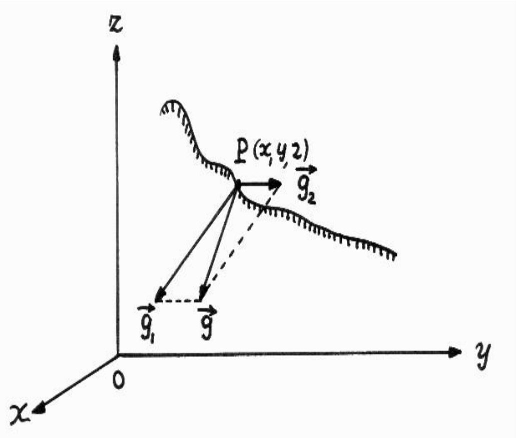
Geodetic astronomy

- Why astronomy ?
- Concepts of spherical astronomy
  - Celestial sphere
  - Motion of the Sun
  - Celestial coordinates of celestial bodies (e.g. stars)
- Concepts of time: Sidereal time, solar time, TAI/UTC/GPST
- Astronomical positioning
- *Earth rotation: polar motion, nutation and precession*

Why do we need astronomy ?

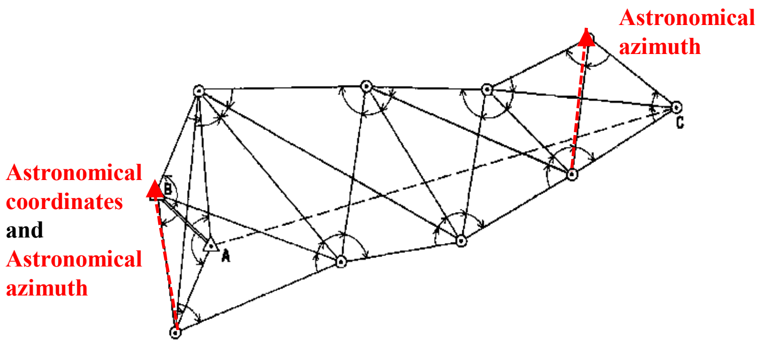
- Astronomical positioning:
  Find absolute positions of ground points
  - Astronomical latitude Φ
  - Astronomical longitude Λ
  - Astronomical azimuth A
- Astronomical concepts still in use
  - Describe satellites or radio sources in the universe
  - Changes in Earth rotation
  - Effect of Earth tides

Astronomical coordinates



- Astronomical latitude Φ
- Astronomical meridian plane and meridian
- Astronomical longitude Λ
- Astronomical azimuth A

Principle of astro-geodetic triangulation



From astronomical to geodetic coordinates

$$\phi = \Phi - \xi$$
$$\lambda = \Lambda - \eta / \cos\phi$$

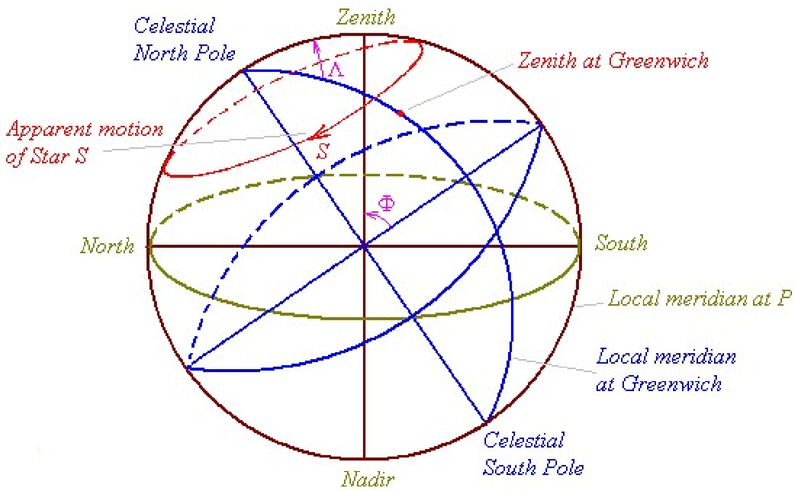
$$\alpha_{ab} = A_{ab} - \Delta A_1 - \Delta A_2$$

$$\Delta A_1 = \eta \; tg\phi = (\Lambda - \lambda)\sin\phi$$
$$\Delta A_2 = (\xi \sin\alpha_{ab} - \eta\cos\alpha_{ab})\cot z_{ab}$$

