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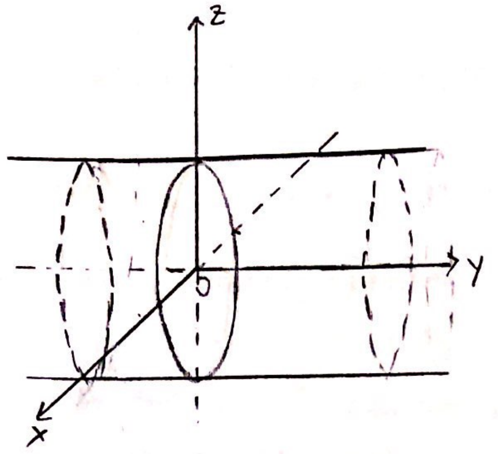
Math023-HW01

12C.1S.

8. Solution.

Description: $x^2+z^2=9$ in $\mathbb{R}^3$ is a cylinder shell.



10. Solution:

$|PQ| = \sqrt{(4-2)^2+(1+1)^2+(1-0)^2} = 3.$

$|PR| = \sqrt{(4-2)^2+(5-1)^2+(4-0)^2} = 6$

$|QR| = \sqrt{(4-4)^2+(1+5)^2+(1-4)^2} = 3\sqrt{5}.$

So $|PQ|^2+|PR|^2=|QR|^2$

$\triangle PQR$ is a right triangle.

14. Solution:

Its equation is: $(x-2)^2+(y+6)^2+(z-4)^2=25.$

① its intersection with xy-plain:

Let $z=0$: $(x-2)^2+(y+6)^2=9$. So its intersection with xy-plain is a circle with radius being 3.

② its intersection with xz-plain:

Let $y=0$: $(x-2)^2+(z-4)^2=-11$. So. this equation DNE in $\mathbb{R}^3$, so there is no intersection with xz-plai

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③ its intersection with yz-plain:

Let $x=0$,

$(y+6)^2+(z-4)^2=21$.

So its intersection with yz-plain is a circle with radius being $\sqrt{21}$

18. $x^2+y^2+z^2+8x-6y+2z+17=0$.

$x^2+8x+16+y^2-6y+9+z^2+2z+1+17=16+9+1$

$(x+4)^2+(y-3)^2+(z+1)^2=26-17$

$(x+4)^2+(y-3)^2+(z+1)^2=9$.

So the circle's center is $(-4,3,-1)$ and its radius is 3.

22. Solution 1:

the circle's equation is:

$(x-5)(x-1)+(y-4)(y-6)+(z-3)(z+9)=0$.

$x^2-6x+5+y^2-10y+24+z^2+6z-27=0$.

$(x-3)^2+(y-5)^2+(z+3)^2=41$.

Solution 2:

The center of the circle is: $\left(\frac{5+1}{2}, \frac{4+6}{2}, \frac{3-9}{2}\right)$

$=(3,5,-3)$.

And its diameter is: $\sqrt{(5-1)^2+(4-6)^2+(3+9)^2}=\sqrt{164}=2\sqrt{41}$

so its radius is $2\sqrt{41}$

So its equation is:

$(x-3)^2+(y-5)^2+(z+3)^2=41$

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44. Solution: Let the coordinator of P be (x_0, y_0, z_0)

so. $|PA|=2|PB| \iff |PA|^2=4|PB|^2 \iff (x_0+1)^2+(y_0-5)^2+(z_0-3)^2=4[(x_0-6)^2+(y_0-2)^2+(z_0+2)^2]$

So. $(x_0-13)(3x_0-11)+(y_0+1)(3y_0-9)+(z_0+7)(3z_0+1)=0$

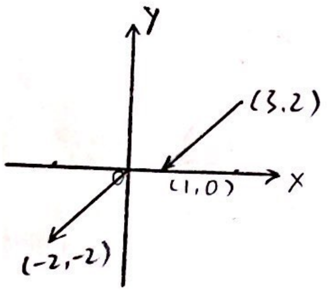
So we could have a circle equation. with $A(13,-1,-7), B(\frac{11}{3},3,-\frac{1}{3})$ as the end points of diameter.

