Manuscript

A Histogram Equalization Algorithm Based on Building a Grey Level Binary Tree Dynamically

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Abstract: Histogram equalization algorithms may produce artifacts when the image is enhanced. Complex filtering algorithms are used for good results, or end-to-end deep learning networks are implemented for image enhancement. The competitive algorithm proposed in this paper uses a binary tree structure to remap grayscale and suppress artifacts. The algorithm can get different degrees of image enhancement results by changing unique variables. The proposed competitive algorithm can also be used to expand the dataset of deep learning tasks. Code and figures are available at https://github.com/F-Quasimo/DEBTHE.

1. Introduction

Histogram equalization (hereinafter referred to as HE) is an effective method to enhance image. The principle is to map the original uneven grey level histogram to the dynamic range of the overall grey-scale evenly by remap the grey scale according to the probability distribution of the original grey level histogram [1]. As is well known, traditional histogram equalization algorithm could sometimes cause vast combination of grey scale, which could result in loss of image detail, as shown by the red circled in Fig. 1. To improve HE effect, various modified algorithms have been put forward subsequently; most of them are based on histogram clipping scheme [2] and multiple-histogram scheme, the latter of which could be classified further into Bi-HE [3, 4] and Multi-HE [5]. For example, Kim put forward "Brightness Preserving Bi-Histogram Equalization" (BBHE) [3] which has improved control of image brightness; Wang raised "Dualistic sub-image histogram equalization" (DSIHE) [4], which performs far better than BBHE in terms of preserving both brightness and entropy of the image. In addition, "Recursive Mean-Separate Histogram Equalization" (RMSHE) [6] and "Recursive Sub-Image Histogram Equalization" (RSIHE) [7] have realized control of image enhancement by controlling the times of recursive partitioning of histogram. "Entropy-based dynamic sub-HE" (EDSHE) [8] obtained the image with high-information entropy by treating image histogram through computing image information entropy. "HE with variable enhancement degree" (HEwVED) [9] combined the original image with the classic HE algorithm in linearity. All the modified methods mentioned above have optimized the effects of histogram equalization to various degrees partially or wholly. Traditional mathematics is not always suitable for dealing with discrete grey-scale mapping; therefore, it has been put forward by some scholars that the methods of non-linear mapping [10-12] and packet processing are more applicable for image processing under discrete condition. Huynh-The [13] et al. has achieved better effect by adopting dynamic regulation method combining the peculiar characteristics of histogram. Also, some other scholars have put forward new histogram equalization method, most of which are with pertinence. Paul [14] control the histogram with a plateau limit to avoid over enhancement. Subramani [15] uses histogram equalization techniques to optimize medical images DHE [16]partition the image histogram based on local minima.. In this essay, several representative algorithms are selected from the research field of global histogram for comparison.

Enlightened by Huffman tree [17], the algorithm proposed in this essay realizes histogram equalization effect by designing an algorithm constructing dynamically binary tree to obtain the grey-scale mapping relation of "original grey scale - mapping grey-scale". Since combination of the nodes of binary trees could not result in directly combination of nodes but to project the information contained by the nodes to deep layer of the binary tree. The final results of the grey scale mapping shall be determined by these binary tresses. By appointing the times of combining binary trees, the proposed algorithm could achieve image enhancement to varying degrees.

2. Related works

The histogram equalization algorithm can maintain the brightness relationship of the scene compared to the bilateral filtering[18] algorithm. In the deep learning task of images, image enhancement is used for data pre-processing and data set expansion. Colour mapping can quickly improve the image effect. High dynamic range images are synthesized by multiple LDR images[19]. The machine learning model proposed by Yue[20] gets better mapping rules; joint filtering[21] methods are also starting to be implemented using deep learning. This section shows the artifacts generated during the histogram equalization process. More implementations are available at https://github.com/F-Quasimo/BBHE_DSIHE_HEwVED_RMSHE_RSIHE_EDSHE_VS2017. Algorithms that do not rely on data sets are expected to be proposed

General steps of histogram equalization algorithm are as follows:

(1) Count up the distribution of gradation of the original image;

(2) Calculate the mapping scope from the input grey scale to the output grey scale based on probability distribution;

(3) Remap the image according to the grey-scale mapping relation.

The mathematical principle adopted by the histogram equalization algorithm is shown as follows:

$$S_k = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{M \times N}\sum_{j=0}^{k} n_j \quad k = 0,1,2...,L-1 \tag{1}[9]$$

In the formula, r_j represents the jth grey level in the original image with the size of $M \times N$, and L refers to the quantity of discrete grey level { r_0, r_1,…, r_{L-1} }. In addition, n_j denotes the number of pixels whose gradation is r_j in the original image; $p_r(r_j)$ indicates the percentage of r_j in original image; S_k is the mapped gradation of r_j.

