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T1 ________________	**45538**	F1 ________________
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T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**E**	F4 ________________

2016
Mathematical Contest in Modeling (MCM/ICM) Summary Sheet

Summary

Providing a water resources strategy for the International Clean water Movement (ICM) poses a severe challenge. World is experiencing its severe water shortage, while we need to make more use of what we have. Thus, understanding the changes of water resources is greatly helpful in analyzing the impacts of environmental and social drivers, formulating plans for utilization of water resources, and generating water resource strategy.

This paper explores the method of comprehensive evaluation of water resources providing ability, and sets up an evaluation model applying the fuzzy comprehensive evaluation method. Built on the data of domestic, industrial, agriculture use of water resources in India, we evaluate the water resources providing ability of India by means of the model.

Using authoritative history data, we use Grey Prediction Model to assist in predicting total demand for water through a statistical view and establish the Logistic Retarded Growth Model to predict the population in India in 15 years.

Considering limited water resources and inadequate sewage disposal systems, we design a sensible intervention plan. By using applied mathematics method, we establish the Sewage Disposal Price System, Sewage Disposal Quantity System and Psychological Prediction Model to discuss the impact on the surrounding areas, as well as the entire water ecosystem.

Through our models, water will become a critical issue in India around 2017 with no intervention, while water scarcity will be postponed by 2 years with our intervention. If water resources exploitation extent reaches 90% in India, this severe problem won't happen until 2024.

Analysis and Optimization of Water Resources: with India for example

45538

February 2, 2016

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Keywords: water resources; India; Grey Forecasting Model; Logistic Retarded Growth Model

Contents

1 Introduction

Last year witnessed *The Drinkable Book* made the headlines - this new technology has been tested with over 25 different water sources in 5 countries, and they are starting to pilot in a few villages in Africa and Asia. *"They've completed an important step and there are more to go through."* Dr Lantagne said. Our group wants to try our best to help solve the world's water problem.

Water is the source of life. It is a critical natural resource upon which all social and economic activities and ecosystem functions depend. As it's known to all, world's fresh water resource situation is not optimistic, and there are two primary causes at present. One is environmental (physical) scarcity, and the other one is social (economic) scarcity. One must not only understand the environmental constraints on water supply, but also how social factors influence availability and distribution of clean water.

1.1 Precondition

Water resources development and utilization are the activity of planning, developing, distributing and managing the use of water resources. It is a sub-set of water cycle management. [1] Ideally, water resources development and utilization quotiety are expected to be almost stable.

Another aspect regarding to our issue is the demand for water. Mainly, water requirement comes from primary (agriculture), secondary (production and construction), and tertiary (service) sector of the economy, ecological environment and domestic. The water consumption of tertiary industry and ecological environment consumption account for less than 1 per cent of the total demand, thus could be disregarded, which leaves 3 major influencing factors. Domestic demand is equivalent to the product of water demand per capita, and the population. Gross domestic product (GDP) is a monetary measure of the value of all final goods and services produced in a period (quarterly or yearly). [2] In our case, we take agriculture and industry into account.

1.2 Parameters

Parameters involved in this model are shown as follows:

- Quantity of a region's water resources α
- Quantity of sewage β
- Water resources utilization level λ
- Water resources exploitation extent μ
- Supply of water γ
- Industrial water requirement ω

[1] Wikipedia, the free encyclopedia. Water resource management. *Wikipedia*, January 2016.
[2] Wikipedia, the free encyclopedia. Gross domestic product. *Wikipedia*, January 2016.

- Agricultural water consumption φ
- Water demand per capita ε
- Population θ
- Relation between water supply and demand ρ

1.3 Evaluation Model

We can get α by look-up table, which is related to the region; we can also obtain β by look-up table, which is a social factor; we assume that the social factor λ and μ are constant values. Considering α refers to quantity of a region's water resources, while β refers to the quantity of sewage, we have γ, the social factor

$$\gamma = \alpha \cdot \lambda \cdot \mu - \beta \tag{1}$$

ω are social factors connected with economic development; we can get ε by look-up table, which is connected with the environment as well as the economic development of the region; θ can also be got by look-up table.

Several parameters are related to time, which are β, ω, φ, ε and θ.

According to the parameters shown above, we have the model

$$\rho = \frac{\gamma}{\omega + \varphi + \varepsilon \cdot \theta} \tag{2}$$

The increasing of ρ indicates that the region excessively supplies clean water; and vice versa.

2 Analysis of India water scarcity

Clean water is so desperately needed in India. The Analysis of Indian water scarcity is the test of our model. We pick India whose water is moderately overloaded. And explain both the environmental and social drivers of this status.