So the coordinator of its center is: $\left(\frac{13+\frac{11}{3}}{2}, \frac{-1+3}{2}, \frac{-7-\frac{1}{3}}{2}\right)=\left(\frac{25}{3},1,-\frac{11}{3}\right)$

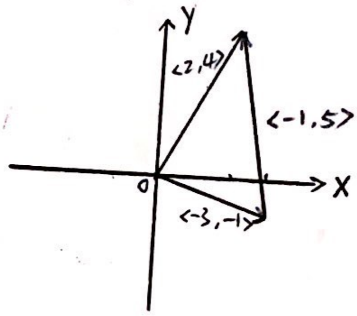
And its radius is: $\frac{1}{2}|AB|=\frac{1}{2}\sqrt{(13-\frac{11}{3})^2+(-1-3)^2+(-7+\frac{1}{3})^2}=\frac{2}{3}\sqrt{83}$.

C12. S2

12. $\vec{AB}=\langle -2,-2\rangle$



15. $\langle 3,-1\rangle, \langle -1,5\rangle$



20. $\vec a=5\vec i+3\vec j, \quad \vec b=-\vec i-2\vec j$

so $\vec a+\vec b=4\vec i+\vec j, \quad 4\vec a+2\vec b=4(5\vec i+3\vec j)+2(-\vec i-2\vec j)=18\vec i+8\vec j$

$|\vec a|=\sqrt{5^2+3^2}=\sqrt{34}$,

$|\vec a-\vec b|$: since $(\vec a-\vec b)=(5+1)\vec i+(3+2)\vec j=6\vec i+5\vec j$

$|\vec a-\vec b|=\sqrt{6^2+5^2}=\sqrt{61}$

24. Let $\vec u$ be the unit vector.

so $\vec u=\frac{-5\vec i+3\vec j-\vec k}{\sqrt{5^2+3^2+1^2}}=-\frac{5}{\sqrt{35}}\vec i+\frac{3}{\sqrt{35}}\vec j+\frac{-1}{\sqrt{35}}\vec k=-\frac{\sqrt{35}}{7}\vec i+\frac{3\sqrt{35}}{35}\vec j-\frac{\sqrt{35}}{35}\vec k$.

28. Let the vector be $\vec a$.

so the angle θ s.t. $\vec a\cdot\vec i=\cos\theta|\vec a|\cdot|\vec i|$

so $8|\vec i|^2=\cos\theta\sqrt{8^2+6^2}$

$\cos\theta=\frac{4}{5}$.

so $\theta=\arccos\frac{4}{5}\approx 37^\circ$

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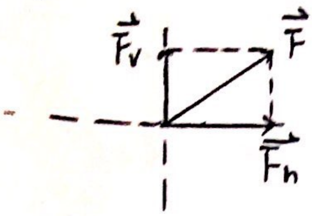
30.



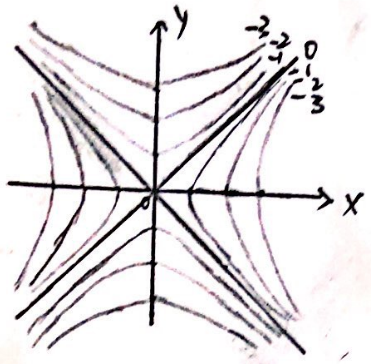
Let $\vec{F_v}$ be tho force on the vertical direction, and $\vec{F_h}$ be the force on the horizontal direction

so $|\vec{F_v}| = (\sin 38^\circ)|\vec{F}|$, $\vec{F_v} = \langle 0, |\vec{F}|\sin 38^\circ \rangle$
$= \langle 0, 50 \cdot \sin 38^\circ \rangle$

