES\_ERR\_INV\_SK

Invalid status key code.

### MSK\_RES\_ERR\_INV\_CONE\_TYPE\_STR

Invalid cone type string encountered.

#### MSK\_RES\_ERR\_INV\_CONE\_TYPE

Invalid cone type code is encountered.

#### MSK\_RES\_ERR\_INVALID\_SURPLUS

Invalid surplus.

### MSK\_RES\_ERR\_INV\_NAME\_ITEM

An invalid name item code is used.

### MSK\_RES\_ERR\_PRO\_ITEM

An invalid problem is used.

### MSK\_RES\_ERR\_INVALID\_FORMAT\_TYPE

Invalid format type.

#### MSK\_RES\_ERR\_FIRSTI

Invalid firsti.

### MSK\_RES\_ERR\_LASTI

Invalid lasti.

### MSK\_RES\_ERR\_FIRSTJ

Invalid firstj.

### MSK\_RES\_ERR\_LASTJ

Invalid lastj.

### MSK\_RES\_ERR\_MAX\_LEN\_IS\_TOO\_SMALL

An maximum length that is too small has been specified.

### MSK\_RES\_ERR\_NONLINEAR\_EQUALITY

The model contains a nonlinear equality which defines a nonconvex set.

### MSK\_RES\_ERR\_NONCONVEX

The optimization problem is nonconvex.

#### MSK\_RES\_ERR\_NONLINEAR\_RANGED

Nonlinear constraints with finite lower and upper bound always define a nonconvex feasible set.

### MSK\_RES\_ERR\_CON\_Q\_NOT\_PSD

The quadratic constraint matrix is not positive semidefinite as expected for a constraint with finite upper bound. This results in a nonconvex problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

#### MSK\_RES\_ERR\_CON\_Q\_NOT\_NSD

The quadratic constraint matrix is not negative semidefinite as expected for a constraint with finite lower bound. This results in a nonconvex problem. The parameter <code>MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL</code> can be used to relax the convexity check.

#### MSK\_RES\_ERR\_OBJ\_Q\_NOT\_PSD

The quadratic coefficient matrix in the objective is not positive semidefinite as expected for a minimization problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

### MSK\_RES\_ERR\_OBJ\_Q\_NOT\_NSD

The quadratic coefficient matrix in the objective is not negative semidefinite as expected for a maximization problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

### MSK\_RES\_ERR\_ARGUMENT\_PERM\_ARRAY

An invalid permutation array is specified.

### MSK\_RES\_ERR\_CONE\_INDEX

An index of a non-existing cone has been specified.

# MSK\_RES\_ERR\_CONE\_SIZE

A cone with too few members is specified.

# MSK\_RES\_ERR\_CONE\_OVERLAP

One or more of the variables in the cone to be added is already member of another cone. Now assume the variable is  $x_j$  then add a new variable say  $x_k$  and the constraint

$$x_j = x_k$$

and then let  $x_k$  be member of the cone to be appended.

### MSK\_RES\_ERR\_CONE\_REP\_VAR

A variable is included multiple times in the cone.

### MSK\_RES\_ERR\_MAXNUMCONE

The value specified for maxnumcone is too small.

#### MSK\_RES\_ERR\_CONE\_TYPE

Invalid cone type specified.

### MSK\_RES\_ERR\_CONE\_TYPE\_STR

Invalid cone type specified.

#### MSK\_RES\_ERR\_CONE\_OVERLAP\_APPEND

The cone to be appended has one variable which is already member of another cone.

# MSK\_RES\_ERR\_REMOVE\_CONE\_VARIABLE

A variable cannot be removed because it will make a cone invalid.

# MSK\_RES\_ERR\_SOL\_FILE\_INVALID\_NUMBER

An invalid number is specified in a solution file.

### MSK\_RES\_ERR\_HUGE\_C

A huge value in absolute size is specified for one  $c_j$ .

# MSK\_RES\_ERR\_HUGE\_AIJ

A numerically huge value is specified for an  $a_{i,j}$  element in A. The parameter  $MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE$  controls when an  $a_{i,j}$  is considered huge.

### MSK\_RES\_ERR\_DUPLICATE\_AIJ

An element in the A matrix is specified twice.

# MSK\_RES\_ERR\_LOWER\_BOUND\_IS\_A\_NAN

The lower bound specified is not a number (nan).

### MSK\_RES\_ERR\_UPPER\_BOUND\_IS\_A\_NAN

The upper bound specified is not a number (nan).

### MSK\_RES\_ERR\_INFINITE\_BOUND

A numerically huge bound value is specified.

#### MSK\_RES\_ERR\_INV\_QOBJ\_SUBI

Invalid value in qosubi.

### MSK\_RES\_ERR\_INV\_QOBJ\_SUBJ

Invalid value in qosubj.

### MSK\_RES\_ERR\_INV\_QOBJ\_VAL

Invalid value in qoval.

#### MSK\_RES\_ERR\_INV\_QCON\_SUBK

Invalid value in qcsubk.

### MSK\_RES\_ERR\_INV\_QCON\_SUBI

Invalid value in qcsubi.

### MSK\_RES\_ERR\_INV\_QCON\_SUBJ

Invalid value in qcsubj.

#### MSK\_RES\_ERR\_INV\_QCON\_VAL

Invalid value in qcval.

### MSK\_RES\_ERR\_QCON\_SUBI\_TOO\_SMALL

Invalid value in qcsubi.

#### MSK\_RES\_ERR\_QCON\_SUBI\_TOO\_LARGE

Invalid value in qcsubi.

### MSK\_RES\_ERR\_QOBJ\_UPPER\_TRIANGLE

An element in the upper triangle of  $Q^o$  is specified. Only elements in the lower triangle should be specified.

### MSK\_RES\_ERR\_QCON\_UPPER\_TRIANGLE

An element in the upper triangle of a  $Q^k$  is specified. Only elements in the lower triangle should be specified.

### MSK\_RES\_ERR\_FIXED\_BOUND\_VALUES

A fixed constraint/variable has been specified using the bound keys but the numerical value of the lower and upper bound is different.

### MSK\_RES\_ERR\_NONLINEAR\_FUNCTIONS\_NOT\_ALLOWED

An operation that is invalid for problems with nonlinear functions defined has been attempted.

### MSK\_RES\_ERR\_USER\_FUNC\_RET

An user function reported an error.

# MSK\_RES\_ERR\_USER\_FUNC\_RET\_DATA

An user function returned invalid data.

### MSK\_RES\_ERR\_USER\_NLO\_FUNC

The user-defined nonlinear function reported an error.

### MSK\_RES\_ERR\_USER\_NLO\_EVAL

The user-defined nonlinear function reported an error.

# MSK\_RES\_ERR\_USER\_NLO\_EVAL\_HESSUBI

The user-defined nonlinear function reported an invalid subscript in the Hessian.

# MSK\_RES\_ERR\_USER\_NLO\_EVAL\_HESSUBJ

The user-defined nonlinear function reported an invalid subscript in the Hessian.

# MSK\_RES\_ERR\_INVALID\_OBJECTIVE\_SENSE

An invalid objective sense is specified.

# MSK\_RES\_ERR\_UNDEFINED\_OBJECTIVE\_SENSE

The objective sense has not been specified before the optimization.

#### MSK\_RES\_ERR\_Y\_IS\_UNDEFINED

The solution item y is undefined.

#### MSK\_RES\_ERR\_NAN\_IN\_DOUBLE\_DATA

An invalid floating point value was used in some double data.

#### MSK\_RES\_ERR\_NAN\_IN\_BLC

 $l^c$  contains an invalid floating point value, i.e. a NaN.

#### MSK\_RES\_ERR\_NAN\_IN\_BUC

 $u^c$  contains an invalid floating point value, i.e. a NaN.

#### MSK\_RES\_ERR\_NAN\_IN\_C

c contains an invalid floating point value, i.e. a NaN.

#### MSK\_RES\_ERR\_NAN\_IN\_BLX

 $l^x$  contains an invalid floating point value, i.e. a NaN.

### MSK\_RES\_ERR\_NAN\_IN\_BUX

 $u^x$  contains an invalid floating point value, i.e. a NaN.

#### MSK\_RES\_ERR\_INVALID\_AIJ

 $a_{i,j}$  contains an invalid floating point value, i.e. a NaN or an infinite value.

#### MSK\_RES\_ERR\_SYM\_MAT\_INVALID

A symmetric matrix contains an invalid floating point value, i.e. a NaN or an infinite value.

#### MSK\_RES\_ERR\_SYM\_MAT\_HUGE

A symmetric matrix contains a huge value in absolute size. The parameter  $MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE$  controls when an  $e_{i,j}$  is considered huge.

#### MSK\_RES\_ERR\_INV\_PROBLEM

Invalid problem type. Probably a nonconvex problem has been specified.

### MSK\_RES\_ERR\_MIXED\_CONIC\_AND\_NL

The problem contains nonlinear terms conic constraints. The requested operation cannot be applied to this type of problem.

### MSK\_RES\_ERR\_GLOBAL\_INV\_CONIC\_PROBLEM

The global optimizer can only be applied to problems without semidefinite variables.

#### MSK\_RES\_ERR\_INV\_OPTIMIZER

An invalid optimizer has been chosen for the problem. This means that the simplex or the conic optimizer is chosen to optimize a nonlinear problem.

### MSK\_RES\_ERR\_MIO\_NO\_OPTIMIZER

No optimizer is available for the current class of integer optimization problems.

### MSK\_RES\_ERR\_NO\_OPTIMIZER\_VAR\_TYPE

No optimizer is available for this class of optimization problems.

### MSK\_RES\_ERR\_FINAL\_SOLUTION

An error occurred during the solution finalization.

### MSK\_RES\_ERR\_POSTSOLVE

An error occurred during the postsolve. Please contact MOSEK support.

### MSK\_RES\_ERR\_OVERFLOW

A computation produced an overflow i.e. a very large number.

# MSK\_RES\_ERR\_NO\_BASIS\_SOL

No basic solution is defined.

# MSK\_RES\_ERR\_BASIS\_FACTOR

The factorization of the basis is invalid.

# MSK\_RES\_ERR\_BASIS\_SINGULAR

The basis is singular and hence cannot be factored.

#### MSK\_RES\_ERR\_FACTOR

An error occurred while factorizing a matrix.

#### MSK\_RES\_ERR\_FEASREPAIR\_CANNOT\_RELAX

An optimization problem cannot be relaxed. This is the case e.g. for general nonlinear optimization problems.

#### MSK\_RES\_ERR\_FEASREPAIR\_SOLVING\_RELAXED

The relaxed problem could not be solved to optimality. Please consult the log file for further details.

# MSK\_RES\_ERR\_FEASREPAIR\_INCONSISTENT\_BOUND

The upper bound is less than the lower bound for a variable or a constraint. Please correct this before running the feasibility repair.

### MSK\_RES\_ERR\_REPAIR\_INVALID\_PROBLEM

The feasibility repair does not support the specified problem type.

#### MSK\_RES\_ERR\_REPAIR\_OPTIMIZATION\_FAILED

Computation the optimal relaxation failed. The cause may have been numerical problems.

#### MSK\_RES\_ERR\_NAME\_MAX\_LEN

A name is longer than the buffer that is supposed to hold it.

### MSK\_RES\_ERR\_NAME\_IS\_NULL

The name buffer is a NULL pointer.

### MSK\_RES\_ERR\_INVALID\_COMPRESSION

Invalid compression type.

### MSK\_RES\_ERR\_INVALID\_IOMODE

Invalid io mode.

### MSK\_RES\_ERR\_NO\_PRIMAL\_INFEAS\_CER

A certificate of primal infeasibility is not available.

### MSK\_RES\_ERR\_NO\_DUAL\_INFEAS\_CER

A certificate of infeasibility is not available.

# MSK\_RES\_ERR\_NO\_SOLUTION\_IN\_CALLBACK

The required solution is not available.

# ${\tt MSK\_RES\_ERR\_INV\_MARKI}$

Invalid value in marki.

# MSK\_RES\_ERR\_INV\_MARKJ

Invalid value in markj.

# MSK\_RES\_ERR\_INV\_NUMI

Invalid numi.

# MSK\_RES\_ERR\_INV\_NUMJ

Invalid numj.

# MSK\_RES\_ERR\_CANNOT\_CLONE\_NL

A task with a nonlinear function callback cannot be cloned.

### MSK\_RES\_ERR\_CANNOT\_HANDLE\_NL

A function cannot handle a task with nonlinear function callbacks.

### MSK\_RES\_ERR\_INVALID\_ACCMODE

An invalid access mode is specified.

#### MSK\_RES\_ERR\_TASK\_INCOMPATIBLE

The Task file is incompatible with this platform. This results from reading a file on a 32 bit platform generated on a 64 bit platform.

# MSK\_RES\_ERR\_TASK\_INVALID

The Task file is invalid.

#### MSK\_RES\_ERR\_TASK\_WRITE

Failed to write the task file.

#### MSK\_RES\_ERR\_LU\_MAX\_NUM\_TRIES

Could not compute the LU factors of the matrix within the maximum number of allowed tries.

#### MSK\_RES\_ERR\_INVALID\_UTF8

An invalid UTF8 string is encountered.

### MSK\_RES\_ERR\_INVALID\_WCHAR

An invalid wchar string is encountered.

#### MSK\_RES\_ERR\_NO\_DUAL\_FOR\_ITG\_SOL

No dual information is available for the integer solution.

### MSK\_RES\_ERR\_NO\_SNX\_FOR\_BAS\_SOL

 $s_n^x$  is not available for the basis solution.

### MSK\_RES\_ERR\_INTERNAL

An internal error occurred. Please report this problem.

### MSK\_RES\_ERR\_API\_ARRAY\_TOO\_SMALL

An input array was too short.

### MSK\_RES\_ERR\_API\_CB\_CONNECT

Failed to connect a callback object.

### MSK\_RES\_ERR\_API\_FATAL\_ERROR

An internal error occurred in the API. Please report this problem.

### MSK\_RES\_ERR\_API\_INTERNAL

An internal fatal error occurred in an interface function.

#### MSK\_RES\_ERR\_SEN\_FORMAT

Syntax error in sensitivity analysis file.

#### MSK\_RES\_ERR\_SEN\_UNDEF\_NAME

An undefined name was encountered in the sensitivity analysis file.

### MSK\_RES\_ERR\_SEN\_INDEX\_RANGE

Index out of range in the sensitivity analysis file.

### MSK\_RES\_ERR\_SEN\_BOUND\_INVALID\_UP

Analysis of upper bound requested for an index, where no upper bound exists.

### MSK\_RES\_ERR\_SEN\_BOUND\_INVALID\_LO

Analysis of lower bound requested for an index, where no lower bound exists.

### MSK\_RES\_ERR\_SEN\_INDEX\_INVALID

Invalid range given in the sensitivity file.

### MSK\_RES\_ERR\_SEN\_INVALID\_REGEXP

Syntax error in regexp or regexp longer than 1024.

# MSK\_RES\_ERR\_SEN\_SOLUTION\_STATUS

No optimal solution found to the original problem given for sensitivity analysis.

# MSK\_RES\_ERR\_SEN\_NUMERICAL

Numerical difficulties encountered performing the sensitivity analysis.

# MSK\_RES\_ERR\_SEN\_UNHANDLED\_PROBLEM\_TYPE

Sensitivity analysis cannot be performed for the specified problem. Sensitivity analysis is only possible for linear problems.

### MSK\_RES\_ERR\_UNB\_STEP\_SIZE

A step size in an optimizer was unexpectedly unbounded. For instance, if the step-size becomes unbounded in phase 1 of the simplex algorithm then an error occurs. Normally this will happen only if the problem is badly formulated. Please contact **MOSEK** support if this error occurs.

#### MSK\_RES\_ERR\_IDENTICAL\_TASKS

Some tasks related to this function call were identical. Unique tasks were expected.

#### MSK\_RES\_ERR\_AD\_INVALID\_CODELIST

The code list data was invalid.

### MSK\_RES\_ERR\_INTERNAL\_TEST\_FAILED

An internal unit test function failed.

### MSK\_RES\_ERR\_XML\_INVALID\_PROBLEM\_TYPE

The problem type is not supported by the XML format.

#### MSK\_RES\_ERR\_INVALID\_AMPL\_STUB

Invalid AMPL stub.

### MSK\_RES\_ERR\_INT64\_TO\_INT32\_CAST

An 32 bit integer could not cast to a 64 bit integer.

### MSK\_RES\_ERR\_SIZE\_LICENSE\_NUMCORES

The computer contains more cpu cores than the license allows for.

### MSK\_RES\_ERR\_INFEAS\_UNDEFINED

The requested value is not defined for this solution type.

### MSK\_RES\_ERR\_NO\_BARX\_FOR\_SOLUTION

There is no  $\overline{X}$  available for the solution specified. In particular note there are no  $\overline{X}$  defined for the basic and integer solutions.

### MSK\_RES\_ERR\_NO\_BARS\_FOR\_SOLUTION

There is no  $\bar{s}$  available for the solution specified. In particular note there are no  $\bar{s}$  defined for the basic and integer solutions.

#### MSK\_RES\_ERR\_BAR\_VAR\_DIM

The dimension of a symmetric matrix variable has to greater than 0.

### MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_ROW\_INDEX

A row index specified for sparse symmetric matrix is invalid.

### MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_COL\_INDEX

A column index specified for sparse symmetric matrix is invalid.

#### MSK\_RES\_ERR\_SYM\_MAT\_NOT\_LOWER\_TRINGULAR

Only the lower triangular part of sparse symmetric matrix should be specified.

# MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_VALUE

The numerical value specified in a sparse symmetric matrix is not a value floating value.

### MSK\_RES\_ERR\_SYM\_MAT\_DUPLICATE

A value in a symmetric matric as been specified more than once.

# MSK\_RES\_ERR\_INVALID\_SYM\_MAT\_DIM

A sparse symmetric matrix of invalid dimension is specified.

# MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_SYM\_MAT

The file format does not support a problem with symmetric matrix variables.

# MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_CONES

The file format does not support a problem with conic constraints.

### MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_GENERAL\_NL

The file format does not support a problem with general nonlinear terms.

#### MSK\_RES\_ERR\_DUPLICATE\_CONSTRAINT\_NAMES

Two constraint names are identical.

### MSK\_RES\_ERR\_DUPLICATE\_VARIABLE\_NAMES

Two variable names are identical.

#### MSK\_RES\_ERR\_DUPLICATE\_BARVARIABLE\_NAMES

Two barvariable names are identical.

#### MSK\_RES\_ERR\_DUPLICATE\_CONE\_NAMES

Two cone names are identical.

# MSK\_RES\_ERR\_NON\_UNIQUE\_ARRAY

An array does not contain unique elements.

# MSK\_RES\_ERR\_ARGUMENT\_IS\_TOO\_LARGE

The value of a function argument is too large.

#### MSK\_RES\_ERR\_MIO\_INTERNAL

A fatal error occurred in the mixed integer optimizer. Please contact MOSEK support.

### MSK\_RES\_ERR\_INVALID\_PROBLEM\_TYPE

An invalid problem type.

### MSK\_RES\_ERR\_UNHANDLED\_SOLUTION\_STATUS

Unhandled solution status.

### MSK\_RES\_ERR\_UPPER\_TRIANGLE

An element in the upper triangle of a lower triangular matrix is specified.

# MSK\_RES\_ERR\_LAU\_SINGULAR\_MATRIX

A matrix is singular.

### MSK\_RES\_ERR\_LAU\_NOT\_POSITIVE\_DEFINITE

A matrix is not positive definite.

### MSK\_RES\_ERR\_LAU\_INVALID\_LOWER\_TRIANGULAR\_MATRIX

An invalid lower triangular matrix.

### MSK\_RES\_ERR\_LAU\_UNKNOWN

An unknown error.

#### MSK\_RES\_ERR\_LAU\_ARG\_M

Invalid argument m.

#### MSK\_RES\_ERR\_LAU\_ARG\_N

Invalid argument n.

### MSK\_RES\_ERR\_LAU\_ARG\_K

Invalid argument k.

### MSK\_RES\_ERR\_LAU\_ARG\_TRANSA

Invalid argument transa.

### MSK\_RES\_ERR\_LAU\_ARG\_TRANSB

Invalid argument transb.

### MSK\_RES\_ERR\_LAU\_ARG\_UPLO

Invalid argument uplo.

# MSK\_RES\_ERR\_LAU\_ARG\_TRANS

Invalid argument trans.

# MSK\_RES\_ERR\_LAU\_INVALID\_SPARSE\_SYMMETRIC\_MATRIX

An invalid sparse symmetric matrix is specified. Note only the lower triangular part with no duplicates is specified.

### MSK\_RES\_ERR\_CBF\_PARSE

An error occurred while parsing an CBF file.

### MSK\_RES\_ERR\_CBF\_OBJ\_SENSE

An invalid objective sense is specified.

# MSK\_RES\_ERR\_CBF\_NO\_VARIABLES

No variables are specified.

- MSK\_RES\_ERR\_CBF\_TOO\_MANY\_CONSTRAINTS Too many constraints specified.
- MSK\_RES\_ERR\_CBF\_TOO\_MANY\_VARIABLES
  Too many variables specified.
- MSK\_RES\_ERR\_CBF\_NO\_VERSION\_SPECIFIED No version specified.
- MSK\_RES\_ERR\_CBF\_SYNTAX Invalid syntax.
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_OBJ Duplicate OBJ keyword.
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_CON Duplicate CON keyword.
- $\begin{tabular}{ll} MSK\_RES\_ERR\_CBF\_DUPLICATE\_VAR\\ Duplicate VAR keyword. \end{tabular}$
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_INT Duplicate INT keyword.
- MSK\_RES\_ERR\_CBF\_INVALID\_VAR\_TYPE Invalid variable type.
- MSK\_RES\_ERR\_CBF\_INVALID\_CON\_TYPE Invalid constraint type.
- MSK\_RES\_ERR\_CBF\_INVALID\_DOMAIN\_DIMENSION Invalid domain dimension.
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_OBJACOORD Duplicate index in OBJCOORD.
- $$\label{eq:msk_res_err_cbf_duplicate} \begin{split} \texttt{MSK\_RES\_ERR\_CBF\_DUPLICATE\_BCOORD} \\ & \text{Duplicate index in BCOORD.} \end{split}$$
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_ACOORD Duplicate index in ACOORD.
- MSK\_RES\_ERR\_CBF\_T00\_FEW\_VARIABLES
  Too few variables defined.
- MSK\_RES\_ERR\_CBF\_T00\_FEW\_CONSTRAINTS Too few constraints defined.
- MSK\_RES\_ERR\_CBF\_T00\_FEW\_INTS Too few ints are specified.
- MSK\_RES\_ERR\_CBF\_TOO\_MANY\_INTS Too many ints are specified.
- MSK\_RES\_ERR\_CBF\_INVALID\_INT\_INDEX Invalid INT index.
- $\begin{array}{c} {\tt MSK\_RES\_ERR\_CBF\_UNSUPPORTED} \\ {\tt Unsupported\ feature\ is\ present.} \end{array}$
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_PSDVAR Duplicate PSDVAR keyword.
- MSK\_RES\_ERR\_CBF\_INVALID\_PSDVAR\_DIMENSION Invalid PSDVAR dimmension.
- MSK\_RES\_ERR\_CBF\_T00\_FEW\_PSDVAR Too few variables defined.

### MSK\_RES\_ERR\_MIO\_INVALID\_ROOT\_OPTIMIZER

An invalid root optimizer was selected for the problem type.

#### MSK\_RES\_ERR\_MIO\_INVALID\_NODE\_OPTIMIZER

An invalid node optimizer was selected for the problem type.

# MSK\_RES\_ERR\_TOCONIC\_CONSTR\_Q\_NOT\_PSD

The matrix defining the quadratric part of constraint is not positive semidefinite.

### MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_FX

The quadratic constraint is an equality, thus not convex.

### MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_RA

The quadratic constraint has finite lower and upper bound, and therefore it is not convex.

# MSK\_RES\_ERR\_TOCONIC\_CONSTR\_NOT\_CONIC

The constraint is not conic representable.

### MSK\_RES\_ERR\_TOCONIC\_OBJECTIVE\_NOT\_PSD

The matrix defining the quadratric part of the objective function is not positive semidefinite.

#### MSK\_RES\_ERR\_SERVER\_CONNECT

Failed to connect to remote solver server. The server string or the port string were invalid, or the server did not accept connection.

### MSK\_RES\_ERR\_SERVER\_PROTOCOL

Unexpected message or data from solver server.

#### MSK\_RES\_ERR\_SERVER\_STATUS

Server returned non-ok HTTP status code

#### MSK\_RES\_ERR\_SERVER\_TOKEN

The job ID specified is incorrect or invalid

# 13.4 Constants

# 13.4.1 Language selection constants

### MSK\_LANG\_ENG

English language selection

#### MSK\_LANG\_DAN

Danish language selection

# 13.4.2 Constraint or variable access modes. All functions using this enum are deprecated. Use separate functions for rows/columns instead.

# MSK\_ACC\_VAR

Access data by columns (variable oriented)

### MSK\_ACC\_CON

Access data by rows (constraint oriented)

# 13.4.3 Basis identification

### MSK\_BI\_NEVER

Never do basis identification.

#### MSK\_BI\_ALWAYS

Basis identification is always performed even if the interior-point optimizer terminates abnormally.

### MSK\_BI\_NO\_ERROR

Basis identification is performed if the interior-point optimizer terminates without an error.

#### MSK\_BI\_IF\_FEASIBLE

Basis identification is not performed if the interior-point optimizer terminates with a problem status saying that the problem is primal or dual infeasible.

### MSK\_BI\_RESERVERED

Not currently in use.

# 13.4.4 Bound keys

#### MSK\_BK\_LO

The constraint or variable has a finite lower bound and an infinite upper bound.

#### MSK\_BK\_UP

The constraint or variable has an infinite lower bound and an finite upper bound.

#### MSK BK FX

The constraint or variable is fixed.

#### MSK\_BK\_FR

The constraint or variable is free.

#### MSK\_BK\_RA

The constraint or variable is ranged.

### 13.4.5 Mark

### MSK\_MARK\_LO

The lower bound is selected for sensitivity analysis.

### MSK\_MARK\_UP

The upper bound is selected for sensitivity analysis.

# 13.4.6 Degeneracy strategies

#### MSK\_SIM\_DEGEN\_NONE

The simplex optimizer should use no degeneration strategy.

# MSK\_SIM\_DEGEN\_FREE

The simplex optimizer chooses the degeneration strategy.

### MSK\_SIM\_DEGEN\_AGGRESSIVE

The simplex optimizer should use an aggressive degeneration strategy.

### MSK\_SIM\_DEGEN\_MODERATE

The simplex optimizer should use a moderate degeneration strategy.

# MSK\_SIM\_DEGEN\_MINIMUM

The simplex optimizer should use a minimum degeneration strategy.

# 13.4.7 Transposed matrix.

#### MSK\_TRANSPOSE\_NO

No transpose is applied.

# MSK\_TRANSPOSE\_YES

A transpose is applied.

# 13.4.8 Triangular part of a symmetric matrix.

### MSK\_UPLO\_LO

Lower part.

### MSK\_UPLO\_UP

Upper part

### 13.4.9 Problem reformulation.

# MSK\_SIM\_REFORMULATION\_ON

Allow the simplex optimizer to reformulate the problem.

#### MSK\_SIM\_REFORMULATION\_OFF

Disallow the simplex optimizer to reformulate the problem.

### MSK\_SIM\_REFORMULATION\_FREE

The simplex optimizer can choose freely.

# MSK\_SIM\_REFORMULATION\_AGGRESSIVE

The simplex optimizer should use an aggressive reformulation strategy.

# 13.4.10 Exploit duplicate columns.

# MSK\_SIM\_EXPLOIT\_DUPVEC\_ON

Allow the simplex optimizer to exploit duplicated columns.

### MSK\_SIM\_EXPLOIT\_DUPVEC\_OFF

Disallow the simplex optimizer to exploit duplicated columns.

# MSK\_SIM\_EXPLOIT\_DUPVEC\_FREE

The simplex optimizer can choose freely.

# 13.4.11 Hot-start type employed by the simplex optimizer

#### MSK\_SIM\_HOTSTART\_NONE

The simplex optimizer performs a coldstart.

### MSK\_SIM\_HOTSTART\_FREE

The simplex optimize chooses the hot-start type.

### MSK\_SIM\_HOTSTART\_STATUS\_KEYS

Only the status keys of the constraints and variables are used to choose the type of hot-start.

# 13.4.12 Hot-start type employed by the interior-point optimizers.

### MSK\_INTPNT\_HOTSTART\_NONE

The interior-point optimizer performs a coldstart.

### MSK\_INTPNT\_HOTSTART\_PRIMAL

The interior-point optimizer exploits the primal solution only.

# MSK\_INTPNT\_HOTSTART\_DUAL

The interior-point optimizer exploits the dual solution only.

# MSK\_INTPNT\_HOTSTART\_PRIMAL\_DUAL

The interior-point optimizer exploits both the primal and dual solution.

# 13.4.13 Progress callback codes

### MSK\_CALLBACK\_BEGIN\_BI

The basis identification procedure has been started.

### MSK\_CALLBACK\_BEGIN\_CONIC

The callback function is called when the conic optimizer is started.

#### MSK\_CALLBACK\_BEGIN\_DUAL\_BI

The callback function is called from within the basis identification procedure when the dual phase is started.

#### MSK\_CALLBACK\_BEGIN\_DUAL\_SENSITIVITY

Dual sensitivity analysis is started.

### MSK\_CALLBACK\_BEGIN\_DUAL\_SETUP\_BI

The callback function is called when the dual BI phase is started.

### MSK\_CALLBACK\_BEGIN\_DUAL\_SIMPLEX

The callback function is called when the dual simplex optimizer started.

#### MSK\_CALLBACK\_BEGIN\_DUAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the dual simplex clean-up phase is started.

#### MSK\_CALLBACK\_BEGIN\_FULL\_CONVEXITY\_CHECK

Begin full convexity check.

# MSK\_CALLBACK\_BEGIN\_INFEAS\_ANA

The callback function is called when the infeasibility analyzer is started.

#### MSK\_CALLBACK\_BEGIN\_INTPNT

The callback function is called when the interior-point optimizer is started.

# MSK\_CALLBACK\_BEGIN\_LICENSE\_WAIT

Begin waiting for license.

# MSK\_CALLBACK\_BEGIN\_MIO

The callback function is called when the mixed-integer optimizer is started.

# MSK\_CALLBACK\_BEGIN\_OPTIMIZER

The callback function is called when the optimizer is started.

### MSK\_CALLBACK\_BEGIN\_PRESOLVE

The callback function is called when the presolve is started.

### MSK\_CALLBACK\_BEGIN\_PRIMAL\_BI

The callback function is called from within the basis identification procedure when the primal phase is started.

# MSK\_CALLBACK\_BEGIN\_PRIMAL\_REPAIR

Begin primal feasibility repair.

# MSK\_CALLBACK\_BEGIN\_PRIMAL\_SENSITIVITY

Primal sensitivity analysis is started.

### MSK\_CALLBACK\_BEGIN\_PRIMAL\_SETUP\_BI

The callback function is called when the primal BI setup is started.

### MSK\_CALLBACK\_BEGIN\_PRIMAL\_SIMPLEX

The callback function is called when the primal simplex optimizer is started.

### MSK\_CALLBACK\_BEGIN\_PRIMAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the primal simplex clean-up phase is started.

# MSK\_CALLBACK\_BEGIN\_QCQO\_REFORMULATE

Begin QCQO reformulation.

#### MSK\_CALLBACK\_BEGIN\_READ

MOSEK has started reading a problem file.

#### MSK\_CALLBACK\_BEGIN\_ROOT\_CUTGEN

The callback function is called when root cut generation is started.

#### MSK\_CALLBACK\_BEGIN\_SIMPLEX

The callback function is called when the simplex optimizer is started.

#### MSK\_CALLBACK\_BEGIN\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the simplex clean-up phase is started.

#### MSK\_CALLBACK\_BEGIN\_TO\_CONIC

Begin conic reformulation.

#### MSK\_CALLBACK\_BEGIN\_WRITE

**MOSEK** has started writing a problem file.

#### MSK\_CALLBACK\_CONIC

The callback function is called from within the conic optimizer after the information database has been updated.

### MSK\_CALLBACK\_DUAL\_SIMPLEX

The callback function is called from within the dual simplex optimizer.

#### MSK\_CALLBACK\_END\_BI

The callback function is called when the basis identification procedure is terminated.

### MSK\_CALLBACK\_END\_CONIC

The callback function is called when the conic optimizer is terminated.

#### MSK\_CALLBACK\_END\_DUAL\_BI

The callback function is called from within the basis identification procedure when the dual phase is terminated.

# MSK\_CALLBACK\_END\_DUAL\_SENSITIVITY

Dual sensitivity analysis is terminated.

### MSK\_CALLBACK\_END\_DUAL\_SETUP\_BI

The callback function is called when the dual BI phase is terminated.

### MSK\_CALLBACK\_END\_DUAL\_SIMPLEX

The callback function is called when the dual simplex optimizer is terminated.

#### MSK\_CALLBACK\_END\_DUAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the dual clean-up phase is terminated.

### MSK\_CALLBACK\_END\_FULL\_CONVEXITY\_CHECK

End full convexity check.

#### MSK\_CALLBACK\_END\_INFEAS\_ANA

The callback function is called when the infeasibility analyzer is terminated.

#### MSK\_CALLBACK\_END\_INTPNT

The callback function is called when the interior-point optimizer is terminated.

#### MSK\_CALLBACK\_END\_LICENSE\_WAIT

End waiting for license.

# MSK\_CALLBACK\_END\_MIO

The callback function is called when the mixed-integer optimizer is terminated.

### MSK\_CALLBACK\_END\_OPTIMIZER

The callback function is called when the optimizer is terminated.

### MSK\_CALLBACK\_END\_PRESOLVE

The callback function is called when the presolve is completed.

#### MSK\_CALLBACK\_END\_PRIMAL\_BI

The callback function is called from within the basis identification procedure when the primal phase is terminated.

#### MSK\_CALLBACK\_END\_PRIMAL\_REPAIR

End primal feasibility repair.

#### MSK\_CALLBACK\_END\_PRIMAL\_SENSITIVITY

Primal sensitivity analysis is terminated.

# MSK\_CALLBACK\_END\_PRIMAL\_SETUP\_BI

The callback function is called when the primal BI setup is terminated.

#### MSK\_CALLBACK\_END\_PRIMAL\_SIMPLEX

The callback function is called when the primal simplex optimizer is terminated.

#### MSK\_CALLBACK\_END\_PRIMAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the primal clean-up phase is terminated.

### MSK\_CALLBACK\_END\_QCQO\_REFORMULATE

End QCQO reformulation.

### MSK\_CALLBACK\_END\_READ

MOSEK has finished reading a problem file.

### MSK\_CALLBACK\_END\_ROOT\_CUTGEN

The callback function is called when root cut generation is is terminated.

#### MSK\_CALLBACK\_END\_SIMPLEX

The callback function is called when the simplex optimizer is terminated.

#### MSK\_CALLBACK\_END\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the simplex clean-up phase is terminated.

# MSK\_CALLBACK\_END\_TO\_CONIC

End conic reformulation.

### MSK\_CALLBACK\_END\_WRITE

MOSEK has finished writing a problem file.

#### MSK\_CALLBACK\_IM\_BI

The callback function is called from within the basis identification procedure at an intermediate point.

# MSK\_CALLBACK\_IM\_CONIC

The callback function is called at an intermediate stage within the conic optimizer where the information database has not been updated.

### MSK\_CALLBACK\_IM\_DUAL\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

#### MSK\_CALLBACK\_IM\_DUAL\_SENSIVITY

The callback function is called at an intermediate stage of the dual sensitivity analysis.

### MSK\_CALLBACK\_IM\_DUAL\_SIMPLEX

The callback function is called at an intermediate point in the dual simplex optimizer.

### MSK\_CALLBACK\_IM\_FULL\_CONVEXITY\_CHECK

The callback function is called at an intermediate stage of the full convexity check.