2.1 Results Of Evaluation Model

As with Table 1 [3] [4] , this table on water condition of India has appeared in some form.

The quantity of water in India could supply every year γ like Equation (1). Through searching the related resources data, we have $\lambda = 60.0\%$, $\mu = 53.6\%$, thus, supply of water $\gamma = 600.578 km^3/yr$.

The same procedure may be easily adapted to obtain our models for other state parameters, which means ρ = 78.887%.

[3]UN FAO, AQUASTAT database. Food and Agriculture Organization of the United Nations. 2013.

[4]Wikipedia, the free encyclopedia. Water pollution in India Water pollution in India. *Wikipedia*, December 2015.

Parameter	Value	Unit
α	1,911.0	km^3/yr
β	13.99921	km^3/yr
ω/θ	14	$m^3/p/yr$
φ/θ	567	$m^3/p/yr$
ε	46	$m^3/p/yr$
θ	1,214.46	millions

Table 1: Water condition of India

2.2 Explanation On Evaluation Model

Depending on the result of the Evaluation Model, water is heavily overloaded in India. There is not sufficient water for all uses, whether agricultural, industrial or domestic. It has been proposed that when annual per capita renewable freshwater availability is less than 1,700 cubic meters, countries begin to experience periodic or regular water stress. Below 1,000 cubic meters, water scarcity begins to hamper economic development and human health and well-being. Unfortunately, this figure towards India is approximately 1,103 cubic meters, which is close to water scarcity warning line.

- **Environmental drivers**

  Geography

  The Indian climate is heavily influenced by the Himalayas and the Thar Desert, both of which drive the economically and culturally pivotal summer and winter monsoons. The Himalayas prevent cold Central Asian katabatic winds from blowing in, keeping the bulk of the Indian subcontinent warmer than most locations at similar latitudes. The Thar Desert plays a bital role in attracting the moisture-laden south-west summer monsoon winds that, between June and October, provide the majority of India's rainfall. Four major climatic groupings predominate in India: tropical wet, tropical dry, subtropical humid, and montane.

  Climate change

  Climate change could have significant impacts on water resources in India because of the intimate connections between the climate and the hydrological cycle. Rising temperatures will increase evaporation and lead to increases in precipitation. Both droughts and floods may become more frequent in India. Possible impacts include increased eutrophication. Climate change could also mean an increase in demand for farm irrigation, garden sprinklers. There is currently ample evidence that increased hydrology variability and change in climate has and will continue to have a profound impact on the water sector through the hydrology cycle, water availability and water demand.

- **Social drivers**

  Population growth

  The Indian population was 1295.29 million in 2014. Thus, water demand will increase unless there are corresponding increases in water conservation and recycling of this vital resource. Access to water for producing food will be one of the main challenges in the decade to come, which will need to be balanced with importance of managing water itself in a sustainable way, taking into account the impact of climate change and other environmental and social variables.

Expansion of business activity

Business activity ranging from industrialization to services such as tourism and entertainment in India continues to expand rapidly. This expansion requires increased water services including both supply and sanitation, which can lead to more pressure on water resources and the natural ecosystem.

Depletion of aquifers

Due to the expanding human population, competition for water is growing such that many of the world's major aquifers are becoming depleted. This is due both for direct human consumption as well as agricultural irrigation by groundwater.

Pollution

Water pollution is among the main concerns of India today. Many pollutants threaten water supplies, but the discharge of raw sewage into natural waters is the most common method in India. Sewage, sludge, garbage, and even toxic pollutants are all dumped into the water. Even if sewage is treated, problems still arise. In addition to sewage, non point source pollution such as agricultural runoff is a significant source of pollution in India, along with urban stormwater runoff and chemical wastes dumped by industries and governments. [5]

Given the seriousness of India water supply and demand imbalances, India is trapped in a vicious cycle of economic and social development - lacking degree of inadequacy of India water resources aggravation - water contradiction acute, with industrial and agricultural water requirement seize ecological environment water demand - ecological and environmental deterioration - vegetation degradation - economic and social development blocked. Water resources in India are scarce and systems for allocating water resources are scarce too.

3 Grey Forecasting Model

What is happening in India, which has too many people in places where there is not enough water, is a foretaste of what is to come. In a thirsty region, prediction of water by 2031 is necessary for envisioning future scenarios and helping pro-active adaptation strategy.

In this section, we make a prediction comprising total supply and demand tooling GM(1,1) and Logistic Retarded Growth Model.

3.1 Assumptions

Based on Evaluation Model, the ratio of ability to provide clean water is for two figures to decide, which are the total supply and demand of water.

[5]Wikipedia, the free encyclopedia. Water resources. *Wikipedia*, February 2016.

