

$$1) \quad \begin{array}{ccc} P_x(\omega) & \xrightarrow{SLT} & P_y(\omega) \\ S_x(\omega) & h(t) & S_y(\omega) \end{array}$$

$$\mu_y = 0,2V = E[Y]$$

$$a) \quad \mu_x = 0,8V = E[X]$$

$$\mu_y = \mu_x \int_{-\infty}^{\infty} h(t) dt = \mu_x H(0)$$

Pela transformada:

$$h(t) \leftrightarrow H(\omega)$$

$$e^{-at} u(t) \leftrightarrow \frac{1}{a+j\omega} \Rightarrow e^{-4t} u(t) \leftrightarrow \frac{1}{4+j\omega}$$

$$\therefore H(\omega) = \frac{1}{4+j\omega} \Rightarrow H(0) = \frac{1}{4} \Rightarrow \frac{\mu_y}{H(0)} = \mu_x \Rightarrow \mu_x = 4 \cdot 0,2$$

$$\therefore \mu_x = 0,8V$$

Pela integral

$$\int_0^{\infty} e^{-4t} dt \frac{(-4)}{(1-4)} = -\frac{1}{4} e^{-4t} \Big|_0^{\infty} = -\frac{1}{4} (0 - 1) \therefore \int_{-\infty}^{\infty} h(t) dt = \frac{1}{4} \rightarrow \text{que leva ao mesmo resultado}$$

b) Saída $\Rightarrow y(t)$

$$DEP: S_y(\omega) = S_x(\omega) \cdot |H(\omega)|^2$$

$$S_x(\omega) = \mathcal{F}[e^{-5|t|}] = \frac{2 \cdot 5}{5^2 + \omega^2} \therefore S_x(\omega) = \frac{10}{25 + \omega^2}$$

$$|H(\omega)| = \left| \frac{1}{4+j\omega} \right| = \frac{1}{|4+j\omega|} = \frac{1}{\sqrt{4^2 + \omega^2}} = \frac{1}{\sqrt{16 + \omega^2}} \Rightarrow |H(\omega)|^2 = \frac{1}{16 + \omega^2}$$

$$\hookrightarrow \text{módulo de um n.º complexo: } |Z| = \sqrt{(\text{Re}[Z])^2 + (\text{Im}[Z])^2}$$

$$\therefore S_y(\omega) = \frac{10}{(25 + \omega^2)(16 + \omega^2)} \quad [\omega/\text{rad/s}]$$

Como $\omega = 2\pi f \Rightarrow S_y(f) = \frac{10}{((2\pi f)^2 + 25)(16 + (2\pi f)^2)} \left[\frac{W}{H_z} \right]$

c) $R_y(\tau) = \mathcal{F}^{-1}[S_y(\omega)]$

Manipulando: Se $x = \omega^2$

$$S_y(x) = \frac{10}{(25+x)(16+x)} = \frac{A}{25+x} + \frac{B}{16+x} = \frac{A(16+x) + B(25+x)}{(25+x)(16+x)}$$

$$\Rightarrow A(16+x) + B(25+x) = 10$$

$$p(x) = -25 \Rightarrow -9A = 10 \therefore A = -\frac{10}{9}$$

$$p(x) = -16 \Rightarrow 9B = 10 \therefore B = \frac{10}{9}$$

$$\Rightarrow S_y(\omega) = -\frac{10}{9} \cdot \frac{1}{(25+\omega^2)} + \frac{10}{9} \cdot \frac{1}{(16+\omega^2)}$$

$$R_y(\tau) = -\frac{10}{9} \cdot \mathcal{F}^{-1}\left[\frac{1}{25+\omega^2}\right] + \frac{10}{9} \cdot \mathcal{F}^{-1}\left[\frac{1}{16+\omega^2}\right]$$

em *: $\frac{1}{5^2+\omega^2} \cdot \frac{2 \cdot 5}{2 \cdot 5} = \frac{1}{10} \cdot \frac{2 \cdot 5}{5^2+\omega^2} \longleftrightarrow \frac{1}{10} e^{-5|\tau|}$

em *: $\frac{1}{4^2+\omega^2} \cdot \frac{2 \cdot 4}{2 \cdot 4} = \frac{1}{8} \cdot \frac{2 \cdot 4}{4^2+\omega^2} \longleftrightarrow \frac{1}{8} e^{-4|\tau|}$

$$\therefore R_y(\tau) = -\frac{1}{9} e^{-5|\tau|} + \frac{5}{36} e^{-4|\tau|}$$

d) $P_x = R_x(0) = e^{-5 \cdot 0} \cdot P_x = 1W$

$$P_y = R_y(0) = -\frac{1}{9} \cdot e^{-5 \cdot 0} + \frac{5}{36} e^{-4 \cdot 0} \therefore P_y = \frac{1}{36} \text{ W}$$

2) $X(t)$ é WSS e $\mu_x = 0$
 $Y(t) = X(at), a > 1$

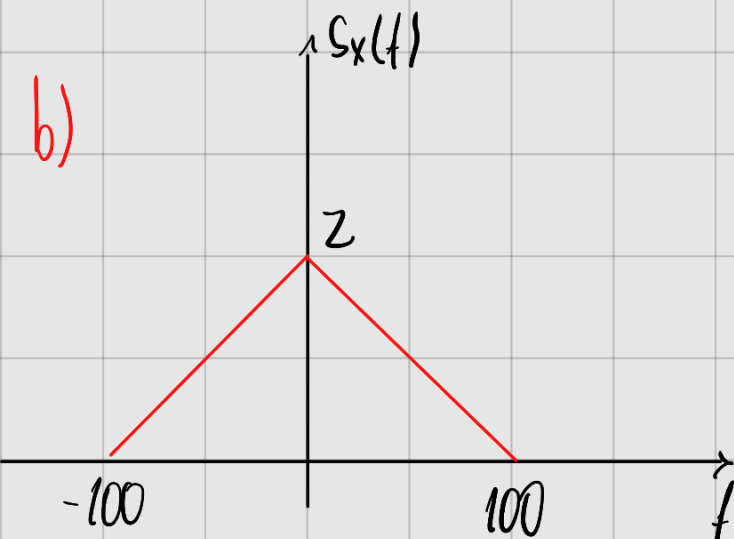
$$R_x(t, t+\tau) = \overline{X(t) X(t+\tau)} \Rightarrow \underline{R_x(at, a(t+\tau))} = \overline{X(at) X(a(t+\tau))} = \underline{\overline{X(at) X(at+a\tau)}} = R_x(a\tau)$$

O intervalo de tempo entre as duas parcelas é de $a\tau \Rightarrow R_x(a\tau)$

$$R_y(t, t+\tau) = \overline{Y(t) Y(t+\tau)} = \overline{X(at) X(a(t+\tau))} = R_x(a\tau)$$

$$\therefore \underline{R_y(\tau) = R_x(a\tau)}$$

b)



$$R_y(\tau) = R_x(a\tau) \Rightarrow S_y(f) = \frac{1}{|a|} S_x\left(\frac{f}{a}\right)$$

Como $a > 1 \Rightarrow$ amplitude dividida por a
 \Rightarrow freq. estendida de a

Compressão no tempo \Rightarrow extensão na frequência



$$c) P_y = \int_{-\infty}^{\infty} S_y(f) df = 2 \cdot 100 \mu \cdot \frac{2}{\pi} \cdot \frac{1}{2} \therefore P_y = 200 \mu W$$