Judges' Commentary: Searching for Submersibles

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Introduction

This year's Problem B of the Mathematical Contest in Modeling (MCM®) was focused on helping a fictitious company develop safety procedures for operating submersibles as part of a tourist adventure program.

Background

Maritime **C**ruises **M**ini-Submarines (MCMS), a company that is based in Greece, wishes to use submersibles to take tourists on adventures exploring the bottom of the Ionian Sea for sunken shipwrecks. They need approval from regulators for safety procedures in case of loss of communication to the host ship and possible mechanical problems in the submersible. In particular, they need a model to predict the location of the submersible over time.

The Problem

Specifically, teams were required to do the following:

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- **Locate:** Develop a model to predict the location of the submersible over time.
- **Prepare:** Specify search equipment to be carried on the host ship.
- **Search:** Recommend initial points of deployment and search patterns to locate a lost submersible and estimate probability of success.
- **Extrapolate:** Expand the models to other global locations and to consider multiple submersibles in the same area.

Introduction and Overview

The authors' vision for this problem was to push teams to consider all the interrelated elements associated with designing a set of safety procedures/equipment for a submersible capable of carrying humans to the deepest parts of the ocean . The authors intended to have teams develop a process/procedure for increasing the probability of quickly finding locating a disabled submersible but also to create a set of equipment that would increase the safety of the submersible's crew and passengers [Beecher and Erickson 2024]. This year's problem is highly complex, with multiple competing requirements to satisfy the safety regulators for approval. Critical information was deliberately missing from the problem statement so as to get modeling teams to work through a deliberate modeling process and establish a set of essential decisions/assumptions.

This problem is not just complex; it's *incredibly* complex and requires the full extent of the modeling team's expertise. The team must consider multiple interrelated elements and tackle the core mathematical challenge of a set of interrelated three-dimensional location and search problems. Their task is to develop a model(s) that predicts the location of the submersible over time and another model that will use information from the location model(s) to recommend initial points of deployment and search patterns for the equipment. The predicted submersible location and initial search deployment point will help minimize the searcher's time to locate a lost submersible, underscoring the intellectual challenge and the need for their expertise in this process.

We start this commentary with a short review of the mechanics of this year's judging process and follow the mechanics with a discussion and observations from the judging on various elements of the problem. We then discuss the importance of sensitivity analysis, assumptions and identifying the strengths and weaknesses of a developed model and finish by addressing some points concerning communication.

The Process

We believe that it is beneficial for teams to understand the judging process; so similar to previous commentaries,we once again review the basic process for this year's Problem B.

Triage

Every paper is read by at least two judges during the triage round to determine if the paper contains all of the required and necessary elements that make it a candidate for further recognition. If a paper addresses all of the question components/issues and appears to have a reasonable model, then judges are likely to identify it as a paper that deserves more attention.

It is imperative for a paper to address all critical elements, since this is a key factor in the judging process. A comprehensive summary that provides a brief overview of the problem, the paper's structure, and specific results stated clearly and concisely, is a significant step towards gaining recognition. Many papers struggle to make an impact during the initial evaluation because they fail to address all elements of the question thoroughly. This lack can lead a judge to conclude that a team's efforts may not measure up to the better papers. For instance, this year, many teams overlooked a critical issue in identifying additional equipment that a rescue vessel might need to bring in to assist if necessary. Moreover, several teams failed to acknowledge or address all the factors identified in the background of the problem.

Developing all required modeling elements is critical and often overlooked. For example, sensitivity analysis remains one of the weakest elements and is usually entirely missing in some papers. Historically, these papers do not do well during the triage.

In addition, the team must express their general approach and results as clearly and concisely as possible in the memorandum to the Greek government discussing their proposed safety procedures. Doing so means providing a broad overview of the problem, strategy, and specific results in concise, concise, nontechnical terms. Ask yourself if someone without an education in mathematics or a background in the problem can read and understand the memorandum. Most teams provided a memorandum, but quite a few failed to adequately provide the results/solution of their modeling process in the memorandum. In many cases, it seemed to the judges that the memorandum (and executive summary) appeared to be written by the team before they completed the modeling effort. This shortcoming did not hurt papers during the triage process but became more of a factor as papers advanced through the judging process.

Clear and concise writing is a significant factor in the success of a paper during the triage round, since the best models and the best effort are not effective if the results are not adequately communicated. It is crucial to remember that this is a modeling competition, and that effective communication is a critical part of the modeling process. Your role in this process is vital, and your ability to effectively communicate your results can significantly impact the success of your paper.