### MSK\_CALLBACK\_IM\_INTPNT

The callback function is called at an intermediate stage within the interior-point optimizer where the information database has not been updated.

#### MSK\_CALLBACK\_IM\_LICENSE\_WAIT

**MOSEK** is waiting for a license.

#### MSK\_CALLBACK\_IM\_LU

The callback function is called from within the LU factorization procedure at an intermediate point.

#### MSK\_CALLBACK\_IM\_MIO

The callback function is called at an intermediate point in the mixed-integer optimizer.

### MSK\_CALLBACK\_IM\_MIO\_DUAL\_SIMPLEX

The callback function is called at an intermediate point in the mixed-integer optimizer while running the dual simplex optimizer.

#### MSK\_CALLBACK\_IM\_MIO\_INTPNT

The callback function is called at an intermediate point in the mixed-integer optimizer while running the interior-point optimizer.

#### MSK\_CALLBACK\_IM\_MIO\_PRIMAL\_SIMPLEX

The callback function is called at an intermediate point in the mixed-integer optimizer while running the primal simplex optimizer.

### MSK\_CALLBACK\_IM\_ORDER

The callback function is called from within the matrix ordering procedure at an intermediate point.

### MSK\_CALLBACK\_IM\_PRESOLVE

The callback function is called from within the presolve procedure at an intermediate stage.

#### MSK\_CALLBACK\_IM\_PRIMAL\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

#### MSK\_CALLBACK\_IM\_PRIMAL\_SENSIVITY

The callback function is called at an intermediate stage of the primal sensitivity analysis.

### MSK\_CALLBACK\_IM\_PRIMAL\_SIMPLEX

The callback function is called at an intermediate point in the primal simplex optimizer.

### MSK\_CALLBACK\_IM\_QO\_REFORMULATE

The callback function is called at an intermediate stage of the conic quadratic reformulation.

# MSK\_CALLBACK\_IM\_READ

Intermediate stage in reading.

# MSK\_CALLBACK\_IM\_ROOT\_CUTGEN

The callback is called from within root cut generation at an intermediate stage.

### MSK\_CALLBACK\_IM\_SIMPLEX

The callback function is called from within the simplex optimizer at an intermediate point.

### MSK\_CALLBACK\_IM\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the simplex clean-up phase. The frequency of the callbacks is controlled by the  $MSK\_IPAR\_LOG\_SIM\_FREQ$  parameter.

#### MSK\_CALLBACK\_INTPNT

The callback function is called from within the interior-point optimizer after the information database has been updated.

### MSK\_CALLBACK\_NEW\_INT\_MIO

The callback function is called after a new integer solution has been located by the mixed-integer optimizer.

# MSK\_CALLBACK\_PRIMAL\_SIMPLEX

The callback function is called from within the primal simplex optimizer.

# MSK\_CALLBACK\_READ\_OPF

The callback function is called from the OPF reader.

#### MSK\_CALLBACK\_READ\_OPF\_SECTION

A chunk of Q non-zeros has been read from a problem file.

#### MSK\_CALLBACK\_SOLVING\_REMOTE

The callback function is called while the task is being solved on a remote server.

#### MSK\_CALLBACK\_UPDATE\_DUAL\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

# MSK\_CALLBACK\_UPDATE\_DUAL\_SIMPLEX

The callback function is called in the dual simplex optimizer.

#### MSK\_CALLBACK\_UPDATE\_DUAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the dual simplex clean-up phase. The frequency of the callbacks is controlled by the  $MSK\_IPAR\_LOG\_SIM\_FREQ$  parameter.

### MSK\_CALLBACK\_UPDATE\_PRESOLVE

The callback function is called from within the presolve procedure.

#### MSK\_CALLBACK\_UPDATE\_PRIMAL\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

### MSK\_CALLBACK\_UPDATE\_PRIMAL\_SIMPLEX

The callback function is called in the primal simplex optimizer.

# MSK\_CALLBACK\_UPDATE\_PRIMAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the primal simplex clean-up phase. The frequency of the callbacks is controlled by the  $MSK\_IPAR\_LOG\_SIM\_FREQ$  parameter.

### MSK\_CALLBACK\_WRITE\_OPF

The callback function is called from the OPF writer.

# 13.4.14 Types of convexity checks.

### MSK\_CHECK\_CONVEXITY\_NONE

No convexity check.

### MSK\_CHECK\_CONVEXITY\_SIMPLE

Perform simple and fast convexity check.

### MSK\_CHECK\_CONVEXITY\_FULL

Perform a full convexity check.

# 13.4.15 Compression types

### MSK\_COMPRESS\_NONE

No compression is used.

# MSK\_COMPRESS\_FREE

The type of compression used is chosen automatically.

### MSK\_COMPRESS\_GZIP

The type of compression used is gzip compatible.

# 13.4.16 Cone types

### MSK\_CT\_QUAD

The cone is a quadratic cone.

# MSK\_CT\_RQUAD

The cone is a rotated quadratic cone.

# 13.4.17 Name types

### MSK\_NAME\_TYPE\_GEN

General names. However, no duplicate and blank names are allowed.

### MSK\_NAME\_TYPE\_MPS

MPS type names.

# MSK\_NAME\_TYPE\_LP

LP type names.

# 13.4.18 Cone types

# MSK\_SYMMAT\_TYPE\_SPARSE

Sparse symmetric matrix.

# 13.4.19 Data format types

### MSK\_DATA\_FORMAT\_EXTENSION

The file extension is used to determine the data file format.

### MSK\_DATA\_FORMAT\_MPS

The data file is MPS formatted.

# MSK\_DATA\_FORMAT\_LP

The data file is LP formatted.

### MSK\_DATA\_FORMAT\_OP

The data file is an optimization problem formatted file.

### MSK\_DATA\_FORMAT\_XML

The data file is an XML formatted file.

# MSK\_DATA\_FORMAT\_FREE\_MPS

The data a free MPS formatted file.

# ${\tt MSK\_DATA\_FORMAT\_TASK}$

Generic task dump file.

## MSK\_DATA\_FORMAT\_CB

Conic benchmark format,

# MSK\_DATA\_FORMAT\_JSON\_TASK

JSON based task format.

# 13.4.20 Double information items

### MSK\_DINF\_BI\_CLEAN\_DUAL\_TIME

Time spent within the dual clean-up optimizer of the basis identification procedure since its invocation.

# MSK\_DINF\_BI\_CLEAN\_PRIMAL\_TIME

Time spent within the primal clean-up optimizer of the basis identification procedure since its invocation.

# MSK\_DINF\_BI\_CLEAN\_TIME

Time spent within the clean-up phase of the basis identification procedure since its invocation.

#### MSK\_DINF\_BI\_DUAL\_TIME

Time spent within the dual phase basis identification procedure since its invocation.

#### MSK\_DINF\_BI\_PRIMAL\_TIME

Time spent within the primal phase of the basis identification procedure since its invocation.

#### MSK\_DINF\_BI\_TIME

Time spent within the basis identification procedure since its invocation.

### MSK\_DINF\_INTPNT\_DUAL\_FEAS

Dual feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed.)

### MSK\_DINF\_INTPNT\_DUAL\_OBJ

Dual objective value reported by the interior-point optimizer.

#### MSK\_DINF\_INTPNT\_FACTOR\_NUM\_FLOPS

An estimate of the number of flops used in the factorization.

#### MSK\_DINF\_INTPNT\_OPT\_STATUS

A measure of optimality of the solution. It should converge to +1 if the problem has a primal-dual optimal solution, and converge to -1 if the problem is (strictly) primal or dual infeasible. If the measure converges to another constant, or fails to settle, the problem is usually ill-posed.

#### MSK\_DINF\_INTPNT\_ORDER\_TIME

Order time (in seconds).

# MSK\_DINF\_INTPNT\_PRIMAL\_FEAS

Primal feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed).

### MSK\_DINF\_INTPNT\_PRIMAL\_OBJ

Primal objective value reported by the interior-point optimizer.

### MSK\_DINF\_INTPNT\_TIME

Time spent within the interior-point optimizer since its invocation.

### MSK\_DINF\_MIO\_CLIQUE\_SEPARATION\_TIME

Separation time for clique cuts.

# MSK\_DINF\_MIO\_CMIR\_SEPARATION\_TIME

Separation time for CMIR cuts.

#### MSK\_DINF\_MIO\_CONSTRUCT\_SOLUTION\_OBJ

If MOSEK has successfully constructed an integer feasible solution, then this item contains the optimal objective value corresponding to the feasible solution.

# MSK\_DINF\_MIO\_DUAL\_BOUND\_AFTER\_PRESOLVE

Value of the dual bound after presolve but before cut generation.

# MSK\_DINF\_MIO\_GMI\_SEPARATION\_TIME

Separation time for GMI cuts.

# MSK\_DINF\_MIO\_HEURISTIC\_TIME

Total time spent in the optimizer.

### MSK\_DINF\_MIO\_IMPLIED\_BOUND\_TIME

Seperation time for implied bound cuts.

#### MSK\_DINF\_MIO\_KNAPSACK\_COVER\_SEPARATION\_TIME

Seperation time for knapsack cover.

### MSK\_DINF\_MIO\_OBJ\_ABS\_GAP

Given the mixed-integer optimizer has computed a feasible solution and a bound on the optimal

objective value, then this item contains the absolute gap defined by

|(objective value of feasible solution) – (objective bound)|.

Otherwise it has the value -1.0.

#### MSK\_DINF\_MIO\_OBJ\_BOUND

The best known bound on the objective function. This value is undefined until at least one relaxation has been solved: To see if this is the case check that  $MSK\_IINF\_MIO\_NUM\_RELAX$  is strictly positive.

### MSK\_DINF\_MIO\_OBJ\_INT

The primal objective value corresponding to the best integer feasible solution. Please note that at least one integer feasible solution must have been located i.e. check  $MSK\_IINF\_MIO\_NUM\_INT\_SOLUTIONS$ .

# MSK\_DINF\_MIO\_OBJ\_REL\_GAP

Given that the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the relative gap defined by

```
\frac{|(\text{objective value of feasible solution}) - (\text{objective bound})|}{\max(\delta, |(\text{objective value of feasible solution})|)}
```

where  $\delta$  is given by the parameter  $MSK\_DPAR\_MIO\_REL\_GAP\_CONST$ . Otherwise it has the value -1.0.

#### MSK\_DINF\_MIO\_OPTIMIZER\_TIME

Total time spent in the optimizer.

# MSK\_DINF\_MIO\_PROBING\_TIME

Total time for probing.

#### MSK\_DINF\_MIO\_ROOT\_CUTGEN\_TIME

Total time for cut generation.

### MSK\_DINF\_MIO\_ROOT\_OPTIMIZER\_TIME

Time spent in the optimizer while solving the root relaxation.

# MSK\_DINF\_MIO\_ROOT\_PRESOLVE\_TIME

Time spent in while presolving the root relaxation.

### MSK\_DINF\_MIO\_TIME

Time spent in the mixed-integer optimizer.

#### MSK\_DINF\_MIO\_USER\_OBJ\_CUT

If the objective cut is used, then this information item has the value of the cut.

#### MSK\_DINF\_OPTIMIZER\_TIME

Total time spent in the optimizer since it was invoked.

#### MSK\_DINF\_PRESOLVE\_ELI\_TIME

Total time spent in the eliminator since the presolve was invoked.

### MSK\_DINF\_PRESOLVE\_LINDEP\_TIME

Total time spent in the linear dependency checker since the presolve was invoked.

### MSK\_DINF\_PRESOLVE\_TIME

Total time (in seconds) spent in the presolve since it was invoked.

# MSK\_DINF\_PRIMAL\_REPAIR\_PENALTY\_OBJ

The optimal objective value of the penalty function.

# MSK\_DINF\_QCQO\_REFORMULATE\_MAX\_PERTURBATION

Maximum absolute diagonal perturbation occurring during the QCQO reformulation.

### MSK\_DINF\_QCQO\_REFORMULATE\_TIME

Time spent with conic quadratic reformulation.

# MSK\_DINF\_QCQO\_REFORMULATE\_WORST\_CHOLESKY\_COLUMN\_SCALING Worst Cholesky column scaling.

# MSK\_DINF\_QCQO\_REFORMULATE\_WORST\_CHOLESKY\_DIAG\_SCALING Worst Cholesky diagonal scaling.

### MSK\_DINF\_RD\_TIME

Time spent reading the data file.

### MSK\_DINF\_SIM\_DUAL\_TIME

Time spent in the dual simplex optimizer since invoking it.

#### MSK\_DINF\_SIM\_FEAS

Feasibility measure reported by the simplex optimizer.

### MSK\_DINF\_SIM\_OBJ

Objective value reported by the simplex optimizer.

### MSK\_DINF\_SIM\_PRIMAL\_TIME

Time spent in the primal simplex optimizer since invoking it.

#### MSK\_DINF\_SIM\_TIME

Time spent in the simplex optimizer since invoking it.

### MSK\_DINF\_SOL\_BAS\_DUAL\_OBJ

Dual objective value of the basic solution.

### MSK\_DINF\_SOL\_BAS\_DVIOLCON

Maximal dual bound violation for  $x^c$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_DVIOLVAR

Maximal dual bound violation for  $x^x$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_NRM\_BARX

Infinity norm of  $\overline{X}$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_NRM\_SLC

Infinity norm of  $s_l^c$  in the basic solution.

#### MSK\_DINF\_SOL\_BAS\_NRM\_SLX

Infinity norm of  $s_l^x$  in the basic solution.

# ${\tt MSK\_DINF\_SOL\_BAS\_NRM\_SUC}$

Infinity norm of  $s_u^c$  in the basic solution.

# ${\tt MSK\_DINF\_SOL\_BAS\_NRM\_SUX}$

Infinity norm of  $s_u^X$  in the basic solution.

# MSK\_DINF\_SOL\_BAS\_NRM\_XC

Infinity norm of  $x^c$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_NRM\_XX

Infinity norm of  $x^x$  in the basic solution.

# MSK\_DINF\_SOL\_BAS\_NRM\_Y

Infinity norm of y in the basic solution.

# MSK\_DINF\_SOL\_BAS\_PRIMAL\_OBJ

Primal objective value of the basic solution.

# MSK\_DINF\_SOL\_BAS\_PVIOLCON

Maximal primal bound violation for  $x^c$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_PVIOLVAR

Maximal primal bound violation for  $x^x$  in the basic solution.

### MSK\_DINF\_SOL\_ITG\_NRM\_BARX

Infinity norm of  $\overline{X}$  in the integer solution.

#### MSK\_DINF\_SOL\_ITG\_NRM\_XC

Infinity norm of  $x^c$  in the integer solution.

#### MSK\_DINF\_SOL\_ITG\_NRM\_XX

Infinity norm of  $x^x$  in the integer solution.

### MSK\_DINF\_SOL\_ITG\_PRIMAL\_OBJ

Primal objective value of the integer solution.

### MSK\_DINF\_SOL\_ITG\_PVIOLBARVAR

Maximal primal bound violation for  $\overline{X}$  in the integer solution.

# MSK\_DINF\_SOL\_ITG\_PVIOLCON

Maximal primal bound violation for  $x^c$  in the integer solution.

### MSK\_DINF\_SOL\_ITG\_PVIOLCONES

Maximal primal violation for primal conic constraints in the integer solution.

### MSK\_DINF\_SOL\_ITG\_PVIOLITG

Maximal violation for the integer constraints in the integer solution.

#### MSK\_DINF\_SOL\_ITG\_PVIOLVAR

Maximal primal bound violation for  $x^x$  in the integer solution.

#### MSK\_DINF\_SOL\_ITR\_DUAL\_OBJ

Dual objective value of the interior-point solution.

#### MSK\_DINF\_SOL\_ITR\_DVIOLBARVAR

Maximal dual bound violation for  $\overline{X}$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_DVIOLCON

Maximal dual bound violation for  $x^c$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_DVIOLCONES

Maximal dual violation for dual conic constraints in the interior-point solution.

#### MSK\_DINF\_SOL\_ITR\_DVIOLVAR

Maximal dual bound violation for  $x^x$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_NRM\_BARS

Infinity norm of  $\overline{S}$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_BARX

Infinity norm of  $\overline{X}$  in the interior-point solution.

# ${\tt MSK\_DINF\_SOL\_ITR\_NRM\_SLC}$

Infinity norm of  $s_l^c$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_NRM\_SLX

Infinity norm of  $s_l^x$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_NRM\_SNX

Infinity norm of  $s_n^x$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SUC

Infinity norm of  $s_u^c$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SUX

Infinity norm of  $s_u^X$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_XC

Infinity norm of  $x^c$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_NRM\_XX

Infinity norm of  $x^x$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_NRM\_Y

Infinity norm of y in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_PRIMAL\_OBJ

Primal objective value of the interior-point solution.

#### MSK\_DINF\_SOL\_ITR\_PVIOLBARVAR

Maximal primal bound violation for  $\overline{X}$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_PVIOLCON

Maximal primal bound violation for  $x^c$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_PVIOLCONES

Maximal primal violation for primal conic constraints in the interior-point solution.

#### MSK\_DINF\_SOL\_ITR\_PVIOLVAR

Maximal primal bound violation for  $x^x$  in the interior-point solution.

### MSK\_DINF\_TO\_CONIC\_TIME

Time spent in the last to conic reformulation.

# 13.4.21 License feature

#### MSK\_FEATURE\_PTS

Base system.

#### MSK\_FEATURE\_PTON

Nonlinear extension.

# 13.4.22 Long integer information items.

### MSK\_LIINF\_BI\_CLEAN\_DUAL\_DEG\_ITER

Number of dual degenerate clean iterations performed in the basis identification.

### MSK\_LIINF\_BI\_CLEAN\_DUAL\_ITER

Number of dual clean iterations performed in the basis identification.

### MSK\_LIINF\_BI\_CLEAN\_PRIMAL\_DEG\_ITER

Number of primal degenerate clean iterations performed in the basis identification.

### MSK\_LIINF\_BI\_CLEAN\_PRIMAL\_ITER

Number of primal clean iterations performed in the basis identification.

# MSK\_LIINF\_BI\_DUAL\_ITER

Number of dual pivots performed in the basis identification.

# MSK\_LIINF\_BI\_PRIMAL\_ITER

Number of primal pivots performed in the basis identification.

# MSK\_LIINF\_INTPNT\_FACTOR\_NUM\_NZ

Number of non-zeros in factorization.

### MSK\_LIINF\_MIO\_INTPNT\_ITER

Number of interior-point iterations performed by the mixed-integer optimizer.

### MSK\_LIINF\_MIO\_PRESOLVED\_ANZ

Number of non-zero entries in the constraint matrix of presolved problem.

### MSK\_LIINF\_MIO\_SIM\_MAXITER\_SETBACKS

Number of times the the simplex optimizer has hit the maximum iteration limit when re-optimizing.

### MSK\_LIINF\_MIO\_SIMPLEX\_ITER

Number of simplex iterations performed by the mixed-integer optimizer.

#### MSK\_LIINF\_RD\_NUMANZ

Number of non-zeros in A that is read.

#### MSK\_LIINF\_RD\_NUMQNZ

Number of Q non-zeros.

# 13.4.23 Integer information items.

#### MSK\_IINF\_ANA\_PRO\_NUM\_CON

Number of constraints in the problem.

### MSK\_IINF\_ANA\_PRO\_NUM\_CON\_EQ

Number of equality constraints.

# MSK\_IINF\_ANA\_PRO\_NUM\_CON\_FR

Number of unbounded constraints.

#### MSK\_IINF\_ANA\_PRO\_NUM\_CON\_LO

Number of constraints with a lower bound and an infinite upper bound.

### MSK\_IINF\_ANA\_PRO\_NUM\_CON\_RA

Number of constraints with finite lower and upper bounds.

### MSK\_IINF\_ANA\_PRO\_NUM\_CON\_UP

Number of constraints with an upper bound and an infinite lower bound.

### MSK\_IINF\_ANA\_PRO\_NUM\_VAR

Number of variables in the problem.

### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_BIN

Number of binary (0-1) variables.

# MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_CONT

Number of continuous variables.

# MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_EQ

Number of fixed variables.

### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_FR

Number of free variables.

### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_INT

Number of general integer variables.

# MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_LO

Number of variables with a lower bound and an infinite upper bound.

# MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_RA

Number of variables with finite lower and upper bounds.

# MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_UP

Number of variables with an upper bound and an infinite lower bound. This value is set by

# MSK\_IINF\_INTPNT\_FACTOR\_DIM\_DENSE

Dimension of the dense sub system in factorization.

# MSK\_IINF\_INTPNT\_ITER

Number of interior-point iterations since invoking the interior-point optimizer.

# MSK\_IINF\_INTPNT\_NUM\_THREADS

Number of threads that the interior-point optimizer is using.

### MSK\_IINF\_INTPNT\_SOLVE\_DUAL

Non-zero if the interior-point optimizer is solving the dual problem.

### MSK\_IINF\_MIO\_ABSGAP\_SATISFIED

Non-zero if absolute gap is within tolerances.

#### MSK\_IINF\_MIO\_CLIQUE\_TABLE\_SIZE

Size of the clique table.

#### MSK\_IINF\_MIO\_CONSTRUCT\_NUM\_ROUNDINGS

Number of values in the integer solution that is rounded to an integer value.

#### MSK\_IINF\_MIO\_CONSTRUCT\_SOLUTION

If this item has the value 0, then **MOSEK** did not try to construct an initial integer feasible solution. If the item has a positive value, then **MOSEK** successfully constructed an initial integer feasible solution.

### MSK\_IINF\_MIO\_INITIAL\_SOLUTION

Is non-zero if an initial integer solution is specified.

### MSK\_IINF\_MIO\_NEAR\_ABSGAP\_SATISFIED

Non-zero if absolute gap is within relaxed tolerances.

#### MSK\_IINF\_MIO\_NEAR\_RELGAP\_SATISFIED

Non-zero if relative gap is within relaxed tolerances.

#### MSK\_IINF\_MIO\_NODE\_DEPTH

Depth of the last node solved.

### MSK\_IINF\_MIO\_NUM\_ACTIVE\_NODES

Number of active branch bound nodes.

### MSK\_IINF\_MIO\_NUM\_BRANCH

Number of branches performed during the optimization.

#### MSK\_IINF\_MIO\_NUM\_CLIQUE\_CUTS

Number of clique cuts.

### MSK\_IINF\_MIO\_NUM\_CMIR\_CUTS

Number of Complemented Mixed Integer Rounding (CMIR) cuts.

#### MSK\_IINF\_MIO\_NUM\_GOMORY\_CUTS

Number of Gomory cuts.

### MSK\_IINF\_MIO\_NUM\_IMPLIED\_BOUND\_CUTS

Number of implied bound cuts.

### MSK\_IINF\_MIO\_NUM\_INT\_SOLUTIONS

Number of integer feasible solutions that has been found.

### MSK\_IINF\_MIO\_NUM\_KNAPSACK\_COVER\_CUTS

Number of clique cuts.

# MSK\_IINF\_MIO\_NUM\_RELAX

Number of relaxations solved during the optimization.

# MSK\_IINF\_MIO\_NUM\_REPEATED\_PRESOLVE

Number of times presolve was repeated at root.

# MSK\_IINF\_MIO\_NUMCON

Number of constraints in the problem solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMINT

Number of integer variables in the problem solved be the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMVAR

Number of variables in the problem solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_OBJ\_BOUND\_DEFINED

Non-zero if a valid objective bound has been found, otherwise zero.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMBIN

Number of binary variables in the problem solved be the mixed-integer optimizer.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMCON

Number of constraints in the presolved problem.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONT

Number of continuous variables in the problem solved be the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMINT

Number of integer variables in the presolved problem.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMVAR

Number of variables in the presolved problem.

# MSK\_IINF\_MIO\_RELGAP\_SATISFIED

Non-zero if relative gap is within tolerances.

# MSK\_IINF\_MIO\_TOTAL\_NUM\_CUTS

Total number of cuts generated by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_USER\_OBJ\_CUT

If it is non-zero, then the objective cut is used.

### MSK\_IINF\_OPT\_NUMCON

Number of constraints in the problem solved when the optimizer is called.

#### MSK\_IINF\_OPT\_NUMVAR

Number of variables in the problem solved when the optimizer is called

### MSK\_IINF\_OPTIMIZE\_RESPONSE

The response code returned by optimize.

#### MSK\_IINF\_RD\_NUMBARVAR

Number of variables read.

# MSK\_IINF\_RD\_NUMCON

Number of constraints read.

#### MSK\_IINF\_RD\_NUMCONE

Number of conic constraints read.

#### MSK\_IINF\_RD\_NUMINTVAR

Number of integer-constrained variables read.

#### MSK\_IINF\_RD\_NUMQ

Number of nonempty Q matrices read.

### MSK\_IINF\_RD\_NUMVAR

Number of variables read.

### MSK\_IINF\_RD\_PROTYPE

Problem type.

### MSK\_IINF\_SIM\_DUAL\_DEG\_ITER

The number of dual degenerate iterations.

### MSK\_IINF\_SIM\_DUAL\_HOTSTART

If 1 then the dual simplex algorithm is solving from an advanced basis.

# MSK\_IINF\_SIM\_DUAL\_HOTSTART\_LU

If 1 then a valid basis factorization of full rank was located and used by the dual simplex algorithm.

### MSK\_IINF\_SIM\_DUAL\_INF\_ITER

The number of iterations taken with dual infeasibility.

### MSK\_IINF\_SIM\_DUAL\_ITER

Number of dual simplex iterations during the last optimization.

# MSK\_IINF\_SIM\_NUMCON

Number of constraints in the problem solved by the simplex optimizer.

### MSK\_IINF\_SIM\_NUMVAR

Number of variables in the problem solved by the simplex optimizer.

### MSK\_IINF\_SIM\_PRIMAL\_DEG\_ITER

The number of primal degenerate iterations.

#### MSK\_IINF\_SIM\_PRIMAL\_HOTSTART

If 1 then the primal simplex algorithm is solving from an advanced basis.

# MSK\_IINF\_SIM\_PRIMAL\_HOTSTART\_LU

If 1 then a valid basis factorization of full rank was located and used by the primal simplex algorithm.

# MSK\_IINF\_SIM\_PRIMAL\_INF\_ITER

The number of iterations taken with primal infeasibility.

#### MSK\_IINF\_SIM\_PRIMAL\_ITER

Number of primal simplex iterations during the last optimization.

#### MSK\_IINF\_SIM\_SOLVE\_DUAL

Is non-zero if dual problem is solved.

#### MSK\_IINF\_SOL\_BAS\_PROSTA

Problem status of the basic solution. Updated after each optimization.

### MSK\_IINF\_SOL\_BAS\_SOLSTA

Solution status of the basic solution. Updated after each optimization.

### MSK\_IINF\_SOL\_ITG\_PROSTA

Problem status of the integer solution. Updated after each optimization.

### MSK\_IINF\_SOL\_ITG\_SOLSTA

Solution status of the integer solution. Updated after each optimization.

### MSK\_IINF\_SOL\_ITR\_PROSTA

Problem status of the interior-point solution. Updated after each optimization.

### MSK\_IINF\_SOL\_ITR\_SOLSTA

Solution status of the interior-point solution. Updated after each optimization.

# MSK\_IINF\_STO\_NUM\_A\_REALLOC

Number of times the storage for storing A has been changed. A large value may indicates that memory fragmentation may occur.

# 13.4.24 Information item types

# MSK\_INF\_DOU\_TYPE

Is a double information type.

# MSK\_INF\_INT\_TYPE

Is an integer.

### MSK\_INF\_LINT\_TYPE

Is a long integer.

# 13.4.25 Input/output modes

### MSK\_IOMODE\_READ

The file is read-only.

### MSK\_IOMODE\_WRITE

The file is write-only. If the file exists then it is truncated when it is opened. Otherwise it is created when it is opened.

### MSK\_IOMODE\_READWRITE

The file is to read and written.

# 13.4.26 Specifies the branching direction.

### MSK\_BRANCH\_DIR\_FREE

The mixed-integer optimizer decides which branch to choose.

#### MSK\_BRANCH\_DIR\_UP

The mixed-integer optimizer always chooses the up branch first.

#### MSK\_BRANCH\_DIR\_DOWN

The mixed-integer optimizer always chooses the down branch first.

#### MSK\_BRANCH\_DIR\_NEAR

Branch in direction nearest to selected fractional variable.

### MSK\_BRANCH\_DIR\_FAR

Branch in direction farthest from selected fractional variable.

#### MSK\_BRANCH\_DIR\_ROOT\_LP

Chose direction based on root lp value of selected variable.

#### MSK\_BRANCH\_DIR\_GUIDED

Branch in direction of current incumbent.

### MSK\_BRANCH\_DIR\_PSEUDOCOST

Branch based on the pseudocost of the variable.

# 13.4.27 Continuous mixed-integer solution type

### MSK\_MIO\_CONT\_SOL\_NONE

No interior-point or basic solution are reported when the mixed-integer optimizer is used.

### MSK\_MIO\_CONT\_SOL\_ROOT

The reported interior-point and basic solutions are a solution to the root node problem when mixed-integer optimizer is used.

### MSK\_MIO\_CONT\_SOL\_ITG

The reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. A solution is only reported in case the problem has a primal feasible solution.

### MSK\_MIO\_CONT\_SOL\_ITG\_REL

In case the problem is primal feasible then the reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. If the problem is primal infeasible, then the solution to the root node problem is reported.

# 13.4.28 Integer restrictions

#### MSK\_MIO\_MODE\_IGNORED

The integer constraints are ignored and the problem is solved as a continuous problem.

### MSK\_MIO\_MODE\_SATISFIED

Integer restrictions should be satisfied.

# 13.4.29 Mixed-integer node selection types

# MSK\_MIO\_NODE\_SELECTION\_FREE

The optimizer decides the node selection strategy.

# MSK\_MIO\_NODE\_SELECTION\_FIRST

The optimizer employs a depth first node selection strategy.

### MSK\_MIO\_NODE\_SELECTION\_BEST

The optimizer employs a best bound node selection strategy.

### MSK\_MIO\_NODE\_SELECTION\_WORST

The optimizer employs a worst bound node selection strategy.

# MSK\_MIO\_NODE\_SELECTION\_HYBRID

The optimizer employs a hybrid strategy.

### MSK\_MIO\_NODE\_SELECTION\_PSEUDO

The optimizer employs selects the node based on a pseudo cost estimate.

# 13.4.30 MPS file format type

### MSK\_MPS\_FORMAT\_STRICT

It is assumed that the input file satisfies the MPS format strictly.

#### MSK\_MPS\_FORMAT\_RELAXED

It is assumed that the input file satisfies a slightly relaxed version of the MPS format.

### MSK\_MPS\_FORMAT\_FREE

It is assumed that the input file satisfies the free MPS format. This implies that spaces are not allowed in names. Otherwise the format is free.

### MSK\_MPS\_FORMAT\_CPLEX

The CPLEX compatible version of the MPS format is employed.

# 13.4.31 Objective sense types

### MSK\_OBJECTIVE\_SENSE\_MINIMIZE

The problem should be minimized.

### MSK\_OBJECTIVE\_SENSE\_MAXIMIZE

The problem should be maximized.

# 13.4.32 On/off

#### MSK\_ON

Switch the option on.

# MSK\_OFF

Switch the option off.

# 13.4.33 Optimizer types

# MSK\_OPTIMIZER\_CONIC

The optimizer for problems having conic constraints.

# MSK\_OPTIMIZER\_DUAL\_SIMPLEX

The dual simplex optimizer is used.

### MSK\_OPTIMIZER\_FREE

The optimizer is chosen automatically.

### MSK\_OPTIMIZER\_FREE\_SIMPLEX

One of the simplex optimizers is used.

# MSK\_OPTIMIZER\_INTPNT

The interior-point optimizer is used.

#### MSK\_OPTIMIZER\_MIXED\_INT

The mixed-integer optimizer.

### MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX

The primal simplex optimizer is used.

# 13.4.34 Ordering strategies

### MSK\_ORDER\_METHOD\_FREE

The ordering method is chosen automatically.

# MSK\_ORDER\_METHOD\_APPMINLOC

Approximate minimum local fill-in ordering is employed.

### MSK\_ORDER\_METHOD\_EXPERIMENTAL

This option should not be used.

# MSK\_ORDER\_METHOD\_TRY\_GRAPHPAR

Always try the graph partitioning based ordering.

# ${\tt MSK\_ORDER\_METHOD\_FORCE\_GRAPHPAR}$

Always use the graph partitioning based ordering even if it is worse than the approximate minimum local fill ordering.

### MSK\_ORDER\_METHOD\_NONE

No ordering is used.

### 13.4.35 Presolve method.

# MSK\_PRESOLVE\_MODE\_OFF

The problem is not presolved before it is optimized.

# MSK\_PRESOLVE\_MODE\_ON

The problem is presolved before it is optimized.

### MSK\_PRESOLVE\_MODE\_FREE

It is decided automatically whether to presolve before the problem is optimized.

# 13.4.36 Parameter type

### MSK\_PAR\_INVALID\_TYPE

Not a valid parameter.

### MSK\_PAR\_DOU\_TYPE

Is a double parameter.

# MSK\_PAR\_INT\_TYPE

Is an integer parameter.

# MSK\_PAR\_STR\_TYPE

Is a string parameter.

# 13.4.37 Problem data items

#### MSK\_PI\_VAR

Item is a variable.

# MSK\_PI\_CON

Item is a constraint.

### MSK\_PI\_CONE

Item is a cone.

# 13.4.38 Problem types

### MSK\_PROBTYPE\_LO

The problem is a linear optimization problem.

#### MSK\_PROBTYPE\_QO

The problem is a quadratic optimization problem.

### MSK\_PROBTYPE\_QCQO

The problem is a quadratically constrained optimization problem.

### MSK\_PROBTYPE\_GECO

General convex optimization.

### MSK\_PROBTYPE\_CONIC

A conic optimization.

### MSK\_PROBTYPE\_MIXED

General nonlinear constraints and conic constraints. This combination can not be solved by MOSEK.

# 13.4.39 Problem status keys

### MSK\_PRO\_STA\_UNKNOWN

Unknown problem status.

# MSK\_PRO\_STA\_PRIM\_AND\_DUAL\_FEAS

The problem is primal and dual feasible.

### MSK\_PRO\_STA\_PRIM\_FEAS

The problem is primal feasible.

### MSK\_PRO\_STA\_DUAL\_FEAS

The problem is dual feasible.

### MSK\_PRO\_STA\_NEAR\_PRIM\_AND\_DUAL\_FEAS

The problem is at least nearly primal and dual feasible.

### MSK\_PRO\_STA\_NEAR\_PRIM\_FEAS

The problem is at least nearly primal feasible.

### MSK\_PRO\_STA\_NEAR\_DUAL\_FEAS

The problem is at least nearly dual feasible.

### MSK\_PRO\_STA\_PRIM\_INFEAS

The problem is primal infeasible.

### MSK\_PRO\_STA\_DUAL\_INFEAS

The problem is dual infeasible.

# MSK\_PRO\_STA\_PRIM\_AND\_DUAL\_INFEAS

The problem is primal and dual infeasible.

# MSK\_PRO\_STA\_ILL\_POSED

The problem is ill-posed. For example, it may be primal and dual feasible but have a positive duality gap.

### MSK\_PRO\_STA\_PRIM\_INFEAS\_OR\_UNBOUNDED

The problem is either primal infeasible or unbounded. This may occur for mixed-integer problems.

# 13.4.40 XML writer output mode

### MSK\_WRITE\_XML\_MODE\_ROW

Write in row order.

# MSK\_WRITE\_XML\_MODE\_COL

Write in column order.

# 13.4.41 Response code type

### MSK\_RESPONSE\_OK

The response code is OK.

#### MSK\_RESPONSE\_WRN

The response code is a warning.

#### MSK\_RESPONSE\_TRM

The response code is an optimizer termination status.

# MSK\_RESPONSE\_ERR

The response code is an error.

# MSK\_RESPONSE\_UNK

The response code does not belong to any class.

# 13.4.42 Scaling type

### MSK\_SCALING\_FREE

The optimizer chooses the scaling heuristic.

# MSK\_SCALING\_NONE

No scaling is performed.