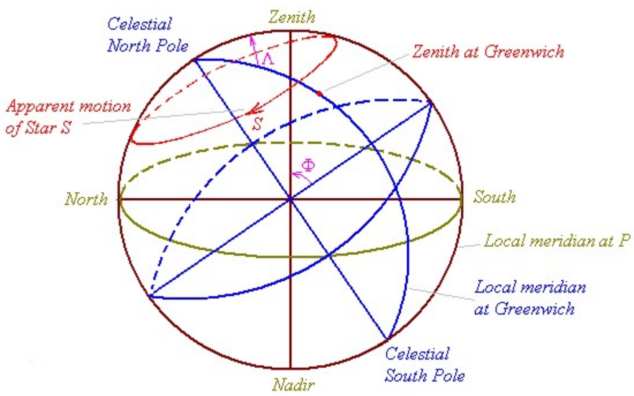
Elements on the celestial sphere

- Elements on the sphere
  - zenith, horizon
  - celestial North pole, celestial equator
  - local meridian
  - zenith and meridian at Greenwich
  - astronomical latitude and longitude
  - apparent motion
- The Sun and the ecliptic
  - ecliptic, obliquity
  - Sun’s apparent motion
  - Vernal vs Autumn equinox
  - polar circle

Celestial sphere



Celestial sphere



- Elements on the celestial sphere
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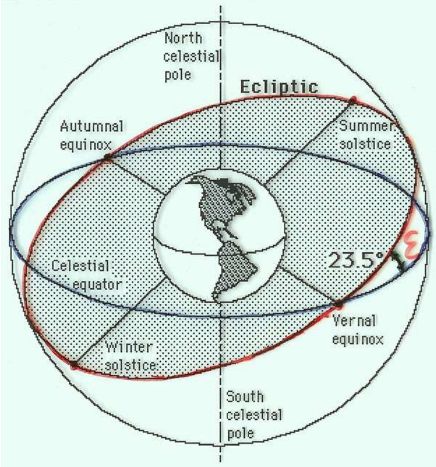
The Sun and the ecliptic



- The Sun and the ecliptic
  - ecliptic, obliquity
  - Sun's apparent motion
  - Vernal vs Autumn equinox
  - polar circle

The Ecliptic

$$\varepsilon = \varepsilon_0 + \Delta\varepsilon$$

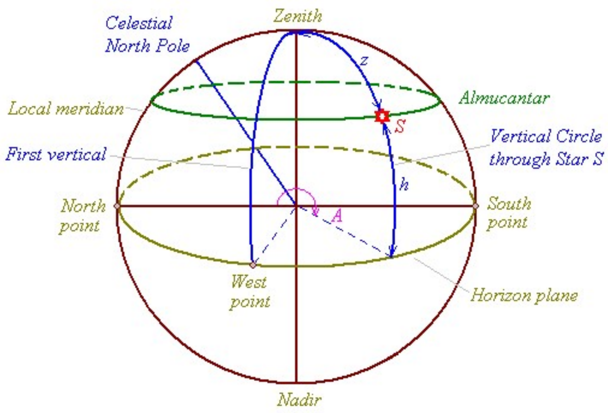
$$\varepsilon_0 = 23^\circ\ 26'\ 21.''448 - 46.''815 \cdot T - 0.''000059\ T^2 + 0.''001813 \cdot T^3$$

$$\Delta\varepsilon \approx 9.''2 \cdot \cos\alpha_m + \ldots\ldots$$

Celestial coordinates of celestial bodies

- Horizon coordinate system
  - zenith distance vs height angle
  - azimuth
- Equatorial coordinate system
  - Declination
  - right ascension vs hour angle
- Ecliptic coordinate system
  - Latitude
  - longitude
- Astronomical triangle
  - sides, angles
  - what changes due to apparent motion ???
  - trigonometrical relationships

Horizon coordinates A and z (or h)



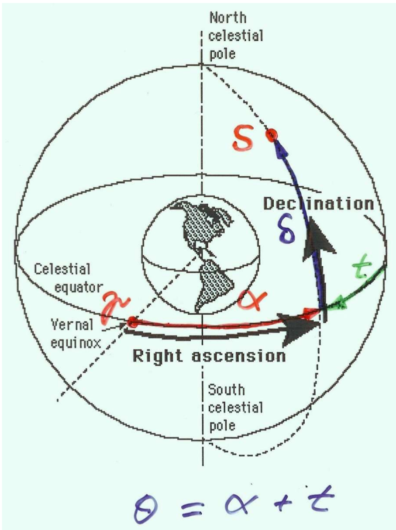
Azimuth: A

Zenith distance: z

Height angle: h

$$z = 90 - h$$

Equatorial coordinates α and δ (or t)