Final Judging

Final judging consists of multiple rounds of judging over several days. As the rounds progress, the judging criteria shift from identifying papers that warrant further consideration to identifying the best papers.

The first round of the final judging process begins with each judge reading a set of papers, and then all judges meet to discuss the critical aspects of the question and what should be included in a "good" paper. This year, these aspects included identifying and linking the types of equipment and information that a submersible needed as part of the search and location aspects of the problem. The judges expected teams to explicitly state the criteria that they used as part of both the location part and the search part of the problem and demonstrate how they were linked.

As final judging progresses, each paper is read by multiple judges, with the final set of papers read by all judges. In these last rounds, the modeling process and the mathematical integrity of a paper play a significant role in identifying the outstanding papers in the competition.

Critical Elements of the Modeling Process

The MCM is designed to be an interesting and challenging modeling process but one achievable in the available time. Successful teams must pay attention to all elements of the modeling process to produce a successful solution and paper. Many teams focus a great deal of time on developing a fantastic model to address the first couple of questions; but they need more time to fully address all problem elements, and they end up fully shortchanging many critical elements. Typically, these include adequately addressing all problem components, developing a clear and concise summary and non-technical report, and failing to test their model. We will address a couple of these areas in this section.

Components of the Problem

The final judges collectively believed that this year's problem was fairly complex because of its open-ended nature. The judges were looking for modeling to capture the essence of four interrelated modeling aspects:

- a location process for the disabled submersible,

- the identification of potential search-and-rescue equipment to increase the probability of finding the submersible,
- a search mechanism to minimize the time to find a disabled submersible, and
- how the team's model performs if they extend it to another region of the world.

Many teams approached the required elements of the problem linearly (the order given). The judges saw modeling efforts that attempted to develop some aspects of interrelated requirements into a single model as a better approach.

Locate

The team's effort to develop a location model is the core of the problem, and judges expected that it would receive the team's most attention. The goal of the location model is to identify the submersible's location over time and how it can facilitate the searchers if it breaks down. The authors deliberately left this as an open-ended problem to allow the teams to follow the mathematical modeling process. Because of the open-ended nature, the judges expected to see a wide range of potential modeling solutions, which meant that teams needed to establish a set of critical assumptions. The judges expected to see at least three major identified areas:

- currents of the submersible's location,
- differing water densities, and
- geography of the sea floor.

In addition, teams needed to assess these three major areas. Teams used a wide range of approaches to address the location aspect, including everything from Newton's second law, energy equations, fluid dynamics, and simple mechanical models. The most common approaches used some version of fluid dynamics. However, no matter what approach the team used for the problem, judges expected to see a modeling approach that captured the three-dimensional nature of the problem.

Based on page counts, many teams focused their efforts on their model to predict the submersible's location. While most teams did capture the three-dimensional nature of the problem, the judges highly value a more comprehensive understanding that adequately addresses the subcomponents of the problem. This includes addressing the uncertainties of their predictions and what information the submersible could send to decrease this level of uncertainty. Such a perspective was seen as a discriminator for better papers, challenging teams to strive for a more-thorough approach.

Better approaches that were viewed favorably by the judges involved the development of interconnected models focused on currents, density, and topography of the ocean floor. These approaches included using Autoregressive Integrated Moving Average (ARIMA) time series models and statistical techniques that leveraged time series data to understand the dataset better or predict future trends. Another approach that judges observed in better papers was the use of computational fluid dynamics, a branch of fluid mechanics that employs numerical analysis and data structures to analyze and solve problems related to fluid flows. To tackle the uncertainties, some teams utilized the Kalman filtering technique, which infers parameters of interest from indirect, inaccurate, and uncertain observations. The judges acknowledge and appreciate these efforts, and they serve as a benchmark for other teams.

Prepare

The authors expected teams to identify a set of search-and-rescue equipment, using information from Locate part of the problem, to aid in location and recovery of a disabled submersible. Teams were allowed to consider different types of equipment but were required to consider costs associated with the availability, maintenance, readiness, and usage of this equipment. The cost consideration typically resulted in teams developing an optimization process. Once again, the open-ended nature of this part of the problem meant that teams needed to decide on an initial set of search-and-rescue equipment. After an internet search, most teams created a list of 3 to 20 pieces of equipment. The judges were not looking for a specific number of pieces of equipment. Still, they hoped to see equipment that addressed both the search-and-rescue aspects of the problem and were somewhat related to insights from their location modeling. Many teams focused on the search aspect of the equipment with less attention to the rescue side of the problem.