- Water resources exploitation extent was stable in the recent past years. We assume it invariant, which leads total supply of water basically unchanged.
- As we said above in Section 1.1, non-domestic water supply is influenced by agricultural and industrial demand. We believe constant-price GDP is closely related to water consumption. That is non-domestic water supply equals divide constant-price GDP by water productivity.

  We mention seveval theorems here.

  Theorem 3.1. *Constant-price GDP also called real GDP; it is inflation-adjusted GDP. Constant-price GDP measures the value of a country's goods and services in relation to a base year.* [6]

  Theorem 3.2. *Inflation as measured by the annual growth rate of the GDP implicit deflator shows the rate of price change in the economy as a whole. The GDP implicit deflator is the ratio of GDP in current local currency to GDP in constant local currency.* [7]

  Table 2 [8] is on inflation, GDP deflator of India.

  Figure 1 describes the inflation, GDP deflator of India. We can tell that GDP deflator has discrete variation, and a variety of factors involving thorny issues like world economy affect it, thus this index mark is among the hard-to-predict extreme ones. However, in recent years, GDP deflator of India remains anchored to 2% to 10%stability. So, we treat GDP deflator η as a constant, and we will not give value.

[6] Investing Answers. Constant-Price GDP. *Financial Dictionary*, 2016.
[7] The World Bank. Inflation, GDP deflator (annual %). *World Development Indicators*, December 2015.
[8] The World Bank. Inflation, GDP deflator (annual %). *World Development Indicators*, December 2015.

Year	Inflation (annual %)	Year	Inflation	Year	Inflation
1961	2.14542764259727	1962	4.40561674267815	1963	8.35362386003059
1964	8.55167697678436	1965	8.30036942444605	1966	13.2707065560506
1967	8.61620470798519	1968	2.41538358224665	1969	3.34336433990501
1970	1.5622434894957	1971	5.32484104024424	1972	10.8396026988096
1973	17.8297156461729	1974	16.6675157302367	1975	-1.64868154554671
1976	5.98185933336551	1977	5.6372293455353	1978	2.46028231547959
1979	15.7280432071898	1980	11.5083208127639	1981	10.8275819783816
1982	8.09586309896582	1983	8.55285960515417	1984	7.92323284632677
1985	7.19378544706504	1986	6.78940045157084	1987	9.3278933052373
1988	8.23251536615848	1989	8.43680887306273	1990	10.6683038465274
1991	13.7518189440416	1992	8.96515235934562	1993	9.86178285382972
1994	9.98004477534052	1995	9.06270221988214	1996	7.57501828846144
1997	6.47627126322838	1998	8.01016752360506	1999	3.06839552032511
2000	3.644970161058	2001	3.21561601740324	2002	3.71568377657489
2003	3.86779808618248	2004	5.72541322722107	2005	4.23692511895295
2006	6.42259004900228	2007	5.75624346624512	2008	8.66466534681875
2009	6.06382663975592	2010	8.9838126852477	2011	6.39886144585687
2012	7.62581432706857	2013	6.25028547668308	2014	3.03592252416092

Table 2: Inflation, GDP deflator of India

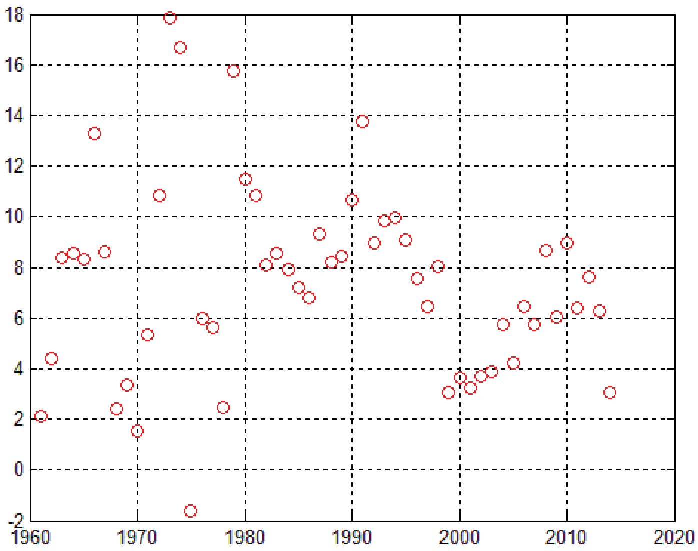

Figure 1: Inflation, GDP deflator of India

Theorem 3.3. *Water productivity is calculated as GDP in constant prices divided by annual total water withdrawal.* [9]

Table 3 [10] is on water productivity on India.