# MSK\_SCALING\_MODERATE

A conservative scaling is performed.

# MSK\_SCALING\_AGGRESSIVE

A very aggressive scaling is performed.

# 13.4.43 Scaling method

### MSK\_SCALING\_METHOD\_POW2

Scales only with power of 2 leaving the mantissa untouched.

### MSK\_SCALING\_METHOD\_FREE

The optimizer chooses the scaling heuristic.

# 13.4.44 Sensitivity types

# MSK\_SENSITIVITY\_TYPE\_BASIS

Basis sensitivity analysis is performed.

# MSK\_SENSITIVITY\_TYPE\_OPTIMAL\_PARTITION

Optimal partition sensitivity analysis is performed.

# 13.4.45 Simplex selection strategy

### MSK\_SIM\_SELECTION\_FREE

The optimizer chooses the pricing strategy.

### MSK\_SIM\_SELECTION\_FULL

The optimizer uses full pricing.

### MSK\_SIM\_SELECTION\_ASE

The optimizer uses approximate steepest-edge pricing.

#### MSK\_SIM\_SELECTION\_DEVEX

The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steep-edge selection).

# MSK\_SIM\_SELECTION\_SE

The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

### MSK\_SIM\_SELECTION\_PARTIAL

The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.

# 13.4.46 Solution items

### MSK\_SOL\_ITEM\_XC

Solution for the constraints.

### MSK\_SOL\_ITEM\_XX

Variable solution.

#### MSK\_SOL\_ITEM\_Y

Lagrange multipliers for equations.

#### MSK\_SOL\_ITEM\_SLC

Lagrange multipliers for lower bounds on the constraints.

### MSK\_SOL\_ITEM\_SUC

Lagrange multipliers for upper bounds on the constraints.

### MSK\_SOL\_ITEM\_SLX

Lagrange multipliers for lower bounds on the variables.

# MSK\_SOL\_ITEM\_SUX

Lagrange multipliers for upper bounds on the variables.

# MSK\_SOL\_ITEM\_SNX

Lagrange multipliers corresponding to the conic constraints on the variables.

# 13.4.47 Solution status keys

# MSK\_SOL\_STA\_UNKNOWN

Status of the solution is unknown.

### MSK\_SOL\_STA\_OPTIMAL

The solution is optimal.

#### MSK\_SOL\_STA\_PRIM\_FEAS

The solution is primal feasible.

# MSK\_SOL\_STA\_DUAL\_FEAS

The solution is dual feasible.

### MSK\_SOL\_STA\_PRIM\_AND\_DUAL\_FEAS

The solution is both primal and dual feasible.

### MSK\_SOL\_STA\_NEAR\_OPTIMAL

The solution is nearly optimal.

### MSK\_SOL\_STA\_NEAR\_PRIM\_FEAS

The solution is nearly primal feasible.

# MSK\_SOL\_STA\_NEAR\_DUAL\_FEAS

The solution is nearly dual feasible.

### MSK\_SOL\_STA\_NEAR\_PRIM\_AND\_DUAL\_FEAS

The solution is nearly both primal and dual feasible.

# MSK\_SOL\_STA\_PRIM\_INFEAS\_CER

The solution is a certificate of primal infeasibility.

### MSK\_SOL\_STA\_DUAL\_INFEAS\_CER

The solution is a certificate of dual infeasibility.

### MSK\_SOL\_STA\_NEAR\_PRIM\_INFEAS\_CER

The solution is almost a certificate of primal infeasibility.

### MSK\_SOL\_STA\_NEAR\_DUAL\_INFEAS\_CER

The solution is almost a certificate of dual infeasibility.

### MSK\_SOL\_STA\_PRIM\_ILLPOSED\_CER

The solution is a certificate that the primal problem is illposed.

### MSK\_SOL\_STA\_DUAL\_ILLPOSED\_CER

The solution is a certificate that the dual problem is illposed.

### MSK\_SOL\_STA\_INTEGER\_OPTIMAL

The primal solution is integer optimal.

#### MSK\_SOL\_STA\_NEAR\_INTEGER\_OPTIMAL

The primal solution is near integer optimal.

# 13.4.48 Solution types

### MSK\_SOL\_BAS

The basic solution.

# MSK\_SOL\_ITR

The interior solution.

# MSK\_SOL\_ITG

The integer solution.

# 13.4.49 Solve primal or dual form

### MSK\_SOLVE\_FREE

The optimizer is free to solve either the primal or the dual problem.

### MSK\_SOLVE\_PRIMAL

The optimizer should solve the primal problem.

### MSK\_SOLVE\_DUAL

The optimizer should solve the dual problem.

# 13.4.50 Status keys

#### MSK\_SK\_UNK

The status for the constraint or variable is unknown.

#### MSK\_SK\_BAS

The constraint or variable is in the basis.

#### MSK\_SK\_SUPBAS

The constraint or variable is super basic.

#### MSK\_SK\_LOW

The constraint or variable is at its lower bound.

### MSK\_SK\_UPR

The constraint or variable is at its upper bound.

### MSK\_SK\_FIX

The constraint or variable is fixed.

#### MSK SK INF

The constraint or variable is infeasible in the bounds.

# 13.4.51 Starting point types

### MSK\_STARTING\_POINT\_FREE

The starting point is chosen automatically.

### MSK\_STARTING\_POINT\_GUESS

The optimizer guesses a starting point.

### MSK\_STARTING\_POINT\_CONSTANT

The optimizer constructs a starting point by assigning a constant value to all primal and dual variables. This starting point is normally robust.

# MSK\_STARTING\_POINT\_SATISFY\_BOUNDS

The starting point is chosen to satisfy all the simple bounds on nonlinear variables. If this starting point is employed, then more care than usual should employed when choosing the bounds on the nonlinear variables. In particular very tight bounds should be avoided.

# 13.4.52 Stream types

# MSK\_STREAM\_LOG

Log stream. Contains the aggregated contents of all other streams. This means that a message written to any other stream will also be written to this stream.

### MSK\_STREAM\_MSG

Message stream. Log information relating to performance and progress of the optimization is written to this stream.

#### MSK\_STREAM\_ERR

Error stream. Error messages are written to this stream.

#### MSK\_STREAM\_WRN

Warning stream. Warning messages are written to this stream.

# 13.4.53 Integer values

# MSK\_MAX\_STR\_LEN

Maximum string length allowed in MOSEK.

# ${\tt MSK\_LICENSE\_BUFFER\_LENGTH}$

The length of a license key buffer.

# 13.4.54 Variable types

MSK\_VAR\_TYPE\_CONT Is a continuous variable.

MSK\_VAR\_TYPE\_INT

Is an integer variable.

# SUPPORTED FILE FORMATS

MOSEK supports a range of problem and solution formats listed in Table 14.1 and Table 14.2. The **Task** format is MOSEK's native binary format and it supports all features that MOSEK supports. The **OPF** format is MOSEK's human-readable alternative that supports nearly all features (everything except semidefinite problems). In general, text formats are significantly slower to read, but can be examined and edited directly in any text editor.

### **Problem formats**

See Table 14.1.

Table 14.1: List of supported file formats for optimization problems

Format Type	Ext.	Binary/Text	LP	QO	CQO	SDP
LP	lp	plain text	X	X		
MPS	mps	plain text	X	X		
OPF	opf	plain text	X	X	X	
CBF	cbf	plain text	X		X	X
OSiL	xml	xml text	X	X		
Task format	task	binary	X	X	X	X
Jtask format	jtask	text	X	X	X	X

## **Solution formats**

See Table 14.2.

Table 14.2: List of supported solution formats.

Format Type	Ext.	Binary/Text	Description
SOL	sol	plain text	Interior Solution
	bas	plain text	Basic Solution
	$_{ m int}$	plain text	Integer
Jsol format	jsol	text	Solution

### Compression

MOSEK supports GZIP compression of files. Problem files with an additional .gz extension are assumed to be compressed when read, and are automatically compressed when written. For example, a file called

problem.mps.gz

will be considered as a GZIP compressed MPS file.

# 14.1 The LP File Format

MOSEK supports the LP file format with some extensions. The LP format is not a completely well-defined standard and hence different optimization packages may interpret the same LP file in slightly different ways. MOSEK tries to emulate as closely as possible CPLEX's behavior, but tries to stay backward compatible.

The LP file format can specify problems on the form

$$\begin{array}{lll} \text{minimize/maximize} & & c^Tx + \frac{1}{2}q^o(x) \\ \text{subject to} & l^c & \leq & Ax + \frac{1}{2}q(x) & \leq & u^c, \\ l^x & \leq & x & \leq & u^x, \\ & & & x_{\mathcal{J}} \text{ integer,} \end{array}$$

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear term in the objective.
- $q^o :\in \mathbb{R}^n \to \mathbb{R}$  is the quadratic term in the objective where

$$q^o(x) = x^T Q^o x$$

and it is assumed that

$$Q^o = (Q^o)^T.$$

- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T$$
.

•  $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer constrained variables.

# 14.1.1 File Sections

An LP formatted file contains a number of sections specifying the objective, constraints, variable bounds, and variable types. The section keywords may be any mix of upper and lower case letters.

# **Objective Function**

The first section beginning with one of the keywords

max
maximum
maximize
min
minimum
minimize

defines the objective sense and the objective function, i.e.

$$c^T x + \frac{1}{2} x^T Q^o x.$$

The objective may be given a name by writing

#### myname:

before the expressions. If no name is given, then the objective is named obj.

The objective function contains linear and quadratic terms. The linear terms are written as:

```
4 x1 + x2 - 0.1 x3
```

and so forth. The quadratic terms are written in square brackets ([]) and are either squared or multiplied as in the examples

```
x1^2
```

and

```
x1 * x2
```

There may be zero or more pairs of brackets containing quadratic expressions.

An example of an objective section is

```
minimize
myobj: 4 x1 + x2 - 0.1 x3 + [ x1^2 + 2.1 x1 * x2 ]/2
```

Please note that the quadratic expressions are multiplied with  $\frac{1}{2}$ , so that the above expression means

minimize 
$$4x_1 + x_2 - 0.1 \cdot x_3 + \frac{1}{2}(x_1^2 + 2.1 \cdot x_1 \cdot x_2)$$

If the same variable occurs more than once in the linear part, the coefficients are added, so that  $4 \times 1 + 2 \times 1$  is equivalent to  $6 \times 1$ . In the quadratic expressions  $\times 1 \times 2$  is equivalent to  $\times 2 \times 1$  and, as in the linear part, if the same variables multiplied or squared occur several times their coefficients are added.

### **Constraints**

The second section beginning with one of the keywords

```
subj to
subject to
s.t.
st
```

defines the linear constraint matrix A and the quadratic matrices  $Q^i$ .

A constraint contains a name (optional), expressions adhering to the same rules as in the objective and a bound:

```
subject to
con1: x1 + x2 + [ x3^2 ]/2 <= 5.1</pre>
```

The bound type (here <=) may be any of <, <=, =, >, >= (< and <= mean the same), and the bound may be any number.

In the standard LP format it is not possible to define more than one bound, but **MOSEK** supports defining ranged constraints by using double-colon (::) instead of a single-colon (:) after the constraint name, i.e.

$$-5 \le x_1 + x_2 \le 5 \tag{14.1}$$

may be written as

```
con:: -5 < x_1 + x_2 < 5
```

By default **MOSEK** writes ranged constraints this way.

If the files must adhere to the LP standard, ranged constraints must either be split into upper bounded and lower bounded constraints or be written as an equality with a slack variable. For example the expression (14.1) may be written as

$$x_1 + x_2 - sl_1 = 0, -5 \le sl_1 \le 5.$$

#### **Bounds**

Bounds on the variables can be specified in the bound section beginning with one of the keywords

```
bound bounds
```

The bounds section is optional but should, if present, follow the subject to section. All variables listed in the bounds section must occur in either the objective or a constraint.

The default lower and upper bounds are 0 and  $+\infty$ . A variable may be declared free with the keyword free, which means that the lower bound is  $-\infty$  and the upper bound is  $+\infty$ . Furthermore it may be assigned a finite lower and upper bound. The bound definitions for a given variable may be written in one or two lines, and bounds can be any number or  $\pm\infty$  (written as  $+\inf/-\inf/+\inf\inf\inf_{-\inf}$ ) as in the example

```
bounds

x1 free

x2 <= 5

0.1 <= x2

x3 = 42

2 <= x4 < +inf
```

### Variable Types

The final two sections are optional and must begin with one of the keywords

```
bin
binaries
binary
```

and

```
gen
general
```

Under general all integer variables are listed, and under binary all binary (integer variables with bounds 0 and 1) are listed:

```
general
x1 x2
binary
x3 x4
```

Again, all variables listed in the binary or general sections must occur in either the objective or a constraint.

### **Terminating Section**

Finally, an LP formatted file must be terminated with the keyword

```
end
```

# 14.1.2 LP File Examples

### Linear example 1o1.1p

```
\ File: lo1.lp
maximize
obj: 3 x1 + x2 + 5 x3 + x4
subject to
c1: 3 x1 + x2 + 2 x3 = 30
c2: 2 x1 + x2 + 3 x3 + x4 >= 15
c3: 2 x2 + 3 x4 <= 25
bounds
0 <= x1 <= +infinity
0 <= x2 <= 10
0 <= x3 <= +infinity
0 <= x4 <= +infinity
end</pre>
```

### Mixed integer example milo1.lp

```
maximize
obj: x1 + 6.4e-01 x2
subject to
c1: 5e+01 x1 + 3.1e+01 x2 <= 2.5e+02
c2: 3e+00 x1 - 2e+00 x2 >= -4e+00
bounds
0 <= x1 <= +infinity
0 <= x2 <= +infinity
general
x1 x2
end
```

# 14.1.3 LP Format peculiarities

#### Comments

Anything on a line after a \ is ignored and is treated as a comment.

#### **Names**

A name for an objective, a constraint or a variable may contain the letters a-z, A-Z, the digits  $\theta$ - $\theta$  and the characters

```
!"#$%&()/,.;?@_'`|~
```

The first character in a name must not be a number, a period or the letter e or E. Keywords must not be used as names.

**MOSEK** accepts any character as valid for names, except \0. A name that is not allowed in LP file will be changed and a warning will be issued.

The algorithm for making names LP valid works as follows: The name is interpreted as an  $\mathtt{utf-8}$  string. For a unicode character  $\mathtt{c}$ :

- If c==\_ (underscore), the output is \_\_ (two underscores).
- If c is a valid LP name character, the output is just c.
- If c is another character in the ASCII range, the output is \_XX, where XX is the hexadecimal code for the character.
- If c is a character in the range 127-65535, the output is \_uxxxx, where xxxx is the hexadecimal code for the character.
- If c is a character above 65535, the output is \_UXXXXXXXX, where XXXXXXXX is the hexadecimal code for the character.

Invalid  $\mathtt{utf-8}$  substrings are escaped as  $\mathtt{XX'}$ , and if a name starts with a period, e or E, that character is escaped as  $\mathtt{XX}$ .

#### Variable Bounds

Specifying several upper or lower bounds on one variable is possible but **MOSEK** uses only the tightest bounds. If a variable is fixed (with =), then it is considered the tightest bound.

### MOSEK Extensions to the LP Format

Some optimization software packages employ a more strict definition of the LP format than the one used by **MOSEK**. The limitations imposed by the strict LP format are the following:

- Quadratic terms in the constraints are not allowed.
- Names can be only 16 characters long.
- Lines must not exceed 255 characters in length.

If an LP formatted file created by MOSEK should satisfy the strict definition, then the parameter

• MSK\_IPAR\_WRITE\_LP\_STRICT\_FORMAT

should be set; note, however, that some problems cannot be written correctly as a strict LP formatted file. For instance, all names are truncated to 16 characters and hence they may loose their uniqueness and change the problem.

To get around some of the inconveniences converting from other problem formats, **MOSEK** allows lines to contain 1024 characters and names may have any length (shorter than the 1024 characters).

Internally in **MOSEK** names may contain any (printable) character, many of which cannot be used in LP names. Setting the parameters

- MSK\_IPAR\_READ\_LP\_QUOTED\_NAMES and
- MSK\_IPAR\_WRITE\_LP\_QUOTED\_NAMES

allows MOSEK to use quoted names. The first parameter tells MOSEK to remove quotes from quoted names e.g, "x1", when reading LP formatted files. The second parameter tells MOSEK to put quotes around any semi-illegal name (names beginning with a number or a period) and fully illegal name (containing illegal characters). As double quote is a legal character in the LP format, quoting semi-illegal names makes them legal in the pure LP format as long as they are still shorter than 16 characters. Fully illegal names are still illegal in a pure LP file.

### 14.1.4 The strict LP format

The LP format is not a formal standard and different vendors have slightly different interpretations of the LP format. To make **MOSEK**'s definition of the LP format more compatible with the definitions of other vendors, use the parameter setting

• MSK\_IPAR\_WRITE\_LP\_STRICT\_FORMAT = MSK\_ON

This setting may lead to truncation of some names and hence to an invalid LP file. The simple solution to this problem is to use the parameter setting

• MSK\_IPAR\_WRITE\_GENERIC\_NAMES = MSK\_ON

which will cause all names to be renamed systematically in the output file.

## 14.1.5 Formatting of an LP File

A few parameters control the visual formatting of LP files written by **MOSEK** in order to make it easier to read the files. These parameters are

- MSK\_IPAR\_WRITE\_LP\_LINE\_WIDTH
- MSK\_IPAR\_WRITE\_LP\_TERMS\_PER\_LINE

The first parameter sets the maximum number of characters on a single line. The default value is 80 corresponding roughly to the width of a standard text document.

The second parameter sets the maximum number of terms per line; a term means a sign, a coefficient, and a name (for example + 42 elephants). The default value is 0, meaning that there is no maximum.

### **Unnamed Constraints**

Reading and writing an LP file with **MOSEK** may change it superficially. If an LP file contains unnamed constraints or objective these are given their generic names when the file is read (however unnamed constraints in **MOSEK** are written without names).

# 14.2 The MPS File Format

**MOSEK** supports the standard MPS format with some extensions. For a detailed description of the MPS format see the book by Nazareth [Naz87].

## 14.2.1 MPS File Structure

The version of the MPS format supported by  $\mathbf{MOSEK}$  allows specification of an optimization problem of the form

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = \frac{1}{2}x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T.$$

Please note the explicit  $\frac{1}{2}$  in the quadratic term and that  $Q^i$  is required to be symmetric.

- $\mathcal{K}$  is a convex cone.
- $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer-constrained variables.

An MPS file with one row and one column can be illustrated like this:

```
*23456789012345678901234567890123456789012345678901234567890
NAME
OBJSENSE
[objsense]
OBJNAME
[objname]
ROWS
? [cname1]
COLUMNS
[vname1]
          [cname1]
                        [value1]
                                      [vname3]
                                                 [value2]
RHS
           [cname1]
                        [value1]
                                      [cname2]
                                                 [value2]
[name]
RANGES
[name]
           [cname1]
                        [value1]
                                      [cname2]
                                                 [value2]
QSECTION
               [cname1]
                                      [vname3]
                                                 [value2]
[vname1]
           [vname2]
                        [value1]
QMATRIX
                        [value1]
[vname1]
           [vname2]
QUADOBJ
           [vname2]
                        [value1]
[vname1]
QCMATRIX
               [cname1]
           [vname2]
[vname1]
                        [value1]
BOUNDS
?? [name]
              [vname1]
                           [value1]
CSECTION
               [kname1]
                            [value1]
                                          [ktype]
[vname1]
ENDATA
```

Here the names in capitals are keywords of the MPS format and names in brackets are custom defined names or values. A couple of notes on the structure:

• Fields: All items surrounded by brackets appear in *fields*. The fields named "valueN" are numerical values. Hence, they must have the format

```
[+|-]XXXXXXX.XXXXX[[e|E][+|-]XXX]
where
```

```
.. code-block:: text
X = [0|1|2|3|4|5|6|7|8|9].
```

- Sections: The MPS file consists of several sections where the names in capitals indicate the beginning of a new section. For example, COLUMNS denotes the beginning of the columns section.
- Comments: Lines starting with an \* are comment lines and are ignored by MOSEK.
- Keys: The question marks represent keys to be specified later.
- Extensions: The sections QSECTION and CSECTION are specific MOSEK extensions of the MPS format. The sections QMATRIX, QUADOBJ and QCMATRIX are included for sake of compatibility with other vendors extensions to the MPS format.

The standard MPS format is a fixed format, i.e. everything in the MPS file must be within certain fixed positions. **MOSEK** also supports a *free format*. See Sec. 14.2.9 for details.

#### Linear example lo1.mps

A concrete example of a MPS file is presented below:

```
* File: lo1.mps
NAME
               101
OBJSENSE
    MAX
ROWS
 N obj
 E c1
 G c2
 L c3
COLUMNS
                          3
    x1
               obj
                          3
    x1
               c1
               c2
                          2
    x1
               obj
    x2
                          1
    x2
               c1
                          1
    x2
               c2
                          1
    x2
               сЗ
                          2
    xЗ
               obj
                          5
    xЗ
               c1
                          2
    хЗ
               c2
                          3
    x4
               obj
                          1
    x4
               c2
                          1
    x4
               сЗ
                          3
RHS
                          30
    rhs
               c1
               c2
                          15
    rhs
               сЗ
                          25
    rhs
RANGES
BOUNDS
UP bound
               x2
                          10
ENDATA
```

Subsequently each individual section in the MPS format is discussed.

### Section NAME

In this section a name ([name]) is assigned to the problem.

#### OBJSENSE (optional)

This is an optional section that can be used to specify the sense of the objective function. The OBJSENSE section contains one line at most which can be one of the following

MIN
MINIMIZE
MAX
MAXIMIZE

It should be obvious what the implication is of each of these four lines.

### OBJNAME (optional)

This is an optional section that can be used to specify the name of the row that is used as objective function. The OBJNAME section contains one line at most which has the form

objname

objname should be a valid row name.

#### ROWS

A record in the ROWS section has the form

# ? [cname1]

where the requirements for the fields are as follows:

Field	Starting Position	Max Width	required	Description
?	2	1	Yes	Constraint key
[cname1]	5	8	Yes	Constraint name

Hence, in this section each constraint is assigned an unique name denoted by [cname1]. Please note that [cname1] starts in position 5 and the field can be at most 8 characters wide. An initial key ? must be present to specify the type of the constraint. The key can have the values E, G, L, or N with the following interpretation:

Constraint type	$l_i^c$	$u_i^c$
E	finite	$l_i^c$
G	finite	$\infty$
L	$-\infty$	finite
N	$-\infty$	$\infty$

In the MPS format an objective vector is not specified explicitly, but one of the constraints having the key  $\mathbb N$  will be used as the objective vector c. In general, if multiple  $\mathbb N$  type constraints are specified, then the first will be used as the objective vector c.

#### COLUMNS

In this section the elements of A are specified using one or more records having the form:

|--|

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

Hence, a record specifies one or two elements  $a_{ij}$  of A using the principle that [vname1] and [cname1] determines j and i respectively. Please note that [cname1] must be a constraint name specified in the ROWS section. Finally, [value1] denotes the numerical value of  $a_{ij}$ . Another optional element is specified by [cname2], and [value2] for the variable specified by [vname1]. Some important comments are:

- All elements belonging to one variable must be grouped together.
- Zero elements of A should not be specified.
- At least one element for each variable should be specified.

#### RHS (optional)

A record in this section has the format

|--|--|--|

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RHS vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The interpretation of a record is that [name] is the name of the RHS vector to be specified. In general, several vectors can be specified. [cname1] denotes a constraint name previously specified in the ROWS section. Now, assume that this name has been assigned to the i th constraint and  $v_1$  denotes the value specified by [value1], then the interpretation of  $v_1$  is:

Constraint	$l_i^c$	$u_i^c$
type		
E	$v_1$	$v_1$
G	$v_1$	
L		$v_1$
N		

An optional second element is specified by [cname2] and [value2] and is interpreted in the same way. Please note that it is not necessary to specify zero elements, because elements are assumed to be zero.

### RANGES (optional)

A record in this section has the form

value2]	[value1] [cname2]	[cname1]	[name]
---------	-------------------	----------	--------

where the requirements for each fields are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RANGE vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The records in this section are used to modify the bound vectors for the constraints, i.e. the values in  $l^c$  and  $u^c$ . A record has the following interpretation: [name] is the name of the RANGE vector and [cname1] is a valid constraint name. Assume that [cname1] is assigned to the i th constraint and let  $v_1$  be the value specified by [value1], then a record has the interpretation:

Constraint type	Sign of $v_1$	$l_i^c$	$u_i^c$
E	_	$u_i^c + v_1$	
E	+		$l_i^c + v_1$
G	- or +	$l_i^c +  v_1 $	
L	- or +	$u_i^c -  v_1 $	
N			

### QSECTION (optional)

Within the QSECTION the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

lue2]	[7	:3]	vname3]	[vna	1]	[value1	2]	[vname2	vname1]	[·
-------	----	-----	---------	------	----	---------	----	---------	---------	----

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value
[vname3]	40	8	No	Variable name
[value2]	50	12	No	Numerical value

A record specifies one or two elements in the lower triangular part of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k th and j th variable, then  $Q^i_{kj}$  is assigned the value given by [value1] An optional second element is specified in the same way by the fields [vname1], [vname3], and [value2].

The example

$$\begin{array}{ll} \text{minimize} & -x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & x \geq 0 & \end{array}$$

has the following MPS file representation

```
* File: qo1.mps
NAME qo1
ROWS
N obj
G c1
COLUMNS
```

x1	c1	1.0
x2	obj	-1.0
x2	c1	1.0
х3	c1	1.0
RHS		
rhs	c1	1.0
QSECTION	ob	oj
x1	x1	2.0
x1	x3	-1.0
x2	x2	0.2
x3	xЗ	2.0
ENDATA		

Regarding the QSECTIONs please note that:

- Only one QSECTION is allowed for each constraint.
- The QSECTIONs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- ullet All entries specified in a QSECTION are assumed to belong to the lower triangular part of the quadratic term of Q.

### QMATRIX/QUADOBJ (optional)

The QMATRIX and QUADOBJ sections allow to define the quadratic term of the objective function. They differ in how the quadratic term of the objective function is stored:

- ullet QMATRIX It stores all the nonzeros coefficients, without taking advantage of the symmetry of the Q matrix.
- ullet QUADOBJ It only store the upper diagonal nonzero elements of the Q matrix.

A record in both sections has the form:

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies one elements of the Q matrix in the objective function. Hence, if the names [vname1] and [vname2] have been assigned to the k th and j th variable, then  $Q_{kj}$  is assigned the value given by [value1]. Note that a line must apper for each off-diagonal coefficient if using a QMATRIX section, while only one entry is required in a QUADOBJ section. The quadratic part of the objective function will be evaluated as  $1/2x^TQx$ .

The example

minimize 
$$-x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2)$$
 subject to  $x_1 + x_2 + x_3 \ge 1$ ,  $x > 0$ 

has the following MPS file representation using QMATRIX

```
* File: qo1_matrix.mps
NAME qo1_qmatrix
ROWS
```

```
N obj
 G c1
COLUMNS
    x1
              c1
                         1.0
                         -1.0
    x2
              obj
                         1.0
    x2
              c1
    xЗ
                         1.0
              c1
RHS
              c1
                         1.0
    rhs
QMATRIX
                         2.0
    x1
              x1
                         -1.0
    x1
              хЗ
    xЗ
              x1
                         -1.0
    x2
              x2
                         0.2
    xЗ
              хЗ
                         2.0
ENDATA
```

or the following using QUADOBJ

```
* File: qo1_quadobj.mps
NAME
              qo1_quadobj
ROWS
N obj
G c1
COLUMNS
    x1
              c1
                         1.0
    x2
              obj
                        -1.0
    x2
              c1
                         1.0
    xЗ
              c1
                         1.0
RHS
                         1.0
   rhs
              c1
QUADOBJ
              x1
                         2.0
    x1
              хЗ
                         -1.0
    x1
              x2
                         0.2
    x2
    xЗ
              хЗ
                         2.0
ENDATA
```

Please also note that:

- ullet A QMATRIX/QUADOBJ section can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QMATRIX/QUADOBJ section must already be specified in the COLUMNS section.

# 14.2.2 QCMATRIX (optional)

A QCMATRIX section allows to specify the quadratic part of a given constraints. Within the QCMATRIX the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

|--|--|

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies an entry of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k th and j th variable, then  $Q^i_{kj}$  is assigned the value given by [value1]. Moreover, the quadratic term is represented as  $1/2x^TQx$ .

The example

minimize 
$$\begin{array}{ccc} x_2 \\ \text{subject to} & x_1+x_2+x_3 & \geq & 1, \\ & \frac{1}{2}(-2x_1x_3+0.2x_2^2+2x_3^2) & \leq & 10, \\ & x \geq 0 \end{array}$$

has the following MPS file representation

```
* File: qo1.mps
NAME
ROWS
N obj
G c1
L q1
COLUMNS
                         1.0
    x1
              c1
              obj
    x2
                         -1.0
                         1.0
    x2
              c1
                         1.0
RHS
                         1.0
    rhs
              c1
    rhs
              q1
                         10.0
QCMATRIX
              q1
                         2.0
    x1
              x1
                         -1.0
    x1
              xЗ
    xЗ
              x1
                         -1.0
    x2
              x2
                         0.2
    хЗ
              xЗ
                         2.0
ENDATA
```

Regarding the QCMATRIXs please note that:

- Only one QCMATRIX is allowed for each constraint.
- The QCMATRIXs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- A QCMATRIX does not exploit the symmetry of Q: an off-diagonal entry (i,j) should appear twice.

## 14.2.3 BOUNDS (optional)

In the BOUNDS section changes to the default bounds vectors  $l^x$  and  $u^x$  are specified. The default bounds vectors are  $l^x=0$  and  $u^x=\infty$ . Moreover, it is possible to specify several sets of bound vectors. A record in this section has the form

where the requirements for each field are:

Field	Starting Position	Max Width	Required	Description
??	2	2	Yes	Bound key
[name]	5	8	Yes	Name of the BOUNDS vector
[vname1]	15	8	Yes	Variable name
[value1]	25	12	No	Numerical value

Hence, a record in the BOUNDS section has the following interpretation: [name] is the name of the bound vector and [vname1] is the name of the variable which bounds are modified by the record. ?? and [value1] are used to modify the bound vectors according to the following table:

??	$l_j^x$	$u_j^x$	Made integer (added to ${\mathcal J}$ )
FR	$-\infty$	$\infty$	No
FX	$v_1$	$v_1$	No
LO	$v_1$	unchanged	No
MI	$-\infty$	unchanged	No
PL	unchanged	$\infty$	No
UP	unchanged	$v_1$	No
BV	0	1	Yes
LI	$\lceil v_1 \rceil$	unchanged	Yes
UI	unchanged	$\lfloor v_1 \rfloor$	Yes

 $v_1$  is the value specified by [value1].

# 14.2.4 CSECTION (optional)

The purpose of the CSECTION is to specify the constraint

$$x \in \mathcal{K}$$
.

in (14.2). It is assumed that K satisfies the following requirements. Let

$$x^t \in \mathbb{R}^{n^t}, \quad t = 1, \dots, k$$

be vectors comprised of parts of the decision variables x so that each decision variable is a member of exactly **one** vector  $x^t$ , for example

$$x^1 = \begin{bmatrix} x_1 \\ x_4 \\ x_7 \end{bmatrix}$$
 and  $x^2 = \begin{bmatrix} x_6 \\ x_5 \\ x_3 \\ x_2 \end{bmatrix}$ .

Next define

$$\mathcal{K} := \left\{ x \in \mathbb{R}^n : \quad x^t \in \mathcal{K}_t, \quad t = 1, \dots, k \right\}$$

where  $\mathcal{K}_t$  must have one of the following forms

• R set:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} \right\}.$$

• Quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : x_1 \ge \sqrt{\sum_{j=2}^{n^t} x_j^2} \right\}. \tag{14.3}$$

• Rotated quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : 2x_1 x_2 \ge \sum_{j=3}^{n^t} x_j^2, \quad x_1, x_2 \ge 0 \right\}.$$
 (14.4)

In general, only quadratic and rotated quadratic cones are specified in the MPS file whereas membership of the  $\mathbb{R}$  set is not. If a variable is not a member of any other cone then it is assumed to be a member of an  $\mathbb{R}$  cone.

Next, let us study an example. Assume that the quadratic cone

$$x_4 \ge \sqrt{x_5^2 + x_8^2}$$

and the rotated quadratic cone

$$x_3x_7 \ge x_1^2 + x_0^2, \quad x_3, x_7 \ge 0,$$

should be specified in the MPS file. One CSECTION is required for each cone and they are specified as follows:

*	1	2	3	4	5	6
*2345678	90123456	67890123	45678901234	567890123456	78901234	567890
CSECTION	k	onea	0.0	QUAD		
x4						
x5						
x8						
CSECTION	k	oneb	0.0	RQUAD		
x7						
x3						
x1						
x0						

This first CSECTION specifies the cone (14.3) which is given the name konea. This is a quadratic cone which is specified by the keyword QUAD in the CSECTION header. The 0.0 value in the CSECTION header is not used by the QUAD cone.

The second CSECTION specifies the rotated quadratic cone (14.4). Please note the keyword RQUAD in the CSECTION which is used to specify that the cone is a rotated quadratic cone instead of a quadratic cone. The 0.0 value in the CSECTION header is not used by the RQUAD cone.

In general, a CSECTION header has the format

|--|--|--|

where the requirement for each field are as follows:

Field	Starting Position	Max Width	Required	Description
[kname1]	5	8	Yes	Name of the cone
[value1]	15	12	No	Cone parameter
[ktype]	25		Yes	Type of the cone.

The possible cone type keys are:

Cone type key	Members	Interpretation.	
QUAD	$\leq 1$	Quadratic cone i.e. (14.3).	
RQUAD	$\leq 2$	Rotated quadratic cone i.e. (14.4).	

Please note that a quadratic cone must have at least one member whereas a rotated quadratic cone must have at least two members. A record in the CSECTION has the format

[vname1]	[vname1]				
----------	----------	--	--	--	--

where the requirements for each field are

Field	Starting Position	Max Width	required	Description
[vname1]	2	8	Yes	A valid variable name

The most important restriction with respect to the CSECTION is that a variable must occur in only one CSECTION.

#### **14.2.5** ENDATA

This keyword denotes the end of the MPS file.

# 14.2.6 Integer Variables

Using special bound keys in the BOUNDS section it is possible to specify that some or all of the variables should be integer-constrained i.e. be members of  $\mathcal{J}$ . However, an alternative method is available.

This method is available only for backward compatibility and we recommend that it is not used. This method requires that markers are placed in the COLUMNS section as in the example:

```
COLUMNS
x1
           obj
                      -10.0
                                       c1
                                                   0.7
x1
           c2
                      0.5
                                       с3
                                                   1.0
x1
           c4
                      0.1
* Start of integer-constrained variables.
                                       'INTORG'
MARK000
           'MARKER'
                      -9.0
                                                   1.0
x2
           obj
                                       c1
                                                   0.6666667
                      0.8333333333
x2
           c2
                                       с3
x2
                      0.25
           c4
x3
                      1.0
                                       с6
                                                   2.0
           obj
MARKO01
           'MARKER'
                                       'INTEND'
```

• End of integer-constrained variables.

Please note that special marker lines are used to indicate the start and the end of the integer variables. Furthermore be aware of the following

- IMPORTANT: All variables between the markers are assigned a default lower bound of 0 and a default upper bound of 1. **This may not be what is intended.** If it is not intended, the correct bounds should be defined in the BOUNDS section of the MPS formatted file.
- MOSEK ignores field 1, i.e. MARKO001 and MARKO01, however, other optimization systems require them
- Field 2, i.e. MARKER, must be specified including the single quotes. This implies that no row can be assigned the name MARKER.
- Field 3 is ignored and should be left blank.
- Field 4, i.e. INTORG and INTEND, must be specified.
- It is possible to specify several such integer marker sections within the COLUMNS section.

### 14.2.7 General Limitations

• An MPS file should be an ASCII file.