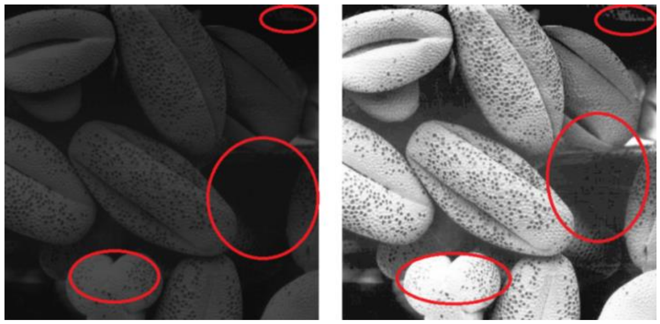

Fig.1. *seed src image and HE result*

In this essay an image with 8 grey scales (L=8，[19,4,5,6,7]) and 200 pixels is processed according to equation (1) to discuss the reasons for information loss in image process with histogram equalization algorithm (HE). Changes of the variables in equation (1) during the processing are as shown in Table (1). Where, G_{num} is the grey scale quantity used in the image (the number of grey scales which n_j is not 0). It could be seen from Table (1) there are 6 grey scales whose corresponding pixels are not 0, respectively [18,4,5]. Processed with histogram equalization algorithm, G_{num} becomes 5, respectively [20,7], when n_{jk} is the pixel quantity of grey scale r_j in image processed. Treated with histogram equalization algorithm, one grey level pixel is combined into other grey levels. In Table (1), it is represented that grey level 2 and 3 are combined into grey level 5. Combination of grey scales suggests G_{num} decreases, which could cause loss of image information. Actually, in Table 1, data in row n_j are the grey scale histogram of the original image and data in row n_{jk} are the grey scale histogram of processed image.

Table 1 HE algorithm steps

r_j /ID	0	1	2	3	4	5	6	7
n_j	14	76	61	7	20	22	0	0
$p_r(r_j)$	0.07	0.38	0.305	0.035	0.1	0.11	0	0
$\sum p_r$	0.07	0.45	0.755	0.79	0.89	1	1	1
S_k	0	3	5	5	6	7	7	7
n_k	14	0	0	76	0	68	20	22

3. Approach proposed

In this section, histogram equalization algorithm based on constructing dynamically binary tree (hereinafter referred to as DEBTHE) is illustrated in detail. It has been proven that multi-histogram approach is a good way to improve [3, 4, 6-9].

Histogram equalization algorithm could enhance the image by remapping grey scale, so it is inevitable for various grey scales to be mapped to the same grey scale, which could result in loss of image detail. Huffman encoding could express the information appearing frequently more conveniently with short codes, which performed in Huffman tree as node with heavily weight being placed in the superficial layer while node with light weight being placed in the deep layer. Inspired by Huffman tree, this essay puts forward histogram equalization algorithm by constructing sequential binary tree dynamically.

3.1 Dynamically constructing a binary tree

Grey scales whose n_j is not 0 are screened out, which are respectively $\{r_1, r_2, ..., r_j\}$. Then these grey scales are ranked into J sequential binary trees containing only root nodes. The weight of these binary trees is n_j. Fig. 2 demonstrates the binary tree sharing the same data as Part 2.

New binary trees are constructed by combining neighbouring binary trees. The rules to construct binary tree are as the follows:

(a) Two combined binary trees shall be adjacent in sequence;
(b) Two binary trees constitute the left and right subtrees of the newly combined binary tree with the order unchanged.
(c) The weight of the new binary tree is the sum of the weights of these two binary trees to be combined;
(d) The binary tree combination shall be the one with the minimum binary tree weight;
(e) Only a group of (two) binary trees could be combined once.

Fig.3. shows the binary trees after merging for one time. Fig.4. shows the binary trees after merging for 2 to 5 times.

3.2 Calculation conducted for remapping table

After the construction of binary trees, n binary trees could be left finally and L grey levels of the original image could be divided averagely into n parts to be allocated to all the binary trees. So, each binary tree has a dynamic scope with the length of G_{savg}, which could be represented by equation (2).

$$G_{savg} = \frac{L-1}{n} \tag{2}$$

Hence, the dynamic range $Range_f$ assigned to each binary tree is listed as below:

$$Range_f = \left[(m-1)\times G_{savg},\ \mathrm{m}\times G_{savg}\right) \quad m = 1,2,3...n \tag{3}$$

After grey scale scopes are allocated to all the binary trees, the grey scales ($Range = [a,b)$) of the parent node are allocated respectively to the left and right subtrees. W_l is the weight of the left subtree and W_r is the weight of the right subtree. $[a,b)$ is the grey scale range allocated to the binary tree. $[a,b)$ is assigned to the left subtree according to equation 4 and to the right subtree according to equation 5. Recursively assign $[a,b)$ from the root node to the leaf node through each subtree. Besides, the leaf node takes the mid-value of the allocated $Range$ as the grey scale value (M_k) which is mapped finally to the node.

$$Range_l = [a, \frac{W_l}{W_l + W_r}(b-a)+a) \tag{4}$$

$$Range_r = [\frac{W_r}{W_l + W_r}(b-a)+a, b) \tag{5}$$

Fig. 2 shows the initial state, that is, all binary trees contain only root node; while Table 2 is about the mapping of grey scale under such condition. Corresponding to S_k, M_k could be obtained after r_j is treated with DEBTHE. Considering Table 2 and Table 1, the value G_{num} of the image obtained by the proposed algorithm does not become less than G_{num} of the original image. Fig. 3 shows detailed mapping process of the proposed algorithm after the binary trees are combined once. Similarly, the G_{num} of the image treated with the proposed algorithm does not change; however, this could not explain all the conditions and the proposed algorithm could just avoid decrease of G_{num} in the original image as far as possible.

Different mapping tables could be obtained by combining binary trees for various times. More combinations of binary trees see Fig. 4. Times for combining binary trees is a parameter required to be defined in the proposed algorithm.

By constructing dynamically binary trees, grey scales with larger n_j are placed in the superficial layer while those with smaller n_j are placed in deep layer of the tree so that grey scales with larger proportion of pixel quantity in the image could obtain larger range during the allocation stage. This algorithm combines grey scales with smaller n_j so that the influences of combining grey scales could be weakened.