Many teams used multi-attribute decision-making techniques, such as the Analytic Hierarchy Process, or the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), or principal component analysis, to identify their final set of equipment. Additional approaches that judges observed in better papers included cost-benefit models and multi-objective optimization models that often looked at minimizing costs, maximizing availability, and minimizing prep time. The judges saw interesting techniques, such as genetic algorithms, to favorably solve these multi-objective optimization problems. In addition, papers that included more than the three specified requirements of availability, maintenance, and readiness tended to do better in the judging process.

Although judges decided to refrain from penalizing any team for taking a linear approach to solving the required elements, papers that attempted to link insights gained from the Locate modeling effort were higher in quality. Better modeling approaches tended to link elements from that part, including the current, water density, or potential depth of the search area as part of the equipment decision criteria. Better modeling approaches also included looking at environmental factors that could impact the performance of the equipment. In addition, these approaches specifically addressed the question of what equipment the host ship should deploy and what a rescue vessel might need to bring.

Search

The judges were particularly interested in a team's selection of search equipment and how it influenced the selected search pattern. They wanted to see teams develop a search model integrating location predictions from their Locate model. They expected the search model to provide optimal deployment points for the search-and-rescue equipment and a recommended search pattern to minimize the time to find a disabled submersible. Many teams started with a fixed set of search patterns, identified the starting location for the search, and then determined the time needed to locate the submersible based on a particular search pattern. The most common patterns included the grid search, constant-range search, box search, and circular search. The judges did not favor one search pattern technique over another but were looking for indications of how the team's selection of search equipment, and the uncertainty of the submersible's location, influenced the selected search pattern. Judges viewed papers that made this connection with more favor.

The judges were deeply impressed by the ingenuity demonstrated by most teams in developing their optimization approaches. The team's ability to identify initial search starting points to minimize the time needed to find the submersible was commendable. Better papers tended to link their search model with the uncertainty analysis from their Locate model and further incorporated some distribution, such as Poisson or Gaussian, to update probabilities of locations within the search area. In addition, these teams utilized some approaches, such as Bayesian analysis, to transform their prior probabilities into posterior probabilities. There is an expectation that the initial point selection will be based on some method to minimize the time to locate the submersible. Teams that made this connection were viewed with more favor among the judges.

The judges were especially interested in teams' use of meta-heuristics, such as ant colony or Tabu search, to identify the best initial search point and search pattern. Such an approach resonated with the judges, making these papers stand out. In addition, these papers tended to develop three-dimensional models to account for multiple environmental factors that they covered in their Locate part.

Extrapolate

How might your model be expanded to account for other tourist destinations, such as the Caribbean Sea? The judges expected to see some mention of how the model would be changed or updated to account for additional tourist destinations. The destination selected by the teams did not matter. Still, papers with a destination that contained factor assumptions, such as ocean currents, seafloor landscape, and seawater density, that were different from those considered in their models, were viewed with more favor by the judges. It was not enough to describe how the conditions differed but to explain how this difference impacted their set of models. For example, many teams selected the Caribbean Sea because the problem prompt mentioned it. Most teams acknowledged that the ocean current in the Caribbean Sea may be more intense than in the Mediterranean Sea. However, they did not describe how this impacted their models, the length of their search, or the probability of finding the submersible. Better papers included a discussion of the impact of environmental changes on their model and how they might need to adjust their model. The judges viewed multi-objective efforts as a superior approach to addressing this problem element.

One element of the Extrapolate-part that was overlooked by many teams was addressing how they might need to change their model to account for multiple submersibles moving in the same general vicinity. The judges saw this problem element as a discriminator for better papers. Better papers also looked at the impact of having a second submersible in the water and the ability of each vessel to cooperate regarding reporting and updating positions. This approach resonated with the judges' evaluation criteria, making these papers stand out.

Executive Summary

The executive summary is a critical element of any document. The executive summary is always read first and sets the tone for the rest of the paper. The executive summary must include methods and results in a concise manner. It appeared to many judges that the executive summary was written before a team completed their modeling effort, yet the team did not come back to update it after completing their effort. Many executive summaries addressed how a team planned to model and solve the problem but failed to include any results.

One successful approach is to break the executive summary into the following four sections as a guide for writing it:

- **Background:** A short synopsis of the problem the paper is attempting to address and why the reader should care.
- **Modeling method:** How you plan to gain insight into the problem.

- **Results:** Provide actual findings from the modeling effort.
- **Conclusion:** What you recommend.

The judges considered a clear, concise and fully developed executive summary as a critical criterion for "good" papers. Your work is important and should be presented in the best possible way.