# 14.2.8 Interpretation of the MPS Format

Several issues related to the MPS format are not well-defined by the industry standard. However, **MOSEK** uses the following interpretation:

• If a matrix element in the COLUMNS section is specified multiple times, then the multiple entries are added together.

• If a matrix element in a QSECTION section is specified multiple times, then the multiple entries are added together.

# 14.2.9 The Free MPS Format

**MOSEK** supports a free format variation of the MPS format. The free format is similar to the MPS file format but less restrictive, e.g. it allows longer names. However, it also presents two main limitations:

- A name must not contain any blanks.
- By default a line in the MPS file must not contain more than 1024 characters. However, by modifying the parameter MSK\_IPAR\_READ\_MPS\_WIDTH an arbitrary large line width will be accepted.

To use the free MPS format instead of the default MPS format the  $\mathbf{MOSEK}$  parameter  $MSK\_IPAR\_READ\_MPS\_FORMAT$  should be changed.

# 14.3 The OPF Format

The Optimization Problem Format (OPF) is an alternative to LP and MPS files for specifying optimization problems. It is row-oriented, inspired by the CPLEX LP format.

Apart from containing objective, constraints, bounds etc. it may contain complete or partial solutions, comments and extra information relevant for solving the problem. It is designed to be easily read and modified by hand and to be forward compatible with possible future extensions.

#### Intended use

The OPF file format is meant to replace several other files:

- The LP file format: Any problem that can be written as an LP file can be written as an OPF file too; furthermore it naturally accommodates ranged constraints and variables as well as arbitrary characters in names, fixed expressions in the objective, empty constraints, and conic constraints.
- Parameter files: It is possible to specify integer, double and string parameters along with the problem (or in a separate OPF file).
- Solution files: It is possible to store a full or a partial solution in an OPF file and later reload it.

### 14.3.1 The File Format

The format uses tags to structure data. A simple example with the basic sections may look like this:

```
[comment]
This is a comment. You may write almost anything here...
[/comment]
# This is a single-line comment.

[objective min 'myobj']
x + 3 y + x^2 + 3 y^2 + z + 1
[/objective]

[constraints]
[con 'con01'] 4 <= x + y [/con]
[/constraints]
[bounds]
[b] -10 <= x,y <= 10 [/b]</pre>
```

```
[cone quad] x,y,z [/cone] [/bounds]
```

A scope is opened by a tag of the form [tag] and closed by a tag of the form [/tag]. An opening tag may accept a list of unnamed and named arguments, for examples:

```
[tag value] tag with one unnamed argument [/tag]
[tag arg=value] tag with one named argument in quotes [/tag]
```

Unnamed arguments are identified by their order, while named arguments may appear in any order, but never before an unnamed argument. The value can be a quoted, single-quoted or double-quoted text string, i.e.

```
[tag 'value'] single-quoted value [/tag]
[tag arg='value'] single-quoted value [/tag]
[tag "value"] double-quoted value [/tag]
[tag arg="value"] double-quoted value [/tag]
```

#### **Sections**

The recognized tags are

#### [comment]

A comment section. This can contain *almost* any text: Between single quotes (') or double quotes (") any text may appear. Outside quotes the markup characters ([ and ]) must be prefixed by backslashes. Both single and double quotes may appear alone or inside a pair of quotes if it is prefixed by a backslash.

# [objective]

The objective function: This accepts one or two parameters, where the first one (in the above example min) is either min or max (regardless of case) and defines the objective sense, and the second one (above myobj), if present, is the objective name. The section may contain linear and quadratic expressions. If several objectives are specified, all but the last are ignored.

### [constraints]

This does not directly contain any data, but may contain the subsection con defining a linear constraint.

[con] defines a single constraint; if an argument is present ([con NAME]) this is used as the name of the constraint, otherwise it is given a null-name. The section contains a constraint definition written as linear and quadratic expressions with a lower bound, an upper bound, with both or with an equality. Examples:

Constraint names are unique. If a constraint is specified which has the same name as a previously defined constraint, the new constraint replaces the existing one.

#### [bounds]

This does not directly contain any data, but may contain the subsections b (linear bounds on variables) and cone (quadratic cone).

[b]. Bound definition on one or several variables separated by comma (,). An upper or lower bound on a variable replaces any earlier defined bound on that variable. If only one bound (upper or lower) is given only this bound is replaced. This means that upper and lower bounds can be specified separately. So the OPF bound definition:

```
[b] x,y \ge -10 [/b]
[b] x,y \le 10 [/b]
```

results in the bound  $-10 \le x, y \le 10$ .

[cone]. currently supports the quadratic cone and the rotated quadratic cone.

A conic constraint is defined as a set of variables which belong to a single unique cone.

• A quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^2 \ge \sum_{i=2}^n x_i^2, \quad x_1 \ge 0.$$

• A rotated quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$2x_1x_2 \ge \sum_{i=3}^n x_i^2, \quad x_1, x_2 \ge 0.$$

A [bounds]-section example:

By default all variables are free.

### [variables]

This defines an ordering of variables as they should appear in the problem. This is simply a space-separated list of variable names. Optionally, an attribute can be added [variables disallow\_new\_variables] indicating that if any variable not listed here occurs later in the file it is an error.

### [integer]

This contains a space-separated list of variables and defines the constraint that the listed variables must be integer values.

#### [hints]

This may contain only non-essential data; for example estimates of the number of variables, constraints and non-zeros. Placed before all other sections containing data this may reduce the time spent reading the file.

In the hints section, any subsection which is not recognized by MOSEK is simply ignored. In this section a hint in a subsection is defined as follows:

```
[hint ITEM] value [/hint]
```

where ITEM may be replaced by numvar (number of variables), numcon (number of linear/quadratic constraints), numanz (number of linear non-zeros in constraints) and numqnz (number of quadratic non-zeros in constraints).

#### [solutions]

This section can contain a set of full or partial solutions to a problem. Each solution must be specified using a [solution]-section, i.e.

```
[solutions]
[solution]...[/solution] #solution 1
[solution]...[/solution] #solution 2
#other solutions....
[solution]...[/solution] #solution n
[/solutions]
```

Note that a [solution]-section must be always specified inside a [solutions]-section. The syntax of a [solution]-section is the following:

```
[solution SOLTYPE status=STATUS]...[/solution]
```

where SOLTYPE is one of the strings

- interior, a non-basic solution,
- basic, a basic solution,
- integer, an integer solution,

and STATUS is one of the strings

- UNKNOWN,
- OPTIMAL,
- INTEGER\_OPTIMAL,
- PRIM\_FEAS,
- DUAL\_FEAS,
- PRIM\_AND\_DUAL\_FEAS,
- NEAR\_OPTIMAL,
- NEAR\_PRIM\_FEAS,
- NEAR\_DUAL\_FEAS,
- NEAR\_PRIM\_AND\_DUAL\_FEAS,
- PRIM\_INFEAS\_CER,
- DUAL\_INFEAS\_CER,
- NEAR\_PRIM\_INFEAS\_CER,

- NEAR\_DUAL\_INFEAS\_CER,
- NEAR\_INTEGER\_OPTIMAL.

Most of these values are irrelevant for input solutions; when constructing a solution for simplex hot-start or an initial solution for a mixed integer problem the safe setting is UNKNOWN.

A [solution]-section contains [con] and [var] sections. Each [con] and [var] section defines solution information for a single variable or constraint, specified as list of KEYWORD/value pairs, in any order, written as

#### KEYWORD=value

Allowed keywords are as follows:

- sk. The status of the item, where the value is one of the following strings:
  - LOW, the item is on its lower bound.
  - UPR, the item is on its upper bound.
  - FIX, it is a fixed item.
  - BAS, the item is in the basis.
  - SUPBAS, the item is super basic.
  - UNK, the status is unknown.
  - INF, the item is outside its bounds (infeasible).
- 1vl Defines the level of the item.
- s1 Defines the level of the dual variable associated with its lower bound.
- su Defines the level of the dual variable associated with its upper bound.
- sn Defines the level of the variable associated with its cone.
- y Defines the level of the corresponding dual variable (for constraints only).

A [var] section should always contain the items sk, lvl, sl and su. Items sl and su are not required for integer solutions.

A [con] section should always contain sk, lvl, sl, su and y.

An example of a solution section

• [vendor] This contains solver/vendor specific data. It accepts one argument, which is a vendor ID – for MOSEK the ID is simply mosek – and the section contains the subsection parameters defining solver parameters. When reading a vendor section, any unknown vendor can be safely ignored. This is described later.

Comments using the # may appear anywhere in the file. Between the # and the following line-break any text may be written, including markup characters.

#### **Numbers**

Numbers, when used for parameter values or coefficients, are written in the usual way by the printf function. That is, they may be prefixed by a sign (+ or -) and may contain an integer part, decimal part and an exponent. The decimal point is always . (a dot). Some examples are

```
1
1.0
.0
.0
1.
1e10
1e+10
1e-10
```

Some invalid examples are

```
e10 # invalid, must contain either integer or decimal part
. # invalid
.e10 # invalid
```

More formally, the following standard regular expression describes numbers as used:

```
[+|-]?([0-9]+[.][0-9]*|[.][0-9]+)([eE][+|-]?[0-9]+)?
```

#### **Names**

Variable names, constraint names and objective name may contain arbitrary characters, which in some cases must be enclosed by quotes (single or double) that in turn must be preceded by a backslash. Unquoted names must begin with a letter (a-z or A-Z) and contain only the following characters: the letters a-z and A-Z, the digits 0-9, braces ({ and }) and underscore (\_).

Some examples of legal names:

```
an_unquoted_name
another_name{123}
'single quoted name'
"double quoted name"
"name with \\"quote\\" in it"
"name with []s in it"
```

### 14.3.2 Parameters Section

In the vendor section solver parameters are defined inside the parameters subsection. Each parameter is written as

```
[p PARAMETER_NAME] value [/p]
```

where PARAMETER\_NAME is replaced by a MOSEK parameter name, usually of the form MSK\_IPAR\_..., MSK\_DPAR\_... or MSK\_SPAR\_..., and the value is replaced by the value of that parameter; both integer values and named values may be used. Some simple examples are

# 14.3.3 Writing OPF Files from MOSEK

To write an OPF file set the parameter  $MSK\_IPAR\_WRITE\_DATA\_FORMAT$  to  $MSK\_DATA\_FORMAT\_OP$  as this ensures that OPF format is used.

Then modify the following parameters to define what the file should contain:

MSK_IPAR_OPF_WRITE_SOL_BAS	F_WRITE_SOL_BAS   Include basic solution, if defined.	
MSK_IPAR_OPF_WRITE_SOL_ITG	Include integer solution, if defined.	
MSK_IPAR_OPF_WRITE_SOL_ITR	Include interior solution, if defined.	
MSK_IPAR_OPF_WRITE_SOLUTION	SInclude solutions if they are defined. If this is off, no solutions are	
	included.	
MSK_IPAR_OPF_WRITE_HEADER	Include a small header with comments.	
MSK_IPAR_OPF_WRITE_PROBLEM	Include the problem itself — objective, constraints and bounds.	
MSK_IPAR_OPF_WRITE_PARAMETER Include all parameter settings.		
MSK_IPAR_OPF_WRITE_HINTS	Include hints about the size of the problem.	

# 14.3.4 Examples

This section contains a set of small examples written in OPF and describing how to formulate linear, quadratic and conic problems.

### Linear Example 1o1.opf

Consider the example:

having the bounds

In the OPF format the example is displayed as shown in Listing 14.1.

Listing 14.1: Example of an OPF file for a linear problem.

```
[comment]
  The lo1 example in OPF format
[/comment]

[hints]
  [hint NUMVAR] 4 [/hint]
  [hint NUMCON] 3 [/hint]
  [hint NUMANZ] 9 [/hint]
[/hints]

[variables disallow_new_variables]
    x1 x2 x3 x4
[/variables]

[objective maximize 'obj']
    3 x1 + x2 + 5 x3 + x4
[/objective]
```

```
[constraints]
[con 'c1'] 3 x1 + x2 + 2 x3 = 30 [/con]
[con 'c2'] 2 x1 + x2 + 3 x3 + x4 >= 15 [/con]
[con 'c3'] 2 x2 + 3 x4 <= 25 [/con]
[/constraints]

[bounds]
[b] 0 <= * [/b]
[b] 0 <= x2 <= 10 [/b]
[/bounds]
```

### Quadratic Example qo1.opf

An example of a quadratic optimization problem is

minimize 
$$x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2$$
  
subject to  $1 \le x_1 + x_2 + x_3, x > 0.$ 

This can be formulated in opf as shown below.

Listing 14.2: Example of an OPF file for a quadratic problem.

```
[comment]
 The qo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 3 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
  [hint NUMQNZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3
[/variables]
[objective minimize 'obj']
 # The quadratic terms are often written with a factor of 1/2 as here,
 # but this is not required.
   - x2 + 0.5 ( 2.0 x1 ^ 2 - 2.0 x3 * x1 + 0.2 x2 ^ 2 + 2.0 x3 ^ 2 )
[/objective]
[constraints]
 [con 'c1'] 1.0 \le x1 + x2 + x3 [/con]
[/constraints]
[bounds]
 [b] 0 <= * [/b]
[/bounds]
```

### Conic Quadratic Example cqo1.opf

Consider the example:

$$\begin{array}{lll} \text{minimize} & x_3 + x_4 + x_5 \\ \text{subject to} & x_0 + x_1 + 2x_2 & = & 1, \\ & x_0, x_1, x_2 & \geq & 0, \\ & x_3 \geq \sqrt{x_0^2 + x_1^2}, \\ & 2x_4x_5 \geq x_2^2. \end{array}$$

Please note that the type of the cones is defined by the parameter to [cone ...]; the content of the cone-section is the names of variables that belong to the cone. The resulting OPF file is in Listing 14.3.

Listing 14.3: Example of an OPF file for a conic quadratic problem.

```
[comment]
 The cqo1 example in OPF format.
[/comment]
[hints]
  [hint NUMVAR] 6 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4 x5 x6
[/variables]
[objective minimize 'obj']
  x4 + x5 + x6
[/objective]
[constraints]
  [con 'c1'] x1 + x2 + 2e+00 x3 = 1e+00 [/con]
[/constraints]
[bounds]
  # We let all variables default to the positive orthant
  [b] 0 \ll * [/b]
  # ...and change those that differ from the default
  [b] x4,x5,x6 free [/b]
  # Define quadratic cone: x4 \ge sqrt(x1^2 + x2^2)
  [cone quad 'k1'] x4, x1, x2 [/cone]
  # Define rotated quadratic cone: 2 x5 x6 >= x3^2
  [cone rquad 'k2'] x5, x6, x3 [/cone]
[/bounds]
```

### Mixed Integer Example milo1.opf

Consider the mixed integer problem:

This can be implemented in OPF with the file in Listing 14.4.

Listing 14.4: Example of an OPF file for a mixed-integer linear problem.

```
[comment]
 The milo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 2 [/hint]
  [hint NUMCON] 2 [/hint]
  [hint NUMANZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2
[/variables]
[objective maximize 'obj']
  x1 + 6.4e-1 x2
[/objective]
[constraints]
  [con 'c1'] 5e+1 x1 + 3.1e+1 x2 \le 2.5e+2 [/con]
  [con 'c2'] -4 \le 3 x1 - 2 x2 [/con]
[/constraints]
[bounds]
  [b] 0 \ll * [/b]
[/bounds]
[integer]
 x1 x2
[/integer]
```

# 14.4 The CBF Format

This document constitutes the technical reference manual of the *Conic Benchmark Format* with file extension: .cbf or .CBF. It unifies linear, second-order cone (also known as conic quadratic) and semidefinite optimization with mixed-integer variables. The format has been designed with benchmark libraries in mind, and therefore focuses on compact and easily parsable representations. The problem structure is separated from the problem data, and the format moreover facilitates benchmarking of hotstart capability through sequences of changes.

### 14.4.1 How Instances Are Specified

This section defines the spectrum of conic optimization problems that can be formulated in terms of the keywords of the CBF format.

In the CBF format, conic optimization problems are considered in the following form:

$$\min / \max \quad g^{obj} 
g_i \in \mathcal{K}_i, \quad i \in \mathcal{I}, 
\text{s.t.} \quad G_i \in \mathcal{K}_i, \quad i \in \mathcal{I}^{PSD}, 
x_j \in \mathcal{K}_j, \quad j \in \mathcal{J}, 
\overline{X}_j \in \mathcal{K}_j, \quad j \in \mathcal{J}^{PSD}.$$
(14.5)

• Variables are either scalar variables,  $x_j$  for  $j \in \mathcal{J}$ , or variables,  $\overline{X}_j$  for  $j \in \mathcal{J}^{PSD}$ . Scalar variables can also be declared as integer.

• Constraints are affine expressions of the variables, either scalar-valued  $g_i$  for  $i \in \mathcal{I}$ , or matrix-valued  $G_i$  for  $i \in \mathcal{I}^{PSD}$ 

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$
$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i.$$

• The **objective function** is a scalar-valued affine expression of the variables, either to be minimized or maximized. We refer to this expression as  $g^{obj}$ 

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj}.$$

CBF format can represent the following cones  $\mathcal{K}$ :

• Free domain - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n\}$$
, for  $n \ge 1$ .

• Positive orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_j \ge 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \ge 1.$$

• Negative orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i \leq 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \geq 1.$$

• Fixpoint zero - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i = 0 \text{ for } j = 1, \dots, n\}, \text{ for } n > 1.$$

• Quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \left( \begin{array}{c} p \\ x \end{array} \right) \in \mathbb{R} \times \mathbb{R}^{n-1}, \ p^2 \ge x^T x, \ p \ge 0 \right\}, \ \text{for } n \ge 2.$$

• Rotated quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \begin{pmatrix} p \\ q \\ x \end{pmatrix} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{n-2}, \ 2pq \ge x^T x, \ p \ge 0, \ q \ge 0 \right\}, \text{ for } n \ge 3.$$

# 14.4.2 The Structure of CBF Files

This section defines how information is written in the CBF format, without being specific about the type of information being communicated.

All information items belong to exactly one of the three groups of information. These information groups, and the order they must appear in, are:

- 1. File format.
- 2. Problem structure.
- 3. Problem data.

The first group, file format, provides information on how to interpret the file. The second group, problem structure, provides the information needed to deduce the type and size of the problem instance. Finally, the third group, problem data, specifies the coefficients and constants of the problem instance.

#### Information items

The format is composed as a list of information items. The first line of an information item is the KEYWORD, revealing the type of information provided. The second line - of some keywords only - is the HEADER, typically revealing the size of information that follows. The remaining lines are the BODY holding the actual information to be specified.

```
KEYWORD
BODY

KEYWORD
HEADER
BODY
```

The KEYWORD determines how each line in the HEADER and BODY is structured. Moreover, the number of lines in the BODY follows either from the KEYWORD, the HEADER, or from another information item required to precede it.

#### **Embedded hotstart-sequences**

A sequence of problem instances, based on the same problem structure, is within a single file. This is facilitated via the CHANGE within the problem data information group, as a separator between the information items of each instance. The information items following a CHANGE keyword are appending to, or changing (e.g., setting coefficients back to their default value of zero), the problem data of the preceding instance.

The sequence is intended for benchmarking of hotstart capability, where the solvers can reuse their internal state and solution (subject to the achieved accuracy) as warmpoint for the succeeding instance. Whenever this feature is unsupported or undesired, the keyword CHANGE should be interpreted as the end of file.

#### File encoding and line width restrictions

The format is based on the US-ASCII printable character set with two extensions as listed below. Note, by definition, that none of these extensions can be misinterpreted as printable US-ASCII characters:

- A line feed marks the end of a line, carriage returns are ignored.
- Comment-lines may contain unicode characters in UTF-8 encoding.

The line width is restricted to 512 bytes, with 3 bytes reserved for the potential carriage return, line feed and null-terminator.

Integers and floating point numbers must follow the ISO C decimal string representation in the standard C locale. The format does not impose restrictions on the magnitude of, or number of significant digits in numeric data, but the use of 64-bit integers and 64-bit IEEE 754 floating point numbers should be sufficient to avoid loss of precision.

### Comment-line and whitespace rules

The format allows single-line comments respecting the following rule:

• Lines having first byte equal to '#' (US-ASCII 35) are comments, and should be ignored. Comments are only allowed between information items.

Given that a line is not a comment-line, whitespace characters should be handled according to the following rules:

- Leading and trailing whitespace characters should be ignored.
  - The seperator between multiple pieces of information on one line, is either one or more whitespace characters.
- Lines containing only whitespace characters are empty, and should be ignored. Empty lines are only allowed between information items.

# 14.4.3 Problem Specification

#### The problem structure

The problem structure defines the objective sense, whether it is minimization and maximization. It also defines the index sets,  $\mathcal{J}$ ,  $\mathcal{J}^{PSD}$ ,  $\mathcal{I}$  and  $\mathcal{I}^{PSD}$ , which are all numbered from zero,  $\{0, 1, \ldots\}$ , and empty until explicitly constructed.

• Scalar variables are constructed in vectors restricted to a conic domain, such as  $(x_0, x_1) \in \mathbb{R}^2_+$ ,  $(x_2, x_3, x_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$x \in \mathcal{K}_1^{n_1} \times \mathcal{K}_2^{n_2} \times \dots \times \mathcal{K}_k^{n_k}$$

which in the CBF format becomes:

```
VAR
n k
K1 n1
K2 n2
...
Kk nk
```

where  $\sum_{i} n_{i} = n$  is the total number of scalar variables. The list of supported cones is found in Table 14.3. Integrality of scalar variables can be specified afterwards.

• **PSD variables** are constructed one-by-one. That is,  $X_j \succeq \mathbf{0}^{n_j \times n_j}$  for  $j \in \mathcal{J}^{PSD}$ , constructs a matrix-valued variable of size  $n_j \times n_j$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes:

```
PSDVAR N n1 n2 ... nN
```

where N is the total number of PSD variables.

• Scalar constraints are constructed in vectors restricted to a conic domain, such as  $(g_0, g_1) \in \mathbb{R}^2_+$ ,  $(g_2, g_3, g_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$g \in \mathcal{K}_1^{m_1} \times \mathcal{K}_2^{m_2} \times \dots \times \mathcal{K}_k^{m_k}$$

which in the CBF format becomes:

14.4. The CBF Format

CON
m k
K1 m1
K2 m2
..
Kk mk

where  $\sum_{i} m_{i} = m$  is the total number of scalar constraints. The list of supported cones is found in Table 14.3.

• **PSD constraints** are constructed one-by-one. That is,  $G_i \succeq \mathbf{0}^{m_i \times m_i}$  for  $i \in \mathcal{I}^{PSD}$ , constructs a matrix-valued affine expressions of size  $m_i \times m_i$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes

```
PSDCON
M m1
m2
..
mM
```

where M is the total number of PSD constraints.

With the objective sense, variables (with integer indications) and constraints, the definitions of the many affine expressions follow in problem data.

#### Problem data

The problem data defines the coefficients and constants of the affine expressions of the problem instance. These are considered zero until explicitly defined, implying that instances with no keywords from this information group are, in fact, valid. Duplicating or conflicting information is a failure to comply with the standard. Consequently, two coefficients written to the same position in a matrix (or to transposed positions in a symmetric matrix) is an error.

The affine expressions of the objective,  $g^{obj}$ , of the scalar constraints,  $g_i$ , and of the PSD constraints,  $G_i$ , are defined separately. The following notation uses the standard trace inner product for matrices,  $\langle X, Y \rangle = \sum_{i,j} X_{ij} Y_{ij}$ .

• The affine expression of the objective is defined as

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj},$$

in terms of the symmetric matrices,  $F_j^{obj}$ , and scalars,  $a_j^{obj}$  and  $b^{obj}$ .

• The affine expressions of the scalar constraints are defined, for  $i \in \mathcal{I}$ , as

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$

in terms of the symmetric matrices,  $F_{ij}$ , and scalars,  $a_{ij}$  and  $b_i$ .

• The affine expressions of the PSD constraints are defined, for  $i \in \mathcal{I}^{PSD}$ , as

$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i,$$

in terms of the symmetric matrices,  $H_{ij}$  and  $D_i$ .

#### List of cones

The format uses an explicit syntax for symmetric positive semidefinite cones as shown above. For scalar variables and constraints, constructed in vectors, the supported conic domains and their minimum sizes are given as follows.

Table 14.3: Cones available in the CBF format

Name	CBF keyword	Cone family
Free domain	F	linear
Positive orthant	L+	linear
Negative orthant	L-	linear
Fixpoint zero	L=	linear
Quadratic cone	Q	second-order
Rotated quadratic cone	QR	second-order

# 14.4.4 File Format Keywords

#### **VER**

Description: The version of the Conic Benchmark Format used to write the file.

**HEADER:** None

BODY: One line formatted as:

INT

This is the version number.

Must appear exactly once in a file, as the first keyword.

### **OBJSENSE**

Description: Define the objective sense.

**HEADER:** None

BODY: One line formatted as:

STR

having MIN indicates minimize, and MAX indicates maximize. Capital letters are required.

Must appear exactly once in a file.

## **PSDVAR**

Description: Construct the PSD variables.

**HEADER**: One line formatted as:

INT

This is the number of PSD variables in the problem.

BODY: A list of lines formatted as:

INT

This indicates the number of rows (equal to the number of columns) in the matrix-valued PSD variable. The number of lines should match the number stated in the header.

#### **VAR**

Description: Construct the scalar variables.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar variables, followed by the number of conic domains they are restricted to.

BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 14.3), and the number of scalar variables restricted to this cone. These numbers should add up to the number of scalar variables stated first in the header. The number of lines should match the second number stated in the header.

#### INT

Description: Declare integer requirements on a selected subset of scalar variables.

**HEADER**: one line formatted as:

INT

This is the number of integer scalar variables in the problem.

BODY: a list of lines formatted as:

INT

This indicates the scalar variable index  $j \in \mathcal{J}$ . The number of lines should match the number stated in the header.

Can only be used after the keyword VAR.

### **PSDCON**

Description: Construct the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of PSD constraints in the problem.

BODY: A list of lines formatted as:

TNT

This indicates the number of rows (equal to the number of columns) in the matrix-valued affine expression of the PSD constraint. The number of lines should match the number stated in the header.

Can only be used after these keywords: PSDVAR, VAR.

#### CON

Description: Construct the scalar constraints.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar constraints, followed by the number of conic domains they restrict to.

BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 14.3), and the number of affine expressions restricted to this cone. These numbers should add up to the number of scalar constraints stated first in the header. The number of lines should match the second number stated in the header.

Can only be used after these keywords: PSDVAR, VAR

### **OBJFCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices  $F_j^{obj}$ , as used in the objective.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT REAL

This indicates the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

### **OBJACOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $a_i^{obj}$ , as used in the objective.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

### **OBJBCOORD**

Description: Input the scalar,  $b^{obj}$ , as used in the objective.

HEADER: None.

BODY: One line formatted as:

#### REAL

This indicates the coefficient value.

### **FCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $F_{ij}$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **ACOORD**

Description: Input sparse coordinates (triplets) to define the scalars,  $a_{ij}$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

#### **BCOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $b_i$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$  and the coefficient value. The number of lines should match the number stated in the header.

#### **HCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $H_{ij}$ , as used in the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as

INT INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the scalar variable index  $j \in \mathcal{J}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **DCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices,  $D_i$ , as used in the PSD constraints.

**HEADER**: One line formatted as

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **CHANGE**

Start of a new instance specification based on changes to the previous. Can be interpreted as the end of file when the hotstart-sequence is unsupported or undesired.

BODY: None Header: None

## 14.4.5 CBF Format Examples

## Minimal Working Example

The conic optimization problem (14.6), has three variables in a quadratic cone - first one is integer - and an affine expression in domain 0 (equality constraint).

minimize 
$$5.1 x_0$$
  
subject to  $6.2 x_1 + 7.3 x_2 - 8.4 \in \{0\}$   
 $x \in \mathcal{Q}^3, x_0 \in \mathbb{Z}.$  (14.6)

Its formulation in the Conic Benchmark Format begins with the version of the CBF format used, to safeguard against later revisions.

```
VER
1
```

Next follows the problem structure, consisting of the objective sense, the number and domain of variables, the indices of integer variables, and the number and domain of scalar-valued affine expressions (i.e., the equality constraint).

```
OBJSENSE
MIN

VAR
3 1
Q 3

INT
1
0

CON
1 1
L= 1
```

Finally follows the problem data, consisting of the coefficients of the objective, the coefficients of the constraints, and the constant terms of the constraints. All data is specified on a sparse coordinate form.

```
OBJACOORD

1
0 5.1

ACOORD
2
0 1 6.2
0 2 7.3

BCOORD
1
0 -8.4
```

This concludes the example.

#### Mixing Linear, Second-order and Semidefinite Cones

The conic optimization problem (14.7), has a semidefinite cone, a quadratic cone over unordered subindices, and two equality constraints.

The equality constraints are easily rewritten to the conic form,  $(g_0, g_1) \in \{0\}^2$ , by moving constants such that the right-hand-side becomes zero. The quadratic cone does not fit under the VAR keyword in this variable permutation. Instead, it takes a scalar constraint  $(g_2, g_3, g_4) = (x_1, x_0, x_2) \in \mathcal{Q}^3$ , with scalar

variables constructed as  $(x_0, x_1, x_2) \in \mathbb{R}^3$ . Its formulation in the CBF format is reported in the following list

```
\mbox{\tt\#} File written using this version of the Conic Benchmark Format:
#
     | Version 1.
VER
1
# The sense of the objective is:
    | Minimize.
OBJSENSE
MIN
# One PSD variable of this size:
# | Three times three.
PSDVAR
1
# Three scalar variables in this one conic domain:
     | Three are free.
VAR
3 1
F 3
\ensuremath{\mathtt{\#}} Five scalar constraints with affine expressions in two conic domains:
# | Two are fixed to zero.
      | Three are in conic quadratic domain.
CON
5 2
L= 2
Q3
# Five coordinates in F^{obj}_j coefficients:
# | F^{obj}[0][0,0] = 2.0
     | F^{obj}[0][1,0] = 1.0
     and more...
OBJFCOORD
0 0 0 2.0
0 1 0 1.0
0 1 1 2.0
0 2 1 1.0
0 2 2 2.0
# One coordinate in a^{obj}_j coefficients:
\# | a^{obj}[1] = 1.0
OBJACOORD
1 1.0
# Nine coordinates in F_ij coefficients:
    | F[0,0][0,0] = 1.0
     | F[0,0][1,1] = 1.0
# | and more...
FCOORD
0 0 0 0 1.0
0 0 1 1 1.0
0 0 2 2 1.0
1 0 0 0 1.0
1 0 1 0 1.0
1 0 2 0 1.0
```

```
1 0 1 1 1.0
1 0 2 1 1.0
1 0 2 2 1.0
# Six coordinates in a_ij coefficients:
     | a[0,1] = 1.0
      | a[1,0] = 1.0
      | and more...
ACOORD
0 1 1.0
1 0 1.0
1 2 1.0
2 1 1.0
3 0 1.0
4 2 1.0
# Two coordinates in b_i coefficients:
   | b[0] = -1.0
      | b[1] = -0.5
BCOORD
0 -1.0
1 -0.5
```

### Mixing Semidefinite Variables and Linear Matrix Inequalities

The standard forms in semidefinite optimization are usually based either on semidefinite variables or linear matrix inequalities. In the CBF format, both forms are supported and can even be mixed as shown in.

minimize 
$$\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle + x_1 + x_2 + 1$$
  
subject to  $\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle - x_1 - x_2 \qquad \geq 0.0,$   
 $x_1 \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} + x_2 \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \succeq \mathbf{0},$   
 $X_1 \succeq \mathbf{0}.$  (14.8)

Its formulation in the CBF format is written in what follows

```
# File written using this version of the Conic Benchmark Format:
#
      | Version 1.
VER
1
# The sense of the objective is:
    | Minimize.
OBJSENSE
# One PSD variable of this size:
# | Two times two.
PSDVAR
1
2
# Two scalar variables in this one conic domain:
      | Two are free.
VAR
2 1
```

```
# One PSD constraint of this size:
# | Two times two.
PSDCON
1
2
\mbox{\tt\#} One scalar constraint with an affine expression in this one conic domain:
     | One is greater than or equal to zero.
CON
1 1
L+ 1
# Two coordinates in F^{obj}_j coefficients:
# | F^{obj}[0][0,0] = 1.0
#
    | F^{obj}[0][1,1] = 1.0
OBJFCOORD
0 0 0 1.0
0 1 1 1.0
# Two coordinates in a^{obj}_j coefficients:
# | a^{obj}[0] = 1.0
#
     | a^{obj}[1] = 1.0
OBJACOORD
0 1.0
1 1.0
# One coordinate in b^{obj} coefficient:
# | b^{obj} = 1.0
OBJBCOORD
1.0
# One coordinate in F_ij coefficients:
# | F[0,0][1,0] = 1.0
FCOORD
1
0 0 1 0 1.0
# Two coordinates in a_ij coefficients:
     | a[0,0] = -1.0
     | a[0,1] = -1.0
#
ACOORD
0 0 -1.0
0 1 -1.0
# Four coordinates in H_ij coefficients:
# | H[0,0][1,0] = 1.0
     | H[0,0][1,1] = 3.0
     and more...
HCOORD
0 0 1 0 1.0
0 0 1 1 3.0
0 1 0 0 3.0
0 1 1 0 1.0
# Two coordinates in D_i coefficients:
     | D[0][0,0] = -1.0
     | D[0][1,1] = -1.0
```

```
DCOORD
2
0 0 0 -1.0
0 1 1 -1.0
```

### Optimization Over a Sequence of Objectives

The linear optimization problem (14.9), is defined for a sequence of objectives such that hotstarting from one to the next might be advantages.

$$\begin{array}{llll} \text{maximize}_k & g_k^{obj} \\ \text{subject to} & 50 \, x_0 + 31 & \leq & 250 \,, \\ & 3 \, x_0 - 2 x_1 & \geq & -4 \,, \\ & x \in \mathbb{R}_+^2, \end{array} \tag{14.9}$$

given,

```
1. g_0^{obj} = x_0 + 0.64x_1.

2. g_1^{obj} = 1.11x_0 + 0.76x_1.

3. g_2^{obj} = 1.11x_0 + 0.85x_1.
```

Its formulation in the CBF format is reported in Listing 14.5.

Listing 14.5: Problem (14.9) in CBF format.

```
# File written using this version of the Conic Benchmark Format:
#
      | Version 1.
VER
1
# The sense of the objective is:
  | Maximize.
OBJSENSE
MAX
# Two scalar variables in this one conic domain:
     | Two are nonnegative.
VAR
2 1
L+ 2
# Two scalar constraints with affine expressions in these two conic domains:
     | One is in the nonpositive domain.
      | One is in the nonnegative domain.
CON
2 2
L- 1
L+ 1
# Two coordinates in a^{obj}_j coefficients:
     | a^{obj}[0] = 1.0
      | a^{obj}[1] = 0.64
OBJACOORD
0 1.0
1 0.64
# Four coordinates in a_ij coefficients:
      | a[0,0] = 50.0
      | a[1,0] = 3.0
```

```
and more...
ACOORD
0 0 50.0
1 0 3.0
0 1 31.0
1 1 -2.0
# Two coordinates in b_i coefficients:
      | b[0] = -250.0
      | b[1] = 4.0
BCOORD
0 -250.0
1 4.0
# New problem instance defined in terms of changes.
CHANGE
# Two coordinate changes in a^{obj}_j coefficients. Now it is:
      | a^{obj}[0] = 1.11
      | a^{obj}[1] = 0.76
OBJACOORD
0 1.11
1 0.76
# New problem instance defined in terms of changes.
# One coordinate change in a^{obj}_j coefficients. Now it is:
      | a^{obj}[0] = 1.11
      | a^{obj}[1] = 0.85
OBJACOORD
1 0.85
```

# 14.5 The XML (OSiL) Format

 $\mathbf{MOSEK}$  can write data in the standard OSiL xml format. For a definition of the OSiL format please see  $\mathbf{http://www.optimizationservices.org/.}$ 

Only linear constraints (possibly with integer variables) are supported. By default output files with the extension .xml are written in the OSiL format.