Moreover, the grey-scale distribution of the original image is further expanded to the entire dynamic range

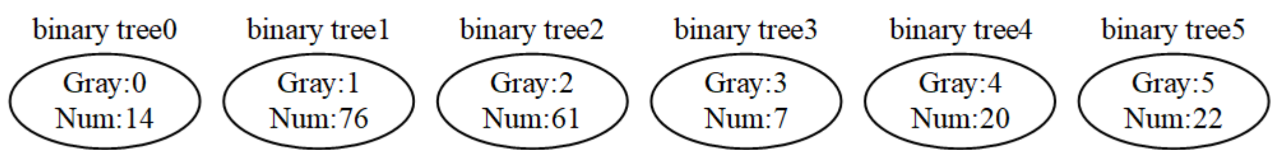

Fig.2. *Initial binary trees*

Table 2 Grey scale mapping calculation table

r_j /ID	0	1	2	3	4	5	6	7
n_j	14	76	61	7	20	22	0	0
S_k	0	1	2	4	5	6		
n_k	14	76	61	7	20	22		

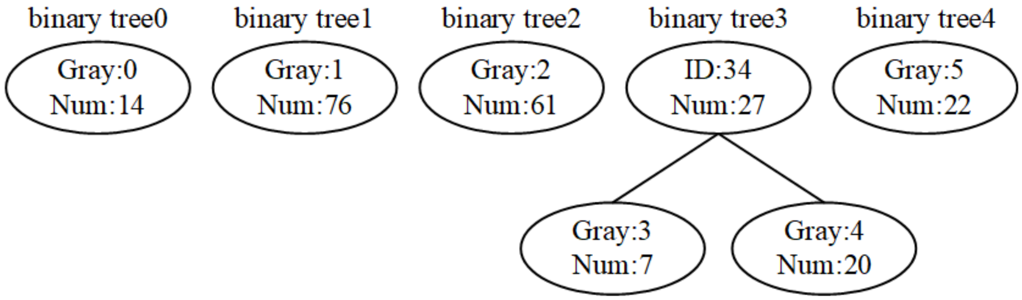

Fig.3. *Binary trees merged once*

Table 3 Grey scale mapping calculation table for binary trees merged once

r_j	0	1	2	3	4	5	6	7
n_j	14	76	61	7	20	22	0	0
r_j /ID	0	1	2	34		5	6	7
Num	14	76	61	27		22	0	0
Range	[0, 1.4)	[1.4,2.8)	[2.8,4.2)	[4.2,5.6)		[5.6,7]		
$Range_{sub}$	[0, 1.4)	[1.4,2.8)	[2.8,4.2)	[4.2,4.56)	[4.56,5.6)	[5.6,7]		
S_k	0	2	3	4	5	6		
n_k	14	76	61	7	20	22	0	0