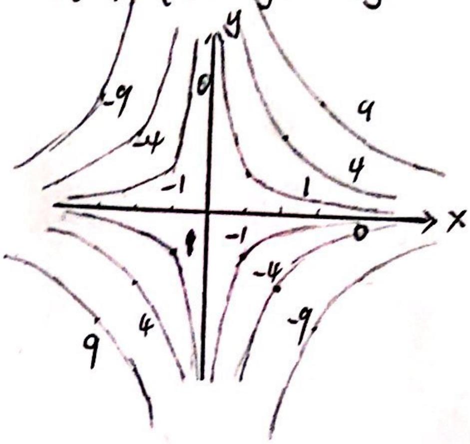
The same reason $\vec{F_h} = \langle 50\cos 38^\circ, 0 \rangle$

C14.S1.

45. $f(x,y) = x^2 - y^2$.

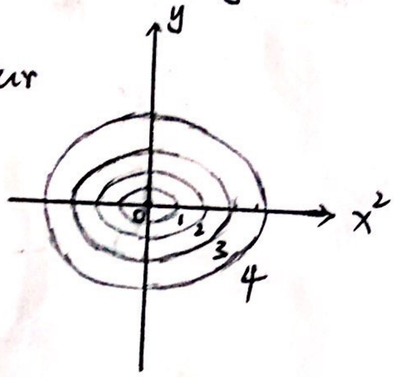


46. $f(x,y) = xy$

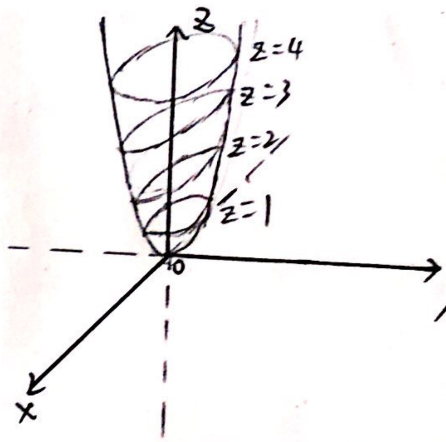


53. $f(x,y) = x^2 + 9y^2$

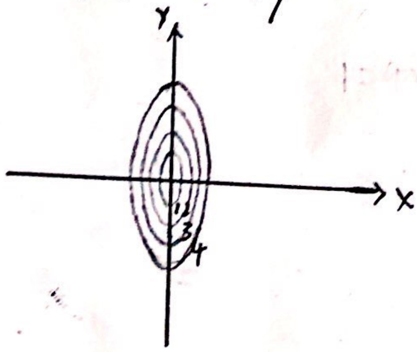
Contour map:



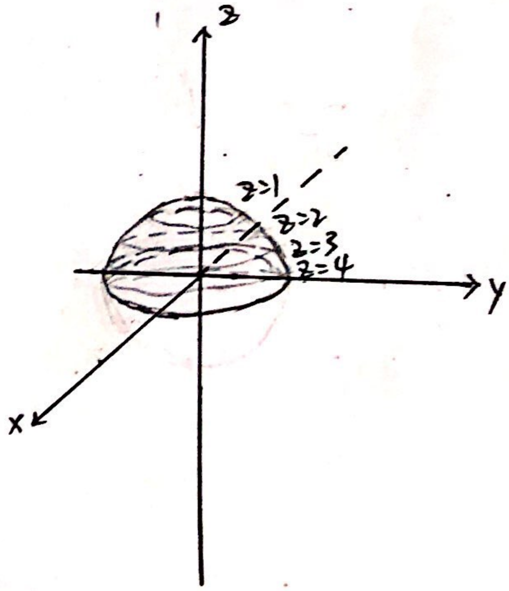
graph:



54. $f(x,y) = \sqrt{36 - 9x^2 - 4y^2}$



graph:



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69. $f(x,y,z) = y^2+z^2$

Solution:

Description:

It's a family of circular cylinders with axis the x-axis $(k>0)$

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