We use curve-fitting, and out put figure 2 describes and predicts the water productivity of India in 15 years.

Since the water productivity of India hasn't changed much in recent 20 years, we take water productivity of India ζ as constant. That is $\zeta = 5 \ US\$/m^3$.

- In consideration of basic living needs for water, we assign $60.955 \ m^3/p/yr$ to ε.

[9] The World Bank. Water productivity. *World Development Indicators*, December 2015.
[10] The World Bank. Water productivity. *World Development Indicators*, December 2015.

Year	Water productivity (constant 2005 US$ GDP per cubic meter of total freshwater withdrawal)	Year	Water productivity
1977	0.502009010450901	1982	0.510462079856866
1987	0.574461186551227	1992	0.746694560746423
2001	1.07430530471852	2012	1.83130887829215
2013	1.95765485601753		

Table 3: Water productivity of India

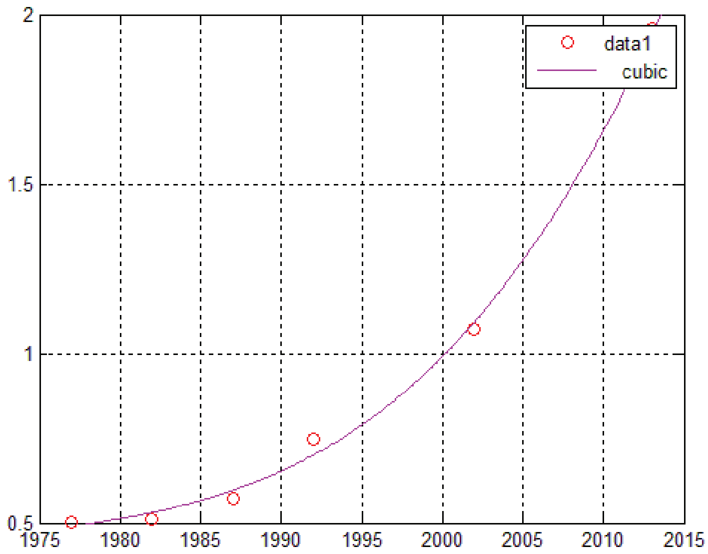

Figure 2: Water Productivity of India in 15 years

3.2 Grey Forecasting Model

Grey theory deals with systems that are characterized by poor information or for which information is lacking. The section presents a Grey Forecasting Model, GM(1,1), using a technique that forecast GDP at market prices (current US$) of India in 15 years.

Assume an original data series to be

$$X^{(0)}(k) = x^0(1), x^0(2), ..., x^0(n) \tag{3}$$

The Accumulated Generation Operation (AGO) is expressed as

$$X^{(1)}(k) = \sum_{n=1}^{k} x^0(n) \tag{4}$$

The first-order differential equation of GM(1,1) model is then give as

$$\frac{dX^{(1)}}{dT} + aX^{(1)} = b \tag{5}$$

The solution for Equation (5) is

$$\widehat{x}^{(1)}(k) = (x^{(0)}(1) - \frac{b}{a})(1 - e^a)e^{-a(k-1)}, k = 2, 3, ..., n \tag{6}$$

Where $x^{(1)}(1) = x^{(0)}(1)$ and the coefficients a and b are called developing and grey input coefficient, respectively. By least-square method, they can be obtained as

$$\begin{pmatrix} a \\ b \end{pmatrix} = (A^T A)^{-1} A^T Y_n \tag{7}$$

Where

$$A = \begin{pmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{pmatrix} \tag{8}$$

$$Y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{9}$$

Create the function document (see Appendix A).

Table 4 [11] is on GDP at market prices (current US$) of India.

Run Result

$G = 133045.15$

which $G = 133045.15 \times 10^8 \;\; US\$$ is estimate of GDP at market prices of India in 2031.

Figure 3 describes and predicts the GDP of India in 15 years.

[11] The World Bank. GDP at market prices (current US$). *World Development Indicators*, December 2015.