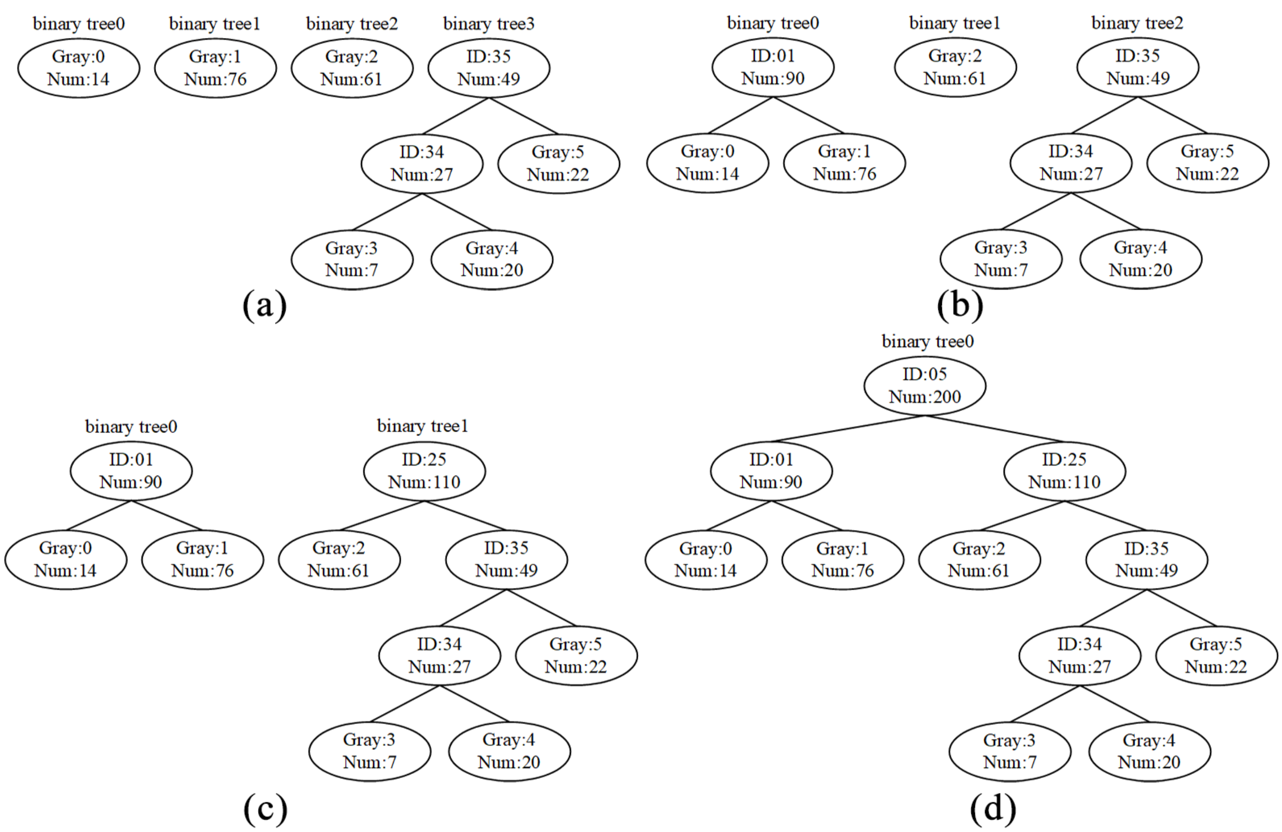

Fig.4.*Binary Trees*

(a) Binary trees merged twice, ***(b)*** Binary trees merged 3 times, ***(c)*** Binary trees merged 4 times, ***(d)*** Binary tree merged 5 times

4. Experimental results and analysis

Fig. 5 shows the "skeleton" image and the results after treatments by different algorithms. Besides the algorithm proposed in this essay, some algorithms mentioned in Part 1 are also adopted for comparison. In this essay, m is the time for combining binary trees. m need to be specified before the algorithm run.

It could be seen from Fig. 5 that output of histogram equalization which requires no specified parameter is unique. After treatment by some algorithms, artefacts appeared in the image. Histogram equalization algorithm with parameters could reduce loss of the details by adjusting the parameters.

From Fig.5 and Fig. 6, it could be seen that DEBTHE could provide more options than other algorithms. With the increase of the parameters, different enhancing effects could be represented accordingly.

Fig. 7 shows the relations between the image G_{num} (y-axis) and the parameter (x-axis). Those algorithms that do not need to specify input parameters are output with unique results, which is represented in the image as horizontal line. Meanwhile, the value of G_{num} about the result image obtained by these algorithms is usually small. DEBTHE and HEwVED share the same parameter adjustment range, but the histogram obtained by DEBTHE is more even. Comparing with RMSHE and RSIHE, DEBTHE provides more optional ranges. With the increase of r, G_{num} of the result image obtained by RMSHE and RSIHE approach quickly towards L. It could be seen from Fig. 5, when $r > 3$, the result images obtained by these two algorithms are close to the original image. The actual valid options for these two algorithms are $r = 2$ and $r = 3$ [8]. As shown in Fig. 7(c), with the increase of the times for combining binary trees, G_{num} of DEBTHE result image decreases slowly. Combining Fig.5, it could be seen that when $m = 140$, DEBTHE has higher G_{num} value while achieving better image enhancement. That is to say, while enhancing the image, DEBTHE could avoid merge of grey scales to the greatest extent while enhancing the image.

Grey scales mapping relation between the grey scales of the original image and the result image is as shown in Fig.8. Since HEwVED has no mapping relation between the grey scales, it would not be discussed here. X-axis is the grey scale of the original image and the corresponding Y-axis is the new grey scales it is mapped.

It could be seen from Fig. 8 that the mapping relations obtained by algorithms without parameters are unique and similar to one another. After $r > 3$ The mapping relation between RMSHE and RSIHE approach $y = x$, which means the algorithm change scarcely the original image. Different mapping relations could be obtained by the proposed algorithm when different parameters are given. Varying degrees of image enhancing effects could be achieved by DEBTHE.

