KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDCATION (SCIENCE)

SPH 402: QUANTUM MECHANICS I

DATE: MONDAY 2ND APRIL 2012 **TIME:** 2.00 P.M – 4.00 P.M

INSTRUCTIONS

Answer question ONE and any other TWO questions. Question One is 30 marks while other questions are 20 marks each.

Q1.

a. i). What is a spherically symmetric potential? Give one example of such potential. (3 marks)

ii). Show that the classical potential energy of a particle of mass m_e in angular motion in a spherically symmetric potential V(r), can be expressed as (6 marks)

$$V = V(r) + \frac{L^2}{2m_e r^2}$$ the symbols have their usual meanings

b. i). Write the differential R(r) equation derived from the Schrodinger equation, in terms of the quantum number l and the other terms as usual. Define terms in the equation. (2 marks)

ii). Solve the R(r) equation for the quantum numbers n =1, l = 0, assume free particle and positive value of the energy E giving your solution in the exponential form. (5 marks)

iii). Discuss the physical significance of the solution(s) obtained in b(ii). (2 marks)

iv). Describe, using a graphical sketch how R(r) varies with r, from 0 - ∞ (2 marks)

c).i). What is Zeeman effect? Explain why it occurs. (3 marks)

ii). Describe "Normal Zeeman effect" and state condition(s) for their observation. (3 marks)

d). i).What is a magnetic dipole and how does it come into existence in an atom (2 marks)

ii). Explain how the vector of a magnetic dipole moment μ relates to that of the orbital angular momentum **L**. (2 marks)

Q2.

a).i). Discuss the difference in the quantization of the orbital angular momentum of electron in an atom, as expressed in the Bohr postulate and the quantum mechanics. (4 marks)

ii). Explain how the difference has since been resolved/harmonized. (2 marks)

iii).Determine the z-component (L_z) of the orbital angular momentum of electron in circular motion at a distance r, if linear momentum is given as $\mathbf{p} = -i\hbar\nabla_{(xyz)}$. (3 marks)

b).The quantum number m comes as a separation constant which can be determined from one of the differential equations of the Schrodinger equation in spherical coordinates.

i).Write the separated equation involving m as the only separation constant (1 marks)

ii).Solve the equation in (i), giving exponential form of solution. (3 marks)

iii).Discuss the physical significance of the solution (2 marks)

iv).Show how the acceptable quantum numbers of m are obtained from the solution in (ii), stating clearly the condition considered. (4 marks)

Q3.

a).i).Explain why the introduction of external magnetic field leads to the absorption of energy by an atom. (3 marks)

ii).Write expression for the potential energy associated with magnetic dipole moment μ, in an external magnetic field strength **B**. Hence, or otherwise show that the energy absorbed by atom in an external magnetic field of strength is quantized in terms of the quantum number m_l. (5 marks)

iii).Write expression for the expected number of spectral lines(m-degenerate states) likely to result from the effect of external magnetic field on the atomic spectra, in terms of the quantum number l. (1 mark)

iv).Explain how the spectral lines are expected to split for the following $l = 0$, 1 and 2. (3 marks)

b). i). Describe the Stern-Gerlach experiment, stating its aim and the observed outcome. (3 marks)

ii). Explain how the observed results of the Stern-Gerlach experiment is a departure from the expected effect of external magnetic field. (2 marks)

iii). Explain how this departure has since been explained. (3 mark)

Q4.

a). i). Starting from the differential R(r) equation, of question 1b(i), show that the quantum mechanical expression for the potential energy of a particle in a spherically symmetric potential takes the form. (4 marks)

$$V = V(r) + \frac{\hbar^2}{2m_e r^2} l(l+1)$$ where symbols have their usual meanings.

ii).Explain how the conclusion to associate orbital angular momentum with the quantum number l is arrived at from the expression in (i). (3marks)

iii) Show, on orbital angular momentum level diagram, the l-degenerate states corresponding to $n = 3$ and on it show the corresponding location of the Bohr's orbital momentum. (4 marks)

b). i). What is *magnetic dipole moment* of an atom. State factors influencing it. (2 marks)

ii). Write expression for the Bohr's magneton and determine its magnitude. (3 marks)

iii). Show that when the magnetic dipole in an external magnetic field is turned from the antiparallel to parallel direction the energy stored in it would be $\Delta U = 2\mu B$. Discuss one mechanism by which the stored energy can be increased in this particular case. (4 marks)

Q5.

a). i). What do you understand by the term "*commuting operators*". (2 marks)

ii). Explain how the concept of commuting operators can be used to determine whether or not a pair of variables can be measured simultaneously according to the Heisenberg's uncertainty principle. (3 marks)

iii). State a condition which any two operators should satisfy in order to commute. (1 marks)

iv). Name any two operators encountered in quantum mechanics which are likely to commute and explain why this is so. (2 marks)

b). i). State the Pauli's exclusion principle. (2 marks)

ii). In Bohr's case the state of an electron can be described fully by just stating the orbital radius or energy level of electron. If any, what problem would this cause about many electron atoms? (3 marks)

iii). Pauli established a rule for determining how electrons should be placed in a system. The rule can be adopted and tested on the wavefunctions for particles in a system. Name of this rule and explain how it should apply to wavefunctions of different particles. (3marks)

iv. State the number of quantum sates l and corresponding number of orbital angular momentum (L) that would arise from the energy state n. (1 marks)

II). Explain the physical meanings of states represented as

a) 1s, 2p b). 1s, 2s, 2p (3 marks)