1.1 $\tilde{l} = 53.76$ m

a) $\hat{\sigma}_0^2 = \dfrac{\sum_{i=1}^{10} p_i \varepsilon_i^2}{10} \approx 0.004138 m^2 \Rightarrow \hat{\sigma}_0 = \sqrt{\hat{\sigma}_0^2} \approx 0.06618$ m

$= 66.2$ mm

i	ε_i	p_i
1	-0.01	2
2	-0.04	2
3	0.01	2
4	0.08	2
5	-0.02	2
6	0.12	1
7	0.00	1
8	-0.07	1
9	-0.08	1
10	0.03	1

b) l_4 har högre vikt än l_7 och därmed sannolikt högre precision. l_4 & l_6 har samma vikt.

$\sigma_i^2 = \dfrac{\sigma_0^2}{p_i} \Rightarrow$ högre $p_i \Rightarrow$ lägre varians.

1.2 $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$, $\quad C_{xx} = \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix}$ (mm^2)

$\sigma_0^2 = 2 mm^2$

a) $Q_{xx} = \dfrac{1}{\sigma_0^2} \cdot C_{xx} = \dfrac{1}{2} \cdot \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

σ_2^2

b) $p_i = \dfrac{\sigma_0^2}{\sigma_2^2} = \dfrac{2}{6} = \dfrac{1}{3}$

c) $P_{xx} = (Q_{xx})^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array}\right] \frac{1}{2} \sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ -1 & 3 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} & 1 \end{array}\right] \frac{2}{5}$$

$$\sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{5} & \frac{2}{5} \end{array}\right] \frac{1}{2} \sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & \frac{1}{5} \\ 0 & 1 & \frac{1}{5} & \frac{2}{5} \end{array}\right]$$

P_{xx}

d) från varians-covariansmatrisen ser vi att icke-diagonalvärdena är skilda från 0 vilket innebär att de är beroende eller korrelerade. Därmed kan viktmatrisen P vara fel.

Därav olika värden $\frac{1}{3}$ & $\frac{2}{5}$

1.3) $\bar{h} = \frac{h_1 - h_2}{2}$, $\sigma_{\bar{h}} = 3$ mm

$\bar{h} = \frac{1}{2}h_1 - \frac{1}{2}h_2$

$\sigma_{\bar{h}}^2 = \left(\frac{1}{2}\right)^2 \sigma_1^2 + \left(-\frac{1}{2}\right)^2 \sigma_2^2 = 9$ $\quad \sigma_1^2 = \sigma_2^2$

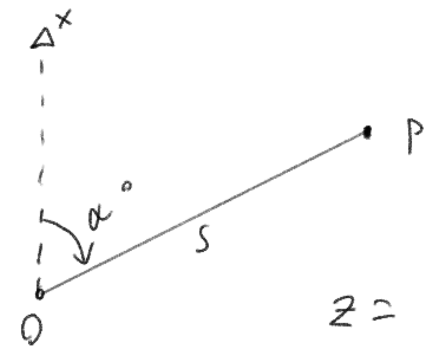
$\Rightarrow \frac{1}{2}\sigma^2 = 9 \Rightarrow \sigma = \sqrt{18}$

$d = h_1 + h_2$

$\sigma_d^2 = \sigma_1^2 + \sigma_2^2 = (\sqrt{18})^2 + (\sqrt{18})^2 = 36$

$\sigma_d = 6$ mm

1.4



$\alpha = 60°$ $\quad \sigma_\alpha = 3''$

$s = 100\,m$ $\quad \sigma_s = 3\,cm$

$$z = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 + s \cdot \cos\alpha \\ y_0 + s \cdot \sin\alpha \end{bmatrix} = \begin{bmatrix} f_1(s,\alpha) \\ f_2(s,\alpha) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \left\{ \begin{array}{l} a_{11} = \dfrac{\partial f_1}{\partial s} = \cos\alpha = \cos 60 = 1/2 \\ a_{12} = \dfrac{\partial f_1}{\partial \alpha} = -s\sin\alpha = -100000 \cdot \sin 60 = -\sqrt{3}\,5000 \\ a_{21} = \dfrac{\partial f_2}{\partial s} = \sin\alpha = \sin 60 = \dfrac{\sqrt{3}}{2} \\ a_{22} = \dfrac{\partial f_2}{\partial \alpha} = s\cos\alpha = 10\,000 \cdot \cos 60 = 5000 \end{array} \right.$$

