Judges' Commentary: Resource Availability and Sex Ratios

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Introduction

The goal of this year's Problem A on Resource Availability and Sex Ratios was to develop and examine models that provide insights into the interactions within an ecosystem where the sex ratio of sea lampreys can vary based on external conditions.

The teams were asked to develop a mathematical model that captures the intricate dynamics of the lamprey population and addresses:

- the impact on the larger ecological system when the population of lampreys can alter its sex ratio,
- the advantages and disadvantages to the population of lampreys,
- the impact on the stability of the ecosystem of changes in the sex ratio of lampreys, and
- whether an ecosystem with variable sex ratios in the lamprey population offers advantages to other organisms in the ecosystem, such as parasites.

We highlight the key aspects that distinguished the Outstanding submissions.

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The Modeling Problem

The problem tasked teams to explore the dynamics of sea lampreys, a species known for its ability to adjust its sex ratio in response to environmental conditions. Unlike many other species that have fixed male-to-female ratios, sea lampreys can vary their sex ratio based on factors such as food availability and larval growth rates. This adaptive behavior presents intriguing challenges and opportunities for ecological modeling.

Teams were required to develop mathematical models that capture the complexities of the lamprey life cycle and the implications of variable sex ratios on ecosystem dynamics. Key aspects of the modeling problem included:

- **Life Cycle Dynamics:** Sea lampreys undergo a complex life cycle involving multiple stages from egg to adult. The determination of sex occurs during the larval stage and is thought to be influenced by larval growth rate, which in turn depends on external circumstances such as resource availability. Understanding these stages and their dynamics was essential for modeling how sex ratios change over time.
- **Parameter Estimation:** Teams had the option to use real-world data, particularly from regions where sea lampreys are well-studied, such as the Great Lakes, to estimate model parameters. Alternatively, theoretical parameters could be used if justified within the context of the problem.
- **Model Complexity:** Balancing model realism with computational feasibility was crucial. Teams had to decide on the appropriate level of detail to capture the biological intricacies of lamprey behavior while ensuring that their models remained practical for analysis.
- **Ecosystem Impacts:** Beyond lamprey population dynamics, models had to explore how changes in lamprey sex ratios affect other species and the overall ecosystem stability. This included examining predator-prey relationships, and the impact on biodiversity and ecosystem stability in the face of environmental changes.

Models

- **Population Dynamics Models**, such as the classic Lotka-Volterra model, were a common choice among teams. These models focused on simulating changes in the sea lamprey population over time, considering factors such as birth rates, mortality rates, sex ratio dynamics, and species interactions. By capturing the life cycle stages of lampreys and their interactions with environmental variables such as food availability and other species, these models provided insights into how population size and composition respond to changing conditions.

- **Stage-Structured Models** were another common approach. These models divided the lamprey life cycle into distinct developmental stages, from egg to adult. Each stage was characterized by specific growth rates, survival probabilities, and sex determination processes. Teams used differential equations or matrix-based approaches to model transitions between stages and to analyze how variations in sex ratios at different stages influenced overall population dynamics.
- **Ecological Network Models:** Some teams chose to use ecological network models to incorporate interactions with other species and ecosystem components. These models represented food webs, predator-prey relationships, and competitive interactions within aquatic ecosystems. By examining the cascading effects of variable lamprey sex ratios on biodiversity, ecosystem stability, and resource dynamics, these models provided insights into the ecological consequences of lamprey adaptations.
- **Statistical and Data-Driven Models** were also a common choice. Due to the invasive species designation of lampreys in the Great Lakes region, there is much publicly available data on the lamprey population, their sex distribution, and resource availability. Many teams focused on this geographic region as their area of study, utilizing the abundance of data available for this area to estimate various parameters in their models and validate model outputs. These models involved regression analyses, time series modeling, or Bayesian approaches to quantify relationships between environmental variables and lamprey population dynamics. By integrating observational data with modeling techniques, teams improved the accuracy and predictive capabilities of their models.
- **Hybrid and Integrated Models:** Often, teams used hybrid and integrated models, combining elements of multiple modeling approaches to achieve comprehensive insights into lamprey dynamics. For example, integrating statistical models with stage-structured frameworks enabled teams to incorporate empirical data while maintaining biological realism in model structures.