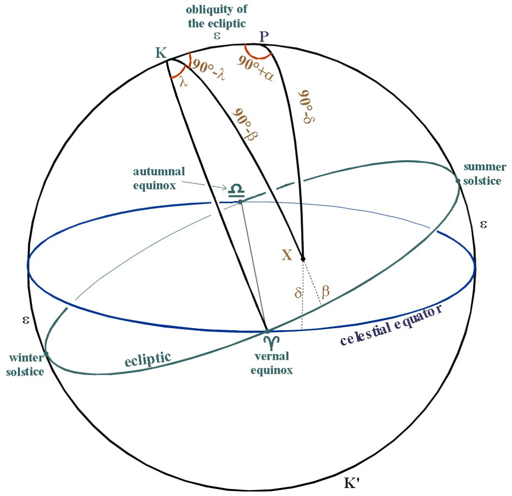


Declination: δ

Right ascension: α

Hour angle: t

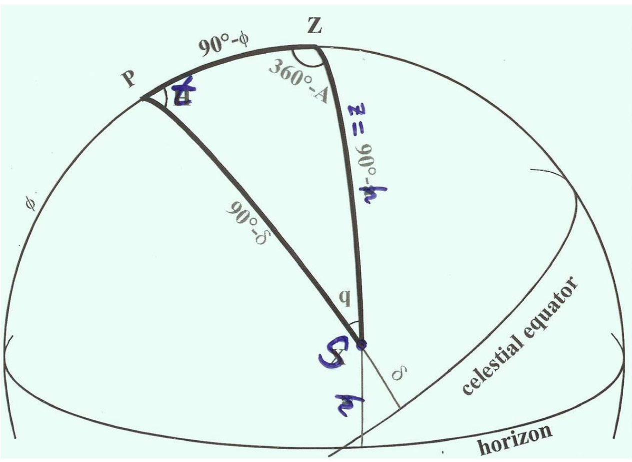
Ecliptic coordinates



Latitude: φ

Longitude: λ

Astronomical triangle



- What are the sides and angles ?
- Which quantity changes with time due to apparent motion ?

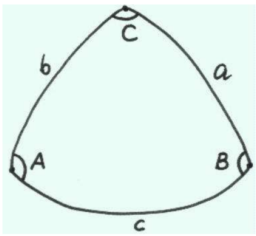
Trigonometric formulas for the astronomical triangle

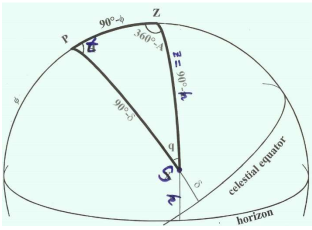
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$
$$\sin a \cos C = \cos c \sin b - \sin c \cos b \cos A$$
$$\sin A \cos b = \cos B \sin C + \sin B \cos C \cos a$$
$$\sin A \cos c = \cos C \sin B + \sin C \cos B \cos a$$





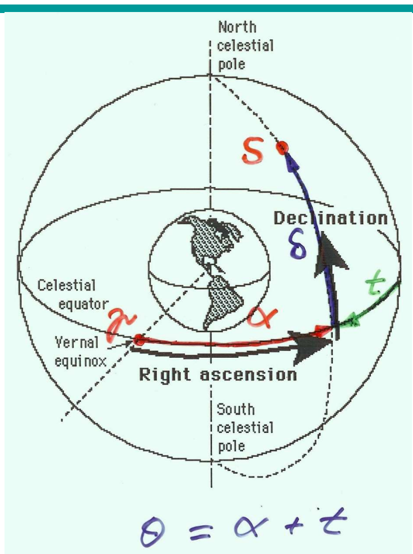
$$\sin\delta = \sin\Phi \sin h + \cos\Phi \cos h \cos A$$
$$\cos\delta \sin t = -\cos h \sin A$$
$$\cos\delta \cos t = \cos\Phi \sin h - \sin\Phi \cos h \cos A$$

$$\sin h = \sin\Phi \sin\delta + \cos\Phi \cos\delta \cos t$$
$$\cos h \sin A = -\cos\delta \sin t$$
$$\cos h \cos A = \cos\Phi \sin\delta - \sin\Phi \cos\delta \cos t$$