The parameter  $MSK\_IPAR\_WRITE\_XML\_MODE$  controls if the linear coefficients in the A matrix are written in row or column order.

## 14.6 The Task Format

The Task format is **MOSEK**'s native binary format. It contains a complete image of a **MOSEK** task, i.e.

- Problem data: Linear, conic quadratic, semidefinite and quadratic data
- Problem item names: Variable names, constraints names, cone names etc.
- Parameter settings
- Solutions

There are a few things to be aware of:

- The task format *does not* support General Convex problems since these are defined by arbitrary user-defined functions.
- Status of a solution read from a file will always be unknown.
- Parameter settings in a task file *always override* any parameters set on the command line or in a parameter file.

The format is based on the TAR (USTar) file format. This means that the individual pieces of data in a .task file can be examined by unpacking it as a TAR file. Please note that the inverse may not work: Creating a file using TAR will most probably not create a valid **MOSEK** Task file since the order of the entries is important.

## 14.7 The JSON Format

MOSEK provides the possibility to read/write problems in valid JSON format.

JSON (JavaScript Object Notation) is a lightweight data-interchange format. It is easy for humans to read and write. It is easy for machines to parse and generate. It is based on a subset of the JavaScript Programming Language, Standard ECMA-262 3rd Edition - December 1999. JSON is a text format that is completely language independent but uses conventions that are familiar to programmers of the C-family of languages, including C, C++, C#, Java, JavaScript, Perl, Python, and many others. These properties make JSON an ideal data-interchange language.

The official JSON website http://www.json.org provides plenty of information along with the format definition.

MOSEK defines two JSON-like formats:

- jtask
- jsol

Warning: Despite being text-based human-readable formats, *jtask* and *jsol* files will include no indentation and no new-lines, in order to keep the files as compact as possible. We therefore strongly advise to use JSON viewer tools to inspect *jtask* and *jsol* files.

### 14.7.1 jtask format

It stores a problem instance. The *jtask* format contains the same information as a *task format*.

Even though a jtask file is human-readable, we do not recommend users to create it by hand, but to rely on **MOSEK**.

## 14.7.2 jsol format

It stores a problem solution. The jsol format contains all solutions and information items.

## 14.7.3 A jtask example

In Listing 14.6 we present a file in the *jtask* format that corresponds to the sample problem from lo1.1p. The listing has been formatted for readability.

Listing 14.6: A formatted *jtask* file for the lol.lp example.

```
{
    "$schema": "http://mosek.com/json/schema#",
    "Task/INFO":{
        "taskname": "lo1",
        "numvar":4,
        "numcon":3,
        "numcone":0,
        "numbarvar":0,
        "numanz":9,
        "numsymmat":0,
        "mosekver":[
            8,
            0,
            0,
            9
    },
    "Task/data":{
        "var":{
            "name":[
                 "x1",
                 "x2",
                 "x3",
                 "x4"
            ],
             "bk":[
                 "lo",
                 "ra",
                 "lo",
                 "lo"
            ],
             "bl":[
                0.0,
                 0.0,
                 0.0,
                 0.0
            ],
             "bu":[
                1e+30,
                1e+1,
                 1e+30,
                 1e+30
            ],
             "type":[
                 "cont",
                 "cont",
                 "cont",
                 "cont"
            ]
        },
        "con":{
             "name":[
                 "c1",
                 "c2",
                 "c3"
            ],
             "bk":[
                 "fx",
                 "lo",
                 "up"
```

```
],
    "bl":[
        3e+1,
        1.5e+1,
            -1e+30
    ],
    "bu":[
        3e+1,
        1e+30,
        2.5e+1
},
"objective":{
    "sense":"max",
    "name":"obj",
    "c":{
        "subj":[
           0,
            1,
            2,
            3
        ],
        "val":[
            3e+0,
            1e+0,
            5e+0,
            1e+0
        ]
    },
    "cfix":0.0
},
"A":{
    "subi":[
       0,
        0,
        Ο,
        1,
        1,
        1,
        1,
        2,
        2
    ],
    "subj":[
       0,
        1,
        2,
        Ο,
        1,
        2,
        3,
        1,
        3
    ],
"val":[
        3e+0,
        1e+0,
        2e+0,
        2e+0,
        1e+0,
        3e+0,
        1e+0,
        2e+0,
```

```
]
   }
"Task/parameters":{
    "iparam":{
        "ANA_SOL_BASIS":"ON",
        "ANA_SOL_PRINT_VIOLATED": "OFF",
        "AUTO_SORT_A_BEFORE_OPT": "OFF",
        "AUTO_UPDATE_SOL_INFO": "OFF",
        "BASIS_SOLVE_USE_PLUS_ONE": "OFF",
        "BI_CLEAN_OPTIMIZER": "OPTIMIZER_FREE",
        "BI_IGNORE_MAX_ITER":"OFF",
        "BI_IGNORE_NUM_ERROR": "OFF",
        "BI_MAX_ITERATIONS":1000000,
        "CACHE_LICENSE": "ON",
        "CHECK_CONVEXITY": "CHECK_CONVEXITY_FULL",
        "COMPRESS_STATFILE": "ON",
        "CONCURRENT_NUM_OPTIMIZERS":2,
        "CONCURRENT_PRIORITY_DUAL_SIMPLEX":2,
        "CONCURRENT_PRIORITY_FREE_SIMPLEX":3,
        "CONCURRENT_PRIORITY_INTPNT":4,
        "CONCURRENT_PRIORITY_PRIMAL_SIMPLEX":1,
        "FEASREPAIR_OPTIMIZE": "FEASREPAIR_OPTIMIZE_NONE",
        "INFEAS_GENERIC_NAMES":"OFF",
        "INFEAS_PREFER_PRIMAL":"ON",
        "INFEAS_REPORT_AUTO":"OFF",
        "INFEAS_REPORT_LEVEL":1,
        "INTPNT_BASIS": "BI_ALWAYS",
        "INTPNT_DIFF_STEP": "ON",
        "INTPNT_FACTOR_DEBUG_LVL":0,
        "INTPNT_FACTOR_METHOD":0,
        "INTPNT_HOTSTART": "INTPNT_HOTSTART_NONE",
        "INTPNT_MAX_ITERATIONS":400,
        "INTPNT_MAX_NUM_COR":-1,
        "INTPNT_MAX_NUM_REFINEMENT_STEPS":-1,
        "INTPNT_OFF_COL_TRH":40,
        "INTPNT_ORDER_METHOD": "ORDER_METHOD_FREE",
        "INTPNT_REGULARIZATION_USE":"ON",
        "INTPNT_SCALING": "SCALING_FREE",
        "INTPNT_SOLVE_FORM": "SOLVE_FREE",
        "INTPNT_STARTING_POINT": "STARTING_POINT_FREE",
        "LIC_TRH_EXPIRY_WRN":7,
        "LICENSE_DEBUG": "OFF",
        "LICENSE_PAUSE_TIME":0,
        "LICENSE_SUPPRESS_EXPIRE_WRNS": "OFF",
        "LICENSE_WAIT": "OFF",
        "LOG":10,
        "LOG_ANA_PRO":1,
        "LOG_BI":4,
        "LOG_BI_FREQ":2500,
        "LOG_CHECK_CONVEXITY":0,
        "LOG_CONCURRENT":1,
        "LOG_CUT_SECOND_OPT":1,
        "LOG_EXPAND":0,
        "LOG_FACTOR":1,
        "LOG_FEAS_REPAIR":1,
        "LOG_FILE":1,
        "LOG_HEAD":1,
        "LOG_INFEAS_ANA":1,
        "LOG_INTPNT":4,
        "LOG MIO":4.
        "LOG_MIO_FREQ":1000,
```

```
"LOG_OPTIMIZER":1,
"LOG_ORDER":1,
"LOG_PRESOLVE":1,
"LOG_RESPONSE":0,
"LOG_SENSITIVITY":1,
"LOG_SENSITIVITY_OPT":0,
"LOG_SIM":4,
"LOG_SIM_FREQ":1000,
"LOG SIM MINOR":1.
"LOG_STORAGE":1,
"MAX_NUM_WARNINGS":10,
"MIO_BRANCH_DIR": "BRANCH_DIR_FREE",
"MIO_CONSTRUCT_SOL": "OFF",
"MIO_CUT_CLIQUE": "ON",
"MIO_CUT_CMIR": "ON",
"MIO_CUT_GMI": "ON",
"MIO_CUT_KNAPSACK_COVER": "OFF",
"MIO_HEURISTIC_LEVEL":-1,
"MIO_MAX_NUM_BRANCHES":-1,
"MIO_MAX_NUM_RELAXS":-1,
"MIO_MAX_NUM_SOLUTIONS":-1,
"MIO_MODE": "MIO_MODE_SATISFIED",
"MIO_MT_USER_CB":"ON",
"MIO_NODE_OPTIMIZER": "OPTIMIZER_FREE",
"MIO_NODE_SELECTION": "MIO_NODE_SELECTION_FREE",
"MIO_PERSPECTIVE_REFORMULATE":"ON",
"MIO_PROBING_LEVEL":-1,
"MIO_RINS_MAX_NODES":-1,
"MIO_ROOT_OPTIMIZER": "OPTIMIZER_FREE",
"MIO_ROOT_REPEAT_PRESOLVE_LEVEL":-1,
"MT_SPINCOUNT":0,
"NUM_THREADS":0,
"OPF_MAX_TERMS_PER_LINE":5,
"OPF_WRITE_HEADER": "ON",
"OPF_WRITE_HINTS": "ON",
"OPF_WRITE_PARAMETERS": "OFF",
"OPF_WRITE_PROBLEM": "ON",
"OPF_WRITE_SOL_BAS":"ON",
"OPF_WRITE_SOL_ITG":"ON",
"OPF_WRITE_SOL_ITR":"ON",
"OPF_WRITE_SOLUTIONS": "OFF",
"OPTIMIZER": "OPTIMIZER_FREE",
"PARAM_READ_CASE_NAME": "ON",
"PARAM_READ_IGN_ERROR":"OFF"
"PRESOLVE_ELIMINATOR_MAX_FILL":-1,
"PRESOLVE_ELIMINATOR_MAX_NUM_TRIES":-1,
"PRESOLVE_LEVEL":-1,
"PRESOLVE_LINDEP_ABS_WORK_TRH":100,
"PRESOLVE_LINDEP_REL_WORK_TRH":100,
"PRESOLVE_LINDEP_USE": "ON",
"PRESOLVE_MAX_NUM_REDUCTIONS":-1,
"PRESOLVE_USE": "PRESOLVE_MODE_FREE",
"PRIMAL_REPAIR_OPTIMIZER": "OPTIMIZER_FREE",
"QO_SEPARABLE_REFORMULATION": "OFF",
"READ_DATA_COMPRESSED": "COMPRESS_FREE",
"READ_DATA_FORMAT": "DATA_FORMAT_EXTENSION",
"READ_DEBUG": "OFF",
"READ_KEEP_FREE_CON":"OFF",
"READ_LP_DROP_NEW_VARS_IN_BOU":"OFF",
"READ_LP_QUOTED_NAMES":"ON",
"READ_MPS_FORMAT": "MPS_FORMAT_FREE",
"READ_MPS_WIDTH": 1024,
"READ_TASK_IGNORE_PARAM":"OFF"
```

```
"SENSITIVITY_ALL": "OFF",
"SENSITIVITY_OPTIMIZER": "OPTIMIZER_FREE_SIMPLEX",
"SENSITIVITY_TYPE": "SENSITIVITY_TYPE_BASIS",
"SIM_BASIS_FACTOR_USE":"ON",
"SIM_DEGEN": "SIM_DEGEN_FREE",
"SIM_DUAL_CRASH":90,
"SIM_DUAL_PHASEONE_METHOD":0,
"SIM_DUAL_RESTRICT_SELECTION":50,
"SIM_DUAL_SELECTION": "SIM_SELECTION_FREE",
"SIM_EXPLOIT_DUPVEC": "SIM_EXPLOIT_DUPVEC_OFF",
"SIM_HOTSTART": "SIM_HOTSTART_FREE",
"SIM_HOTSTART_LU": "ON",
"SIM_INTEGER":0,
"SIM_MAX_ITERATIONS":10000000,
"SIM_MAX_NUM_SETBACKS":250,
"SIM_NON_SINGULAR": "ON",
"SIM_PRIMAL_CRASH":90,
"SIM_PRIMAL_PHASEONE_METHOD":0,
"SIM_PRIMAL_RESTRICT_SELECTION":50,
"SIM_PRIMAL_SELECTION": "SIM_SELECTION_FREE",
"SIM_REFACTOR_FREQ":0,
"SIM_REFORMULATION": "SIM_REFORMULATION_OFF",
"SIM_SAVE_LU":"OFF",
"SIM_SCALING": "SCALING_FREE",
"SIM_SCALING_METHOD": "SCALING_METHOD_POW2",
"SIM_SOLVE_FORM": "SOLVE_FREE",
"SIM_STABILITY_PRIORITY":50,
"SIM_SWITCH_OPTIMIZER":"OFF",
"SOL_FILTER_KEEP_BASIC": "OFF",
"SOL_FILTER_KEEP_RANGED": "OFF",
"SOL_READ_NAME_WIDTH":-1,
"SOL_READ_WIDTH": 1024,
"SOLUTION_CALLBACK": "OFF",
"TIMING_LEVEL":1,
"WRITE_BAS_CONSTRAINTS": "ON",
"WRITE_BAS_HEAD": "ON",
"WRITE_BAS_VARIABLES": "ON",
"WRITE_DATA_COMPRESSED":0,
"WRITE_DATA_FORMAT": "DATA_FORMAT_EXTENSION",
"WRITE_DATA_PARAM": "OFF",
"WRITE_FREE_CON": "OFF",
"WRITE_GENERIC_NAMES": "OFF",
"WRITE_GENERIC_NAMES_IO":1,
"WRITE_IGNORE_INCOMPATIBLE_CONIC_ITEMS": "OFF",
"WRITE_IGNORE_INCOMPATIBLE_ITEMS": "OFF",
"WRITE_IGNORE_INCOMPATIBLE_NL_ITEMS": "OFF"
"WRITE_IGNORE_INCOMPATIBLE_PSD_ITEMS":"OFF",
"WRITE_INT_CONSTRAINTS":"ON",
"WRITE_INT_HEAD":"ON",
"WRITE_INT_VARIABLES": "ON",
"WRITE_LP_FULL_OBJ": "ON",
"WRITE_LP_LINE_WIDTH":80,
"WRITE_LP_QUOTED_NAMES": "ON",
"WRITE_LP_STRICT_FORMAT": "OFF",
"WRITE_LP_TERMS_PER_LINE":10,
"WRITE_MPS_FORMAT": "MPS_FORMAT_FREE",
"WRITE_MPS_INT":"ON",
"WRITE_PRECISION":15,
"WRITE_SOL_BARVARIABLES": "ON",
"WRITE_SOL_CONSTRAINTS": "ON",
"WRITE_SOL_HEAD": "ON",
"WRITE_SOL_IGNORE_INVALID_NAMES": "OFF",
"WRITE_SOL_VARIABLES": "ON",
```

```
"WRITE_TASK_INC_SOL": "ON",
    "WRITE_XML_MODE": "WRITE_XML_MODE_ROW"
},
"dparam":{
    "ANA_SOL_INFEAS_TOL":1e-6,
    "BASIS_REL_TOL_S":1e-12,
    "BASIS_TOL_S":1e-6,
    "BASIS_TOL_X":1e-6,
    "CHECK_CONVEXITY_REL_TOL":1e-10,
    "DATA_TOL_AIJ":1e-12,
    "DATA_TOL_AIJ_HUGE":1e+20,
    "DATA_TOL_AIJ_LARGE":1e+10,
    "DATA_TOL_BOUND_INF":1e+16,
    "DATA_TOL_BOUND_WRN":1e+8,
    "DATA_TOL_C_HUGE":1e+16,
    "DATA_TOL_CJ_LARGE":1e+8,
    "DATA_TOL_QIJ":1e-16,
    "DATA_TOL_X":1e-8,
    "FEASREPAIR_TOL":1e-10,
    "INTPNT_CO_TOL_DFEAS":1e-8,
    "INTPNT_CO_TOL_INFEAS":1e-10,
    "INTPNT_CO_TOL_MU_RED":1e-8,
    "INTPNT_CO_TOL_NEAR_REL":1e+3,
    "INTPNT_CO_TOL_PFEAS":1e-8,
    "INTPNT_CO_TOL_REL_GAP":1e-7,
    "INTPNT_NL_MERIT_BAL":1e-4,
    "INTPNT_NL_TOL_DFEAS":1e-8,
    "INTPNT_NL_TOL_MU_RED":1e-12,
    "INTPNT_NL_TOL_NEAR_REL":1e+3,
    "INTPNT_NL_TOL_PFEAS":1e-8,
    "INTPNT_NL_TOL_REL_GAP":1e-6,
    "INTPNT_NL_TOL_REL_STEP":9.95e-1,
    "INTPNT_QO_TOL_DFEAS":1e-8,
    "INTPNT_QO_TOL_INFEAS":1e-10,
    "INTPNT_QO_TOL_MU_RED":1e-8,
    "INTPNT_QO_TOL_NEAR_REL":1e+3,
    "INTPNT_QO_TOL_PFEAS":1e-8,
    "INTPNT_QO_TOL_REL_GAP":1e-8,
    "INTPNT_TOL_DFEAS":1e-8,
    "INTPNT_TOL_DSAFE":1e+0,
    "INTPNT_TOL_INFEAS":1e-10,
    "INTPNT_TOL_MU_RED":1e-16,
    "INTPNT_TOL_PATH":1e-8,
    "INTPNT_TOL_PFEAS":1e-8,
    "INTPNT_TOL_PSAFE":1e+0,
    "INTPNT_TOL_REL_GAP":1e-8,
    "INTPNT_TOL_REL_STEP":9.999e-1,
    "INTPNT_TOL_STEP_SIZE":1e-6,
    "LOWER_OBJ_CUT":-1e+30,
    "LOWER_OBJ_CUT_FINITE_TRH":-5e+29,
    "MIO_DISABLE_TERM_TIME":-1e+0,
    "MIO_MAX_TIME":-1e+0,
    "MIO_MAX_TIME_APRX_OPT":6e+1,
    "MIO_NEAR_TOL_ABS_GAP":0.0,
    "MIO_NEAR_TOL_REL_GAP":1e-3,
    "MIO_REL_GAP_CONST":1e-10,
    "MIO_TOL_ABS_GAP":0.0,
    "MIO_TOL_ABS_RELAX_INT":1e-5,
    "MIO_TOL_FEAS":1e-6,
    "MIO_TOL_REL_DUAL_BOUND_IMPROVEMENT":0.0,
    "MIO_TOL_REL_GAP":1e-4,
    "MIO_TOL_X":1e-6,
    "OPTIMIZER_MAX_TIME":-1e+0,
```

```
"PRESOLVE_TOL_ABS_LINDEP": 1e-6,
            "PRESOLVE_TOL_AIJ":1e-12,
            "PRESOLVE_TOL_REL_LINDEP":1e-10,
            "PRESOLVE_TOL_S":1e-8,
            "PRESOLVE_TOL_X":1e-8,
            "QCQO_REFORMULATE_REL_DROP_TOL":1e-15,
            "SEMIDEFINITE_TOL_APPROX":1e-10,
            "SIM_LU_TOL_REL_PIV":1e-2,
            "SIMPLEX_ABS_TOL_PIV":1e-7,
            "UPPER_OBJ_CUT":1e+30,
            "UPPER_OBJ_CUT_FINITE_TRH":5e+29
        "sparam":{
            "BAS_SOL_FILE_NAME":"",
            "DATA_FILE_NAME": "examples/tools/data/lo1.mps",
            "DEBUG_FILE_NAME": "",
            "INT_SOL_FILE_NAME":""
            "ITR_SOL_FILE_NAME":"",
            "MIO_DEBUG_STRING":"",
            "PARAM_COMMENT_SIGN":"%%",
            "PARAM_READ_FILE_NAME":"",
            "PARAM_WRITE_FILE_NAME":"",
            "READ_MPS_BOU_NAME":"",
            "READ_MPS_OBJ_NAME":"",
            "READ_MPS_RAN_NAME":"",
            "READ_MPS_RHS_NAME":"",
            "SENSITIVITY_FILE_NAME":"",
            "SENSITIVITY_RES_FILE_NAME":"",
            "SOL_FILTER_XC_LOW":"",
            "SOL_FILTER_XC_UPR":"",
            "SOL_FILTER_XX_LOW":"",
            "SOL_FILTER_XX_UPR":"",
            "STAT_FILE_NAME":"",
            "STAT_KEY":"",
            "STAT_NAME":""
            "WRITE_LP_GEN_VAR_NAME": "XMSKGEN"
        }
   }
}
```

## 14.8 The Solution File Format

MOSEK provides several solution files depending on the problem type and the optimizer used:

- basis solution file (extension .bas) if the problem is optimized using the simplex optimizer or basis identification is performed,
- interior solution file (extension .sol) if a problem is optimized using the interior-point optimizer and no basis identification is required,
- integer solution file (extension .int) if the problem contains integer constrained variables.

All solution files have the format:

INDEX ?	NAME <name></name>	AT ACTIVITY ?? <a value=""></a>	LOWER LIMIT <a value=""></a>	UPPER LIMIT <a value=""></a>	DUAL LOWER <a value=""></a>	DUAL UPPER <a value=""></a>	
VARIAB INDEX → DUAL	NAME	AT ACTIVITY	LOWER LIMIT	UPPER LIMIT	DUAL LOWER	DUAL UPPER	CONIC
?	<name></name>	?? <a value=""></a>	<a value=""></a>	<a value=""></a>	<a value=""></a>	<a value=""></a>	<a value=""></a>

In the example the fields ? and <> will be filled with problem and solution specific information. As can be observed a solution report consists of three sections, i.e.

- HEADER In this section, first the name of the problem is listed and afterwards the problem and solution status are shown. Next the primal and dual objective values are displayed.
- ullet CONSTRAINTS For each constraint i of the form

$$l_i^c \le \sum_{j=1}^n a_{ij} x_j \le u_i^c, \tag{14.10}$$

the following information is listed:

- INDEX: A sequential index assigned to the constraint by MOSEK
- NAME: The name of the constraint assigned by the user.
- AT: The status of the constraint. In Table 14.4 the possible values of the status keys and their interpretation are shown.

	Table 11.1. Status Reys.
Status key	Interpretation
UN	Unknown status
BS	Is basic
SB	Is superbasic
LL	Is at the lower limit (bound)
UL	Is at the upper limit (bound)
EQ	Lower limit is identical to upper limit
**	Is infeasible i.e. the lower limit is greater than the upper limit.

Table 14.4: Status keys.

- ACTIVITY: the quantity  $\sum_{j=1}^{n} a_{ij}x_{j}^{*}$ , where  $x^{*}$  is the value of the primal solution.
- LOWER LIMIT: the quantity  $l_i^c$  (see (14.10).)
- UPPER LIMIT: the quantity  $u_i^c$  (see (14.10).)
- DUAL LOWER: the dual multiplier corresponding to the lower limit on the constraint.
- DUAL UPPER: the dual multiplier corresponding to the upper limit on the constraint.
- VARIABLES The last section of the solution report lists information about the variables. This information has a similar interpretation as for the constraints. However, the column with the header CONIC DUAL is included for problems having one or more conic constraints. This column shows the dual variables corresponding to the conic constraints.

### Example: lo1.sol

228

In Listing 14.7 we show the solution file for the lol.opf problem.

Listing 14.7: An example of .sol file.

```
NAME :
PROBLEM STATUS : PRIMAL_AND_DUAL_FEASIBLE
SOLUTION STATUS : OPTIMAL
OBJECTIVE NAME : obj
```

	8.33333333e+01			
DUAL OBJECTIVE :	8.33333332e+01			
CONSTRAINTS				
INDEX NAME	AT ACTIVITY	LOWER LIMIT	UPPER LIMIT	ш
→DUAL LOWER	DUAL UPPER			
0 c1	EQ 3.0000000000000e+01	3.00000000e+01	3.00000000e+01	-0.
→0000000000000e+00	-2.49999999741654e+00			
1 c2	SB 5.3333333349188e+01	1.50000000e+01	NONE	2.
→09157603759397e-10	-0.000000000000e+00			
2 c3	UL 2.4999999842049e+01	NONE	2.50000000e+01	-0.
→0000000000000e+00	-3.33333332895110e-01			
VARIABLES				
INDEX NAME	AT ACTIVITY	LOWER LIMIT	UPPER LIMIT	⊔
→DUAL LOWER	DUAL UPPER	0.000000000		
0 x1	LL 1.67020427073508e-09	0.0000000e+00	NONE	-4.
→49999999528055e+00	-0.0000000000000e+00		4 00000000 104	
1 x2	LL 2.93510446280504e-09	0.0000000e+00	1.00000000e+01	-2.
→16666666494916e+00	6.20863861687316e-10			
2 x3	SB 1.49999999899425e+01	0.0000000e+00	NONE	-8.
→79123177454657e-10	-0.0000000000000e+00	0.00000000		
3 x4	SB 8.33333332273116e+00	0.0000000e+00	NONE	-1.
→69795978899185e-09	-0.000000000000e+00			

# **FIFTEEN**

# LIST OF EXAMPLES

List of examples shipped in the distribution of Command Line Tools:

Table 15.1: List of distributed examples

File	Description
25fv47.mps	A large linear problem from the Netlib library
ampl1.res	Interfacing MOSEK from AMPL
ampl2.res	Interfacing MOSEK from AMPL
ampl3.res	Interfacing MOSEK from AMPL
cqo1.mps	A simple conic quadratic problem
dgo.f	Nonlinear part of a geometric optimization example dgo.mps
dgo.mps	Linear part of a geometric optimization example
diet.dat	Data for the diet example diet.mod
diet.mod	A diet balancing AMPL example
dinfeas.lp	A simple dual infeasible linear problem
expopt1.eo	Data file with an exponential optimization problem
feasrepair.lp	An example demonstrating repair of infeasible problems
infeas.lp	A simple primal infeasible problem
lo1.mps	A simple linear problem
qo1.mps	A simple quadratic problem
sensitivity.	Sensitivity analysis specification for transport.lp
ssp	
transport.lp	A linear problem in the sensitivity analysis example

Additional examples can be found on the  $\mathbf{MOSEK}$  website and in other  $\mathbf{MOSEK}$  publications.

## SIXTEEN

## **INTERFACE CHANGES**

The section show interface-specific changes to the **MOSEK** Command Line Tools in version 8. See the release notes for general changes and new features of the **MOSEK** Optimization Suite.

# 16.1 Compatibility

## 16.2 Parameters

#### Added

- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL
- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE
- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE
- MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP
- MSK\_DPAR\_SEMIDEFINITE\_TOL\_APPROX
- MSK\_IPAR\_INTPNT\_MULTI\_THREAD
- MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN
- MSK\_IPAR\_LOG\_ANA\_PRO
- MSK\_IPAR\_MIO\_CUT\_CLIQUE
- MSK\_IPAR\_MIO\_CUT\_GMI
- MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND
- MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER
- MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL
- MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE
- MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL
- MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL
- MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_FILL

- MSK\_IPAR\_REMOVE\_UNUSED\_SOLUTIONS
- MSK\_IPAR\_WRITE\_LP\_FULL\_OBJ
- MSK\_IPAR\_WRITE\_MPS\_FORMAT
- MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN

#### Removed

- MSK\_DPAR\_FEASREPAIR\_TOL
- MSK\_DPAR\_MIO\_HEURISTIC\_TIME
- MSK\_DPAR\_MIO\_MAX\_TIME\_APRX\_OPT
- MSK\_DPAR\_MIO\_REL\_ADD\_CUT\_LIMITED
- MSK\_DPAR\_MIO\_TOL\_MAX\_CUT\_FRAC\_RHS
- MSK\_DPAR\_MIO\_TOL\_MIN\_CUT\_FRAC\_RHS
- MSK\_DPAR\_MIO\_TOL\_REL\_RELAX\_INT
- MSK\_DPAR\_MIO\_TOL\_X
- MSK\_DPAR\_NONCONVEX\_TOL\_FEAS
- MSK\_DPAR\_NONCONVEX\_TOL\_OPT
- MSK\_IPAR\_ALLOC\_ADD\_QNZ
- MSK\_IPAR\_CONCURRENT\_NUM\_OPTIMIZERS
- MSK\_IPAR\_CONCURRENT\_PRIORITY\_DUAL\_SIMPLEX
- MSK\_IPAR\_CONCURRENT\_PRIORITY\_FREE\_SIMPLEX
- MSK\_IPAR\_CONCURRENT\_PRIORITY\_INTPNT
- MSK\_IPAR\_CONCURRENT\_PRIORITY\_PRIMAL\_SIMPLEX
- MSK\_IPAR\_FEASREPAIR\_OPTIMIZE
- MSK\_IPAR\_INTPNT\_FACTOR\_DEBUG\_LVL
- MSK\_IPAR\_INTPNT\_FACTOR\_METHOD
- MSK\_IPAR\_LIC\_TRH\_EXPIRY\_WRN
- MSK\_IPAR\_LOG\_CONCURRENT
- MSK\_IPAR\_LOG\_FACTOR
- MSK\_IPAR\_LOG\_HEAD
- MSK\_IPAR\_LOG\_NONCONVEX
- MSK\_IPAR\_LOG\_OPTIMIZER
- MSK\_IPAR\_LOG\_PARAM
- MSK\_IPAR\_LOG\_SIM\_NETWORK\_FREQ
- MSK\_IPAR\_MIO\_BRANCH\_PRIORITIES\_USE
- MSK\_IPAR\_MIO\_CONT\_SOL
- MSK\_IPAR\_MIO\_CUT\_CG
- MSK\_IPAR\_MIO\_CUT\_LEVEL\_ROOT
- MSK\_IPAR\_MIO\_CUT\_LEVEL\_TREE
- MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL

- MSK\_IPAR\_MIO\_HOTSTART
- MSK\_IPAR\_MIO\_KEEP\_BASIS
- MSK\_IPAR\_MIO\_LOCAL\_BRANCH\_NUMBER
- MSK\_IPAR\_MIO\_OPTIMIZER\_MODE
- MSK\_IPAR\_MIO\_PRESOLVE\_AGGREGATE
- MSK\_IPAR\_MIO\_PRESOLVE\_PROBING
- MSK\_IPAR\_MIO\_PRESOLVE\_USE
- MSK\_IPAR\_MIO\_STRONG\_BRANCH
- MSK\_IPAR\_MIO\_USE\_MULTITHREADED\_OPTIMIZER
- MSK\_IPAR\_NONCONVEX\_MAX\_ITERATIONS
- MSK\_IPAR\_PRESOLVE\_ELIM\_FILL
- MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_USE
- MSK\_IPAR\_QO\_SEPARABLE\_REFORMULATION
- MSK\_IPAR\_READ\_ANZ
- MSK\_IPAR\_READ\_CON
- MSK\_IPAR\_READ\_CONE
- MSK\_IPAR\_READ\_MPS\_KEEP\_INT
- MSK\_IPAR\_READ\_MPS\_OBJ\_SENSE
- MSK\_IPAR\_READ\_MPS\_RELAX
- MSK\_IPAR\_READ\_QNZ
- MSK\_IPAR\_READ\_VAR
- MSK\_IPAR\_SIM\_INTEGER
- MSK\_IPAR\_WARNING\_LEVEL
- MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_CONIC\_ITEMS
- MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_NL\_ITEMS
- MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_PSD\_ITEMS
- MSK\_SPAR\_FEASREPAIR\_NAME\_PREFIX
- MSK\_SPAR\_FEASREPAIR\_NAME\_SEPARATOR
- MSK\_SPAR\_FEASREPAIR\_NAME\_WSUMVIOL

# 16.3 Constants

## **Added**

- MSK\_BRANCH\_DIR\_FAR
- MSK\_BRANCH\_DIR\_GUIDED
- MSK\_BRANCH\_DIR\_NEAR
- MSK\_BRANCH\_DIR\_PSEUDOCOST
- MSK\_BRANCH\_DIR\_ROOT\_LP

16.3. Constants 235

- MSK\_CALLBACK\_BEGIN\_ROOT\_CUTGEN
- MSK\_CALLBACK\_BEGIN\_TO\_CONIC
- MSK\_CALLBACK\_END\_ROOT\_CUTGEN
- MSK\_CALLBACK\_END\_TO\_CONIC
- MSK\_CALLBACK\_IM\_ROOT\_CUTGEN
- MSK\_CALLBACK\_SOLVING\_REMOTE
- MSK\_DATA\_FORMAT\_JSON\_TASK
- MSK\_DINF\_MIO\_CLIQUE\_SEPARATION\_TIME
- MSK\_DINF\_MIO\_CMIR\_SEPARATION\_TIME
- MSK\_DINF\_MIO\_GMI\_SEPARATION\_TIME
- MSK\_DINF\_MIO\_IMPLIED\_BOUND\_TIME
- MSK\_DINF\_MIO\_KNAPSACK\_COVER\_SEPARATION\_TIME
- MSK\_DINF\_QCQO\_REFORMULATE\_MAX\_PERTURBATION
- MSK\_DINF\_QCQO\_REFORMULATE\_WORST\_CHOLESKY\_COLUMN\_SCALING
- MSK\_DINF\_QCQO\_REFORMULATE\_WORST\_CHOLESKY\_DIAG\_SCALING
- MSK\_DINF\_SOL\_BAS\_NRM\_BARX
- MSK\_DINF\_SOL\_BAS\_NRM\_SLC
- MSK\_DINF\_SOL\_BAS\_NRM\_SLX
- MSK\_DINF\_SOL\_BAS\_NRM\_SUC
- MSK\_DINF\_SOL\_BAS\_NRM\_SUX
- MSK\_DINF\_SOL\_BAS\_NRM\_XC
- MSK\_DINF\_SOL\_BAS\_NRM\_XX
- MSK\_DINF\_SOL\_BAS\_NRM\_Y
- MSK\_DINF\_SOL\_ITG\_NRM\_BARX
- MSK\_DINF\_SOL\_ITG\_NRM\_XC
- MSK\_DINF\_SOL\_ITG\_NRM\_XX
- MSK\_DINF\_SOL\_ITR\_NRM\_BARS
- MSK\_DINF\_SOL\_ITR\_NRM\_BARX
- MSK\_DINF\_SOL\_ITR\_NRM\_SLC
- MSK\_DINF\_SOL\_ITR\_NRM\_SLX
- MSK\_DINF\_SOL\_ITR\_NRM\_SNX
- MSK\_DINF\_SOL\_ITR\_NRM\_SUC
- MSK\_DINF\_SOL\_ITR\_NRM\_SUX
- MSK\_DINF\_SOL\_ITR\_NRM\_XC
- MSK\_DINF\_SOL\_ITR\_NRM\_XX
- MSK\_DINF\_SOL\_ITR\_NRM\_Y
- MSK\_DINF\_TO\_CONIC\_TIME
- MSK\_IINF\_MIO\_ABSGAP\_SATISFIED
- MSK\_IINF\_MIO\_CLIQUE\_TABLE\_SIZE

- MSK\_IINF\_MIO\_NEAR\_ABSGAP\_SATISFIED
- MSK\_IINF\_MIO\_NEAR\_RELGAP\_SATISFIED
- MSK\_IINF\_MIO\_NODE\_DEPTH
- MSK\_IINF\_MIO\_NUM\_CMIR\_CUTS
- MSK\_IINF\_MIO\_NUM\_IMPLIED\_BOUND\_CUTS
- MSK\_IINF\_MIO\_NUM\_KNAPSACK\_COVER\_CUTS
- MSK\_IINF\_MIO\_NUM\_REPEATED\_PRESOLVE
- MSK\_IINF\_MIO\_PRESOLVED\_NUMBIN
- MSK\_IINF\_MIO\_PRESOLVED\_NUMCON
- MSK\_IINF\_MIO\_PRESOLVED\_NUMCONT
- MSK\_IINF\_MIO\_PRESOLVED\_NUMINT
- MSK\_IINF\_MIO\_PRESOLVED\_NUMVAR
- MSK\_IINF\_MIO\_RELGAP\_SATISFIED
- MSK\_LIINF\_MIO\_PRESOLVED\_ANZ
- MSK\_LIINF\_MIO\_SIM\_MAXITER\_SETBACKS
- MSK\_MPS\_FORMAT\_CPLEX
- MSK\_SOL\_STA\_DUAL\_ILLPOSED\_CER
- MSK\_SOL\_STA\_PRIM\_ILLPOSED\_CER