It's crucial to remember that the main body of the report should align with what the team has written in the executive summary and in the memorandum. Be careful here! Many teams appear to write these before their modeling effort is complete, as if crafting a "best-case wish list" of what they hope to achieve. When the main body of the report does not coincide with what is written, judges immediately have a negative impression.

Memorandum

The two-page memorandum is a critical element of the modeling process for communicating analysis results. A memorandum typically contains much of the same information as an executive summary but with a slightly different style. It is an act of communication designed to translate the technical aspects of the modeling process into clear, concise, and nontechnical terms for the reader. The memorandum is an expanded version of the executive summary that provides more details for management. Like the executive summary, the memorandum must contain he background of the problem, the modeling approach, results for all elements of the problem, and recommendations/conclusions

The judges considered a clear, concise, and fully-developed memorandum a critical criterion for "good" papers. Unfortunately, many teams did not provide a satisfactory memorandum. Most teams would benefit from focusing more on the nontechnical communication required in their report! If the memorandum indicates potential novel problem-solving approaches, surprising and well-described results, and clear (brief) explanations of implications, there is a good chance that senior management will want to examine the accompanying report in detail.

Assumptions

Assumptions drive model-building and use of a model. The common models used by many teams, such as regression, principal component analysis, and linear and nonlinear program approaches, all have inherent assumptions that the team should address in the paper. The better papers provided a concise list of relevant assumptions with a corresponding justification. For any model, there is always a set of assumptions about when and how to use it, which the team should include in the paper. Many papers essentially repeated the assumptions from the information in the problem statement and perhaps a few additional assumptions. One common fault was a list of assumptions that were not addressed or used in the model.

In proper mathematical modeling, assumptions and their justification drive a team's model-building choices, thereby representing a compromise that must be in place for the team to continue its efforts. As mentioned in previous Judges' Commentaries in *The UMAP Journal*, while typically presented up front in a technical report, assumptions arise as needed throughout the modeling effort. Be careful here! Many teams appear to write the assumptions before their modeling effort is complete and do not update them as they develop the models. You should update your assumption section as you build your models. Removing unused assumptions strengthens analysis and results and, in addition to parameter testing, becomes a critical part of sensitivity analysis: "What if this assumption did not hold? How would it affect the results [the team] is showing?"

Here are examples of acceptable assumptions from one of the more successful papers:

Assumption: Negligible mass of people on board the submersible.
Justification: The effect of the mass of the person on the submersible is a dynamic load on the submersible. The calculation of the dynamic load is more complicated. Still, for a submersible traveling in the water, the effect of seawater on the submersible is much greater than the effect of the person in the submersible. Therefore, we assume that the person's mass can be ignored, i.e., the person's effect on the submersible can be ignored.

Assumption: The volume of the submersible remained consistent over the study period.
Justification: The volume of the submersible largely determines the buoyancy and dive capacity, maneuvering and maneuverability, load carrying capacity, and pressure resistance of the submersible, which in this case is not self-supporting, so we assume that the submersible is not able to change its volume for the duration of the study.

Assumption: Climate in the Ionian Sea remained constant over the study period.
Justification: The climate type of the Ionian Sea is Mediterranean, influenced by the subtropical high and the westerly wind belt, the salinity of the seawater, the air pressure, and the seawater temperature are roughly stable, so that the size of the wind, the strength of the ocean currents and the density of the seawater are approximately the same.

Sensitivity

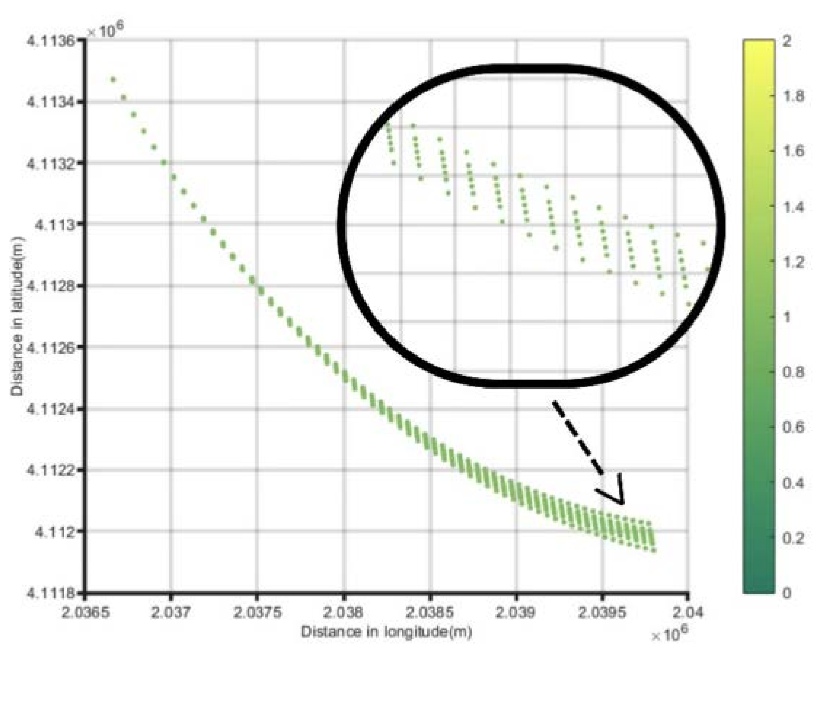
Sensitivity analysis is the most significant natural discriminator between papers moving on to later judging rounds and those eliminated by judges for further consideration. The judges did not move a paper forward to the final rounds that did not have a good sensitivity analysis (error analysis and testing are included here). In sensitivity analysis, the judges expect the teams to indicate if the results change based on changes in the input values. In many papers, if present, the sensitivity analysis appeared more as a token analysis only to meet the spirit of the requirement rather than being a substantial analysis effort.