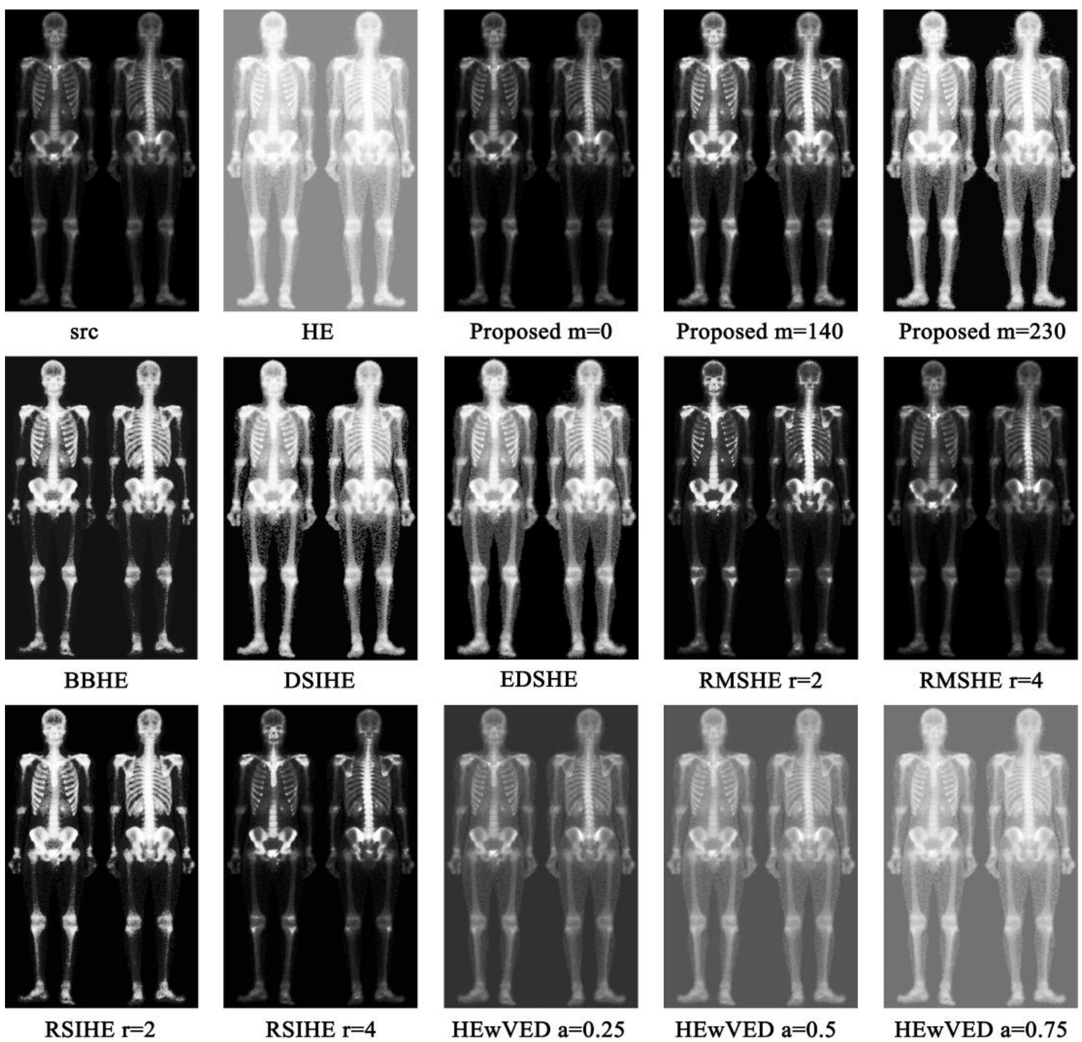

Fig. 5. *Result of skeleton and processed skeleton*

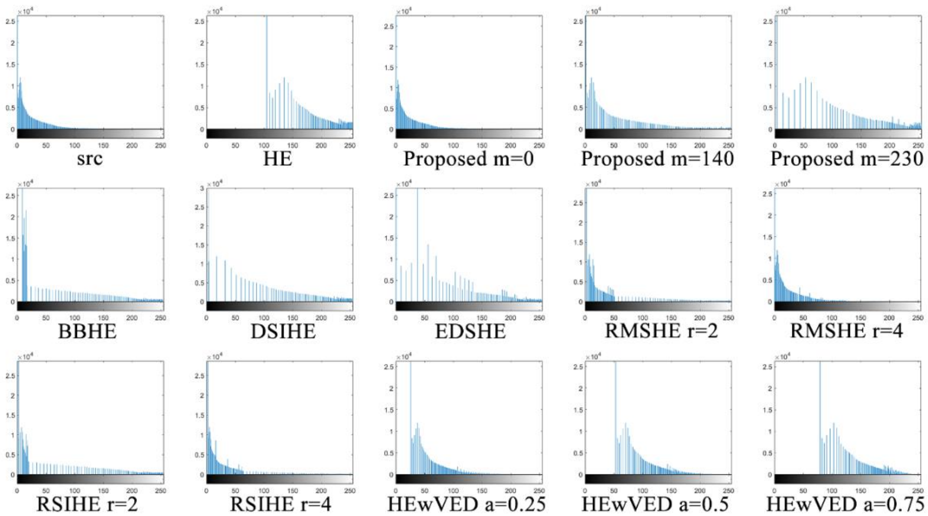

__Fig.6.__ Skeleton histogram.

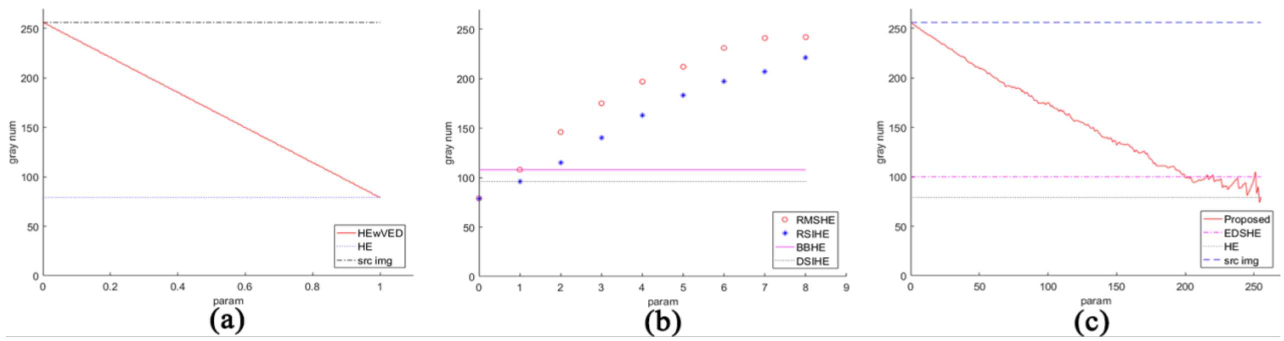

__Fig.7.__ Skeleton.

(a) HEwVED/HE algorithm parameter-grey scale number curve, ***(b)*** RMSHE/RSIHE/BBHE/DSIHE algorithm parameter-grey scale number curve, ***(c)*** Proposed/EDSHE algorithm parameter-grey scale number curve

