d) $\sigma_0 = 0.15$ $\quad \hat{\sigma}_0 \approx 0.126414368$

$$(n-2)\frac{\hat{\sigma}_0^2}{\sigma_0^2} \sim \chi^2_\alpha(n-2) \Rightarrow 8\cdot\frac{\hat{\sigma}_0^2}{\sigma_0^2} \approx 5.7$$

$H_0: \hat{\sigma}_0 = \sigma_0$

$H_1: \hat{\sigma}_0 \neq \sigma_0$

$\chi^2_{0.05}(8) \approx 15.5$

$5.7 < 15.5 \Rightarrow$ Do not reject null hypothesis

$\Rightarrow$ can't conclude $H_1: \hat{\sigma} \neq \sigma_0$

regression line significant

3.1 $\quad p_1 = p_2 = 2 \quad p_3 = 1$

a) $\underset{3\cdot 1}{L'} = \begin{bmatrix} 60^\circ\ 00'\ 03'' \\ 60^\circ\ 00'\ 03'' \\ 59^\circ\ 59'\ 51'' \end{bmatrix}$ $\qquad \underset{3\cdot 3}{P} = \begin{bmatrix} 2 & & \\ & 2 & \\ & & 1 \end{bmatrix}$

$\underset{3\cdot 1}{L} - \underset{3\cdot 1}{\varepsilon} = \underset{3\cdot 2}{A}\ \underset{3\cdot 1}{X}$ $\qquad$ where,

$$\underset{3\cdot 1}{L} = L' - c = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 180^\circ \end{bmatrix} = \begin{bmatrix} 60^\circ\ 00'\ 03'' \\ 60^\circ\ 00'\ 03'' \\ -120^\circ\ 00'\ 09'' \end{bmatrix}$$

$$\underset{3\cdot 1}{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

$$\underset{3\cdot 2}{A} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}, \qquad \underset{2\cdot 1}{\underline{X}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b) $$A^T P A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & & \\ & 2 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$(A^T P A)^{-1} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\hat{\underline{X}} = (A^T P A)^{-1} A^T P L = \frac{1}{8} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & & \\ & 2 & \\ & & 2 \end{bmatrix} L$$

$$= \frac{1}{8} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \end{bmatrix} L =$$

$$= \frac{1}{4} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \end{bmatrix} \begin{bmatrix} 60^\circ\ 00'\ 03'' \\ 60^\circ\ 00'\ 03'' \\ -120^\circ\ 00'\ 09'' \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 180^\circ - 60 + 120 + \quad 9'' - 3'' + 9'' \\ -60^\circ + 180^\circ + 120^\circ \quad -3'' + 9'' + 9'' \end{bmatrix}$$

$$= \begin{bmatrix} 60^\circ\ 00'\ (15/4)'' \\ 60^\circ\ 00'\ (15/4)'' \end{bmatrix}$$

$$\hat{L} = A\hat{\underline{X}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 60^\circ\ 00'\ (15/4)'' \\ 60^\circ\ 00'\ (15/4)'' \end{bmatrix} = \begin{bmatrix} 60^\circ\ 00'\ 3.75'' \\ 60^\circ\ 00'\ 3.75'' \\ -120^\circ\ 00'\ 7.5'' \end{bmatrix}$$

$$\hat{L}' = \hat{L} + c = \begin{bmatrix} 60^\circ\ 00'\ 3.75'' \\ 60^\circ\ 00'\ 3.75'' \\ -120^\circ\ 00'\ 7.5'' \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 180 \end{bmatrix} = \begin{bmatrix} 60^\circ\ 00'\ 3.75'' \\ 60^\circ\ 00'\ 3.75'' \\ 59^\circ\ 59'\ 52.5'' \end{bmatrix}$$

$$\hat{\varepsilon} = L - A\hat{X} = \begin{bmatrix} 60^\circ\ 00'\ 03'' \\ 60^\circ\ 00'\ 03'' \\ -120^\circ\ 00'\ 09'' \end{bmatrix} - \begin{bmatrix} 60^\circ\ 00'\ 3.75'' \\ 60^\circ\ 00'\ 3.75'' \\ -120^\circ\ 00'\ 7.5'' \end{bmatrix}$$

$$= \begin{bmatrix} -0.75'' \\ -0.75'' \\ -1.5'' \end{bmatrix}$$

c) $$\hat{\sigma}_0^2 = \frac{\hat{\varepsilon}^T P \hat{\varepsilon}}{n-m} = \frac{1}{3-2} \sum_{i=1}^{3} (P_i \hat{\varepsilon}_i \hat{\varepsilon}_i) = 2 \cdot \left(-\frac{3}{4}\right)^2 + 2 \cdot \left(-\frac{3}{4}\right)^2 + 1 \cdot \left(\frac{3}{2}\right)^2$$

$$= \frac{18}{16} + \frac{18}{16} + \frac{9}{4}$$

$$= \frac{36}{16} + \frac{36}{16} = \frac{72}{16} = \left(\frac{9}{2}\right)''$$

$$\hat{\sigma}_0 = \sqrt{\frac{9}{2}} = \pm \frac{3}{\sqrt{2}}\ ''$$

d) $$Q_{\hat{x}\hat{x}} = (A^T P A)^{-1} = \frac{1}{8} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

$$C_{\hat{x}\hat{x}} = \hat{\sigma}_0^2 \, Q_{\hat{x}\hat{x}} = \frac{9}{2} \cdot \frac{1}{8} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} = \frac{9}{16} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

$$Q_{\hat{L}\hat{L}} = A(A^T P A)^{-1} A^T = A \, \frac{1}{8} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} \frac{1}{8} \begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -2 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -2 \\ -2 & -2 & 4 \end{bmatrix}$$

$$C_{\hat{l}\hat{l}} = \hat{\sigma}_0^2 \, Q_{\hat{l}\hat{l}} = \frac{9}{2} \cdot \frac{1}{8} \begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -2 \\ -2 & -2 & 4 \end{bmatrix} = \frac{9}{16} \begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -2 \\ -2 & -2 & 4 \end{bmatrix}$$

$$Q_{\hat{\varepsilon}\hat{\varepsilon}} = P^{-1} - Q_{\hat{l}\hat{l}} = \frac{1}{2} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 2 \end{bmatrix} - \frac{1}{8} \begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -2 \\ -2 & -2 & 4 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

$$C_{\hat{\varepsilon}\hat{\varepsilon}} = \hat{\sigma}_0^2 \, Q_{\hat{\varepsilon}\hat{\varepsilon}} = \frac{2}{9} \cdot \frac{1}{8} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{bmatrix} = \frac{9}{16} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

3.2