Abgabe - Übungsblatt [6]

Angewandte Mathematik: Numerik

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17. Dezember 2020

Aufgabe 1

Here comes your text ...

Aufgabe 2

And some more text ...

Aufgabe 3

$$A = \begin{pmatrix} -2.000 & -2.000 & -2.000 \\ -2.000 & -1.000 & -1.000 \\ 1.000 & 0.000 & -1.000 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -2.000 & -2.000 & -2.000 \\ -2.000 & -1.000 & -1.000 \\ 1.000 & 0.000 & -1.000 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -2.000 \\ -2.000 \\ 1.000 \end{pmatrix}$$

$$||a_1|| = \sqrt{(-2)^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$$

$$v_1 = a_1 + sign(A_{11})||a_1||e_1 = \begin{pmatrix} -2.000 \\ -2.000 \\ 1.000 \end{pmatrix} - 3 \times \begin{pmatrix} 1.000 \\ 0.000 \\ 0.000 \end{pmatrix} = \begin{pmatrix} -5.000 \\ -2.000 \\ 1.000 \end{pmatrix}$$

$$H_1 = I - 2 \cdot \frac{v_1 \cdot v_1^T}{v_1^T \cdot v_1} = \begin{pmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{pmatrix} - \frac{2}{30} \cdot \begin{pmatrix} -5.000 \\ -2.000 \\ 1.000 \end{pmatrix} \cdot \begin{pmatrix} -5.000 & -2.000 & 1.000 \end{pmatrix} =$$

$$\begin{pmatrix} -0.667 & -0.667 & 0.333 \\ -0.667 & 0.733 & 0.133 \\ 0.333 & 0.133 & 0.933 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0.600 & 0.467 \\ -0.800 & -1.733 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.600 \\ -0.800 \end{pmatrix}$$

$$||a_2|| = \sqrt{0.6^2 + (-0.8^2} = \sqrt{1} = 1$$

$$v_2 = a_2 + sign(A_{22})||a_2||e_2 = \begin{pmatrix} 0.600 \\ -0.800 \end{pmatrix} + 1 \times \begin{pmatrix} 1.000 \\ 0.000 \end{pmatrix} = \begin{pmatrix} 1.600 \\ -0.800 \end{pmatrix}$$

$H_2 = I - 2 \cdot \frac{v_2 \cdot v_2^T}{v_2^T \cdot v_2} = \begin{pmatrix} 1.000 & 0.000 \\ 0.000 & 1.000 \end{pmatrix} - \frac{2}{80} \cdot \begin{pmatrix} 1.600 \\ -0.800 \end{pmatrix} \cdot \begin{pmatrix} 1.600 & -0.800 \end{pmatrix} = \begin{pmatrix} -0.600 & 0.800 \\ 0.800 & 0.600 \end{pmatrix}$

$A_3 = \begin{pmatrix} -0.667 \end{pmatrix}$

$a_3 = \begin{pmatrix} -0.667 \end{pmatrix}$

$||a_3|| = \sqrt{(-\frac{2}{3})^2} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

$v_3 = a_3 + sign(A_{33})||a_3||e_3 = \begin{pmatrix} -0.667 \end{pmatrix} - \frac{2}{3} \times \begin{pmatrix} 1.000 \end{pmatrix} = \begin{pmatrix} -1.333 \end{pmatrix}$

$H_3 = I - 2 \cdot \frac{v_3 \cdot v_3^T}{v_3^T \cdot v_3} = \begin{pmatrix} 1.000 \end{pmatrix} - \frac{2}{144} \cdot \begin{pmatrix} -1.333 \end{pmatrix} \cdot \begin{pmatrix} -1.333 \end{pmatrix} = \begin{pmatrix} -1.000 \end{pmatrix}$

$R = H_3 H_2 H_1 A = \begin{pmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & -1.000 \end{pmatrix} \times \begin{pmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & -0.600 & 0.800 \\ 0.000 & 0.800 & 0.600 \end{pmatrix} \times \begin{pmatrix} -0.667 & -0.667 & 0.333 \\ -0.667 & 0.733 & 0.133 \\ 0.333 & 0.133 & 0.933 \end{pmatrix} \times \begin{pmatrix} -2.000 & -2.000 & -2.000 \\ -2.000 & -1.000 & -1.000 \\ 1.000 & 0.000 & -1.000 \end{pmatrix} = \begin{pmatrix} 3.000 & 2.000 & 1.667 \\ 0.000 & -1.000 & -1.667 \\ 0.000 & -0.000 & 0.667 \end{pmatrix}$

$Q = H_1 H_2 H_3 = \begin{pmatrix} -0.667 & -0.667 & 0.333 \\ -0.667 & 0.733 & 0.133 \\ 0.333 & 0.133 & 0.933 \end{pmatrix} \times \begin{pmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & -0.600 & 0.800 \\ 0.000 & 0.800 & 0.600 \end{pmatrix} \times \begin{pmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & -1.000 \end{pmatrix} = \begin{pmatrix} -0.667 & 0.667 & 0.333 \\ -0.667 & -0.333 & -0.667 \\ 0.333 & 0.667 & -0.667 \end{pmatrix}$

Aufgabe 4

Vorgegebenes Gerüst verändert

```
import numpy as np
from numpy import linalg as la
from typing import Union

def HouseholderQR(x: np.ndarray) -> Union[np.ndarray, int
    ]:
    m, n = A.shape
    Q = np.eye(m)
    R = A.copy()

    for j in range(n):
        x = R[j:, j]
        normx = np.linalg.norm(x)
        rho = -np.sign(x[0])
        u1 = x[0] - rho * normx
        u = x / u1
        u[0] = 1
        beta = -rho * u1 / normx

        R[j:, :] = R[j:, :] - beta * np.outer(u, u).dot(R
            [j:, :])
```

```
            Q[:, j:] = Q[:, j:] - beta * Q[:, j:].dot(np.
                outer(u, u))

    return Q, R


if(__name__=="__main__"):
    A=np.random.rand(4,3)
    Q,R=HouseholderQR(A)
    print("The following matrix should be upper
        triangular:")
    print(R)
    print("If the solution consitutes a decomposition,
        the following is near zero:")
    print(la.norm(A-np.dot(Q,R)))
    print("If Q is unitary, the following is near zero:")
    print(la.norm(np.dot(Q.T,Q)-np.eye(Q.shape[0])))
```