### Changed

- MSK\_SOL\_STA\_INTEGER\_OPTIMAL
- MSK\_SOL\_STA\_NEAR\_DUAL\_FEAS
- MSK\_SOL\_STA\_NEAR\_DUAL\_INFEAS\_CER
- MSK\_SOL\_STA\_NEAR\_INTEGER\_OPTIMAL
- MSK\_SOL\_STA\_NEAR\_OPTIMAL
- MSK\_SOL\_STA\_NEAR\_PRIM\_AND\_DUAL\_FEAS
- MSK\_SOL\_STA\_NEAR\_PRIM\_FEAS
- MSK\_SOL\_STA\_NEAR\_PRIM\_INFEAS\_CER
- MSK\_LICENSE\_BUFFER\_LENGTH

### Removed

- MSK\_CALLBACKCODE\_BEGIN\_CONCURRENT
- MSK\_CALLBACKCODE\_BEGIN\_NETWORK\_DUAL\_SIMPLEX
- MSK\_CALLBACKCODE\_BEGIN\_NETWORK\_PRIMAL\_SIMPLEX
- MSK\_CALLBACKCODE\_BEGIN\_NETWORK\_SIMPLEX
- MSK\_CALLBACKCODE\_BEGIN\_NONCONVEX
- MSK\_CALLBACKCODE\_BEGIN\_PRIMAL\_DUAL\_SIMPLEX
- MSK\_CALLBACKCODE\_BEGIN\_PRIMAL\_DUAL\_SIMPLEX\_BI

16.3. Constants 237

- MSK\_CALLBACKCODE\_BEGIN\_SIMPLEX\_NETWORK\_DETECT
- MSK\_CALLBACKCODE\_END\_CONCURRENT
- MSK\_CALLBACKCODE\_END\_NETWORK\_DUAL\_SIMPLEX
- MSK\_CALLBACKCODE\_END\_NETWORK\_PRIMAL\_SIMPLEX
- MSK\_CALLBACKCODE\_END\_NETWORK\_SIMPLEX
- MSK\_CALLBACKCODE\_END\_NONCONVEX
- MSK\_CALLBACKCODE\_END\_PRIMAL\_DUAL\_SIMPLEX
- MSK\_CALLBACKCODE\_END\_PRIMAL\_DUAL\_SIMPLEX\_BI
- MSK\_CALLBACKCODE\_END\_SIMPLEX\_NETWORK\_DETECT
- MSK\_CALLBACKCODE\_IM\_MIO\_PRESOLVE
- MSK\_CALLBACKCODE\_IM\_NETWORK\_DUAL\_SIMPLEX
- MSK\_CALLBACKCODE\_IM\_NETWORK\_PRIMAL\_SIMPLEX
- MSK\_CALLBACKCODE\_IM\_NONCONVEX
- MSK\_CALLBACKCODE\_IM\_PRIMAL\_DUAL\_SIMPLEX
- MSK\_CALLBACKCODE\_NONCOVEX
- MSK\_CALLBACKCODE\_UPDATE\_NETWORK\_DUAL\_SIMPLEX
- MSK\_CALLBACKCODE\_UPDATE\_NETWORK\_PRIMAL\_SIMPLEX
- MSK\_CALLBACKCODE\_UPDATE\_NONCONVEX
- MSK\_CALLBACKCODE\_UPDATE\_PRIMAL\_DUAL\_SIMPLEX
- MSK\_CALLBACKCODE\_UPDATE\_PRIMAL\_DUAL\_SIMPLEX\_BI
- MSK\_DINFITEM\_BI\_CLEAN\_PRIMAL\_DUAL\_TIME
- MSK\_DINFITEM\_CONCURRENT\_TIME
- MSK\_DINFITEM\_MIO\_CG\_SEPERATION\_TIME
- MSK\_DINFITEM\_MIO\_CMIR\_SEPERATION\_TIME
- MSK\_DINFITEM\_SIM\_NETWORK\_DUAL\_TIME
- MSK\_DINFITEM\_SIM\_NETWORK\_PRIMAL\_TIME
- MSK\_DINFITEM\_SIM\_NETWORK\_TIME
- MSK\_DINFITEM\_SIM\_PRIMAL\_DUAL\_TIME
- MSK\_FEATURE\_PTOM
- MSK\_FEATURE\_PTOX
- MSK\_IINFITEM\_CONCURRENT\_FASTEST\_OPTIMIZER
- MSK\_IINFITEM\_MIO\_NUM\_BASIS\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_CARDGUB\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_COEF\_REDC\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_CONTRA\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_DISAGG\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_FLOW\_COVER\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_GCD\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_GUB\_COVER\_CUTS

- MSK\_IINFITEM\_MIO\_NUM\_KNAPSUR\_COVER\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_LATTICE\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_LIFT\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_OBJ\_CUTS
- MSK\_IINFITEM\_MIO\_NUM\_PLAN\_LOC\_CUTS
- MSK\_IINFITEM\_SIM\_NETWORK\_DUAL\_DEG\_ITER
- MSK\_IINFITEM\_SIM\_NETWORK\_DUAL\_HOTSTART
- MSK\_IINFITEM\_SIM\_NETWORK\_DUAL\_HOTSTART\_LU
- MSK\_IINFITEM\_SIM\_NETWORK\_DUAL\_INF\_ITER
- MSK\_IINFITEM\_SIM\_NETWORK\_DUAL\_ITER
- MSK\_IINFITEM\_SIM\_NETWORK\_PRIMAL\_DEG\_ITER
- MSK\_IINFITEM\_SIM\_NETWORK\_PRIMAL\_HOTSTART
- MSK\_IINFITEM\_SIM\_NETWORK\_PRIMAL\_HOTSTART\_LU
- MSK\_IINFITEM\_SIM\_NETWORK\_PRIMAL\_INF\_ITER
- MSK\_IINFITEM\_SIM\_NETWORK\_PRIMAL\_ITER
- MSK\_IINFITEM\_SIM\_PRIMAL\_DUAL\_DEG\_ITER
- MSK\_IINFITEM\_SIM\_PRIMAL\_DUAL\_HOTSTART
- MSK\_IINFITEM\_SIM\_PRIMAL\_DUAL\_HOTSTART\_LU
- MSK\_IINFITEM\_SIM\_PRIMAL\_DUAL\_INF\_ITER
- MSK\_IINFITEM\_SIM\_PRIMAL\_DUAL\_ITER
- MSK\_IINFITEM\_SOL\_INT\_PROSTA
- MSK\_IINFITEM\_SOL\_INT\_SOLSTA
- MSK\_IINFITEM\_STO\_NUM\_A\_CACHE\_FLUSHES
- MSK\_IINFITEM\_STO\_NUM\_A\_TRANSPOSES
- MSK\_LIINFITEM\_BI\_CLEAN\_PRIMAL\_DUAL\_DEG\_ITER
- MSK\_LIINFITEM\_BI\_CLEAN\_PRIMAL\_DUAL\_ITER
- MSK\_LIINFITEM\_BI\_CLEAN\_PRIMAL\_DUAL\_SUB\_ITER
- MSK\_MIOMODE\_LAZY
- MSK\_OPTIMIZERTYPE\_CONCURRENT
- MSK\_OPTIMIZERTYPE\_MIXED\_INT\_CONIC
- MSK\_OPTIMIZERTYPE\_NETWORK\_PRIMAL\_SIMPLEX
- MSK\_OPTIMIZERTYPE\_NONCONVEX
- MSK\_OPTIMIZERTYPE\_PRIMAL\_DUAL\_SIMPLEX

# 16.4 Response Codes

### Added

• MSK\_RES\_ERR\_CBF\_DUPLICATE\_PSDVAR

- MSK\_RES\_ERR\_CBF\_INVALID\_PSDVAR\_DIMENSION
- MSK\_RES\_ERR\_CBF\_TOO\_FEW\_PSDVAR
- MSK\_RES\_ERR\_DUPLICATE\_AIJ
- MSK\_RES\_ERR\_FINAL\_SOLUTION
- MSK\_RES\_ERR\_JSON\_DATA
- MSK\_RES\_ERR\_JSON\_FORMAT
- MSK\_RES\_ERR\_JSON\_MISSING\_DATA
- MSK\_RES\_ERR\_JSON\_NUMBER\_OVERFLOW
- MSK\_RES\_ERR\_JSON\_STRING
- MSK\_RES\_ERR\_JSON\_SYNTAX
- MSK\_RES\_ERR\_LAU\_INVALID\_LOWER\_TRIANGULAR\_MATRIX
- MSK\_RES\_ERR\_LAU\_INVALID\_SPARSE\_SYMMETRIC\_MATRIX
- MSK\_RES\_ERR\_LAU\_NOT\_POSITIVE\_DEFINITE
- MSK\_RES\_ERR\_MIXED\_CONIC\_AND\_NL
- MSK\_RES\_ERR\_SERVER\_CONNECT
- MSK\_RES\_ERR\_SERVER\_PROTOCOL
- MSK\_RES\_ERR\_SERVER\_STATUS
- MSK\_RES\_ERR\_SERVER\_TOKEN
- MSK\_RES\_ERR\_SYM\_MAT\_HUGE
- MSK\_RES\_ERR\_SYM\_MAT\_INVALID
- MSK\_RES\_ERR\_TASK\_WRITE
- MSK\_RES\_ERR\_TOCONIC\_CONSTR\_NOT\_CONIC
- MSK\_RES\_ERR\_TOCONIC\_CONSTR\_Q\_NOT\_PSD
- MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_FX
- MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_RA
- MSK\_RES\_ERR\_TOCONIC\_OBJECTIVE\_NOT\_PSD
- MSK\_RES\_WRN\_SYM\_MAT\_LARGE

#### Removed

- MSK\_RES\_ERR\_AD\_INVALID\_OPERAND
- MSK\_RES\_ERR\_AD\_INVALID\_OPERATOR
- MSK\_RES\_ERR\_AD\_MISSING\_OPERAND
- MSK\_RES\_ERR\_AD\_MISSING\_RETURN
- MSK\_RES\_ERR\_CONCURRENT\_OPTIMIZER
- MSK\_RES\_ERR\_INV\_CONIC\_PROBLEM
- MSK\_RES\_ERR\_INVALID\_BRANCH\_DIRECTION
- MSK\_RES\_ERR\_INVALID\_BRANCH\_PRIORITY
- MSK\_RES\_ERR\_INVALID\_NETWORK\_PROBLEM
- MSK\_RES\_ERR\_MBT\_INCOMPATIBLE

- MSK\_RES\_ERR\_MBT\_INVALID
- MSK\_RES\_ERR\_MIO\_NOT\_LOADED
- MSK\_RES\_ERR\_MIXED\_PROBLEM
- MSK\_RES\_ERR\_NO\_DUAL\_INFO\_FOR\_ITG\_SOL
- MSK\_RES\_ERR\_ORD\_INVALID
- MSK\_RES\_ERR\_ORD\_INVALID\_BRANCH\_DIR
- MSK\_RES\_ERR\_TOCONIC\_CONVERSION\_FAIL
- MSK\_RES\_ERR\_TOO\_MANY\_CONCURRENT\_TASKS
- MSK\_RES\_WRN\_TOO\_MANY\_THREADS\_CONCURRENT

## **BIBLIOGRAPHY**

- [AA95] E. D. Andersen and K. D. Andersen. Presolving in linear programming. *Math. Programming*, 71(2):221–245, 1995.
- [AGMX96] E. D. Andersen, J. Gondzio, Cs. Mészáros, and X. Xu. Implementation of interior point methods for large scale linear programming. In T. Terlaky, editor, *Interior-point methods of mathematical programming*, pages 189–252. Kluwer Academic Publishers, 1996.
- [ART03] E. D. Andersen, C. Roos, and T. Terlaky. On implementing a primal-dual interior-point method for conic quadratic optimization. *Math. Programming*, February 2003.
- [AY96] E. D. Andersen and Y. Ye. Combining interior-point and pivoting algorithms. *Management Sci.*, 42(12):1719–1731, December 1996.
- [AY98] E. D. Andersen and Y. Ye. A computational study of the homogeneous algorithm for large-scale convex optimization. *Computational Optimization and Applications*, 10:243–269, 1998.
- [AY99] E. D. Andersen and Y. Ye. On a homogeneous algorithm for the monotone complementarity problem. *Math. Programming*, 84(2):375–399, February 1999.
- [And09] Erling D. Andersen. The homogeneous and self-dual model and algorithm for linear optimization. Technical Report TR-1-2009, MOSEK ApS, 2009. URL: http://docs.mosek.com/whitepapers/homolo.pdf.
- [And13] Erling D. Andersen. On formulating quadratic functions in optimization models. Technical Report TR-1-2013, MOSEK ApS, 2013. Last revised 23-feb-2016. URL: http://docs.mosek.com/whitepapers/qmodel.pdf.
- [Chv83] V. Chvátal. Linear programming. W.H. Freeman and Company, 1983.
- [FGK03] R. Fourer, D. M. Gay, and B. W. Kernighan. AMPL. A modeling language for mathematical programming. Thomson, 2nd edition, 2003.
- [Naz87] J. L. Nazareth. Computer Solution of Linear Programs. Oxford University Press, New York, 1987.
- [RTV97] C. Roos, T. Terlaky, and J. -Ph. Vial. Theory and algorithms for linear optimization: an interior point approach. John Wiley and Sons, New York, 1997.
- [Wal00] S. W. Wallace. Decision making under uncertainty: is sensitivity of any use. *Oper. Res.*, 48(1):20–25, January 2000.
- [Wol98] L. A. Wolsey. Integer programming. John Wiley and Sons, 1998.
- [MOSEKApS12] MOSEK ApS. *The MOSEK Modeling Cookbook*. MOSEK ApS, Fruebjergvej 3, Boks 16, 2100 Copenhagen O, 2012. URL: https://docs.mosek.com/modeling-cookbook/index.html.

244 Bibliography

# **SYMBOL INDEX**

Functions	MSK_DPAR_INTPNT_TOL_PSAFE, 96		
Parameters	MSK_DPAR_INTPNT_TOL_REL_GAP, 97 MSK_DPAR_INTPNT_TOL_REL_STEP, 97		
Double parameters, 91	MSK_DPAR_INTPNT_TOL_STEP_SIZE, 97		
MSK_DPAR_ANA_SOL_INFEAS_TOL, 91	MSK_DPAR_LOWER_OBJ_CUT, 97		
MSK_DPAR_BASIS_REL_TOL_S, 91	MSK_DPAR_LOWER_OBJ_CUT_FINITE_TRH, 97		
MSK_DPAR_BASIS_TOL_S, 91	MSK_DPAR_MIO_DISABLE_TERM_TIME, 97		
MSK_DPAR_BASIS_TOL_X, 91	MSK_DPAR_MIO_MAX_TIME, 98		
MSK_DPAR_CHECK_CONVEXITY_REL_TOL, 91	MSK_DPAR_MIO_NEAR_TOL_ABS_GAP, 98		
MSK_DPAR_DATA_SYM_MAT_TOL, 91	MSK_DPAR_MIO_NEAR_TOL_REL_GAP, 98		
MSK_DPAR_DATA_SYM_MAT_TOL_HUGE, 92	MSK_DPAR_MIO_REL_GAP_CONST, 98		
MSK_DPAR_DATA_SYM_MAT_TOL_LARGE, 92	MSK_DPAR_MIO_TOL_ABS_GAP, 98		
MSK_DPAR_DATA_TOL_AIJ, 92	MSK_DPAR_MIO_TOL_ABS_RELAX_INT, 98		
MSK_DPAR_DATA_TOL_AIJ_HUGE, 92	MSK_DPAR_MIO_TOL_FEAS, 99		
MSK_DPAR_DATA_TOL_AIJ_LARGE, 92	${\tt MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT},$		
MSK_DPAR_DATA_TOL_BOUND_INF, 92	99		
MSK_DPAR_DATA_TOL_BOUND_WRN, 92	MSK_DPAR_MIO_TOL_REL_GAP, 99		
MSK_DPAR_DATA_TOL_C_HUGE, 92	MSK_DPAR_OPTIMIZER_MAX_TIME, 99		
MSK_DPAR_DATA_TOL_CJ_LARGE, 93	MSK_DPAR_PRESOLVE_TOL_ABS_LINDEP, 99		
MSK_DPAR_DATA_TOL_QIJ, 93	MSK_DPAR_PRESOLVE_TOL_AIJ, 99		
MSK_DPAR_DATA_TOL_X, 93	MSK_DPAR_PRESOLVE_TOL_REL_LINDEP, 99		
MSK_DPAR_INTPNT_CO_TOL_DFEAS, 93	MSK_DPAR_PRESOLVE_TOL_S, 99		
MSK_DPAR_INTPNT_CO_TOL_INFEAS, 93	MSK_DPAR_PRESOLVE_TOL_X, 100		
MSK_DPAR_INTPNT_CO_TOL_MU_RED, 93	MSK_DPAR_QCQO_REFORMULATE_REL_DROP_TOL, 100		
MSK_DPAR_INTPNT_CO_TOL_NEAR_REL, 93	MSK_DPAR_SEMIDEFINITE_TOL_APPROX, 100		
MSK_DPAR_INTPNT_CO_TOL_PFEAS, 94	MSK_DPAR_SIM_LU_TOL_REL_PIV, 100		
MSK_DPAR_INTPNT_CO_TOL_REL_GAP, 94	MSK_DPAR_SIMPLEX_ABS_TOL_PIV, 100		
MSK_DPAR_INTPNT_NL_MERIT_BAL, 94	MSK_DPAR_UPPER_OBJ_CUT, 100		
MSK_DPAR_INTPNT_NL_TOL_DFEAS, 94	MSK_DPAR_UPPER_OBJ_CUT_FINITE_TRH, 100		
MSK_DPAR_INTPNT_NL_TOL_MU_RED, 94	Integer parameters, 101		
MSK_DPAR_INTPNT_NL_TOL_NEAR_REL, 94	MSK_IPAR_ANA_SOL_BASIS, 101		
MSK_DPAR_INTPNT_NL_TOL_PFEAS, 94	MSK_IPAR_ANA_SOL_PRINT_VIOLATED, 101		
MSK_DPAR_INTPNT_NL_TOL_REL_GAP, 95	MSK_IPAR_AUTO_SORT_A_BEFORE_OPT, 101		
MSK_DPAR_INTPNT_NL_TOL_REL_STEP, 95	MSK_IPAR_AUTO_UPDATE_SOL_INFO, 101		
MSK_DPAR_INTPNT_QO_TOL_DFEAS, 95	MSK_IPAR_BASIS_SOLVE_USE_PLUS_ONE, 101		
MSK_DPAR_INTPNT_QO_TOL_INFEAS, 95	MSK_IPAR_BI_CLEAN_OPTIMIZER, 101		
MSK_DPAR_INTPNT_QO_TOL_MU_RED, 95	MSK_IPAR_BI_IGNORE_MAX_ITER, 101		
MSK_DPAR_INTPNT_QO_TOL_NEAR_REL, 95	MSK_IPAR_BI_IGNORE_NUM_ERROR, 102		
MSK_DPAR_INTPNT_QO_TOL_PFEAS, 95	MSK_IPAR_BI_MAX_ITERATIONS, 102		
MSK_DPAR_INTPNT_QO_TOL_REL_GAP, 96	MSK_IPAR_CACHE_LICENSE, 102		
MSK_DPAR_INTPNT_TOL_DFEAS, 96	MSK_IPAR_CHECK_CONVEXITY, 102		
MSK_DPAR_INTPNT_TOL_DSAFE, 96	MSK_IPAR_COMPRESS_STATFILE, 102		
MSK_DPAR_INTPNT_TOL_INFEAS, 96	MSK_IPAR_INFEAS_GENERIC_NAMES, 102		
MSK_DPAR_INTPNT_TOL_MU_RED, 96	MSK_IPAR_INFEAS_PREFER_PRIMAL, 102		
MSK_DPAR_INTPNT_TOL_PATH, 96	MSK_IPAR_INFEAS_REPORT_AUTO, 103		
MSK_DPAR_INTPNT_TOL_PFEAS, 96	MSK_IPAR_INFEAS_REPORT_LEVEL, 103		