Year	GDP at market prices (current US$)	Year	GDP at market prices	Year	GDP at market prices
1960	37679274491.2745	1961	39920452403.4524	1962	42900864969.865
1963	49271095508.0955	1964	57470781032.781	1965	60599264111.7178
1966	46669801600	1967	51014155360	1968	54016411986.6667
1969	59472993626.6667	1970	63517182000	1971	68532271313.2286
1972	72716595884.3124	1973	87014945186.3156	1974	101271489826.241
1975	100199514365.231	1976	104518118776.804	1977	123617837582.482
1978	139708688961.553	1979	155674337010.019	1980	189594121351.874
1981	196883474523.345	1982	204234366470.486	1983	222090283347.207
1984	215878233650.679	1985	236589100981.279	1986	253352444883.275
1987	283926977522.458	1988	301790951204.236	1989	301233728792.841
1990	326608014285.316	1991	274842161318.346	1992	293262722482.422
1993	284194018792.066	1994	333014993709.73	1995	366600193391.349
1996	399787263892.645	1997	423160799040.86	1998	428740690379.961
1999	466866720520.974	2000	476609148165.173	2001	493954161367.563
2002	523968381476.715	2003	618356467437.027	2004	721584805204.777
2005	834214699568.14	2006	949116769619.215	2007	1238699170077.86
2008	1224097069460.46	2009	1365371474048.93	2010	1708458876830.18
2011	1835814449585.35	2012	1831781515472.09	2013	1861801615477.85
2014	2048517438873.54	2015			

Table 4: GDP at market prices of India

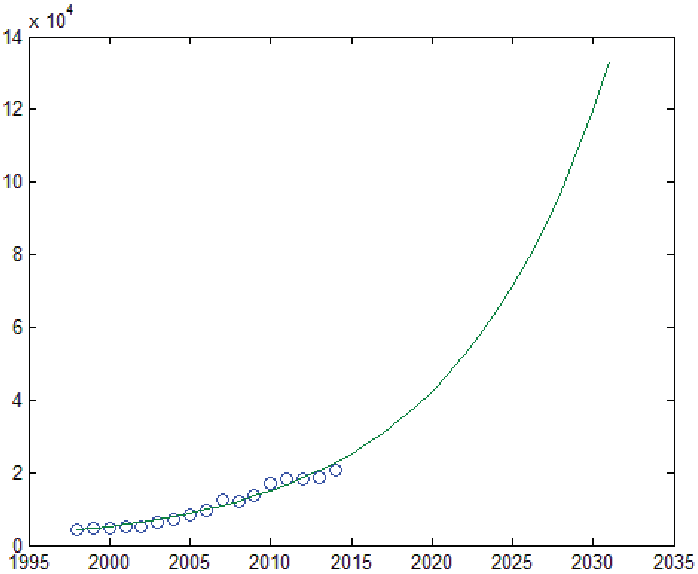

Figure 3: GDP at market prices of India in 15 years

3.3 Logistic Retarded Growth Model

Because of the inhibition of factors such as resource and environment population growth, population increases slow down as the population approaches a certain quantity. We assume that the growth rate of population is decreasing function of x. Set

$$r(x) = r(1 - \frac{x}{x_m}) \tag{10}$$

Which r is inherent growth rate (when x is small), x_m is population carrying capacity (the maximum population size that the resource and environment can sustain indefinitely), so we have the following differential equation:

$$\begin{cases} \frac{dx}{dt} = r \cdot x(1 - \frac{x}{x_m}) \\ x(0) = x_0 \end{cases} \tag{11}$$

Create the function document (see Appendix B).

Table 5 [12] is on the population of India.

Run Result

$a = 29.2225 \quad 0.0280281$

$y1 = 16.6703$

Which a_1 and a_2 represent x_m and r in $x(t) = \frac{x_m}{1+(\frac{x_m}{x_0}-1)\cdot e^{-r\cdot t}}$ respectively, while $y1 = 1.66703 \ \ billion$ is estimate of the population of India in 2031.

Figure 4 describes and predicts the population of India in 15 years.

[12] The World Bank. Population, total. *World Development Indicators*, December 2015.

Year	Population	Year	Population	Year	Population
1960	449661874	1961	458691457	1962	468054145
1963	477729958	1964	487690114	1965	497920270
1966	508402908	1967	519162069	1968	530274729
1969	541844848	1970	553943226	1971	566605402
1972	579800632	1973	593451889	1974	607446519
1975	621703641	1976	636182810	1977	650907559
1978	665936435	1979	681358553	1980	697229745
1981	713561406	1982	730303461	1983	747374856
1984	764664278	1985	782085127	1986	799607235
1987	817232241	1988	834944397	1989	852736160
1990	870601776	1991	888513869	1992	906461358
1993	924475633	1994	942604211	1995	960874982
1996	979290432	1997	997817250	1998	1016402907
1999	1034976626	2000	1053481072	2001	1071888190
2002	1090189358	2003	1108369577	2004	1126419321
2005	1144326293	2006	1162088305	2007	1179685631
2008	1197070109	2009	1214182182	2010	1230984504
2011	1247446011	2012	1263589639	2013	1279498874
2014	1295291543	2015			

