Homework #2

Math 263, Spring 2022
Due Thursday, February 10.

1. A certain class has 10 students, 2 of whom are sisters. The class needs to be split across three rooms for a socially-distanced exam, where room A can seat 2 students, room B 3 students, and room C 5 students. How many ways can the students be divided across the rooms, if the 2 sisters are not allowed to be in the same room (say, for academic integrity)?

2. Suppose you roll a 4-sided die (with sides numbered 1, 2, 3, and 4), and then you flip a coin n times, where n is the number you rolled.

   (a) Describe (mathematically) the sample space of this experiment.

   (b) How large is the sample space? (In particular, the sample space is a finite set. How many elements does it have?)

   (c) Describe the event that, in the course of the experiment, you see exactly 3 heads.

   (d) How many events (corresponding to this experiment) are there?

3. Suppose that a number x is to be randomly selected from the real line, and 3 people each bet on an interval where the number will be. Ann bets that x will be in $A = [2, 9]$, Bill bets that x will be in $B = (5, 11]$, and Chris bets that x will be in $C = (-\infty, 0)$. Describe each of the following events as a set of real numbers:

   (a) $A \cup B$

   (b) the event that Ann wins her bet but Bill loses his bet

   (c) the event that none of the three wins their bet

4. Suppose that, at a large state school, 50% of sophomores take a math course and 35% of sophomores take a physics course.

   (a) If this is all that you know, what are the largest possible and smallest possible percentages of sophomores that take at least one math or physics course?

   (b) Now suppose that the registrar tells you that, in fact, 60% of sophomores take at least one math or physics course. What percentage of sophomores take both a math course and a physics course?

5. There are 5 white, 7 black, and 8 red balls in a urn. You randomly pull out 4 balls.

   (a) What is the probability that at least 3 of the balls are red?

   (b) What is the probability that all 4 balls are the same color?