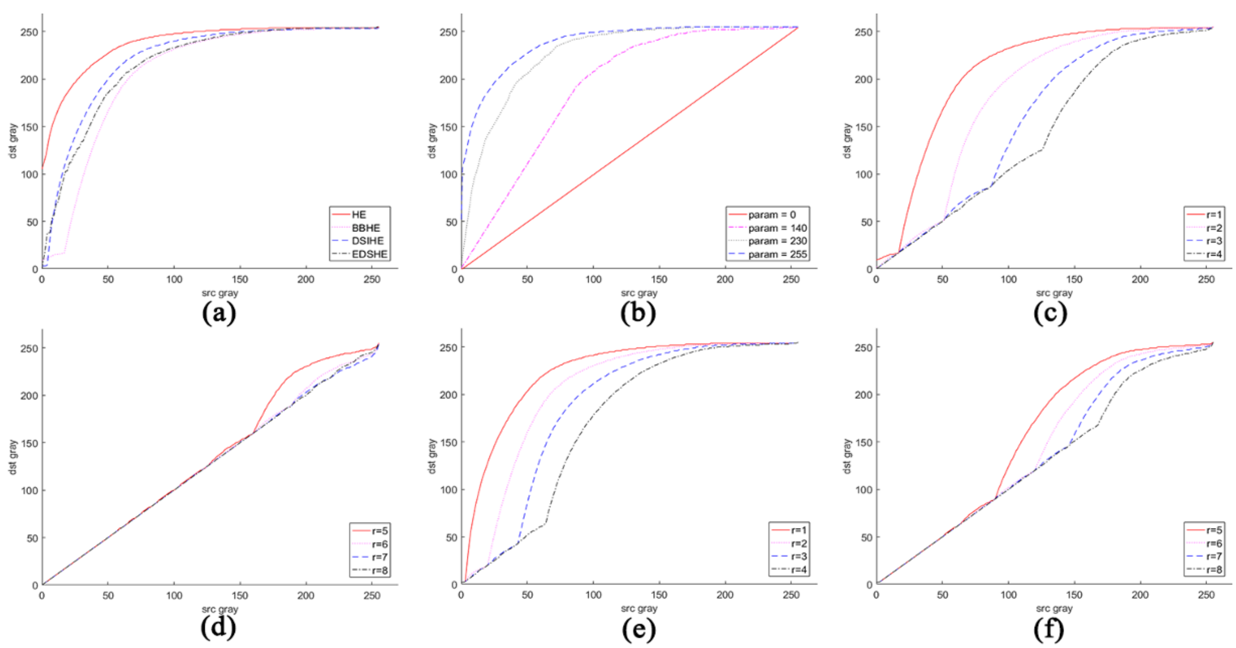

__Fig.8.__ Skeleton.

(a)HE/BBHE/DSIHE/EDSHE grey scale mapping curve, ***(b)*** Proposed algorithm grey scale mapping curve under different parameters, ***(c)*** RMSHE grey scale mapping curve with the parameters of 1 to 4, ***(d)*** RMSHE grey scale mapping curve with the parameters of 5 to 8, ***(e)*** RSIHE grey scale mapping curve with the parameters of 1 to 4, ***(f)*** RSIHE grey scale mapping curve with the parameters of 5 to 8

Fig.9 shows the image of 'Einstein' and its contrast enhanced versions obtained from different algorithms. In addition, Fig.10 to Fig.12 demonstrates the conclusions obtained previously once more. As shown in Fig. 9, HE, Proposed, BBHE, DSIHE and EDSHE show better results compared with other methods. When $m \leq 40$, DEBTHE exhibits better effect in details such as hands and hair. As shown in Fig.10(c), the image treated by DEBTHE has higher G_{num} than that by other algorithms. When G_{num} of the original image is smaller, DEBTHE could exhibit excellent performance more easily. Fig. 13 shows the application of DEBTHE in colourful image after the colour model is change.