Table 5: Population of India

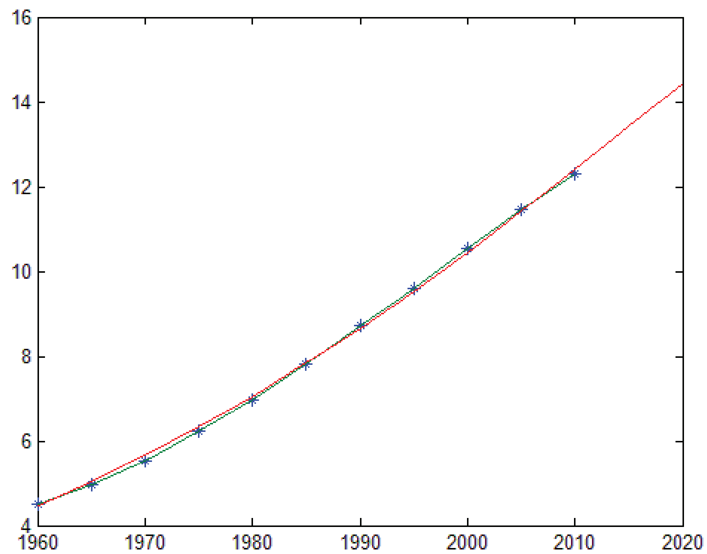

Figure 4: Population of India in 15 years

3.4 Results Of Prediction Model

For constant-price GDP G_{let}

Lemma 3.4.

$$G_{let} = \frac{G}{\eta + 1} \tag{12}$$

when $G = 133045.15 \times 10^8$, and $2\% \leqslant \eta \leqslant 10\%$, we have $G_{let} = \frac{133045.15 \times 10^8}{\eta+1}$, thus $1.2095014 \times 10^{13} \leqslant G_{let} \leqslant 1.3043642 \times 10^{13}$

Lemma 3.5.

$$\omega = \frac{G_{let}}{\zeta} \tag{13}$$

When $\zeta = 5\ US\$/m^3$ as we mentioned before, we have

$$2491.0028\ km^3/yr \leqslant \varphi + \omega \leqslant 2608.7284\ km^3/yr \tag{14}$$

Domestic water demand is

$$\theta \cdot \varepsilon = 16.6703 \times 10^8 \cdot 60.955 = 101.6138137\ km^3/yr \tag{15}$$

Like Equation 2, we have

$$\rho = from\ 23.02\%\ to\ 24.83\% \tag{16}$$

Given the water resources utilization level and water resources exploitation extent basically unchanged, with the population dramatically increasing and huge consumption for GDP growth in 15 years, the relation between water supply and demand will drop from 78.887% to about 24%, which means that water scarcity becomes even tense. Even if in an ideal world, water resources exploitation extent increases to 100%, the relation between water supply and demand is merely 45%, which cannot meet the needs for water of its citizens, and leads to massively shortage of water.

3.5 Impact On the Lives Of the Citizens Of India

Apart from inadequate supply of water, 27% of the villages and 4% to 6% urban population in India lack access to drinking water. There is a genuine concern about the quality of water, which is severely affecting the health. It is reported that over 70% of the water consumed by the rural population in India does not meet the WHO standards. It has been reported that 80% of rural illnesses, 21% of transmissible diseases and 20% of deaths among children in the age group of 5 years, are directly linked to consumption of unsafe water.

In 2031, a large number of people are suffering from fluorosis after consuming water containing more than 1.5 ppm fluoride. Poor sanitation both in rural and urban areas, is another reason for pollution of drinking water sources. Nitrates and harmful germs from human excreta flow and percolate down to contaminate the water tanks and open wells.

Most of the well water used for drinking in irrigated areas is polluted. Excessive irrigation has also been causing further damage to soil productivity. As the water reaching lower layers of soil and the salts present in this region are dissolved in water. Subsequently, these salts come to the top soil through capillary action. Such soils with high concentrations turn into sodic wastelands, unfit for agricultural production. As the people living in these villages are helplessly consuming such hard water, the incidences of illnesses are high.

4 The intervention plan

Since the total availability of water resources is invariable, with one central task of India is at developing economy, boosting GDP should never stagnate. Consumption for GDP growth is essential; domestic water is also requisite; growth of population is difficult to control. Under such circumstances, a sensible intervention plan to remit water scarcity is to sewage disposal. We find that the investigation of the governments makes a great difference to development of sewage disposal.

In this section, we establish the Sewage Disposal Price System, Sewage Disposal Quantity System and Psychological Prediction Model with proportion of investment of the governments as parameters.