Important Aspects of Model Development

- **Literature Review:** Even if a team chooses not to include a literature review in their report, reviewing available literature on the subject could be helpful in understanding the advantages and limitations of the various modeling approaches used to tackle similar problems.
- **Assumptions** form the cornerstone of the model development process. Teams should refrain from using as assumptions "facts" that do not directly relate to their models, and focus on assumptions that inform the

various decisions in model formulation. While setting out clear assumptions is one of the first steps in model formulation and development, it might be a good practice for teams to write up their assumptions as the last step in preparing report, to make sure that the list is comprehensive. When formulating assumptions, teams should not only provide justifications, but also include appropriate citations.

- **Variables:** It is important to carefully define all variables and explain what they mean in the context of the model. It is a good practice not only to have a list of all variables, but also to refer to them and indicate their meaning in the main body of the report. If a team chooses to use a particular functional form for a certain aspect of their model, it is important to justify the choice and elaborate on its significance.

- **Parametrization:** When determining parameter values, teams should explain the rationale behind the chosen values, avoiding the usage of arbitrary values. The choices of parameters should be well-motivated, justified, and supported with appropriate citations. When estimating parameter values from data, it is important to cite not only the data source but also the program or software used to estimate the parameters, mention the fitting method used, and comment on the results of the fitting (such as goodness of fit). If a team chooses theoretical values, it is important to justify the choice and provide a rationale for the chosen values in the context of the problem. Reproducibility is an important aspect of modeling, and teams that are clear and transparent with their choices of parameters are favorably considered by the judges.

- **Algorithm Description and Numerical Simulations:** It is important to clearly describe the algorithm used to arrive at a solution. If a team uses numerical simulations, it is important to run the simulations for an adequate time span and use an appropriate step size to provide smooth results.

- **Sensitivity Analysis** is an integral part of model development and can help determine the robustness of the model to changes in parameter values. The team should justify their selection of parameters for sensitivity analysis. For example, if for a particular parameter the team is uncertain about its value, the value tends to vary a lot in the field, or the parameter is thought to have a large effect on the model result, that parameter might be a good candidate for sensitivity analysis.

- **Strengths and Weaknesses:** A brief reflection on the model and results, including discussion of strengths and weaknesses of the modeling approach, can not only help the teams highlight the strong points and advantages of their methods but also provide an opportunity for teams to consider potential limitations and issues. The ability to think critically about the modeling process and provide a balanced and comprehensive evaluation of the modeling approach speaks to the academic maturity of the team and is considered favorably by the judges.

Model Considerations

We highlight some select aspects and challenges of this year's problem and also discuss some general comments about organization and presentation of results. For a complete overview of how to better prepare for next year's contest, we encourage teams to also refer to the previous-year commentaries listed in the References, since they contain additional insights and guidance not covered here.

- **Discrete-time vs. Continuous Time:** One of the fundamental choices when formulating a model is whether to use discrete time or continuous time. When making such a choice, teams should carefully consider the nature of the dynamics for the species at hand. In general, discrete-time models are best suited with species that have distinct non-overlapping generations whose population changes at specific intervals, such as breeding seasons or cycles. Continuous-time models are better suited for species with overlapping generations and continuous reproduction. While both continuous- and discrete-time models were used this year, judges particularly appreciated teams that thought carefully about their decision and justified their choice of a particular model type.
- **Incorporation of Age Structure:** Lampreys have a complex life cycle and undergo the transition to adults in a series of distinct life stages. Since the sex determination in particular is thought to be dependent on the larval growth rate, teams that undertook a careful treatment of the age structure were particularly well regarded by the judges.
- **The Analytic Hierarchy Process (AHP)** is a well-established technique for decision-making, used to facilitate decision-making when multiple criteria are involved. Every year, this appears as a popular choice used by many teams. A central element of this technique is the judgment matrix, consisting of pairwise comparisons of elements of interest in terms of their relative importance. It is important that teams explain their reasoning in regard to the coefficients of this matrix, since the AHP result relies heavily on a carefully formulated judgment matrix.
- **Model Simplicity and Cohesion:** To answer the various questions presented in the problem, it is advised that teams make the modeling process cohesive and use models that can build on each other. A complicated model is not necessarily better, since a simple model can be easily modified to address additional questions. For this year's problems, many teams chose to start out with a simple model of the lamprey population and modified it gradually by adding other species. Doing this

allowed teams to consider the impact of the lampreys' variable sex ratio on the larger ecosystem and other organisms around them, as well as to investigate the stability of the ecosystem as a whole in light of the variable sex ratio.

