Math 163: Homework 2

Due Thursday, February 10th at 11:59pm

- This homework must be submitted to Gradescope and typed in LaTeXfor full credit. (Be sure to select the pages of your problem numbers in Gradescope.)

- Use complete sentences, and carefully justify **every line** of your proofs and other answers. You will be graded on the clarity of your proofs as well as their accuracy, so proofread your homework as you would an essay for an English class.

- Limit your assumptions about the integers to the list of properties on page 23 of your text (also available on Coursesite).

- Problems from your text are marked with and L, and end with the problem number. Thus L 1.2.3 refers section 1.2, exercise 3 (for example).

- Write out English words, rather than using symbols, unless you are in the middle of an equation or set.

- You're welcome to work on your homework with another student in the class. Make sure you complete your write up independently. You may not post homework questions online asking for help, but are encouraged to seek help from your instructor and your TA. If you use an outside resource other than your text, reference it in your response.

1. (a) Prove that the product of an even number and an integer is an even number.

   (b) Prove that the product of two odd numbers is odd.

2. Prove: If $a|b$ and $a|c$, then $a|(2b + 3c)$.

3. (a) If a, b are integers, then $(ab)^2 = a^2b^2$. Rigorously prove this fact using only the Basic Properties of Integers on page 23 of your text. Hint: This will only require a few lines; don't overthink it.

   (b) Similarly, state and prove a theorem which gives the expansion of $(a + b)^2$ using only the Basic Properties of Integers.

   (c) Imagine a group of aliens land on earth, and ask to speak with Earth's greatest mathematicians. After an exchange, it is established that in the aliens' number system, the following properties are true:

   - Closure under $+$ and $\cdot$.

- Associativity of addition and multiplication.
- Communitivity of addition.
- A left and a right distributive law: $a(b+c) = ab + ac$ and $(b+c)a = (ba+ca)$.
- Existance of an additive and multiplicative identity.

In particular, the alien's number system does NOT have a commutative law of multiplication. Using only these properties, state (and justify, as above) the expansion of $(ab)^2$ and $(a+b)^2$ is in the alien's number system. Reflect on why the alien's number system has two distributive laws rather than one.

4. L 1.1.3.

5. L 1.1.4. Be sure to justify every answer. In particular, to show something is a tautology or a contradiction, you must show that every possible value of True or False for every statement gives the same result. (Usually, students do this with a truth table.) To show that it is neither, you need to give one example where the result is true, and one example where the result is false. (Thus you need not give the entire table in this case.)