Concepts of time

- Sidereal time: time based on earth rotation
  with reference to stars
- Solar time: time based on earth rotation **and** the motion of the earth around the Sun
  (one round = 1 year)
- Dynamic time: dynamic time, TDB, TDT
  time realized by atomic clocks
  TAI, UTC, GPST

Definition of Sidereal time



One complete round of earth rotation is one ***sidereal day*** (24h)

True **sidereal time** (θ) is defined as the hour angle of the *true* vernal equinox

For any star with α at time epoch t :

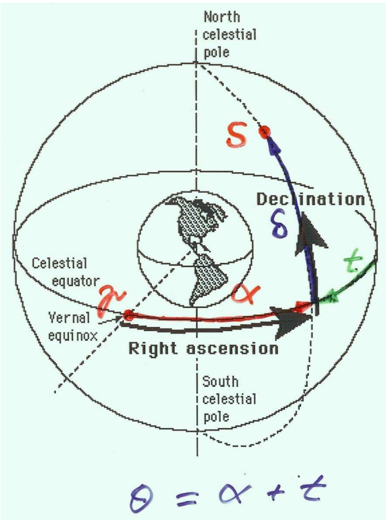
$$\theta = \alpha + t$$

Sidereal time

- Sidereal time is dependent on the observer's meridian
  - Local Apparent Sidereal Time (LAST) at point P
  - Greenwich Apparent Sidereal Time (GAST)
  - LAST - GAST = Λ_P
- Variations in earth rotation $\rightarrow$ variation in Υ
- Mean position of Υ $\rightarrow$ Mean sidereal time
  - Local Mean Sidereal Time (LMST) at point P
  - Greenwich Mean Sidereal Time (GMST)
  - LMST - GMST = Λ_P
- Equation of the Equinox
  = true sidereal time – mean sidereal time

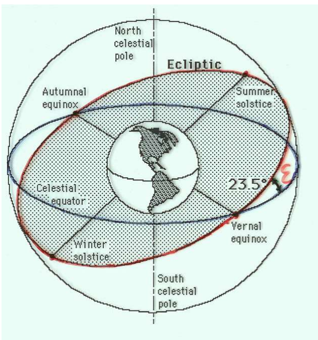
$Eq.E$ = LAST - LMST = GAST – GMST = $\Delta\Psi \cos \varepsilon_0$ (< 1 s)

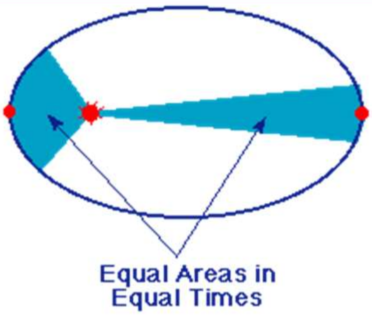
Definition of *True* Solar time



- *True* solar time is defined as the hour angle of the *true* Sun + 12h
- Solar time is dependent on the observer's meridian
- Local true Solar Time (LST) at P
- Greenwich true Solar Time: GST
- LST - GST = Λ_P

Non-uniform motion of the Earth





Kepler's 2nd Law:
The time derivative of the area swept by the earth is a constant.

Solar time based on the true Sun is not uniform !!! ← → Equal areas in equal time intervals imply different angular velocities