MSK_IPAR_INTPNT_DIFS_TIFER_103 MSK_IPAR_INTPNT_DIFS_TIFER_103 MSK_IPAR_INTPNT_MAX_NUK_COR, 103 MSK_IPAR_INTPNT_MAX_NUK_COR, 104 MSK_IPAR_INTPNT_OFF_COL_TRH, 105 MSK_		
MSK_IPAR_INTPNT_MAX_NUM_COR, 103 MSK_IPAR_INTPNT_MAX_NUM_COR, 103 MSK_IPAR_INTPNT_MAX_NUM_CREPIREMENT_STEPS, 104 MSK_IPAR_INTPNT_MAX_NUM_REPIREMENT_STEPS, 104 MSK_IPAR_INTPNT_MIX_NUM_REPIREMENT_STEPS, 104 MSK_IPAR_INTPNT_MIX_NUM_REPIREMENT_STEPS, 104 MSK_IPAR_INTPNT_MIX_NUM_REPIREMENT_STEPS, 104 MSK_IPAR_INTPNT_GOL_TRR, 104 MSK_IPAR_INTPNT_GOL_TRR, 104 MSK_IPAR_INTPNT_GOL_TRR, 104 MSK_IPAR_INTPNT_GOL_TRR, 104 MSK_IPAR_INTPNT_SOLVE_FORM, 104 MSK_IPAR_INTPNT_SOLVE_FORM, 104 MSK_IPAR_INTPNT_SOLVE_FORM, 104 MSK_IPAR_INTPNT_SOLVE_FORM, 104 MSK_IPAR_INTPNT_SOLVE_FORM, 105 M	MSK_IPAR_INTPNT_BASIS, 103	MSK_IPAR_MIO_PERSPECTIVE_REFORMULATE, 111
MSK_IPAR_INTPNT_MAX_NUM_REFINEMENT_STEPS,   104   MSK_IPAR_INTPNT_MAX_NUM_REFINEMENT_STEPS,   104   MSK_IPAR_INTPNT_MULTI_THREAD, 104   MSK_IPAR_INTPNT_MULTI_THREAD, 104   MSK_IPAR_INTPNT_MULTI_THREAD, 104   MSK_IPAR_INTPNT_ORDER_METHOD, 104   MSK_IPAR_INTPNT_ORDER_METHOD, 104   MSK_IPAR_INTPNT_GROUGH_REFINEMENT_STEPS,   105   MSK_IPAR_INTPNT_SGULAN_INTER_NOS_   105   MSK_IPAR_INTPNT_SGULAN_INTER_NOS_   105   MSK_IPAR_INTPNT_SGULAN_INTER_NOS_   105   MSK_IPAR_INTER_STEP_NOS_   105   MSK_IPAR_INTER_SCULAN_INTER_NOS_   106	MSK_IPAR_INTPNT_DIFF_STEP, 103	MSK_IPAR_MIO_PROBING_LEVEL, 111
MSK_IPAR_INTPNT_MAX_NUM_REFINEMENT_STEPS,   104	MSK_IPAR_INTPNT_HOTSTART, 103	MSK_IPAR_MIO_RINS_MAX_NODES, 112
MSK_IPAR_INTPNT_MAX_NUM_CGR. 103   MSK_IPAR_INTPNT_MAX_NUM_REFINEMENT_STEPS, 104   MSK_IPAR_INTPNT_MINIT_ITREAD, 104   MSK_IPAR_INTPNT_OFF_COL_TEH, 104   MSK_IPAR_INTPNT_SOLING, 104   MSK_IPAR_INTPNT_SOLING, 104   MSK_IPAR_INTPNT_SOLING, 105   MSK_IPAR_INTPNT_SOLING, 105   MSK_IPAR_INTPNT_SOLING, 106   MSK_IPAR_INTENT_STARTING_POINT, 105   MSK_IPAR_INTENT_STARTING_POINT, 105   MSK_IPAR_INTENT_SOLING, 106   MSK_IPAR_INTENT_SOLING, 107   MSK_IPAR_INTENT_SOLING, 106   MSK_IPAR_INTENT_SOLING, 107   MSK_IPAR_	MSK_IPAR_INTPNT_MAX_ITERATIONS, 103	
INSK_IPAR_INTPNT_MAX_NUM_REFINEMENT_STEPS,   104		
MSK_IPAR_INTPNT_MULTI_THREAD, 104   MSK_IPAR_INTPNT_MULTI_THREAD, 104   MSK_IPAR_INTPNT_OFF_COL_TRI, 104   MSK_IPAR_INTPNT_OFF_COL_TRI, 104   MSK_IPAR_INTPNT_OFF_COL_TRI, 104   MSK_IPAR_INTPNT_OFF_COL_TRI, 104   MSK_IPAR_INTPNT_GOBER_METHOD, 104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_104   MSK_IPAR_INTPNT_SOLENG_FORN_105   MSK_IPAR_ICCENSE_DEBUG_I.05   MSK_IPAR_ICCENSE_DEBUG_I.05   MSK_IPAR_ICCENSE_DEBUG_I.05   MSK_IPAR_ICCENSE_DEBUG_I.05   MSK_IPAR_ICCENSE_DEBUG_I.05   MSK_IPAR_ICCENSE_DEBUG_I.06   MSK_IPAR_LOG_ONER_MATT, 105   MSK_IPAR_LOG_ONER_MATT, 105   MSK_IPAR_LOG_ONER_MATT, 105   MSK_IPAR_LOG_ONER_MATT, 105   MSK_IPAR_LOG_ONER_MATT, 105   MSK_IPAR_LOG_ONER_MATT, 105   MSK_IPAR_DOF_WRITE_SOL_BAS, 113   MSK_IPAR_OFF_WRITE_SOL_BAS, 113   MSK_IPAR_DOFF_WRITE_SOL_BAS, 113   MSK_IPAR_MOFF_WRITE_SOL_BAS, 113   MSK_IPAR_MOFF_WRITE_SOL_BAS, 113   MSK_IPAR_MAN_TREAS_IND_MAN_IND_ARCAS_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN_INDMAN		
MSK_IPAR_INTPNT_UNLTI_THREAD, 104		
MSK_IPAR_INTPNT_OFF_COL_TRH, 104   MSK_IPAR_INTPNT_REGULARIZATION_USE, 104   MSK_IPAR_INTPNT_SCALING, 104   MSK_IPAR_OFF_WRITE_PARAMETERS, 113   MSK_IPAR_LOFF_WRITE_SCALING, 113   MSK_IPAR_LOFF_WRITE_SCALING, 113   MSK_IPAR_LOFF_WRITE_SCALING, 113   MSK_IPAR_LOFF_WRITE_SCALING, 113   MSK_IPAR_OFF_WRITE_SCALING, 113   MSK_IPAR_MREAD_OFF_WRITE_SCALING, 113   MSK_IPAR_MREAD_OFF_WRITE_SCALING, 113   MSK_IPAR_MREAD_ING, 114		
MSK_IPAR_INTPNT_GROURE,METHOD, 104   MSK_IPAR_OPF_MAX_TERMS_PER_LINE, 113		
MSK_IPAR_INTPNT_REGULARIZATION_USE, 104   MSK_IPAR_INTPNT_SCALING, 104   MSK_IPAR_INTPNT_SCALING, 104   MSK_IPAR_INTPNT_SCALING, 104   MSK_IPAR_INTPNT_SCALING, 104   MSK_IPAR_OPF_WRITE_PARAMETERS, 113   MSK_IPAR_OPF_WRITE_PARAMETERS, 113   MSK_IPAR_OPF_WRITE_PARAMETERS, 113   MSK_IPAR_OPF_WRITE_PARAMETERS, 113   MSK_IPAR_OPF_WRITE_PARAMETERS, 113   MSK_IPAR_OPF_WRITE_SCAL_DAS, 114   MSK_IPAR_DAS, 114   MSK_IPAR_DAS, 114   MSK_IPAR_DAS, 114   MSK_IPAR_DAS, 114   MSK_IPAR_PARAM_READ_CASE_MAN, 114   MSK_IPAR_PARAM_READ_CASE_MAN, 114   MSK_IPAR_PARAM_READ_CASE_MAN, 115   MSK_IPAR_PARAM_READ_CASE_MAN, 115   MSK_IPAR_PARESOLVE_LINDEP_ABS_WORK_TH, 115   MSK_IPAR_PARESOLVE_LINDEP_MAN, 115   MSK_IPAR_PARESOLVE_LINDEP_MAN, 115   MSK_IPAR_PARESOLVE_LINDEP_MAN, 115   MSK_IPAR_PARESOLVE_LINDEP_MAN, 115   MSK_IPAR_PARESOLVE_LINDEP_MAN, 115   MSK_IPAR_PARESOLVE_LINDEP_MAN, 115   MSK_IPAR_READ_DAS, 115   MSK_IPAR_READ_DAS, 115   MSK_IPAR_READ_DAS, 115   MSK_IPAR_READ_DAS, 115   MSK_IPAR_READ_DAS, 115   MSK_IPAR_READ_DAS, 115   MSK_IPAR_MAN, 115   MSK_IPAR_MID_CUT_CIDUE, 110   MSK_IPAR_MID_CUT_CIDUE, 110   MSK_IPAR_MID_CUT_CIDUE, 110		
MSK_IPAR_INTPHT_SCALING, 104   MSK_IPAR_OPF_WRITE_PARAMETERS, 113		
MSK_IPAR_INTPHT_SOLVE_FORM. 104         MSK_IPAR_INTPHT_SOLVE_FORM. 104         MSK_IPAR_INTPHT_STARTING_POINT, 105         MSK_IPAR_LICENSE_DEBUG, 105         MSK_IPAR_LICENSE_DEBUG, 105         MSK_IPAR_LICENSE_DEBUG, 105         MSK_IPAR_OPF_WRITE_SOL_BAS, 113         MSK_IPAR_LICENSE_DAUSE_TIME_IO5         MSK_IPAR_OPF_WRITE_SOL_DAS, 113         MSK_IPAR_DAS, 113         MSK_IPAR_DAS, 114		
MSK_IPAR_LICENSE_DEBUG, 105	MSK_IPAR_INTPNT_SCALING, 104	MSK_IPAR_OPF_WRITE_HINTS, 113
MSK_IPAR_LICENSE_DAUSE_TIME, 105	MSK_IPAR_INTPNT_SOLVE_FORM, 104	MSK_IPAR_OPF_WRITE_PARAMETERS, 113
MSK_IPAR_LICENSE_PAUSE_TINE, 105         MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS, 105         MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS, 105         MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS, 105         MSK_IPAR_LICENSE_THAL_EXPIRY_WRN, 105         MSK_IPAR_DOF_WRITE_SOL_ITR, 114         MSK_IPAR_LOG_CITENT, 104         MSK_IPAR_DOF_WRITE_SOL_TITS, 114         MSK_IPAR_LOG_CITS.         MSK_IPAR_DOF_WRITE_SOL_TITS, 114         MSK_IPAR_PRESOLVE_LINDS, 114         MSK_IPAR_PRESOLVE_LINDS, 114         MSK_IPAR_PRESOLVE_LINDS, 115         MSK_IPAR_PRESOLVE_LINDSP_ABS_WORK_TRH, 115         MSK_IPAR_PRES	MSK_IPAR_INTPNT_STARTING_POINT, 105	MSK_IPAR_OPF_WRITE_PROBLEM, 113
MSK_IPAR_LICENSE_PAUSE_TINE, 105         MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS, 105         MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS, 105         MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRNS, 105         MSK_IPAR_LICENSE_THAL_EXPIRY_WRN, 105         MSK_IPAR_DOF_WRITE_SOL_ITR, 114         MSK_IPAR_LOG_CITENT, 104         MSK_IPAR_DOF_WRITE_SOL_TITS, 114         MSK_IPAR_LOG_CITS.         MSK_IPAR_DOF_WRITE_SOL_TITS, 114         MSK_IPAR_PRESOLVE_LINDS, 114         MSK_IPAR_PRESOLVE_LINDS, 114         MSK_IPAR_PRESOLVE_LINDS, 115         MSK_IPAR_PRESOLVE_LINDSP_ABS_WORK_TRH, 115         MSK_IPAR_PRES	MSK_IPAR_LICENSE_DEBUG, 105	MSK_IPAR_OPF_WRITE_SOL_BAS, 113
MSK_IPAR_LICENSE_SUPPRESS_EXPIRE_WRN, 105         MSK_IPAR_OFF_WRITE_SOLL_ITR, 114           MSK_IPAR_LICENSE_TRH_EXPIRY_WRN, 105         MSK_IPAR_OFF_WRITE_SOLLUTIONS, 114           MSK_IPAR_LOG, 105         MSK_IPAR_POFF_WRITE_SOLLUTIONS, 114           MSK_IPAR_LOG_ANA_PRO, 106         MSK_IPAR_PRESOLVE_CASE_NAME, 114           MSK_IPAR_LOG_BI, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL, 114           MSK_IPAR_LOG_CBI_FREQ, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL, 114           MSK_IPAR_LOG_CHECK_CONVEXITY, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL, 114           MSK_IPAR_LOG_COVI_SECOND_OPT, 106         MSK_IPAR_PRESOLVE_LIMINATOR_MAX_NUM_TRIES,           MSK_IPAR_LOG_COVI_SECOND_OPT, 106         MSK_IPAR_PRESOLVE_LIMINATOR_MAX_NUM_TRIES,           MSK_IPAR_LOG_GEREARIR, 106         MSK_IPAR_PRESOLVE_LIMINATOR_MAX_NUM_TRIES,           MSK_IPAR_LOG_GEREARIR, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_IPARE, 107         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_INITER, 107         MSK_IPAR_PRESOLVE_USE, 115           MSK_IPAR_LOG_MIO, 107         MSK_IPAR_READ_CORNAT, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LOG_	MSK_IPAR_LICENSE_PAUSE_TIME, 105	MSK_IPAR_OPF_WRITE_SOL_ITG, 113
MSK_IPAR_LICENSE_TRH_EXPLITY_WRN, 105		
MSK_IPAR_LICENSE_WAIT, 105         MSK_IPAR_DG, 105         MSK_IPAR_DG, 105         MSK_IPAR_DG, 105         MSK_IPAR_PARAM_READ_CASE_NAME, 114           MSK_IPAR_LOG_BI, 106         MSK_IPAR_PARAM_READ_IGA_ERROR, 114         MSK_IPAR_LOG_BI, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL, 114           MSK_IPAR_LOG_CHEK_CONVEXITY, 106         MSK_IPAR_LOG_CUT_SECOND_OPT, 106         MSK_IPAR_PRESOLVE_LINDEP_ABX_NUM_TRIES, 114           MSK_IPAR_LOG_CUT_SECOND_OPT, 106         MSK_IPAR_PRESOLVE_LINDEP_ABX_WORK_TRIH, 115           MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_PRESOLVE_LINDEP_ABX_WORK_TRIH, 115           MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_PRESOLVE_LINDEP_ABX_WORK_TRIH, 115           MSK_IPAR_LOG_INTEAS_ANA, 107         MSK_IPAR_PRESOLVE_LINDEP_ABX_WORK_TRIH, 115           MSK_IPAR_LOG_INTEAS_ANA, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_INTENTY, 107         MSK_IPAR_PRESOLVE_USE, 115           MSK_IPAR_LOG_ONDER, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_ONDER, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 109         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_READ_LOG_SIM_NINGR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR		
MSK_IPAR_LOG, 105         MSK_IPAR_LOG_ANA_PRO, 106         MSK_IPAR_LOG_ANA_PRO, 106         MSK_IPAR_LOG_BI_GN_ERROR, 114           MSK_IPAR_LOG_BI_FREQ, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL, 114           MSK_IPAR_LOG_CUT_SECOND_OPT, 106         MSK_IPAR_LOG_CUT_SECOND_OPT, 106         MSK_IPAR_LOG_CUT_SECOND_OPT, 106           MSK_IPAR_LOG_EXPAND, 106         MSK_IPAR_PRESOLVE_LEVEL, 114           MSK_IPAR_LOG_EXPAND, 106         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK, IPAR_LOG_STAS_REPAIR, 106         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK, IPAR_LOG_FILE, 107         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_INTEAS_ANA, 107         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_PRESOLVE_USS, 115           MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_PRESOLVE_USS, 115           MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_DEBUG, 116           MSK, IPAR_LOG_SENSITIVITY, 109         MSK_IPAR_READ_LEP_GROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_LEP_GROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_LEP_GROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_READ_LOS_SIM		
MSK_IPAR_LOG_ANA_PRO, 106         MSK_IPAR_PARAM_READ_ION_ERROR, 114           MSK_IPAR_LOG_BI_FREQ, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL, 114           MSK_IPAR_LOG_CHECK_CONVEXITY, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_NUM_TRIES,           MSK_IPAR_LOG_CHECK_CONVEXITY, 106         MSK_IPAR_PRESOLVE_LEVEL, 114           MSK_IPAR_LOG_ERSOND_OPT, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_FILE, 107         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_INTENT, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_INTENT, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_MIO, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_MIO, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_SESONSE, 108         MSK_IPAR_READ_DATA_COMPATA, 115           MSK_IPAR_LOG_SENSITIVITY, 018         MSK_IPAR_READ_DATA_COMPATA, 116           MSK_IPAR_LOG_SENSITIVITY, 018         MSK_IPAR_READ_LOBOUN, 116           MSK_IPAR_READ_DATA_COMPATA, 115         MSK_IPAR_READ_LOBOUN, 116           MSK_IPAR_READ_LOS SENSITIVITY, 018         MSK_IPAR_READ_LOBOUND, 116           MSK_IPAR_READ_LOS SENSITIVITY,		
MSK_IPAR_LOG_BI, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_FILL, 114           MSK_IPAR_LOG_BI_FREQ, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_NUM_TRIES,           MSK_IPAR_LOG_CONVEXITY, 106         MSK_IPAR_LOG_COT_SECOND_OPT, 106         MSK_IPAR_LOG_ECXPAND, 106         MSK_IPAR_LOG_ECXPAND, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_FILE, 107         MSK_IPAR_LOG_FILE, 107         MSK_IPAR_PRESOLVE_LINDEP_USE, 115         MSK_IPAR_LOG_INTPONT, 107         MSK_IPAR_PRESOLVE_USE, 115         MSK_IPAR_PRESOLVE_USE, 116         MSK_IPAR_PRESOLVE_USE, 116         MSK_IPAR_PRESOLVE_USE, 116         MSK_IPAR_PRESOLVE_USE, 116         MSK_IPAR_PRESOLVE_USE, 116 <td></td> <td></td>		
MSK_IPAR_LOG_BI_FREQ, 106         MSK_IPAR_PRESOLVE_ELIMINATOR_MAX_NUM_TRIES,           MSK_IPAR_LOG_CCHECK_CONVEXITY, 106         114           MSK_IPAR_LOG_CCT_SECOND_OPT, 106         MSK_IPAR_PRESOLVE_LEVEL, 114           MSK_IPAR_LOG_EXPAND, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_EXPAND, 106         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_FILE, 107         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_MIND, 107         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_MIND, 107         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_SESPONSE, 108         MSK_IPAR_READ_LEP_SEDG, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LEP_SED_PREE_CON, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MSE_MIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MSE_MIDTH, 116           MSK_IPAR_READ_MSS_COUTIONS, 117		
MSK_IPAR_LOG_CHECK_CONVEXITY, 106         114           MSK_IPAR_LOG_CUT_SECOND_OPT, 106         MSK_IPAR_PRESOLVE_LEVEL, 114           MSK_IPAR_LOG_EXPAND, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_FILE, 107         MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115           MSK_IPAR_LOG_INFEAS_ANA, 107         MSK_IPAR_PRESOLVE_LINDEP_USE, 115           MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REBOUCTIONS, 115           MSK_IPAR_LOG_MIO, 107         MSK_IPAR_PRESOLVE_USE, 115           MSK_IPAR_LOG_MIO, 107         MSK_IPAR_PRESOLVE_USE, 115           MSK_IPAR_LOG_MIO, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_DORDER, 107         MSK_IPAR_READ_DATA_FORMAT, 115           MSK_IPAR_LOG_STOBER, 107         MSK_IPAR_READ_DATA_FORMAT, 115           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_CORMAT, 116           MSK_IPAR_MIO_GENSTRUTCT_SOL, 109         MSK_IPAR_SENSITIVITY_TYPE, 117           MSK_IPAR_		
MSK_IPAR_LOG_CUT_SECOND_OPT, 106         MSK_IPAR_PRESOLVE_LEVEL, 114           MSK_IPAR_LOG_EXPAND, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_EXPAND, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115           MSK_IPAR_LOG_FILE, 107         MSK_IPAR_PRESOLVE_LINDEP_USE, 115           MSK_IPAR_LOG_INFEAS_ANA, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_MID_FREQ, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_MID_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_PRESOLVE, 107         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LEP_QUOTED_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LEP_QUOTED_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_MIO_GSIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_MIO_BRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_TALL, 117           MSK_IPAR_MIO_COUT_COUT_GOUT_MID_MAS_MID_MAS_MID_MAS_MID_		
MSK_IPAR_LOG_EXPAND, 106         MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115         MSK_IPAR_LOG_FILE, 107         MSK_IPAR_PRESOLVE_LINDEP_USE, 115         MSK_IPAR_LOG_INTEPNT, 107         MSK_IPAR_PRESOLVE_LINDEP_USE, 115         MSK_IPAR_LOG_INTEPNT, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115         MSK_IPAR_LOG_MIO, 107         MSK_IPAR_PRESOLVE_USE, 115         MSK_IPAR_LOG_MIO, 107         MSK_IPAR_PRESOLVE_USE, 115         MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_COWRESSED, 115         MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_FORMAT, 115         MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DEBUG, 116         MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_DEBUG, 116         MSK_IPAR_RE		
MSK_IPAR_LOG_FEAS_REPAIR, 106         MSK_IPAR_LOG_FILE, 107         MSK_IPAR_LOG_FILE, 107         MSK_IPAR_RESOLVE_LINDEP_USE, 115           MSK_IPAR_LOG_INFEAS_ANA, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_INFEAS_ANA, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_MIO, 107         MSK_IPAR_PRESOLVE_USE, 115           MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_DATA_COMPRESSED, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_DEPOUPOTED_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CUT_CLIQUE, 109         MSK_IPAR_SIM_DUAL_CRASH, 117 <tr< td=""><td>MSK_IPAR_LOG_CUT_SECOND_OPT, 106</td><td>MSK_IPAR_PRESOLVE_LEVEL, 114</td></tr<>	MSK_IPAR_LOG_CUT_SECOND_OPT, 106	MSK_IPAR_PRESOLVE_LEVEL, 114
MSK_IPAR_LOG_FILE, 107         MSK_IPAR_LOG_INFEAS_ANA, 107         MSK_IPAR_PRESOLVE_LINDEP_USE, 115           MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_PRESOLVE_USE, 115           MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_BEAD_DATA_FORMAT, 115           MSK_IPAR_LOG_PRESOLVE, 107         MSK_IPAR_BEAD_DEBUG, 116           MSK_IPAR_LOG_SENSONSE, 108         MSK_IPAR_DEBUD_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_NINOR, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_MPS_COLUTIONS, 117           MSK_IPAR_MIO_GONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CUT_CUT_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_PRASCONE_METHOD, 117           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_SELECTION, 118           MSK_IPAR_MIO_M	MSK_IPAR_LOG_EXPAND, 106	MSK_IPAR_PRESOLVE_LINDEP_ABS_WORK_TRH, 115
MSK_IPAR_LOG_INFEAS_ANA, 107         MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115           MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_PRESOLVE_USE, 115           MSK_IPAR_LOG_MIO, 107         MSK_IPAR_PRESOLVE_USE, 115           MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_DATA_FORMAT, 115           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_LED_CON, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LEP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SIM, 108         MSK_IPAR_READ_LP_GUOTED_NAMES, 116           MSK_IPAR_LOG_SIM, 108         MSK_IPAR_READ_LP_GUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_READ_TASK_IGNORE_PARAM, 116         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_MIO_GOSIM_MINOR, 108         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_TYPE, 117           MSK_IPAR_MIO_CUT_CLIQUE, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_CRAS	MSK_IPAR_LOG_FEAS_REPAIR, 106	MSK_IPAR_PRESOLVE_LINDEP_REL_WORK_TRH, 115
MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_LOG_USE, 115           MSK_IPAR_LOG_MIO_107         MSK_IPAR_PRIMAL_REPAIR_OPTIMIZER, 115           MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_FORMAT, 115           MSK_IPAR_LOG_RESOLVE, 107         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LEP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_NOS         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_MIO_COSTORAGE, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_MIO_DEBRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CUT_CUTIQIT, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_CRASH, 117	MSK_IPAR_LOG_FILE, 107	MSK_IPAR_PRESOLVE_LINDEP_USE, 115
MSK_IPAR_LOG_INTPNT, 107         MSK_IPAR_LOG_USE, 115           MSK_IPAR_LOG_MIO_107         MSK_IPAR_REAL_REPAIR_OPTIMIZER, 115           MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_FORMAT, 115           MSK_IPAR_LOG_RESOLVE, 107         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LEP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_TOR         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FIREQ, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_MIO_OSTRUCTS.OL, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_KNP	MSK_IPAR_LOG_INFEAS_ANA, 107	MSK_IPAR_PRESOLVE_MAX_NUM_REDUCTIONS, 115
MSK_IPAR_LOG_MIO, 107         MSK_IPAR_PRIMAL_REPAIR_OPTIMIZER, 115           MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_FORMAT, 115           MSK_IPAR_LOG_PRESOLVE, 107         MSK_IPAR_READ_LOEDUG, 116           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_KEEP_FREE_CON, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM_108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_READ_MPS_WIDTH, 116         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK	MSK_IPAR_LOG_INTPNT, 107	
MSK_IPAR_LOG_MIO_FREQ, 107         MSK_IPAR_READ_DATA_COMPRESSED, 115           MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_FORMAT, 115           MSK_IPAR_LOG_PRESOLVE, 107         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_KEEP_FREE_CON, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, 117           MSK_IPAR_MAX_NUM_WARNINGS, 109         MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, 117           MSK_IPAR_MIO_COTSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_COTSTRUCT_SOL, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111		
MSK_IPAR_LOG_ORDER, 107         MSK_IPAR_READ_DATA_FORMAT, 115           MSK_IPAR_LOG_PRESOLVE, 107         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_KEEP_FREE_CON, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_MIO_CUT_MARAM, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_CUT_CUT_CUT_GOL, 109         MSK_IPAR_SIM_DAL_CRASH, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118		
MSK_IPAR_LOG_PRESOLVE, 107         MSK_IPAR_READ_DEBUG, 116           MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_KEEP_FREE_CON, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SIM_SITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_MAMES, 116           MSK_IPAR_LOG_SIM_TREQ, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_MAX_NUM_WARNINGS, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_BRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_TYPE, 117           MSK_IPAR_MIO_CUT_CLIQUE, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DUGL_CRASH, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_HEURISTIC_LEVEL, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111		
MSK_IPAR_LOG_RESPONSE, 108         MSK_IPAR_READ_KEEP_FREE_CON, 116           MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_MOS_UNUSED_SOLUTIONS, 117         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_MOS_UNUSED_SOLUTIONS, 117         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_READ_TASK_IGNORE_PARAM, 116         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_READ_TASK_IGNORE_PARAM, 116         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_READ_MPS_WIDTH, 116         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_READ_MPS_WIDTH, 116         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_CUT_CUT_CUTE, 109         MSK_IPAR_SENSITIVITY_TYPE, 117           MSK_IPAR_MIO_CUT_GMI, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_MAX_NUM_		
MSK_IPAR_LOG_SENSITIVITY, 108         MSK_IPAR_READ_LP_DROP_NEW_VARS_IN_BOU, 116           MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_SENSITIVITY, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CUT_CLIQUE, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_GMI, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_HOURISTIC_LEVEL, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_RELAXS, 110         MSK_IPAR_SIM_HOTSTART_LU, 118		
MSK_IPAR_LOG_SENSITIVITY_OPT, 108         MSK_IPAR_READ_LP_QUOTED_NAMES, 116           MSK_IPAR_LOG_SIM, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, 117           MSK_IPAR_MAX_NUM_WARNINGS, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_BRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_TYPE, 117           MSK_IPAR_MIO_CUT_COMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_SELECTION, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_RELAXS, 110         MSK_IPAR_SIM_HOTSTART_LU, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_MAX_ITERATIONS, 118           MSK_IPAR_MIO_ODDE_OPTIMIZER, 111         MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_LOG_SIM, 108         MSK_IPAR_READ_MPS_FORMAT, 116           MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, 117           MSK_IPAR_MINO_STORAGE, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_BRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CUT_CLIQUE, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_GMI, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_HEURISTIC_LEVEL, 110         MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_MAX_ITERATIONS, 118           MSK_IPAR_MIO_ONDE, 111         MSK_IPAR_SIM_NON_SINGULAR, 119           MSK_IPAR_SIM_OND_SINGULAR, 119         MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_LOG_SIM_FREQ, 108         MSK_IPAR_READ_MPS_WIDTH, 116           MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_MIO_BRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_SIN_LOCUT_CONSTRUCT_SOL, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_COMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_SIM_DUAL_CRASH, 117         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_MAX_ITERATIONS, 118           MSK_IPAR_MIO_MAX_NUM_SETBACKS, 118         MSK_IPAR_SIM_NON_SINGULAR, 119           MSK_IPAR_MIO_NODE_OPTIMIZ		
MSK_IPAR_LOG_SIM_MINOR, 108         MSK_IPAR_READ_TASK_IGNORE_PARAM, 116           MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, 117           MSK_IPAR_MAX_NUM_WARNINGS, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_BRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_TYPE, 117           MSK_IPAR_MIO_CUT_CLIQUE, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_SELECTION, 118           MSK_IPAR_MIO_HEURISTIC_LEVEL, 110         MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_MAX_ITERATIONS, 118           MSK_IPAR_MIO_MODE, 111         MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118           MSK_IPAR_MIO_NODE_OPTIMIZER, 111         MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_LOG_STORAGE, 108         MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, 117           MSK_IPAR_MAX_NUM_WARNINGS, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_BRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_TYPE, 117           MSK_IPAR_MIO_CUT_CLIQUE, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_GMI, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_HEURISTIC_LEVEL, 110         MSK_IPAR_SIM_DUAL_SELECTION, 118           MSK_IPAR_MIO_HEURISTIC_LEVEL, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART_LU, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_MAX_ITERATIONS, 118           MSK_IPAR_MIO_MODE, 111         MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118           MSK_IPAR_MIO_NODE_OPTIMIZER, 111         MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MAX_NUM_WARNINGS, 109         MSK_IPAR_SENSITIVITY_ALL, 117           MSK_IPAR_MIO_BRANCH_DIR, 109         MSK_IPAR_SENSITIVITY_OPTIMIZER, 117           MSK_IPAR_MIO_CONSTRUCT_SOL, 109         MSK_IPAR_SENSITIVITY_TYPE, 117           MSK_IPAR_MIO_CUT_CLIQUE, 109         MSK_IPAR_SIM_BASIS_FACTOR_USE, 117           MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_GMI, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_SELECTION, 118           MSK_IPAR_MIO_HEURISTIC_LEVEL, 110         MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_HOTSTART_LU, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_MAX_ITERATIONS, 118           MSK_IPAR_MIO_MODE, 111         MSK_IPAR_SIM_NON_SINGULAR, 119           MSK_IPAR_SIM_PRIMAL_CRASH, 119         MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_BRANCH_DIR, 109 MSK_IPAR_SENSITIVITY_OPTIMIZER, 117 MSK_IPAR_MIO_CONSTRUCT_SOL, 109 MSK_IPAR_SENSITIVITY_TYPE, 117 MSK_IPAR_MIO_CUT_CLIQUE, 109 MSK_IPAR_SIM_BASIS_FACTOR_USE, 117 MSK_IPAR_MIO_CUT_CMIR, 109 MSK_IPAR_SIM_DEGEN, 117 MSK_IPAR_MIO_CUT_GMI, 109 MSK_IPAR_SIM_DUAL_CRASH, 117 MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109 MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117 MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110 MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118 MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110 MSK_IPAR_SIM_DUAL_SELECTION, 118 MSK_IPAR_MIO_HEURISTIC_LEVEL, 110 MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118 MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110 MSK_IPAR_SIM_HOTSTART, 118 MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111 MSK_IPAR_SIM_HOTSTART_LU, 118 MSK_IPAR_SIM_HOTSTART_LU, 118 MSK_IPAR_SIM_MAX_ITERATIONS, 118 MSK_IPAR_MIO_MODE, 111 MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118 MSK_IPAR_MIO_NODE_OPTIMIZER, 111 MSK_IPAR_SIM_PRIMAL_CRASH, 119		MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, 117
MSK_IPAR_MIO_CONSTRUCT_SOL, 109 MSK_IPAR_MIO_CUT_CLIQUE, 109 MSK_IPAR_SIM_BASIS_FACTOR_USE, 117 MSK_IPAR_MIO_CUT_CMIR, 109 MSK_IPAR_SIM_DEGEN, 117 MSK_IPAR_MIO_CUT_GMI, 109 MSK_IPAR_SIM_DUAL_CRASH, 117 MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109 MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117 MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110 MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118 MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110 MSK_IPAR_SIM_DUAL_SELECTION, 118 MSK_IPAR_MIO_HEURISTIC_LEVEL, 110 MSK_IPAR_SIM_DUAL_SELECTION, 118 MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110 MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118 MSK_IPAR_MIO_MAX_NUM_RELAXS, 110 MSK_IPAR_SIM_HOTSTART, 118 MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111 MSK_IPAR_SIM_HOTSTART_LU, 118 MSK_IPAR_SIM_MAX_ITERATIONS, 118 MSK_IPAR_MIO_MODE, 111 MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118 MSK_IPAR_MIO_MT_USER_CB, 111 MSK_IPAR_SIM_NON_SINGULAR, 119 MSK_IPAR_SIM_PRIMAL_CRASH, 119	MSK_IPAR_MAX_NUM_WARNINGS, 109	MSK_IPAR_SENSITIVITY_ALL, 117
MSK_IPAR_MIO_CUT_CLIQUE, 109 MSK_IPAR_SIM_BASIS_FACTOR_USE, 117 MSK_IPAR_MIO_CUT_CMIR, 109 MSK_IPAR_SIM_DEGEN, 117 MSK_IPAR_MIO_CUT_GMI, 109 MSK_IPAR_SIM_DUAL_CRASH, 117 MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109 MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117 MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110 MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118 MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110 MSK_IPAR_SIM_DUAL_SELECTION, 118 MSK_IPAR_MIO_HEURISTIC_LEVEL, 110 MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118 MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110 MSK_IPAR_SIM_HOTSTART, 118 MSK_IPAR_MIO_MAX_NUM_RELAXS, 110 MSK_IPAR_SIM_HOTSTART_LU, 118 MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111 MSK_IPAR_SIM_MAX_ITERATIONS, 118 MSK_IPAR_MIO_MODE, 111 MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118 MSK_IPAR_MIO_MODE_OPTIMIZER, 111 MSK_IPAR_SIM_PRIMAL_CRASH, 119	MSK_IPAR_MIO_BRANCH_DIR, 109	MSK_IPAR_SENSITIVITY_OPTIMIZER, 117
MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_GMI, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_SELECTION, 118           MSK_IPAR_MIO_HEURISTIC_LEVEL, 110         MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_RELAXS, 110         MSK_IPAR_SIM_HOTSTART_LU, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_MAX_ITERATIONS, 118           MSK_IPAR_MIO_MODE, 111         MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118           MSK_IPAR_MIO_NODE_OPTIMIZER, 111         MSK_IPAR_SIM_NON_SINGULAR, 119           MSK_IPAR_SIM_PRIMAL_CRASH, 119	MSK_IPAR_MIO_CONSTRUCT_SOL, 109	MSK_IPAR_SENSITIVITY_TYPE, 117
MSK_IPAR_MIO_CUT_CMIR, 109         MSK_IPAR_SIM_DEGEN, 117           MSK_IPAR_MIO_CUT_GMI, 109         MSK_IPAR_SIM_DUAL_CRASH, 117           MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109         MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117           MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110         MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118           MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110         MSK_IPAR_SIM_DUAL_SELECTION, 118           MSK_IPAR_MIO_HEURISTIC_LEVEL, 110         MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118           MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110         MSK_IPAR_SIM_HOTSTART, 118           MSK_IPAR_MIO_MAX_NUM_RELAXS, 110         MSK_IPAR_SIM_HOTSTART_LU, 118           MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111         MSK_IPAR_SIM_MAX_ITERATIONS, 118           MSK_IPAR_MIO_MODE, 111         MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118           MSK_IPAR_MIO_NODE_OPTIMIZER, 111         MSK_IPAR_SIM_NON_SINGULAR, 119           MSK_IPAR_SIM_PRIMAL_CRASH, 119	MSK_IPAR_MIO_CUT_CLIQUE, 109	MSK_IPAR_SIM_BASIS_FACTOR_USE, 117
MSK_IPAR_MIO_CUT_GMI, 109 MSK_IPAR_SIM_DUAL_CRASH, 117 MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109 MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117 MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110 MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118 MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110 MSK_IPAR_SIM_DUAL_SELECTION, 118 MSK_IPAR_MIO_HEURISTIC_LEVEL, 110 MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118 MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110 MSK_IPAR_SIM_HOTSTART, 118 MSK_IPAR_MIO_MAX_NUM_RELAXS, 110 MSK_IPAR_SIM_HOTSTART_LU, 118 MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111 MSK_IPAR_SIM_MAX_ITERATIONS, 118 MSK_IPAR_MIO_MODE, 111 MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118 MSK_IPAR_MIO_MT_USER_CB, 111 MSK_IPAR_SIM_NON_SINGULAR, 119 MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_CUT_IMPLIED_BOUND, 109 MSK_IPAR_SIM_DUAL_PHASEONE_METHOD, 117 MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110 MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110 MSK_IPAR_MIO_HEURISTIC_LEVEL, 110 MSK_IPAR_MIO_HEURISTIC_LEVEL, 110 MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110 MSK_IPAR_SIM_HOTSTART, 118 MSK_IPAR_MIO_MAX_NUM_RELAXS, 110 MSK_IPAR_SIM_HOTSTART_LU, 118 MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111 MSK_IPAR_MIO_MODE, 111 MSK_IPAR_SIM_MAX_ITERATIONS, 118 MSK_IPAR_MIO_MODE, 111 MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118 MSK_IPAR_MIO_MODE_OPTIMIZER, 111 MSK_IPAR_SIM_PRIMAL_CRASH, 119		· · · · · · · · · · · · · · · · · · ·
MSK_IPAR_MIO_CUT_KNAPSACK_COVER, 110  MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110  MSK_IPAR_SIM_DUAL_RESTRICT_SELECTION, 118  MSK_IPAR_MIO_HEURISTIC_LEVEL, 110  MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118  MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110  MSK_IPAR_SIM_HOTSTART, 118  MSK_IPAR_MIO_MAX_NUM_RELAXS, 110  MSK_IPAR_SIM_HOTSTART_LU, 118  MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111  MSK_IPAR_SIM_MAX_ITERATIONS, 118  MSK_IPAR_MIO_MODE, 111  MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118  MSK_IPAR_SIM_ONDE_OPTIMIZER, 111  MSK_IPAR_SIM_PRIMAL_CRASH, 119		,
MSK_IPAR_MIO_CUT_SELECTION_LEVEL, 110  MSK_IPAR_SIM_DUAL_SELECTION, 118  MSK_IPAR_MIO_HEURISTIC_LEVEL, 110  MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118  MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110  MSK_IPAR_SIM_HOTSTART, 118  MSK_IPAR_MIO_MAX_NUM_RELAXS, 110  MSK_IPAR_SIM_HOTSTART_LU, 118  MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111  MSK_IPAR_SIM_MAX_ITERATIONS, 118  MSK_IPAR_MIO_MODE, 111  MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118  MSK_IPAR_MIO_MT_USER_CB, 111  MSK_IPAR_SIM_NON_SINGULAR, 119  MSK_IPAR_MIO_NODE_OPTIMIZER, 111  MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_HEURISTIC_LEVEL, 110  MSK_IPAR_SIM_EXPLOIT_DUPVEC, 118  MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110  MSK_IPAR_SIM_HOTSTART, 118  MSK_IPAR_MIO_MAX_NUM_RELAXS, 110  MSK_IPAR_SIM_HOTSTART_LU, 118  MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111  MSK_IPAR_SIM_MAX_ITERATIONS, 118  MSK_IPAR_MIO_MODE, 111  MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118  MSK_IPAR_MIO_MT_USER_CB, 111  MSK_IPAR_SIM_NON_SINGULAR, 119  MSK_IPAR_MIO_NODE_OPTIMIZER, 111  MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_MAX_NUM_BRANCHES, 110  MSK_IPAR_SIM_HOTSTART, 118  MSK_IPAR_MIO_MAX_NUM_RELAXS, 110  MSK_IPAR_SIM_HOTSTART_LU, 118  MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111  MSK_IPAR_SIM_MAX_ITERATIONS, 118  MSK_IPAR_MIO_MODE, 111  MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118  MSK_IPAR_MIO_MT_USER_CB, 111  MSK_IPAR_SIM_NON_SINGULAR, 119  MSK_IPAR_MIO_NODE_OPTIMIZER, 111  MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_MAX_NUM_RELAXS, 110 MSK_IPAR_SIM_HOTSTART_LU, 118 MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111 MSK_IPAR_SIM_MAX_ITERATIONS, 118 MSK_IPAR_MIO_MODE, 111 MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118 MSK_IPAR_MIO_MT_USER_CB, 111 MSK_IPAR_SIM_NON_SINGULAR, 119 MSK_IPAR_MIO_NODE_OPTIMIZER, 111 MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_MAX_NUM_SOLUTIONS, 111  MSK_IPAR_SIM_MAX_ITERATIONS, 118  MSK_IPAR_MIO_MODE, 111  MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118  MSK_IPAR_SIM_NON_SINGULAR, 119  MSK_IPAR_MIO_NODE_OPTIMIZER, 111  MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_MODE, 111 MSK_IPAR_SIM_MAX_NUM_SETBACKS, 118 MSK_IPAR_MIO_MT_USER_CB, 111 MSK_IPAR_SIM_NON_SINGULAR, 119 MSK_IPAR_MIO_NODE_OPTIMIZER, 111 MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_MT_USER_CB, 111 MSK_IPAR_SIM_NON_SINGULAR, 119 MSK_IPAR_MIO_NODE_OPTIMIZER, 111 MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_NODE_OPTIMIZER, 111 MSK_IPAR_SIM_PRIMAL_CRASH, 119		
MSK_IPAR_MIO_NODE_SELECTION, 111 MSK_IPAR_SIM_PRIMAL_PHASEONE_METHOD, 119		
	MSK_IPAR_MIO_NODE_SELECTION, 111	MSK_IPAR_SIM_PRIMAL_PHASEONE_METHOD, 119

```
MSK_IPAR_SIM_PRIMAL_RESTRICT_SELECTION, 119
                                              MSK_SPAR_READ_MPS_RAN_NAME, 126
MSK_IPAR_SIM_PRIMAL_SELECTION, 119
                                              MSK_SPAR_READ_MPS_RHS_NAME, 126
MSK_IPAR_SIM_REFACTOR_FREQ, 119
                                              MSK_SPAR_REMOTE_ACCESS_TOKEN, 126
MSK_IPAR_SIM_REFORMULATION, 120
                                              MSK_SPAR_SENSITIVITY_FILE_NAME, 126
MSK_IPAR_SIM_SAVE_LU, 120
                                              MSK_SPAR_SENSITIVITY_RES_FILE_NAME, 127
{\tt MSK\_IPAR\_SIM\_SCALING},\ 120
                                              MSK_SPAR_SOL_FILTER_XC_LOW, 127
MSK_IPAR_SIM_SCALING_METHOD, 120
                                              MSK_SPAR_SOL_FILTER_XC_UPR, 127
MSK_IPAR_SIM_SOLVE_FORM, 120
                                              MSK_SPAR_SOL_FILTER_XX_LOW, 127
MSK_IPAR_SIM_STABILITY_PRIORITY, 120
                                              MSK_SPAR_SOL_FILTER_XX_UPR, 127
MSK_IPAR_SIM_SWITCH_OPTIMIZER, 120
                                              MSK_SPAR_STAT_FILE_NAME, 127
MSK_IPAR_SOL_FILTER_KEEP_BASIC, 120
                                              MSK_SPAR_STAT_KEY, 127
MSK_IPAR_SOL_FILTER_KEEP_RANGED, 121
                                              MSK_SPAR_STAT_NAME, 127
                                              MSK_SPAR_WRITE_LP_GEN_VAR_NAME, 127
MSK_IPAR_SOL_READ_NAME_WIDTH, 121
MSK_IPAR_SOL_READ_WIDTH, 121
                                              Response codes
MSK_IPAR_SOLUTION_CALLBACK, 121
MSK_IPAR_TIMING_LEVEL, 121
                                              Termination, 128
MSK_IPAR_WRITE_BAS_CONSTRAINTS, 121
                                              MSK_RES_OK, 128
MSK_IPAR_WRITE_BAS_HEAD, 121
                                              MSK_RES_TRM_INTERNAL, 129
MSK_IPAR_WRITE_BAS_VARIABLES, 122
                                              MSK_RES_TRM_INTERNAL_STOP, 129
MSK_IPAR_WRITE_DATA_COMPRESSED, 122
                                              MSK_RES_TRM_MAX_ITERATIONS, 128
MSK_IPAR_WRITE_DATA_FORMAT, 122
                                              MSK_RES_TRM_MAX_NUM_SETBACKS, 129
MSK_IPAR_WRITE_DATA_PARAM, 122
                                              MSK_RES_TRM_MAX_TIME, 128
MSK_IPAR_WRITE_FREE_CON, 122
                                              MSK_RES_TRM_MIO_NEAR_ABS_GAP, 128
MSK_IPAR_WRITE_GENERIC_NAMES, 122
                                              MSK_RES_TRM_MIO_NEAR_REL_GAP, 128
MSK_IPAR_WRITE_GENERIC_NAMES_IO, 122
                                              {\tt MSK\_RES\_TRM\_MIO\_NUM\_BRANCHES},~128
MSK_IPAR_WRITE_IGNORE_INCOMPATIBLE_ITEMS,
                                              MSK_RES_TRM_MIO_NUM_RELAXS, 128
        122
                                              MSK_RES_TRM_NUM_MAX_NUM_INT_SOLUTIONS, 128
MSK_IPAR_WRITE_INT_CONSTRAINTS, 123
                                              MSK_RES_TRM_NUMERICAL_PROBLEM, 129
MSK_IPAR_WRITE_INT_HEAD, 123
                                              MSK_RES_TRM_OBJECTIVE_RANGE, 128
MSK_IPAR_WRITE_INT_VARIABLES, 123
                                              MSK_RES_TRM_STALL, 128
MSK_IPAR_WRITE_LP_FULL_OBJ, 123
                                              MSK_RES_TRM_USER_CALLBACK, 129
MSK_IPAR_WRITE_LP_LINE_WIDTH, 123
                                              Warnings, 129
MSK_IPAR_WRITE_LP_QUOTED_NAMES, 123
                                              MSK_RES_WRN_ANA_ALMOST_INT_BOUNDS, 131
MSK_IPAR_WRITE_LP_STRICT_FORMAT, 123
                                              MSK_RES_WRN_ANA_C_ZERO, 131
MSK_IPAR_WRITE_LP_TERMS_PER_LINE, 124
                                              MSK_RES_WRN_ANA_CLOSE_BOUNDS, 131
{\tt MSK\_IPAR\_WRITE\_MPS\_FORMAT},\,124
                                              MSK_RES_WRN_ANA_EMPTY_COLS, 131
MSK_IPAR_WRITE_MPS_INT, 124
                                              MSK_RES_WRN_ANA_LARGE_BOUNDS, 131
MSK_IPAR_WRITE_PRECISION, 124
                                              MSK_RES_WRN_CONSTRUCT_INVALID_SOL_ITG, 131
MSK_IPAR_WRITE_SOL_BARVARIABLES, 124
                                              MSK_RES_WRN_CONSTRUCT_NO_SOL_ITG, 131
MSK_IPAR_WRITE_SOL_CONSTRAINTS, 124
                                              MSK_RES_WRN_CONSTRUCT_SOLUTION_INFEAS, 131
MSK_IPAR_WRITE_SOL_HEAD, 124
                                              MSK_RES_WRN_DROPPED_NZ_QOBJ, 129
MSK_IPAR_WRITE_SOL_IGNORE_INVALID_NAMES,
                                              MSK_RES_WRN_DUPLICATE_BARVARIABLE_NAMES,
                                                      131
MSK_IPAR_WRITE_SOL_VARIABLES, 125
                                              MSK_RES_WRN_DUPLICATE_CONE_NAMES, 131
MSK_IPAR_WRITE_TASK_INC_SOL, 125
                                              MSK_RES_WRN_DUPLICATE_CONSTRAINT_NAMES, 131
MSK_IPAR_WRITE_XML_MODE, 125
                                              MSK_RES_WRN_DUPLICATE_VARIABLE_NAMES, 131
String parameters, 125
                                              MSK_RES_WRN_ELIMINATOR_SPACE, 131
MSK_SPAR_BAS_SOL_FILE_NAME, 125
                                              MSK_RES_WRN_EMPTY_NAME, 130
MSK_SPAR_DATA_FILE_NAME, 125
                                              MSK_RES_WRN_IGNORE_INTEGER, 130
MSK_SPAR_DEBUG_FILE_NAME, 125
                                              MSK_RES_WRN_INCOMPLETE_LINEAR_DEPENDENCY_CHECK,
MSK_SPAR_INT_SOL_FILE_NAME, 125
MSK_SPAR_ITR_SOL_FILE_NAME, 125
                                              MSK_RES_WRN_LARGE_AIJ, 129
MSK_SPAR_MIO_DEBUG_STRING, 125
                                              MSK_RES_WRN_LARGE_BOUND, 129
MSK_SPAR_PARAM_COMMENT_SIGN, 126
                                              MSK_RES_WRN_LARGE_CJ, 129
MSK_SPAR_PARAM_READ_FILE_NAME, 126
                                              MSK_RES_WRN_LARGE_CON_FX, 129
MSK_SPAR_PARAM_WRITE_FILE_NAME, 126
                                              MSK_RES_WRN_LARGE_LO_BOUND, 129
MSK_SPAR_READ_MPS_BOU_NAME, 126
                                              MSK_RES_WRN_LARGE_UP_BOUND, 129
MSK_SPAR_READ_MPS_OBJ_NAME, 126
                                              MSK_RES_WRN_LICENSE_EXPIRE, 130
```

```
MSK_RES_WRN_LICENSE_FEATURE_EXPIRE, 130
                                              MSK_RES_ERR_CANNOT_HANDLE_NL, 144
MSK_RES_WRN_LICENSE_SERVER, 130
                                              MSK_RES_ERR_CBF_DUPLICATE_ACOORD, 148
MSK_RES_WRN_LP_DROP_VARIABLE, 129
                                              MSK_RES_ERR_CBF_DUPLICATE_BCOORD, 148
MSK_RES_WRN_LP_OLD_QUAD_FORMAT, 129
                                              MSK_RES_ERR_CBF_DUPLICATE_CON, 148
                                              MSK_RES_ERR_CBF_DUPLICATE_INT, 148
MSK_RES_WRN_MIO_INFEASIBLE_FINAL, 130
{\tt MSK\_RES\_WRN\_MPS\_SPLIT\_BOU\_VECTOR},\ 129
                                              MSK_RES_ERR_CBF_DUPLICATE_OBJ, 148
                                              MSK_RES_ERR_CBF_DUPLICATE_OBJACOORD, 148
MSK_RES_WRN_MPS_SPLIT_RAN_VECTOR, 129
MSK_RES_WRN_MPS_SPLIT_RHS_VECTOR, 129
                                              MSK_RES_ERR_CBF_DUPLICATE_PSDVAR, 148
MSK_RES_WRN_NAME_MAX_LEN, 129
                                              MSK_RES_ERR_CBF_DUPLICATE_VAR, 148
{\tt MSK\_RES\_WRN\_NO\_DUALIZER},\ 132
                                              MSK_RES_ERR_CBF_INVALID_CON_TYPE, 148
MSK_RES_WRN_NO_GLOBAL_OPTIMIZER, 130
                                              MSK_RES_ERR_CBF_INVALID_DOMAIN_DIMENSION,
MSK_RES_WRN_NO_NONLINEAR_FUNCTION_WRITE,
        130
                                              MSK_RES_ERR_CBF_INVALID_INT_INDEX, 148
MSK_RES_WRN_NZ_IN_UPR_TRI, 129
                                              MSK_RES_ERR_CBF_INVALID_PSDVAR_DIMENSION,
MSK_RES_WRN_OPEN_PARAM_FILE, 129
                                                      148
MSK_RES_WRN_PARAM_IGNORED_CMIO, 130
                                              MSK_RES_ERR_CBF_INVALID_VAR_TYPE, 148
                                              MSK_RES_ERR_CBF_NO_VARIABLES, 147
MSK_RES_WRN_PARAM_NAME_DOU, 130
MSK_RES_WRN_PARAM_NAME_INT, 130
                                              MSK_RES_ERR_CBF_NO_VERSION_SPECIFIED, 148
MSK_RES_WRN_PARAM_NAME_STR, 130
                                              MSK_RES_ERR_CBF_OBJ_SENSE, 147
MSK_RES_WRN_PARAM_STR_VALUE, 130
                                              MSK_RES_ERR_CBF_PARSE, 147
                                              {\tt MSK\_RES\_ERR\_CBF\_SYNTAX},\ 148
MSK_RES_WRN_PRESOLVE_OUTOFSPACE, 131
MSK_RES_WRN_QUAD_CONES_WITH_ROOT_FIXED_AT_ZENGK_RES_ERR_CBF_TOO_FEW_CONSTRAINTS, 148
                                              MSK_RES_ERR_CBF_TOO_FEW_INTS, 148
MSK_RES_WRN_RQUAD_CONES_WITH_ROOT_FIXED_AT_ZEEK_RES_ERR_CBF_TOO_FEW_PSDVAR, 148
        132
                                              MSK_RES_ERR_CBF_TOO_FEW_VARIABLES, 148
MSK_RES_WRN_SOL_FILE_IGNORED_CON, 130
                                              MSK_RES_ERR_CBF_TOO_MANY_CONSTRAINTS, 147
MSK_RES_WRN_SOL_FILE_IGNORED_VAR, 130
                                              MSK_RES_ERR_CBF_TOO_MANY_INTS, 148
MSK_RES_WRN_SOL_FILTER, 130
                                              MSK_RES_ERR_CBF_TOO_MANY_VARIABLES, 148
MSK_RES_WRN_SPAR_MAX_LEN, 129
                                              MSK_RES_ERR_CBF_UNSUPPORTED, 148
MSK_RES_WRN_SYM_MAT_LARGE, 132
                                              MSK_RES_ERR_CON_Q_NOT_NSD, 141
MSK_RES_WRN_TOO_FEW_BASIS_VARS, 130
                                              MSK_RES_ERR_CON_Q_NOT_PSD, 140
MSK_RES_WRN_TOO_MANY_BASIS_VARS, 130
                                              MSK_RES_ERR_CONE_INDEX, 141
MSK_RES_WRN_UNDEF_SOL_FILE_NAME, 130
                                              MSK_RES_ERR_CONE_OVERLAP, 141
                                              MSK_RES_ERR_CONE_OVERLAP_APPEND, 141
MSK_RES_WRN_USING_GENERIC_NAMES, 130
MSK_RES_WRN_WRITE_CHANGED_NAMES, 131
                                              MSK_RES_ERR_CONE_REP_VAR, 141
MSK_RES_WRN_WRITE_DISCARDED_CFIX, 131
                                              MSK_RES_ERR_CONE_SIZE, 141
MSK_RES_WRN_ZERO_AIJ, 129
                                              MSK_RES_ERR_CONE_TYPE, 141
MSK_RES_WRN_ZEROS_IN_SPARSE_COL, 131
                                              MSK_RES_ERR_CONE_TYPE_STR, 141
MSK_RES_WRN_ZEROS_IN_SPARSE_ROW, 130
                                              MSK_RES_ERR_DATA_FILE_EXT, 134
Errors, 132
                                              MSK_RES_ERR_DUP_NAME, 134
MSK_RES_ERR_AD_INVALID_CODELIST, 146
                                              MSK_RES_ERR_DUPLICATE_AIJ, 141
MSK_RES_ERR_API_ARRAY_TOO_SMALL, 145
                                              MSK_RES_ERR_DUPLICATE_BARVARIABLE_NAMES,
MSK_RES_ERR_API_CB_CONNECT, 145
                                              MSK_RES_ERR_DUPLICATE_CONE_NAMES, 147
MSK_RES_ERR_API_FATAL_ERROR, 145
MSK_RES_ERR_API_INTERNAL, 145
                                              MSK_RES_ERR_DUPLICATE_CONSTRAINT_NAMES, 146
MSK_RES_ERR_ARG_IS_TOO_LARGE, 138
                                              MSK_RES_ERR_DUPLICATE_VARIABLE_NAMES, 146
MSK_RES_ERR_ARG_IS_TOO_SMALL, 138
                                              MSK_RES_ERR_END_OF_FILE, 134
MSK_RES_ERR_ARGUMENT_DIMENSION, 137
                                              MSK_RES_ERR_FACTOR, 143
                                              MSK_RES_ERR_FEASREPAIR_CANNOT_RELAX, 144
MSK_RES_ERR_ARGUMENT_IS_TOO_LARGE, 147
MSK_RES_ERR_ARGUMENT_LENNEQ, 137
                                              MSK_RES_ERR_FEASREPAIR_INCONSISTENT_BOUND,
MSK_RES_ERR_ARGUMENT_PERM_ARRAY, 141
MSK_RES_ERR_ARGUMENT_TYPE, 137
                                              MSK_RES_ERR_FEASREPAIR_SOLVING_RELAXED, 144
MSK_RES_ERR_BAR_VAR_DIM, 146
                                              MSK_RES_ERR_FILE_LICENSE, 132
MSK_RES_ERR_BASIS, 140
                                              MSK_RES_ERR_FILE_OPEN, 134
MSK_RES_ERR_BASIS_FACTOR, 143
                                              MSK_RES_ERR_FILE_READ, 134
MSK_RES_ERR_BASIS_SINGULAR, 143
                                              MSK_RES_ERR_FILE_WRITE, 134
MSK_RES_ERR_BLANK_NAME, 134
                                              MSK_RES_ERR_FINAL_SOLUTION, 143
MSK_RES_ERR_CANNOT_CLONE_NL, 144
                                              MSK_RES_ERR_FIRST, 139
```

```
MSK_RES_ERR_FIRSTI, 140
                                              MSK_RES_ERR_INVALID_CONE_NAME, 134
MSK_RES_ERR_FIRSTJ, 140
                                              MSK_RES_ERR_INVALID_FILE_FORMAT_FOR_CONES,
MSK_RES_ERR_FIXED_BOUND_VALUES, 142
MSK_RES_ERR_FLEXLM, 132
                                              MSK_RES_ERR_INVALID_FILE_FORMAT_FOR_GENERAL_NL,
MSK_RES_ERR_GLOBAL_INV_CONIC_PROBLEM, 143
{\tt MSK\_RES\_ERR\_HUGE\_AIJ},\,141
                                              MSK_RES_ERR_INVALID_FILE_FORMAT_FOR_SYM_MAT,
MSK_RES_ERR_HUGE_C, 141
MSK_RES_ERR_IDENTICAL_TASKS, 145
                                              MSK_RES_ERR_INVALID_FILE_NAME, 134
MSK_RES_ERR_IN_ARGUMENT, 137
                                              MSK_RES_ERR_INVALID_FORMAT_TYPE, 140
MSK_RES_ERR_INDEX, 138
                                              MSK_RES_ERR_INVALID_IDX, 139
MSK_RES_ERR_INDEX_ARR_IS_TOO_LARGE, 138
                                              MSK_RES_ERR_INVALID_IOMODE, 144
MSK_RES_ERR_INDEX_ARR_IS_TOO_SMALL, 138
                                              MSK_RES_ERR_INVALID_MAX_NUM, 139
MSK_RES_ERR_INDEX_IS_TOO_LARGE, 137
                                              MSK_RES_ERR_INVALID_NAME_IN_SOL_FILE, 137
MSK_RES_ERR_INDEX_IS_TOO_SMALL, 137
                                              MSK_RES_ERR_INVALID_OBJ_NAME, 134
                                              MSK_RES_ERR_INVALID_OBJECTIVE_SENSE, 142
MSK_RES_ERR_INF_DOU_INDEX, 138
MSK_RES_ERR_INF_DOU_NAME, 138
                                              MSK_RES_ERR_INVALID_PROBLEM_TYPE, 147
MSK_RES_ERR_INF_INT_INDEX, 138
                                              MSK_RES_ERR_INVALID_SOL_FILE_NAME, 134
                                              MSK_RES_ERR_INVALID_STREAM, 134
MSK_RES_ERR_INF_INT_NAME, 138
MSK_RES_ERR_INF_LINT_INDEX, 138
                                              MSK_RES_ERR_INVALID_SURPLUS, 140
MSK_RES_ERR_INF_LINT_NAME, 138
                                              MSK_RES_ERR_INVALID_SYM_MAT_DIM, 146
MSK_RES_ERR_INF_TYPE, 138
                                              MSK_RES_ERR_INVALID_TASK, 134
MSK_RES_ERR_INFEAS_UNDEFINED, 146
                                              MSK_RES_ERR_INVALID_UTF8, 145
MSK_RES_ERR_INFINITE_BOUND, 142
                                              MSK_RES_ERR_INVALID_VAR_NAME, 134
MSK_RES_ERR_INT64_TO_INT32_CAST, 146
                                              MSK_RES_ERR_INVALID_WCHAR, 145
{\tt MSK\_RES\_ERR\_INTERNAL},\ 145
                                              MSK_RES_ERR_INVALID_WHICHSOL, 138
MSK_RES_ERR_INTERNAL_TEST_FAILED, 146
                                              MSK_RES_ERR_JSON_DATA, 137
MSK_RES_ERR_INV_APTRE, 139
                                              MSK_RES_ERR_JSON_FORMAT, 137
MSK_RES_ERR_INV_BK, 139
                                              MSK_RES_ERR_JSON_MISSING_DATA, 137
                                              MSK_RES_ERR_JSON_NUMBER_OVERFLOW, 137
MSK_RES_ERR_INV_BKC, 139
MSK_RES_ERR_INV_BKX, 139
                                              MSK_RES_ERR_JSON_STRING, 137
MSK_RES_ERR_INV_CONE_TYPE, 140
                                              MSK_RES_ERR_JSON_SYNTAX, 137
MSK_RES_ERR_INV_CONE_TYPE_STR, 140
                                              MSK_RES_ERR_LAST, 139
MSK_RES_ERR_INV_MARKI, 144
                                              MSK_RES_ERR_LASTI, 140
MSK_RES_ERR_INV_MARKJ, 144
                                              MSK_RES_ERR_LASTJ, 140
MSK_RES_ERR_INV_NAME_ITEM, 140
                                              MSK_RES_ERR_LAU_ARG_K, 147
MSK_RES_ERR_INV_NUMI, 144
                                              MSK_RES_ERR_LAU_ARG_M, 147
MSK_RES_ERR_INV_NUMJ, 144
                                              MSK_RES_ERR_LAU_ARG_N, 147
MSK_RES_ERR_INV_OPTIMIZER, 143
                                              MSK_RES_ERR_LAU_ARG_TRANS, 147
                                              {\tt MSK\_RES\_ERR\_LAU\_ARG\_TRANSA},\,147
MSK_RES_ERR_INV_PROBLEM, 143
MSK_RES_ERR_INV_QCON_SUBI, 142
                                              MSK_RES_ERR_LAU_ARG_TRANSB, 147
MSK_RES_ERR_INV_QCON_SUBJ, 142
                                              MSK_RES_ERR_LAU_ARG_UPLO, 147
MSK_RES_ERR_INV_QCON_SUBK, 142
                                              MSK_RES_ERR_LAU_INVALID_LOWER_TRIANGULAR_MATRIX,
MSK_RES_ERR_INV_QCON_VAL, 142
MSK_RES_ERR_INV_QOBJ_SUBI, 142
                                              MSK_RES_ERR_LAU_INVALID_SPARSE_SYMMETRIC_MATRIX,
MSK_RES_ERR_INV_QOBJ_SUBJ, 142
                                                       147
MSK_RES_ERR_INV_QOBJ_VAL, 142
                                              MSK_RES_ERR_LAU_NOT_POSITIVE_DEFINITE, 147
MSK_RES_ERR_INV_SK, 140
                                              MSK_RES_ERR_LAU_SINGULAR_MATRIX, 147
MSK_RES_ERR_INV_SK_STR, 140
                                              MSK_RES_ERR_LAU_UNKNOWN, 147
MSK_RES_ERR_INV_SKC, 140
                                              MSK_RES_ERR_LICENSE, 132
MSK_RES_ERR_INV_SKN, 140
                                              MSK_RES_ERR_LICENSE_CANNOT_ALLOCATE, 133
                                              {\tt MSK\_RES\_ERR\_LICENSE\_CANNOT\_CONNECT,\ 133}
MSK_RES_ERR_INV_SKX, 140
MSK_RES_ERR_INV_VAR_TYPE, 139
                                              MSK_RES_ERR_LICENSE_EXPIRED, 132
MSK_RES_ERR_INVALID_ACCMODE, 144
                                              MSK_RES_ERR_LICENSE_FEATURE, 132
MSK_RES_ERR_INVALID_AIJ, 143
                                              MSK_RES_ERR_LICENSE_INVALID_HOSTID, 133
MSK_RES_ERR_INVALID_AMPL_STUB, 146
                                              MSK_RES_ERR_LICENSE_MAX, 132
MSK_RES_ERR_INVALID_BARVAR_NAME, 134
                                              MSK_RES_ERR_LICENSE_MOSEKLM_DAEMON, 132
                                              MSK_RES_ERR_LICENSE_NO_SERVER_LINE, 133
MSK_RES_ERR_INVALID_COMPRESSION, 144
MSK_RES_ERR_INVALID_CON_NAME, 134
                                              MSK_RES_ERR_LICENSE_NO_SERVER_SUPPORT, 133
```

```
MSK_RES_ERR_LICENSE_SERVER, 132
                                              MSK_RES_ERR_NAME_MAX_LEN, 144
                                              MSK_RES_ERR_NAN_IN_BLC, 143
MSK_RES_ERR_LICENSE_SERVER_VERSION, 133
MSK_RES_ERR_LICENSE_VERSION, 132
                                              MSK_RES_ERR_NAN_IN_BLX, 143
MSK_RES_ERR_LINK_FILE_DLL, 133
                                              MSK_RES_ERR_NAN_IN_BUC, 143
MSK_RES_ERR_LIVING_TASKS, 134
                                              MSK_RES_ERR_NAN_IN_BUX, 143
                                              {\tt MSK\_RES\_ERR\_NAN\_IN\_C,\ 143}
MSK_RES_ERR_LOWER_BOUND_IS_A_NAN, 141
                                              MSK_RES_ERR_NAN_IN_DOUBLE_DATA, 143
MSK_RES_ERR_LP_DUP_SLACK_NAME, 136
MSK_RES_ERR_LP_EMPTY, 136
                                              MSK_RES_ERR_NEGATIVE_APPEND, 139
MSK_RES_ERR_LP_FILE_FORMAT, 136
                                              MSK_RES_ERR_NEGATIVE_SURPLUS, 139
MSK_RES_ERR_LP_FORMAT, 136
                                              MSK_RES_ERR_NEWER_DLL, 133
MSK_RES_ERR_LP_FREE_CONSTRAINT, 136
                                              MSK_RES_ERR_NO_BARS_FOR_SOLUTION, 146
MSK_RES_ERR_LP_INCOMPATIBLE, 136
                                              MSK_RES_ERR_NO_BARX_FOR_SOLUTION, 146
MSK_RES_ERR_LP_INVALID_CON_NAME, 137
                                              MSK_RES_ERR_NO_BASIS_SOL, 143
MSK_RES_ERR_LP_INVALID_VAR_NAME, 136
                                              {\tt MSK\_RES\_ERR\_NO\_DUAL\_FOR\_ITG\_SOL},\ 145
MSK_RES_ERR_LP_WRITE_CONIC_PROBLEM, 136
                                              MSK_RES_ERR_NO_DUAL_INFEAS_CER, 144
MSK_RES_ERR_LP_WRITE_GECO_PROBLEM, 136
                                              MSK_RES_ERR_NO_INIT_ENV, 134
MSK_RES_ERR_LU_MAX_NUM_TRIES, 145
                                              MSK_RES_ERR_NO_OPTIMIZER_VAR_TYPE, 143
MSK_RES_ERR_MAX_LEN_IS_TOO_SMALL, 140
                                              MSK_RES_ERR_NO_PRIMAL_INFEAS_CER, 144
MSK_RES_ERR_MAXNUMBARVAR, 138
                                              MSK_RES_ERR_NO_SNX_FOR_BAS_SOL, 145
MSK_RES_ERR_MAXNUMCON, 138
                                              MSK_RES_ERR_NO_SOLUTION_IN_CALLBACK, 144
MSK_RES_ERR_MAXNUMCONE, 141
                                              MSK_RES_ERR_NON_UNIQUE_ARRAY, 147
MSK_RES_ERR_MAXNUMQNZ, 138
                                              MSK_RES_ERR_NONCONVEX, 140
MSK_RES_ERR_MAXNUMVAR, 138
                                              MSK_RES_ERR_NONLINEAR_EQUALITY, 140
                                              MSK_RES_ERR_NONLINEAR_FUNCTIONS_NOT_ALLOWED,
MSK_RES_ERR_MIO_INTERNAL, 147
MSK_RES_ERR_MIO_INVALID_NODE_OPTIMIZER, 149
                                                       142
MSK_RES_ERR_MIO_INVALID_ROOT_OPTIMIZER, 148
                                              MSK_RES_ERR_NONLINEAR_RANGED, 140
MSK_RES_ERR_MIO_NO_OPTIMIZER, 143
                                              MSK_RES_ERR_NR_ARGUMENTS, 137
MSK_RES_ERR_MISSING_LICENSE_FILE, 132
                                              MSK_RES_ERR_NULL_ENV, 134
MSK_RES_ERR_MIXED_CONIC_AND_NL, 143
                                              MSK_RES_ERR_NULL_POINTER, 134
                                              MSK_RES_ERR_NULL_TASK, 134
MSK_RES_ERR_MPS_CONE_OVERLAP, 135
MSK_RES_ERR_MPS_CONE_REPEAT, 135
                                              MSK_RES_ERR_NUMCONLIM, 139
MSK_RES_ERR_MPS_CONE_TYPE, 135
                                              MSK_RES_ERR_NUMVARLIM, 139
MSK_RES_ERR_MPS_DUPLICATE_Q_ELEMENT, 135
                                              MSK_RES_ERR_OBJ_Q_NOT_NSD, 141
MSK_RES_ERR_MPS_FILE, 135
                                              MSK_RES_ERR_OBJ_Q_NOT_PSD, 141
MSK_RES_ERR_MPS_INV_BOUND_KEY, 135
                                              MSK_RES_ERR_OBJECTIVE_RANGE, 139
MSK_RES_ERR_MPS_INV_CON_KEY, 135
                                              MSK_RES_ERR_OLDER_DLL, 133
MSK_RES_ERR_MPS_INV_FIELD, 135
                                              MSK_RES_ERR_OPEN_DL, 133
MSK_RES_ERR_MPS_INV_MARKER, 135
                                              MSK_RES_ERR_OPF_FORMAT, 136
MSK_RES_ERR_MPS_INV_SEC_NAME, 135
                                              MSK_RES_ERR_OPF_NEW_VARIABLE, 136
MSK_RES_ERR_MPS_INV_SEC_ORDER, 135
                                              MSK_RES_ERR_OPF_PREMATURE_EOF, 137
MSK_RES_ERR_MPS_INVALID_OBJ_NAME, 136
                                              MSK_RES_ERR_OPTIMIZER_LICENSE, 132
MSK_RES_ERR_MPS_INVALID_OBJSENSE, 136
                                              MSK_RES_ERR_OVERFLOW, 143
                                              MSK_RES_ERR_PARAM_INDEX, 137
MSK_RES_ERR_MPS_MUL_CON_NAME, 135
{\tt MSK\_RES\_ERR\_MPS\_MUL\_CSEC},\ 135
                                              MSK_RES_ERR_PARAM_IS_TOO_LARGE, 137
MSK_RES_ERR_MPS_MUL_QOBJ, 135
                                              MSK_RES_ERR_PARAM_IS_TOO_SMALL, 137
MSK_RES_ERR_MPS_MUL_QSEC, 135
                                              MSK_RES_ERR_PARAM_NAME, 137
MSK_RES_ERR_MPS_NO_OBJECTIVE, 135
                                              MSK_RES_ERR_PARAM_NAME_DOU, 137
MSK_RES_ERR_MPS_NON_SYMMETRIC_Q, 135
                                              MSK_RES_ERR_PARAM_NAME_INT, 137
MSK_RES_ERR_MPS_NULL_CON_NAME, 135
                                              MSK_RES_ERR_PARAM_NAME_STR, 137
MSK_RES_ERR_MPS_NULL_VAR_NAME, 135
                                              MSK_RES_ERR_PARAM_TYPE, 138
MSK_RES_ERR_MPS_SPLITTED_VAR, 135
                                              MSK_RES_ERR_PARAM_VALUE_STR, 138
                                              {\tt MSK\_RES\_ERR\_PLATFORM\_NOT\_LICENSED},\ 133
MSK_RES_ERR_MPS_TAB_IN_FIELD2, 136
MSK_RES_ERR_MPS_TAB_IN_FIELD3, 136
                                              MSK_RES_ERR_POSTSOLVE, 143
MSK_RES_ERR_MPS_TAB_IN_FIELD5, 136
                                              MSK_RES_ERR_PRO_ITEM, 140
MSK_RES_ERR_MPS_UNDEF_CON_NAME, 135
                                              MSK_RES_ERR_PROB_LICENSE, 132
MSK_RES_ERR_MPS_UNDEF_VAR_NAME, 135
                                              MSK_RES_ERR_QCON_SUBI_TOO_LARGE, 142
MSK_RES_ERR_MUL_A_ELEMENT, 139
                                              MSK_RES_ERR_QCON_SUBI_TOO_SMALL, 142
MSK_RES_ERR_NAME_IS_NULL, 144
                                              MSK_RES_ERR_QCON_UPPER_TRIANGLE, 142
```

```
MSK_RES_ERR_QOBJ_UPPER_TRIANGLE, 142
MSK_RES_ERR_READ_FORMAT, 135
MSK_RES_ERR_READ_LP_MISSING_END_TAG, 136
MSK_RES_ERR_READ_LP_NONEXISTING_NAME, 136
MSK_RES_ERR_REMOVE_CONE_VARIABLE, 141
MSK_RES_ERR_REPAIR_INVALID_PROBLEM, 144
MSK_RES_ERR_REPAIR_OPTIMIZATION_FAILED, 144
MSK_RES_ERR_SEN_BOUND_INVALID_LO, 145
MSK_RES_ERR_SEN_BOUND_INVALID_UP, 145
MSK_RES_ERR_SEN_FORMAT, 145
MSK_RES_ERR_SEN_INDEX_INVALID, 145
MSK_RES_ERR_SEN_INDEX_RANGE, 145
MSK_RES_ERR_SEN_INVALID_REGEXP, 145
MSK_RES_ERR_SEN_NUMERICAL, 145
MSK_RES_ERR_SEN_SOLUTION_STATUS, 145
MSK_RES_ERR_SEN_UNDEF_NAME, 145
MSK_RES_ERR_SEN_UNHANDLED_PROBLEM_TYPE, 145
MSK_RES_ERR_SERVER_CONNECT, 149
MSK_RES_ERR_SERVER_PROTOCOL, 149
MSK_RES_ERR_SERVER_STATUS, 149
{\tt MSK\_RES\_ERR\_SERVER\_TOKEN},\ 149
MSK_RES_ERR_SIZE_LICENSE, 132
MSK_RES_ERR_SIZE_LICENSE_CON, 132
MSK_RES_ERR_SIZE_LICENSE_INTVAR, 132
MSK_RES_ERR_SIZE_LICENSE_NUMCORES, 146
MSK_RES_ERR_SIZE_LICENSE_VAR, 132
MSK_RES_ERR_SOL_FILE_INVALID_NUMBER, 141
MSK_RES_ERR_SOLITEM, 138
MSK_RES_ERR_SOLVER_PROBTYPE, 139
MSK_RES_ERR_SPACE, 133
MSK_RES_ERR_SPACE_LEAKING, 134
MSK_RES_ERR_SPACE_NO_INFO, 134
MSK_RES_ERR_SYM_MAT_DUPLICATE, 146
MSK_RES_ERR_SYM_MAT_HUGE, 143
MSK_RES_ERR_SYM_MAT_INVALID, 143
MSK_RES_ERR_SYM_MAT_INVALID_COL_INDEX, 146
MSK_RES_ERR_SYM_MAT_INVALID_ROW_INDEX, 146
MSK_RES_ERR_SYM_MAT_INVALID_VALUE, 146
MSK_RES_ERR_SYM_MAT_NOT_LOWER_TRINGULAR,
        146
MSK_RES_ERR_TASK_INCOMPATIBLE, 144
MSK_RES_ERR_TASK_INVALID, 144
MSK_RES_ERR_TASK_WRITE, 144
MSK_RES_ERR_THREAD_COND_INIT, 133
MSK_RES_ERR_THREAD_CREATE, 133
MSK_RES_ERR_THREAD_MUTEX_INIT, 133
MSK_RES_ERR_THREAD_MUTEX_LOCK, 133
MSK_RES_ERR_THREAD_MUTEX_UNLOCK, 133
MSK_RES_ERR_TOCONIC_CONSTR_NOT_CONIC, 149
MSK_RES_ERR_TOCONIC_CONSTR_Q_NOT_PSD, 149
MSK_RES_ERR_TOCONIC_CONSTRAINT_FX, 149
{\tt MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_RA},\,149
MSK_RES_ERR_TOCONIC_OBJECTIVE_NOT_PSD, 149
MSK_RES_ERR_TOO_SMALL_MAX_NUM_NZ, 139
MSK_RES_ERR_TOO_SMALL_MAXNUMANZ, 139
{\tt MSK\_RES\_ERR\_UNB\_STEP\_SIZE},\ 145
MSK_RES_ERR_UNDEF_SOLUTION, 139
MSK_RES_ERR_UNDEFINED_OBJECTIVE_SENSE, 142
```

```
MSK_RES_ERR_UNHANDLED_SOLUTION_STATUS, 147
MSK_RES_ERR_UNKNOWN, 133
MSK_RES_ERR_UPPER_BOUND_IS_A_NAN, 141
MSK_RES_ERR_UPPER_TRIANGLE, 147
MSK_RES_ERR_USER_FUNC_RET, 142
MSK_RES_ERR_USER_FUNC_RET_DATA, 142
MSK_RES_ERR_USER_NLO_EVAL, 142
MSK_RES_ERR_USER_NLO_EVAL_HESSUBI, 142
MSK_RES_ERR_USER_NLO_EVAL_HESSUBJ, 142
MSK_RES_ERR_USER_NLO_FUNC, 142
MSK_RES_ERR_WHICHITEM_NOT_ALLOWED, 138
MSK_RES_ERR_WHICHSOL, 138
MSK_RES_ERR_WRITE_LP_FORMAT, 136
MSK_RES_ERR_WRITE_LP_NON_UNIQUE_NAME, 136
MSK_RES_ERR_WRITE_MPS_INVALID_NAME, 136
MSK_RES_ERR_WRITE_OPF_INVALID_VAR_NAME, 136
MSK_RES_ERR_WRITING_FILE, 136
MSK_RES_ERR_XML_INVALID_PROBLEM_TYPE, 146
MSK_RES_ERR_Y_IS_UNDEFINED, 142
```