- **Assessing Stability:** To investigate the impact of the variable sex ratios of lampreys on the overall stability of the ecosystem, many teams chose to consider the degree of biodiversity in the ecosystem, as a proxy measure of stability. In addition, teams also considered the ecosystem's stability in terms of both resilience (ecosystem's ability to recover after a disturbance) and resistance (ecosystem's ability to remain unchanged under disturbances). While resistance and resilience are similar concepts, taking a more in-depth view of stability in terms of these aspects allows for a more comprehensive picture. Teams that took a multi-faceted look at stability were regarded particularly favorably by the judges.

Writing and Presentation of Results

Executive Summary

The executive summary is a chance for teams to showcase their approach, results, and conclusions. Since this is often the first part that judges read, it is important that it includes a clear motivation, main approaches, results, and conclusions. A successful executive summary should be clear and easy to understand. Teams should focus on the main points and key takeaways, with a clear explanation of the problem. Submissions with a well-organized executive summary that clearly presented the approach and main results stood out to the judges.

Citations to References

In addition to providing a list of all references used in the model development, teams should also include citations in the main body of the text for all research papers, data sets, images, and parameter values from outside sources. If teams use a well-known model, they do not need to show model derivations that have been done before but can simply cite the previous work. It is more important for teams to focus on the justifications for their approach and provide reasoning behind making model choices.

Figures and Tables

Teams should think carefully about how best to showcase their main results through figures and tables, and how to weave those into the main narrative. For complicated figures that show multiple pieces of information, teams should think carefully about how to best convey the information to the reader. Teams should make sure that all figures are legible, with appropriately-sized and accurately-labeled axes, including both the quantities displayed and their units, to help the reader interpret the results. A figure caption could be a good way to bring clarity and summarize the main results. In addition, it is important to integrate all figures and tables into the narrative, citing them in the text and interpreting what they are supposed to show. Do not expect the reader to have to try to figure out what a table or figure says!

The Very Last Step

Finally, it is important to do a careful and thorough read over the entire report to make sure that it is clear, well-organized, and devoid of errors and inconsistencies that may have been overlooked during the initial drafting process. This review helps to not only demonstrate the teams' attention to detail, but also makes it easier for the audience to read and understand the work that was done.

Conclusion

The 2024 MCM Problem A provided a platform for teams to explore the adaptive behaviors of sea lampreys and their ecological consequences through mathematical modeling. The participants presented diverse approaches and insights, highlighting the interplay between biological dynamics and environmental factors in shaping ecosystems. The judges once again were impressed with the accomplishments of the teams, especially considering the scope of the questions that teams had to address and the time constraints involved.

This year's Problem A underscored the importance of interdisciplinary collaboration in addressing complex ecological challenges. By integrating biological principles with mathematical modeling, participants not only advanced their understanding of lamprey behavior but also their understanding of the various mathematical tools for studying and managing species interactions in natural environments.

References

Bieri, Joanna. 2019. Judges' commentary: Game of ecology. *The UMAP Journal* 40 (4): 325–334.

________. 2020. Judges' commentary: Moving north. *The UMAP Journal* 41 (4): 387–395.

________. 2021. Judges' commentary: Fungi. *The UMAP Journal* 42 (4): 419–428.

________, and Kelly Black. 2018. Judges' commentary: Multi-hop HF radio propagation. *The UMAP Journal* 39 (3): 287-294.

Sunkula, Mahesh, and Sofya Zaytseva. 2023. Judges' commentary: Drought-stricken plant communities. *The UMAP Journal* 44 (4) (2023): 375–384.

U.S. Geological Survey. 2017. Sex-shifting fish: Growth rate could determine sea lamprey sex. https://www.usgs.gov/news/national-news-release/sex-shifting-fish-growth-rate-could-determine-sea-lamprey-sex.

Zaytseva, Sofya. 2022. Judges' commentary: Power profile of a cyclist. *The UMAP Journal* 43 (4): 391–402.

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