Definition of Mean Solar time

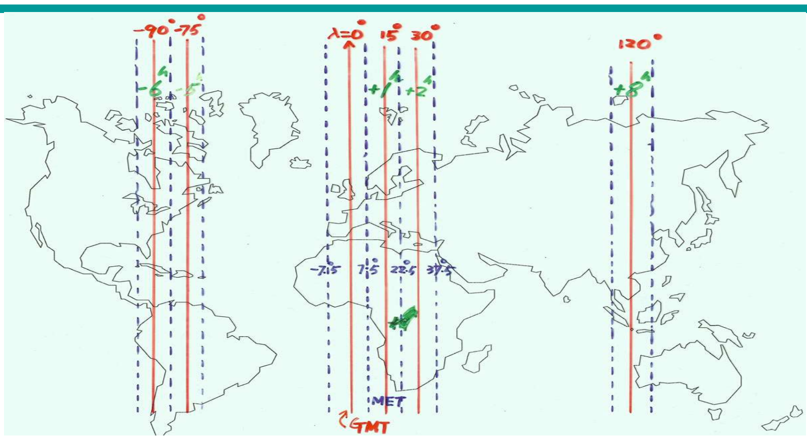
- Definition of *"Mean Sun"* :
  - Moves along the celestial equator at the constant angular velocity
  - Moves one complete round on the equator just as the true Sun moves a complete round along the ecliptic
- *Mean* solar time = hour angle of *mean* Sun + 12h
- *Mean* solar time depends on the observer's meridian
  - Local Mean solar Time (LMT) at point P
  - Greenwich Mean solar Time (GMT)
  - LMT - GMT = Λ_P
- Equation of Time (e)

e = true solar time – mean solar time (-14 min,+16 min)

Universal Time (UT)

- Universal Time (UT) is based on Greenwich Mean Time (GMT)
- UT1 = UT + correction due to polar motion
  = astronmically defined mean solar time
- UT in time zones:
  - ✓ the world is divided into a number of time zones
  - ✓ Each time zone uses the mean solar time referred to the local (central) meridian

Zone time (mean solar time)



- Zone time: mean solar time referred to the mid-meridian of the zone. Longitude difference of 15 degrees imply time diffeence of 1 hour
- SNT (Swedish Normal Time) = CET = UT + 1h

From *UT1* to *GAST*: Julian date

- Julian date (JD) is a consecutive number for every epoch in the human history of written records
  - JD = 0.0 on 1 January, 4713 BC, UT1=12h
  - JD = 2 451 545.0 on 1 January, 2000, UT1=12h (JD2000.0)
- Julian Modified Julian date (MJD) = JD – 2 400 000.5
- Calculation of JD for epoch UT1, year (Y), month (M), day (D)

$$
\begin{array}{llll}
y = Y - 1 & and & m = M + 12 & if\ M \leq 2 \\
y = Y & and & m = M & if\ M > 2
\end{array}
$$

$$JD = INT(365.25 \cdot y) + INT(30.6001 \cdot (m+1)) + D + \frac{UT1}{24} + 1\,720\,981.5$$

Greenwich Mean Sidereal Time $(GMST)_0$ at mid-night (UT1=0)

- Epoch: year (Y), month (M), day (D), UT1=0
- JD for the epoch: $JD_{UT1=0}$
- T_0: time interval between J2000.0 and the epoch (mid-night), measured in the Julian century of 365 25 mean solar days

$$T_0 = \frac{JD_{UT=0} - 2\,451\,545.0}{36525}$$

- Greenwich Mean Solar Time $(GMST)_0$ at mid-night:

$$(GMST)_0 = 24\,110.54841^s + 8\,640\,184.812866^s\ T_0 + 0.093\,104^s\ T_0^2 - 6.2^s \cdot 10^{-6} T_0^3$$

Conversion from Mean Solar Time UT1 to Greenwich Mean Sidereal Time GAST

- A tropical year is the time for the *true* Sun to move a complete round along the ecliptic

A tropical year = 365.24219 mean solar days

= 366.24219 sidereal days

$\longrightarrow$ 1 mean solar time unit = **1.002 737 909 35** sidereal time unit

1 mean solar day = 1 sidereal day + 3 sidereal minutes + 56.555 sidereal seconds

- Conversion from UT1 to GAST

Y, M, D, UT1 $\rightarrow T_0 \rightarrow JD_{UT1=0} \rightarrow (GMST)_0 \rightarrow GMST \rightarrow GAST$

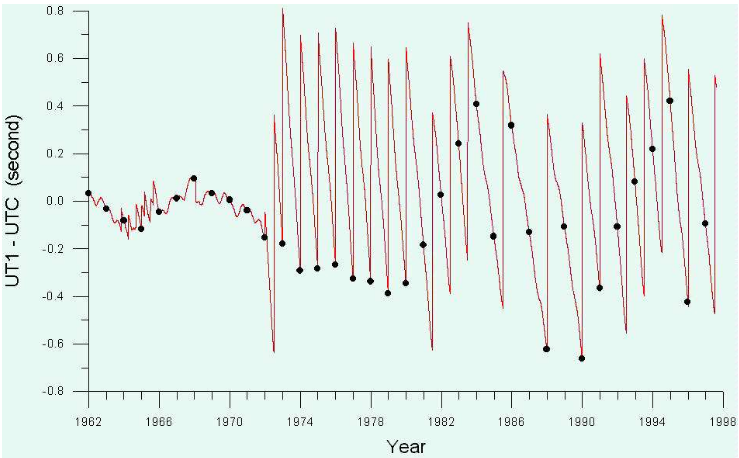
GMST = $(GMST)_0$ + **1.002 737 909 35** · UT1

