Math 163: Homework 1

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Theorem 1. *The sum of two odd integers is an even integer.*

Proof. Let x and y be odd integers
Then there is an integer a, such that $x = 2a + 1$
Then there is an integer b, such that $y = 2b + 1$
Then,

$$x + y = (2a + 1) + (2b + 1)$$

By commutativity:

$$x + y = 2a + 2b + 2$$

By distributive law:

$$x + y = 2(a + b + 1)$$

Because $a, b, 1 \in \mathbb{Z}$
Therefore $a + b + 1 \in \mathbb{Z}$ (by closure under addition of the integers)
Thus, $x + y$ is an even integer by definition of even. □