$$A = \begin{bmatrix} 1/2 & -\sqrt{3}\,5000 \\ \frac{\sqrt{3}}{2} & 5000 \end{bmatrix}$$

$$C_{\varepsilon\varepsilon} = \begin{bmatrix} \sigma_s^2 & 0 \\ 0 & \sigma_\alpha^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & \left(\frac{3}{\rho''}\right)^2 \end{bmatrix}$$

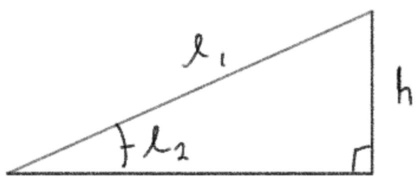
$$C_p = A \cdot C_{\varepsilon\varepsilon} \cdot A^T = \begin{bmatrix} 1/2 & -\sqrt{3}\,5000 \\ \frac{\sqrt{3}}{2} & 5000 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & \frac{9}{\rho''^2} \end{bmatrix} \begin{bmatrix} 1/2 & \frac{\sqrt{3}}{2} \\ -\sqrt{3}\,5000 & 5000 \end{bmatrix}$$

$$= \begin{bmatrix} 4.5 & \frac{-\sqrt{3}\,5000 \cdot 9}{\rho''^2} \\ \frac{\sqrt{3} \cdot 9}{2} & \frac{5000 \cdot 9}{\rho''^2} \end{bmatrix} \begin{bmatrix} 1/2 & \frac{\sqrt{3}}{2} \\ -\sqrt{3}\,5000 & 5000 \end{bmatrix} =$$

$$= \begin{bmatrix} 2.25 + \frac{(\sqrt{3}5000)^2 \cdot 9}{\rho''^2} & \frac{4.5 \cdot \sqrt{3}}{2} - \frac{\sqrt{3} \cdot 5000^2 \cdot 9}{\rho''^2} \\ \frac{\sqrt{3} \cdot 9}{4} - \frac{\sqrt{3} \cdot 9 \cdot 5000^2}{\rho''^2} & \frac{3 \cdot 9}{4} + \frac{5000^2 \cdot 9}{\rho''^2} \end{bmatrix} =$$

$$= \begin{bmatrix} 2.26587 & 3.88795 \\ 3.88795 & 6.75529 \end{bmatrix} \quad (cm^2)$$

1.5



measured:

$l_1 = 4125.300$ m $\qquad \sigma_1 = 8$ mm

$l_2 = 20.0000°$ $\qquad \sigma_2 = 6''$

Scale error: 2 ppm

a) $h = l_1 \sin l_2$

b) $\varepsilon_h = \frac{\partial f}{\partial l_1} \varepsilon_1 + \frac{\partial f}{\partial l_2} \varepsilon_2$

$\frac{\partial f}{\partial l_1} = \sin l_2 = \sin 20°$

$\frac{\partial f}{\partial l_2} = l_1 \cos l_2 = 4\ 125\ 300 \cdot \cos 20°$

$$\sigma_h = \sqrt{\left(\frac{\partial f}{\partial \ell_1}\right)^2 \cdot \sigma_1^2 + \left(\frac{\partial f}{\partial \lambda_2}\right)^2 \sigma_2^2} =$$

$$= \sqrt{\sin^2 20^\circ \cdot 8^2 + (4\ 125\ 300 \cdot \cos 20^\circ)^2 \cdot \left(\frac{6}{\rho''}\right)^2}$$

$$\approx 112.796 \text{ mm}$$

c) $\sin^2 20^\circ \cdot 8^2 \approx 7.5$ $(a_1$

$(4\ 125\ 300 \cdot \cos 20)^2 \cdot \left(\frac{6}{\rho''}\right)^2 \approx 12\ 700$ (a_2)

$a_2 \gg a_1 \Rightarrow$ standardfel påverkar mest