GAST = GMST + Eq.E

Universal Time Coordinated (UTC)

- Dynamic time
  - time in an inertial reference frame for description of motions
  - Barycentric dynamic time (TDB) and Terrestrial dynamic time (TDT)
  -
- International Atomic Time (TAI) – a realization of TDT
  - 1 second is the duration of 9 192 631 770 periods of radiation of Celsium 133
  -
- **Universal Time Coordinated (UTC)**
  - runs at the rate of TAI
  - UTC = TAI on 1958-01-01 = TAI – 19s (1980-01-06 0h)
  - subjects to a one-second adjustment (leap second) so that UT1-UTC ≤±0.9s
  - UTC = TAI – 37s on 2016-12-31 (**last** new leap second !)
- GPS Time (GPST): GPST = UTC (1980-01-06 0h) = TAI – 19s

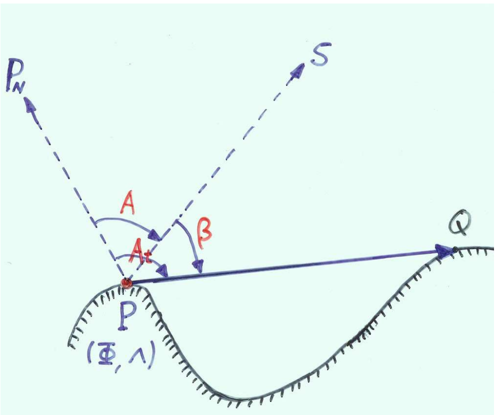
Variation of UT1-UTC



Astronomical Positioning

- Azimuth determination from a known point
  - measure time + angle between Polaris and the object
- Latitude determination
  - measure star's zenith distance at meridian passage
- Longitude determination
  - measure time at star's meridian passage
- Simultaneous determination of latitude & longitude
  - measure 2 or more stars' zenith distances and times

Azimuth determination with Polaris



- Assume (Φ, Λ) approx. known
- Measure angle β at epoch UT1
- Compute azimuth A of Polaris

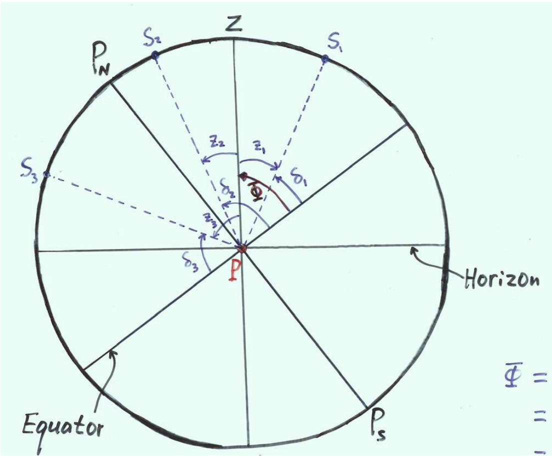
$$\cot A = \frac{\sin\Phi\cos t - \cos\Phi\tan\delta}{\sin t}$$

$$t = \theta - \alpha = LAST - \alpha = GAST + \Lambda - \alpha$$

- Find azimuth A_t from P to Q

  $A_t = A + \beta$
- Polaris moves very slowly. (δ≈89º)
- Accuracy: 0.3-0.7" after 3 nights

Latitude determination



- Star S in the south

  $\Phi = \delta_1 + z_1$

- Star S in the North

  $\Phi = \delta_2 - z_2$

- Star S below the Pole

  $\Phi = 180^\circ - (\delta_3 + z_3)$

Simultaneous determination of (Φ, Λ)

Observing the zenith distances of 3 different stars at 3 different epoches

$$\cos z = \sin \Phi \ \sin \delta + \cos \Phi \ \cos \delta \cos t$$

$$t = \theta - \alpha = LAST - \alpha = GAST + \Lambda - \alpha$$