# **INDEX**

Symbols	mosek command line option, 14
-=	-paro <name></name>
mosek command line option, 14	mosek command line option, 14
-a	-q < name >
mosek command line option, 13	mosek command line option, 14
-anapro	-r
mosek command line option, 13	mosek command line option, 14
-anasoli <name></name>	-removeitg
mosek command line option, 13	mosek command line option, 14
-anasolo <name></name>	-rout < name >
mosek command line option, 13	mosek command line option, 14
-basi <name></name>	-sen < file >
mosek command line option, 13	mosek command line option, 14
-baso <name></name>	-silent
mosek command line option, 13	mosek command line option, 14
-d <name> <value></value></name>	-toconic
mosek command line option, 13	mosek command line option, 14
-dbgmem <name></name>	-V
mosek command line option, 13	mosek command line option, 14
-f	-W
mosek command line option, 13	mosek command line option, 14
-h, -?	-X
mosek command line option, 13	mosek command line option, 14
-info <name></name>	Λ
mosek command line option, 14	Α
-infrepo <name></name>	AMPL
mosek command line option, 14	outlev, 18
-inti <name></name>	wantsol, 18
mosek command line option, 13	arguments
-into <name></name>	command line tool, 9
mosek command line option, 13	
-itri <name></name>	В
mosek command line option, 13	basis identification, 41
-itro <name></name>	basis type
mosek command line option, 13	sensitivity analysis, 71
-l,-L <dir></dir>	bound
mosek command line option, 14	constraint, 25
-max	variable, 25
mosek command line option, 14	,
-min	C
mosek command line option, 14	CBF format, 204
-n	certificate
mosek command line option, 14	dual, 27, 30, 31, 33
-out <name></name>	primal, 27, 29, 31
mosek command line option, 14	command line tool
-p <name>, -pari <name></name></name>	arguments, 9
r	G

complementarity, 26	duality, 26
cone	Ц
dual, 29	Н
quadratic, 28	hot-start, 43
rotated quadratic, 28	I
semidefinite, 30	I
conic optimization, 28	infeasibility, 27, 29, 31
infeasibility, 29	conic optimization, 29
interior-point, 45	linear optimization, 27
termination criteria, 46	semidefinite, 31
constraint	infeasible, 58
bound, 25	dual, 26, 27, 30, 31, 33
matrix, 25, 33	primal, 25, 27, 29, 31, 39, 46
quadratic, 32	problem, 25, 27, 29, 31
constraints	infeasible problems, 58
lower limit, 33	installation, 6
upper limit, 33	requirements, 6
convex interior-point	troubleshooting, 6
optimizers, 49	integer
cut, 51	optimizer, 50
D	integer feasible
	solution, 52
decision	integer optimization, 50
variables, 33	$\operatorname{cut}$ , 51
determinism, 38	delayed termination criteria, 52
dual	objective bound, 51
certificate, 27, 30, 31, 33	optimality gap, 53
cone, 29	relaxation, 51
feasible, 26	termination criteria, 52
infeasible, 26, 27, 30, 31, 33	tolerance, $52$
problem, 25, 29, 30	integer optimizer
variable, 26, 29	logging, 53
duality	interior-point
conic, 29	conic optimization, 45
gap, $26$ linear, $25$	linear optimization, 38
semidefinite, 30	logging, 42, 48
dualizer, 36	optimizer, 38, 45
dualizer, 50	termination criteria, 40, 46
E	interior-point optimizer, 49
eliminator, 36	1
	json format, 220
F	Json format, 220
feasible	L
dual, 26	linear dependency, 36
primal, 25, 39, 46	linear optimization, 25
problem, 25	infeasibility, 27
format	interior-point, 38
CBF, 204	simplex, 43
json, 220	termination criteria, 40, 43
LP, 178	linearity interval, 70
MPS, 183	logging
OPF, 195	integer optimizer, 53
OSiL, 219	interior-point, 42, 48
sol, 227	optimizer, 42, 44, 48
task, 219	simplex, 44
G	lower limit
	constraints, 33
gap	constraints, oo

254 Index

variables, 34	OPF format, 195
LP format, 178	optimal
	solution, 26
M	optimality gap, 53
matrix	optimization
constraint, 25, 33	conic quadratic, 28
MIP, see integer optimization	linear, 25
mixed-integer, see integer	semidefinite, 30
mosek command line option	optimizer
-=, 14	determinism, 38
-a, 13	integer, 50
-anapro, 13	interior-point, 38, 45
-anasoli <name>, 13</name>	logging, 42, 44, 48
-anasolo <name>, 13</name>	parallelization, 37
-basi <name>, 13</name>	selection, 36, 38
-baso <name>, 13</name>	simplex, 43
-d <name> <value>, 13</value></name>	optimizers
-dbgmem <name>, 13</name>	convex interior-point, 49
-f, 13	OSiL format, 219
-1, 13 -h, -?, 13	outlev
	AMPL, 18
-info <name>, 14</name>	AMI L, 10
-infrepo <name>, 14</name>	Р
-inti <name>, 13</name>	·
-into <name>, 13</name>	pair sensitivity analysis
-itri <name>, 13</name>	optimal partition type, 71
-itro <name>, 13</name>	parallelization, 37
-l,-L <dir>, 14</dir>	parameter
-max, 14	simplex, 44
-min, 14	parameter file, 12
-n, 14	presolve, 35
-out $\langle \text{name} \rangle$ , 14	eliminator, 36
-p <name>, -pari <name>, 14</name></name>	linear dependency check, 36
-paro <name>, 14</name>	numerical issues, 36
-q <name>, 14</name>	primal
-r, 14	certificate, 27, 29, 31
-removeitg, 14	feasible, 25, 39, 46
-rout <name>, 14</name>	infeasible, 25, 27, 29, 31, 39, 46
-sen <file>, 14</file>	problem, 25, 29, 30
-silent, 14	solution, 25
-toconic, 14	primal-dual
-v, 14	problem, 38, 45
-w, 14	solution, 26
-x, 14	problem
MPS format, 183	dual, 25, 29, 30
free, 195	feasible, 25
NI	infeasible, 25, 27, 29, 31
N	primal, 25, 29, 30
near-optimal	primal-dual, 38, 45
solution, 41, 48, 52	unbounded, 27
numerical issues	
presolve, 36	Q
scaling, 37	quadratic
simplex, 44	constraint, 32
	quadratic cone, 28
O	quadratic optimization, 32
objective, 25	quality
objective bound, 51	solution, 53
objective vector, 33	,

Index 255

R		dual, 26, 29
relaxation, 51 rotated quadratic cone, 28	varia	semidefinite, 30 ables
S		decision, 33 lower limit, 34
scaling, 37		upper limit, 34
semidefinite	W	
cone, 30	wan	taol
infeasibility, 31	wan	AMPL, 18
variable, 30		Tivii L, 10
semidefinite optimization, 30		
sensitivity analysis, 69 basis type, 71		
shadow price, 70		
simplex		
linear optimization, 43		
logging, 44		
numerical issues, 44		
optimizer, 43		
parameter, 44		
termination criteria, 43		
sol format, 227		
solution		
file format, 227 integer feasible, 52		
near-optimal, 41, 48, 52		
optimal, 26		
primal, 25		
primal-dual, 26		
quality, 53		
Т		
task format, 219		
termination criteria		
conic optimization, 46		
delayed, 52		
integer optimization, 52		
interior-point, 40, 46		
linear optimization, 40, 43		
simplex, 43		
tolerance, 41, 48, 52		
thread, 37 tolerance		
integer optimization, 52		
termination criteria, 41, 48, 52		
troubleshooting		
installation, 6		
U		
unbounded		
problem, 27		
upper limit		
constraints, 33		
variables, 34		
V		
variable, 25		
bound, 25		

256 Index