KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

INSTITUTE OF OPEN, DISTANCE AND E-LEARNING

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SPH 426: STATISTICAL MECHANICS

DATE: Thursday 7th July, 2011 **TIME:** 4.30 p.m. – 6.30 p.m.

INSTRUCTIONS: **Answer question ONE and any other TWO (2) questions. Question ONE carries 30 marks while each of the others carries 20 marks.**

1. (a) Define the following

(i) Γ space

(ii) μ space

(iii) Ensemble

(iv) Microcanonical ensemble (8 marks)

(b) Both Fermi-Dirac and Bose-Einstein statistics apply to indistinguishable particles. But they are different. Why? (3 marks)

(c) Justify the definition of entropy as $S = k \ln \Omega$. The terms have their usual meanings. (6 marks)

(d) Find the value of the partition function defined as $Z = \sum_j e^{-\varepsilon_j/kT}$, for an ideal classical gas. (7 marks)

(e) Consider the free electron theory of metals and use Fermi-Dirac statistics to show that the total energy E at absolute zero (T=0) is given as $E = \frac{3}{5} N\varepsilon_F$, where ε_F is the Fermi energy. (6 marks)

2. (a) Use the value of partition function obtained in Q.1(d) to show that for a classical ideal gas the number of particles having energies between ε and $\varepsilon + \Delta\varepsilon$ is given as

$$\Delta n = \frac{N\varepsilon^{1/2} e^{-\varepsilon/kT}}{\sqrt{\frac{\pi}{4}}(kT)^{3/2}} \Delta\varepsilon$$. (Use Maxwll-Boltzmann distribution law.) (15 marks)

(b) Use the result obtained above to find the Maxwell distribution law of velocities. (5 marks)

3. (a) Explain how the Bose-Einstein distribution formula

$$n_j = \frac{1}{e^{-\alpha} e^{\beta\varepsilon_j} - 1}$$

reduces to

$$n_j = \frac{1}{e^{\beta\varepsilon_j} - 1}$$

when one considers the blackbody radiation in the form of radiation in a cavity. (5 marks)

(b) Use the above Bose-Einstein distribution formula to obtain the Planck's radiation formula

$$u(\omega, T) d\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3 d\omega}{e^{\beta\hbar\omega} - 1} .$$ (15 marks)

4. (a) Obtain the Fermi-Dirac distribution law

$$n_j = \frac{1}{e^{-\alpha} e^{\beta\varepsilon_j} + 1} .$$

The terms have their usual meanings. (10 marks)

(b) Use the above distribution law in the free electron theory of metals to show that at T = 0 the Fermi energy ε_F is given as

$$\varepsilon_F = \left(\frac{3N\pi^2}{V}\right)^{2/3} \frac{\hbar^2}{2m} \qquad \text{(10 marks)}$$

5. (a) Show that the Fermi-Dirac distribution formula

$$n_j = \frac{1}{e^{-\alpha}e^{\beta\varepsilon_j} + 1}$$

is in conformity with the Pauli's exclusion principle. (5 marks)

(b) Consider the case of Boltzmann gas and use the relation $E = \sum_j n_j \varepsilon_j$ to show that the value of Lagrange multiplier $\beta = \dfrac{1}{kT}$. (15 marks)

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