Here are two sensitivity analysis examples from this year's set of papers.

> We experimented with varying initial positions and velocities, and we found consistent predictions for nearly all regions of the Ionian Sea.

This statement is good, but there are no details regarding the actual changes or predictions. This leaves the reader simply having to accept the authors' statement of conducting the analysis and their results. The inclusion of the data, even in an appendix, is critical for the reader to understand precisely what the author is doing during this process.

> [A] sensitivity analysis is conducted on these three drag coefficients. By varying them by $\pm 1\%$, $\pm 2\%$, and $\pm 4\%$, while keeping other conditions constant, the trajectories for these six scenarios are simulated. A comparison is made between these simulated trajectories and the original trajectory, resulting in the two graphs presented below [we show below only the first graph].



> From the graph, it can be observed that after 10 hours, ... the deviation of the trajectory does not exceed 400 meters in any direction. This is within an acceptable range, indicating that it does not significantly affect the main direction of the trajectory.

This was a much better statement in the sensitivity analysis section. The team clearly stated what they would explore, provided actual data, illustrated the sensitivity with a figure, and provided in the text a conclusion to be drawn from the data display. The judges view this level of information more favorably when reviewing papers.

COMAP continues to stress the importance of a team considering, in reflection, what their model can answer and what it cannot. Such understanding prevents teams from overstating implications—"... our model solves every problem of this type ..."—or failing to see the model's potential for making a more significant impact on the problem described.

Communication

Clarity and style will always be challenging for international contests, and the MCM is no exception. Judges realize that teams relying on Google and other translation software are slightly disadvantaged, so the judges do not penalize teams for minor grammatical issues and small spelling errors. Since judging is blind as to where papers originate, this policy applies equally to all papers. However, as these errors increase, it becomes difficult for judges to ignore—and even more challenging to completely understand—some nuances of a team's approach and results. All judges look at the References section to see the breadth and depth of a team's basis for modeling, and this is no different from what professional journal reviewers do when considering papers for publication.

The judges focus on the quality of the writing, with particular attention to the summary and the memorandum. The quality of writing continues to improve, but many teams need to focus more on their writing to advance further in the judging process. Basic spelling and grammar errors in a paper distract the reader. We suggest setting aside some time at the end of your modeling process to ensure that you have time to proofread your paper before submission.

Figures

Too many figures are lifted and cut and pasted into team entries, without attribution to the source. Having no figure is better that using one "borrowed" without attribution. This year, the judges decided to call such behavior "poor referencing." The judges recognize that it is a time-constrained contest and creating original figures might not be practical; but make sure always to reference the source for the figure. Borrowing work form others without attribution is not a good practice and sets a poor tone for your paper.

Final Thoughts

Participation in the contest and learning from the process is winning. Learning to work as a team to accomplish your team's goals and objectives is a success. As a reminder, the COMAP award (Successful Participant, Honorable Mention, Meritorious, Finalist, and Outstanding) is based on a comparison to other papers and the judges' expectations. The Outstanding teams modeled and presented all the aspects of the problem described in the problem statement, including the fully-developed standard elements (assumptions, sensitivity analysis, strengths, and weaknesses, etc.), created an effective model, explained the modeling choices made, and were clearly and concisely written. Every year, the judges continue to be impressed with the quality of the submissions, especially considering the time constraints.

Reference

Beecher, Amanda, and Keith A. Erickson. 2024. Authors' commentary: Searching for submersibles. *The UMAP Journal* 45 (4): 363–367.

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