4.1 Sewage Disposal Model

Parameters involved in this model are shown as follows:

- Sewage disposal price per unit volume $p\ \ US\$/m^3$
- The proportion of investment of the government x
- The volume of sewage treated per year v
- For a sewage plant, its construction capital to sewage treated quantity per day ratio is approximately constant, which defined as m.
- We assume that the lifetime of a sewage is 20 years. To attract social capitals, we need to provide investors with yield r. The common industry yield is from 8% to 12%. Here, we assign 10% to it.
- The daily operating costs of a sewage plant $z = 0.1\ \ US\$/m^3$

So, the ideal prediction model of P is

$$p = \frac{m}{20} \cdot (1 - x) \cdot r + z \tag{17}$$

4.2 Phychological Prediction Model

Meanwhile, we assume that sewage plant investors are rational. That is they pursue interest. As long as the government invests more, the investors invest more. We establish

Psychological Prediction Model with x and v:

$$v = \beta \cdot (1 - e^{-x^2}) \tag{18}$$

Figure 5 describes the relationship between x and v, also x and p.

By comprehensive evaluation of Figure 5, to make p as small as possible, while v as large as possible, $0.7 \leqslant x \leqslant 0.9$, which means the most optimal proportion of investment of the government is between 70% to 90%.

4.3 Analysis Of the Intervention Plan

In this section, we describe the impact of our intervention plan on India, which is also the strengths and weaknesses of the plan.

- **Strengths**

  Environment

  Sewage plants collect wastewater, make the wastewater meet the emission standard, and recycle it. Its series of treatments not only increase the utilization of freshwater, but also reduce the pollutant, which have an important protection for water, soil and air resources.

  Sewage plants protect groundwater resources from polluting for wastewater seeps into the ground in most cases. They also protect soil and plants downstream.

- **Weaknesses**

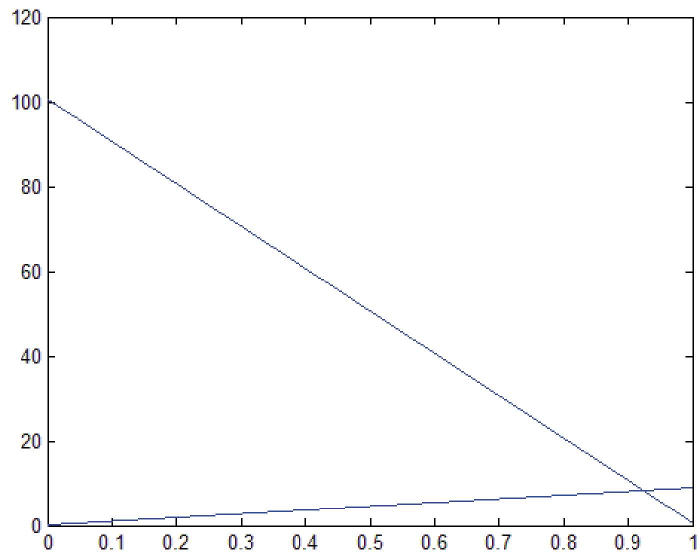

Figure 5: Phychological Prediction Model

Environment

At present, sewage treatment is complicated. It needs sewage pump, wastewater treatment facility-integral equipment which will produce noise pollution. The noise pollution from sewage plants may reduce quality of life.

In another aspect, sewage has complex origins and multiple elements, mainly including sulphur compounds such as H_2S, SO_2, mercaptan and thiophene; nitrogen compounds such as NH_3 and amine; hydrocarbon compound such as alkane, olefin and arene; oxygen-contained compounds such as alcohol and aldehyde. These gases and vapours will spread air pollution.

Economy

In our Sewage Disposal Model, sewage treatment is mainly funded by society, while the investment of government plays a supplementary role. Sewage treatment is not profitable, so sewage project is always at a loss.

5 Water Scarcity Prediction Model

We use the recognized water scarcity warning line, annual per capita renewable freshwater availability is 1,000 cubic meters, as standard. We convert annual per capita freshwater availability to relation between water supply and demand, that means when ρ reaches over 63.1%, the ability of India to provide clean water to meet the minimum needs for water.

We assume that sewage plants in India process 90% to 100% sewage. According to our intervention plan, we have

$$\rho = \frac{\gamma}{\frac{G_{let}}{(\eta+1)\cdot\zeta+\varepsilon\cdot\theta}} \geqslant 63.1\% \tag{19}$$

We integrate data into Table 6.

5.1 Results Of Water Scarcity Prediction Model

In this section, we explain water scarcity in India with no intervention, our intervention, and in an ideal situation.

