Numerical Relativity Homework 2

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June 2024

1 Sod Shock Tube Problem

The Sod shock tube problem consists in a one-dimensional Riemann problem with Initial discontinuities in density and pressure. The time evolution of the system can be computed by solving the Euler equations. Since the solution of this problem can be computed exactly, it is useful to test the accuracy of numerical codes. In this exercise we solve the Sod problem with the Einstein Toolkit (ETK) using different resolutions and we compare the results with the exacts solution.

The solution to the problem is described by three characteristics, each of them related to the propagation speed of the fluid in different regions. Those can be associated to either a rarefaction wave, a shock wave, or a contact discontinuity. Specifically, the pressure and the velocity of the fluid develops a rarefaction wave and a shock wave, while the density develops also a contact discontinuity. The exact solution along with the initial conditions is shown in Figure 1.

For the numerical solutions the HLLE Riemann solver has been used, along with the Minmod slope limiter. The domain extends in the range $[-0.5, 0.5]$ and the evolutions proceeds up to time $t = 0.4$. The used grid spacing are $\{0.005, 0.0025, 0.00125, 0.000625\}$, corresponding respectively to $\{200, 400, 800, 1600\}$ grid points.

1.1 Higher Resolution: 1600 points

We will use the results obtained with the highest resolutions to showcase how the numerical solutions look like. Figure 2 shows some snapshots of the profiles of density, pressure and velocity of the fluid, including the initial and the final ones. As can be seen, S and CD waves travel in opposite direction with respect to the R waves. We point out that the fact that the initial profile in Figure 2 doesn't perfectly resemble a step (as it should) is just due to the way the numerical results are interpolated on the chosen uniform grid $[-0.45, 0.45]$. Figure 3 compares the interpolated initial data along with the raw initial data actually used by the ETK.

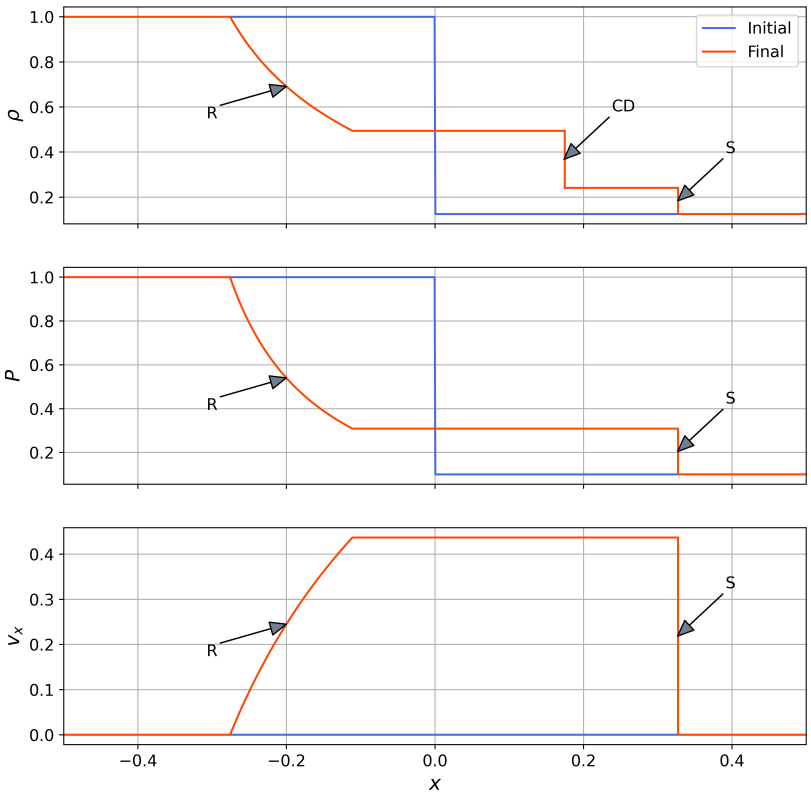

Figure 1: Exact Solution; Initial and final conditions; The arrows points at the different types of waves: R (Rarefaction), S (Shock) and CD (Contact Discontinuity).

