1) $\lambda x.(M\,N) =_\beta S\,(\lambda x.M)\,(\lambda x.N)$

$S = \lambda x f g x.\ f\,x\,(g\,x)$

$S\,(\lambda x.M)\,(\lambda x.N) =_\beta$

$\lambda x.\ (\lambda x.M)\,x\,((\lambda x.N)\,x) =_\beta$

$\lambda x.\ (M\,N)$

2) $xor = \lambda a b x y.\ \underset{}{a\,(b\,y\,x)\,(b\,x\,y)}$ ~~$a\,(b\,x\,y)\,(b\,y\,x)$~~

Let's prove that it is xor.

a	b	result
false	false	$\lambda x y.\ y = false$
false	true	$\lambda x y.\ x = true$
true	false	$\lambda x y.\ x = true$
true	true	$\lambda x y.\ y = false$

For commutativity

~~$\lambda a b x y.\ a\,(b\,x\,y)\,(\ \lambda b a x y =$~~

$\lambda b a x y.\ b\,(a\,y\,x)\,(a\,x\,y)$

the table is:

a	b	result
false	false	$\lambda xy. y$ = false
false	true	$\lambda xy. x$ = true
true	false	$\lambda xy. x$ = true
true	true	$\lambda xy. y$ = false.

If a then result is not b
If not a then resut is b.

5) mult = $\lambda mnf. m(nf)$
  1 = id
  mult 1 n = $\lambda f. id(nf) = \lambda f. nf = n.$
  mult m 1 = $\lambda f. m(idf) = \lambda f. mf = m.$
In a way, we ~~of~~ iterate function
n times and then m more, m·n in
total.

4) pair = $\lambda xyf. fxy$
  fst = $\lambda p. p$ true
  snd = $\lambda p. p$ false.
  succ = $\lambda nfx. f(nfx)$
  factorial = ~~n (λp. pair succ λn. snd~~
(n (λp. pair (succ (fst p)) ~~(fst p (snd p))~~
(pair ~~false id~~ id id)))   mult (fst p)(snd p)

we iterate n times each time
multiplying second element by number of itera
tion and then taking second element as
a result.
factorial 3 = snd (3 (λp. pair (succ fst
(fst p)) ~~(fst p (snd p))~~ (mult (fst p) (snd p)))
(pair id id)) =

$= (\lambda p. p \; \overset{(\lambda xy.y)}{\cancel{false}}) \; ((\lambda f x. f(f(f x))))$

$(\lambda p f. f \; (succ \; \lambda f x. f_1(\lambda p(p \; (\lambda xy. x_1)) \; f_1$
~~$(\lambda f. f \; (\lambda p.p)(\lambda p.p))$~~

$(\lambda f_1. (p \; (\lambda xy. x))((p(\lambda xy.y)) f_1)$

$(\lambda f_1. f_1 (\lambda p_1. p_1) \; (\lambda p_1. p_1))$

3) ~~pred n = λn. snd (n~~
~~pred = $\lambda n. n$ (succ λx. succ x) false~~