Year	GDP	Population	Demand Of Water
1995	366600193391	960874982	136.098
2000	476609148165	1053481072	195.841
2005	834214699568	1144326293	291.595
2010	1708458876830	1230984504	448.738
2015	3448924517813	1341058433	708.881
2020	5796817144582	1443169380	1142.000
2025	9743063043035	1545456036	1865.738
2030	11992172785919	1646920019	2280.857

Table 6: Demand of Water in India in 15 years

- **With no intervention**

  Like Equation 1, $\gamma = 600.578\ km^3/yr$, so, the maximum demand for water is $951.787\ km^3/yr$. According to Table 6, we estimate that water scarcity will occur around 2017.

- **With our intervention plan**

  β tends be 0, so $\gamma = 614.5776\ km^3/yr$, the maximum demand for water is $973.974\ km^3/yr$. According to Table 6, we estimate that water scarcity will occur around 2018.

- **Best-case scenario**

  β tends to be 0, while λ is tend to be 100%, so $\gamma = 1146.6\ km^3/yr$. so, the maximum demand for water is $1817.115\ km^3/yr$. According to Table 6, we estimate that water scarcity will occur around 2024.

Considering backward technique of sewage treatment and huge amount of sewage in India, there is no doubt that sewage disposal is a feasible scheme at present. Our Sewage Disposal Model learns from sustainable strategy, aiming at the least capital from the government and the most effective wastewater treatment, mitigating water scarcity.

Using our intervention plan, water scarcity will be postponed 1 year after no intervention. In an ideal situation, it will be postponed 7 years after no intervention. Thus, it proves that our intervention plan is valuable.

In conclusion, India can become less susceptible to waters scarcity. It is a pity that water become a critical issue sooner or later.

6 Conclusions

In this section, we detail the strengths and weaknesses of our models.

6.1 Evaluation Of Grey Forecasting Model

- **Strengths**

  Grey Forecasting Model needs small amounts of data, whose prediction is relatively accurate. The sample data doesn't need regularity of distribution. The model is simple in computation and checked conveniently.

  Grey Forecasting Model is applied to medium long term prediction. It has its superiority in predicting GDP.

- **Weaknesses**

  With enough data, the prediction is an exact science, while in the other way, it has an accuracy deviation.

  In our paper, we have no comprehensive and perfect statistics on data of India. Thus our results produce some error inevitably.

6.2 Evaluation Of Logistic Retarded Growth Model

- **Strengths**

  We use Logistic Model for prediction of population, and we also take the inhibition into account. The prediction is in line with objective laws and credible for the authoritative history data.

- **Weaknesses**

  The Logistic Model neglects some uncontrollable influences from policy, such as the birth control in China. As a result, the prediction is about population explosion without intervention, whose figures increase dramatically.

6.3 Evaluation Of Psychological Prediction Model

- **Strengths**

  Psychological Prediction Model is estimation for social investors' investment psychology. It analyses the possibility of social investment under the change of the government's fund. It's an effective discretization algorithm for psychological change.

- **Weaknesses**

  Psychological Prediction Model is based on the investors for-profit purpose, neglecting purpose for public interest or advertising.

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Appendices

Appendix A Grey Forecasting Model

Here is the function definition programme for we used in our Grey Forecasting Model as follow.

Input Matlab source:

```
syms a u;
c=[a u]';
A=[4287,4669,4766,4940,5240,6184,7216,8342,9491,
12387,12241,13654,17085,18358,18318,18618,20669];
Ago=cumsum(A);
n=length(A);
for i=1:(n-1)
C(i)=(Ago(i)+Ago(i+1))/2;
end
Yn=A;Yn(1)=[];
Yn=Yn';
E=[-C;ones(1,n-1)];
   c=inv(E*E')*E*Yn;
c=c';
a=c(1);u=c(2);
F=[];F(1)=A(1);
for i=2:(n+17)
F(i)=(A(1)-u/a)/exp(a*(i-1))+u/a ;
end
G=[];G(1)=A(1);
for i=2:(n+17)
G(i)=F(i)-F(i-1);
end
t1=1998:2014;
t2=1998:2031;
G
plot(t1,A,'o',t2,G)
```

Appendix B Logistic Retarded Growth Model

Here is the function definition programme for we used in our Logistic Retarded Growth Model as follow.

Input Matlab source:

```
x=1960:5:2014;
y=[4.4966 4.9792 5.5394 6.2170 6.9722 7.8208 8.7060 9.6087 10.5348 11.4432 12.3098];
plot(x,y,'*',x,y);
hold on;
a0=[0.01,0.001];
a=lsqcurvefit('curvefit_fun2',a0,x,y);
disp(['a=' num2str(a)]);
xi=1960:5:2020;
```

```
yi=curvefit_fun2(a,xi);
plot(xi,yi,'r');
x1=2031;
y1=curvefit_fun2(a,x1)
hold off
```