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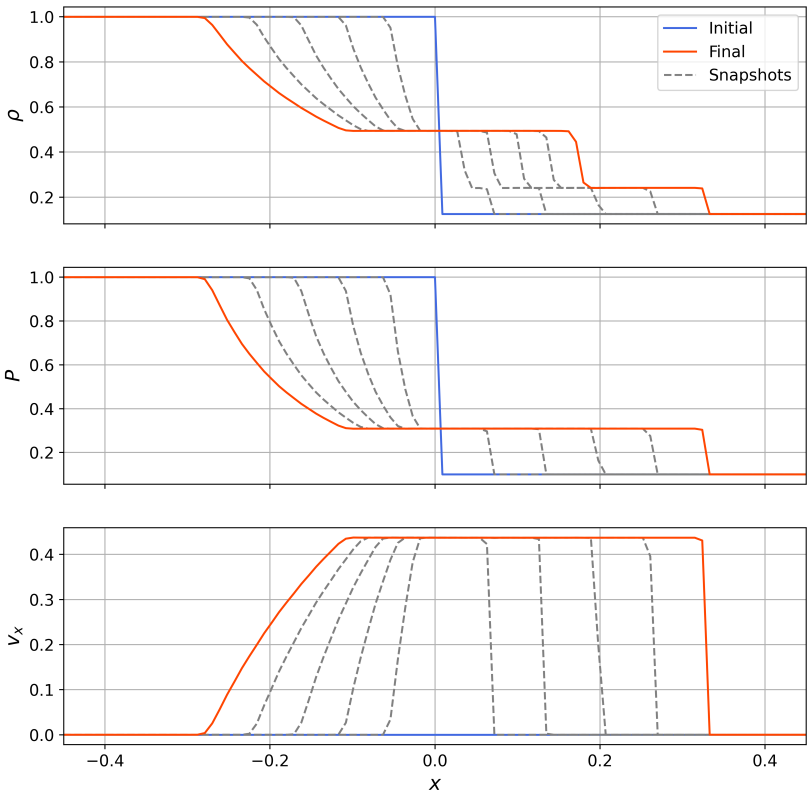

Figure 2: Numerical Solution; 1600 points; Snapshots. {fig:all_1600_snapsho

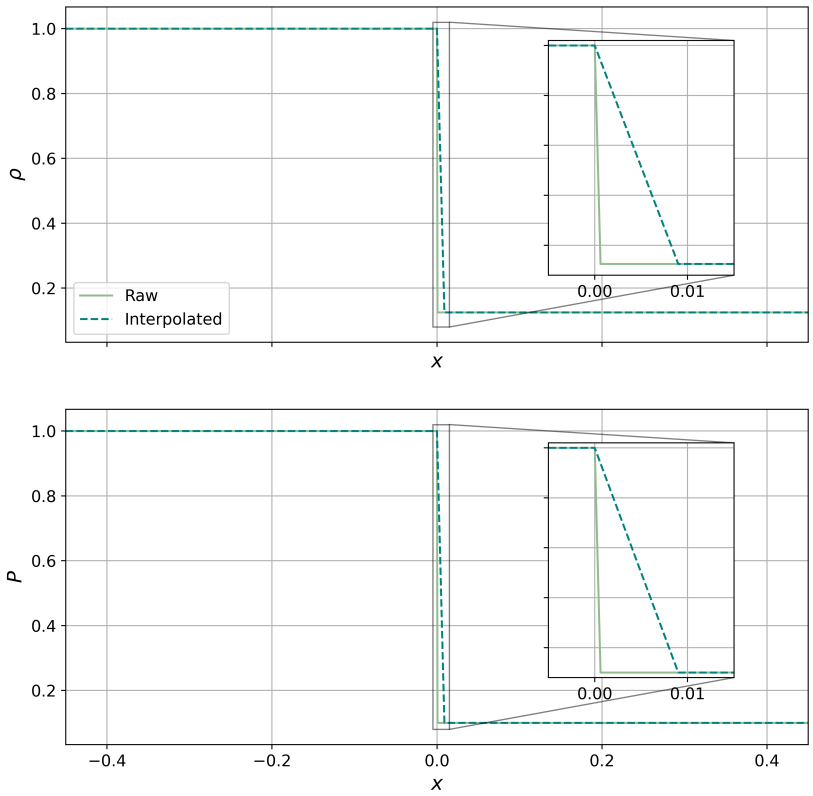

Figure 3: Initial data; 1600 points; Raw and interpolated. {fig:all_1600_initial