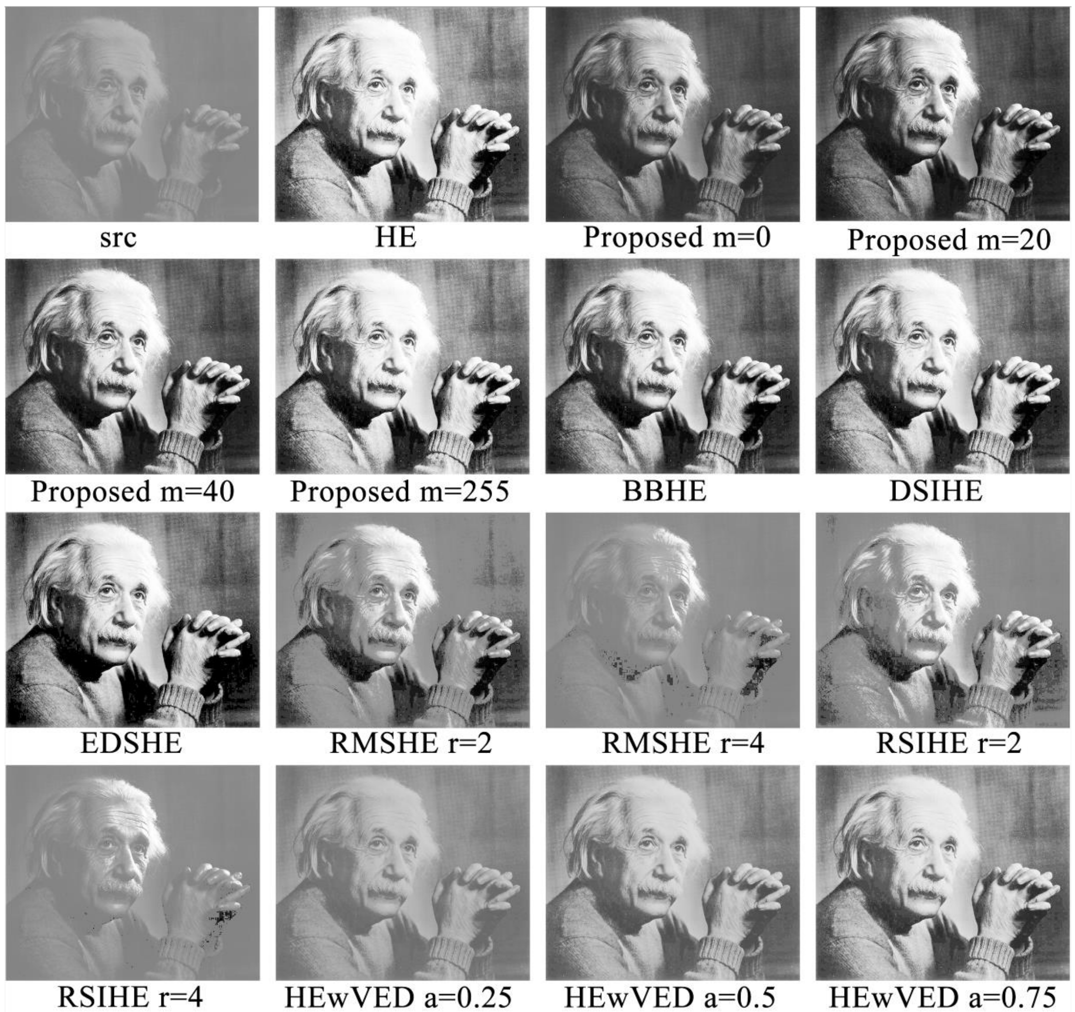

Fig.9. *Results of Einstein and processed Einstein*

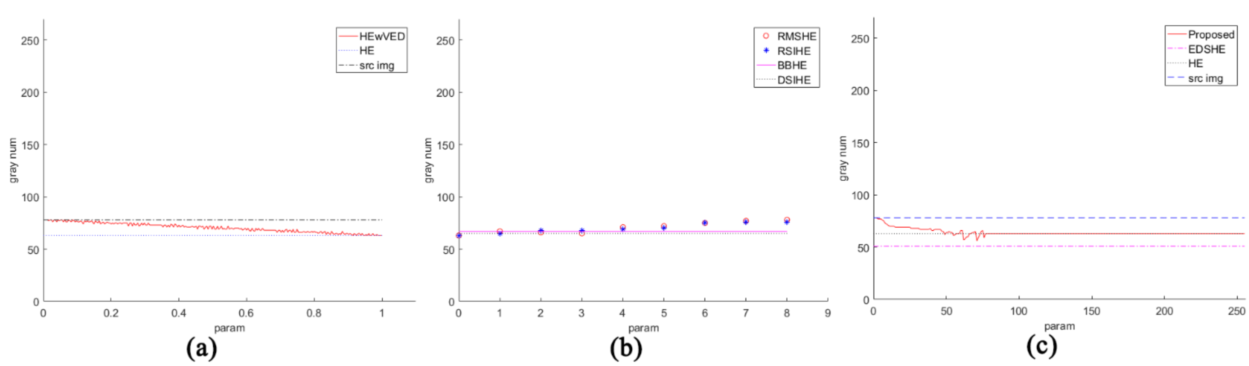

Fig.10. *Einstein.*

(a) HEwVED/HE algorithm parameter-grey scale number curve, ***(b)*** RMSHE/RSIHE/BBHE/DSIHE algorithm parameter-grey scale number curve, ***(c)*** Proposed/EDSHE algorithm parameter-grey scale number curve

