Math 263: Homework 2

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1. The number of ways are:
$$\binom{10}{2}\binom{8}{3} - \binom{8}{3} - \binom{8}{2}\binom{6}{1} - \binom{8}{2}\binom{6}{3} = 1736$$

2. Let 1 be the head, and 0 be the tail, and S be the sample space.
   (a) The sample space is:
   $$\bigcup_{j=1}^{4} \{(S_1, \ldots, S_j) : S_i \in 1, 0\}$$
   (b) The size of the sample space is:
   $$\sum_{i=1}^{4} 2^n = 2 + 4 + 8 + 16 = 30$$
   (c) Let the event set be E, so the event:
   $$E = \{(1,1,1,0), (1,1,0,1), (1,0,1,1), (0,1,1,1), (1,1,1)\}$$
   (d) Recall: Any subset E of the sample space is known as an event. The number of this events is 2^{30}.

3. (a) Let S_a be the events:
   $$S_a = \{x : x \in [2, 11]\}$$
   (b) Let S_b be the events, so $S_b = A \cup B^c$
   $$S_b = \{x : x \in [2, 5]\}$$
   (c) Let S_c be the events, so $S_c = (A \cup B \cup C)^c$
   $$S_c = \{x : x \in [0, 2) \cup (11, +\infty]\}$$

4. Let the group of sophomores taking math be A, and the group of sophomores taking physics be B.
   (a) The largest possibility can be: $A \cup B = \varnothing$, so it can be $50\% + 35\% = 85\%$.
   The smallest possibility can be: $A \supset B$, so it can be 50%.
   (b) $A \cup B = A + B - A \cap B$, so $A \cap B = 35\% + 50\% - 60\% = 25\%$.
   Thus, the percentage of sophomores taking both a math and a physics course is 25%.

5. Let the numbers of the total events be S, Then $S = \binom{20}{4} = 4845$
   (a) Let the number of this events be $S_{r \geqslant 3}$.
   Then, $S_{r \geqslant 3} = \binom{8}{3}\binom{12}{1} + \binom{8}{4} = 742$
   Thus, the probability $= \frac{S_{r \geqslant 3}}{S} = \frac{742}{4845}$
   (b) Let the number of this event be S_4.
   Then, $S_4 = \binom{5}{4} + \binom{7}{4} + \binom{8}{4} = 110$
   Thus, the probability $= \frac{S_4}{S} = \frac{22}{969}$.