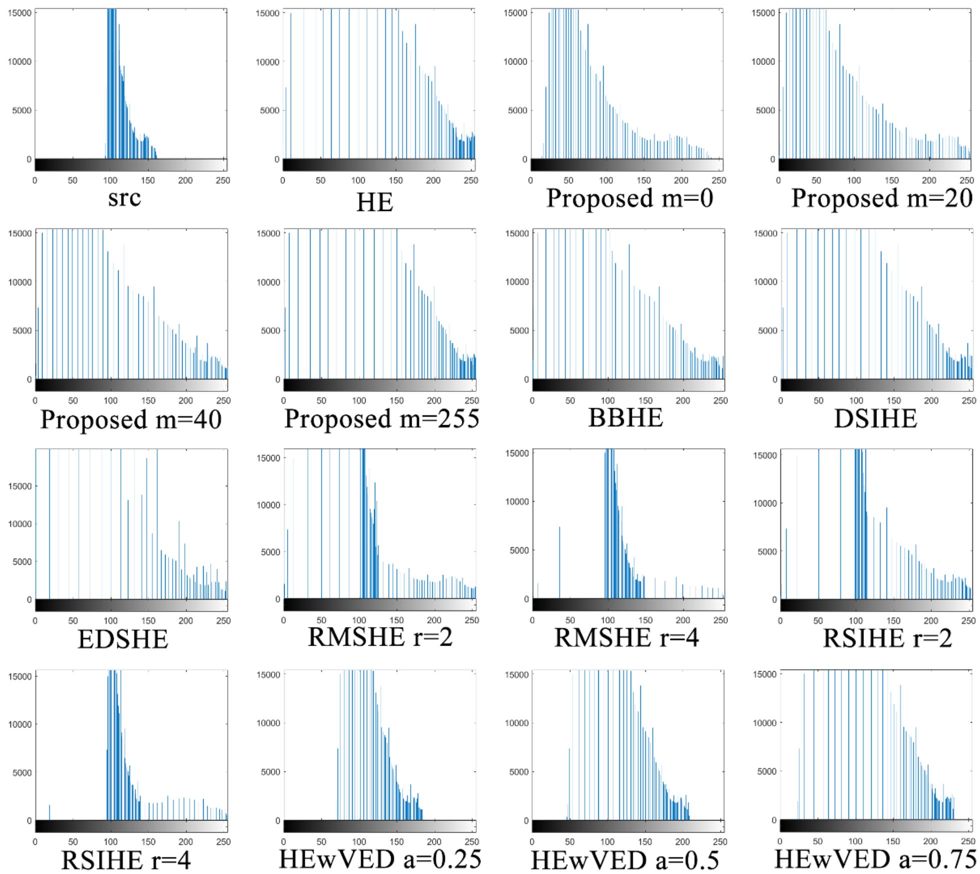

Fig.11. *Einstein histogram*

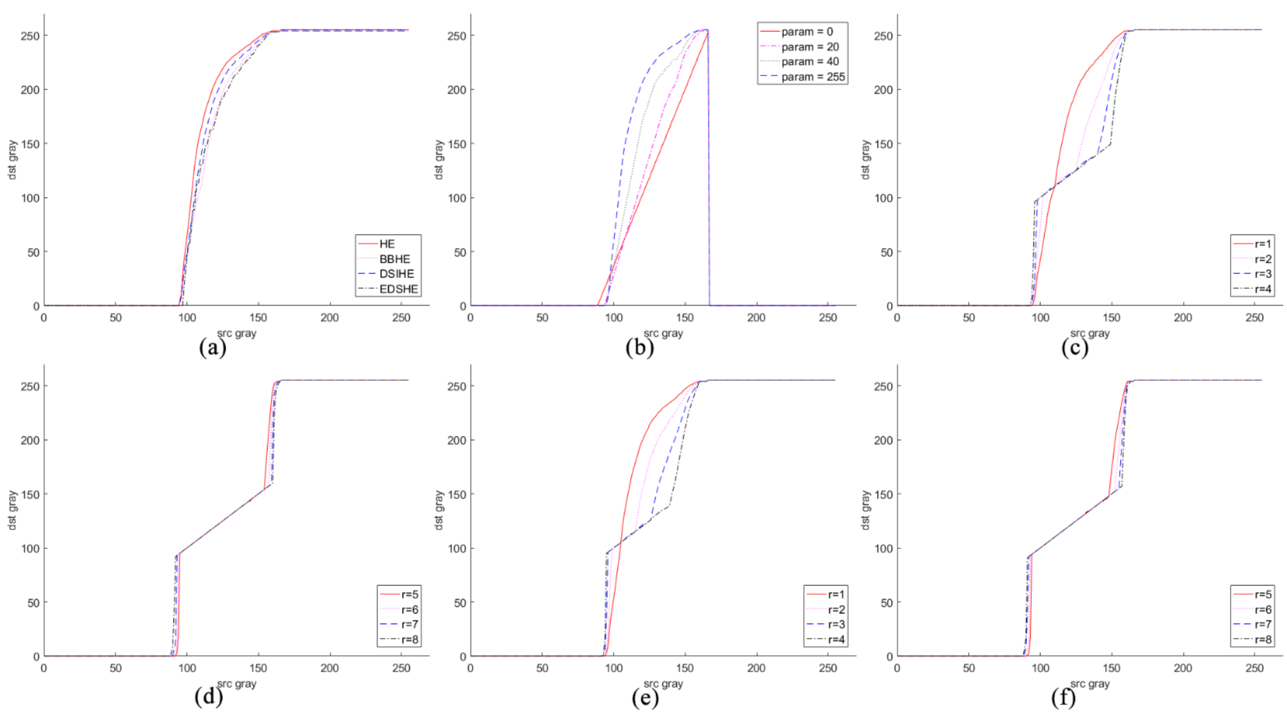

Fig.12. *Einstein.*

(a)HE/BBHE/DSIHE/EDSHE grey scale mapping curve, ***(b)*** Proposed algorithm grey scale mapping curve under different parameters, ***(c)*** RMSHE grey scale mapping curve with the parameters of 1 to 4, ***(d)*** RMSHE grey scale mapping curve with the parameters of 5 to 8, ***(e)*** RSIHE grey scale mapping curve with the parameters of 1 to 4, ***(f)*** RSIHE grey scale mapping curve with the parameters of 5 to 8

(a) (b) (c)

Fig.13. *Colour image.*
(a)src image ***(b)***Proposed m=50 ***(c)***Proposed m=100

5. Conclusions

In this essay, a new method of dynamic histogram equalization for image enhancement is put forward, which is also known as a kind of global based algorithm. The propose algorithm requires to define an input parameter, which could be set manually or calculated automatically with other methods as required by the scenes. With different input parameters, the proposed algorithm could achieve image enhancement to varying degrees. It is inevitable that the algorithm of histogram equalization could generate multiple grey scale mappings to the same grey scale. The proposed algorithm selects the adjacent grey levels with the smallest number of merged pixels when merging the grey levels, which correspondingly minimizes the artefacts generated by the merged grey levels. Verified by experiments, it could be concluded that this method can be applied not only to grey-scale images, but also to colour images after the transformation of colour model. The algorithm proposed in this paper can get different forms of results, and provides an optional solution to expand the data set for deep learning tasks.

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